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On Two-Way Communication in Cellular Automata with a Fixed Number of Cells

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Abstract

The effect of adding two-way communication to k cells one-way cellular automata (kC-OCAs) on their size of description is studied. kC-OCAs are a parallel model for the regular languages that consists of an array of k identical deterministic finite automata (DFAs), called cells, operating in parallel. Each cell gets information from its right neighbor only. In this paper, two models with different amounts of two-way communication are investigated. Both models always achieve quadratic savings when compared to DFAs. When compared to a one-way cellular model, the result is that minimum two-way communication can achieve at most quadratic savings whereas maximum two-way communication may provide savings bounded by a polynomial of degree k.

1 Introduction

The descriptional complexity of abstract machines is a field of theoretical computer science in which the size of description of certain objects is studied. One main question is how the size of description varies when an object is described by several descriptional systems. One early and basic result is from Meyer and Fischer in [9] who proved that there exists an infinite sequence of regular languages $(L_n)_{n\geq 1}$ such that each L_n is recognized by an n-state nondeterministic finite automaton (NFA) and each equivalent deterministic finite automaton (DFA) needs at least 2^n states. Since an NFA can be converted to a DFA with at most 2^n states by the subset construction, their result shows that there is a tight exponential trade-off between NFAs and DFAs. In [9] it is additionally proven that the trade-off between two descriptional systems may not be bounded by any recursive function. They showed such a non-recursive trade-off between context-free grammars and DFAs.

In preceding papers some research on the descriptional complexity of cellular automata was started. In [5] it is proven that there are non-recursive trade-offs between different models of unrestricted cellular automata. A cellular automaton can be described as a set of many identical DFAs, called cells, which are arranged in a line. The next state of each cell depends on the current state of the cell and the current states of a bounded number of neighboring cells. The transition rule is applied synchronously to each cell at the same time. One simple model is the realtime one-way cellular automaton (realtime-OCA). Here the local transition rule depends only on the state of the cell and the neighboring cell to the right. Furthermore, the input is processed in realtime. It is known [5] that for unrestricted cellular automata almost all decidability questions are undecidable and not even semidecidable and that there exist neither pumping lemmas nor minimization algorithms for these automata. Thus, it is obvious to restrict the general model. This is done in [6] where a one-way cellular automaton with a fixed number k of cells (kC-OCA) is studied whereas an unrestricted OCA is provided with as many cells as the input is long. The generative capacity of the restricted model is then reduced to the set of regular languages. The trade-off between kC-OCAs and DFAs is bounded by a polynomial of degree k. The upper bound when converting an n-state DFA to a kC-OCA is $n+1$ and this upper bound is known to be tight [7]. That is, there are regular languages which are "inherently sequential," since both a sequential and a parallel model need almost the same number of states.

In this paper, we want to continue the study of kC-OCAs and we look at the effect of adding two-way communication on the size of description. We investigate two generalized models. In the one model only the rightmost cell is allowed to communicate with its left neighbor (kC-OCA$_t$) and in the other model all cells are allowed to communicate with their left neighbors (kC-CA). The main results are as follows. Even one two-way communication cell is sufficient so that every n-state DFA can be equivalently converted to a kC-OCA$_t$ with $O(\sqrt{n})$ states. Hence, "two-way is always better than one-way". The "inherently sequential" languages mentioned above can be accepted by a kC-CA with n states whereas any DFA or kC-OCA needs at least $O(n^k)$ states. Thus, two-way communication always provides quadratical savings when compared with DFAs and may provide polynomial savings of degree k when compared with kC-OCAs.

2 Preliminaries and Definitions

Let Σ^* denote the set of all strings over the finite alphabet Σ, ϵ the empty string, and $\Sigma^+ = \Sigma^* \setminus \{\epsilon\}$. By $|w|$ we denote the length of a string w and by $|M|$ the number of states of a DFA M. Let REG denote the family of regular languages. In this paper we do not distinguish whether a language L contains the empty string ϵ or not. In other words, we identify L with $L \setminus \{\epsilon\}$. We assume that the reader is familiar with the common notions of formal language theory as presented in [3]. We say that two automata are equivalent if both accept the same language. Concerning the notations and definitions for kC-OCAs, kC-OCA$_t$s, and kC-CAs we adapt the notations of the unrestricted model as introduced in [4] to our needs. More detailed information on unrestricted cellular automata can be found in [4], more detailed information on kC-OCAs may be found in [6].

Definition: A k cells one-way cellular automaton (kC-OCA) A is a tuple $A = (Q, \Sigma, \sqcup, \triangledown, k, \delta_r, \delta, F)$, where

1. $Q \neq \emptyset$ is the finite set of cell states,
2. Σ is the input alphabet,
3. $\sqcup \notin Q \cup \Sigma$ is the quiescent state,

4. $\triangledown \notin Q \cup \Sigma$ is the end-of-input symbol,
5. k is the number of cells,
6. $F \subseteq Q$ the set of accepting cell states and
7. $\delta_r : (Q \cup \{\sqcup\}) \times (\Sigma \cup \{\triangledown\}) \to Q \cup \{\sqcup\}$ is the local transition function for the rightmost cell. We require that only the pair $(\sqcup, \triangledown)$ is mapped to $\sqcup$.
8. $\delta : (Q \cup \{\sqcup\}) \times (Q \cup \{\sqcup\}) \to Q \cup \{\sqcup\}$ is the local transition function for the other cells. We require that only the pair $(\sqcup, \sqcup)$ is mapped to $\sqcup$.

A k cells one-way cellular automaton with two-way communication cell (kC-OCA$_t$) A is identical to a kC-OCA except that 7. is redefined as follows.

7'. $\delta_r : (Q \cup \{\sqcup\}) \times (Q \cup \{\sqcup\}) \times (\Sigma \cup \{\triangledown\}) \to Q \cup \{\sqcup\}$ is the local transition function for the rightmost cell. We require that only the tuple $(\sqcup, \sqcup, \triangledown)$ is mapped to $\sqcup$.

A k cells two-way cellular automaton (kC-CA) A is identical to a kC-OCA except that 7. and 8. are redefined. Since the leftmost cell has no left neighbor, an additional boundary state $\# \notin Q \cup \Sigma$ is needed.

7". $\delta_r : (Q \cup \{\sqcup, \#\}) \times (Q \cup \{\sqcup\}) \times (\Sigma \cup \{\triangledown\}) \to Q \cup \{\sqcup\}$ is the local transition function for the rightmost cell. We require that only the tuples $(\sqcup, \sqcup, \triangledown)$ and $(\#, \sqcup, \triangledown)$ are mapped to $\sqcup$.

8". $\delta : (Q \cup \{\sqcup, \#\}) \times (Q \cup \{\sqcup\}) \times (Q \cup \{\sqcup\}) \to Q \cup \{\sqcup\}$ is the local transition function for the other cells. We require that only the tuples $(\sqcup, \sqcup, \sqcup)$ and $(\#, \sqcup, \sqcup)$ are mapped to $\sqcup$.

The restricted models work similar to the unrestricted model. The next state of each cell depends on the current state of the cell itself and its right neighbor. The next state of the rightmost cell in kC-OCA$_t$s and all cells in kC-CAs additionally depend on the state of the left neighboring cell. The transition rule is applied synchronously to each cell at the same time. In contrast to unrestricted cellular automata the input is processed as follows. In the beginning all cells are in the quiescent state. The rightmost cell is the communicating cell to the input. At every time step one input symbol is processed by the rightmost cell. All other cells behave as described. The input is accepted, if the leftmost cell enters an accepting state. Since the minimal time to read the input and to send all information from the rightmost cell to the leftmost cell is the length of the input plus k, we input a special end-of-input symbol $\triangledown$ to the rightmost cell after reading the input. The size of an automaton $A = (Q, \Sigma, \sqcup, \triangledown, k, \delta_r, \delta, F)$ is defined as the number of states in Q, i.e. $|A| = |Q|$. To simplify matters we identify the cells by positive integers.

A configuration of a kC-OCA (kC-OCA$_t$, kC-CA) at some time step $t \geq 0$ is a pair (c_t, w_t) where $w_t \in \Sigma^*$ denotes the remaining input and c_t is a description of the k cell states, formally a mapping $c_t : \{1, \ldots, k\} \to Q \cup \{\sqcup\}$. We consider the input string $u = u_1 \ldots u_n$: The initial configuration at time 0 is defined by $c_0(i) = \sqcup$, $1 \leq i \leq k$ and $w_0 = u$.

During a computation the kC-OCA (kC-OCA$_t$, kC-CA) steps through a sequence of configurations whereby successor configurations are computed according to the global transition function Δ: Let (c_t, w_t), $t \geq 0$, be a configuration, then its successor configuration is defined as follows:

For kC-OCAs:

$$
\begin{aligned}
(c_{t+1}, w_{t+1}) \;=\; & \Delta(c_t, w_t) \iff \\
& c_{t+1}(i) = \delta(c_t(i), c_t(i+1)), i \in \{1, \dots, k-1\} \\
& c_{t+1}(k) = \delta_r(c_t(k), x)
\end{aligned}
$$

For kC-OCA$_t$s:

$$
\begin{aligned}
(c_{t+1}, w_{t+1}) \;=\; & \Delta(c_t, w_t) \iff \\
& c_{t+1}(i) = \delta(c_t(i), c_t(i+1)), i \in \{1, \dots, k-1\} \\
& c_{t+1}(k) = \delta_r(c_t(k-1), c_t(k), x)
\end{aligned}
$$

For kC-CAs:

$$
\begin{aligned}
(c_{t+1}, w_{t+1}) \;=\; & \Delta(c_t, w_t) \iff \\
& c_{t+1}(1) = \delta(\#, c_t(1), c_t(2)) \\
& c_{t+1}(i) = \delta(c_t(i-1), c_t(i), c_t(i+1)), i \in \{2, \dots, k-1\} \\
& c_{t+1}(k) = \delta_r(c_t(k-1), c_t(k), x)
\end{aligned}
$$

where $x = \nabla$ and $w_{t+1} = \epsilon$ if $w_t = \epsilon$, and $x = x_1$ and $w_{t+1} = x_2 \dots x_n$ if $w_t = x_1 x_2 \dots x_n$. Thus, Δ is induced by δ_r and δ.

An input string u is accepted by a kC-OCA (kC-OCA$_t$, kC-CA) if at some time step during its computation the leftmost cell enters an accepting state from the set of accepting states $F \subseteq Q$.

Definition: Let $A = (Q, \Sigma, \sqcup, \nabla, k, \delta_r, \delta, F)$ be a kC-OCA (kC-OCA$_t$, kC-CA).

1. A string $u \in \Sigma^+$ is accepted by A if there exists a time step $i \in \mathbb{N}$ such that $c_i(1) \in F$ holds for the configuration $(c_i, w_i) = \Delta^i((c_0, u))$.
2. $T(A) = \{u \in \Sigma^+ \mid u \text{ is accepted by } A\}$ is the language accepted by A.
3. If all $u \in T(A)$ are accepted within $|u|+k$ time steps, we say that A accepts in realtime. $\mathcal{L}(k\text{C-OCA}) = \{L \mid L \text{ is accepted by a realtime-}k\text{C-OCA}\}$. $\mathcal{L}(k\text{C-OCA}_t)$ and $\mathcal{L}(k\text{C-CA})$ are defined analogously.

We investigate in this paper the descriptional systems DFA, kC-OCA, kC-OCA$_t$, and kC-CA. As descriptional complexity measure for these automata we count the number of states. Since a kC-OCA (kC-OCA$_t$, kC-CA) is composed of k identical cells, this measure is reasonable. The definitions of upper and lower bounds follow the presentation in [1].

We say that a function $f : \mathbb{N} \to \mathbb{N}$, $f(n) \geq n$ is an *upper bound* for the blow-up in complexity when changing from one descriptional system D_1 to another system D_2, if every description $M \in D_1$ of size n has an equivalent description $M' \in D_2$ of size at most $f(n)$.

We say that a function $g : \mathbb{N} \to \mathbb{N}$, $g(n) \geq n$ is a *lower bound* for the trade-off between two descriptional systems D_1 and D_2, if there is an infinite sequence $(L_i)_{i \in \mathbb{N}}$ of pairwise distinct languages L_i such that for all $i \in \mathbb{N}$ there is a description $M \in D_1$ for L_i of size n and every description $M' \in D_2$ for L_i is at least of size $g(n)$.

3 Comparing kC-CAs and kC-OCA$_t$ with DFAs

In [6], Lemma 2 it is shown that every n-state kC-OCA can be converted to an equivalent DFA having $O(n^k)$ states. The idea of the construction is to build the Cartesian product of k cells and to define the accepting states suitably. It can be easily observed that the same construction can be used for kC-CAs and kC-OCA$_t$s. The only difference is to define the transition function of the DFA according to the new transition functions δ and δ_r. However, this can be realized without increasing the number of states. Lemma 4 in [6] shows that every kC-OCA accepting $L_{n,k} = \{a^m \mid m \geq n^k + n^{k-1} + 1\}$ needs at least $n+1$ states. The essential observation is that $n^k + n^{k-1} + 1$ distinct configurations on k cells have to be distinguished which requires $n+1$ states. The same reasoning holds for kC-CAs and kC-OCA$_t$, because the ability of cells to see its left neighbor obviously does not reduce the number of configurations which have to be distinguished. Hence we obtain the following two lemmas:

Lemma 1 *Every n-state kC-CA (kC-OCA$_t$) can be converted to an equivalent DFA having $O(n^k)$ states.*

Lemma 2 *Every kC-CA (kC-OCA$_t$) accepting $L_{n,k} = \{a^m \mid m \geq n^k + n^{k-1} + 1\}$ needs at least $n+1$ states.*

It is known [6] that an n-state DFA can be converted to an equivalent $(n+1)$-state kC-OCA. This bound is known to be tight, i.e., there are languages where the parallelism provided in terms of additional cells does not help to reduce the size of description. This situation changes in case of kC-CAs and kC-OCA$_t$s. Here, these models can always achieve savings in size when compared to DFAs.

Lemma 3 *Every n-state DFA M can be converted to a kC-OCA$_t$ (kC-CA) A such that $T(A) = T(M)$ and $|A| \leq \lceil\sqrt{n}\rceil(|\Sigma|+1) + 2|\Sigma| + 1 = O(\sqrt{n})$ where $T(M) \subseteq \Sigma^*$.*

Proof: Let M be an n-state DFA accepting a language over the alphabet Σ. Let Q denote the set of states, $F \subseteq Q$ the set of accepting states, 0 the initial state, and δ the transition function. We construct a kC-OCA$_t$ by simulating M in the last two cells from the right. In detail, the state set Q is encoded by two bits of a $\lceil\sqrt{n}\rceil$-ary alphabet. This encoding is then used to compute the first bit of M's actual state in the last but one cell and the second bit in the rightmost cell, respectively. After reading the input, we check whether an accepting state of M has been computed in the last two cells and we then send an accepting state with maximum speed to the left. Otherwise, the computation is blocked.

Let $Q = \{0, 1, 2, \ldots, n-1\}$ and $\ell = \lceil\sqrt{n}\rceil - 1$. Let

$$Q_c = \{00, 01, 02, \ldots, 0\ell, 10, 11, 12, \ldots, 1\ell, \ldots, \ell 0, \ell 1, \ell 2, \ldots, \ell\ell\}$$

If $|Q_c| > |Q|$, we delete some endmost elements of Q_c so that $|Q_c| = |Q|$. Let $\phi : Q \to Q_c$ be any, but fixed bijection between Q and Q_c such that $\phi(0) = 00$. Let

$[\cdot]_1, [\cdot]_2 : Q \to \{0, 1, 2, \ldots, \ell\}$ define projections from states $q \in Q$ on the first and second bits of their encodings, respectively. E.g.: $q = 1, \phi(q) = 01, [q]_1 = 0, [q]_2 = 1$.

Now, $Q' = (\{0, 1, 2, \ldots, \ell\} \times (\Sigma \cup \{h\})) \cup \{g\} \cup \Sigma \cup \Sigma'$ where Σ' is a "primed" version of Σ and $\{g, h\} \cap (\{0, 1, 2, \ldots, \ell\} \cup \Sigma \cup \Sigma') = \emptyset$. Let $\psi : \Sigma \to \Sigma'$ be the bijection defined by $\psi(\sigma) = \sigma'$ for all $\sigma \in \Sigma$.

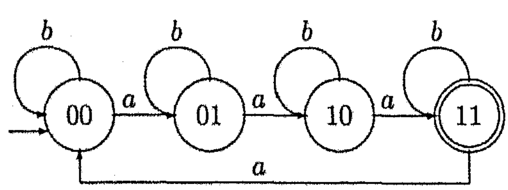
Let $A = (Q', \Sigma, \sqcup, \triangledown, k, \delta'_r, \delta', \{g\})$ and for $i, j \in \{0, 1, 2, \ldots, \ell\}$, $\sigma, \tau, \tau_l, \tau_r \in \Sigma$, and $\tau', \tau'_r \in \Sigma'$:

$$\begin{aligned}
\delta'_r(\sqcup, \sqcup, \sigma) &= \sigma && (1)\\
\delta'_r(\sqcup, \tau, \sigma) &= \psi(\sigma) && (2)\\
\delta'_r((i, \tau_l), \tau', \sigma) &= ([\delta(0, \tau_l)]_2, \sigma) && (3)\\
\delta'_r((j, \tau_l), (i, \tau), \sigma) &= ([\delta(\phi^{-1}(j, i), \tau_l)]_2, \sigma) && (4)\\
\delta'_r(\sqcup, \tau, \triangledown) &= \begin{cases} g & : \ \delta(0, \tau) \in F \\ \tau & : \ \text{otherwise} \end{cases} && (5)\\
\delta'_r((i, \tau_l), \tau', \triangledown) &= ([\delta(\phi^{-1}(i, 0), \tau_l)]_2, h) && (6)\\
\delta'_r((j, \tau_l), (i, \tau), \triangledown) &= ([\delta(\phi^{-1}(j, i), \tau_l)]_2, h) && (7)\\
\delta'(\sqcup, \tau) &= (0, \tau) && (8)\\
\delta'((0, \tau), \tau'_r) &= ([\delta(0, \tau)]_1, \psi^{-1}(\tau'_r)) && (9)\\
\delta'((i, \tau), (j, \tau_r)) &= ([\delta(\phi^{-1}(i, j), \tau)]_1, \tau_r) && (10)\\
\delta'((i, \tau), (j, h)) &= \begin{cases} g & : \ \delta(\phi^{-1}(i, j), \tau) \in F \\ (i, \tau) & : \ \text{otherwise} \end{cases} && (11)\\
\delta'(q, g)) &= g; \ q \in Q' \cup \{\sqcup\} && (12)\\
\delta'(\sqcup, (i, \tau_r)) &= (i, \tau_r) && (13)
\end{aligned}$$

The remaining transitions are undefined. Rather than to give a formal proof of $T(A) = T(M)$, we refer to the following Example 1. The number of states in Q' may be estimated as: $|Q'| \leq \lceil \sqrt{n} \rceil (|\Sigma| + 1) + 2|\Sigma| + 1 = O(\sqrt{n})$.

The construction for kC-CAs is essentially the same by ignoring any information from the left in all cells except the rightmost cell. $\square$

Example 1: Consider the following DFA M:



A 4C-OCA$_t$ works as follows. The last two cells are initialized in the first two time steps. Then the binary encoding of M's states can be found in the first component of the last two cells. After reading the end-of-input symbol for the first time the rightmost cell is marked with a special symbol h. In the next time step the last but one cell processes the last input symbol stored in its second component and sends an accepting state g to the left if an accepting state of M has been computed; otherwise it remains in its state and the computation is blocked. The processing of $aaab$ and $aababa$ may be found in the following tables.

$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$aaab$
$\sqcup$	$\sqcup$	$\sqcup$	a	aab
$\sqcup$	$\sqcup$	$(0,a)$	a'	ab
$\sqcup$	$(0,a)$	$(0,a)$	$(1,a)$	b
$(0,a)$	$(0,a)$	$(1,a)$	$(0,b)$	ϵ
$(0,a)$	$(1,a)$	$(1,b)$	$(1,h)$	ϵ
$(1,a)$	$(0,b)$	g	$(1,h)$	ϵ
$(1,b)$	g	g	$(1,h)$	ϵ
g	g	g	$(1,h)$	ϵ

$\sqcup$	$\sqcup$	$\sqcup$	$\sqcup$	$aababa$
$\sqcup$	$\sqcup$	$\sqcup$	a	$ababa$
$\sqcup$	$\sqcup$	$(0,a)$	a'	$baba$
$\sqcup$	$(0,a)$	$(0,a)$	$(1,b)$	aba
$(0,a)$	$(0,a)$	$(1,b)$	$(0,a)$	ba
$(0,a)$	$(1,b)$	$(1,a)$	$(0,b)$	a
$(1,b)$	$(1,a)$	$(1,b)$	$(1,a)$	ϵ
$(1,a)$	$(0,b)$	$(1,a)$	$(1,h)$	ϵ
$(1,b)$	$(0,a)$	$(1,a)$	$(1,h)$	ϵ
$(1,a)$	$(1,a)$	$(1,a)$	$(1,h)$	ϵ
$(0,a)$	$(0,a)$	$(1,a)$	$(1,h)$	ϵ

The next lemma says that the construction given in Lemma 3 is in a way optimal for kC-OCA$_t$s, since the upper bound is proven to be nearly tight. The same reasoning provides a lower bound for kC-CAs. Surprisingly, the lower bound does not depend on the number of cells. That is, whatever number of cells is provided, there are regular languages where at most savings of $O(\sqrt[3]{n/\log n})$ can be achieved. It is an open question whether the construction of Lemma 3 can be improved to $O(\sqrt[3]{n})$ or, alternatively, whether the proof for kC-CAs can be refined to show a lower bound of $O(\sqrt{n})$.

Lemma 4 *There is an infinite sequence of languages $(L_n)_{n \in \mathbb{N}}$ such that each L_n is accepted by an $(n+1)$-state DFA. Every kC-OCA$_t$ accepting L_n needs at least $\Omega(\sqrt{n/\log n})$ states and every kC-CA accepting L_n needs at least $\Omega(\sqrt[3]{n/\log n})$ states.*

Proof: We show that there is a constant c depending on k such that for every $n \in \mathbb{N}$ with $n \geq c$ holds: there is a singleton language $L_n = \{x\}$ with $|x| = n$ that is accepted by a DFA having $n+1$ states and every kC-OCA$_t$ accepting L_n needs at least m states with $m + 1 \geq \sqrt{\frac{n}{(\log n)(|\Sigma|+1)}}$.

In the following we are using an incompressibility argument. More general information on Kolmogorov complexity and the incompressibility method may be found in [8]. Let L_n be a singleton of length n and A a kC-OCA$_t$ accepting L_n. Then $C(L_n|n)$ denotes the minimal size of a program describing L_n and knowing the length n. It is easy to see that the size of this minimal description is lower than or equal to the size of a certain encoding *cod*(A) of A and the size $|P|$ of a program P which describes how a kC-OCA$_t$ is encoded and how a kC-OCA$_t$ describes L_n. Obviously, $|P|$ does not depend on L_n, A, and n. An encoding *cod*(A) of A consists of encodings *cod*(δ) and *cod*(δ_r) of its transition functions, an encoding *cod*(F) of the accepting states, and an encoding *cod*(k) of k.

We choose $n \in \mathbb{N}$ such that $|P| \leq (1/8)n$ and $\log k \leq (1/8)n$. According to Theorem 2.3. of [8] we know that there exists a string x of length n such that $C(x|n) \geq n$.

We set $L_n = \{x\}$ and assume by way of contradiction that there exists a kC-OCA$_t$ A accepting L_n with m states and $m+1 < \sqrt{\frac{n}{(\log n)(|\Sigma|+1)}}$.

Since $|\Sigma| \geq 2$, we can conclude the following two inequalities which will be needed below.

$$(m+1)^2 \log m \leq (1/3)(m+1)^2(|\Sigma|+1)\log m,$$
$$(m+1)\log m \leq (1/6)(m+1)^2(|\Sigma|+1)\log m$$

$$\begin{aligned}
|cod(A)| + |P| &\leq \underbrace{\log m^{(m+1)m}}_{|cod(\delta)|} + \underbrace{\log m^{(m+1)^2(|\Sigma|+1)}}_{|cod(\delta_r)|} + \underbrace{m \log m}_{|cod(F)|} + \underbrace{\log k}_{|cod(k)|} + |P| \\
&\leq (m+1)m\log m + (m+1)^2(|\Sigma|+1)\log m + m\log m + \frac{1}{4}n \\
&\leq (m+1)^2\log m + (m+1)^2(|\Sigma|+1)\log m + (m+1)\log m + \frac{1}{4}n \\
&\leq (\frac{1}{3} + 1 + \frac{1}{6})(m+1)^2(|\Sigma|+1)\log m + \frac{1}{4}n \\
&= \frac{3}{2}(m+1)^2(|\Sigma|+1)\log m + \frac{1}{4}n \\
&< \frac{3}{2}\frac{n}{(\log n)(|\Sigma|+1)}(|\Sigma|+1)\log\left(\left(\frac{n}{(\log n)(|\Sigma|+1)}\right)^{1/2}\right) + \frac{1}{4}n \\
&= \frac{3}{4}\frac{n}{\log n}\log\left(\frac{n}{(\log n)(|\Sigma|+1)}\right) + \frac{1}{4}n \\
&= \frac{3}{4}\frac{n}{\log n}(\log n - \log\log n - \log(|\Sigma|+1)) + \frac{1}{4}n \\
&= \frac{3}{4}n\left(1 - \frac{\log\log n}{\log n} - \frac{\log(|\Sigma|+1)}{\log n}\right) + \frac{1}{4}n \\
&\leq \frac{3}{4}n + \frac{1}{4}n = n
\end{aligned}$$

Hence, $C(L_n|n) \leq |cod(A)| + |P| < n$ which is a contradiction to $C(L_n|n) \geq n$.

The reasoning for kC-CAs is quite the same by considering $m+1 < \sqrt[3]{\frac{n}{(\log n)(|\Sigma|+1)}}$, estimating $|cod(\delta)| \leq \log m^{(m+1)^2 m}$, and replacing $(m+1)^2$ by $(m+1)^3$. □

As consequence of Lemma 1 and Lemma 3 we obtain:

Theorem 1 $\mathcal{L}(kC\text{-}OCA_t) = \mathcal{L}(kC\text{-}CA) = REG$.

4 Comparing kC-OCAs, kC-OCA$_t$s and kC-CAs

4.1 Embedding

Lemma 5 *Every n-state kC-OCA can be converted to an equivalent n-state kC-CA or kC-OCA$_t$. Every n-state kC-OCA$_t$ can be converted to an equivalent n-state kC-CA. All bounds are tight.*

Proof: Obviously, a kC-CA or kC-OCA$_t$ can simulate a kC-OCA without increasing the number of states. By the same token, a kC-CA can simulate a kC-OCA$_t$. In Lemma 2 it is shown that any kC-OCA accepting $L_{n,k}$ needs at least $n+1$ states, since $n^k + n^{k-1} + 1$ states have to be distinguished with k cells. The same reasoning holds for kC-OCA$_t$s and kC-CAs, respectively. Hence, every kC-OCA$_t$ and kC-CA accepting $L_{n,k}$ needs at least $n+1$ states. □

4.2 Upper Bounds

Lemma 6

(a) Every n-state kC-CA can be converted to a kC-OCA with at most $O(n^k)$ states.

(b) Every n-state kC-OCA$_t$ can be converted to a kC-OCA with at most n^2+n states.

(c) Every n-state kC-CA can be converted to a kC-OCA$_t$ with at most $O(n^{\lceil k/2 \rceil})$ states.

Proof: (a) An n-state kC-CA can be converted to a DFA having $O(n^k)$ states due to Lemma 1. This DFA can be converted to a kC-OCA having $O(n^k)$ states applying Lemma 1 from [6]. (b) Let A be an n-state kC-OCA$_t$. We construct a kC-OCA A' which is essentially the same as A except that the rightmost cell has to simulate the last but one cell. Therefore, we consider the Cartesian product of two cells in the rightmost cell. A' works the same way as A except that the last but one cell is additionally simulated in the first component of the rightmost cell. It is easy to see that $T(A') = T(A)$ and $|A'| \leq n^2 + n$. (c) Let A be an n-state kC-CA. To construct a kC-OCA$_t$ A', we have to simulate k cells in the last two cells. Therefore, we consider the Cartesian product of $\lceil k/2 \rceil$ cells in every cell. In the rightmost cell of A', the last $\lceil k/2 \rceil$ cells are simulated and the first $k - \lceil k/2 \rceil$ cells in the last but one cell. Now, A' works the same way as A until the end-of-input symbol is read the first time by the rightmost cell. At this moment, A' checks whether the actual configuration of A, which is encoded in the last two cells, leads to an accepting state in the first cell within the next k time steps when computed in A. If so, an accepting state is sent with maximum speed to the left, otherwise the computation is blocked. It is easy to verify that $T(A') = T(A)$ and $|A'| \leq (n+1)^{\lceil k/2 \rceil} + 2 = O(n^{\lceil k/2 \rceil})$. □

4.3 Lower Bounds

In this section we consider the languages $L_p = \{a^n \mid n \equiv 0 \mod p\}$ where p is a prime number. It is shown in [7] that every kC-OCA accepting $L_p \cdot \{a\}$ needs at least $p+1$ states. The first part of the proof can be easily adapted to show that every kC-OCA accepting L_p needs at least p states.

Lemma 7 *Every kC-OCA accepting L_p needs at least p states.*

The following result from the theory of numbers may be found in [2]:

Theorem 2 (Bertrand's Postulate) *If $n \geq 1$, there is at least one prime p such that $n < p \leq 2n$.*

Let $(n_m)_{m\in\mathbb{N}}$ be an infinite sequence of natural numbers such that $2n_i < n_{i+1}$ for all $i \geq 1$. This implies $n_i^k < 2n_i^k < n_{i+1}^k$ for all $i \geq 1$. Due to Theorem 2, there exists a prime number p_i such that $n_i^k < p_i < 2n_i^k < n_{i+1}^k$ for all $i \geq 1$. Thus, there exists an infinite sequence of prime numbers $(p_m)_{m\in\mathbb{N}}$ such that $n_i^k < p_i < 2n_i^k < n_{i+1}^k$ for all $i \geq 1$ and $k \geq 2$.

Lemma 8 *Every language L_{p_i} can be accepted by a kC-CA having $O(n_i)$ states.*

Proof: We know that $n_i^k < p_i < 2n_i^k$ due to the above considerations. Let $p_i = n_i^k + r$ with $1 \leq r < n_i^k$. The rough idea is as follows. We construct an n_i-ary counter. After $n_i^k + k - 1$ time steps, the leftmost cell gets a carry-over. Then, at every time step, the leftmost cell starts a signal from left to right that checks whether r has been counted. If so, the counter is reset and the next counting of $n_i^k + r$ starts; otherwise the signal is canceled. If the input is read and $n_i^k + r$ is counted, the input is accepted, otherwise the input is rejected. To be more precise, let A be an n_i-ary counter. A construction may be found in [6]. Let $c_t(j)$ denote the state of the j-th cell after reading a^t. We now construct a kC-CA A' where each cell is split into two subcells, so we can speak of two tracks. On the first track we install an n_i-ary counter. Let $c^1(j)$ denote the state of the first track of the j-th cell. We have to distinguish two cases. At first, we consider the case $r \geq 2k-1$: After $n_i^k + k - 1$ steps the first cell gets a carry-over; from this time the second track is used to check whether r has been counted. In detail, a signal is started which successively checks whether $c^1(1) = c_{n_i^k+r-(k-1)}(1)$, $c^1(2) = c_{n_i^k+r-(k-1)+1}(2)$, $c^1(3) = c_{n_i^k+r-(k-1)+2}(3)$, $\ldots, c^1(k) = c_{n_i^k+r-(k-1)+(k-1)}(k) = c_{n_i^k+r}(k)$. As soon as one of the above equations does not hold, the signal is stopped. If all equations hold, then the signal has arrived at the rightmost cell and r has been counted. If the next input is $\triangledown$, then an accepting state is sent with maximum speed to the left, otherwise we send a signal with maximum speed to the left which resets the counter on the first track and stops the emitting of signals from the leftmost cell. Then, the automaton works as described and starts the next counting of $n_i^k + r$.

The case $r < 2k - 1$ is more complicated. The construction is identical to the above construction until the leftmost cell gets a carry-over. Then a signal R from left to right is started at every time step successively checking whether $c^1(1) = c_{n_i^k}(1)$, $c^1(2) = c_{n_i^k+1}(2)$, $\ldots, c^1(k) = c_{n_i^k+2k-1}(k)$. If R arrives at the rightmost cell and the next input is $\triangledown$, then an accepting state is sent with maximum speed to the left, otherwise we send a signal I with maximum speed to the left which initializes the counter with $2k - r$ and starts the next computation. One special case remains to be treated. If the rightmost cell reads the first end-of-input symbol at some time t before R has been arriving at the rightmost cell, then a signal L from right to left is initialized successively checking the following equations: $c_t^1(k) = c_{n_i^k+r}(k)$, $c_{t+1}^1(k-1) = c_{n_i^k+r+1}(k-1)$, $\ldots, c_{t+k}^1(1) = c_{n_i^k+r+k-1}(1)$. L is canceled as soon as one of these equations does not hold. If L meets R, then an accepting state is sent with maximum speed to the left.

We now have to sum up the number of states used in the construction. The counting can be realized with $n_i + 1$ states and the signals R, L, and I need at most $k + 1$ states each. Thus, the size of the kC-CA is at most $(n_i+1)(2k+2)+k+2 = O(n_i)$. □

Lemma 9

(a) Every kC-OCA accepting L_{p_i} needs at least $\Omega(n_i^k)$ states.

(b) Every kC-OCA$_t$ accepting L_{p_i} needs at least $\Omega(n_i^{k/2})$ states.

Proof: Due to Lemma 7 we know that every kC-OCA accepting L_{p_i} needs at least $p_i = n_i^k + r$ states with $1 \leq r \leq n_i^k$. Hence, $p_i = \Omega(n_i^k)$. L_{p_i} is accepted by a kC-CA with $O(n_i)$ states. Assume by way of contradiction that L_{p_i} is accepted by a kC-OCA$_t$ with $n = o(n_i^{k/2})$ states. Due to the construction of Lemma 6(b), then there exists a kC-OCA accepting L_{p_i} with $n^2 + n = o(n_i^k)$ states which is a contradiction to (a). □

Theorem 3 *The following table holds. An entry in column A and row B describes the upper and lower bounds when converting type-A automata to type-B automata.*

	DFA	*kC-OCA*	*kC-OCA$_t$*	*kC-CA*
DFA	—	$O(n^k)$ (a) $\Omega(n^k)$	$O(n^k)$ (a) $\Omega(n^k)$	$O(n^k)$ (a) $\Omega(n^k)$
kC-OCA	$\leq n+1$ (b) $\geq n+1$	—	n^2+n (c) $\Omega(n^2)$	$O(n^k)$ (d) $\Omega(n^k)$
kC-OCA$_t$	$O(\sqrt{n})$ (e) $\Omega(\sqrt{n/\log n})$	$\leq n$ (f) $\geq n$	—	$O(n^{\lceil k/2\rceil})$ (d) $\Omega(n^{k/2})$
kC-CA	$O(\sqrt{n})$ (e) $\Omega(\sqrt[3]{n/\log n})$	$\leq n$ (f) $\geq n$	$\leq n$ (f) $\geq n$	—

Proof: (*a*) follows from Lemma 1 and Lemma 2. (*b*) follows from Lemma 1 in [7] and Lemma 7. (*d*) follows from Lemma 6(a), Lemma 8(a), Lemma 9(a). (*e*) follows from Lemma 3 and Lemma 4. (*f*) follows from Lemma 5. The upper bound in (*c*) follows from Lemma 6(b). Combining Lemma 8 and Lemma 3, we can construct a kC-OCA$_t$ accepting L_{p_i} with $n = O(n_i^{k/2})$ states. An equivalent kC-OCA needs $p_i = \Omega(n^2)$ states. □

5 Conclusion

We studied the descriptional complexity of cellular automata with a fixed number of cells and several amounts of two-way communication ranging from no two-way communication (kC-OCAs) to maximum two-way communication (kC-CAs). kC-OCA$_t$s are an intermediate model with minimum two-way communication. All models describe the regular languages. We showed that the conversion to DFAs implies a polynomial blow-up of degree k and this upper bound was shown to be tight. On the other hand, the conversion of DFAs to cellular automata may provide no savings in case of one-way communication, but always provides quadratic savings in case of two-way communication. The latter bound was additionally shown to be nearly tight for kC-OCA$_t$s. Furthermore, we showed bounds which are tight in order of magnitude when converting kC-OCA$_t$s or kC-CAs to kC-OCAs. In the latter case, maximum two-way communication may provide polynomial savings of degree k in contrast to one-way communication. Some open problems result from our considerations. Lemma 4 showed that there are languages such that kC-CAs permit at most cubic savings. It should be investigated whether the upper bound can be improved such that kC-CAs always achieve cubic savings. Since we have studied here two models with a minimum and a maximum amount of two-way communication, it could be interesting to investigate how the size of description of a language varies when gradually more and more cells are provided with two-way communication.

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