Tobias Berg | Florian Heider

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Tobias Berg† Florian Heider‡

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Abstract

This paper examines the dynamic relationship between firm leverage and risk-taking. We embed the traditional agency problem of asset substitution within a multi-period model, revealing a U-shaped relationship between leverage and risk-taking, evident in data from both the U.S. and Europe. Firms with medium leverage avoid risk to preserve the option of issuing safe debt in the future. This option is valuable because safe debt does not incur the expected cost of bankruptcy, anticipated by debt-holders due to future risk-taking incentives. Our model offers new insights on the interaction between companies' debt financing and their risk profiles.

Keywords: leverage, risk-taking incentives, dynamic model
JEL: G3, G31, G32, G33

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†Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt, Germany. berg@finance.uni-frankfurt.de.

‡Goethe University Frankfurt, SAFE, and CEPR. Theodor-W.-Adorno-Platz 3, 60323 Frankfurt, Germany. heider@safe-frankfurt.de.

Introduction

The relationship between leverage and risk-taking is a central question in corporate finance, with significant implications for both theory and practice. This paper examines how leverage influences risk-taking behavior within a dynamic model, addressing the well-known agency problem of asset-substitution where insiders (owners or managers acting in the interest of owners) choose the risk profile of a firm at the expense of debt holders.

The static intuition of how the agency problem shapes the relationship is well-understood, powerful, and ubiquitous. Higher levels of debt induce risk-taking because with limited liability and risk choices that cannot be fully controlled by outsiders, the insider benefits more from the upside of risk-taking than he suffers from the downside (Jensen and Meckling, 1976).

Despite the strong theoretical intuition, evidence on the relationship between leverage and risk-taking is mixed. In a survey of the empirical literature in corporate finance, we find that only one-third of the papers support the notion that higher leverage leads to increased risk-taking, while another third finds the opposite, and the remaining third finds evidence for a U-shaped relationship (see Table 1 for more details).[1]

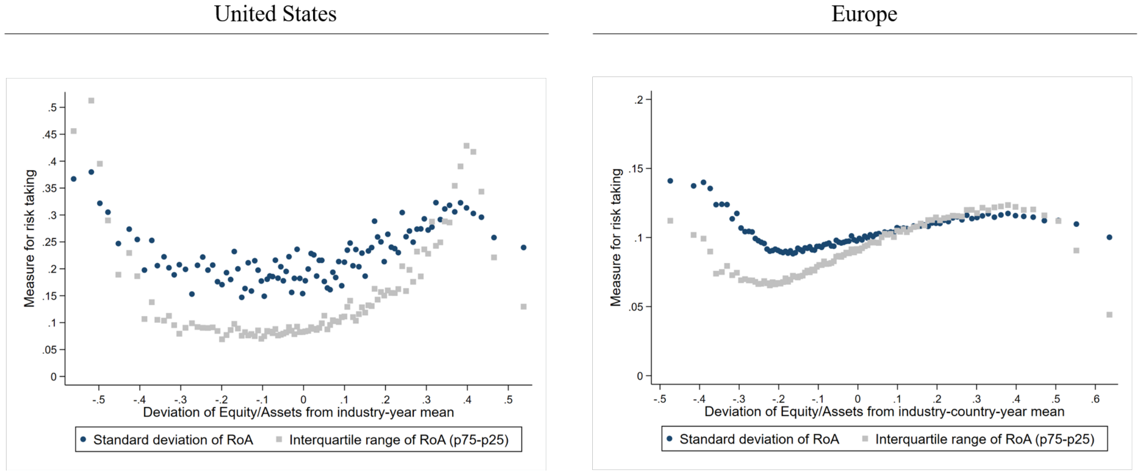
Our own empirical analysis of the broad relationship between leverage and risk-taking indeed shows support for a U-shaped relationship (Figure 1). Both in the U.S. (public firms) and in Europe (public and private firms), having more debt is correlated first with *lower* firm risk before it eventually correlates with higher firm risk. The U-shaped relationship is robust (see Appendix B for details) and economically significant. Measures of risk-taking are only half for medium-levered firms compared to high- or low-levered firms.

Our paper addresses the inconsistency between the powerful but static intuition and actual evidence by embedding the agency problem in a dynamic framework. In each period, the insider makes an unobservable choice between a safe and a risky project. The risky project adds a mean-preserving spread to the safe project (so they have the same net-present value, NPV). To finance the project, the insider raises debt from outside

[1] Additional studies for the banking industry include Esty (1997) and Landier, Sraer, and Thesmar (2015). However, banking is special due to explicit and implicit government guarantees and regulations on leverage.

Figure 1: **The empirical relationship between risk-taking and leverage is U-shaped**

This figure depicts the empirical relationship between risk-taking and leverage. The left-hand figure is based on data from publicly listed firms in the U.S. over the 2010-2019 period, and the right-hand figure is based on data from both publicly listed and private firms in Europe over the same period. Risk-taking is measured as either the standard deviation or interquartile range of the return on assets. Leverage is measured as equity/assets relative to the industry(-country)-year mean. Details of the sample and the construction of the variables are provided in Appendix B.



investors. If the insider cannot repay the debt, the outsiders bear a bankruptcy cost while the insider is protected by limited liability.

The reason why the static intuition is not robust in a dynamic setting can be explained with just two periods. In the second (last) period, the static intuition applies: because of limited liability, the insider will choose the risky project if the amount of debt issued is sufficiently high. Outsiders anticipate this behavior and charge a higher interest rate to recoup the expected cost of bankruptcy. This scenario is costly for the insider, but there is nothing he can do since he cannot commit to choosing the safe project when the firms leverage is high.

Although the insider cannot commit to choosing the safe project in the second period, he can choose the safe project in the first period. To understand why he might prefer to avoid risk, suppose that he does choose the risky project in the first period. If the project fails, the value of the firm decreases and hence, the leverage of the firm may be high enough that there is risk taking in the second period. In that case, the insider has to bear the expected cost of bankruptcy from the second period risky debt. If instead the insider chooses the safe project in the first period, leverage does not increase and he can issue safe, cheap debt in the second period.

The central insight of our dynamic model is that there is an intermediate level of leverage where the insider prefers to avoid risk because it preserves the option to issue safe debt in the future. This option is valuable because safe debt does not bear the expected cost of bankruptcy that arises because debt-holders anticipate future risk-taking. If leverage is outside the medium range, the static logic reasserts itself. With high leverage, the insider chooses the risky project in the first period, as the option to issue safe debt is already lost. With low leverage, the insider is indifferent between the safe and the risky project, as the latter is a mean-preserving spread of the former, and the option to issue safe debt holds no value.

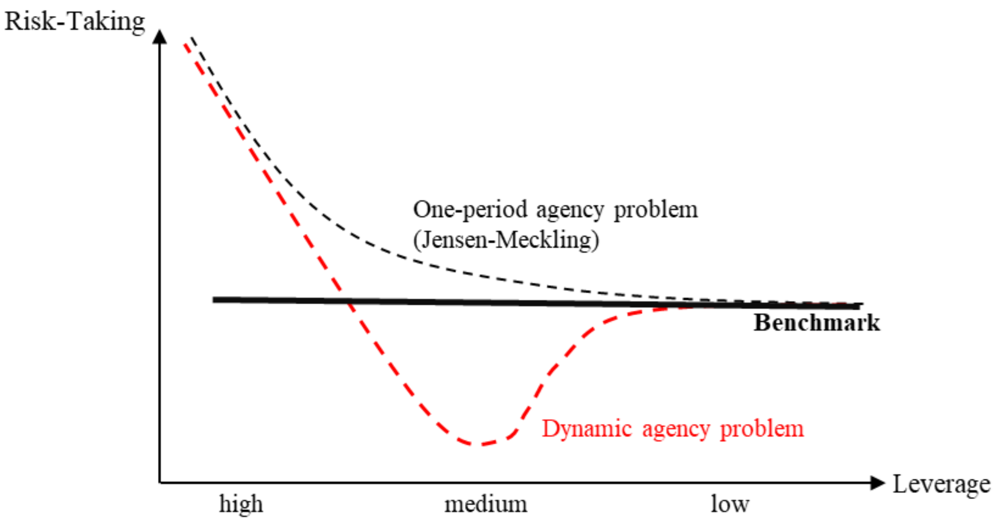
The U-shaped relationship between leverage and risk-taking in our dynamic model (illustrated in Figure 2) aligns with the U-shaped relationship observed in the empirical data (Figure 1). The horizontal line in Figure 2 represents the scenario where there is no agency problem between firm insiders and outsiders. In this benchmark case, there is no relationship between a firm's risk profile and its level of debt. The dashed black line depicts the static, one-period agency problem where higher leverage induces more risk-taking. The dashed red line illustrates the scenario where the one-period agency problem is repeated across periods. Firms with medium leverage actively avoid risk to prevent the costs imposed by debt holders, which arise from the inability to commit to not engaging in risk-taking in the future.

The risk avoidance behavior of firms with medium leverage offers a new perspective on the low-leverage puzzle, which refers to the empirical observation that some firms carry less debt than what standard frictions (such as the tax shield versus bankruptcy costs) would predict (Miller, 1977; Graham, 2000; Korteweg, 2010). One possible explanation is that the NPV of distress is larger due to risk premia – bankruptcy tends to occur more frequently in bad times (Almeida and Philippon, 2007). Our model suggests that the effective NPV of safe projects is larger because the option to issue safe, cheap debt in the future holds significant value.

The intuition (the option to issue safe debt is valuable) and the main result (firms with medium leverage avoid risk) extend to the N-period case through backward induction.

Figure 2: **In our dynamic model, the relationship between risk-taking and leverage is U-shaped**

This figure illustrates the relationship between risk-taking and leverage. The thick black line illustrates a benchmark situation where risk-taking is independent of leverage. This occurs when there is no agency problem (and no other frictions). The dashed black line illustrates the relationship between risk-taking and leverage in a static framework as in Jensen and Meckling (1976) where an agency problem induces high-levered firms to take more risk, which is inefficient. The dashed red line illustrates the relationship between leverage and risk-taking in our dynamic model, which connects successive one-period, Jensen-and-Meckling-type agency problems. The relationship becomes U-shaped because medium-levered firms engage in inefficient risk-avoidance.



To ensure the model's tractability, we make two assumptions. First, we assume the net present value (NPV) of both the safe and risky projects is zero. This prevents the familiar complication in dynamic settings of "growing out of the problem." If the insider accumulates too much equity, leverage decreases, and the agency problem disappears. Second, we assume that early liquidation of the firm results in a complete loss of value for the insider. This assumption prevents the reduction of the agency problem through shrinking the firm. In summary, the insider invests to maintain the firm's value, refinances all maturing debt with new debt, and defaults if refinancing is no longer possible.

As we aim to explore the effect of leverage on risk-taking, our baseline model assumes that outsiders' financing comes in the form of a debt claim. The asset substitution agency problem could possibly be avoided by using equity funding. In an extension that includes a tax shield on debt (Appendix C), we demonstrate that debt is used in equilibrium, and that risk-avoiding behavior persists for firms with medium leverage, provided the tax shield is sufficiently high.

Short-term debt is necessary for the result that medium-leverage firms engage in risk avoidance, but issuing short-term debt is not always optimal. By issuing short-term debt and thus needing to issue new debt in the future, the insider exposes himself to expected bankruptcy costs due to the incentive to engage in future risk-taking. This exposure induces the insider to choose the safe project and avoid these costs. With long-term debt, the exposure to expected bankruptcy costs occurs only once, at the beginning, making the situation akin to the one-period problem where there is no risk avoidance. When leverage is high and the option to issue safe debt is lost, the insider prefers to issue long-term debt. There is less default with long-term debt and the now-unavoidable risk-taking, because early project failures can be offset by subsequent project successes.

Finally, risk avoidance, like risk-taking, is inefficient but for a different reason. Risk-taking is inefficient because the insider has to bear the expected cost of bankruptcy. It is this inefficiency that, in a dynamic model, induces the insider to avoid risk. However, risk avoidance is also inefficient. We show that with medium leverage, the insider prefers the safe project over the risky one even when the safe project has a lower NPV because it preserves the valuable option to issue safe, cheap debt in the future.

We next discuss the related literature. In Section 1, we illustrate how repeating the one-period agency problem once gives rise to risk avoidance with a simple numerical example. Section 2 presents the static, one-period benchmark. Section 3 solves the two-period model with short-term (one period) debt. Section 4 considers long-term (two-period debt) and Section 5 examines the insider's choice between short-term and long-term debt. Section 6 extends the model to N periods. Section 7 offers concluding remarks. All proofs are in Appendix A.

Related literature

There is a large literature that revisits the positive relationship between firm risk and leverage in the context of an agency problem. However, the key difference in our study is that we do not introduce new assumptions to the one-period agency problem of asset substitution, except to repeat it over multiple periods. This approach is natural because firms make repeated decisions about their risk profile over time.

A first, early strand of the literature finds a negative relationship between firm risk and leverage by assuming the absence of an agency problem, i.e., no conflict of interest between debt-holders and equity-holders. The implication is that, in the presence of bankruptcy costs, a highly leveraged firm maximizes firm value by choosing low-risk projects (Mayers and Smith, 1982; Smith and Stulz, 1985; Mayers and Smith, 1987).

In our model, where there is a conflict of interest, only medium-leveraged firms have an incentive to avoid risk. These medium-leveraged firms do not avoid risk to prevent bankruptcy in the subsequent period, but rather to avoid the cost associated with issuing debt that accounts for the insider's future risk-shifting incentives. Consequently, risk avoidance occurs before bankruptcy becomes imminent.

A second strand of the literature introduces additional frictions such as managerial risk aversion and career concerns (Amihud and Lev, 1981; Mayers and Smith, 1982; Smith and Stulz, 1985; Mayers and Smith, 1987; Holmstrom, 1999; Eckbo and Thorburn, 2003; Bertrand and Mullainathan, 2003), or the (exogenous) existence of future positive NPV projects or charter values that incentivize firms to survive the initial period (Froot, Scharfstein, and Stein, 1993; Almeida, Campello, and Weisbach, 2011).[2]

In our model, there are no additional frictions. We simply repeat the asset-substitution agency problem. In a dynamic setting, medium-leveraged firms avoid risk to preserve the option to issue safe debt in the future. The option value arises because debt-holders anticipate future potential risk-taking incentives. The same friction that induces highly leveraged firms to take risks leads medium-leveraged firms to avoid risk.

Finally, a third strand of the literature explores how variations in the design of debt contracts, particularly the use of covenants (Smith and Warner, 1979; Amihud and Lev, 1981; Leland, 1994; Acharya, Amihud, and Litov, 2011) or short-term debt (Barnea, Haugen, and Senbet, 1980; Calomiris and Kahn, 1991; Flannery, 1994; Diamond and Rajan, 2001), affect firms' risk choices.

Our model does not require covenants to create a preference for safe projects. While our model can also feature short-term debt in equilibrium, it makes an entirely different

[2]For general charter value models in banking, see Marcus (1984) and Keeley (1990).

prediction. In existing models, short-term debt makes shareholders internalize the risk-shifting costs, and, in the limit, the risk-shifting problem disappears (see solid black line "Benchmark" in Figure 2). In our paper, the relationship between leverage and risk-taking is U-shaped, and the risk-shifting problem does not disappear with short-term debt. Instead, it creates incentives for highly leveraged firms to take risks and for medium-leveraged firms to avoid risk (see the dashed red line in Figure 2). Moreover, this risk avoidance is inefficient, as medium-leverage firms prefer safe projects even when they have lower NPV than risky ones.

Our model contributes to the growing literature that analyzes dynamic inefficiencies in corporate finance and revisits basic findings from static models of corporate behavior. Early papers explore the optimal capital structure by trading off bankruptcy costs and taxes (Leland, 1994; Leland and Toft, 1996). Subsequent models examine optimal debt maturity in dynamic settings (Diamond and He, 2014; Dangl and Zechner, 2018; Huang, Oehmke, and Zhong, 2019; Geelen, 2019), as well as leverage dynamics of firms that cannot commit to a debt issuance policy ex ante (Brunnermeier and Oehmke, 2013; He and Milbradt, 2016; Admati, DeMarzo, Hellwig, and Pfleiderer, 2018; DeMarzo and He, 2021).

Similar to these papers, our model highlights that findings from static models do not necessarily carry over to a dynamic setting. In a static model, borrowers have to bear the cost of inefficient risk-shifting. In a dynamic model, the decision not to risk-shift today allows equity holders to avoid the cost of inefficient risk-shifting tomorrow. This dynamic behavior leads to a U-shaped relationship between risk-taking and leverage in our model.

A small number of papers explore risk-taking in a continuous-time model. Leland (1994) provides comparative statics for the value of equity as a function of asset risk. He concludes that equity holders have an incentive to decrease risk if covenants are strict enough, and equity holders might be better off ex ante issuing debt with restrictive covenants. However, the paper does not solve for the optimal level of risk-taking. Leland (1998) and Ericsson (2000) examine the joint determination of capital structure and investment risk in a continuous-time model. Both papers restrict the level of investment risk to two discrete values with a single switching point.

Our dynamic discrete-time model allows for a flexible form of the relationship between risk-taking and leverage, and it is tractable enough to solve for the optimal level of risk-taking as a function of leverage. We do not make any assumption about the relationship between leverage and risk-taking or that risk-taking is irreversible.[3] In contrast to these models, we predict a U-shaped relationship between risk and leverage.

1 Illustrative example

1.1 One period

The following example illustrates the well-known incentive of equity holders with limited liability to risk-shift (Jensen and Meckling (1976)). A firm has two projects, a safe one and a risky one (Figure 3). Both require an investment of 80. The safe project pays off 80 and the risky project adds a mean-preserving spread of ± 20 to the safe project (the zero net present value of projects is not important). The (inside) equity is 10 so the equity holder raises debt worth 70 from lenders. The equity holder's project choice is non-contractible, she has limited liability, and bankruptcy incurs a fixed cost of 10.

The equity holder has an incentive to choose the risky project. To see this, suppose she goes for the safe project. Lenders require debt with face value of 70, which generates a payoff of 10 to the equity holder. However, with a face value of debt of 70, the equity holder prefers the risky project because it generates an expected payoff of 15 ($0.5 \cdot (100-70) + 0.5 \cdot 0$ – there is default when the payoff is 60).

Choosing the risky project hurts the equity holder because she has to bear the expected bankruptcy costs from risk-shifting. Lenders anticipate the equity holder's incentive to risk-shift and require a face value of debt of 90 (because $0.5 \cdot 90 + 0.5 \cdot (60 - 10) = 70$). The expected payoff to the equity holder drops to 5 ($0.5 \cdot (100 - 90) + 0.5 \cdot 0$). The difference between this payoff and initial equity of 10 with a zero NPV investment is the expected

[3]To keep the analysis simple, Leland (1998) imposes a positive relation between leverage and risk-taking by assumption: firms with high leverage are assumed to choose high investment risk, firms with low leverage are assumed to choose low investment risk, and the model solves for the optimal switching point (see pages 1219/1220 in Leland (1998)). Ericsson (2000) imposes the following constraint on risk-taking by assumption: firms start with a low level of risk and can switch to a higher level of risk subsequently. Once high risk is chosen, it is assumed to be irreversible (see Section 2.2 on pages 12/13 in Ericsson (2000)).

bankruptcy cost ($0.5 \cdot 10 = 5$).

1.2 Two periods

Our two-period example overturns the one-period intuition. In each period, there is the choice between the safe and the risky investment project. As in the one-period example, the payoff of the safe project is equal to the investment, and the risky project adds a mean-preserving spread of ± 20. In the first period, the investment need is 100. Like in any standard binominal tree, the second period starts where the first period ends (with 80, 100 or 120 depending on the outcome of the first period). Panel (a) of Figure 4 shows the entire payoff tree, while Panel (b) shows all possible paths in the first period and only the equilibrium paths in the second period.

The following example depends on two key assumptions: first, we assume one-period debt. In our formal model the debt maturity is endogenous, but simply assuming one-period debt simplifies exposition. Second, the equity holder cannot simply pay back the first-period debt by liquidating (part of) the assets, but needs to refinance the first-period debt. This assumption is important: if liquidation were costless, the equity holder could always avoid agency costs by liquidating the entire firm. We discuss liquidation costs in more detail in Section 2.

We assume that the initial value of equity is 30, so the equity holder raises debt worth 70 in the first period. First-period debt is safe because a face value of 70 always allows the equity holder to refinance debt in the interim period, even with the worst possible first-period payoff (80).

If the equity holder chooses the risky project in the first period and the payoff is 120 then she can easily refinance the 70 first-period debt. Second-period debt is safe, she is indifferent between choosing the safe project and the risky project in the second period, and the payoff to the equity holder after the second period is 50.[4] If, on the other hand, the payoff of the risky project in the first period is 80, then the second period is exactly as in the previous one-period example. The equity holder needs to obtain new debt worth 70 ($80 - (80 - 70)$) and has an incentive to choose the risky project in the

[4] The expected value of equity is $120 - 70 = 50$, or $0.5 \cdot (100 - 70) + 0.5 \cdot (140 - 70) = 50$.

second period. Lenders anticipate this, require a face value of 90 in the second period and the equity holder's expected payoff for the second period is 5. The overall expected payoff to the equity holder from choosing the risky project in the first period therefore is 27.5 $(0.5 \cdot 50 + 0.5 \cdot 5)$.

The equity holder avoids risk in the first period because this preserves the option to issue safe debt in the second period. When she chooses the safe project in the first period, the payoff is 100, which makes it possible to issue safe debt to finance the second-period investment of 70 $(100 - (100 - 70))$. Second-period debt is safe because a face value of 70 is below the lowest payoff at the end of the second period (80). The overall expected pay-off to the equity holder from choosing the safe project in the first period is therefore 30 $(1 \cdot 30)$. In the first period the equity holder prefers the safe project (payoff 30) to the risky project (payoff 27.5) because otherwise she may have an incentive to risk-shift in the second period. Possibly not being able to issue safe debt in second period, the equity holder bears an expected costs of bankruptcy of 2.5 $(0.5 \cdot 0.5 \cdot 10)$. The equity holder prefers the safe project in the first period precisely because she does not want to end up in the risk-shifting region in the second period.

2 One-period model

There is one period, starting at $t = 1$ and ending at $t = 2$. We depict the key elements of the timeline in Figure 5. All agents are risk-neutral and the discount rate is 0%. The insider has access to a set of investment opportunities (described in more detail below) that have an expected payoff of I_1 in t=2 and require an amount of $I_1 - E_1$ of outside financing at t=1. E_1 is the expected value of the insiders claim in absence of any frictions that might make external financing costly (described in more detail below). If, for example, the expected payoff is 100 and the insider needs to raise 80 of outside financing, then the insiders claim is worth 20 in the absence of any frictions makes external financing costly. As we are interested in exploring the effect of leverage on risk-taking, we assume that outsiders' financing comes in the form of a debt claim with notional F_2. If the value of the assets in $t = 2$ is not enough to meet the repayment, then the lenders have the right to seize the assets and liquidate them. Liquidation incurs fixed bankruptcy costs of $b > 0$.[5]

After the financing has happened at $t = 1$, the insider can choose between two investment options $c_1 \in \{s_1, r_1\}$ at $t = 1\frac{1}{2}$: first, a safe investment s_1 that pays off I_1 at $t = 2$ for sure. Second, a risky investment r_1 that returns $I_1 \pm \gamma$ with 50% probability each, i.e., the second project adds a mean-preserving spread to the safe project. We assume that $I_1 > \gamma$ so that the value of assets is positive. The parameter γ governs the degree of risk-shifting in our model. We assume the project choice is non-contractible. It can be deduced at $t = 2$ from the asset value, but because the insider has limited liability, lenders cannot impose the strategy choice.

The insider can also choose not to raise financing at $t = 1$ and not to invest in $t = 1.5$ in which case her payoff is zero. For the mechanics of our model it is not important that the outside option is zero; all we require is that the investment options have a higher expected payoff than the outside option. Choosing an expected payoff of zero for the outside option serves as a convenient normalization and increases the ease of notation.[6] In the following, we label E_1 the no-frictions value of the insider's equity at $t = 1$ (sometimes also, in short, initial insider's equity E_1). The value of equity can be lower than E_1 if frictions make outside financing costly or impede raising outside financing at all.

Taken together, there are four possible asset values at $t = 2$: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$, refering to the outside option ($A_2 = 0$), the safe investment opportunity ($A_2 = I_1$) and

[5] Our results are robust to using proportional bankruptcy costs. We use fixed costs for two reasons: First, fixed costs ease notational convenience and facilitate the interpretability of the results. Second, proportional bankruptcy costs imply somewhat counterintuitive comparative statics: conditional on default, a firm occurs lower deadweight costs of bankruptcy the worse its performance.

[6] There are three possible interpretations of the value of the outside option: (1) Debt structure in place: An existing firm with a capital structure of E_1 (equity) and $D_1 = I_1 - E_1$ (debt) needs to refinance its debt D_1. The assets in place cannot be liquidated. The outside option of zero is thus a consequence of the illiquidity of the assets in place. (2) An insider is equipped with wealth E_1 and needs to raise $I_1 - E_1$ from outsiders. The NPV of investment inside the firm is zero, while the outside option has a negative NPV. In this case, the negative NPV of the outside option simply serves as a convenient normalization; the alternative is to assume a positive NPV inside the firm and a zero NPV for the outside option. (3) Intangible capital: the insider has intangible capital which is worth zero if the insider is not able to raise sufficient outside financing.

the risky investment opportunity ($A_2 = I_1 \pm \gamma$).[7] The value of the debt claim at $t = 2$ is

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2 \\ A_2 - b & A_2 < F_2 \end{cases} \tag{1}$$

and the value of the equity claim at $t = 2$ is

$$E_2 = \begin{cases} A_2 - F_2 & A_2 \geq F_2 \\ 0 & A_2 < F_2. \end{cases} \tag{2}$$

We make the following assumption about the bankruptcy costs b:

Assumption 1 *The cost of liquidating assets in bankruptcy is not too large:*

$$b < 2\gamma \tag{3}$$

If $b \geq 2\gamma$, then the insider could only issue safe debt or not obtain financing at all. This assumption thus ensures that inefficient risk taking is a possibility in the one-period model.

At $t = 1$, the insider maximizes the expected value of her (equity) claim, $\mathbb{E}_1[E_2]$, by choosing the face value F_2 and an investment strategy c_1 subject to the choice being incentive compatible, and subject to the outsiders' participation constraint, $\mathbb{E}_1[D_2] \geq I_1 - E_1$. The following proposition gives the insider's optimal choice of F_2 and c_1 as a function of the no-frictions value of the insider's equity at $t = 1$.

Proposition 1 *Given the no-frictions value of the insider's equity at $t = 1$, E_1, there are three possible equilibrium outcomes:*

i) When $E_1 \geq \gamma$, then the insider can issue safe debt. She is indifferent between choosing the safe or the risky strategy. The expected value of her equity at $t = 1$ is E_1.

[7]Note that we make a distinction between assets (A_t) and investments (I_t) throughout the paper. The reason is that the insider may choose not to raise financing at $t = 1$ and not to invest in $t = 1.5$ in which case the asset value is zero.

ii) When $\gamma > E_1 \geq \frac{1}{2}b$, then the insider cannot issue safe debt. She chooses the risky strategy. The expected value of her equity at $t = 1$ is $E_1 - \frac{1}{2}b$.

iii) When $E_1 < \frac{1}{2}b$, the insider cannot obtain financing and the expected value of her equity at $t = 1$ is zero.

When the insider has enough initial equity, $E_1 \geq \gamma$, losses cannot exceed the equity amount and safe debt can be issued. There is no default and because the safe and risky strategy have the same expected value, the insider is indifferent between them. The expected value of her equity is equal to the no-frictions value of the insider's equity.

When the insider's initial equity, E_1, is less than γ, but larger than the expected costs of bankruptcy, $\frac{1}{2}b$, the insider cannot issue safe debt and chooses the risky strategy. The face value of debt is too large to make the safe strategy incentive compatible. The expected value of the insider's equity at $t = 2$ reflects the costs of having to compensate the lenders for the expected costs of bankruptcy. Assumption 1 ensures that $\gamma > \frac{1}{2}b$ and thus makes the risk-taking case possible.

When the insider's initial equity, E_1, is not enough to cover the expected costs of bankruptcy, she cannot raise outside financing. In this case, her payoff drops to the outside option, which by assumption is equal to zero.

Figure 6 plots the equilibrium outcome as a function of the insider's initial equity E_1. The results demonstrate the well-known fact: in a one-period model, risk-shifting occurs for high levels of leverage. If lenders rationally anticipate risk-shifting, then the expected costs of risk-shifting are borne by the equity holder in expectation.

The following corollary establishes the fact that the insider can strictly prefer the risky strategy over the safe strategy even if the safe strategy has a higher expected payoff than the risky strategy.

Corollary 1 *(**Higher expected payoff of the safe strategy**) Assume the same setup as in Proposition 1, but the safe strategy's payoff is $I_1 + \Delta$, with $\Delta \in [0, \frac{1}{2}\gamma - \frac{1}{4}b)$ being non-negative but not too large. The insider chooses the risky strategy whenever $E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$.*

Proposition 1 has established the fact that the insider may prefer the risky strategy over the safe strategy if both have the same expected payoff. Corollary 1 establishes that the insider may even prefer the risky strategy over the safe strategy if the safe strategy has a higher expected payoff. The length of the interval where risk-shifting occurs ($E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$) is decreasing in the expected excess payoff Δ of the safe strategy over the risky strategy.

3 Two-period model with short-term debt

The two-period model adds a second period, from $t = 0$ to $t = 1$, in which the same one-period model from the previous section plays out again. The asset purchased in the first period (at $t = 0.5$) is carried onto the second period. The risk choice in the second period (at $t = 1.5$) can therefore be interpreted as a risky versus safe mode to operate the asset purchased in the first period.

Figure 7 shows the timeline. At $t = 0$ the insider has access to a set of investment opportunities that have an expected payoff of I_0 and require an amount $I_0 - E_0$ of outside financing at $t = 0$. E_0 is the expected value of the insiders' claim in absence of any frictions that might make external financing costly. As before, we label this the no-frictions value of the insider's equity (sometimes also, in short, initial insider's equity E_0). We assume that the financing need of $I_0 - E_0$ is financed by raising short-term debt with face value F_1. Depending on the choice of strategy, the asset value at $t = 1$ is $A_1 \in \{0, I_0 - \gamma, I_0, I_0 + \gamma\}$.

To connect the two periods, we assume that the assets in place A_1 cannot be liquidated. This implies that the insider cannot simply pay back the debt by liquidating (part of) the assets. Instead, the insider needs to refinance the first-period debt notional F_1 from new lenders. If the insider is unable to raise F_1 from new lenders then she defaults, lenders receive $\min(F_1, A_1 - b)$ and the insider receives $\max(0, A_1 - b - F_1)$. Depending on the choice of strategy, the asset value at $t = 2$ is $A_2 \in \{0, A_1 - \gamma, A_1, A_1 + \gamma\}$.

Before solving for the equilibrium, some notes about our assumptions are warranted.

First, we assume that the debt issued by firms is short-term, i.e., one-period debt. The implications of using long-term (two-period) debt and the endogenous choice of debt maturity are explored in Sections 4 and 5. Second, we rule out the possibility of renegotiating first-period debt. This means that if the insider cannot raise new debt after the first period, bankruptcy will occur. One way to rationalize this assumption is to consider that debt-holders are short-lived. In this scenario, the first generation of debt-holders finances the first period, while a second generation finances the second period. Alternatively, if debt-holders are dispersed, then it is also difficult to renegotiate first-period debt. Third, we assume that the insider cannot liquidate assets in place (A_1) outside of bankruptcy. This assumption can be relaxed by introducing liquidation costs (l). Under this relaxation, two trade-offs emerge. If liquidation costs are lower than bankruptcy costs ($l < b$), the insider might prefer to liquidate assets to avoid bankruptcy. Similarly, if liquidation costs are lower than the expected agency costs from the second period, the insider might prefer liquidation over continuation.

At $t = 0$, the insider maximizes the expected value of her equity claim, $\mathbb{E}_0[E_2(E_0)]$, which is a function of the no-frictions value of the insider's equity E_0, by choosing debt face values (F_1, F_2) and an investment strategy in both periods (c_0, c_1) subject to the choice being incentive compatible, and subject to the outsiders' participation constraint. The following proposition gives the equilibrium outcome of the two-period model with short-term debt:

Proposition 2 *Given the no-frictions value of the insider's equity at $t = 0$, E_0, and assuming $b < \frac{4}{3}\gamma$, there are four possible equilibrium outcomes:*

i) When $E_0 \geq 2\gamma$, then the insider can issue safe debt in both periods. She is indifferent between the safe and the risky strategy in both periods. The expected value of her equity at $t = 0$ is E_0.

ii) When $2\gamma > E_0 \geq \gamma$, the insider can issue safe debt in the first period and prefers the safe strategy in the first period. In the second period she can issue safe debt and is indifferent between the safe and the risky strategy. The expected value of her equity at $t = 0$ is E_0.

iii) When $\gamma > E_0 \geq \frac{3}{4}b$, then the insider cannot issue safe debt in the first period and chooses the risky strategy in the first period. If the risky strategy is unsuccessful, she defaults. If the risky strategy succeeds, the second period strategy depends on E_0 and she either perfers the risky strategy or is indifferent in the second period. The expected value of her equity at $t = 0$ is $E_0 - \frac{1}{2}b$ or $E_0 - \frac{3}{4}b$.

iv) When $E_0 < \frac{3}{4}b$, then the insider cannot obtain financing. The expected value of her equity at $t = 0$ is zero.

Note that in case iii), default always occurs when the first-period strategy is unsuccessful. Depending on the value of E_0, default may also occur when the first-period strategy is successful and the second-period strategy is unsuccessful. The first case gives rise to expected bankruptcy costs of $\frac{1}{2}b$, while the latter case gives rise to expected bankruptcy costs of $\frac{3}{4}b$. The restriction $b < \frac{4}{3}\gamma$ ensures that case iii) exists. If $b \geq \frac{4}{3}\gamma$, then risk-shifting is so costly that financing with risk-shifting (i.e., case iii)) is no longer possible. For $b \geq \frac{4}{3}\gamma$, case ii) is unaffected and case iv) occurs for $E_0 < \gamma$. See the proof of Proposition 2 for details.

Figure 8 illustrates the equilibrium outcomes as a function of the insider's initial equity at $t = 0$. If the insider's initial equity is abundant, $E_0 \geq 2\gamma$, then the insider can issue safe debt in both periods and she is indifferent between the safe and the risky strategy. This corresponds to case i) of Proposition 1 in the one period case, except that the initial equity has to be large enough to cover failure of the risky strategy in both periods.

When the level of the insider's initial equity is intermediate, $\gamma \leq E_0 < 2\gamma$ (i.e., the equity can cover failure in one period, but not in both), then the key result of our paper occurs. The insider actually prefers the safe strategy over the risky strategy. Doing so allows to issue safe debt again in the second period. The insider could risk-shift in the first period, but this is costly because if the project fails, she is no longer able to issue safe debt in the second period. Not being able to issue safe debt in the second period would imply that the insider engages in risk-shifting in the second period and the lenders would price in the expected costs of bankruptcy.

Case iii) of the two-period model is analogous to case ii) of the one period model.

The insider's initial equity is not high enough to issue safe debt in the first period and inefficient risk-shifting occurs.

Case iv) of the two-period model in turn is analogous to case iii) of the one period model. The insider's initial equity is insufficient to cover the cost of risky debt that prices in the expected costs of bankruptcy. Note that in the two-period model, more initial equity is needed than in the one-period model. The insider factors in that even if she succeeds in the first period, her equity at $t = 1$ (after paying off the first-period lenders) may not be high enough to obtain financing with safe debt from second period lenders.

The following corollary establishes the fact that the insider can strictly prefer the safe strategy over the risky strategy even if the risky strategy has a higher expected payoff than the safe strategy.

Corollary 2 *(***Higher expected payoff of the risky strategy***) Assume the same setup as in Proposition 2, but the risky strategy's payoff in the first period is $I_0 + \Delta \pm \gamma$, with $\Delta \in [0, \frac{1}{4}b)$ being non-negative but not too large. The insider chooses the safe strategy in the first period whenever $E_0 \in [\gamma + \Delta, 2\gamma - \Delta)$.*[8]

Proposition 2 has established the fact that the insider may prefer the safe strategy over the risky strategy if both have the same expected payoff. Corollary 2 establishes that the insider may even prefer the safe strategy over the risky strategy if the risky strategy has a higher expected payoff. The length of the interval where the safe project is preferred over the risky project ($E_0 \in [\gamma + \Delta, 2\gamma - \Delta)$) is decreasing in the expected excess payoff Δ of the risky strategy over the safe strategy. Corollary 2 therefore suggests that investment decisions are not only distorted for highly leveraged firms (who may prefer a risky project to a safe project even if the latter has a higher payoff, see Corollary 1), but also for medium leveraged firms (who may prefer a safe project to a risky project even if the latter has a higher payoff). This finding contributes to the low leverage puzzle, i.e., the empirical observation that firm leverage seems to be too low to be explained by standard frictions.

Note also that the choosing the safe strategy as outlined in Corollary 2 is inefifcient

[8]Note that $\Delta \in [0, \frac{1}{4}b)$ together with Assumption 1 ensures that the interval for E_0 is non-empty.

from a welfare perspective. While reducing risk to avoid bankruptcy costs can be efficient for highly leveraged firms, firms reduce risk too early. If firms could commit to an investment strategy, they would want to commit to choosing the safe strategy when they are highly leveraged. Such a commitment would allow highly leveraged firms to avoid deadweight costs of bankruptcy, and it would allow medium leveraged firms to choose the risky strategy if the risky strategy has a higher payoff. In our model, medium-leveraged firms do not avoid risk to avoid bankruptcy in the subsequent period, but rather to avoid deadweight costs of risk-shifting in the subsequent period. Thus, firms in our model reduce risk inefficiently early.

We have assumed so far that the outsiders' financing comes in the form of a debt claim. The agency costs of debt financing that arise in our model can be avoided by using equity funding. In Appendix C, we illustrate that debt is used in equilibrium – and risk-avoiding behavior continues to hold for firms with medium leverage – if the tax shield is sufficiently high.

4 Two-period model with long-term debt

The set-up is as in the two-period model in Section 3, with the exception that the maturity of the debt is two periods. Section 5 discusses the equilibrium outcome if debt maturity is endogenous, i.e. if the insider can choose between issuing one or two period debt. The following proposition gives the equilibrium outcome of the two-period model with long-term debt:

Proposition 3 *Given the no-frictions value of the insider's equity at $t = 0$, E_0, and assuming $b < \gamma$, there are three possible equilibrium outcomes:*

i) When $E_0 \geq 2\gamma$, then the insider can issue safe long-term debt. She is indifferent between choosing the safe or the risky strategy in both periods. The expected value of her equity at $t = 0$ is E_0.

ii) When $2\gamma > E_0 \geq \frac{3}{4}b$, then the insider cannot issue safe debt. She chooses the risky strategy in both periods. The expected value of her equity at $t = 0$ is $E_0 - \frac{1}{4}b$ or

$E_0 - \frac{3}{4}b$.

iii) When $E_0 < \frac{3}{4}b$, then the insider cannot obtain financing. The expected value of her equity at $t = 0$ is zero.

Note that in case ii), depending on the value of E_0, default may either occur only after two unsuccessful outcomes of the risky strategy, or if the risky strategy is unsuccessful in at least one of the two periods. The first case gives rise to expected bankruptcy costs of $\frac{1}{4}b$, while the latter case gives rise to expected bankruptcy costs of $\frac{3}{4}b$. If $b \geq \gamma$, then the threshold that separates cases ii) and iii) becomes $\frac{1}{2}\gamma + \frac{1}{4}b$ (instead of $\frac{3}{4}b$). See the proof of Proposition 3 for details.

Proposition 3 mimics the results from the one-period model: the insider has an incentive to risk-shift for low levels of initial equity ($E_0 < 2\gamma$) as long as she is able to raise outside debt ($E_0 > \frac{3}{4}b$). The insider bears the costs of risk-shifting, reflected in the terms $-\frac{1}{4}b$ and $-\frac{3}{4}b$ in case ii). In contrast to the one-period model, a higher initial insider's equity is needed to avoid risk-shifting ($E_0 > 2\gamma$ in the two-period model with long-term debt as opposed to $E_0 > \gamma$ in the one-period model).

5 Choice between short-term and long-term debt

Proposition 2 and 3 can be combined to derive the equilibrium outcome when debt maturity is endogenous. The insider simply chooses the debt maturity that maximizes the expected value of her equity claim at $t = 0$.

The following table summarizes the equilibrium strategies in the first period and the resulting expected equity values as a function of the initial insider's equity E_0 for short-term and long-term debt, as derived in Proposition 2 and 3:

		2 x Short-period debt		Long-term debt	
Leverage	E_0	Strategy	$\mathbb{E}[E_2(E_0)]$	Strategy	$\mathbb{E}[E_2(E_0)]$
Low	$[2\gamma, \infty)$	Indifferent	E_0	Indifferent	E_0
Medium	$[\gamma, 2\gamma)$	Risk-avoiding	E_0	Risk-shifting	$E_0 - \frac{1}{4}b$
High	$[\frac{3}{4}b, \gamma)$	Risk-shifting	$E_0 - \frac{1}{2}b$ or $E_0 - \frac{3}{4}b$	Risk-shifting	$E_0 - \frac{1}{4}b$ or $E_0 - \frac{3}{4}b$
Very High	$[0, \frac{3}{4}b)$	No financing	0	No financing	0

Both long-term debt and short-term debt lead to the same expected equity value as long as the initial insider's equity is sufficient to cover two periods of unsuccessful investment with the risky strategy ($E_0 > 2\gamma$). This makes intuitive sense as the insider always fully bears the gains and losses of any strategy that she implements.

The crucial case arises if initial insider's equity is large enough to cover one, but not two, periods of losses, i.e. $E_0 \in [\gamma, 2\gamma)$. In this case, the insider is strictly better off by issuing short-term debt and choosing the safe project in the first period. Thus, the insider has an incentive to issue short-term debt and avoid risk in the first period.

When the initial insider's equity is not sufficient to cover one period of losses, i.e. $E_0 < \gamma$, the insider either has an incentive to risk-shift (resulting in an expected equity value smaller than E_0) or is unable to obtain financing (resulting in an equity value of zero) both with short-term and with long-term debt.

Taken together, this implies that the key result from Proposition 2 carries over to a setting with endogenous debt maturity: a firm with a medium leverage $E_0 \in [\gamma, 2\gamma)$ has an incentive to issue short-term debt, avoid risk and choose the safe strategy over the risky strategy.

6 Extension to N periods

The insights from the two-period model extend to N-periods. The proof is by backward induction. In the final period, the static risk-shifting logic applies. Consequently, in the penultimate period, the insider has an incentive to avoid risk and issue safe debt if leverage is not too high. By doing so, the insider avoids paying the expected bankruptcy cost that debt-holders would require if they anticipated risk-taking in the final period.

The proof also clarifies that what matters is the expected (market) value of equity as a function of the book value of equity (i.e., leverage) under the insider's optimal future choices of safe or risky projects. We show that this function has three properties that carry over across periods. First, for low levels of book equity, the insider cannot issue new debt, and the expected value of equity drops to zero (the insider defaults). Second, if book equity is not large enough to cover one-period losses, the insider can only issue risky debt, leading to a loss in the expected value of equity. Third, if book equity is sufficient to cover one-period losses, the insider can issue safe debt, and the expected value of equity equals the current book value (recall that both the safe and risky projects have zero NPV).

The following proposition gives the equilibrium outcome of the N-period model:

Proposition 4 *Assume the model consists of N periods. In periods* $1, ..., N-1$*, the model contains the same payoffs and potential strategy choices as depicted in the first period of Figure 7. In period* N*, the payoffs and potential strategy choices are as depicted in the last period of Figure 7. Given the no-frictions value of the insider's equity at* $t = n$*,* E_n*, and assuming* $b < \gamma$*, there are four possible equilibrium outcomes:*

i) When $E_n \geq 2\gamma$*, then the insider can issue safe debt in the next period. She is indifferent between the safe and the risky strategy in the next period. The expected value of her equity at* $t = n$ *is* E_n*.*

ii) When $2\gamma > E_n \geq \gamma$*, the insider can issue safe debt in the next period and prefers the safe strategy in all subsequent periods. The expected value of her equity at* $t = n$ *is* E_n*.*

iii) When $\gamma > E_n \geq \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$*, then the insider cannot issue safe debt in the next period and chooses the risky strategy in the next period. The expected value of her equity at* $t = n$ *is smaller than* E_n*.*

iv) When $E_n < \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$*, then the insider cannot obtain financing. The expected value of her equity at* $t = n$ *is zero.*

Note that case ii) is an absorbing state, i.e., for $2\gamma > E_n \geq \gamma$, the insider chooses the safe strategy in all subsequent periods. Choosing the safe strategy preserves the opportunity to issue safe debt and thus avoids incurring (expected) bankruptcy costs. The preference for the safe project perpetuates the situation of medium-leveraged firms: medium leverage implies safe investment choices, but safe investment choices lead to medium leverage in the subsequent period as well.[9]

[9]Case (i) is an absorbing state as well if the insider chooses the safe strategy in case she is indifferent between the safe and the risky strategy.

The minimum amount of equity capital needed to be able to refinance is $E_n^{min} = \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$. Special cases include the one-period model ($n = N - 1$) which yields $E_{N-1}^{min} = \frac{1}{2} b$, and the two-period model ($n = N - 2$) which yields ($E_{N-2}^{min} = \frac{3}{4} b$). The risk-shifting region $\gamma > E_n \geq \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$ decreases as n increases. Thus, the standard risk-shifting result from the one-period model applies to an increasingly smaller interval as the number of periods grows larger. One empirical prediction is that risk-shifting will be less important in practice for business models with a longer-term horizon, that is, where cash flows take longer to materialize (typically interpreted as growth firms). Such growth firms will find it harder to raise outside debt financing, but if they are able to, the region of risk-shifting is smaller than for firms that invest in projects that pay off quickly.

For large N and equity values slightly above b, the insider is trapped in a risk-shifting spiral: failure of the risky project implies bankruptcy, while success implies ending up in a region where agency conflicts are still too large to allow for the issuance of safe debt. The insider either defaults or remains in the risk-shifting region iii).

7 Conclusion

Risk-shifting la Jensen and Meckling features prominently in corporate finance and banking. At first glance, the logic is intuitive: when borrowers have limited liability and their investment choices are unobservable, high leverage provides an incentive to choose a risky investment strategy. If the risky strategy fails, the downside for borrowers is limited. If the risky strategy succeeds, they obtain a larger upside than under a safe investment strategy.

Risk-shifting is a key explanation for why high leverage is harmful. It is inefficient because lenders anticipate borrowers' incentives to choose risky investments and increase the cost of debt accordingly. The higher cost of debt lowers the value of borrowers' equity stake, but there is nothing they can do, so the standard logic goes, because they cannot credibly commit not to risk-shift.

Our paper shows that the risk-shifting logic is not robust in a dynamic model because it ignores the option value of being able to issue cheap, safe debt in the future. While it remains true that firms with high leverage engage in risk-shifting, firms with medium leverage actively avoid risk (and firms with low leverage can always issue safe debt and hence, are not tempted to risk-shift in the first place). Medium-leverage firms prefer safe investment strategies in early periods to avoid the cost of having to issue risky debt in future periods. The logic to avoid risk in a dynamic model relies entirely on the inefficiency of risk-shifting. In a static model, borrowers have to bear this inefficiency. In a dynamic model, the decision not to risk-shift today allows insiders to avoid the cost of inefficient risk-shifting tomorrow.

The valuable option to issue safe debt gives rise to a "safety trap." Firms with medium leverage prefer safe, possibly inefficient projects to risky ones. The choice of safety over risk perpetuates the current value of the firm and its leverage. In the next period, the situation repeats. The insider again prefers the safe project because it preserves the option to issue safe debt, and so on.

When firms are stuck in such a safety trap, there are potentially adverse consequences for the economy as a whole. Empirically, economies only recover slowly after crises Reinhart and Rogoff (2014). It would be interesting to investigate whether the risk avoidance we uncover in our model, where firms aim to maintain the valuable option to issue safe debt in the future, can be a factor in such protracted recoveries.

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Figures and Tables

Figure 3: One-period model / Example

This figure illustrates the payoff structure of the safe project and the risky project in our one-period example. Both projects require an investment of 80 and have zero net present value. The risky project adds a mean-preserving spread to the safe project. Probabilities for the up/down state for the risky project are 0.5 each.

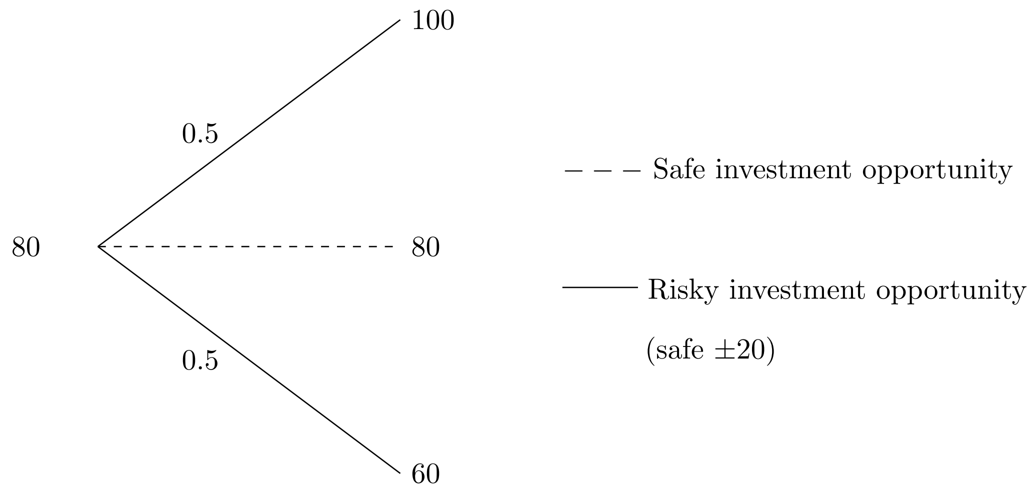
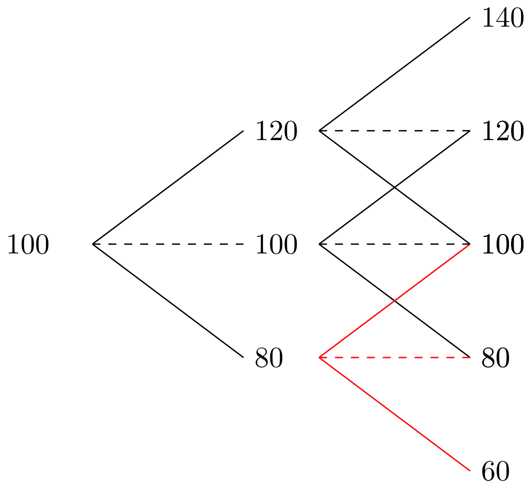
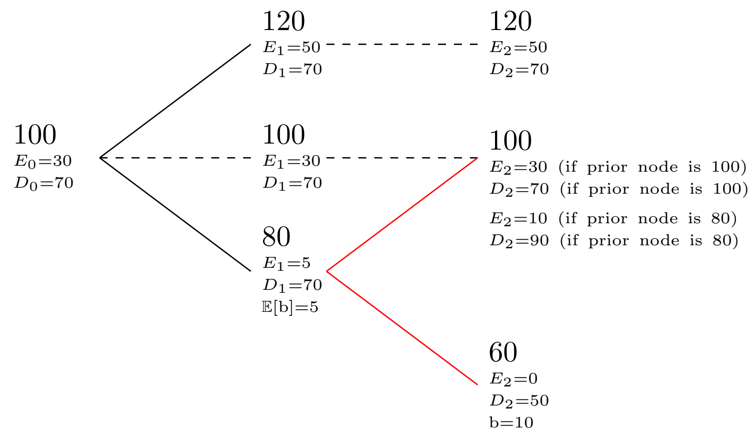


Figure 4: Two-period model / Example

This figure illustrates the payoff structure in our two-period example. Figure (a) depicts all possible outcome paths. Figure (b) depicts all possible paths for period 1 and only the equilibrium paths for period 2. Probabilities for the up/down state for the risky project are 0.5 each. The example assumes a fixed bankruptcy cost of $b = 10$, implying a payoff to lenders of $60 - 10 = 50$ in the down-state. Note that the red part of each figure is equivalent to the one-period game from Figure 3. In Figure (b), we assume without loss of generality that the equity holder chooses the safe project if she is indifferent between the safe and the risky project.



(a) All paths. Red: One-period model from Figure 3.



(b) All paths for period 1, equilibrium paths for period 2 only. Red = One-period model from Figure 3. E_i and D_i refer to expected payoffs for equity and debt at time i in equilibrium.

Figure 5: One-period model / Timeline

This figure illustrates the timeline of events in the one-period model.

Financing

- Initial insider's equity E_1
- Financing $I_1 - E_1$ from outsiders via debt contract with notional F_2

Investment choice

- Strategy $c_1 \in \{s_1, r_1\}$, s_1 yields payoff I_1 (safe project), r_1 yields payoff $I_1 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

Payoff

- Assets: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$
- Equity: $E_2 = max(A_2 - F_2, 0)$
- Debt:

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2, \\ A_2 - b & A_2 < F_2. \end{cases}$$

- Note: $b =$ bankruptcy costs

Figure 6: One-period model / Equilibrium

This figure illustrates the equilibrium in the one-period model. The parameter γ denotes the risk-shifting parameter, $\mathbb{E}(BC) = \frac{1}{2}b$ denotes expected bankruptcy costs.

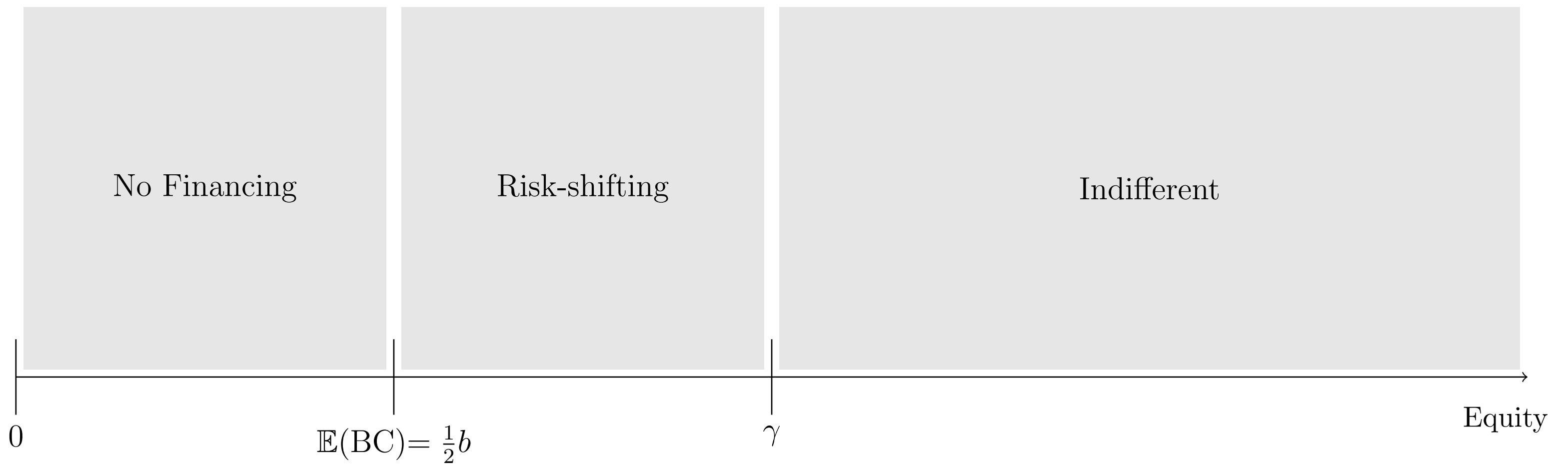


Figure 7: Two-period model / Timeline

This figure illustrates the timeline of events in the two-period model.

t=0

Financing

- Initial insider's equity E_0
- Financing $I_0 - E_0$ from outsiders via debt contract with notional F_1

t=0.5

Investment choice

- Strategy $c_0 \in \{s_0, r_0\}$, s_0 yields payoff I_0 (safe project), r_1 yields payoff $I_0 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

t=1

Value of assets

- Assets: $A_1 \in \{0, I_0 - \gamma, I_0, I_0 + \gamma\}$

Financing

- Refinancing of debt notional F_1 from outsiders via debt contract with notional F_2

Payoff in case that refinancing of debt notional F_1 not successful

- Equity: $E_1 = \max[A_1 - b - F_1, 0]$
- Debt: $D_1 = \min[A_1 - b, F_1]$

t=1.5

Investment choice

- Strategy $c_1 \in \{s_1, r_1\}$, s_1 yields payoff I_1 (safe project), r_1 yields payoff $I_1 \pm \gamma$ (risky project)
- Outside option (no investment) yields payoff 0

t=2

Payoff

- Assets: $A_2 \in \{0, I_1 - \gamma, I_1, I_1 + \gamma\}$
- Equity: $E_2 = \max[A_2 - F_2, 0]$
- Debt:

$$D_2 = \begin{cases} F_2 & A_2 \geq F_2, \\ A_2 - b & A_2 < F_2. \end{cases}$$

Figure 8: Two-period model / Equilibrium

This figure illustrates the equilibrium in the one-period and two-period model. The parameter γ denotes the risk-shifting parameter, $\mathbb{E}(BC)$ denotes expected bankruptcy costs in the one-period model when the insider is just able to receive outside financing ($E_0 = \frac{1}{2}b$), and $\mathbb{E}(BC') = \frac{3}{4}b$ denotes expected bankruptcy costs in the two-period model with risky debt when the insider is just able to receive outside financing ($E_0 = \frac{3}{4}b$). $\mathbb{E}(BC')$ differs from $\mathbb{E}(BC)$ because a firm with risky debt in the two-period model can go bankrupt in either $t = 1$ (with probability $\frac{1}{2}$) or in $t = 2$ (with probability $\frac{1}{4}$) and $\mathbb{E}(BC')$ captures the sum of the expected bankruptcy costs in $t = 1$ and $t = 2$.

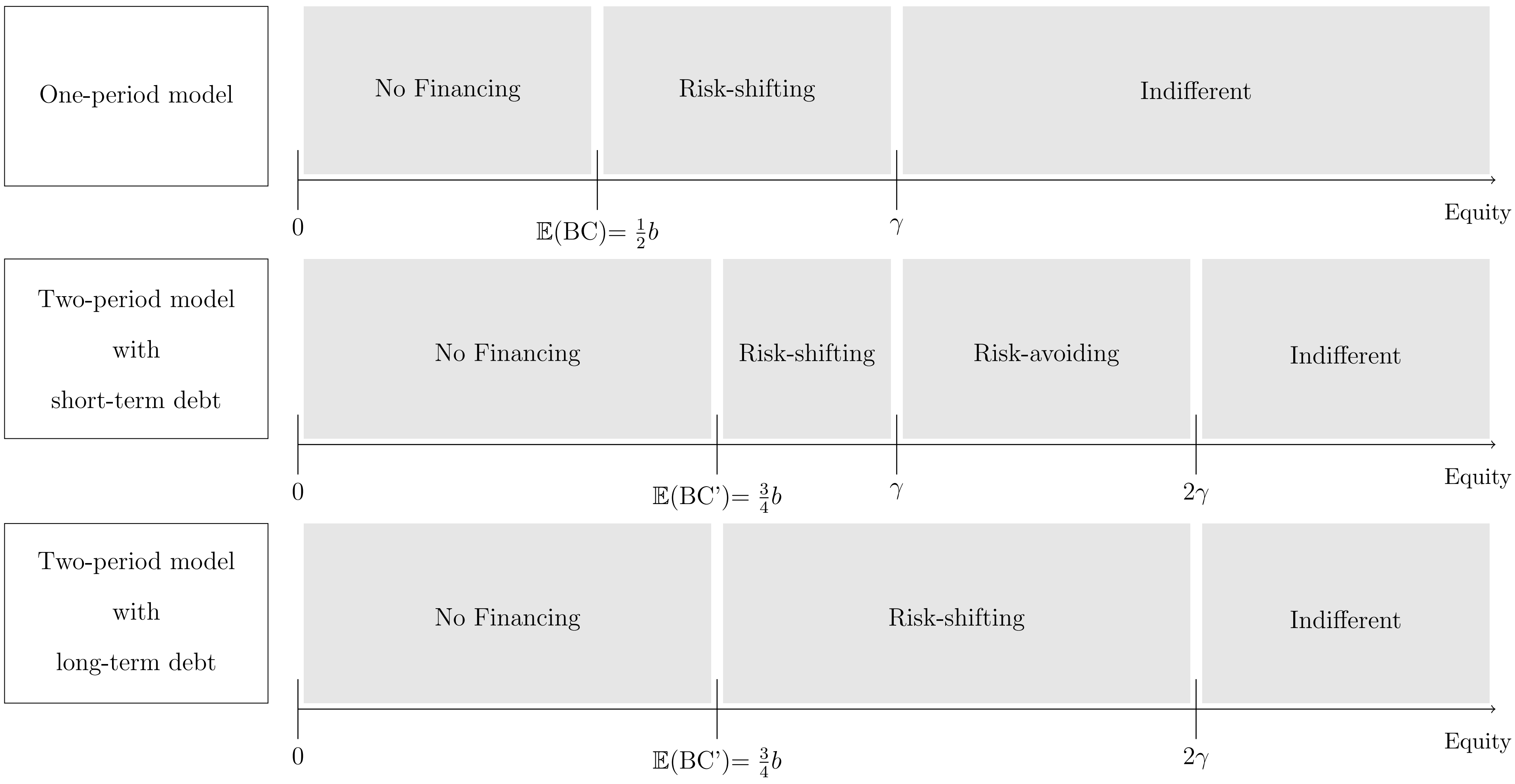


Figure 9: N-period model / Equilibrium

This figure illustrates the equilibrium in the one-period, two-period, and N-period model. The parameter γ denotes the risk-shifting parameter, b denotes bankruptcy costs.

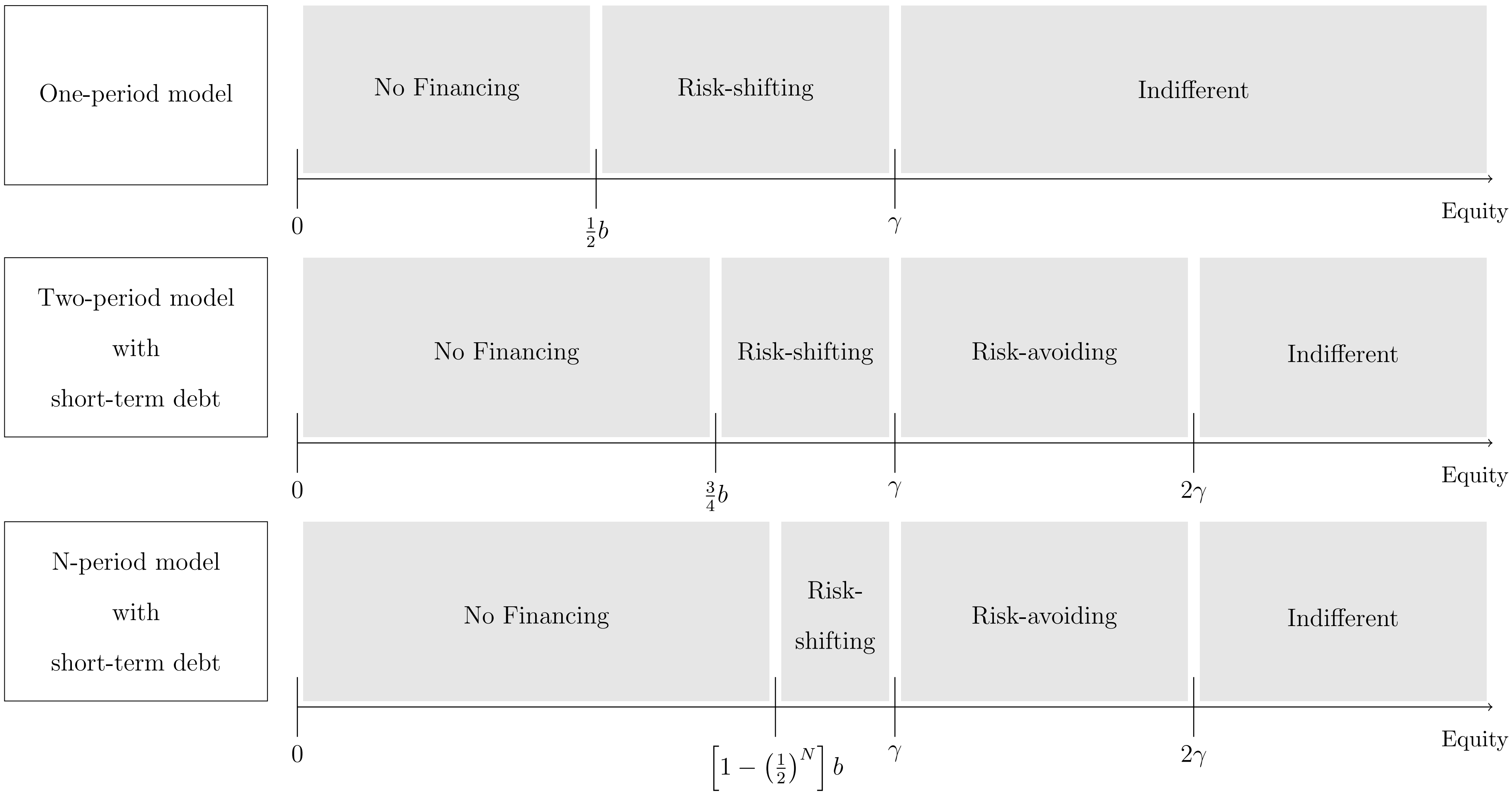


Table 1: Review of the empirical literature on leverage and risk-taking

This table provides an overview of empirical papers that analyze the relationship between leverage and risk-taking. In the first step, we select papers published in the three major finance journals (*Journal of Finance*, *Journal of Financial Economics*, *Review of Financial Studies*) that cite Jensen and Meckling (1976) and whose abstract discusses the relationship between leverage and risk-taking. We expand this list with papers from other journals that are frequently cited in the related literature in the papers identified in the first step.

Authors	Journal (Year)	Relation between leverage and risk-taking	Short summary
Panel A: General			
Haushalter	JF (2000)	Negative	Firms with higher leverage hedge more frequently
Eisdorfer	JF (2008)	Positive	Evidence of risk-shifting behavior in investment decisions of distressed firms
Pichler, Stomper, and Zulehner	RFS (2008)	Positive	Firms engange in risk-shifting by setting lower prices if debt is higher
Purnanandam	JFE (2008)	Non-monotonic	Moderately leveraged firms hedge more than low or high leveraged firms
Becker and Strömberg	RFS (2012)	Positive	Highly levered firms reduce risk when managers face less incentives to favor equity over debt
Gilje	RFS (2016)	Negative	Firms in the oil industry reduce investment risk when approaching financial distress
Aretz, Banerjee, and Pryshchepa	RF (2018)	Non-monotonic	Moderately, but not highly, leveraged firms increase risk-taking following a distress-risk-increasing hurricane
Panel B: Studies of distressed firm behavior			
Asquith, Gertner, and Scharfstein	QJE (1994)	Negative	Firms in financial distress decrease (asset) risk via asset sales
Andrade and Kaplan	JF (1998)	No relation	No evidence of risk-shifting when firms became financially (not economically) distressed
Panel C: Risk-shifting via defined-benefit corporate pension plans			
Cocco and Volpin	FAJ (2007)	Positive	Risk-shifting by leveraged firms in the asset mix of defined benefit pension plans
Rauh	RFS (2009)	Negative	Firms with weak credit ratings allocate a larger share of defined benefit pension fund assets to safer securities
Pedersen	JFQA (2019)	No relation	No evidence of risk-shifting in corporate pension plans following a shock to leverage
Panel D: CEO surveys			
Graham and Harvey	JFE (2001)	No relation	No relationship between leverage and risk-shifting behavior in survey of CEOs

A Proofs

Proof of Proposition 1

We restrict our attention to a face value of debt smaller than the payoff after success of the risky project, i.e. $F_2 \leq I_1 + \gamma$, otherwise the insider is as well off by not investing.

Incentive constraint: The insider chooses the safe strategy iff the expected payoff from the safe strategy is larger or equal to the payoff from the risky strategy:

$$\begin{aligned} & \max[0, I_1 - F_2] && \geq \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}\max[0, I_1 - \gamma - F_2] \\ \Leftrightarrow \quad & I_1 - \gamma - F_2 && \geq 0 \\ \Leftrightarrow \quad & F_2 && \leq I_1 - \gamma \end{aligned} \tag{4}$$

The risky strategy is always incentive compatible. Because the insider has a call option payoff, the risky strategy provides a higher payoff whenever the call option is valuable in at least one state of the world.

Participation constraint: If the insider issues debt with a low face value, $F_2 \leq I_1 - \gamma$, then default does not occur even under the risky strategy, and the outsiders' participation constraint is

$$F_2 \geq I_1 - E_1. \tag{5}$$

If the insider issues debt with a face value such that, $I_1 + \gamma > F_2 > I_1 - \gamma$, then the outsiders knows that the insider chooses the risky strategy, default occurs in the low state and the outsiders' participation constraint is

$$\begin{aligned} \frac{1}{2}F_2 + \frac{1}{2}(I_1 - \gamma - b) \geq I_1 - E_1 \\ \Leftrightarrow F_2 \geq I_1 - 2E_1 + \gamma + b. \end{aligned} \tag{6}$$

Equity value: Now consider the insider's optimal choice of face value F_2 subject to the outsiders' participation constraint and her own incentive compatible strategy choice. Given that the equity value decreases in the face value of debt, the insider has the incentive to reduce the face value of debt as much as possible. If the insider chooses safe debt, $I_1 - \gamma \geq F_2$, then the lowest possible face value of debt is given by the binding participation constraint (5), and the expected value of her equity is

$$\mathbb{E}_1[E_2] = I_1 - F_2 = I_1 - (I_1 - E_1) = E_1 \tag{7}$$

If she chooses risky debt, $I_1 + \gamma > F_2 > I_1 - \gamma$, then the face value of debt is given by the binding participation constraint (6), and the expected value of her equity is

$$\mathbb{E}_1[E_2] = \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}0 = E_1 - \frac{1}{2}b. \tag{8}$$

Equilibrium strategies: To realize the maximum value $\mathbb{E}_1[E_2] = E_1$, it must be that $I_1 - \gamma \geq F_2 = I_1 - E_1$, or $E_1 \geq \gamma$. In this case, the insider can issue safe debt and she is indifferent between choosing the safe or the risky strategy.

If $E_1 < \gamma$, only risky debt $I_1 + \gamma > F_2 > I_1 - \gamma$ is possible with a face value of $F_2 = I_1 - 2E_1 + \gamma + b$. The value of equity with risky debt is given by (8).

When the value of equity $E_1 < \frac{1}{2}b$, then the participation constraint (6) requires $F_2 > I_1 + \gamma$. As this is higher than the payoff after success of the risky project, the insider cannot obtain financing and the value of equity is zero.

Proof of Corollary 1

The incentive constraint (4) becomes $\max[0, I_1 + \Delta - F_2] \geq \frac{1}{2}(I_1 + \gamma - F_2) + \frac{1}{2}\max[0, I_1 - \gamma - F_2] \Leftrightarrow F_2 \leq I_1 - \gamma + 2\Delta$. The participation constraints for safe debt (5) and risky debt (6) remain unchanged. The equity value becomes $\mathbb{E}[E_2] = E_1 + \Delta$ for the safe strategy and remains $\mathbb{E}[E_2] = E_1 - \frac{1}{2}b$ for the risky strategy.

In equilibrium, the safe strategy is choosen for $F_2 \leq I_1 - \gamma + 2\Delta$, which – using the participation constraint with the lowest possible face value $F_2 = I_1 - E_1$ – becomes $E_1 \geq \gamma - 2\Delta$. The risky strategy is choosen for $E_1 < \gamma - 2\Delta$ as long as the insider can obtain funding (governed by the same participation constraint (6) as in the proof of Proposition 1), i.e. for $E_1 \in [\frac{1}{2}b, \gamma - 2\Delta)$. The interval for E_1 is non-empty if $\frac{1}{2}b < \gamma - 2\Delta \Leftrightarrow \Delta < \frac{1}{2}\gamma - \frac{1}{4}b$.

Proof of Proposition 2

Our aim is to derive the optimal first-period strategy $c_0 \in \{safe\,(s_0), risky\,(r_0)\}$, the resulting book value of equity after the first period, E_1, and the corresponding expected equity payoff E_2 at the end of the two period-model. The key variable to derive the optimal c_0 is the book value of equity after the first period, defined as $E_1 := A_1 - F_1$. This book value E_1 determines – via Proposition 1 – the equity holder's ability to roll over debt (and therefore bankruptcy and the participation constraint of the initial debtholders) as well as the equity payoff at the end of the two-period model (and therefore the equity holder's incentive compatibility constraint).

In the following, we denote by $E_1(s_0)$, $E(r_1^+)$, and $E(r_1^-)$ the book value of equity after a safe project, a successful risky project, and an unsuccessful risky project, respectively. In a slight abuse of notation, we denote the equity payoff after the second period with $E_2 := \max(A_2 - F_2, 0)$. Please note that E_1 is a book value and can therefore be negative, while E_2 is always non-negative. Finally, $\mathbb{E}_0[E_2]$ denotes the expected value of E_2 under the equilibrium strategy.

Case 1: $\mathbf{F_1 \leq I_0 - 2\gamma}$ (Book value of equity $\geq \gamma$ after first period)
The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = A_1 - F_1 = I_0 - F_1 \geq 2\gamma \quad \overset{(Prop.1)}{\Rightarrow} \quad E_2(s_0) = I_0 - F_1 \tag{9}$$

$$\left.\begin{array}{ll} E_1(r_0^+) = A_1 - F_1 = I_0 + \gamma - F_1 & \geq 3\gamma \\ E_1(r_0^-) = A_1 - F_1 = I_0 - \gamma - F_1 & \geq \gamma \end{array}\right\} \quad \overset{(Prop.1)}{\Rightarrow} \quad E_2(r_0) = I_0 - F_1 \tag{10}$$

No matter whether the insider chooses the safe or risky project, she ends up in case (i) of Proposition 1 ($E_1 \geq \gamma$). She can always issue safe debt in period 2. The debtholders' binding participation constraint is therefore $F_1 = I_0 - E_0$ which implies $E_2 = I_0 - F_1 = E_0$. The range of feasible values for E_0 is (using $F_1 \leq I_0 - 2\gamma$) $E_0 \geq 2\gamma$.

To sum up, when $F_1 \leq I_0 - 2\gamma$, the insider's first-period investment choice c_0 and the expected value of her equity at $t = 0$, $\mathbb{E}_0[E_2]$, as function of her initial equity E_0, are:

$$c_0, \mathbb{E}_0[E_2] = \Big\{ (s_0, r_0), E_0 \quad \text{if } E_0 \in [2\gamma, \infty) \qquad \text{(indifferent)} \tag{11}$$

Case 2: $\mathbf{F_1} \in (\mathbf{I_0} - \mathbf{2}\gamma, \mathbf{I_0} - \gamma]$ (Book value of equity ≥ 0 after first period)
For $F_1 \in (I_0 - 2\gamma, I_0 - \gamma)$, the book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = A_1 - F_1 = I_0 - F_1 \in (\gamma, 2\gamma) \quad \overset{(Prop.1)}{\Rightarrow} \quad E_2(s_0) = I_0 - F_1 \tag{12}$$

$$\left.\begin{array}{l} E_1(r_0^+) = A_1 - F_1 = I_0 + \gamma - F_1 \quad \in (2\gamma, 3\gamma) \\ E_1(r_0^-) = A_1 - F_1 = I_0 - \gamma - F_1 \quad \in (0, \gamma) \end{array}\right\} \overset{(Prop.1)}{\Rightarrow} \quad E_2(r_0) < I_0 - F_1 \tag{13}$$

Incentive compatibility (Choice of c_0): If the insider chooses the safe project, she ends up in case (i) of Proposition 1 ($E_1 \geq \gamma$). She can always issue safe debt in period 2. If the insider chooses the risky project, she ends up with probability 1/2 in case (i) of Proposition 1 ($E_1 \geq \gamma$) and with probability 1/2 in cases (ii)/(iii) of Proposition 1 ($E_1 \in [0, \gamma)$). In the first case, the payoff in period $t = 2$ is *equal* to $I_0 + \gamma - F_1$, in the latter case it is *smaller* than $I_0 - \gamma - F_1$ due to the deadweight cost of risk-shifting that need to be borne by the equity holder. Thus, the equity holder has an incentive to choose the safe project.

For $F_1 = I_0 - \gamma$, $E_2(r_0) = E_2(s_0) = I_0 - F_1$ (i.e., equation (13) holds with equality) and the insider is thus indifferent between choosing the safe and the risky project. We assume that the insider will choose the safe project as a tie-breaker rule in this situation.

Debtholders' participation constraint: Given that the equity holder has an incentive to choose the safe project, and thus is always able to refinance first-period debt in the intermediate period, the debt is safe. The debtholders' binding participation constraint is therefore $F_1 = I_0 - E_0$.

Expected value of insider's equity: The debtholders' participation constraint $F_1 = I_0 - E_0$ implies that $E_2 = I_0 - F_1 = E_0$. Together with $F_1 \in [I_0 - 2\gamma, I_0 - \gamma)$, the range of feasible values for E_0 is $E_0 \in [\gamma, 2\gamma)$.

To sum up, when $F_1 \in (I_0 - 2\gamma, I_0 - \gamma]$, the insider's first-period investment choice c_0 and the expected value of her equity at $t = 0$, $\mathbb{E}_0[E_2]$, as function of her initial equity E_0, are:

$$c_0, \mathbb{E}_0[E_2] = \Big\{ s_0, E_0 \quad \text{if } E_0 \in [\gamma, 2\gamma) \qquad \text{(safe)} \tag{14}$$

Case 3: $\mathbf{F_1} > \mathbf{I_0} - \gamma$ (Book value of equity can be negative after first period)

Case 3a: $\mathbf{F_1} \in (\mathbf{I_0} - \gamma, \mathbf{I_0}]$
The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = A_1 - F_1 = I_0 - F_1 \in [0, \gamma) \quad \overset{(Prop.1)}{\Rightarrow} \quad E_2(s_0) \leq I_0 - F_1 \tag{15}$$

$$\left.\begin{array}{l} E_1(r_0^+) = A_1 - F_1 = I_0 + \gamma - F_1 \quad \in [\gamma, 2\gamma) \\ E_1(r_0^-) = A_1 - F_1 = I_0 - \gamma - F_1 \quad \in [-\gamma, 0) \end{array}\right\} \overset{(Prop.1)}{\Rightarrow} \quad E_2(r_0) > I_0 - F_1 \tag{16}$$

Incentive compatibility (Choice of c_0): If the insider chooses the risky strategy, she benefits from limited liability: with probability 1/2 book equity in $t = 1$ is negative, she can push part of the losses onto debtholders and she thus receives a payoff of 0 at the end of period 2. With probability 1/2 book equity in $t = 1$ is equal to $I_0 + \gamma - F_1 > \gamma$, she ends up in case (i) of Proposition 1 and receives a payoff of $I_0 + \gamma - F_1$ in $t = 2$. The expected payoff in $t = 2$ from choosing the risky strategy in the first period is therefore $E_2 = \frac{1}{2}0 + \frac{1}{2}(I_0 + \gamma - F_1)$. Using $F_1 > I_0 - \gamma \Leftrightarrow \gamma > I_0 - F_1$ this implies $E_2 > I_0 - F_1$. As the payoff from the safe project is $E_2 \leq I_0 - F_1$, the equity holder has an incentive to choose the risky project.

Debtholders' participation constraint: The debtholders' binding participation constraint is therefore $\frac{1}{2}F_1 + \frac{1}{2}(I_0 - \gamma - b) = I_0 - E_0 \Leftrightarrow F_1 = I_0 - 2E_0 + \gamma + b$.

Expected value of insider's equity: The binding participation constraint $F_1 = I_0 - 2E_0 + \gamma + b$ implies that $E_2 = \frac{1}{2}0 + \frac{1}{2}(I_0 + \gamma - F_1) = E_0 - \frac{1}{2}b$. Together with $F_1 \in (I_0 - \gamma, I_0]$, the range of feasible values for E_0 is $E_0 \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma + \frac{1}{2}b)$.

Case 3b: $\mathbf{F_1 > I_0}$

Without loss of generality, we assume that $F_1 \in (I_0, I_0+\gamma-\frac{1}{2}b)$ [note that if $F_1 \geq I_0+\gamma-\frac{1}{2}b$, then the insider would default even after a successful risky strategy and therefore always receive a paoyff of zero]. The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = A_1 - F_1 = I_0 - F_1 \in [-\gamma, 0) \qquad \overset{(Prop.1)}{\Rightarrow} \quad E_2(s_0) = 0 \tag{17}$$

$$\left.\begin{aligned} E_1(r_0^+) &= A_1 - F_1 = I_0 + \gamma - F_1 &\in (\tfrac{1}{2}b, \gamma) \\ E_1(r_0^-) &= A_1 - F_1 = I_0 - \gamma - F_1 &\in (-2\gamma, -\gamma) \end{aligned}\right\} \overset{(Prop.1)}{\Rightarrow} \quad E_2(r_0) > 0 \tag{18}$$

Incentive compatibility (Choice of c_0): The insider always defaults on the first-period debt when she chooses the safe strategy, but does not always default when choosing the risky strategy. The insider thus chooses the risky strategy.

Debtholders' participation constraint: The debtholders' binding participation constraint is therefore $\frac{1}{2}F_1 + \frac{1}{2}(I_0 - \gamma - b) = I_0 - E_0 \Leftrightarrow F_1 = I_0 - 2E_0 + \gamma + b$.

Expected value of insider's equity: The binding participation constraint $F_1 = I_0-2E_0+\gamma+b$ implies that $E_2 = \frac{1}{2}0+\frac{1}{2}(I_0+\gamma-F_1-\frac{1}{2}b) = E_0-\frac{3}{4}b$. Together with $F_1 \in (I_0, I_0-\gamma+\frac{1}{2}b)$, the range of feasible values for E_0 is $E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b)$.

To sum up, when $F_1 > I_0 - \gamma$, the insider's first-period investment choice c_0 and the expected value of her equity at $t = 0$, $\mathbb{E}_0[E_2]$, as function of his initial equity E_0, are:

$$c_0, \mathbb{E}_0[E_2] = \begin{cases} \text{no inv}, 0 & \text{if } E_0 \in [0, \frac{3}{4}b) \quad \text{(no investment)} \\ r_0, E_0 - \frac{3}{4}b & \text{if } E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b) \quad \text{(risky)} \\ r_0, E_0 - \frac{1}{2}b & \text{if } E_0 \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma + \frac{1}{2}b) \quad \text{(risky)} \end{cases} \tag{19}$$

The equilibrium strategies follow from (11), (14), and (19). If both safe and risky debt is possible (for $E_0 \in [\gamma, \gamma + \frac{1}{2}b)$), then the insider will choose the face value that maximizes equity value, i.e. the safe strategy. For $b < \gamma$, this yields:

$$c_0, \mathbb{E}_0[E_2] = \begin{cases} \text{no inv}, 0 & \text{if } E_0 \in [0, \frac{3}{4}b) & \text{(no investment)} \\ r_0, E_0 - \frac{3}{4}b & \text{if } E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b) & \text{(risky)} \\ r_0, E_0 - \frac{1}{2}b & \text{if } E_0 \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma) & \text{(risky)} \\ s_0, E_0 & \text{if } E_0 \in [\gamma, 2\gamma) & \text{(safe)} \\ (s_0, r_0), E_0 & \text{if } E_0 \in [2\gamma, \infty) & \text{(indifferent)} \end{cases} \tag{20}$$

For $b \geq \gamma$, some of the intervals in (20) are empty. The following table summarizes the strategies, outcomes, and expected equity value as a function of initial equity for various values of b:

$b < \gamma$	E_0				
	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{2}b)$	$[\frac{1}{2}\gamma + \frac{1}{2}b, \gamma)$	$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
1st period	no inv.	risky	risky	safe	indifferent
2nd period	n.a	risky/no inv.	indiff./no inv.	indifferent	indifferent
Default	n.a.	1st or 2nd period failure	1st period failure	never	never
$\mathbb{E}_0[E_2]$	0	$E_0 - \frac{3}{4}b$	$E_0 - \frac{1}{2}b$	E_0	E_0
$\gamma \leq b < \frac{4}{3}\gamma$	E_0				
	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \gamma)$		$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
1st period	no inv.	risky		safe	indifferent
2nd period	n.a	risky/no inv.		indifferent	indifferent
Default	n.a.	1st or 2nd period failure		never	never
$\mathbb{E}_0[E_2]$	0	$E_0 - \frac{3}{4}b$		E_0	E_0
$\frac{4}{3}\gamma \leq b < 2\gamma$	E_0				
	$[0, \gamma)$			$[\gamma, 2\gamma)$	$[2\gamma, \infty)$
1st period	no inv.			safe	indifferent
2nd period	n.a			indifferent	indifferent
Default	n.a.			never	never
$\mathbb{E}_0[E_2]$	0			E_0	E_0

Note that in Proposition 2 we assumed $b < \frac{4}{3}\gamma$ to ensure that case iii) of Proposition 2 exists, see also Note (2.) below Proposition 2.

Proof of Corollary 2

The book value of equity after the risky strategy is now $E_2[E_1(r_0^+)] = I_0 + \Delta + \gamma - F_1$ (in case of success) and $E_2[E_1(r_0^-)] = I_0 + \Delta - \gamma - F_1$ (in case of failure). The book value of equity after the safe strategy is $E_2[E_1(s_0)] = I_0 - F_1$. As in the proof of Proposition 2, we first derive the optimal strategy c_0 given a face value of debt F_1 (incentive compatability), and then use the derive the debtholders' participation constraint to derive the equilibrium face value of debt, the equilibrium strategy c_0 and the equilibrium equity values.

- $F_1 \leq I_0 - 2\gamma + \Delta$: The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = I_0 - F_1 \geq 2\gamma - \Delta \overset{(\Delta<\gamma, Prop.1)}{\Rightarrow} E_2(s_0) = I_0 - F_1 \tag{21}$$

$$\left.\begin{aligned} E_1(r_0^+) &= I_0 + \Delta + \gamma - F_1 &&\geq 3\gamma \\ E_1(r_0^-) &= I_0 + \Delta - \gamma - F_1 &&\geq \gamma \end{aligned}\right\} \overset{(Prop.1)}{\Rightarrow} E_2(r_0) = I_0 + \Delta - F_1 \tag{22}$$

  The risky strategy is therefore preferred over the safe strategy because it provides higher payoff ($+\Delta$). Debt is safe and the binding debtholders' participation constraint is therefore $F_1 = I_0 - E_0$. Thus, this case applies for $E_0 \geq 2\gamma - \Delta$.

- $F_1 \in (I_0 - 2\gamma + \Delta, I_0 - \gamma + \Delta - \frac{1}{2}b]$:[10] The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = I_0 - F_1 \geq \gamma - \Delta + \tfrac{1}{2}b \overset{(\Delta<\frac{1}{4}b)}{\geq} \gamma \overset{(Prop.1)}{\Rightarrow} E_2(s_0) = I_0 - F_1 \tag{23}$$

$$\left.\begin{aligned} E_1(r_0^+) &= I_0 + \Delta + \gamma - F_1 &&\geq 2\gamma + \tfrac{1}{2}b \\ E_1(r_0^-) &= I_0 + \Delta - \gamma - F_1 &&\in [\tfrac{1}{2}b, \gamma) \end{aligned}\right\} \overset{(Prop.1)}{\Rightarrow} E_2(r_0) = I_0 - F_1 + \Delta - \frac{1}{4}b \tag{24}$$

  Because $\Delta < \frac{1}{4}b$ (by assumption), this implies that $E_2[E_1(r_0)] < E_2[E_1(s_0)]$ so that the safe strategy is preferred over the risky strategy. Debt is safe and the binding debtholders' participation constraint is therefore $F_1 = I_0 - E_0$. Thus, this case applies for $E_0 \in [\gamma - \Delta + \frac{1}{2}b, 2\gamma - \Delta)$.

- $F_1 \in (I_0 - \gamma + \Delta - \frac{1}{2}b, I_0 - \gamma - \Delta]$:[11] The book value of equity in $t = 1$ (E_1) and the corresponding expected payoff in $t = 2$ (E_2) are equal to:

$$E_1(s_0) = I_0 - F_1 \geq \gamma + \Delta \overset{(Prop.1)}{\Rightarrow} E_2(s_0) = I_0 - F_1 \tag{25}$$

$$\left.\begin{aligned} E_1(r_0^+) &= I_0 + \Delta + \gamma - F_1 &&\geq 2\gamma + 2\Delta \\ E_1(r_0^-) &= I_0 + \Delta - \gamma - F_1 &&< \tfrac{1}{2}b \end{aligned}\right\} \overset{(Prop.1)}{\Rightarrow} E_2(r_0) \leq I_0 - F_1 \tag{26}$$

  Note that the last inequality follows from $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = \frac{1}{2}(I_0 + \Delta + \gamma - F_1)$ and the fact that $\Delta + \gamma \leq I_0 - F_1$ (the upper bound for F_1 is $F_1 \leq I_0 - \gamma - \Delta$, which is equvalent to $\Delta + \gamma \leq I_0 - F_1$). Thus, $E_2[E_1(r_0)] \leq E_2[E_1(s_0)]$ so that the safe strategy is preferred over the risky strategy (for $\Delta + \gamma < I_0 - F_1$) or has the same expected payoff as the risky strategy (for $\Delta + \gamma = I_0 - F_1$). As a tiebreaker rule, we assume that the safe strategy is choosen. Debt is safe and the binding debtholders' participation constraint is therefore $F_1 = I_0 - E_0$. Thus, this case applies for $E_0 \in [\gamma + \Delta, \gamma - \Delta + \frac{1}{2}b)$.

[10] The term $\frac{1}{2}b$ ensures that $E_1(r_0^-) \geq \frac{1}{2}b$, so that case (ii) of Proposition 1 applies.

[11] Note that by assumption, $\Delta < \frac{1}{4}b$ so that the interval is non-empty.

- $F_1 \in (I_0-\gamma-\Delta, I_0-\gamma]$: The book value of equity in $t=1$ (E_1) and the corresponding expected payoff in $t=2$ (E_2) are equal to:

$$E_1(s_0) = I_0 - F_1 \geq \gamma \qquad \Rightarrow E_2(s_0) = I_0 - F_1 \tag{27}$$

$$\left.\begin{array}{ll} E_1(r_0^+) = I_0 + \Delta + \gamma - F_1 & \geq 2\gamma + \Delta \\ E_1(r_0^-) = I_0 + \Delta - \gamma - F_1 < 2\Delta & \overset{(\Delta<\frac{1}{4}b)}{<} \frac{1}{2}b \end{array}\right\} \Rightarrow E_2(r_0) > I_0 - F_1 \tag{28}$$

Note that the last inequality follows from $E_2[E_1(r_0)] = \frac{1}{2}E_2[E_1(r_0^-)] + \frac{1}{2}E_2[E_1(r_0^+)] = \frac{1}{2}(I_0 + \Delta + \gamma - F_1)$ and the fact that $\Delta + \gamma > I_0 - F_1$ (the lowwer bound for F_1 is $F_1 > I_0 - \gamma - \Delta$, which is equvalent to $\Delta + \gamma > I_0 - F_1$). Thus, $E_2[E_1(r_0)] > E_2[E_1(s_0)]$ so that the risky strategy is preferred over the safe strategy. Debt is risky and the binding debtholders' participation constraint is therefore $F_1 = I_0 - 2E_0 - \Delta + \gamma + b$. Thus, this case applies for $E_0 \in [\gamma - \frac{1}{2}\Delta + \frac{1}{2}b, \gamma + \frac{1}{2}b)$. Note that the lower bound of this interval $\gamma - \frac{1}{2}\Delta + \frac{1}{2}b > \gamma + \frac{3}{2}\Delta$ (using $b > 4\Delta$) is above the lower bar of the interval in the last case ($E_0 \in [\gamma + \Delta, \gamma - \Delta + \frac{1}{2}b)$). The equity holder's payoff is decreasing in the face value of debt F_1 and thus the equity holder is better off by choosing a lower face value of debt $F_1 \leq I_0 - \gamma - \Delta$.

For $F_1 > I_0 - \gamma$, the risky strategy is preferred over the safe strategy for $\Delta = 0$ (see Proposition 2), so that it is also preferred in the case where the risky strategy has a higher expected payoff than the safe strategy (i.e. for $\Delta > 0$).

The lower and upper bound for the feasibility of the risky strategy needs to be recalculated. For the lower bound, note that for risky debt that defaults in case of failure of the risky strategy, the binding participation constraint is $F_1 = I_0 - 2E_0 - \Delta + \gamma + b$. The equity holder's incentive compatibility constraint is $I_0 + \Delta + \gamma - F_1 > \frac{1}{2}b$ (the book value of equity after the first period in case of success must be high enough to allow the equity holder to continue). Combining both yields $E_0 > \frac{3}{4}b - \Delta$. A higher Δ thus allows the equity holder to start with a lower initial level of equity. The upper bound is simply determined by the smallest value for which the safe strategy is feasible, i.e. $E_0 < I_0 + \Delta$.

Taken together, the first-period strategy choice and expected value of equity for $E_0 \in [\gamma, \infty)$ at $t = 0$ are:

$$c_0, \mathbb{E}_0[E_2] = \begin{cases} \text{no inv},\ 0 & \text{if } E_0 \in [0, \frac{3}{4}b - \Delta) \quad \text{(no investment)} \\ r_0,\ \in (0, E_0) & \text{if } E_0 \in [\frac{3}{4}b - \Delta, \gamma + \Delta) \quad c_0 = r_0 \text{ (risky)} \\ s_0,\ E_0 & \text{if } E_0 \in [\gamma + \Delta, 2\gamma - \Delta) \quad c_0 = s_0 \text{ (safe)} \\ r_0,\ E_0 + \Delta & \text{if } E_0 \in [2\gamma - \Delta, \infty) \quad c_0 = r_0 \text{ (risky)} \end{cases} \tag{29}$$

Proof of Proposition 3

We restrict our attention to a face value of debt smaller than the payoff after success of the risky project, i.e. $F_2 \leq I_1 + \gamma$, otherwise the insider is as well off by not investing.

Incentive constraint
The insider can choose six strategies in the two-period model: four deterministic strategies (safe in both periods, risky in both periods, safe/risky, risky/safe), and two strategies that depend on the first-period outcome (risky in the first period and safe after success/risky otherwise, risky in the first period and risky after success/safe otherwise), resulting in payoffs $A_2 \in \{I_0 - 2\gamma, I_0 - \gamma, I_0, I_0 + \gamma, I_0 + 2\gamma\}$. Let p_i denote the probability of a payoff

$A_2^i \in \{I_0 - 2\gamma, I_0 - \gamma, I_0, I_0 + \gamma, I_0 + 2\gamma\}$. The insider chooses the strategy that maximizes the payoff

$$\mathbb{E}_0[E_2] = \sum_i p_i \max[0, A_2^i - F_2] = \underbrace{\sum_i p_i(A_2^i - F_2)}_{I_0 - F_2} + \sum_i p_i \max[F_2 - A_2^i, 0] \tag{30}$$

The first sum in (30) is equal to $I_0 - F_2$ for all strategies and the IC constraint thus hinges on the last sum in (30). The last sum is zero for the safe/safe strategy in any case, but it is larger than zero for all other strategies as long as $F_2 > \min_i A_2^i = I_0 - 2\gamma$. Thus, the safe/safe strategy is only incentive-compatible iff

$$F_2 \leq I_0 - 2\gamma$$

If $F_2 > I_0 - 2\gamma$, it is straightforward to see that the strategy risky/risky maximizes the payoff (30). The strategy risky/risky results in a payoff of $I_0 - 2\gamma$ with probability $\frac{1}{4}$, I_0 with probability $\frac{1}{2}$, and $I_0 + 2\gamma$ with probability $\frac{1}{4}$.

Participation constraint and expected equity value
PC for safe debt: With safe debt ($F_2 \leq I_0 - 2\gamma$), the participation constraint is

$$F_2^{\text{safe}} \geq I_0 - E_0,$$

which, together with $F_2 \leq I_0 - 2\gamma$ implies $E_0 \geq 2\gamma$. The resulting equity value at $t = 0$ is

$$\mathbb{E}_0[E_2(E_0)] = I_0 - F_2^{\text{safe}} = E_0 \ \text{ if } E_0 \in [2\gamma, \infty) \tag{31}$$

PC for risky debt: With risky debt that defaults in the low state ($A_2 = I_0 - 2\gamma$) only, the participation constraint is

$$\begin{aligned} &\frac{3}{4}F_2^{\text{riskyI}} + \frac{1}{4}(I_0 - 2\gamma - b) \geq I_0 - E_0 \\ \Leftrightarrow \quad &F_2^{\text{riskyI}} \geq I_0 - \frac{4}{3}E_0 + \frac{2}{3}\gamma + \frac{1}{3}b \end{aligned}$$

For default to occur in the low state, but not in the medium state, we require $F_2^{\text{riskyI}} \leq I_0$ and $F_2^{\text{riskyI}} > I_0 - 2\gamma$ which implies $E_0 \in [\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma + \frac{1}{4}b)$. The resulting equity value at $t = 0$ is:

$$\begin{aligned} \mathbb{E}_0[E_2(E_0)] &= \frac{1}{4}(I_0 + 2\gamma - F_2^{\text{riskyI}}) + \frac{1}{2}(I_0 - F_2^{\text{riskyI}}) \\ &= E_1 - \frac{1}{4}b \ \text{ if } E_0 \in [\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma + \frac{1}{4}b) \end{aligned} \tag{32}$$

With risky debt that defaults in the low state ($A_2 = I_0 - 2\gamma$) and in the medium state ($A_2 = I_0$), the participation constraint is

$$\begin{aligned} &\frac{1}{4}F_2^{\text{riskyII}} + \frac{1}{2}(I_0 - b) + \frac{1}{4}(I_0 - 2\gamma - b) \geq I_0 - E_0 \\ \Leftrightarrow \quad &F_2^{\text{riskyII}} \geq I_0 - 4E_0 + 3b + 2\gamma \end{aligned}$$

For default to occur in the low and medium state, but not in the high state, we require $F_2^{\text{riskyII}} \leq I_0 + 2\gamma$ and $F_2^{\text{riskyII}} > I_0$ which implies $E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{3}{4}b)$. The resulting equity value at $t = 0$ is:

$$\begin{aligned}\mathbb{E}_0[E_2(E_0)] &= \frac{1}{4}(I_0 + 2\gamma - F_2^{\text{riskyII}}) \\ &= E_0 - \frac{3}{4}b \text{ if } E_0 \in [\frac{3}{4}b, \frac{1}{2}\gamma + \frac{3}{4}b) \qquad (33)\end{aligned}$$

Note that the upper bound of (33) is higher than the lower bound of (32), which suggests that there are values for E_0 where the insider can issue either debt with a face value that defaults in the low and medium state or debt with a face value that defaults in the low state only.

Equilibrium strategies
The equilibrium strategies follow from three constraints. First, (31), (32), and (33) provide the range of values for the initial insider's equity E_0 that make safe debt and the two versions of risky debt (default in the low state only, default in the low and medium state) feasible. Second, if several strategies are feasible, the insider chooses whichever strategy yields the highest expected value $\mathbb{E}_0[E_2(E_0)]$. Third, one has to carefully consider the upper and lower bounds in (31), (32), and (33) to make sure not to make statements about empty intervals. The following table provides the equilibrium strategies, the cases where default occurs, and the expected equity value. Subindex "1" denotes the first period, subindex "2" denotes the second period, and ID_i denotes that the insider is indifferent between the safe and the risky strategy (i.e., indifferent between r_0 and s_0 or indifferent between r_1 and s_1).

	E_0			
	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{4}b)$	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$
Strategy	No_0	r_0, r_1	r_0, r_1	ID_0, ID_1
Default	always	down_0, down_1	down_0, down_1	never
		$\vee$ down_0, up_1		
		$\vee$ up_0, down_1		
Equity value	0	$E_0 - \frac{3}{4}b$	$E_0 - \frac{1}{4}b$	E_0
Non-empty	Always	$b < \gamma$	$b < 6\gamma$	Always
$b \in (0, \gamma)$	$[0, \frac{3}{4}b)$	$[\frac{3}{4}b, \frac{1}{2}\gamma + \frac{1}{4}b)$	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$
$b \in (\gamma, 6\gamma)$	$[0, \frac{1}{2}\gamma + \frac{1}{4}b)$	———	$[\frac{1}{2}\gamma + \frac{1}{4}b, 2\gamma)$	$[2\gamma, \infty)$

Proof of Proposition 4

Outline of the proof

The aim is to characterize the optimal investment choice (no investment, safe or risky investment, or indifference between them) at each moment in time, c_n^* for $n \in \{0, ...N-1\}$, as a function of the book value of equity E_n at that moment in time. Note that the game ends at N and that the last decision period is $N - 1$.

The proof is by induction. First, we start the induction and show what is the optimal choice in the last (decision) period $n = N-1$, c^*_{N-1}. This first step is the same as the one period problem. Moreover, we will characterize the market value of equity as a function of the book equity under the optimal investment choice :

$$f(E_{N-1}) \equiv \mathbb{E}_{N-1}[E^*_N(E_{N-1})]$$

The market value of equity at $N-1$ is the expectation at $N-1$ of the book value of equity at N, where the book value of equity at N depends on the optimal investment choice c^*_{N-1}, which in turn depends on book value of equity at $N-1$.

The function $f(E_{N-1})$ has three properties:

- **P1 (ability to issue safe debt):** If $E_{N-1} \geq \gamma$ then $f(E_{N-1}) = E_{N-1}$.
- **P2 (inability to issue safe debt):** If $E_{N-1} < \gamma$ then $f(E_{N-1}) < E_{N-1}$.
- **P3 (inability to issue debt):** If and only if $E_{N-1} \leq E^{min}_{N-1}$, with $0 \leq E^{min}_{N-1} < \gamma$, then $f(E_{N-1}) = 0$.

Property P1 says that if the book value of equity is above the loss when the risky investment fails, γ, then it is possible to issue safe debt and hence, obtain a market value of equity that is equal to the book value. Property P2 says this is not possible when the value of equity is below γ. Then the manager can only issue risky debt and hence, more expensive debt. Property P3 says that there exists a threshold level of book equity below which the manager cannot obtain funds. In that case, he cannot invest and the market value of equity drops to zero. We also that the manager is protected by limited liability so that the market value of equity cannot be negative even though the book value of equity can be negative, $f(E_{N-1}) \geq 0$.

In the second step of the proof, we link any period n to the next one $n+1$. We show what is the optimal investment choice at $n < N-1$ as a function of E_n assuming that all future choices $c^*_{n+1}, c^*_{n+2}, ..., c^*_{N-1}$ are optimal. Moreover, we assume that the three properties of the function f hold at $n+1$, i.e., they hold for

$$f(E_{n+1}) \equiv \mathbb{E}_{n+1}[E^*_N(E_{n+1})],$$

which is the market value of equity at $n+1$. It is given by the expectation at $n+1$ of the final value of book equity E^*_N under the equilibrium choices as of $n+1$ until the end, $c^*_{n+1}, c^*_{n+2}, ..., c^*_{N-1}$.

We then show that under the optimal investment choice at n, c^*_n, the properties of the function f also hold at n. Together with step one, this then completes the induction.

The key element of the proof is to show that the three properties of f carry over across periods. In the last period $n = N-1$, it is never optimal for the manager to choose the safe investment. It is also easy to see why the manager can issue safe debt if book equity is above γ, which is when he is indifferent between the safe and the risky investment. This is no longer true in all other periods $n < N-1$. The manager may prefer the safe investment. With the safe investment, he can issue safe debt in the future, which in turn enables him to issue safe debt today.

The first step in the backward induction - the choice in the last decision period $n = N - 1$

We can directly apply the solution to the one-period case in Proposition 1, where we replace the subscript 1 with $N-1$. This gives the optimal choice of risk c^*_{N-1} and verifies the three properties of the function f at E_{N-1}.

Connecting successive periods - optimal choice at $n < N-1$ **given optimal choices as of** $n+1$

We have to distinguish three cases depending on how large is the repayment F_{n+1} that needs to be refinanced at $n+1 < N$ with new debt given the assets in place at n, I_n (the value of the initial investment after risk choices at moments up to n).

Case 1: $F_{n+1} \leq I_n - 2\gamma$

In this case, little debt needs to be refinanced. So even if you choose the risky investment at n and fail at $n+1$, the debt the manager raises at $n+1$ is safe because it can be repaid even if there is another failure (at $n+2$).

The next set of equations shows what is the market value of equity at $n+1$ when at n the manager chooses the safe strategy s_n, when he chooses the risky investment and succeeds, r_n^+ (the superscript $^+$ indicates success), and when he chooses the risky investment and fails, r_n^- (the superscript $^-$ indicates failure).

$$
\begin{aligned}
E_{n+1}(s_n) = I_n - F_{n+1} \geq 2\gamma \quad &\overset{P1}{\Rightarrow} \quad f(E_{n+1}(s_n)) = I_n - F_{n+1} \\
E_{n+1}(r_n^+) = I_n + \gamma - F_{n+1} \geq 3\gamma \quad &\overset{P1}{\Rightarrow} \quad f(E_{n+1}(r_n^+)) = I_n + \gamma - F_{n+1} \\
E_{n+1}(r_n^-) = I_n - \gamma - F_{n+1} \geq \gamma \quad &\overset{P1}{\Rightarrow} \quad f(E_{n+1}(r_n^-)) = I_n - \gamma - F_{n+1}
\end{aligned}
$$

For example, in the first line we have the case when the manager chooses the safe strategy at n. The book value of equity at $n+1$ is the payoff from the investment, I_n, minus the debt repayment F_{n+1}. In case 1 we have $I_n - F_{n+1} \geq 2\gamma$ and hence, the book value of equity is high enough for Property 1 to apply: The manager can issue safe debt and the market value of equity is equal to the book value. Importantly, this also holds when the manager chooses the risky strategy and fails.

Knowing the market value of equity at $n+1$, we can derive the incentive compatible risk choice at n, c_n. If the manager chooses the safe investment at n, the expected market value of equity at n is equal to the market value at $n+1$, $I_n - F_{n+1}$. If he chooses the risky investment at n, the expected market value of equity at n is $\frac{1}{2}f(E_{n+1}(r_n^+)) + \frac{1}{2}f(E^{n+1}(r_{n-)})) = I_n - F_{n+1}$. The manager is indifferent.

Knowing the incentive compatible risk choice at n, we can obtain the debt repayment at $n+1$ from the outsiders' participation constraint. This constraint always binds as the manager chooses the lowest possible debt repayment. At n they provide an amount $I_n - E_n$ of financing in return for a debt repayment of F_{n+1}. The risk choice of the manager does not matter (he is indifferent) because he can always issue new safe debt at $n+1$ to repay the maturing debt. As outsiders' always receive the debt repayment, we have $F_{n+1} = I_n - E_n$.

Using the expression for the debt repayment F_{n+1}, we know the book value of equity at n: $E_n = I_n - F_{n+1}$. This is also the market value of equity at n, which we derived earlier.

As we are in Case 1, the book value of equity at n must be large enough, $E_n \geq 2\gamma$. We summarize Case 1 in the following Lemma:

Lemma 1 *If the book value of equity at n is $E_n \geq 2\gamma$ then the manager is indifferent between the safe and the risky investment, $c_n = \{s_n, r_n\}$, and the market value of equity at n is equal to the book value at n.*

Case 2: $I_n - 2\gamma < F_{n+1} \leq I_n - \gamma$

In this case, the refinancing need is higher than in Case 1. If the manager chooses the risky investment and he fails, then the refinancing need is so large he can no longer issue safe debt in the next period.

Again, we show the market value of equity at $n+1$ when at n the manager chooses the safe strategy, when he chooses the risky investment and succeeds, and when he chooses the risky investment and fails.

$$\begin{aligned} E_{n+1}(s_n) = I_n - F_{n+1} \geq \gamma \quad &\overset{P1}{\Rightarrow} \quad f(E_{n+1}(s_n)) = I_n - F_{n+1} \\ E_{n+1}(r_n^+) = I_n + \gamma - F_{n+1} \geq 2\gamma \quad &\overset{P1}{\Rightarrow} \quad f(E^{n+1}(r_n+_)) = I_n + \gamma - F_{n+1} \\ E_{n+1}(r_n^-) = I_n - \gamma - F_{n+1} \in [0,\gamma) \quad &\overset{P2}{\Rightarrow} \quad f(E^{n+1}(r_n-_)) < I_n - \gamma - F_{n+1} \end{aligned} \tag{34}$$

The new element is in the third line. If the manager chooses the risky strategy at n, the book value at $n+1$ falls below γ when there is failure. Then Property 2 applies and the market value of equity at $n+1$ is less than the book value at $n+1$.

If the manager chooses the safe investment at n, the expected market value of equity at n is equal to the market value at $n+1$, $I_n - F_{n+1}$. If he chooses the risky investment at n, the expected market value of equity at n is $\frac{1}{2}f(E_{n+1}(r_n^+)) + \frac{1}{2}f(E_{n+1}(r_n^-)) < I_n - F_{n+1}$. The manager therefore prefers the safe investment at n.

As the manager prefers the safe investment at n, the outsiders know he can issue safe debt at $n+1$ (see (34)), always repay F_{n+1}, and hence, $F_{n+1} = I_n - E_n$.

Using the expression for the debt repayment F_{n+1}, we know the book value of equity at n: $E_n = I_n - F_{n+1}$. This is also the market value of equity at n (all under the incentive-compatible choice of the safe investment).

As we are in Case 2, the book value of equity at n must be such that $E_n \in [\gamma, 2\gamma)$. We summarize Case 2 in the following Lemma:

Lemma 2 *If the book value of equity at n is $E_n \in [\gamma, 2\gamma)$ then the manager chooses the safe investment, $c_n = s_n$, and the market value of equity at n is equal to the book value at n.*

Case 3: $I_n - \gamma < F_{n+1} \leq I_n$

The refinancing need is even higher than in Case 2. Now, the risky investment becomes attractive because if it fails, the manager cannot issue new debt to repay the maturing debt but his downside is limited through limited liability. If the risky investment, however, succeeds, the manager can issue safe debt in the next period.

$$\begin{aligned} E_{n+1}(s_n) = I_n - F_{n+1} \in [0,\gamma) \quad &\overset{P2}{\Rightarrow} \quad f(E_{n+1}(s_n)) < I_n - F_{n+1} \\ E_{n+1}(r_n^+) = I_n + \gamma - F_{n+1} \geq \gamma \quad &\overset{P1}{\Rightarrow} \quad f(E_{n+1}(r_n^+)) = I_n + \gamma - F_{n+1} \\ E_{n+1}(r_n^-) = I_n - \gamma - F_{n+1} < 0 \quad &\overset{P3}{\Rightarrow} \quad f(E_{n+1}(r_n^-)) = 0 \end{aligned}$$

If the manager chooses the safe investment at n, he cannot issue safe debt at $n+1$ and hence, the market value of equity at $n+1$ is below the book value. If he chooses the risky investment and it succeeds, he can issue safe debt at $n+1$ and the market value of equity is equal to the book value of equity. But if the risky investment fails, then the book value of equity at $n+1$ is negative. With a negative book value of equity, the manager is unable to issue any new debt at $n+1$. He cannot repay the maturing debt F_{n+1}, defaults at $n+1$, and the market value of equity at $n+1$ drops to zero (Property 3, which applies because $E^-_{n+1}(r_n) < E^{min}_{n+1}$ for sure).

When the manager chooses the risky investment at n then the market value of equity at n is $\frac{1}{2}f(E_{n+1}(r^+_n)) + \frac{1}{2}f(E_{n+1}(r^-_n)) = \frac{1}{2}(I_n + \gamma - F_{n+1})$. This is larger than $I_n - F_{n+1}$ because in Case 3 we have $F_{n+1} > I_n - \gamma$. Choosing the safe investment at n is no longer incentive compatible for the manager.

Outsiders know that the manager chooses the risky investment at n and hence, know they will not be repaid at $n+1$ if the risky investment fails. The manager cannot raise new debt after failure and therefore defaults. The debt repayment F_{n+1} only occurs after success, when the manager can issue new safe debt. For outsiders to break even, we have $I_n - E_n = \frac{1}{2}F_{n+1} + \frac{1}{2}(I_n - \gamma - b)$ or $F_{n+1} = I_n - 2E_n + \gamma + b$.

Using the expression for the debt repayment F_{n+1}, we know the book value of equity at n is $E_n = \frac{1}{2}(I_n + \gamma - F_{n+1} + b)$ while the market value of equity at n is $\frac{1}{2}(I_n + \gamma - F_{n+1})$. The market value is lower because the manager bears the expected cost of bankruptcy $\frac{1}{2}b$. Note that the market value at n is always strictly positive in Case 3 (at the maximal debt repayment $F_{n+1} = I_n$ the market value is $\frac{1}{2}\gamma$).

As we are in Case 3, the book value of equity at n must be such that $E_n \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma + \frac{1}{2}b)$

Lemma 3 *If the book value of equity at n is $E_n \in [\frac{1}{2}\gamma + \frac{1}{2}b, \gamma + \frac{1}{2}b)$ then the manager chooses the risky investment, $c_n = r_n$, and the market value of equity at n is less than the book value at n (but still strictly positive).*

Case 4: $I_n < F_{n+1}$

In case 4, the repayment is so large that the manager can, possibly, only issue new debt at $n+1$ if he chose the risky investment at n and it succeeded.

$$
\begin{aligned}
&E_{n+1}(s_n) = I_n - F_{n+1} && < 0 && \overset{P3}{\Rightarrow} && f(E_{n+1}(s_n)) = 0 \\
&E_{n+1}(r^+_n) = I_n + \gamma - F_{n+1} && \begin{cases} \in (E^{min}_{n+1}, \gamma) & \overset{P2}{\Rightarrow} \quad f(E_{n+1}(r^+_n)) < I_n + \gamma - F_{n+1} \\ \leq E^{min}_{n+1} & \overset{P3}{\Rightarrow} \quad f(E_{n+1}(r^+_n)) = 0 \end{cases} \\
&E_{n+1}(r^-_n) = I_n - \gamma - F_{n+1} && < 0 && \overset{P3}{\Rightarrow} && f(E_{n+1}(r^-_n)) = 0
\end{aligned}
$$

We need to distinguish two cases. If $I_n + \gamma - F_{n+1} > E^{min}_{n+1}$, the manager prefers the risky strategy. Note that the market value of equity at $n+1$ after success of the risky investment is strictly positive because $E^{min}_{n+1} \geq 0$.

The outsiders know the manager chooses the risky investment and can raise new (risky) debt $n+1$ if he succeeds. For outsiders to break even we have $I_n - E_n = \frac{1}{2}F_{n+1} + \frac{1}{2}(I_n - \gamma - b)$ or $F_{n+1} = I_n - 2E_n + \gamma + b$. The book value of equity at n therefore is $E_n = \frac{1}{2}(I_n + \gamma - F_{n+1} + b)$ while the market value of equity at n is $\frac{1}{2}(I_n + \gamma - F_{n+1}) > 0$. The market value is lower because the manager bears the expected cost of bankruptcy $\frac{1}{2}b$. As we are in Case 4 with $F_{n+1} > I$, we have $E_n < \frac{1}{2}(\gamma + b)$.

If $I_n + \gamma - F_{n+1} \leq E^{min}_{n+1}$, the manager is indifferent between the safe and risky investment. In both cases he is unable to issue new debt at $n+1$ and hence, cannot continue. Note that the manager cannot issue new debt at $n+1$ even though the book value of equity at n can be positive. Outsiders know that the manager will not be able to repay the debt at $n+1$ and hence do not provide funding at n.[12]

Knowing the debt repayment F_{n+1}, the condition $I_n + \gamma - F_{n+1} > E^{min}_{n+1}$ becomes $E_n > \frac{1}{2}(E^{min}_{n+1} + b)$. When the book value of equity at n is equal or below this threshold, then it is not possible to issue debt at n and the market value of equity at n drops to zero. We therefore have

$$E^{min}_n = \frac{1}{2}(E^{min}_{n+1} + b)$$

Furthermore, in last (decision) period $n = N-1$ we have $E_n = \frac{1}{2}b$ (see Proposition 1). In the second to last period $n = N-2$ we then have $E^{min}_n = \frac{1}{2}\left(\frac{1}{2}b + b\right) = \frac{3}{4}b$ and in the third to last period $n = N-3$ we have $E^{min}_n = \frac{1}{2}\left(\frac{3}{4}b + b\right) = \frac{7}{8}b$, and so on. The minimum book value of equity below which a manager cannot issue debt is therefore given by

$$E^{min}_n = \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b \tag{35}$$

Lemma 4 *Case i): If the book value of equity at n is* $E_n \in (E^{min}_{n+1}, \frac{1}{2}\gamma + \frac{1}{2}b)$ *then the manager chooses the risky investment,* $c_n = r_n$*, and the market value of equity at n is less than the book value at n (but still strictly positive). Case ii): If the book value of equity at n is* $E_n \leq E^{min}_{n+1}$ *then the manager cannot obtain financing and stops. The market value of equity at n is zero.*

Comparing the cases: optimal choice c^*_n as a function of E_n

Lemma 1 to 4 describe the optimal choice c_n and the resulting market value of equity as a function of the book value of equity at n. However, the intervals for E_n partially overlap across Lemma 1 to 4.

First, for $E_n \geq 2\gamma$, there is no issue of overlap. Lemma 1 then tells us that the manager is indifferent between the safe and risky investment $c^*_n = \{s, r\}$ and the market value of equity at n (under the optimal risk choice) is $f(E_n) = E_n$.

Second, the interval of Lemma 2 partially overlaps with the one of Lemma 3 because $\gamma < \gamma + \frac{1}{2}b$ (but not with the one of Lemma 4 because our assumption $b < \gamma$ ensures that $\gamma > \frac{1}{2}\gamma + \frac{1}{2}b$). The choice of safe investment (Lemma 2) dominates the choice of the risky investment (Lemma 3) because the market value of equity is higher with safe the investment. Hence, when $E_n \in [\gamma, 2\gamma)$, $c^*_n = s$ and $f(E_n) = E_n$.

Third, in the interval $E_n \in (\left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b, \gamma)$ either Lemma 3 or Lemma 4 (case i) applies. In both cases manager chooses the risky investment $c^*_n = r$ and the market value of equity is less than the book value, $f(E_n) < E_n$.

Finally, if $E_n \leq \left[1 - \left(\frac{1}{2}\right)^{N-n}\right] b$ then Lemma 4 (case ii) applies. The manager cannot obtain financing at n. He has to stop operating and the market value of equity drops to zero $f(E_n) = 0$. Note that only in this case is the market value of equity at n equal to zero.

It is straightforward to see that the three properties of the market value of equity at $n+1$ (as a function of book equity at the same time and optimal investment choices

[12] Outsiders provide $I_n - E_n$ and expect to obtain $I_n - b$. We will show below that $E_n < b$ as required in this case.

from that time onward), $f(E_{n+1})$, carry over to the market value of equity in the previous period n, $f(E_n)$. Moreover, the three properties of the market value of equity hold in the last (decision) period $n = N - 1$, $f(E_{N-1})$. Hence, the three properties always hold, which validates our initial assumption.

B Empirical analysis

In this appendix, we provide details on the empirical analysis to create figure 1.

Data set:
The analysis is based on annual firm-level data for the 2010-2019 period using two different data sets. The U.S. sample is based on Compustat data and covers publicly listed firms. The European sample is based on Amadeus and covers publicly listed and private firms with limited liability from the largest ten European countries (Germany, United Kingdom, France, Italy, Spain, Netherlands, Poland, Sweden, Belgium, Austria). We apply the following filters to both data sets:

i. we require non-missing values for total assets, shareholder funds, and Earnings before Interest and Taxes (EBIT).

ii. we drop financial and public sector firms (Compustat: SIC codes from 6000-6999 and 4900-4949, Amadeus: two-digit NACE codes 64, 70, 81-82, 84-88, 90-92, 94, 97-99).

iii. we restrict the analysis to firms with total assets larger than USD 10mn (Compustat), and to the size categories "very large" and "large" in Amadeus (defined as having either EUR 20mn in assets, or EUR 10mn in sales, or more than 150 employees).

iv. for each firm-year, we only keep one financial statement (we drop unconsolidated statements if consolidated statements are available, and we keep the latest financial statement for the few cases where firms have multiple financial statements in one year).

This results in 36,328 firm-year observations for the U.S. and 1,382,044 firm-year observations for Europe.

Variable definitions:
We define *Equity/Assets* as *Shareholder Funds* (Compustat: *ceq*, Amadeus: *shfd*) divided by *Total Assets* (Compustat: *at*, Amadeus: *toas*). *Return on Assets* is defined as *Earnings before Interest and Taxes* (Compustat and Amadeus: *ebit*) divided by *Total Assets*. We winsorize *Equity/Assets* and *Return on Assets* at the 1% and 99% level.

Creation of figure 1:
For the U.S., we calculate the variable *Deviation of Equity/Assets from industry-year mean* as *Equity/Assets* minus the simple unweighted mean of *Equity/Asset* in the same industry in the same year. The industry is defined using the two-digit SIC code. In Europe, we proceed similarly, with two differences: we use the same industry and country to derive the variable *Deviation of Equity/Assets from industry-country-year mean*, and we define industries using the two-digit NACE code.

We then create percentiles by *Deviation of Equity/Assets from industry-year mean* (U.S) and *Deviation of Equity/Assets from industry-country-year mean* (Europe). For each percentile, we determine the cross-sectional standard deviation as well as the difference between the 75^{th} and the 25^{th} percentile of the subsequent years *Return on Assets*. We use the subsequent years *Return on Assets* so that our measure of leverage precedes our measure of risk-taking.

Robustness:
The U-shaped pattern in figure 1 is robust to: i) including financials and public sector firms, ii) using just the first or the second half of our sample period (2010-2014, 2015-2019), ii) including smaller firms below the USD 10 million threshold (results are noisier when we include very small firms with assets below USD 1 million), iv) controlling for size (log(total assets)), profitability, and age (logarithm of years since IPO for Compustat, logarithm of years since foundation for Amadeus).

C Tax shield of debt

We are interested in exploring the effect of leverage on risk-taking. Thus, our baseline model assumes that the outsiders' financing comes in the form of a debt claim. The agency costs of debt financing that arise in our model can be avoided by using equity funding. In this appendix, we illustrate that debt is used in equilibrium – and risk-avoiding behavior continues to hold for firms with medium leverage – if the tax shield is sufficiently high.

To illustrate this point, we use the example from section 1. We model the tax shield of debt in a reduced form: when outsiders provide financing of X in form of a debt claim, the participation constraint requires an expected payoff to outsiders of X; while an equity contract requires an expected payoff to ousiders of $(1 + \tau) \cdot X$. The larger τ, the larger the tax shield of debt.

We first consider the one period example disucssed in section 1.1. Recall that the insider needs to raise outside financing of 70 to generate a no-frictions value of 80. The safe project yields 80 while the risky projects yields 60 or 100 with equal probability, and bankruptcy costs are 10. With debt, the insider chooses the risky project in equilibrium and the expected payoff to the insider is 5, which reflects the fact that the insider needs to bear the expected bankruptcy costs ($\frac{1}{2} \cdot 10 = 5$). Now assume the outsiders provide equity funding. The outsiders' participation constraint requires an expected payoff of $(1 + \tau)70$, resulting in an insider equity value of $80 - (1 + \tau)70 = 10 - \tau \cdot 70$. Thus, in the one period example, debt will be used if the deadweight costs of bankruptcy (5) are lower than the tax shield of debt $(\tau \cdot 70)$, or $\tau \geq 5/70 = 7.1\%$.

In the two period example, recall that the insider needs to raise outside financing of 30 to generate a no-frictions value of 100. Bankruptcy does not occur on equilibrium with debt financing (see section 1.2) because the insider prefers the safe project in the first period. Thus, debt financing will always be used in the presence of a tax shield of debt when safe and risky projects have the same expected payoff. Now assume that the risky strategy's payoff in the first period increases by Δ, so that the expected payoff of the risky project is $100 + \Delta$ and Δ fulfills the conditions of Corollary 2. If outsiders provide equity financing, the insiders expected payoff is now $100+\Delta-(1-\tau)70 = 30+\Delta-\tau 70$ and there is a trade-off: equity implies more efficient investment $(+\Delta)$, but carries a tax disadvantage $(-\tau \cdot 70)$. Outsiders will provide debt financing in equilibrium if $\tau \cdot 70 \geq \Delta \Leftrightarrow \tau \geq \frac{\Delta}{70}$. Therefore, if the debt tax shield is sufficiently large, debt is used in equilibrium and the risk-avoiding behavior continues to hold for firms with medium leverage.

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