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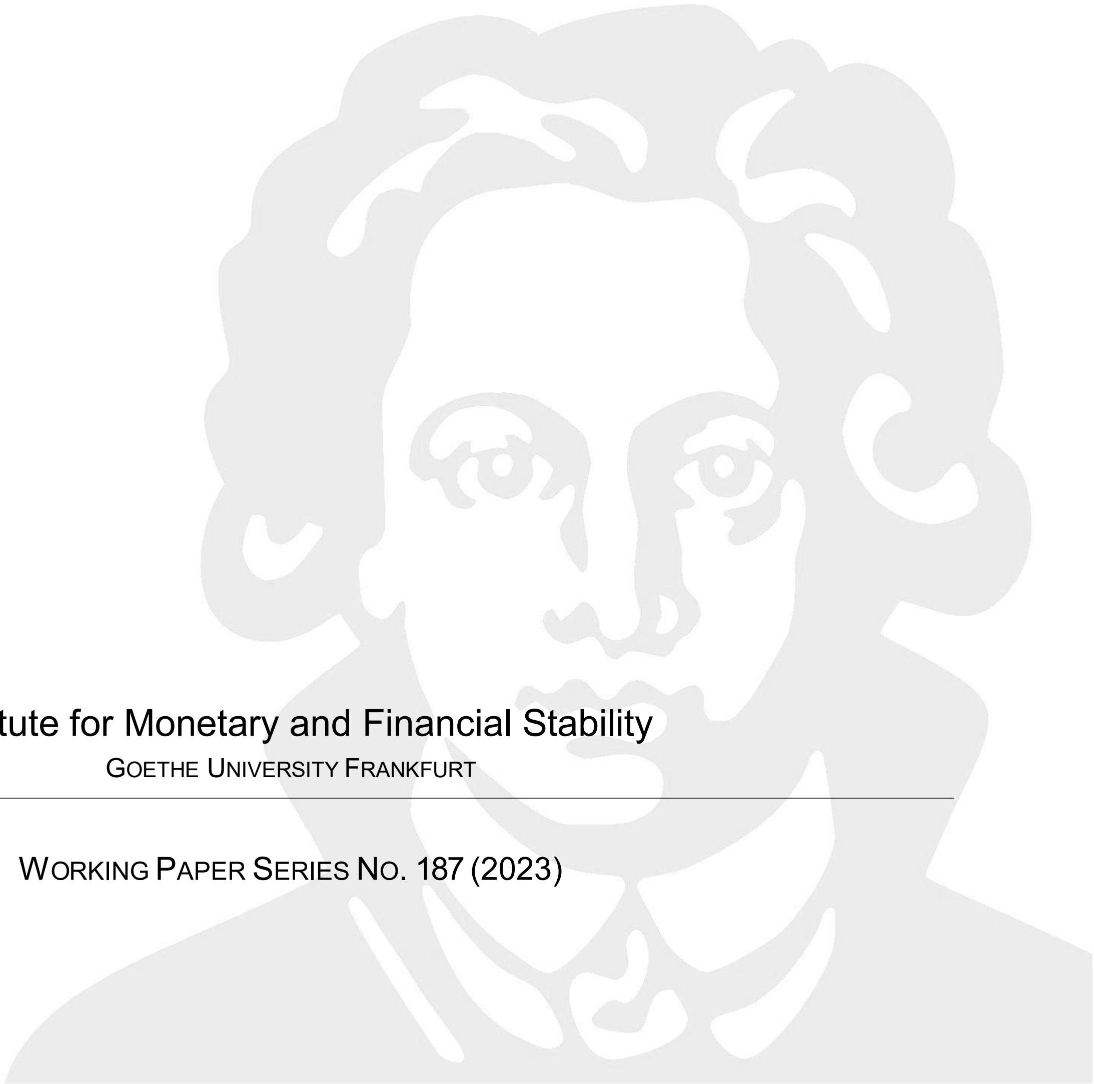

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Transition Risk Uncertainty and Robust Optimal Monetary Policy

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Abstract

Climate change has become one of the most prominent concerns globally. In this paper, we study the transition risk of greenhouse gas emission reduction in structural environmental-macroeconomic DSGE models. First, we analyze the uncertainty in model prediction on the effect of unanticipated and pre-announced carbon price increases. Second, we conduct optimal model-robust policy in different settings. We find that reducing emissions by 40% causes 0.7% - 4% output loss with 2% on average. Pre-announcement of carbon prices affects the inflation dynamics significantly. The central bank should react slightly less to inflation and output growth during the transition risk. With optimal carbon price designs, it should react even less to inflation, and more to output growth.

JEL Codes: Q58, E32, Q54, C11, E17, E52

Keywords: Climate change, Environmental policy, Optimal policy, Transition risk, Model uncertainty, DSGE models

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1 Introduction

Concerns about climate change have turned from a discussion to a problem that calls for prompt resolutions around the world. Hence, these problems have become a burning topic that academics, industry, and policymakers are working intensively to solve with different approaches. The Paris Agreement was adopted with the goal to keep the global temperature increase below 2°C compared to pre-industrial levels. Since then, climate change has evolved to be the major issue of the global economic policy agenda. As suggested by IMF (2022), the greenhouse gas emissions need to be cut by 25 to 50 % to limit global warming under 1.5°C to 2°C. Moreover, both developed and emerging market economies aim for reaching a zero-emission economy by 2050, following the International Energy Agency (IEA) report (Bouckaert et al., 2021). This demands an ambitious and prompt climate policy intervention. To keep the Paris Agreement target, the price of carbon must rise significantly to internalize climate risk. Anyhow, such intervention will affect the macroeconomic environment and cause macroeconomic risk. This kind of risk (called transition risk) refers to unanticipated rapid climate policy. However, climate policy implementation and macroeconomic effects to achieve the mentioned target remain under ongoing discussions.

The main concern of imposing the ambitious climate policy is its impairing effect on economic performance. Hence, the question of how much we have to trade off when we increase the carbon price sufficiently remains ambiguous. Levin, Wieland and Williams (2003) mention model uncertainty in studying the implication of policy due to the diversity in aggregate dynamics. Similarly, the effects of environmental policies might be subject to the same concern. First, the prediction of the effects of transition risk is different among models and depends largely on model structure and dynamics. In this paper, we call it transition risk uncertainty. Second, motivated by using the forward guidance policy of central banks after the global financial crisis, it is highly interesting to see differences in the effect of forward guidance climate policy compared to unanticipated ambitious climate policy which is the main concern of transition risks.

During the transition path, it is natural to question the role of central bank policy. Following the conclusion of its strategic review in 2021, the European Central Bank (ECB) declared that it would become more involved in environmental policy. However, the implementation of environmentally involved policies needs to be studied in depth. Moreover, besides ECB, the question of how central banks around the world react to the issues of climate change through their monetary policy is highly discussed. Traditionally, most central banks around the world commit to stabilizing price levels and promoting a strong economy. Hence, it is natural to ask if the current mandate is sufficient without including environmental targets in their decision-making process and how optimal monetary can be conducted during the transition path.

In this paper, to address the above-mentioned matters, we analyze the macroeconomic effects of several climate policies in the form of a carbon price hike. We include the environmental aspect in 29 structural macroeconomic models which cover most of the prominent features in macroeconomic modeling. To study the effect of our ambitious climate policy, we focus on two scenarios. In the first case, the carbon price is implemented to get a reduction of 40% in the current emission level. In the second case, the carbon price increases linearly with communication from the government until we achieve a zero-emission economy after 30 years. In this context, we document the uncertainty in model predictions through three main cases: unanticipated carbon price hike, forward guidance carbon price hike, and transition path toward a zero-emission economy. Within our set of models, we observe a large variety in the effect of transition risk. Overall, we find that an ambitious carbon price to cut emissions by more than 40% decreases output between 0.7% and 4.2% at peak. Moreover, with 4 periods ahead of communication about the carbon price hike, we observe the drop in output from the first period when the carbon price hike is not implemented. Importantly, we find that pre-announced climate policy has a significant effect on the fluctuation of inflation, but not on the rest of the economy. Our analysis suggests that policymakers should communicate climate policy well to reduce the fluctuations of transition risk in the economy.

As climate change and environmental policies can have serious implications for the macroeconomic environment, this concern cannot be considered in isolation from the monetary policy which is responsible for price and output stabilization. Therefore, we conduct several in-depth exercises on optimal monetary policy and environmental policy. Similar to Dück and Verona (2023), we also search optimal model-robust monetary rules for a large set of structural macroeconomic models. We do it in three scenarios. In the first scenario, we conduct optimal monetary policy during business-cycle shocks. In the second scenario, we study optimal policy given an unanticipated introduction of a carbon price to cut emissions permanently by 40%. In the third scenario, we conduct optimal policy in cooperation between the central bank and the government. Historically, central banks control monetary policy, and fiscal authorities determine the carbon price policy. In our paper, the carbon price is implemented as a form of tax. Hence, the fiscal authority can optimally set the carbon price. Given the increasing importance of climate change, we conduct a joint optimal monetary and climate policy. Therefore, in response to a standard total factor productivity (TFP) shock, we choose the optimal monetary policy response and carbon price reaction to minimize the fluctuations in inflation, output growth, and emissions.

We find that over the business cycle, the central bank should be more restrained in its interest rate setting with respect to inflation and output growth due to model uncertainty. During transition risk, the central bank should be (slightly) more cautious than over the business cycle. In cooperation with the government, the central bank should always react less strongly with respect to inflation and always (significantly) more with respect to output growth, compared to the business cycle and transition risk scenarios. This pattern becomes more pronounced with the increasing relative importance of the variance of emissions. Moreover, model-robust rules always prescribe a more aggressive reaction to output growth than the average of model-specific rules. In all scenarios, the optimal model-robust rule has a slightly higher loss than the optimal model-specific policy rule of the specific model, but it has (on average) a 3-4 times lower loss increase than the model-specific rules in all (other) models.

Our paper belongs to two strands of literature. The first strand is a fast-growing literature on the implications of climate change as well as climate policy in the macroeconomic environment. The second strand of literature is about optimal policy and model uncertainty.

In the development of environmental macroeconomic models, the inclusion of environmental aspects into equilibrium models originated in Integrated Assessment Models (IAMs). Nordhaus (1977) provides a pioneering work with Dynamic Integrated Models of Climate Change and the Economy (DICE). Then, due to the need to include environmental aspects in business cycle models, scholars started to include environmental aspects in Real Business Cycle (RBC) models. Some pioneer works include Fischer and Springborn (2011) and Heutel (2012). The climate aspect is included in the sense that emissions come from production activity. Pollution is accumulated through emissions and has some damage to firm productivity. This set of models allows for studying some climate policies such as carbon price, or cap and trade. Annicchiarico and Di Dio (2015) enrich the New-Keynesian (NK) model with environmental aspects which gives rise to monetary policy analysis. Since then, the literature has grown swiftly. More papers focus on the transition risk in the form of a carbon price hike as well as potential policies to mitigate the risk.

Some notable works on macroprudential and green quantitative easing are Benmir and Roman (2020), Carattini, Heutel and Melkadze (2021), Ferrari and Landi (2021), Le (2023) and others. These papers do the analysis with environmental two sectors models, brown and green sectors, which differ in emissions during production. This setup is aimed to study policies with allocative effects between green and brown sectors to mitigate the effect of transition risk. Notably, Le (2023) additionally finds that using capital control to drive the capital to the green sector proves to be effective, rather than asking for cooperation among countries in Ferrari and Pagliari (2021). However, our paper focuses on the uncertainty about the prediction of transition risk. From the report of FTSE Russel (Russell, 2018), the green sector is relatively small in most economies which is estimated to account for 6% of the global market capitalization. Hence, we believe our analysis can provide well-approximated predictions in transition risk without including the two sectors' setup for the sake of simplicity.

The second set of literature lies on robust optimal monetary policy. The large body of literature on model robustness tries to identify monetary policy rules, which are robust to model uncertainty. Using different sets of models, estimated with data from the United States or Euro Area, Adalid et al. (2005), Kuester and Wieland (2010), Levin and Williams (2003), Orphanides and Wieland (2013), Schmidt and Wieland (2013) and Taylor and Wieland (2012) show that a rule, which is optimized on one specific model, might perform poorly in other models. In contrast, the model-robust rule performs well in most models. Levin, Wieland and Williams (2003) focus on the horizon of forecast-based policy rules and find that increasing the forecast horizon makes a rule less robust and tends to generate indeterminacy in more models. According to Orphanides and Wieland (2013) rules perform better when they use current variables compared to forecasts. Binder et al. (2017) find that optimal model-robust monetary policy rules exhibit weaker responses to inflation and the output gap in the presence of financial frictions. Dück and Verona (2023) find that policymakers should be more restrained in their inflation reaction if they aim at stabilizing specific frequencies (i.e. business-cycle frequency) of inflation and output growth, and even more restrained due to model uncertainty.

The rest of the paper is organized as follows. In section 2 we show how to incorporate climate in a standard New-Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model to transform it into an Environmental-DSGE Model (E-DSGE), before analyzing the effect of various climate policy scenarios on the economy in section 3. Then, we conduct optimal monetary policy in section 4, and we evaluate the policy rules in terms of central bank loss in section 5. Section 6 concludes.

2 A Smets and Wouters (2007) E-DSGE Model

In this part, we briefly describe how to implement the climate aspect into standard NK-DSGE models using a similar framework to Annicchiarico and Di Dio (2015) and Heutel (2012). For demonstration purposes, we present the derivations for the seminal model of Smets and Wouters (2007) with environmental variables similar to the setup of Annicchiarico and Di Dio (2015) and Heutel (2012). For the rest of the models in our analysis, to ensure comparability, all the structural DSGE models used in this paper are enriched by this framework in terms of model structure and environment-related parameters.

2.1 Households

There is a continuum of identical households with utility function and budget constraints given by:

$$U_t = \left[\frac{1}{1-\sigma_c}\,(C_t - \lambda\, C_{t-1})^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\, L_t^{1+\sigma_l}\right) \tag{1}$$

$$C_t + I_t + \frac{B_t}{\epsilon_t^b\, R_t\, P_t} - T_t^{env} \leq \frac{B_{t-1}}{P_t} + \frac{W_t^h\, L_t}{P_t} + \frac{R_t^k\, Z_t\, K_{t-1}}{P_t} - a(Z_t)\, K_{t-1} + \frac{Div_t}{P_t} \tag{2}$$

where C_t is the consumption with external consumption habit parameter, λ. L_t is hours worked with hourly wage W_t^h. T_t^{env} is the lump-sum tax or transfers (including the redistributed government income from carbon emissions) and Div_t is the dividend from the labor union. B_t are nominal bond holdings with return R_t. Households hold capital, K_t, which gives a return R_t^k, and Z_t is capital utilization. The function $a(Z_t)$ describes the physical cost of the use of capital. The Smets and Wouters (2007) model features an exogenous premium to bond return rate, ϵ_t^b. This presents inefficiency in the financial sector and can give rise to the difference between the deposit rate and the risk-free rate.

The capital stock follows a law of motion with the depreciation rate, δ, and a quadratic investment adjustment cost, $S(.)$. ϵ_t^i is an investment-specific price shock. The utilization rate of capital yields the effective amount of capital, K_t^s.

$$K_t = (1-\delta)\, K_{t-1} + \epsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{3}$$

$$K_t^s = Z_t\, K_{t-1} \tag{4}$$

Households choose consumption, labor, bonds, investment, capital, and utilization rate to maximize utility subject to the budget constraint, and the law of motion of capital. The maximization problem and first-order conditions (FOCs) are:

$$\max_{C_t,L_t,B_t,I_t,K_t,Z_t} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \beta^s \, U_{t+s} \tag{5}$$

$$s.t. \quad (1),\ (2),\ (3)$$

$$\frac{\partial}{\partial C_t} = 0 \;\Leftrightarrow\; \Xi_t \;=\; \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\, L_t^{1+\sigma_l}\right)\, (C_t - \lambda\, C_{t-1})^{-\sigma_c} \tag{6}$$

$$\frac{\partial}{\partial L_t} = 0 \;\Leftrightarrow\; \left[\frac{1}{1-\sigma_c}\,(C_t - \lambda\, C_{t-1})^{1-\sigma_c}\right] \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\, L_t^{1+\sigma_l}\right)\,(\sigma_c - 1)\, L_t^{\sigma_l} \;=\; -\,\Xi_t\,\frac{W_t^h}{P_t} \tag{7}$$

$$\frac{\partial}{\partial B_t} = 0 \;\Leftrightarrow\; \Xi_t \;=\; \beta\, \epsilon_t^b\, R_t\, \mathbf{E}_t \left[\frac{\Xi_{t+1}}{\pi_{t+1}}\right] \tag{8}$$

$$\begin{aligned} \frac{\partial}{\partial I_t} = 0 \;\Leftrightarrow\; \Xi_t \;=\; & \Xi_t^k\, \epsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\ & + \beta\, \mathbf{E}_t \left[\Xi_{t+1}^k\, \epsilon_{t+1}^i\, S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{aligned} \tag{9}$$

$$\frac{\partial}{\partial K_t} = 0 \;\Leftrightarrow\; \Xi_t^k \;=\; \beta\, \mathbf{E}_t \left[\Xi_{t+1}\left(\frac{R_{t+1}^k\, Z_{t+1}}{P_{t+1}} - a\,(Z_{t+1})\right) + \Xi_{t+1}^k\,(1-\delta)\right] \tag{10}$$

$$\frac{\partial}{\partial Z_t} = 0 \;\Leftrightarrow\; \frac{R_t^k}{P_t} \;=\; a'(Z_t) \tag{11}$$

Ξ and Ξ^k are the Lagrange multiplier for the budget constraint and the capital accumulation equation, respectively. Inflation is defined as $\pi_t = \frac{P_t}{P_{t-1}}$.

2.2 Labor Union

Households supply labor to a labor union which sets the wage subject to a Calvo scheme. Labor used by the intermediate goods producers L_t is aggregated in the form:

$$L_t \;=\; \left[\int_0^1 L_t(i)^{\frac{1}{1+\lambda_{w,t}}} \, di \right]^{1+\lambda_{w,t}} \tag{12}$$

where $\lambda_{w,t}$ reflects the time-varying (ARMA process) elasticity of substitution among different types of labor (or similar explanation). The labor union maximizes its profit in a perfectly competitive environment subject to labor aggregation.

$$\max_{L_t(i)} \quad W_t \, L_t - \int_0^1 W_t(i) \, L_t(i) \, di \qquad\qquad s.t. \;\; (12) \tag{13}$$

Which yields the following sector-specific labor demand and wage index:

$$L_t(i) \;=\; \left(\frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} \;\; L_t \tag{14}$$

$$W_t \;=\; \left[\int_0^1 W_t(i)^{\frac{1}{\lambda_{w,t}}} \, di \right]^{\lambda_{w,t}} \tag{15}$$

The labor unions take the marginal dis-utility of labor as the cost of the labor services in their negotiations with the labor packer. The markup on the marginal dis-utility is distributed back to the households. However, the union is also subject to nominal rigidity in Calvo (1983) framework. Hence, they can adjust the wage with a probability $1 - \xi_w$ every period. If wages can not be set optimally, they are indexed using past inflation with degree ι_w. Subscript $*$ depicts the steady-state level of corresponding variables. Wages are growing at rate γ, which is the labor-augmenting deterministic growth rate of the economy. The labor union maximizes the wage income in the following problem:[1]

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[1] We display derivations for models using Calvo without inflation indexation in appendix A.1.

$$\max_{\tilde{W}_t(i)} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \xi_w^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, \left[W_{t+s}(i) - W^h_{t+s}\right] \, L_{t+s}(i) \tag{16}$$

$$s.t. \quad L_{t+s}(i) \; = \; \left(\frac{W_{t+s}(i)}{W_{t+s}}\right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} L_{t+s} \tag{17}$$

$$W_{t+s}(i) \; = \; \tilde{W}_t(i) \; \left(\Pi_{l=1}^{s} \, \gamma \, \pi_{t+l-1}^{\iota_w} \, \pi_*^{1-\iota_w}\right) \tag{18}$$

Since all re-optimizing labor unions are identical, the first-order condition with respect to optimal wage becomes:

$$\mathbf{E}_t \sum_{s=0}^{\infty} \xi_w^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, L_{t+s} \, \frac{1}{\lambda_{w,t+s}} \left[(1 + \lambda_{w,t+s}) \, W^h_{t+s} - X^w_{t,s} \, \tilde{W}_t\right] \; = \; 0 \tag{19}$$

$$X^w_{t,s} \; = \; \left\{ \begin{array}{ll} 1 & \text{for} \;\; s \; = \; 0 \\ \\ \left(\Pi_{l=1}^{s} \, \gamma \, \pi_{t+l-1}^{\iota_w} \, \pi_*^{1-\iota_w}\right) & \text{for} \;\; s \; > \; 0 \end{array} \right\} \tag{20}$$

Finally, the aggregate wage expression for every period is:

$$W_t \; = \; \left[(1 - \xi_w) \, \tilde{W}_t^{\frac{1}{\lambda_{w,t}}} + \xi_w \, \left(\gamma \, \pi_{t-1}^{\iota_w} \, \pi_*^{1-\iota_w} \, W_{t-1}\right)^{\frac{1}{\lambda_{w,t}}}\right]^{\lambda_{w,t}} \tag{21}$$

2.3 Production Sectors

In Smets and Wouters (2007) model, the final good Y_t is aggregated using a continuum of intermediate goods $Y_t(i)$ following the framework of Kimball (1995). This gives rise to strategic complementarities in the firm price setting.[2]

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[2] We display derivations for models using CES aggregator in appendix A.2.

$$\max_{Y_t, Y_t(i)} \quad P_t\, Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di \tag{22}$$

$$s.t. \quad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}\right) \;=\; 1 \tag{23}$$

where G(.) is a strictly concave and increasing function characterized by G(1) = 1. As in Kimball (1995), the assumptions on G(.) imply that the demand for input, $Y_t(i)$, is decreasing in its relative price, while the elasticity of demand is a positive function of the relative price (or a negative function of the relative output). This is strategic complementarity in price setting. From the FOCs, we get:

$$Y_t(i) \;=\; Y_t\, G'^{-1}\left[\frac{P_t(i)}{P_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}\right) \frac{Y_t(i)}{Y_t}\, di\right] \tag{24}$$

In the intermediate sectors, the (typically Cobb-Douglas) production function takes the following form:

$$Y_t(i) \;=\; A_t^{env}\, K_t^s(i)^\alpha\, \left(\gamma^t\, L_t(i)\right)^{1-\alpha} - \gamma^t\, \Phi \tag{25}$$

where total factor productivity, A_t^{env}, is a decreasing function of pollution (X_t, the stock of atmospheric carbon), and is an aggregate variable which is assumed to be given to firm i. Φ is the fixed cost in production. Effective capital and hours worked are production inputs. Intuitively, the more carbon stock in the atmosphere, the less productivity for the production sector which is documented in Nordhaus and Boyer (2003) as:

$$A_t^{env} \;=\; \left(1 - \left(d_0 + d_1\, X_t + d_2\, X_t^2\right)\right) \epsilon_t^a \tag{26}$$

$$\ln(\epsilon_t^a) \;=\; (1-\rho_a)\, \ln(\epsilon^a) + \rho_a\, \ln(\epsilon_{t-1}^a) + \eta_t^a \qquad\qquad \eta_t^a \sim \mathcal{N}(0,\, \sigma_a^2) \tag{27}$$

where ϵ_t^a is the total factor productivity in a standard NK-DSGE model without an environmental aspect. Equation 27 describes the law of motion of productivity as an AR(1) process with steady-state ϵ^a, persistence ρ_a and standard deviation σ_a. The carbon stock can be accumulated by total domestic emissions, e_t, and the rest of the world (ROW) emissions, e_t^{row}. For a closed economy, ROW emissions are exogenous. Domestic emissions are emitted through the production of intermediate goods. We assume that a fraction μ_t of emissions are abated by firm i. Thus, the abatement cost (z_t) is defined as an increasing function of the output of firm i. Because all intermediate firms choose the same (optimal) price, inputs, and output, the equilibrium conditions hold without index i. Hence, equation 28 describes total pollution in the economy, while emissions and abatement costs are given by:

$$X_t \;=\; \eta\, X_{t-1} + e_t + e_t^{row} \tag{28}$$

$$e_t \;=\; \gamma_1\,(1-\mu_t)\,Y_t \tag{29}$$

$$z_t \;=\; \theta_1\,\mu_t^{\theta_2}\,Y_t \tag{30}$$

All models in our analysis feature price stickiness to give rise to monetary policy. However, models use either Rotemberg (1982) with a price adjustment cost or Calvo (1983) with a price resetting framework. For both kinds of price setting, the final goods firm chooses labor, capital, and emissions abated to minimize the cost, given the production function and the environment damage function. Since there is no firm-specific tax, the following minimization problem yields the same first-order conditions for all firms, such that we ignore the firm index:

$$\min_{L_t, K_t^s, \mu_t} \quad W_t\,L_t + R_t^k\,K_t^s + \tau_t^e\,e_t + z_t \tag{31}$$

$$s.t. \qquad (29),\ (30),\ (25)$$

$$\frac{\partial}{\partial L_t} = 0 \;\Leftrightarrow\; (1-\alpha)\,\Psi_t\,Y_t \;=\; W_t\,H_t \tag{32}$$

$$\frac{\partial}{\partial K_t^s} = 0 \;\Leftrightarrow\; \alpha \, \Psi_t \, Y_t \;=\; R_t^k \, K_{t-1} \tag{33}$$

$$\frac{\partial}{\partial \mu_t} = 0 \;\Leftrightarrow\; \mu_t \;=\; \left(\frac{\tau_t^e \, \gamma_1}{\theta_1 \, \theta_2} \right)^{\frac{1}{\theta_2 - 1}} \tag{34}$$

where μ_t expresses the optimal abatement level of the firm given the price on carbon emissions, τ_t^e. If there is no price on carbon emissions, $\tau_t^e = 0$, firms do not pay abatement cost, $\mu_t = 0$. Notably, the environment does not change the optimal capital-labor ratio of the firm. In the above equations, Ψ_t determines marginal cost in a standard macroeconomic model as a combination of extra units of capital and labor needed to manufacture the additional unit of output. Hence, it is convenient to express Ψ_t in the following form from the perspective of intermediate firms.

$$\Psi_t \;=\; \frac{1}{\alpha^{\alpha} \, (1-\alpha)^{(1-\alpha)} \, \gamma^{(1-\alpha)\, t} \, A_t^{env}} \, W_t^{(1-\alpha)} \, \left(R_t^k\right)^{\alpha} \tag{35}$$

Different from the standard model, the marginal cost that goes into price setting includes environmental cost, carbon price, and abatement cost. Thus, we can internalize the effect of the environment through market mechanisms given that the damage to the productivity level is relatively small to firms. The following form of marginal cost appears in the New Keynesian Phillips curve (NKPC) regardless of the framework of price setting with nominal rigidity.

$$MC_t \;=\; \Psi_t + \tau_t^e \, (1 - \mu_t) \, \gamma_1 + \theta_1 \, \mu_t^{\theta_2} \tag{36}$$

Under Calvo pricing with partial indexation (notation is the same as for Calvo wage setting), the optimal price set by the firm that is allowed to re-optimize results from the following optimization problem:[3]

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[3] We display derivations for the Rotemberg quadratic price adjustment cost framework in appendix A.3.

$$\max_{\tilde{P}_t(i)} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \left[\tilde{P}_t(i) \, \left(\Pi_{l=1}^{s} \, \pi_{t+l-1}^{\iota_p} \, \pi_*^{1-\iota_p} \right) - MC_{t+s} \right] Y_{t+s}(i) \tag{37}$$

$$s.t \quad Y_{t+s}(i) \; = \; Y_{t+s} \, G'^{-1} \left(\frac{P_t(i) \, X_{t,s}^p}{P_{t+s}} \int_0^1 G' \left(\frac{Y_{t+s}(i)}{Y_{t+s}} \right) \frac{Y_{t+s}(i)}{Y_{t+s}} \, di \right) \tag{38}$$

$$X_{t,s}^p \; = \; \left\{ \begin{matrix} 1 & \text{for } \; s \; = \; 0 \\ \\ \left(\Pi_{l=1}^{s} \, \pi_{t+l-1}^{\iota_p} \, \pi_*^{1-\iota_p} \right) & \text{for } \; s \; > \; 0 \end{matrix} \right\} \tag{39}$$

Since all firms which re-optimize the price are identical, the first-order condition of the intermediate firm and the price index becomes:

$$\mathbf{E}_t \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, Y_{t+s} \left[X_{t,s}^p \, \tilde{P}_t + \left(\tilde{P}_t \, X_{t,s}^p - MC_{t+s} \right) \frac{1}{G'^{-1}(z_{t+s})} \, \frac{G'(x_{t+s})}{G''(x_{t+s})} \right] \; = \; 0 \tag{40}$$

$$P_t \; = \; (1 - \xi_p) \, \tilde{P}_t \, G'^{-1} \left[\frac{\tilde{P}_t \, \tau_t}{P_t} \right] + \xi_p \, \pi_{t+l-1}^{\iota_p} \, \pi_*^{1-\iota_p} \, P_{t-1} \, G'^{-1} \left[\frac{\pi_{t+l-1}^{\iota_p} \, \pi_*^{1-\iota_p} \, P_{t-1} \, \tau_t}{P_t} \right] \tag{41}$$

where they define additional helping variables:

$$\tau_t \; = \; \int_0^1 G' \left(\frac{Y_t(i)}{Y_t} \right) \frac{Y_t(i)}{Y_t} \, di \tag{42}$$

$$z_t \; = \; \frac{P_t(i)}{P_t} \, \tau_t \tag{43}$$

$$x_t \; = \; G'^{-1}(z_t) \tag{44}$$

2.4 Government, Central Bank, and Market Clearing

The government chooses the carbon price for each period, freely. It generates income from that. On the other hand, we assume that the government fully re-distributes income generated by carbon price payments to households. For our analysis, the carbon price follows an

AR(1) process to simulate the transition risk. Consequently, government-related equations become:

$$P_t\, G_t + B_{t-1} \;=\; T_t^{env} + \frac{B_t}{R_t} \tag{45}$$

$$T_t^{env} \;=\; T_t + \tau_t^e\, e_t \tag{46}$$

$$\ln\left(\tau_t^e\right) \;=\; (1-\rho_e)\;\ln\left(\tau^e\right) + \rho_e\;\ln\left(\tau_{t-1}^e\right) + \epsilon_t^{\tau^e} \qquad\qquad \epsilon_t^{\tau^e} \sim \mathcal{N}(0,\, \sigma_{\tau^e}^2) \tag{47}$$

The market clearing condition additionally includes the abatement cost compared to a standard NK-DSGE model.

$$Y_t \;=\; C_t + I_t + G_t + a(Z_t)\; K_{t-1} + \theta_1\; \mu_t^{\theta_2}\; Y_t \tag{48}$$

The central bank conducts monetary policy through a standard Taylor rule. In the Smets and Wouters (2007) model, it is (using our notation):

$$\frac{R_t}{R_*} \;=\; \left(\frac{R_{t-1}}{R_*}\right)^{\rho}\; \left[\left(\frac{\pi_t}{\pi_*}\right)^{\alpha_\pi}\; (Y_t^{gap})^{\alpha_y}\right]^{1-\rho}\; (\Delta Y_t^{gap})^{\alpha_{\Delta y}}\;\; r_t \tag{49}$$

where Δ is the lag operator, ρ is the interest rate smoothing and α_π, α_y, and $\alpha_{\Delta y}$ are the central bank's reaction coefficients. Smets and Wouters (2007) define the output gap Y_t^{gap} as output deviations from the natural output. r_t is a monetary policy shock.

For our model-robustness exercise, we take a common monetary policy (Taylor) rule, as described in the optimal policy section 4. The environmental policy is our main interest when we study the transition risk through a permanent increase in the carbon price (tax). Moreover, in section 4, we also study the optimal response of climate policy under a positive TFP shock.

For the purpose of this study, we take 29 structural DSGE models from the Macroeconomic Model Data Base[4] and enrich them with the environmental framework explained above.

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[4] www.macromodelbase.com/. Wieland et al. (2012) and Wieland et al. (2016) explain database developments over the years and provide several applications.

Our set of models includes most of the prominent features inside the field of macroeconomic models such as financial frictions, job search and matching, and wage and price stickiness, amongst others. The set of models is provided in appendix D. Lastly, the paper focus on the reaction of output, consumption, investment, and price level which are common variables among models.

3 Climate Policy and Transition Path

3.1 Unanticipated Introduction of Climate Policy

In this scenario, we introduce a carbon price increase to match the 40% emission reduction following the IMF suggestion. To do that, we assume that the government decides to introduce the necessary carbon price increase in the 5th period without communicating it to the public. As there are no shock simulations in the first 4 periods, we stay at the deterministic steady state. At the beginning of period 5, there is an introduction of an unanticipated permanent carbon price. We want to draw attention to the transition effect after the introduction of our carbon price.

Figure 1 shows the resulting impulse response functions (IRFs).[5] On average, output decreases by 2% after 12 periods, slowly returning to -1.5% and staying there. Trough output reductions range from -0.7% to 4%. Consumption and investment show a similar pattern decreasing on average by 1.5% and 6%, respectively. However, as stylized facts suggest, consumption is more persistently reduced compared to output and investment is more volatile than output. The carbon price increase has an inflationary effect of 0.4 percentage points after 7 periods. Some models suggest inflation increases of 1 percentage point (and more) or moderate deflation until period 25. Due to inflation and output movements, the interest rate typically peaks at 0.2 percentage points on average after 8 periods. Interest rates fall after 15 periods, slowly returning to 0 from below, afterward. Intuitively, an increase in carbon price increases price levels directly due to their direct effect on the cost structure of firms. Thus, we observe an on-impact increase in inflation when the carbon price policy

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[5] Some models do not feature investment. In this case, they are excluded from this IRF only.

is implemented. The purpose of the introduction of the carbon price is to internalize climate change. Hence, it reduces the demand which decreases output afterwards. Under an unanticipated introduction of the carbon price, the effect from the supply side seems to be dominant in the first few periods where the increase of marginal cost following the increase in carbon price increases the inflation rate. Notably, we also observe a small deflationary effect. This happens when households internalize the permanent effect of the carbon price and the demand reaches its bottom point. Lastly, the simulation is aimed to shift the whole economy to a new steady state.

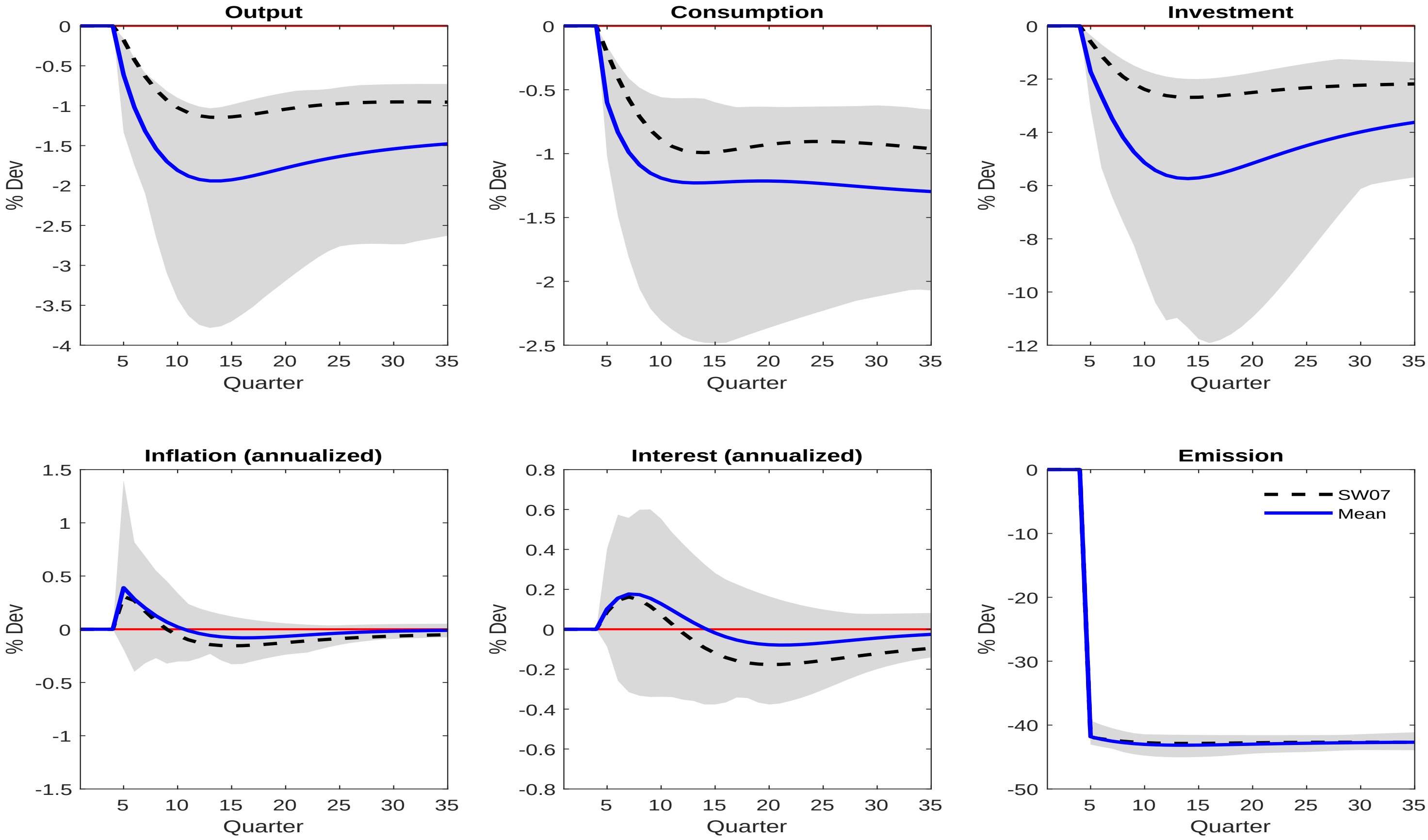

Figure 1: Unanticipated shock in the 5th period
Notes. The blue line depicts the average of all models used, while the grey shaded area is the 90% credible set across models. The dashed black line depicts the model of Smets and Wouters (2007). Time is in quarters. Impulse responses are in % deviation from steady state, while inflation and interest are in percentage point deviation.

3.2 Forward Guidance of Climate Policy

In the previous section, we show the results of a surprise introduction of an ambitious carbon price. This way, we completely neglect the possibility of policymakers communicating this policy to the public, which is closer to reality. There is a vast literature on the effect of forward guidance of monetary policy in which they highlight the signaling effect of central banks. They find a significant effect of forward guidance in all aspects of the macroeconomic environment. Motivated by those studies, this section incorporates forward guidance in carbon price into our analysis. We study forward guidance of climate policy through two scenarios. In the first case, we pre-announce the implementation of carbon prices 5 periods ahead. In the second case, instead of delaying the carbon price, we introduce a linearly increasing path of the carbon price from the first period, so that we also reduce emissions by more than 40% in period 5.

To analyze the effect of forward guidance, we compare the previous IRFs to the case in which the government implements the same carbon price increase after 5 periods but communicates its plans today. Figure 2 shows the resulting IRFs. Due to forward guidance, the macroeconomic variables respond immediately, yet both the mean dynamics and the lower confidence interval suggest a slightly milder recession. More significantly, the effect on inflation and interest rates is less pronounced. Due to forward guidance, the carbon price increase has a deflationary effect at first (around -0.2 percentage points on average), while after 7 periods inflation only spikes to around 0.25 percentage points on average (instead of 0.4). Consequently, interest rates decrease at first and increase only moderately when inflation spikes.

In both cases, we observe the recessionary effect of a carbon price hike. However, households seem to react ahead of the materialization in the carbon price. The demand drops significantly ahead of the introduction of the carbon price when the production sector still can operate with the same cost. This creates a disinflationary effect in the first 4 periods. Although the output drops less compared to figure 1, we get into a recession from the first period. Thus, the average effect on output might be approximately the same. The disinflationary effect creates space for central banks to increase the rate later at a lower cost.

However, further investigation on this matter is out of the scope of this paper.

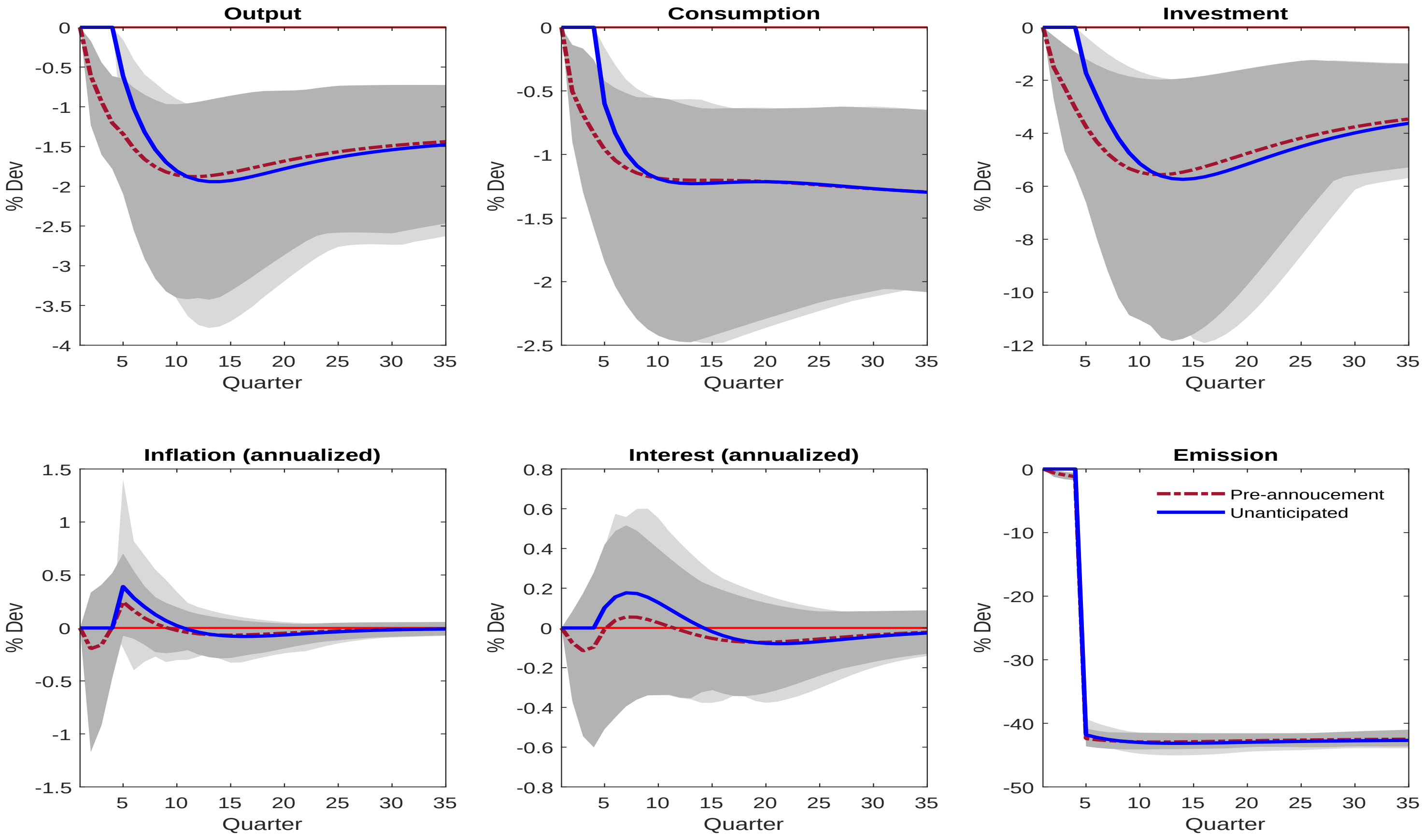

Figure 2: Anticipated shock in the 5th period - pre-announcement today
Notes. The dash-dotted red (solid blue) line depicts the average of all models used, with the light (dark) grey shaded area being the corresponding 90% credible set across models for anticipated (unanticipated), respectively. Time is in quarters. Impulse responses are in % deviation from steady state, while inflation and interest are in percentage point deviation.

In the previous figure, we observe that the pre-announcement of climate policy can affect the macroeconomic environment. It is natural to ask if we should implement the carbon price sooner but with a well-communicated path. In figure 3, we shed some light on that question. We keep our target of 40% reduction in emission but we impose a linearly increasing carbon price for 4 periods to match the emission target in figure 1 in the 5th period. The responses in output, consumption, and investment are similar to the previous scenarios. However, if the government announces a linear price increase from today until period 5, only a few models predict a deflationary effect. Moreover, on average, the inflation rate peaks at less than 0.2 percentage points annually which is about half of the inflation rate peak of the first scenario, and the results show no deflationary effect compared to the second scenario.

This result suggests that climate policy can be implemented immediately with good forward guidance to have less volatility in the inflation rate with approximately the same effect on output.

3.3 Toward Zero-Emission Economy

As mentioned earlier, both developed and emerging market economies aim for reaching a zero-emission economy by 2050. In this part, we show our last scenario, which is reaching the zero-emissions economy after 30 years. As pointed out in the previous part about the effect of communication on climate policy, we impose a perfect foresight linearly increasing carbon price similar to Ferrari and Nispi Landi (2023) so that we reach a zero-emission economy after 30 years.

Figure 4 shows that some models predict a small-scale boost of the economy in the short term, which is most visible for investment. However, on average, output decreases slowly and then accelerates from period 5 onward to -4% after 30 years and then stays constant at this level. Under our analysis, this can be interpreted as a permanent loss of 4% output to transform into a clean economy. Similarly, consumption and investment decrease on average to -3% and -8%, respectively. Compared to the -40% emission scenario, the economy is (obviously) in a deeper recession, but macroeconomic variables do react slower since the firms expect a higher cost in the future and consequently produce more in the short term. In particular, investment reaches -6% after 25 years, instead of 3 years in the -40% scenario. Furthermore, although the confidence interval indicates a deeper recession for output and consumption (nearly doubled), the lower bound CI of investment is nearly at the same magnitude as before. Even after 50 years, no clear recovery is visible. Regarding inflation and interest rates, the zero-emission scenario predicts a small deflationary effect on average (around 0.2 percentage points after 25 years) and a comparable reduction of interest rates. However, both variables seem to return to zero afterward.

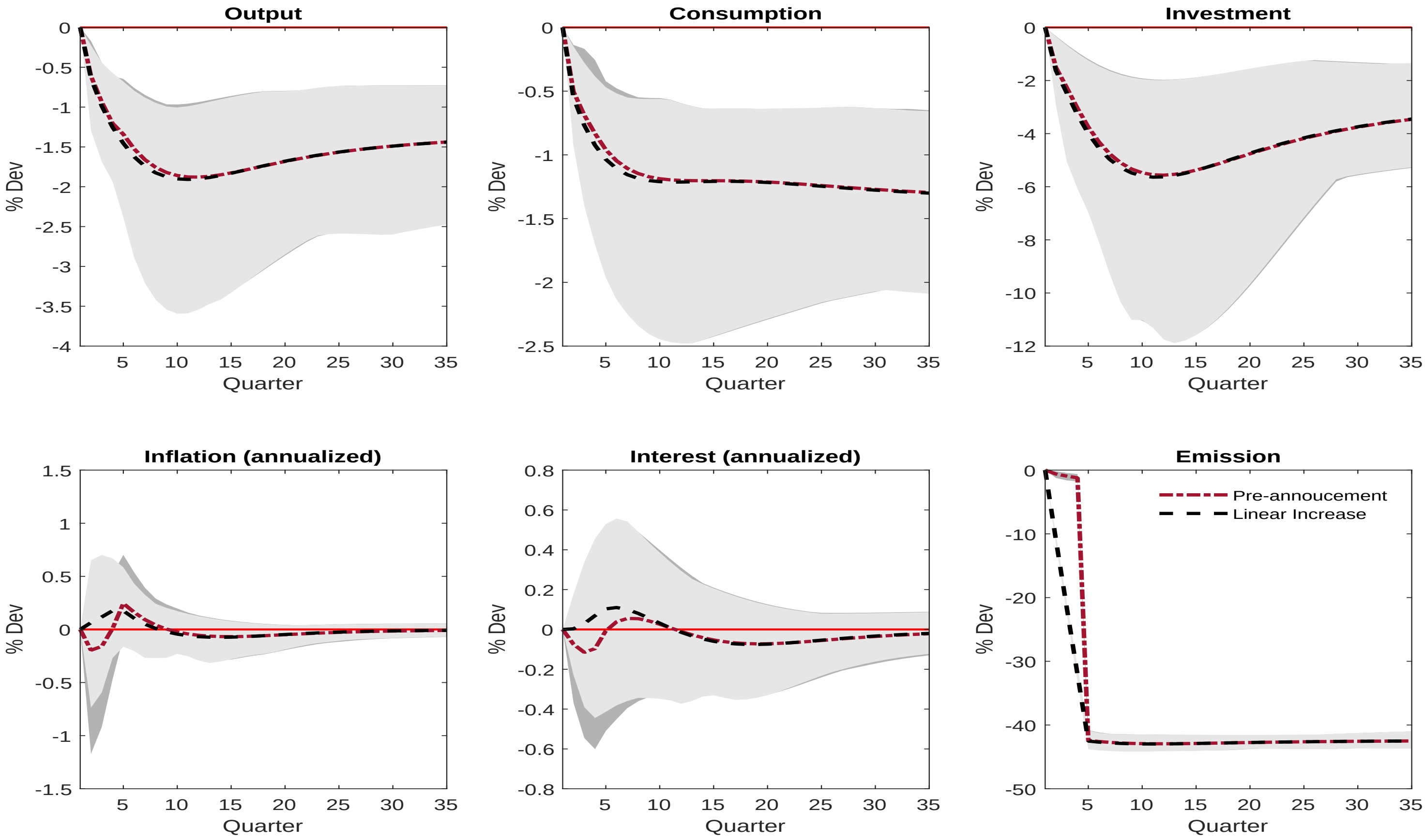

Figure 3: Anticipated shock - linearly increasing for 5 periods
Notes: The dashed black (dash-dotted red) line depicts the average of all models used, with light (dark) grey shaded area being the 90% credible set across models for linear (one-time) increase, respectively. Time is in quarters. Impulse responses are in % deviation from steady state, while inflation and interest are in percentage point deviation.

Our analysis is similar to the exercise in Ferrari and Nispi Landi (2023) where a forward guidance carbon price path is implemented. Our average result among models of inflation confirms the study of the inflationary effect of green transition in Ferrari and Nispi Landi (2022). We observe that if the households believe in the commitment to environmental policy, they take into account the future drop in output and income. This directs to a strong decrease in consumption and investment in the current period. As discussed above, a hike in carbon price will increase marginal cost and raise inflation from the supply side. However, if households anticipate the policy from the beginning, the decrease in demand outweighs the change in marginal cost which decreases the inflation rate. Within our set of models, compared to the other two scenarios, we observe that the design of environmental policy affects the inflation dynamic significantly. With an unanticipated introduction of a carbon price hike, the marginal cost effect seems to dominate as the households' demand takes some time to adjust. In the anticipated one-time introduction, the households cut back their consumption and investment as they anticipate the hike in carbon price in the 5th period. However, if the carbon price does not increase linearly for a long period, the supply side outranks the demand at some point. Thus, we see an inflationary effect before the price level stabilizes.

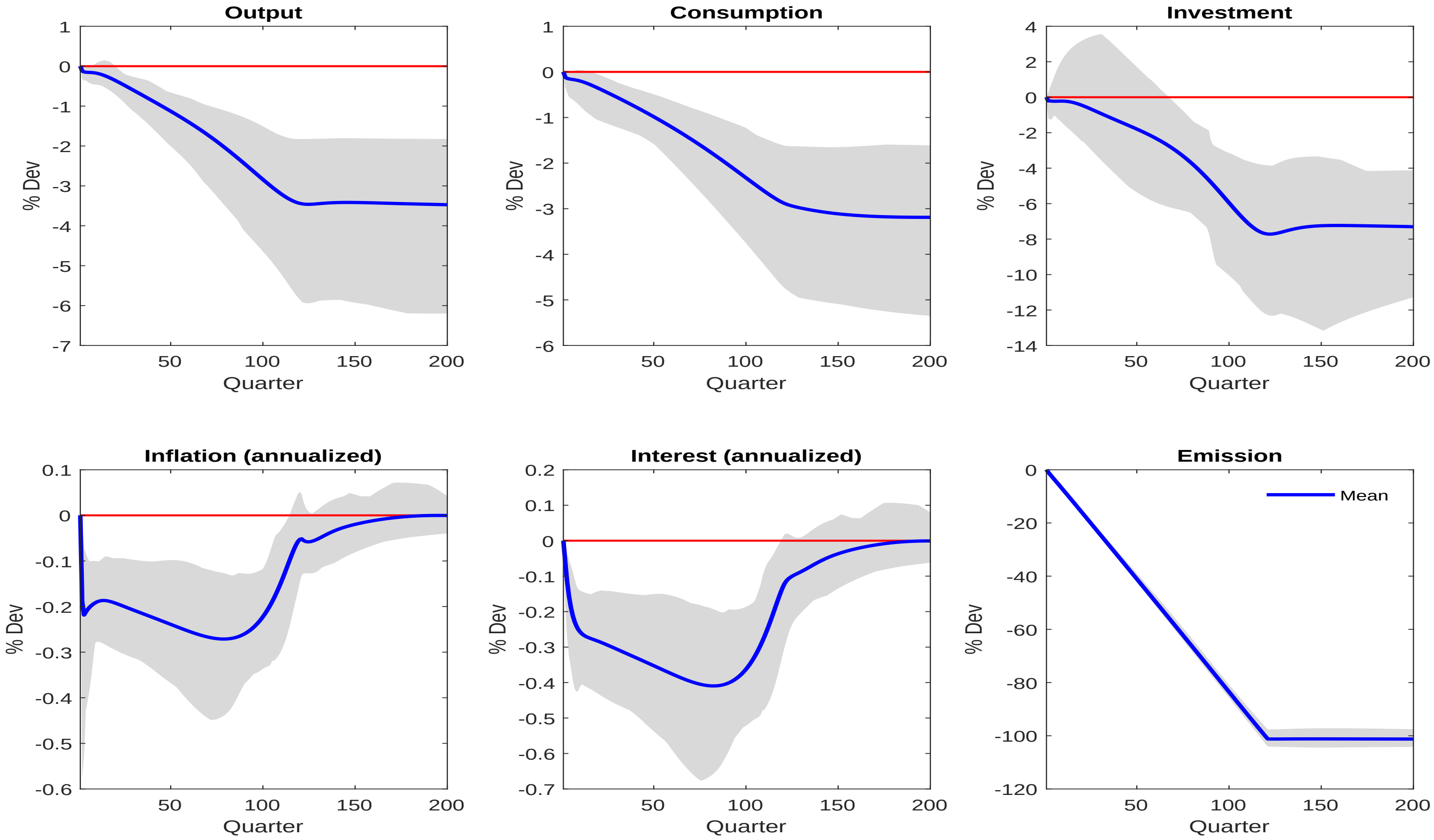

Figure 4: Anticipated shock toward a zero-emission economy
Notes. The blue line depicts the average of all models used, while the grey shaded area is the 90% credible set across models. Time is in quarters. Impulse responses are in % deviation from steady state, while inflation and interest are in percentage point deviation.

4 Optimal Policy

In this section, we study model-robust optimal monetary policy in the sense of setting the coefficients of the Taylor rule to minimize the central bank's loss function. We follow common practice (see Kuester and Wieland, 2010) by choosing a Taylor rule, which depends on current inflation and a measure of real activity. Similar to Dück and Verona (2023), we replace output gap with output growth because the latter is defined consistently across models. The common Taylor rule for all models becomes:[6]

$$\tilde{R}_t \; = \; \rho \, \tilde{R}_{t-1} + (1-\rho) \, \left(\alpha_\pi \, \mathbf{E}_t \left[\tilde{\pi}^q_{t+h} \right] + \alpha_y \, \Delta \tilde{Y}_t \right) \quad \text{for log-linearized models} \tag{50}$$

$$\frac{R_t}{R_*} \; = \; \left(\frac{R_{t-1}}{R_*} \right)^{\rho} \left[\left(\frac{\mathbf{E}_t \left[\pi^q_{t+h} \right]}{\pi_*} \right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}} \right)^{\alpha_y} \right]^{1-\rho} \quad \text{for models written in non-linear form}$$

where R_t is the annualized interest rate with smoothing parameter ρ, π^q_t and ΔY_t being the annualized inflation rate and output growth with central bank reaction coefficients α_π and α_y, respectively. As a baseline, we assume that the central bank sets interest rates according to observed inflation and output growth (h=0). As a robustness check, we use h=2 and 4 to incorporate expectations of future inflation in the policy decisions, as done in Binder et al. (2017).

We follow common practice (see Levin and Williams, 2003) and define the loss function of the central bank as a weighted sum of the unconditional variances of inflation, the change of interest rate, and output growth. Consistently with the Taylor rule, we use output growth instead of the output gap. The central bank loss function for a single model, as well as the unified loss function for all models simultaneously, are given by:

$$L^{CB}_m \; = \; Var_m \left(\tilde{\pi}^q \right) + \lambda_y \, Var_m \left(\Delta \tilde{Y} \right) + \lambda_R \, Var_m \left(\Delta \tilde{R} \right) \tag{51}$$

$$L^{CB} \; = \; \sum_{m=1}^{M} \omega_m \, L^{CB}_m \tag{52}$$

where ω_m is the weight of each model, λ_y and λ_R are the relative importance of output growth and the change of interest rate, compared to inflation, respectively. In this paper, we follow common practice (see Kuester and Wieland, 2010) and use flat priors ($\omega_m = 1/M$ for all models), and $\lambda_y = 1$, $\lambda_R = 0.5$ as relative importance. In appendix B we conduct various robustness tests for all optimal policy scenarios.

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[6] The "tilde" variables is expressed in percentage deviations from steady state.

4.1 Optimal Policy over Business Cycles

The central bank conducts optimal monetary policy over the business cycle by choosing the coefficients of the Taylor rule to minimize its (single or unified) loss function. That is, the central bank considers all model-specific shocks over the business cycle (i.e. temporary), taking the agent's decisions and the carbon price as given. We assume that the government increases carbon price temporarily, such that $\rho^e < 1$ (we set $\rho^e = 0.9$).

To find optimal model-specific policy rules, the central bank considers one model and the single loss function (see equation 51), such that it solves the following minimization problem (in the log-linearized case):

$$\min_{\{\rho,\alpha_\pi,\alpha_y\}} \quad L_m^{CB} \tag{53}$$

$$\begin{aligned} s.t. \quad \tilde{R}_t &= \rho\, \tilde{R}_{t-1} + (1-\rho)\, \left(\alpha_\pi\, \mathbf{E}_t\left[\tilde{\pi}^q_{t+h}\right] + \alpha_y\, \Delta\tilde{Y}_t\right) \\ \tilde{\tau}^e_t &= \rho^e\, \tilde{\tau}^e_{t-1} + \sigma^e \epsilon^e_t \qquad \epsilon^e_t \sim \mathcal{N}(0,1) \\ \mathbf{E}_t&\left[f\left(x_t, x_{t+1}, x_{t-1}, \Theta\right)\right] = 0 \end{aligned}$$

and there exists a unique and stable equilibrium for that model. f is the set of equations besides the policy rule. x and Θ are the endogenous variables and parameters.

The central bank considers all models and the unified loss function (see equation 52) to find the optimal model-robust policy rule, such that it solves the following minimization problem (in the log-linearized case):

$$\min_{\{\rho,\alpha_\pi,\alpha_y\}} \quad L^{CB} \tag{54}$$

$$s.t. \quad \tilde{R}_t = \rho\, \tilde{R}_{t-1} + \alpha_\pi\, \mathbf{E}_t\left[\tilde{\pi}^q_{t+h}\right] + \alpha_y\, \Delta\tilde{Y}_t$$

$$\tilde{\tau}_t^e \;=\; \rho^e \, \tilde{\tau}_{t-1}^e + \sigma^e \epsilon_t^e \qquad \epsilon_t^e \sim \mathcal{N}(0,1)$$

$$\mathbf{E}_t \left[f_m \left(x_t^m, x_{t+1}^m, x_{t-1}^m, z_t, \Theta^m \right) \right] = 0 \quad \forall m \in M$$

and there exists a unique and stable equilibrium for all models. z are the common endogenous variables in all models. The subscript or superscript m depicts model-specific endogenous variables (x^m), equations (f_m), and parameters (Θ^m).

As in the model uncertainty literature (see Dück and Verona, 2023), we define a grid for each policy parameter and check that those unconditional variances of all models used are non-distortive to ensure that the robust policy rule is not driven by a single model. We set the limits for each policy parameter ($\rho \in [0, 0.9]$, $\alpha_\pi \in [1, 5]$, $\alpha_y \in [0, 2]$)[7] and run a grid search (with steps of size 0.1 (0.2) below (above) 1 for all grids) to minimize the loss function.

Row 1 of table 1 shows the average of optimal model-specific coefficients (left) and optimal model-robust coefficients (right). If the central bank reacts to inflation and output growth equally, while penalizing interest rate movements, the degree of interest rate smoothing is large (0.9) for both rules. If the central bank considers model uncertainty, then its response with respect to inflation (and output growth) is significantly reduced from 10.3 to 8 (and from 6.2 to 4). This confirms findings in Dück and Verona (2023), qualitatively. Including a mutual effect of the economy and environment in the models, however, reduces the reaction coefficients both of the model-specific and of the model-robust rule. In the optimal model-robust rule, reaction to inflation (output growth) decreases from 10 (9) (see table 3, panel B, row 5 in Dück and Verona, 2023) to 8 (4).[8] Other minor sources of the quantitative differences might be the slightly different set of models we are using for our analysis and conducting the policy exercise for all regions simultaneously.

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[7] This yields grid boundaries of $\alpha_\pi \leq 50$ and $\alpha_y \leq 20$ for equation 50.

[8] Due to different notations, they display the coefficients as 1 (0.9) and 0.8 (0.4), which results in (after transformation to our notation) 10 (9) and 8 (4).

	Relative weights				Individual models				Model-robust rule			
Scenario	λ_y	λ_R	λ_e	λ_{τ^e}	$\bar{\rho}$	$\bar{\alpha}_\pi$	$\bar{\alpha}_y$	$\bar{\theta}^e$	ρ_r	$\alpha_{\pi,r}$	$\alpha_{y,r}$	θ^e_r
Business cycle	1	0.5			0.9	10.3	6.2		0.9	8	4	
Transition risk	1	0.5			0.89	6.7	3.8		0.9	7	5	
Cooperation	1	0.5	0.5	0.1	0.87	5.1	13.4	0.23	0.9	5	16	0.2
Cooperation	1	0.5	1	0.1	0.83	3.5	14.1	0.49	0.9	4	20	0.55
Cooperation	1	0.5	0	0	0.9	12	6.6	1	0.9	9	5	1

Table 1: Model-specific and model-robust monetary policy rules
Notes. A bar depicts the average across models. In the no-cooperation scenarios, θ^e is not chosen optimally and hence the corresponding values are left blank intentionally.

4.2 Optimal Policy for Transition Risk

We use the same maximization problem as in the equation system 54, but use permanent carbon price shock only to capture optimal policy during the transition risk, such that $\rho^e = 1$.

Row 2 in table 1 suggests that the central bank should be on average more restrained in its reaction to inflation and output growth due to the larger importance of the carbon price. However, the model-robust rule is very similar to the rule optimized over the business cycle. The optimal inflation response is slightly smaller, while the optimal output growth response is slightly larger, despite the average model-specific implying the opposite.

4.3 Optimal Policy in Cooperation with Government

As mentioned earlier, the environment has been the top priority for the government and the carbon price appears to be one of the most prominent tools to tackle this problem. Hence, in this part, we study the case when the central bank cooperates with the government to ‘fight’ climate change. Central banks can only control the monetary policy rule while the fiscal authority is in charge of carbon price policy. Hence, in this setup, the government follows a rule to set carbon prices, which is an increasing function of emissions. In particular, we assume the carbon price rule to be:

$$\tilde{\tau}_t^{\,e} = \theta^e \, \tilde{e}_t \qquad \text{for log-linearized models} \tag{55}$$

$$\frac{\tau_t^e}{\tau_*^e} = \left(\frac{e_t}{e_*}\right)^{\theta^e} \qquad \text{for models written in non-linear form} \tag{56}$$

The loss function becomes the sum of the central bank and government loss functions. The latter is the variance of emissions and the variance of the carbon price which is the main instrument for environmental policy, such that the new (single and unified) policymaker's loss function becomes:

$$L_m^{PM} = L_m^{CB} + \lambda_e \, Var_m\left(\tilde{e}\right) + \lambda_{\tau^e} \, Var\left(\tilde{\tau}^e\right) \tag{57}$$

$$L^{PM} = \sum_{m=1}^{M} \omega_m \, L_m^{PM} \tag{58}$$

We follow Annicchiarico and Di Dio (2015) and use total factor productivity (TFP) shock only for this scenario.[9] As central banks have a dual mandate to control price stability and promote output growth, we assume the government has a target to reduce emissions through its carbon price policy. Since the policymaker can control the carbon price, we assume they do it by choosing the reaction parameter θ^e optimally. Following the design of the central bank loss function, we extend the joint loss function to reflect the environmental target. The joint problem of the policymaker in a cooperative manner to find the optimal model-robust policy rule takes the following form (in the log-linearized case):[10]

$$\min_{\{\rho,\alpha_\pi,\alpha_y,\theta^e\}} \quad L^{PM} \tag{59}$$

$$s.t. \quad \tilde{R}_t = \rho \, \tilde{R}_{t-1} + \alpha_\pi \, \mathbf{E}_t \left[\tilde{\pi}_{t+h}^q\right] + \alpha_y \, \Delta\tilde{Y}_t$$

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[9] For this analysis, we introduce a common a TFP shock with common persistence ($\rho_a = 0.9$) for all models which do not feature a TFP shock, originally.

[10] Analogously to the first two scenarios, the policymaker finds the optimal model-specific policy rules by considering the single loss function (see equation 57) and equations of the corresponding model only.

$$\tilde{\tau}_t^e = \theta^e \, \tilde{e}_t$$

$$\mathbf{E}_t \left[f_m \left(x_t^m, x_{t+1}^m, x_{t-1}^m, z_t, \Theta^m \right) \right] = 0 \quad \forall m \in M$$

We set the grid limits as $\theta^e \in [0, 1]$ with step size 0.1. Rows 3-6 of table 1 show the average of optimal model-specific coefficients (left) and optimal model-robust coefficients (right) for several combinations of penalizing variations of emissions and carbon price. For given relative importance of output growth and interest rate changes, we notice the following pattern. First, model-robust rules mostly prescribe a more restrained reaction to inflation than the average model-specific rules, while the reaction to output growth is more aggressive. Second, compared to the business cycle and transition risk scenarios, the central bank should always react less strongly with respect to inflation and always (significantly) more with respect to output growth. This finding becomes more pronounced, the larger the relative importance of the variance of emissions (compare rows 3 and 4).

If the central bank penalizes variations in emissions more than the variation in carbon price (rows 3-4), then the optimal reaction coefficient typically increases with the difference in the penalties. In particular, row 4 suggests a coefficient of 0.5, while it is 0.2 and 0.1 in the neighboring rows. Moreover, the coefficients are basically equal for the optimal model-specific rules and the model-robust rule (for example 0.23 and 0.2 in row 3). Interest rate smoothing slightly decreases for the model-specific rules but remains at a very high level.

If the policymaker cooperates with the government, but they do not care about emissions (see row 5), we find that the optimal response of the carbon price with respect to emissions is always at the upper bound. Optimal response coefficients become similar to the business cycle scenario, yet the central bank reacts slightly more aggressively.

4.4 Optimal Policy per Region

In this subsection, we compare optimal monetary policy per region. Therefore, we split the models by the data used for estimation or calibration. Table 7 depicts the region for estimated models. Euro Area (United States) models acronyms begin with EA_ (US_), respectively. Model acronym NK_ indicates that the model is calibrated. Only one model (NK_CKL09) is calibrated using EA data. In total, we consider 5 EA models ($M_{EA} = 5$) and 24 US models ($M_{US} = 24$).

Consequently, we repeat the optimal policy analysis of this section (all three scenarios) for each of the two regions, separately. In particular, we use flat priors, setting weights of the other region to zero, such that:

$$\omega_m^{EA} = \begin{cases} \frac{1}{M_{EA}} & \text{for EA models} \\ 0 & \text{otherwise} \end{cases}$$

$$\omega_m^{US} = \begin{cases} \frac{1}{M_{US}} & \text{for US models} \\ 0 & \text{otherwise} \end{cases}$$

As in the baseline case for all models, we set $\lambda_y = 1$, $\lambda_R = 0.5$ (and $\lambda_e = 0.5$, $\lambda_{\tau^e} = 0.1$ for the cooperation scenario). To obtain optimal model-robust policy rules, we use ω_m^{EA} or ω_m^{US} to solve the optimization problem 54 over the business cycle and during transition risk, and optimization problem 59 for cooperation with the government.

Table 2 shows the results of average model-specific and model-robust policy rules for each region. On average, EA models prescribe optimal policy rules with a more restrained reaction to inflation, but a more aggressive reaction to output growth (with the exception of transition risk scenario) than the US models (for loss functions with the same relative importance of variation of output growth, λ_y). Regarding optimal model-robust rules, due to a large number of US models, the model-robust rule considering *all* models follows US model implications. As for model-specific rules, optimal model-robust EA rules imply a more restrained reaction to inflation, but a more aggressive reaction to output growth (with the exception of the transition risk scenario) compared to US rules. Notably, regarding model uncertainty, the typical implications do not hold for EA models over the business cycle and US models during transition risk. The first might be a consequence of the few available EA models used for the analysis.

	Euro Area								United States							
	Individual models				Model-robust rule				Individual models				Model-robust rule			
Scenario	$\bar{\rho}^{EA}$	$\bar{\alpha}_{\pi}^{EA}$	$\bar{\alpha}_{y}^{EA}$	$\bar{\theta}^{e,EA}$	ρ_{r}^{EA}	$\alpha_{\pi,r}^{EA}$	$\alpha_{y,r}^{EA}$	$\theta_{r}^{e,EA}$	$\bar{\rho}^{US}$	$\bar{\alpha}_{\pi}^{US}$	$\bar{\alpha}_{y}^{US}$	$\bar{\theta}^{e,US}$	ρ_{r}^{US}	$\alpha_{\pi,r}^{US}$	$\alpha_{y,r}^{US}$	$\theta_{r}^{e,US}$
Business cycle	0.9	8.2	7.8		0.9	8	8		0.9	10.7	5.8		0.9	8	4	
Transition risk	0.9	5.6	2.8		0.9	4	1		0.88	7	4		0.9	7	5	
Cooperation	0.9	4	15.8	0.2	0.9	2	18	0.2	0.87	5.3	12.9	0.23	0.9	5	16	0.2

Table 2: Model-specific and model-robust monetary policy rules
Notes. A bar depicts the average across models. In the no-cooperation scenarios, θ^{e} is not chosen optimally and hence the corresponding values are left blank intentionally. Superscript EA or US indicates the model regions.

5 Evaluation of Model-robust Monetary Policy

Model-robust rules are policy rules, which perform well across models. Consequently, the optimal model-specific rule optimized on one model might perform poorly (worse than the model-robust rule) in another model. On the other hand, model-robust rules perform worse than the model-specific optimal Taylor rule in this specific model.

We try to quantify these (dis-)advantages of model-robust Taylor rules by calculating and comparing the losses of the following rules in all scenarios of optimal monetary policy. In appendix C we do this using the corresponding policy rules to calculate and compare IRFs.

First, for each model, we use its own optimal model-specific Taylor rule. This depicts the loss of a single model would be true and the policymaker uses it for its optimal policy decision. In this ideal world, if we select a true model randomly, the ideal loss would be, on average:

$$L^{i} = \frac{1}{M} \sum_{m=1}^{M} L_{m} | \left(\rho_{m}, \alpha_{\pi,m}, \alpha_{y,m}\right) \tag{60}$$

Second, to insure against picking the wrong model, the policymaker computes the optimal model-robust Taylor rule, no matter which model is chosen. If we select a model randomly, the model-robust loss would be, on average:

$$L^r = \frac{1}{M} \sum_{m=1}^{M} L_m | \left(\rho_r, \alpha_{\pi,r}, \alpha_{y,r}\right) \tag{61}$$

Table 3 (column 1) shows the percentage loss increases of the optimal model-robust loss compared to the loss when picking the true model. For all scenarios, model-robust rules exhibit larger losses than the model-specific optimal policy rule (4% - 20%). Due to the different settings in each scenario, losses are not comparable between the scenarios, but rather within a scenario.

Last, we analyze the case that the central bank chooses the wrong model for optimal policy. In particular, we calculate the average loss of using the model-specific Taylor rule optimized in one model in all models. We compute the average for all model-specific rules to ensure the stability of the results. The loss of picking the wrong model is then, on average:

$$L^w = \frac{1}{M} \sum_{p=1}^{M} \left[\frac{1}{M} \sum_{m=1}^{M} L_m | \left(\rho_p, \alpha_{\pi,p}, \alpha_{y,p}\right) \right] \tag{62}$$

Table 3 (column 2) shows the percentage loss decreases of the optimal model-robust loss compared to the loss when picking the wrong model. For all scenarios, model-robust losses are 11% - 23% smaller than the average losses of picking the wrong model. To have the same denominator, we compare the loss of picking the wrong and true model (column 3) and find large loss increases of 16% - 56%.

We can confirm that model-robust rules are indeed robust to model uncertainty and insure against picking the wrong model at a comparatively lower cost, i.e. selecting the wrong model causes 3-4 times larger loss increases than relying on the model-robust rule.

Scenario \ Loss	$100\,\frac{L^r - L^i}{L^i}$	$100\,\frac{L^r - L^w}{L^w}$	$100\,\frac{L^w - L^i}{L^i}$
Business cycle	4 %	-11 %	16 %
Transition risk	7 %	-17 %	28 %
Cooperation	20 %	-23 %	56 %

Table 3: Percentage loss deviations

6 Conclusion

Climate change has turned from a highly discussed political concern to many open questions for economists. Hence, many questions remain unanswered on all aspects of the environment and climate change. In this paper, we have investigated the role of the most promising climate policy, a carbon price (tax), in the environment and its cost to the economy. We study this by taking into account the uncertainty of model predictions. Hence, we include environmental aspects into 29 workhorse DSGE models that have been used for studying business cycles around the world for the last decades. Within the predictions in our set of models, we observe uncertainty in transition risk. We also find that well communication and early climate action can mitigate the fluctuation in price levels. Furthermore, we find a deflationary effect of a well-communicated carbon price to transform the economy into a zero-emission economy. This might be of interest to policymakers as this transition path does not create a trade-off problem between output growth and price stability. Hence, the government can act more to mitigate the transition path effect.

In optimal policy exercise, we find that during transition risk, the central bank should react less in their interest rate setting with respect to inflation and output growth and even less due to model uncertainty. In cooperation with the government, the central bank should always react less strongly with respect to inflation and always (significantly) more with respect to output growth, compared to the other scenarios. This pattern becomes more pronounced with the increasing relative importance of the variance of emissions. Moreover, model-robust rules always prescribe a more aggressive reaction to output growth than the average model-specific rules. In all scenarios, the model-robust rule has a slightly higher loss than the optimal model-specific policy rule of the specific model, but it has (on average) a 3-4 times lower loss increase than the model-specific rule in other models.

To the best of our knowledge, this paper is among the first to study the uncertainty in transition risk. Moreover, it opens many venues for future research. It is natural to analyze the role of the green sector during the transition risk. This way, the effect of transition risk and the role of the substitution effect of green sectors can be better approximated. We leave this for future research.

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A Model Variants - Derivations

A.1 Models with Calvo and no Inflation Indexation

In this setting, the intermediate goods producers choose an optimal price, which is not increased by past inflation. Hence, $X^p_{t,s} = 1 \, \forall s$. The intermediate goods firms maximization problem, FOC, and price index become:

$$\max_{\tilde{P}_t(i)} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \left[\tilde{P}_t(i) \, - MC_{t+s} \right] \, Y_{t+s}(i)$$

$$s.t \quad Y_{t+s}(i) \, = \, Y_{t+s} \, G'^{-1} \left(\frac{P_t(i)}{P_{t+s}} \int_0^1 G' \left(\frac{Y_{t+s}(i)}{Y_{t+s}} \right) \frac{Y_{t+s}(i)}{Y_{t+s}} \, di \right)$$

$$\mathbf{E}_t \sum_{s=0}^{\infty} \xi_p^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, Y_{t+s}(i) \left[\tilde{P}_t(i) + \left(\tilde{P}_t(i) - MC_{t+s} \right) \, \frac{1}{G'^{-1}(z_{t+s})} \, \frac{G'(x_{t+s})}{G''(x_{t+s})} \right] \, = \, 0$$

$$P_t \, = \, (1 - \xi_p) \, \tilde{P}_t \, G'^{-1} \left[\frac{\tilde{P}_t \, \tau_t}{P_t} \right] + \xi_p \, P_{t-1} \, G'^{-1} \left[\frac{P_{t-1} \, \tau_t}{P_t} \right]$$

Analogously, the optimal wage setting changes slightly. Note that $X^w_{t,s} = 1 \, \forall s$. Furthermore, $W_{t+s}(i) = \gamma \, \tilde{W}_t(i)$, such that the optimization problem, the FOC, and the wage index simplify to:

$$\max_{\tilde{W}_t(i)} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \xi_w^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, \left[W_{t+s}(i) - W^h_{t+s} \right] \, L_{t+s}(i)$$

$$s.t. \quad L_{t+s}(i) \, = \, \left(\frac{\gamma \, \tilde{W}_t(i)}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} \, L_{t+s}$$

$$\mathbf{E}_t \sum_{s=0}^{\infty} \xi_w^s \, \beta^s \, \frac{\Xi_{t+s} \, P_t}{\Xi_t \, P_{t+s}} \, L_{t+s} \, \frac{1}{\lambda_{w,t+s}} \left[(1 + \lambda_{w,t+s}) \, W^h_{t+s} - \gamma \, \tilde{W}_t \right] \, = \, 0$$

$$W_t \, = \, \left[(1 - \xi_w) \, \tilde{W}_t^{\frac{1}{\lambda_{w,t}}} + \xi_w \, \left(\gamma \, W_{t-1} \right)^{\frac{1}{\lambda_{w,t}}} \right]^{\lambda_{w,t}}$$

A.2 Models with CES Aggregator

Using the CES aggregator, the constraint of intermediate goods firms in their cost minimization problem becomes and replaces equation 23:

$$Y_t \;=\; \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$$

where ϵ refers to the price elasticity of demand for good i. The resulting demand function and price index yield:

$$Y_{t+s}(i) \;=\; \left(\frac{P_t(i)\,\left(\Pi_{l=1}^{s}\,\pi_{t+l-1}^{\iota_p}\,\pi_*^{1-\iota_p}\right)}{P_{t+s}}\right)^{-\epsilon} Y_{t+s}$$

$$P_t \;=\; \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

The optimal price setting of intermediate goods producers is then subject to the demand equation above and yields the FOC and price index:

$$\mathbf{E}_t \sum_{s=0}^{\infty} \xi_p^s\, \beta^s\, \frac{\Xi_{t+s}\, P_t}{\Xi_t\, P_{t+s}}\, Y_{t+s} \left[(1-\epsilon)\,\left(\Pi_{l=1}^{s}\,\pi_{t+l-1}^{\iota_p}\,\pi_*^{1-\iota_p}\right)^{1-\epsilon}\,\tilde{P}_t + \epsilon\,\left(\Pi_{l=1}^{s}\,\pi_{t+l-1}^{\iota_p}\,\pi_*^{1-\iota_p}\right)^{-\epsilon}\, MC_{t+s}\right] \;=\; 0$$

$$P_t^{1-\epsilon} \;=\; (1-\xi_p)\,\tilde{P}_t^{1-\epsilon} + \xi_p\,\left(\pi_{t-1}^{\iota_p}\,\pi_*^{1-\iota_p}\right)^{1-\epsilon}\, P_{t-1}^{1-\epsilon}$$

A.3 Models with Rotemberg Price Adjustment Costs

For Rotemberg (1982) quadratic price adjustment costs, the intermediate good firms still maximize their profits. They can choose the optimal price each period but are subject to quadratic price adjustment costs and as usual the production function as well as the demand function. The maximization problem, and since all firms are identical, the FOC, and price index become:[11]

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[11] For reasons of clarity, we use CES aggregation and note that there is no inflation indexation.

$$\max_{P_t(i)} \quad \mathbf{E}_t \sum_{s=0}^{\infty} \beta^s \, \frac{\Xi_{t+s}}{\Xi_t} \left[\frac{P_t(i) - MC_t(i)}{P_t} \, Y_t(i) - \frac{\phi_p}{2} \, \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi_* \right)^2 \, Y_t \right]$$

$$s.t. \;\; (25), \;\; Y_t(i) \; = \; \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \, Y_t$$

The NKPC takes the following form:

$$\pi_t \; (\pi_t - \pi_*) \; = \; \beta \, \mathbf{E}_t \left(\frac{\Xi_{t+1}}{\Xi_t} \, \pi_{t+1} \; (\pi_{t+1} - \pi_*) \; \frac{Y_t}{Y_{t-1}} \right) + \frac{\epsilon}{\phi_p} \, \left(MC_t - \frac{\epsilon - 1}{\epsilon} \right)$$

Analogously, the optimal wage setting reads, and simplifies to :

$$\max_{w_t(i)} \quad E_t \sum_{t=0}^{\infty} \beta^t \frac{\Xi_t}{\Xi_0} \left[\frac{w_t(i)}{P_t} L_t(i) - \frac{\phi_w}{2} \, \left(\frac{w_t(i)}{w_{t-1}(i)} - \pi_*^w \right)^2 \, Y_t \right]$$

$$s.t. \;\; L_t(i) \; = \; \left(\frac{w_t(i)}{w_t} \right)^{-\epsilon_w} \, L_t$$

$$\pi_t^w \; (\pi_t^w - \pi_*) \; = \; \beta \, \mathbf{E}_t \left(\frac{\Xi_{t+1}}{\Xi_t} \, \pi_{t+1}^w \; (\pi_{t+1}^w - \pi_*) \; \frac{Y_t}{Y_{t-1}} \right) + \left(\frac{1 - \epsilon}{\phi_w} \right)$$

Note that the growth rate of wages is γ in the Smets and Wouters (2007) model, such that the steady state wage inflation is γ. Consequently, $\pi_*^w = \gamma$.

The market clearing condition includes price and wage adjustment costs.

$$Y_t \; = \; C_t + I_t + G_t + a(Z_t) \, K_{t-1} + \theta_1 \, \mu_t^{\theta_2} \, Y_t + \frac{\phi_p}{2} \; (\pi_t - \pi_*)^2 \, Y_t + \frac{\phi_w}{2} \; (\pi_t^w - \pi_*^w)^2 \, Y_t$$

B Robustness Checks

B.1 Optimal Policy over Business Cycles

The optimal policy coefficients of various robustness tests can be seen in table 4.

For the first test, we use h-periods ahead expectation of inflation in the Taylor rule of the central bank (see equation 50) for h=2 and h=4. We find (see rows 1 and 2 of each panel) that the longer the forecast horizon of inflation, the stronger the reaction to inflation while keeping the reaction to output growth (approximately) unchanged. This holds for the model-specific and the model-robust rules. Again, model-robust rules prescribe smaller coefficients than model-specific rules. As pointed out in , indeterminacy increases with an increased forecasting horizon in the Taylor rule. This likely causes the region above $\alpha_\pi = 10$ to become unstable for some US models, such that the next best solution is at a rather small inflation reaction coefficient for this region.

Second, we mimic the single mandate of the ECB of maintaining only price stability by setting the relative importance of output growth in the loss function to zero. Row 3 shows that the ECB should not react (or only barely on average) to output growth while keeping inflation reaction for the robust rule unchanged. This holds for all regions. However, for the average of US models, the inflation coefficient slightly increases.

Lastly, we check the robustness with respect to penalizing changes in the interest rate. If the penalty decreases to 0.1 (row 4), then both coefficients increase (output growth more), while they slightly decrease if the penalty is increased to 1 (row 5). Although the coefficients differ in their magnitude, the model-robust rules typically prescribe smaller coefficients than the model-specific rule (with one exception). Noticeably, even for a small penalty (0.1 in row 4), the interest rate smoothing coefficient remains very large.

			Individual models			Model-robust rule		
λ_y	λ_R	h	$\bar{\rho}$	$\bar{\alpha_\pi}$	$\bar{\alpha_y}$	ρ_r	$\alpha_{\pi,r}$	$\alpha_{y,r}$
1	0.5	2	0.89	21.4	4.9	0.9	20	4
1	0.5	4	0.89	25	5.3	0.9	22	4
0	0.5	0	0.9	11.7	1	0.9	8	0
1	0.1	0	0.9	18.3	13.4	0.9	14	14
1	1	0	0.9	7.8	4.2	0.9	7	3

			Euro Area						United States					
			Individual models			Model-robust rule			Individual models			Model-robust rule		
λ_y	λ_R	h	$\bar{\rho}^{EA}$	$\bar{\alpha}_{\pi}^{EA}$	$\bar{\alpha}_{y}^{EA}$	ρ_r^{EA}	$\alpha_{\pi,r}^{EA}$	$\alpha_{y,r}^{EA}$	$\bar{\rho}^{US}$	$\bar{\alpha}_{\pi}^{US}$	$\bar{\alpha}_{y}^{US}$	ρ_r^{US}	$\alpha_{\pi,r}^{US}$	$\alpha_{y,r}^{US}$
1	0.5	2	0.9	27.2	5	0.9	22	7	0.88	20.3	4.8	0.9	20	4
1	0.5	4	0.9	37.2	6.2	0.9	44	5	0.89	22.4	5.2	0.9	10	5
0	0.5	0	0.9	8	1.8	0.9	8	2	0.9	12.5	0.8	0.9	8	0
1	0.1	0	0.9	15.4	16.4	0.9	14	16	0.9	19	12.8	0.9	14	12
1	1	0	0.9	6.4	5.2	0.9	6	6	0.9	8.1	4	0.9	7	3

Table 4: Model-specific and model-robust monetary policy rules over the business cycle

B.2 Optimal Policy for Transition Risk

For the optimal policy during transition risk, we repeat the first two robustness exercises and display the resulting coefficients in table 5.

Forecast-based Taylor rules for inflation (rows 1-2) call for a more aggressive reaction with respect to inflation for all rules. In contrast to the optimal coefficients over the business cycle, for transition risk, the central bank should be more cautiously adjusting interest rates for observed output growth the larger the forecast horizon is. This holds for model-specific and model-robust rules. EA models imply a very strong reaction to inflation, but less reaction with respect to output growth, compared to US models. Notably, for US models, the average inflation response increases from 2 to 4 quarters forecasting horizon, while model-robust coefficients decrease. This might again indicate increasing instability regions with higher forecast horizon.

Second, the implications for the single mandate of the ECB remain valid for transition risk. In particular, the central bank should on average react substantially more to inflation and less to output growth, while the latter coefficients become very small or zero. This holds for both regions, but not for model-robust rules.

			Individual models			Model-robust rule		
λ_y	λ_R	h	$\bar{\rho}$	$\bar{\alpha}_\pi$	$\bar{\alpha}_y$	ρ_r	$\alpha_{\pi,r}$	$\alpha_{y,r}$
1	0.5	2	0.87	8.9	2.5	0.9	9	3
1	0.5	4	0.9	10.6	2.1	0.9	10	1
0	0.5	0	0.87	16.3	1.3	0.9	8	0

			Euro Area						United States					
			Individual models			Model-robust rule			Individual models			Model-robust rule		
λ_y	λ_R	h	$\bar{\rho}^{EA}$	$\bar{\alpha}_\pi^{EA}$	$\bar{\alpha}_y^{EA}$	ρ_r^{EA}	$\alpha_{\pi,r}^{EA}$	$\alpha_{y,r}^{EA}$	$\bar{\rho}^{US}$	$\bar{\alpha}_\pi^{US}$	$\bar{\alpha}_y^{US}$	ρ_r^{US}	$\alpha_{\pi,r}^{US}$	$\alpha_{y,r}^{US}$
1	0.5	2	0.9	17	1.8	0.9	12	0	0.86	7.2	2.7	0.9	8	3
1	0.5	4	0.9	17	0	0.9	26	0	0.9	9.3	2.6	0.9	7	0
0	0.5	0	0.9	15	3	0.9	8	0	0.86	16.6	1	0.9	8	0

Table 5: Model-specific and model-robust monetary policy rules for transition risk

B.3 Optimal Policy in Cooperation with Government

Table 6 shows optimal policy coefficients for various robustness tests in the cooperation with government scenario.

We check the implications of the different relative importance of variations in emissions and carbon price. If they are weighted equally and not zero (rows 1-2), then the government should not change the carbon price when it observes emissions but rather set the interest rate slightly more aggressively with respect to output growth. The optimal response is 18 instead of 16 (see row 1). Decreasing the importance of variations in emissions and carbon price (see row 2), assimilates the optimal response coefficients with the coefficients optimized over the business cycle, i.e. inflation response is larger and output growth response is smaller. These findings hold for both regions as well.

Forecast-based Taylor rules for inflation (rows 3-4) seem to have no or only s influence on the policymaker's carbon price reaction on emissions. For the robust rules, the coefficient remains 0.2, while on average it increases slightly to up to 0.32. As before, the reaction to inflation becomes larger (on average) if a forecast-based policy rule is used, but it does not increase with the forecast horizon. Notably, this does not hold for model-robust rules. Furthermore, model-robust rules show a typical pattern, neither for inflation nor for output growth. They also do not always exhibit a more cautious reaction to inflation or output growth. Somewhat strangely, the model-robust interest rate smoothing coefficient decreases for h=2 for all (0.6) and for US models (0.8), but it decreases only moderately for all (0.8) and not for the US models if h=4.

Last, we find that the single mandate of the ECB (row 3) calls for the same coefficients as the dual mandate of the FED. This holds independent of the region

For all robustness tests, the model-robust findings of the cooperation scenario hold. In particular, model-robust rules exhibit the same reaction to emissions as the model-specific average, and the policymaker is more cautiously reacting to inflation (with the exception of forecast-based rules) and more aggressive to output growth.

					Individual models				Model-robust rule			
λ_y	λ_R	λ_e	λ_{τ^e}	h	$\bar{\rho}$	$\bar{\alpha_\pi}$	$\bar{\alpha_y}$	$\bar{\theta}^e$	ρ_r	$\alpha_{\pi,r}$	$\alpha_{y,r}$	θ_r^e
1	0.5	0.5	0.5	0	0.87	5	13.3	0.03	0.9	5	18	0
1	0.5	0.1	0.1	0	0.9	8.4	8.6	0.04	0.9	7	10	0
1	0.5	0.5	0.1	2	0.89	9.3	9.6	0.23	0.6	3	4	0.2
1	0.5	0.5	0.1	4	0.88	9.1	8.6	0.25	0.8	5	6	0.2
0	0.5	0.5	0.1	0	0.88	5.1	12.7	0.23	0.9	5	16	0.22

					Euro Area								United States							
					Individual models				Model-robust rule				Individual models				Model-robust rule			
λ_y	λ_R	λ_e	λ_{τ^e}	h	$\bar{\rho}^{EA}$	$\bar{\alpha}_\pi^{EA}$	$\bar{\alpha}_y^{EA}$	$\bar{\theta}^{e,EA}$	ρ_r^{EA}	$\alpha_{\pi,r}^{EA}$	$\alpha_{y,r}^{EA}$	$\theta_r^{e,EA}$	$\bar{\rho}^{US}$	$\bar{\alpha}_\pi^{US}$	$\bar{\alpha}_y^{US}$	$\bar{\theta}^{e,US}$	ρ_r^{US}	$\alpha_{\pi,r}^{US}$	$\alpha_{y,r}^{US}$	$\theta_r^{e,US}$
1	0.5	0.5	0.5	0	0.9	4	15.8	0	0.9	2	18	0	0.87	5.2	12.8	0.04	0.9	5	16	0
1	0.5	0.1	0.1	0	0.9	5.4	8.4	0.04	0.9	3	8	0	0.9	9.1	8.7	0.05	0.9	7	10	0
1	0.5	0.5	0.1	2	0.86	15.8	9	0.24	0.9	2	14	0.2	0.89	7.9	9.7	0.23	0.7	2.3	1	0.2
1	0.5	0.5	0.1	4	0.9	22.8	10.4	0.32	0.7	1.3	1.7	0.2	0.87	6.2	8.3	0.24	0.9	10	16	0.2
0	0.5	0.5	0.1	0	0.9	4.4	15	0.2	0.9	2	18	0.2	0.87	5.2	12.2	0.23	0.9	5	16	0.2

Table 6: Model-specific and model-robust monetary policy rules in cooperation with government

C Evaluation of Model-robust Monetary Policy

Figure 5 displays the IRFs using the policy rules explained in section 5. It can be seen that in terms of output, consumption, and investment, the model-robust rule performs across models nearly as well as the model-specific optimal rules in the corresponding (own) model. However, picking some model-specific optimal policy rule performs, on average around 0.5% worse for output (and half the amount for consumption), while the 90% credible set exhibits significantly less consumption and investment at the lower tale/end. Notably, inflation is very similar for all the rules. This means, picking the model-robust rule decreases the loss in terms of output, consumption, and investment during the transition risk, without impacting inflation.

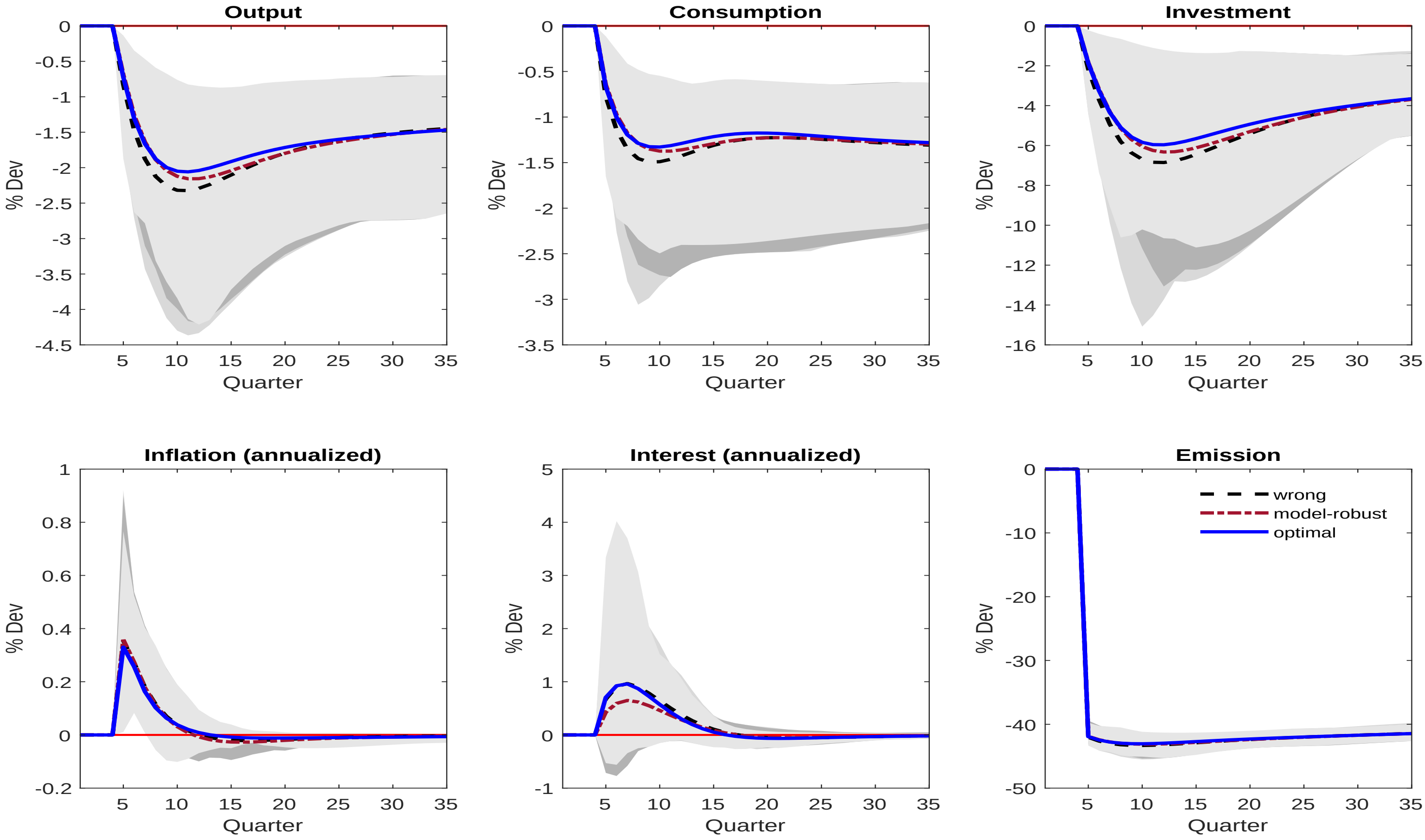

Figure 5: Optimal Monetary Policy Rules - Comparison
Notes: The solid blue, dash-dotted red and dashed black lines depict averages of all models used when using the model-specific rule in each model, the model-robust rule in all rules, and the model-specific rules in all models while the medium, dark and light grey shaded areas display the 90% credible set across models, respectively. The figure displays an unanticipated carbon price increase for the 40% emission reduction scenario. Time is in quarters. Impulse responses are in % deviation from steady state, while inflation and interest are in percentage point deviation.

D List of models

Table 7 shows the acronyms and references of all models used in our work. The detailed description of model specification (sorted by model acronyms) can be found in Macroeconomic Model Data Base by Wieland et al. (2016).[12]

Model acronym	Reference
EA_CKL09	Christoffel, Kuester and Linzert (2009)
EA_SW03	Smets and Wouters (2003)
EA_SWW14	Smets, Warne and Wouters (2014)
EA_VI16bgg	Villa (2016)
NK_BGG99	Bernanke, Gertler and Gilchrist (1999)
NK_CKL09	Christoffel, Kuester and Linzert (2009)
NK_GK09lin	Gertler and Karadi (2011)
NK_KM16	Krause and Moyen (2016)
US_ACELm	Altig et al. (2005)
US_ACELswm	Altig et al. (2005)
US_BKM12	Bils, Klenow and Malin (2012)
US_CD08	Christensen and Dib (2008)
US_CFOP14	Carlstrom et al. (2014)
US_DG08	De Graeve (2008)
US_DNGS15	Del Negro, Giannoni and Schorfheide (2015)
US_DNGS15_SW	Del Negro, Giannoni and Schorfheide (2015)
US_DNGS15_SWpi	Del Negro, Giannoni and Schorfheide (2015)
US_DNGS15_SWSP	Del Negro, Giannoni and Schorfheide (2015)
US_FMS13	Fève, Matheron and Sahuc (2013)
US_FU19	Fratto and Uhlig (2020)
US_JPT11	Justiniano, Primiceri and Tambalotti (2011)
US_KK14	Kliem and Kriwoluzky (2014)
US_KS15	Kriwoluzky and Stoltenberg (2015)
US_LTW17gz	Leeper, Traum and Walker (2017)
US_LTW17nu	Leeper, Traum and Walker (2017)
US_LWY13	Leeper, Walker and Yang (2013)
US_RA07	Rabanal (2007)
US_SW07	Smets and Wouters (2007)
US_VI16bgg	Villa (2016)

Table 7: List of models

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[12] www.macromodelbase.com/files/documentation_source/mmb-model-description.pdf

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