CFS Working Paper Series

No. 714

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Enough Liquidity with Enough Capital—and Vice Versa?*

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Abstract

We study the interplay of capital and liquidity regulation in a general equilibrium setting by focusing on future funding risks. The model consists of a banking sector with long-term illiquid investment opportunities that need to be financed by short-term debt and by issuing equity. Reliance on refinancing long-term investment in the middle of the life-time is risky, since the next generation of potential short-term debt holders may not be willing to provide funding when the return prospects on the long-term investment turn out to be bad. For moderate return risk, equilibria with and without bank default coexist, and bank default is a self-fulfilling prophecy. Capital and liquidity regulation can prevent bank default and may implement the first-best. Yet the former is more powerful in ruling out undesirable equilibria and thus dominates liquidity regulation. Adding liquidity regulation to optimal capital regulation is redundant.

Keywords: financial intermediation, funding risk, bank default, banking regulation, liquidity requirements, capital requirements

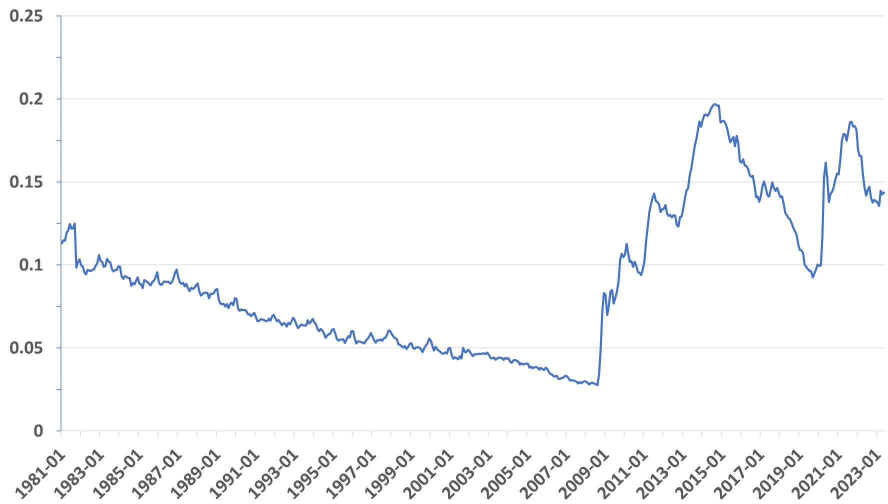
JEL Classification: G21, G33, G38

*We are grateful to Jean-Charles Rochet and seminar participants at ETH Zurich for helpful comments.

1 Introduction

Time scales for production and financing differ. Accordingly, financial intermediaries have specialized in using short-term funds to finance illiquid long-term investments. In the financial crisis of 2007/08, the fragility of many banks with regard to their reliance on short-term funding was extreme, and their equity capital proved insufficient to avoid insolvency—or at least market participants were no longer sure about the banks' solvency (Brunnermeier, 2009; Hellwig, 2009). More recently, the collapse of Silicon Valley Bank (SVB) in the U.S. and the emergency takeover of Credit Suisse by UBS in Switzerland have rocked the international banking system.

Figure 1: *U.S. Commercial Banks' Cash Ratio*



Source: Federal Reserve Release, H.8 data.

As is apparent from Figure 1, banks in the U.S. economized on cash asset holdings after the regulatory focus shifted from liquidity to capital requirements in the early 1980s. Financial institutions started to rely more on wholesale short-term investors to obtain funding (see Adrian and Shin, 2009). Conduits, special investment vehicles, and investment banks performed significant maturity transformations and played the yield curve, thus exposing themselves to large-scale funding risk. When panic withdrawals by such investors set in during the weekend when Lehman Brothers collapsed, banks were forced to sell assets. This triggered asset price declines, caused further distress sales, and undermined access to new financing.

In the aftermath of the financial crisis, renewed awareness of liquidity risks led to calls to introduce stricter liquidity requirements and to limit the banks' appetite for short-term funding. Those regulations have been implemented mainly through a "Liquiditiy Coverage Ratio" (LCR) and a "Net Stable Funding Ratio" (NSFR) in the revised regulatory frameworks known as Basel III. Exceptions and loopholes in the applicability of Basel III, however, enabled mid-sized banks in the U.S. such as SVB to operate under significantly lower regulatory requirements. In March 2023, SVB collapsed after it had failed to raise new equity capital to cover a stated loss of about $1.8bn on its bond portfolio. This caused a loss of confidence and a run on the bank by its deposit holders.

Conceptually, it remains unclear whether liquidity regulation is necessary for a stable banking system or whether adequate capital requirements can also address liquidity problems. After all, liquidity and refinancing problems in the banking sector are often thought to be the consequence of doubts about solvency (see Pierret, 2015 and Schmitz et al., 2019, for empirical evidence on the liquidity-solvency nexus). Thakor (2018) argues that the financial crisis of 2007/08 was an insolvency risk crisis and therefore only capital requirements, not liquidity regulation, should be used to address its causes. When regulatory authorities in the U.S. shut down SVB in March 2023, they cited "inadequate liquidity and insolvency" as the cause, but it appears that liquidity problems became imminent only after the bank's solvency had been called into question. Similarly, the emergency takeover of Credit Suisse by UBS brokered by the Swiss authorities was not fundamentally about liquidity issues, but rather the consequence of a general loss of trust and confidence in the bank's overall viability.

In this paper, we study the interplay of capital and liquidity regulation in a general equilibrium setting by focusing on future funding risks. The model consists of a banking sector with long-term illiquid investment opportunities that need to be financed by short-term debt and by issuing equity. Reliance on refinancing long-term investment in the middle of the life-time is risky, since the next generation of potential short-term debt holders may not be willing to provide funding when the return prospects on the long-term investment turn out to be bad. In this case, banks have to default and there is inefficient early liquidation of their long-term investments.

Our main insights are as follows: For large macroeconomic shocks and in the absence of banking regulation, there is equilibrium bank default in the bad state of the world. For more moderate shocks, bank default is a self-fulfilling prophecy in the sense that there are multiple equilibria: one equilibrium with the possibility of bank default, which is inefficient, and one equilibrium without bank default, which is first-best. Banking regulation can help to avoid bank default and to implement the first-best. More specifically, we consider capital regulation in the form of a minimum ratio of equity capital to risky long-term investment and liquidity regulation in the form of a minimum ratio of liquidity to short-term debt. While the two types of regulation turn out to be equally capable of implementing the first-best as *an* equilibrium outcome, the former is more powerful than the latter in ruling out other inefficient equilibria. Adding liquidity regulation to optimal capital regulation is redundant.

Model summary and detailed results. We consider a three-period economy with a single homogeneous good and risk-neutral agents. Banks are set up via equity financing from owners and they collect deposits from short-term debt holders. The banks can invest their funds in risky long-term investment projects operated by entrepreneurs or in short-term liquid assets. Since each generation of depositors lives only for one period, banks have to refinance their debt in the second period and thus face funding risk. Macroeconomic uncertainty about the returns on the long-term investment is resolved just before the second-generation depositors decide on whether to provide the banks with new funds.

We find that in the presence of small macroeconomic shocks, banks' equity is high enough for them never to default, so no banking regulation is needed to achieve the first-best. In the presence of more severe shocks, however, equilibrium bank equity is insufficient to absorb losses in the bad state of the world. Since bank equity holders are subject to limited liability, the initial short-term debt holders demand a risk premium on any funds they provide to the banks if they consider bank default an eventuality. The fact that a risk premium lets depositors earn higher interest in the good state of the world implies lower expected returns on bank equity, which makes raising equity funding more difficult. Additionally, banks have to hold larger liquidity reserves to repay the initial depositors in the good state of the world. The result is that banks cannot achieve the efficient scale of long-term investment. If it eventually becomes clear that the return prospects on banks' long-term investments are dire, short-term debt holders refuse to refinance banks which they know will not be able to fully repay them later. Hence, banks become illiquid and default. In default, banks are forced to liquidate their long-term investments early, which generates high losses and is socially inefficient.

For moderate macroeconomic shocks, equilibria with and without bank default coexist. Bank default then becomes a self-fulfilling prophecy. If banks and short-term debt holders behave as if there were no possibility of default, meaning in particular that the initial depositors do not demand a risk premium, then banks will not default in any state of the world. If, however, banks and depositors assume default in the bad state of the world and act accordingly, i.e., the initial depositors demand a risk premium and banks act on their limited liability when making investment decisions, then banks will indeed default in the bad state of the world. In this sense, a banking crisis can result solely from the fear of a banking crisis and the attendant behavior of market participants.

Banking regulation helps to counteract market inefficiencies. If macroeconomic risk is not extreme, both liquidity regulation and capital regulation are capable of implementing the first-best.[1] Intuitively, capital requirements can solve the liquidity problem as well. If the banks' required capital ratio is high enough to make it clear that they will never default and will be able to repay the funds provided to them in all states of the world, then they can always refinance. Perhaps more surprisingly, liquidity requirements can, to some extent, also address solvency problems. The reason is that solvency issues arise in states of the world where it becomes clear that the returns from banks' risky long-term investments will not be enough to fully repay depositors. Larger investment in a safe asset, i.e., holding higher liquidity reserves, can make up for losses from risky long-term investment.

Comparing capital regulation to liquidity regulation in more detail, we find that the former is more powerful than the latter. While the two forms of regulation are equivalent in implementing the first-best as *an* equilibrium, capital regulation is more capable of implementing it *uniquely*, i.e., of simultaneously ruling out other inefficient equilibria. To

[1] For extreme shocks, the extent of the safety buffer required to avoid default in the bad state implies that the amount of leftover funds is insufficient to achieve the efficient scale of long-term investment. In this case, liquidity and/or capital requirements cannot implement the first-best.

understand this, we first note that eliminating inefficient equilibria with default requires stricter banking regulation than the one required for implementing the first-best as *an* equilibrium. Stricter regulation to rule out equilibria with default, however, implies that there may be insufficient resources in the economy to still achieve the efficient scale of long-term investment in the remaining equilibrium without default. It follows that there can be macroeconomic circumstances where it is possible to implement the first-best, but it is not possible to do so uniquely. More precisely, under tighter liquidity regulation the amount of total resources available in the economy may fall short of the amount of resources needed to simultaneously meet the liquidity requirements and achieve the first-best amount of long-term investment. Similarly, under tighter capital regulation there may not be enough equity funding available in the economy to simultaneously meet the regulatory capital ratio and achieve the first-best amount of risky long-term investment.

Now capital regulation, however, is more powerful than liquidity regulation because the former has to be strengthened by less than the latter when aiming to rule out the existence of equilibria with default as opposed to implementing the first-best as just *an* equilibrium. With the argument above, this implies that there exist circumstances where it is possible to uniquely implement the first-best through capital regulation, but not through liquidity regulation. That capital regulation has to be strengthened by less than liquidity regulation stems from the fact that the banks' amount of long-term investment in an equilibrium with default is smaller than in the efficient equilibrium. Thus, for any given amount of equity capital, there are more resources for investment in liquid assets in the former equilibrium than in the latter. This means that any additional liquidity holdings required to rule out an equilibrium with default through liquidity regulation do not have to be (fully) matched by more equity capital when ruling out an equilibrium with default through capital regulation.

Related literature. The risk of a collapse of short-term funding (funding risk) plus the risk that assets cannot be converted into cash without substantial losses in original value (liquidity risk), have been major concerns throughout the history of banking. Since the work of Diamond and Dybvig (1983), individual uncertainty about the timing of consumption needs, coupled with financial intermediaries issuing claims that can be called at any time, has been the focus of extensive analysis. Recent contributions analyzing the effects of liquidity and capital requirements in Diamond-Dybvig-style settings are Diamond and Kashyap (2016) and Kashyap et al. (2020). In our model, consumption needs are certain and financial intermediaries do not face the risk of depositors' run. However, they do face funding risk, as future investors may not be willing to refinance banks.

A number of other papers have pursued and applied different approaches to capture the effects of liquidity regulation, also in combination with capital requirements (Vives, 2014; Calomiris et al., 2015; Walther, 2016; Macedo and Vicente, 2017; Cecchetti and Kashyap, 2018; Dewatripont and Tirole, 2018; Kara and Ozsoy, 2019; Carletti et al., 2020).[2] Their findings on whether liquidity and capital requirements are rather substitutes or complements vary and they show that the economic circumstances and the way in which banking regulation is implemented, matter.

Morris and Shin (2016) provide a clear theoretical distinction between insolvency risk and illiquidity risk, the latter meaning that although in the absence of liquidity problems the eventual realization of asset values would be sufficient to service all debt, runs by short-term debt holders and the necessity of fire sales may diminish asset values and bring about bank default. They emphasize that illiquidity risk is not typically independent of insolvency risk: "The conclusion that there is no illiquidity risk without solvency uncertainty is an intuitive one, but not present in many models and much discussion of illiquidity risk" (p. 1137). Our model accounts for that fact: once uncertainty vanishes, illiquidity can occur only as a consequence of looming insolvency.

Quantitative analysis of optimal banking regulation using calibrated dynamic banking models has been provided by Covas and Driscoll (2014), De Nicolo et al. (2014), Clerc et al. (2015), Faia (2019), Van den Heuvel (2019), Cont et al. (2020), Corbae and D'Erasmo (2021), Chiba (2022), and others. Finally, there is a long history of inquiries into liquidity provision and maturity transformation, which we cannot summarize here (see e.g., Freixas and Rochet, 2008).

The paper is organized as follows: Section 2 introduces the model. Section 3 defines

[2]See Allen and Gale (2017) and Bouwman (2019) for reviews of the literature.

equilibrium and provides equilibrium considerations on bank default. Section 4 sets up the Arrow-Debreu benchmark and determines the Pareto frontier. Section 5 solves for equilibrium without banking regulation. Section 6 introduces liquidity and capital requirements into the model. Section 7 concludes. Technical proofs can be found in the appendix.

2 The Model

We consider a simple three-period economy ($t = 1, 2, 3$) with a single physical good. At the center of the analysis are banks that compete for equity and debt and act as financial intermediaries. There are three classes of risk-neutral agents: owners of banks, debt holders, and entrepreneurs running investment projects. The details of the model are set out in the subsections below.

2.1 Technologies

The model encompasses a short-term technology (henceforth "ST") and a long-term technology (henceforth "LT").

Short-term technology. ST yields a constant output $1 + r_s$ ($r_s \geq 0$) in period $t + 1$ for each unit of the physical good invested in period t, i.e., $1 + r_s$ is the productivity of ST. Hence, output for one unit of investment from $t = 1$ to $t = 3$ is given by $(1 + r_s)^2$. The short-term technology can be interpreted in two different ways. First, it may represent a real production technology converting time t-goods into time $t + 1$-goods. Second, ST may represent investment in a money market fund with returns r_s per period.[3]

Long-term technology. LT yields output after two periods. It is subject to macroeconomic risk and exhibits decreasing marginal returns. Specifically, if an aggregate amount K is invested in period $t = 1$, output in period $t = 3$ is $\eta f(K)$, where η is an aggregate shock:

$$\eta = \begin{cases} h \,(\text{high}) & \text{with probability } p, \\ l \;\, (\text{low}) & \text{with probability } 1 - p, \end{cases} \tag{1}$$

[3]The final investment of resources channeled into the fund has to be a real investment of the ST-type or default-free short-term debt in another technology.

where p and $1-p$ are the probabilities of high and low productivity shocks, respectively, and l and h are real numbers satisfying $0 < l < h$. We use m to denote the average aggregate shock:

$$m = ph + (1-p)l.$$

Throughout the paper, we will keep m fixed and vary l and h simultaneously to capture more or less severe shocks. Then, the expected output in $t = 3$ is

$$\mathbb{E}\left(\eta f(K)\right) = f(K)(ph + (1-p)l) = mf(K).$$

While all investors in the economy will have frictionless access to ST, access to LT will be prohibitively costly except for financial intermediaries, who will thus be the sole investors in LT.

Inada conditions. We assume that the long-term technology exhibits decreasing marginal returns, i.e., $f'(K) > 0$, $f''(K) < 0$. Moreover, we assume that $f(K)$ satisfies Inada Conditions, i.e., $\lim_{K\to 0} f'(K) = \infty$ and $\lim_{K\to\infty} f'(K) = 0$. The assumptions on $f(\cdot)$ imply that there exists a uniquely determined $\bar{K} > 0$, such that $f'(\bar{K})m = (1+r_s)^2$. We note that $f'(K)m < (1+r_s)^2$, for all $K > \bar{K}$, and $f'(K)m > (1+r_s)^2$, for all $K < \bar{K}$. We also assume $\left|\frac{f''(K)K}{f'(K)}\right| < 1$, which is fulfilled by all production functions of the type $f(K) = K^\alpha$ with $0 < \alpha < 1$. The production function $f(K) = \ln(1+K)$ meets all assumptions except the Inada condition for $K \to 0$.

Shock realization and early liquidation. The uncertainty about long-term investment returns is resolved in period $t = 2$, i.e., in $t = 2$ all market participants observe η and learn whether it is high or low. We assume that the gross return of LT investment is zero if the investment is liquidated already in $t = 2$, which represents a strong form of illiquidity. This assumption simplifies the analysis, but is not essential for our results.[4] To simplify notation, we denote the marginal productivity of LT as follows: $1 + r_L^h = hf'(K)$, $1 + r_L^l = lf'(K)$ and $1 + r_L^e = p(1 + r_L^h) + (1-p)(1 + r_L^l) = mf'(K)$.

[4]Liquidation will only occur when $\eta = l$. Clearly, LT has a liquidation value of zero if $l = 0$. In case of $l > 0$, output may be less than $lf(K)$ if production ends prematurely. For simplicity, we assume that LT output and salvage value become zero upon early liquidation.

2.2 Owners

There is a continuum of long-term investors of measure 1. They live for three periods from $t = 1$ to $t = 3$. They are risk-neutral and are only concerned with consumption when old in $t = 3$, i.e., they do not consume in $t = 1$ and $t = 2$. Henceforth, we simply call them owners. Each owner is endowed with initial wealth w. The aggregate wealth of owners in $t = 1$ is denoted by W. They have two opportunities to invest their endowment: (i) bank equity capital, (ii) ST.[5] The aggregate amount of investment in ST by owners at $t = 1$ is denoted by L_I.

2.3 Debt holders

There is a continuum of investors with measure 2μ ($\mu > 0$) who are born in $t = 1$ and live for three periods. They are also risk-neutral and are called debt holders. Half of them receive an endowment w in $t = 1$, the other half receive endowment w in $t = 2$. The first half consumes in $t = 2$, the second half consumes in $t = 3$. Wealth can be transferred between periods at the certain interest rate r_s. Hence, we obtain two savings functions $S_t = \mu W$ ($t = 1, 2$). We note that the assumptions on debt holders ensure that banks have access to the same measure of potential resources for short-term debt financing in $t = 1$ and $t = 2$. Debt holders can also invest in ST.

2.4 Entrepreneurs

We assume that LT is operated by a representative third agent called an entrepreneur, who collects the surplus on the risky illiquid investment and who is also risk-neutral. We assume that the market for LT investment is competitive, so investors receive marginal returns on investment.

2.5 Banks

We consider a competitive banking system with n banks indexed by $i = 1, ..., n$ in $t = 1$.

[5]Owners cannot invest in LT directly. As we will establish in Section 2.5, LT investment requires intermediation by banks. There are many ways to justify why banks are needed to channel funds from equity holders and depositors to investment projects (see, e.g., Diamond, 1984; Hellwig, 1998; Gersbach and Uhlig, 2006). One example would be that such investments require access to monitoring technologies that only banks have access to.

Founding of banks. Banks start as a number (or name) only, and investors can offer resources to a bank in exchange for equity contracts. An equity contract specifies that the holder is entitled to obtain a share of dividends at time $t = 3$ in proportion to the resources s/he has given to a particular bank. Equity holders are protected by limited liability. By offering equity, investors thus become owners of the bank. The amount of equity obtained by a bank is determined by the amount of resources offered by owners, which, in turn, depends on the expected return on equity. If banks receive a positive amount of equity from owners, they are set up and can operate.[6]

Liquidation of banks. If banks do not default earlier, they are liquidated in $t = 3$ and equity holders receive the residual value. We use e_1^i and e_3^i to denote the equity capital of bank i in $t = 1$ and $t = 3$, respectively. Aggregate equity in $t = 1$ and $t = 3$ is denoted by E_1 and E_3, respectively.

Deposit contracts. Besides equity contracts, bank i offers short-term deposit contracts in $t = 1$ in a competitive market at the (promised) deposit rate r_{d1} and receives an amount d_1^i of deposits ($i = 1, ..., n$). The aggregate amount of deposits of the entire banking system in the first period is denoted by D_1.

Banks' investment decisions. After receiving funds from debt holders and equity holders, banks make their investment decision. We denote the amount of $t = 1$ investment by bank i in ST and in LT by l_B^i and k^i, respectively. The aggregate investment amounts are given by $L_B = \sum_{i=1}^n l_B^i$ and $K = \sum_{i=1}^n k^i$. Banks offer state-contingent credit contracts to entrepreneurs. Since the market for investment in LT is competitive, a bank i that in $t = 1$ invests an amount k^i in LT receives, depending on the state, either $(1 + r_L^l)k^i = lf'(K)k^i$ or $(1 + r_L^h)k^i = hf'(K)k^i$ in $t = 3$. Since investment in ST has a short economic life-time, it represents liquid assets. The amount l_B^i that banks invest in ST in the absence of regulation is called *voluntary liquidity.*

Refinancing. In $t = 2$, each bank has to pay back its first depositors. To collect new funds, bank i offers the second-generation debt holders new deposit contracts in a competitive market at the deposit rate r_{d2}. The aggregate amount of deposits issued in

[6]In practice, banks need to attract a minimum level of equity capital in order to have the right to operate. Such requirements can easily be introduced into our framework. In fact, the kind of capital regulation proposed in Section 6.2 constitutes such a requirement.

$t = 2$ is denoted by D_2.

Bank default. Upon receiving the second-period deposits, bank i faces one of the following two situations: either it is liquid, is able to pay back the first depositors, and continues to operate, or it cannot fulfill its obligations and defaults. The condition for survival in $t = 2$ is $l_B^i(1 + r_s) + d_2^i \geq d_1^i(1 + r_{d1})$. If bank i survives in $t = 2$, it receives the return on its LT investment in $t = 3$ and again faces one of two possible scenarios: either its final return from LT and ST investment is sufficiently high to be able to pay back the second depositors, or it has to default on its obligations. In the first case, bank i fully repays its depositors and proportionally distributes the remaining funds among equity capital owners. In the second case, bank i becomes insolvent and equity holders receive nothing. The details of the insolvency procedure will be discussed in Section 3.2.

Bank profits. We denote the profits of bank i in $t = 3$ by π_3^i, if it survives until that date. Hence,

$$\pi_3^i = \eta f'(K)k^i - d_2^i(1 + r_{d2}) + \left[l_B^i(1 + r_s) + d_2^i - d_1^i(1 + r_{d1})\right](1 + r_s),$$

if $l_B^i(1 + r_s) + d_2^i \geq d_1^i(1 + r_{d1})$. Accounting for limited liability, the amount of equity of bank i in $t = 3$ is given by

$$e_3^i = \begin{cases} \max\{\pi_3^i, 0\} & \text{if } \; l_B^i(1 + r_s) + d_2^i \geq d_1^i(1 + r_{d1}), \\ 0 & \text{if } \; l_B^i(1 + r_s) + d_2^i < d_1^i(1 + r_{d1}). \end{cases} \tag{2}$$

The objective of banks in the interest of bank owners is to maximize the expected return on equity:[7]

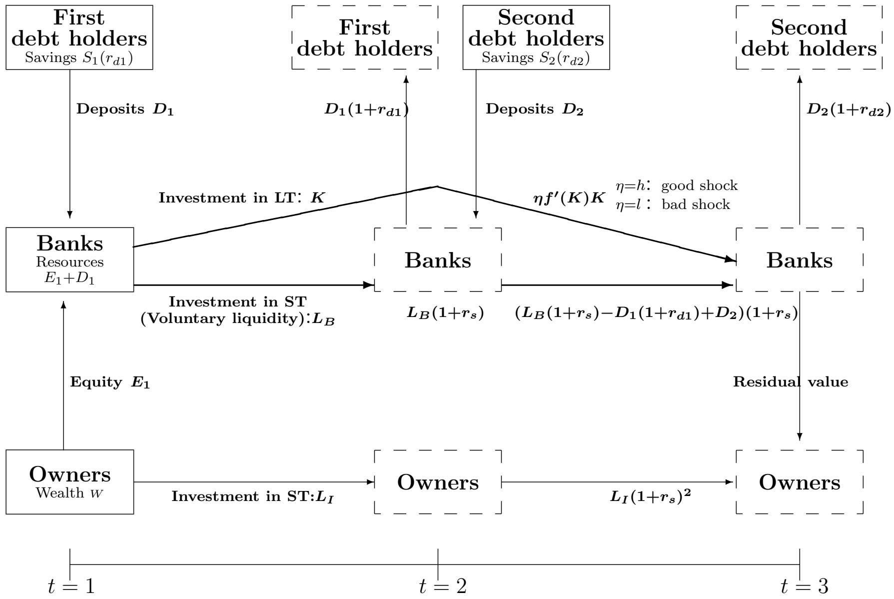
$$\max_{l_B^i, k^i} \mathbb{E}\left[\frac{e_3^i}{e_1^i}\right].$$

Since e_1^i $(i = 1, ..., n)$ is given when investment decisions are made (in $t = 1$ and $t = 2$) and since bank owners are risk-neutral, banks maximize $\mathbb{E}\left[e_3^i\right]$.

We illustrate the economy in Figure 2.

[7]With e_3^i given by Equation (2), this takes into account the limited liability characteristic of equity. We neglect conflicts of interest between bank managers and bank owners.

Figure 2: *The model and sequence of events*



2.6 Regulatory environment and main assumptions

Throughout the paper, we make the following assumption that allows us to focus on the economically interesting cases.

Assumption 1

$$W < \bar{K}, \tag{3}$$

$$W + \mu W > \bar{K} + \frac{1 + r_s - p}{1 + r_s}\mu W. \tag{4}$$

We note that Assumption 1 implies $W < \bar{K} < W + \mu W$. The first part of the assumption states that both owners and debt holders have to invest in LT in order to achieve the amount $\bar{K}$; the wealth of owners alone is not sufficient. The second part of the assumption guarantees that the total aggregate wealth of owners and debt holders is sufficiently high for the banking system to be able to invest an amount $\bar{K}$ in LT and some resources in ST to remain solvent in $t = 2$, at least in the good state of the world. In the basic version of the model, there is no banking regulation.

3 Equilibrium Concept

3.1 Preliminary considerations

For ease of presentation in the derivation of equilibrium, we impose the tie-breaking rule that debt holders invest in bank deposits if they are indifferent between investment in ST and bank deposits. Then, we observe that the aggregate amount of deposits from debt holders in the first and second period is given by $D_1 = S_1$ and $D_2 = S_2$, respectively, if at least one bank survives in $t = 2$ and if expected returns on D_1 and D_2 are at least r_s.

The initial situation of the economy can be described as follows: Banks are initially identical in the eyes of potential shareholders, so each bank receives the same amount of equity in the first period and thus $e_1^i = \frac{E_1}{n}$ $(i = 1, ..., n)$.[8] The same holds for debt holders in the first period, hence, $d_1^i = \frac{D_1}{n}$.

In $t = 1$, bank i chooses the investments k^i and l_B^i, given its available resources $\frac{E_1+D_1}{n}$. We use R_t^η to denote the actual aggregate repayment of the banking system to debt holders in period t $(t = 2, 3)$ in states $\eta = l$ and $\eta = h$. We use $R_t^e = \mathbb{E}\left[R_t^\eta\right]$ to denote the expected aggregate repayment to debt holders.

3.2 Definition of equilibrium

Given these preliminary considerations, we will be looking for competitive symmetric equilibria defined as follows:

Definition 1 (Competitive equilibrium)
A competitive symmetric equilibrium is a sextuplet $\{K, E_1, L_B, L_I, r_{d1}, r_{d2}\}$, *such that the following conditions hold:*

(i) l_B^i *and* k^i *solve* $\max\left\{\mathbb{E}\left[e_3^i\right]\right\}$ *s.t.* $k^i + l_B^i \leq \frac{E_1+D_1}{n}$, *where* $K = \sum_{i=1}^n k^i$, $L_B = \sum_{i=1}^n l_B^i$ *and* $l_B^i = l_B^j$, $k^i = k^j$, *for* $i \neq j$,

(ii) $D_1 + E_1 = K + L_B$,

(iii) $E_1 + L_I = W$,

[8] This holds if one applies a suitable version of the law of large numbers and if owners randomize between banks. It also holds if each shareholder buys an identical amount of equity of each bank or if the set of owners is partitioned into n groups G_i of measure $1/n$, where group G_i only buys shares of bank i.

(iv) $n\mathbb{E}\left[e_3^i\right] \geq E_1(1+r_s)^2$, *if* $E_1 > 0$,

(v) $R_2^e \geq D_1(1+r_s)$ *and* $D_1 = S_1 = \mu W$,

(vi) $R_3^\eta \geq D_2(1+r_s)$ *and* $D_2 = S_2 = \mu W$, *if at least one bank survives in period* $t = 2$, *state* $\eta \in \{l,\, h\}$.

Condition (i) guarantees that given LT return expectations the investment choice by bank i maximizes its expected return on equity. Condition (ii) represents the savings/investment balance of the banking system and of an individual bank, if we divide both sides by the number of banks n. Condition (iii) represents the aggregate budget constraint of owners. Condition (iv) guarantees that investing a positive amount E_1 in equity yields at least the same expected return as investing in ST. Otherwise, banks would not be founded by owners. Conditions (v) and (vi) determine the amount of debt banks can attract in both periods if they can offer sufficiently high expected returns.

3.3 Default and work-out procedures

We next discuss bank default. Since the return of LT becomes known in $t = 2$, it is already clear at that time whether banks will go bankrupt or not. This yields the following property:

Early default. *A bank* i *will not obtain new funding and defaults in* $t = 2$ *if there is no* d_2^i *and deposit rate* $r_{d2} \geq r_s$ *such that* $e_3^i \geq 0$.

Hence, if in $t = 2$ bank i cannot obtain an amount of new deposits at $r_{d2} = r_s$, which guarantees its survival in $t = 3$, it will default in $t = 2$. Possibly, bank i is liquid in $t = 2$ while $\pi_3^i = 0$. In that case, the bank's equity holders are indifferent between defaulting and not defaulting in $t = 2$. For convenience, we assume that the firm does not default in $t = 2$.

We observe that if a bank defaults in $t = 2$ at $r_{d2} = r_s$, it would default at any higher deposit rate $r_{d2} > r_s$. The reason is that long-term investment has been made, returns are known, and in $t = 2$ banks can invest any resources that they do not need to pay out into ST at interest rate r_s. Moreover, as returns are known in $t = 2$, new depositors at

$t = 2$ know whether banks will be able to pay back or not. Given equal funding of banks in $t = 1$, this yields

Lemma 1

In any equilibrium, either all banks default in $t = 2$ and receive no new funds or they do not default and receive new deposits at interest rate $r_{d2} = r_s$.

We next observe that the following condition for investment in voluntary liquidity has to hold:

Lemma 2

In any equilibrium,

$$l_B^i \geq \underline{l}_B \coloneqq \frac{1}{n(1+r_s)} \left[D_1(1+r_{d1}) - D_2 \right]. \tag{5}$$

Lemma 2 derives from the following considerations: First, banks can always ensure $\mathbb{E}\left[e_3^i\right] = \frac{E_1}{n}(1+r_s)^2$ by investing their entire resources in ST. A necessary condition to generate positive payoffs for equity holders in $t = 3$ is that each bank is able to fulfill its obligations towards the first depositors, otherwise bank i would go bankrupt in $t = 2$ in state h as well as in state l. This condition is

$$l_B^i(1+r_s) + d_2^i \geq \frac{D_1}{n}(1+r_{d1}),$$

which is identical to Condition (5) if every bank survives and thus $d_2^i = \frac{D_2}{n}$.

Bank profits. If bank i survives in $t = 2$, its final equity in state η is given by

$$e_3^i = \max \Big\{ \eta f'(K) k^i + \Big(l_B^i(1+r_s) - \frac{D_1}{n}(1+r_{d1}) + d_2^i \Big)(1+r_s) \\ - d_2^i(1+r_{d2}), 0 \Big\}.$$

With $r_{d2} = r_s$, this yields

$$e_3^i = \max \left\{ \eta f'(K) k^i + \left(l_B^i(1+r_s) - \frac{D_1}{n}(1+r_{d1}) \right)(1+r_s), 0 \right\}.$$

We obtain the following lemma:

Lemma 3

In any equilibrium, a bank i will default in $t = 2$ if and only if $\eta = l$ and

$$(1 + r_L^l)k^i + \left(l_B^i(1 + r_s) - \frac{D_1}{n}(1 + r_{d1})\right)(1 + r_s) < 0. \quad (6)$$

Lemma 3 follows from the following logic: A bank i will never default in $\eta = h$ in equilibrium, as otherwise it would also default in $\eta = l$. Then, $\mathbb{E}[e_3^i]$ would be zero. As bank i can always secure $\mathbb{E}[e_3^i] = \frac{E_1}{n}(1 + r_s)^2$ by investing all resources in ST, a default in $\eta = h$ cannot occur in equilibrium. Now, suppose first that Condition (6) holds. Then, bank i would default in $t = 3$, $\eta = l$ for any level of deposits d_2^i it received. Hence, it will already default in $t = 2$, $\eta = l$. Suppose second that Condition (6) does not hold. Then, as $l_B^i \geq \underline{l}_B^i$ by Condition (5), bank i will survive in $t = 2$ for $\eta = l$ and $\eta = h$ for $d_2^i = \frac{D_2}{n}$. Bank i will be able to pay back depositors in both states of the world in $t = 3$ and thus will never default—provided that $r_{d2} = r_s$.

4 Pareto-efficient Allocations

Before we explore the properties of the economy with and without governmental intervention, it is useful to characterize the allocation when no frictions are present. That is, direct investment in both technologies is possible, no financial intermediation is needed, and financial assets are complete in the extended sense of Arrow-Debreu: While not all contingent claims are available at $t = 1$, a combination of a $t = 1$ portfolio and $t = 2$ trade permits the achievement of any feasible allocation, in particular every Pareto-efficient allocation. We proceed in two steps. First, we determine the Pareto-efficient Arrow-Debreu equilibrium allocation. Second, we determine the much larger Pareto frontier of the economy.

4.1 Arrow-Debreu benchmark

To formulate the Arrow-Debreu system (henceforth AD), we assume that all agents can directly invest the physical good in ST, [9] which simplifies the exposition. Entrepreneurs

[9]Equivalently, we could assume that real short-term assets for $t = 1$ (and $t = 2$) are available with a repayment of $1 + r_s$ in $t = 2$ (and $t = 3$) if one unit is bought. The equilibrium price of these assets

issue long-term assets in $t = 1$, the price and amount of which are denoted by q_1 and a_L, respectively. If an agent buys one unit of the asset, s/he receives the following state contingent repayment in $t = 3$:

$$r^{\eta} = \begin{cases} \eta = h : & r^h = \frac{h(1+r_s)^2}{m}, \\ \eta = l \ : & r^l \ = \frac{l(1+r_s)^2}{m}. \end{cases}$$

The expected repayment is $(1 + r_s)^2$ and thus equivalent to investing one unit repeatedly in ST. Long-term assets are traded in $t = 1$ as well as in a secondary market in $t = 2$ after the state η has been revealed. We use q_2^h and q_2^l to denote the state-contingent prices of the asset in period 2 in terms of the consumption good. All individuals are price takers.

Entrepreneurs' optimization problem. An entrepreneur maximizes expected profits and all types of investors maximize expected consumption, which in the latter case is equivalent to maximization of expected wealth for the date at which the agent would like to consume. To maximize expected profits, the entrepreneur solves

$$\max_{a_L} \left\{ p \left[hf(q_1 a_L) - a_L r^h \right] + (1-p) \left[lf(q_1 a_L) - a_L r^l \right] \right\},$$

where r^h and r^l are the state-contingent repayments of the long-term asset. Profit maximization yields $mf'(q_1 a_L) q_1 = (1 + r_s)^2$. We will show below that $q_1 = 1$, which then implies $a_L = \bar{K}$. There is no default risk, since $lf(\bar{K}) > lf'(\bar{K})\bar{K} = r^l \bar{K}$.

Recall that Assumption 1 implies $W < \bar{K} < W + \mu W$ and thus captures the case where contributions by both groups of agents investing in $t = 1$ are needed to finance long-term investment in the amount $\bar{K}$. We use C_2^D, C_3^D, C_3^O, C_3^E to denote the consumption levels of debt holders in periods 2 and 3, owners in period 3, and entrepreneurs in period 3, respectively. We obtain

Proposition 1

The AD equilibrium is characterized by

(i) $\hat{a}_L = \bar{K}$ *and* $\hat{q}_1 = 1$.

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would be unity, and buying such assets is equivalent to directly investing in ST.

(ii)

$$\hat{q}_2^{\eta} = \begin{cases} \eta = h: & \frac{h(1+r_s)}{m}, \\ \eta = l\ : & \frac{l(1+r_s)}{m}. \end{cases}$$

(iii) Owners invest their entire wealth in the long-term asset. They consume

$$\hat{C}_3^O = \begin{cases} \eta = h: & r^h W, \\ \eta = l\ : & r^l W. \end{cases}$$

(iv) Debt holders who invest in $t = 1$ buy $\bar{K} - W$ long-term assets and invest the residual wealth in ST. They consume

$$\hat{C}_2^D = \begin{cases} \eta = h: & (\bar{K} - W)\hat{q}_2^h + (\mu W + W - \bar{K})(1 + r_s), \\ \eta = l\ : & (\bar{K} - W)\hat{q}_2^l + (\mu W + W - \bar{K})(1 + r_s). \end{cases} \tag{7}$$

(v) Debt holders who invest in $t = 2$ do the same. They consume

$$\hat{C}_3^D = \mu W(1 + r_s),$$

independently of the state.

(vi) Entrepreneurs invest all resources they receive in LT and consume

$$\hat{C}_3^e = \begin{cases} \eta = h: & hf(\bar{K}) - r^h \bar{K}, \\ \eta = l\ : & lf(\bar{K}) - r^l \bar{K}. \end{cases}$$

The proof is in Appendix A. We note that the AD solution has two important characteristics: first, an amount $\bar{K}$ is invested in LT in $t = 1$, second, there is no default risk, and thus no investment in LT is liquidated in $t = 2$. Together, these characteristics mean that the AD solution maximizes expected aggregate intertemporal consumption defined by

$$\mathbb{E}\left[C_2 + \frac{1}{1 + r_s} C_3\right],$$

where C_2 and C_3 denote aggregate consumption in periods 2 and 3. The latter is given by

$$C_3 = C_3^D + C_3^O + C_3^E.$$

4.2 Pareto frontier

The AD equilibrium allocation depicted in Proposition 1 is but one of many Pareto-efficient allocations. We now determine the entire Pareto frontier.

Suppose that, in period 1, $K \in [0, W + \mu W]$ is invested in LT and $W + \mu W - K$ is invested in ST. Then, in period 2, $(1+r_s)(W+\mu W-K)+\mu W$ is available for consumption and investment. Let γ_2^η and $1 - \gamma_2^\eta$ denote the fractions allocated to consumption and investment, respectively, in state η. Then, consumption in period 2 is $C_{S2}^\eta = \gamma_2^\eta[(1 + r_s)(W + \mu W - K) + \mu W]$. Define $\gamma_2 := p\gamma_2^h + (1 - p)\gamma_2^l$. The expected available amount Y of goods in period 3 is given by

$$\begin{aligned} Y &= \mathbb{E}\{\eta f(K) + (1 + r_s)(1 - \gamma_2^\eta)[(1 + r_s)(W + \mu W - K) + \mu W]\} \\ &= mf(K) + (1 + r_s)(1 - \gamma_2)[(1 + r_s)(W + \mu W - K) + \mu W]. \end{aligned} \tag{8}$$

Since each type of investor has only one consumption period, we can disregard discounting. In $t = 2$, the short-term investors of period 1 have expected consumption C_{S2}, given by

$$\begin{aligned} C_{S2} &= \mathbb{E}\{\gamma_2^\eta[(1 + r_s)(W + \mu W - K) + \mu W]\} \\ &= \gamma_2[(1 + r_s)(W + \mu W - K) + \mu W]. \end{aligned} \tag{9}$$

In $t = 3$, the short-term investors of period 2 and the long-term investors combined have expected consumption Y.

The expected consumption levels C_{S2} and Y are determined by the choice of (γ_2, K). In particular, the pair $(\gamma_2, K) = (1, 0)$ maximizes C_{S2}, yields $Y = 0$, and is a Pareto-optimal choice. Apart from that, the pair $(\gamma_2, K) = (0, \bar{K})$ maximizes Y, yields $C_{S2} = 0$, and is another Pareto-optimal choice. We obtain

Proposition 2

The Pareto-optimal choices of (γ_2, K) *are* $(1, K)$ *with* $K \leq \bar{K}$ *and* $(\gamma_2, \bar{K})$ *with* $\gamma_2 < 1$.

The proof is in Appendix A. The intuition is the following: (i) Whenever $K > \bar{K}$, shifting resources from LT investment to ST investment while keeping time-2 consumption C_{S2} constant enables an increase of time-3 consumption Y. This is because for $K > \bar{K}$, ST investment yields higher expected returns than LT investment. Hence, $K > \bar{K}$ cannot be Pareto-optimal. (ii) For $K < \bar{K}$ and $\gamma_2 < 1$, a marginal increase in K can be offset by an adequate increase in γ_2 to keep C_{S2} constant, while at the same time Y increases. This is because for $K < \bar{K}$, LT investment yields higher expected returns than ST investment. Hence, pairs (γ_2, K) with $\gamma_2 < 1$ and $K \neq \bar{K}$ cannot be Pareto-optimal. (iii) Finally, pairs (γ_2, K) with $K < \bar{K}$ and $\gamma_2 = 1$ are Pareto-optimal, as it is not possible to jointly vary K and γ_2 in such a way as to increase Y without decreasing C_{S2} or vice versa.

Remarks. Some remarks with regard to Proposition 2 are in order:

Remark 1: Even though state-dependent choices γ_2^l and γ_2^h are possible, risk neutrality implies that only $\gamma_2 = p\gamma_2^h + (1-p)\gamma_2^l$ matters for expected welfare considerations.

Remark 2: The social welfare function $C_{S2} + Y$ is maximal at $(0, \bar{K})$. The social welfare function $C_{S2} + \frac{1}{1+r_s}Y$ is maximal at all $(\gamma_2, \bar{K})$ with $\gamma_2 \in [0, 1]$.

Remark 3: The AD allocation obtained in Proposition 1 is part of the Pareto frontier as described by Proposition 2. The fact that in Proposition 1 $\hat{C}_2^D$ satisfies $0 < \hat{C}_2^D < \mu W + (1 + r_s)(\mu W + W - \bar{K})$, $\eta \in \{l, h\}$, corresponds to $\gamma_2 \in (0, 1)$ in Proposition 2. The AD allocation further requires $K = \bar{K}$.

Remark 4: The characterization of Proposition 2 shows that the equilibrium allocation with bank default that we will obtain in Section 5.2 Proposition 4 is not Pareto-optimal.

5 Equilibria without Regulation

We next characterize equilibrium when no banking regulation is present. For this purpose, we will vary l (and h) while keeping m fixed. This allows us to distinguish between low or high macroeconomic risk, keeping expected output at the scale $\bar{K}$ constant. We distinguish two cases: *no bank default* and *bank default*. Later, we identify the conditions under which these cases are indeed equilibria.

5.1 No bank default

Suppose for the moment that banks do not default in either state of the world. In this case, $r_{d1} = r_s$, as repayment of deposits is certain. Moreover, r_{d2} is also equal to r_s as debt holders will be paid back in both states in $t = 3$.

Banks' optimization problem. The maximization problem of bank i in $t = 1$ is given by

$$\max_{l_B^i, k^i} \left\{ \mathbb{E}\left(e_3^i\right) \right\} = \max_{l_B^i, k^i} \Big\{ (1+r_L^e)k^i + \left(l_B^i(1+r_s) - \frac{D_1}{n}(1+r_s) + d_2^i \right)(1+r_s) \\ - d_2^i(1+r_s) \Big\}. \quad (10)$$

Using $l_B^i = \frac{E_1+D_1}{n} - k^i$, we rewrite Equation (10) as

$$\max_{k^i} \left\{ \mathbb{E}\left(e_3^i\right) \right\} = \max_{k^i} \left\{ (1+r_L^e)k^i + \left(\frac{E_1}{n} - k^i \right)(1+r_s)^2 \right\}. \quad (11)$$

Bank i chooses its investment amount k^i, given $1 + r_L^e = mf'(K)$, recognizing that $l_B^i = \frac{E_1+D_1}{n} - k^i \geq \underline{l}_B$.

We obtain the following lemma:

Lemma 4

In any equilibrium without regulatory intervention in which banks do not default in state l (and h), $1 + r_L^e = (1 + r_s)^2$ and $K = \bar{K}$.

The proof is in Appendix A. In aggregate, banks hold voluntary liquidity L_B at least equal to $\underline{L}_B \,(= n\,\underline{l}_B^i)$, i.e., at least the minimum amount necessary to be able to pay back the first-period depositors in $t = 2$. Owners are indifferent between investing the remaining funds, given by $W + D_1 - \bar{K} - \underline{L}_B$, in ST themselves or through banks. Hence, in equilibrium, $L_B \in [\underline{L}_B, W + D_1 - \bar{K}]$, with

$$\underline{L}_B = D_1 - \frac{D_2}{1+r_s} = \frac{r_s \mu W}{1+r_s}.$$

Existence of a no-bank-default equilibrium. Next, we examine the conditions under which banks do not default ($e_3^i \geq 0$) for $l_B^i = \underline{l}_B$, so that $r_{d1} = r_s$, $k^i = \frac{\bar{K}}{n}$, and $l_B^i \in \left[\underline{l}_B, \frac{W+D_1-\bar{K}}{n}\right]$ do indeed constitute an equilibrium. According to Lemma 3 and

Lemma 4, if bank i chooses minimum liquidity $l_B^i = \underline{l}_B$, it will default in $t = 2$ in state $\eta = l$ if and only if

$$(1 + r_L^l)k^i + \left(\underline{l}_B(1 + r_s) - \frac{D_1}{n}(1 + r_s)\right)(1 + r_s) < 0.$$

That is, bank i will default if the return on investment in LT in state l is sufficiently low and the bank's liquidity is not high enough to absorb the losses, so that the obligations towards the second depositors cannot be fulfilled. In this case, bank i does not obtain new deposits in $t = 2$ and defaults immediately.

We use l^{min} to denote the critical threshold for the size of the negative shock at which a bank only just survives. For the situation described in Lemma 4, it is defined by

$$l^{min} f'(\bar{K})\frac{\bar{K}}{n} - (1 + r_s)\frac{D_2}{n} = 0, \tag{12}$$

which yields

$$l^{min} = \frac{(1 + r_s)D_2}{f'(\bar{K})\bar{K}}. \tag{13}$$

So, if $l \geq l^{min}$ and $K = \bar{K}$, a bank with liquidity holdings $l_B^i = \underline{l}_B$ will never default. Since it will also not default for any liquidity holdings greater than the minimum ones, and since $r_{d1} = r_s$ and $K = \bar{K}$ imply $\mathbb{E}\left(\frac{e_3^i}{e_1^i}\right) = (1 + r_s)^2$ for all $l_B^i > \underline{l}_B$, owners are indifferent between investing any remaining resources in ST themselves or through banks.

Using equality $f'(\bar{K})(ph + (1-p)l) = (1 + r_s)^2$, we obtain the corresponding threshold level for the size of the positive shock, denoted by h^{max}:

$$\begin{aligned} h^{max} &= \frac{(1 + r_s)^2 - (1 - p)l^{min} f'(\bar{K})}{pf'(\bar{K})} \\ &= \frac{(1 + r_s)^2\bar{K} - (1 - p)(1 + r_s)D_2}{pf'(\bar{K})\bar{K}}. \end{aligned} \tag{14}$$

Non-existence of a no-bank-default equilibrium. We note that for $l < l^{min}$, there is no equilibrium in which banks do not default. For suppose there were an equilibrium in which banks do not default and thus $K = \bar{K}$, $r_{d1} = r_s$ and $mf'(\bar{K}) = (1 + r_s)^2$. Clearly, by the definition of l^{min}, banks with $l_B^i = \underline{l}_B$ would default—a contradiction. Now suppose that bank i held sufficient voluntary liquidity $l_B^i = \hat{l}_B^i$ (with $\hat{l}_B^i > \underline{l}_B$) to just avoid default in $\eta = l$.[10] Then its expected profits are given by

$$\mathbb{E}(e_3^i) = p\left[hf'(\bar{K})k^i + \left(\hat{l}_B^i - \underline{l}_B\right)(1+r_s)^2 - \frac{D_2}{n}(1+r_s)\right] + (1-p)\cdot 0, \tag{15}$$

where $e_3^i = 0$ for $\eta = l$, since the bank holds voluntary liquidity to only just survive in the bad state. If instead the bank decides to hold only voluntary liquidity $\underline{l}_B$ and to default in state $\eta = l$, its expected profits are given by

$$\mathbb{E}(e_3^i) = p\left[hf'(\bar{K})\left(k^i + \hat{l}_B^i - \underline{l}_B\right) - \frac{D_2}{n}(1+r_s)\right] + (1-p)\cdot 0, \tag{16}$$

where $e_3^i = 0$ for $\eta = l$, as the bank is protected by limited liability in case of default. Since $hf'(\bar{K}) > mf'(\bar{K}) = (1+r_s)^2$, Expression (16) is greater than Expression (15), and thus the bank prefers default in $\eta = l$. Hence, for $l < l^{min}$ an equilibrium without bank default does not exist.

Summary. We summarize our results within the following proposition:

Proposition 3

A.) Suppose that $l \geq l^{min}$ ($h \leq h^{max}$). Then there exists a no-bank-default equilibrium without regulatory intervention, with

- *(i)* $K = \bar{K}$,
- *(ii)* $L_B \in [\underline{L}_B, W + D_1 - \bar{K}]$,
- *(iii)* $E_1 = \bar{K} + L_B - \mu W$,
- *(iv)* $L_I = W - E_1$,
- *(v)* $r_{d2} = r_s$,
- *(vi)* $r_{d1} = r_s$.

B.) For $l < l^{min}$, there exists no equilibrium in which banks do not default.

We note that for $l \geq l^{min}$ we obtain an equilibrium with a first-best allocation regarding the investment in LT and ST and regarding the expected utility of debt holders, owners,

[10] Note that, for an individual bank, holding voluntary liquidity $l_B^i > \hat{l}_B^i$ is never strictly better than $l_B^i = \hat{l}_B^i$, since investing in LT has to be at least as profitable as investing in ST. We will explore regulatory liquidity requirements in Section 6.1.

and entrepreneurs. The amount of equity banks can attract is sufficient to survive if the negative shock is not too large.

5.2 Bank default

We next examine equilibria with bank default. Suppose for the moment that banks default in state $\eta = l$ in $t = 2$. The time-2 liquidation value of LT is zero. For banks to survive in $t = 2$ in state $\eta = h$, they must be able to fulfill their obligations towards the first depositors. As stated in Lemma 2, this requires liquidity holdings of at least $\underline{l}_B$.

Equilibrium deposit rate and liquidity holdings. In $t = 2$, each bank has funds $l_B^i(1 + r_s)$ from investment in ST in the previous period. In case of a bank default in $t = 2$, $\eta = l$, these funds are distributed among the debt holders. Therefore, the expected aggregate repayment to debt holders is given by

$$R_2^e = p(1 + r_{d1})D_1 + (1 - p)L_B(1 + r_s).$$

The expected repayment on deposits must at least equal $1 + r_s$ for debt holders to invest in deposits in $t = 1$. This yields the following equilibrium condition:

$$\begin{aligned} \frac{R_2^e}{D_1} = 1 + r_s = p(1 + r_{d1}) + (1 - p)\frac{L_B(1 + r_s)}{D_1}; \\ 1 + r_{d1} = \frac{1 + r_s}{p} - \frac{(1 - p)}{p}\frac{L_B(1 + r_s)}{D_1}. \end{aligned} \tag{17}$$

Banks' optimization problem. The expected equity of bank i in $t = 3$ is given by

$$\begin{aligned} \mathbb{E}\left(e_3^i\right) = p\bigg[&(1 + r_L^h)k^i + \left(l_B^i(1 + r_s) - (1 + r_{d1})\frac{D_1}{n} + d_2^i\right)(1 + r_s) \\ &- (1 + r_s)d_2^i\bigg]. \end{aligned} \tag{18}$$

Using $l_B^i = \frac{E_1 + D_1}{n} - k^i$, the optimization problem of bank i is as follows:

$$\begin{aligned} \max_{k^i}\left\{\mathbb{E}\left(e_3^i\right)\right\} = \max_{k^i}\bigg\{p\bigg(&(1 + r_L^h)k^i + \left(\frac{E_1}{n} - k^i\right)(1 + r_s)^2 \\ &+ \frac{D_1}{n}(1 + r_s)(r_s - r_{d1})\bigg)\bigg\}. \end{aligned} \tag{19}$$

Bank i acts competitively and chooses its investment amount k^i, taking returns as given and recognizing that $l_B^i = \frac{E_1+D_1}{n} - k^i \geq \underline{l}_B$.

We obtain the following lemma:

Lemma 5

In any equilibrium without regulatory intervention in which banks default in state $\eta = l$,

(i) $p(1 + r_L^h) = (1 + r_s)^2$ *and*

$$K = (f')^{-1}\left(\frac{(1 + r_s)^2}{ph}\right) =: K^*. \tag{20}$$

(ii) $l_B^i = \frac{\mu W}{n}\left(1 - \frac{p}{1+r_s}\right) =: \underline{l}_B^*$ *and*

$$r_{d1} = r_s + 1 - p =: r_{d1}^*. \tag{21}$$

The proof is in Appendix A. The result in the first part of the lemma is intuitive. In expected terms, banks pay back $(1 + r_s)^2$ to debt holders and equity holders from $t = 1$ to $t = 3$. As long-term investments are liquidated in state l (with zero liquidation value), expected payoff per unit of investment in LT is equal to $phf'(K^*)$. Hence, in equilibrium, $phf'(K^*) = (1 + r_s)^2$. The second part of the lemma establishes that banks hold the minimum amount of liquidity required to survive in $t = 2$, $\eta = h$. We note that equilibrium aggregate equity is then given by

$$E_1 = K^* - \frac{D_2 p}{(1 + r_s)}. \tag{22}$$

Equilibrium scale of LT investment. Now consider the equilibrium scale of LT investment K^* as a function of l, with $K^*(l)$ given by Equation (20). We note that $l < m$ and that, for m constant, $ph = m - (1 - p)l$. The following lemma establishes the properties of $K^*(l)$:

Lemma 6

Suppose there is equilibrium bank default in state $\eta = l$. Then

(i) $K^*(l)$ *exists and is uniquely determined,*

(ii) $K^*(l) < \bar{K}$,

(iii) $\frac{\mathrm{d}K^*(l)}{\mathrm{d}l} < 0$.

The proof is in Appendix A. The fact that, according to Part *(ii)* of the lemma, LT investment is lower when there is bank default in the bad state is intuitive. Since depositors demand a risk premium and equity returns in case of default are zero, an amount of LT investment smaller than $\bar{K}$ ensures sufficiently high marginal returns in the good state to make investment in bank equity attractive compared to ST investment. Part *(iii)* reflects the fact that banks are protected by limited liability, i.e., banks' returns on equity feature a zero lower bound. Thus, varying values of l do not affect them in the bad state if they default anyway. For m constant, however, smaller values of l imply correspondingly higher values of h, which positively affects banks' returns if the good state materializes. This makes investing in bank equity more attractive and increases the scale of LT investment.

Conditions for bank default. We next derive the conditions under which banks will actually default if $\eta = l$, and thus indeed $r_{d1} = r^*_{d1}$, $k^i = \frac{K^*}{n}$, and $l^i_B = \underline{l}^*_B$ in equilibrium. For the situation described in Lemma (5), bank i defaults in $t = 2$ in state $\eta = l$ if

$$e^i_3 = lf'(K^*(l))\frac{K^*(l)}{n} - \frac{D_2}{n}(1 + r_s) < 0. \tag{23}$$

We obtain

Lemma 7

A.) For $l < l^{min}$,

$$s(l) \coloneqq lf'(K^*(l))K^*(l) - D_2(1 + r_s) < 0. \tag{24}$$

B.) It holds that $s(0) < 0$ *and* $s'(0) > 0$. *Further, assume that*

$$\frac{-f''(K)K}{f'(K)} - \frac{f'''(K)K}{-f''(K)} \geq -1. \tag{25}$$

Then, $s(l)$ *has at most one extremum, which constitutes a maximum.*

C.) Assume Condition (25) holds. There is a unique solution to $s(l) = 0$ if and only if

$$K^*(m) - \frac{D_2 p}{1 + r_s} > 0. \tag{26}$$

The proof is in Appendix A. Part A.) of the lemma establishes that for $l < l^{min}$, Condition (23) holds and thus banks default. Part B.) elaborates on the existence and uniqueness of a value for l such that Condition (23) holds with equality. We note that Condition (25) as stated in Part B.) is sufficient but not necessary. It is met by all production functions of the type $f(K) = K^\alpha$ with $0 < \alpha < 1$ as well as by a logarithmic production function $f(K) = \ln(1+K)$. Assumption (25) implies that there are at most two solutions to $s(l) = 0$ within the interval (l^{min}, m).[11] If they exist, we use l^{min*} and l^{min**} ($l^{min} < l^{min*} < l^{min**} < m$) to denote them. If neither l^{min*} nor l^{min**} exist, Condition (23) always holds. If l^{min*} exists but l^{min**} does not, then $s(l) < 0$ for all $l < l^{min*}$, but $s(l) \geq 0$ for all $l \geq l^{min*}$. If both l^{min*} and l^{min**} exist, then $s(l) < 0$ for all $l < l^{min*}$ and $l > l^{min**}$, but $s(l) \geq 0$ for all $l^{min*} \leq l \leq l^{min**}$.

Case selection. To avoid case distinctions, we focus in what follows on the case where l^{min*} exists but l^{min**} does not, i.e., we focus on the case where Condition (26) as stated in Lemma 7 C.) holds.[12] We note that

$$l^{min*} = \frac{(1 + r_s)D_2}{f'(K^*(l^{min*}))K^*(l^{min*})} > l^{min}. \tag{27}$$

We obtain the following proposition:

Proposition 4

A.) Suppose $l < l^{min*}$. Then there exists an equilibrium where banks default in $t = 2$, $\eta = l$, with

(i) $K = K^* = (f')^{-1}\left(\frac{(1+r_s)^2}{ph}\right)$,

(ii) $E_1 = K^* - \frac{D_2 p}{1+r_s} =: E_1^*$,

(iii) $L_B = \underline{L}_B^* = D_1 - \frac{D_2 p}{1+r_s} = \frac{r_{d1}^* \mu W}{1+r_s}$,

[11] We note that, by definition, l cannot exceed m. Hence, we have implicitly assumed $l^{min} < m$. If $l^{min} \geq m$, then there obviously is a unique bank default equilibrium for all l ($l \leq m$).

[12] There are model specifications where this condition obtains, as we show in Section 6.4.

(iv) $L_I = W - K^* + \frac{D_2 p}{1+r_s}$,

(v) $r_{d1} = r_{d1}^* = r_s + 1 - p$.

B.) For $l \geq l^{min*}$, there exists no equilibrium with bank default.

The proof is in Appendix A. In contrast to the no-bank-default equilibrium in Proposition 3, L_B and E_1 in the equilibrium with bank default are uniquely determined. The reason is the following: Because of limited liability, it is optimal for an individual bank to invest only the minimum amount $\underline{l}_B^*$ in liquidity to survive in t=2, $\eta = h$, and to invest the rest in LT. If banks invest more than K^* in LT, however, the expected return on bank equity is lower than the return on investment in ST. Thus, owners provide only $E_1 = K^* + \underline{L}_B^* - \mu W$ of aggregate bank equity.

We note that, in equilibrium, the amount of LT investment is $K = K^* \, (< \bar{K})$ and second generation debt holders invest in ST if $\eta = l$ in $t = 2$. With regard to Proposition 2, we thus have $K < \bar{K}$ and $\gamma_2 < 1$, which implies that the equilibrium allocation is not Pareto-optimal. Two types of inefficiency are associated with bank default: first, there is early liquidation of long-term investment, second, the scale of long-term investment falls short of its optimum $\bar{K}$.

5.3 Summary

We can now summarize the findings from the previous two subsections. This will also allows us to comment on the uniqueness of equilibrium.

Since Proposition 3 B.) established that there is no equilibrium without bank default for $l < l^{min}$ and Proposition 4 A.) established the existence of an equilibrium with bank default for $l < l^{min*}$, and knowing that $l^{min} < l^{min*}$, we can conclude that for $l < l^{min}$, there is a unique equilibrium with bank default. Similarly, Proposition 3 A.) together with Proposition 4 B.) and the fact that $l^{min} < l^{min*}$ implies that for $l \geq l^{min*}$ there is a unique equilibrium without bank default.

For $l^{min} < l < l^{min*}$, equilibria with and without bank default coexist. To see this, first suppose that banks do not default and thus $K = \bar{K}$. Then, according to Condition (12) an individual bank that holds minimum liquidity $\underline{l}_B^i$ will not default, since $l > l^{min}$.

Second, suppose that banks default in state $\eta = l$ and thus $K = K^*$. Then, according to Condition (23) an individual bank will default, since $l < l^{min*}$ implies $s(l) < 0$.

The following theorem summarizes our results:

Theorem 1

There exist critical levels l^{min}, l^{min*}, satisfying

$$l^{min} = \frac{(1 + r_s)D_2}{f'(\bar{K})\bar{K}},$$
$$l^{min*} = \frac{(1 + r_s)D_2}{f'(K^*(l^{min*}))K^*(l^{min*})},$$

with $l^{min} < l^{min*}$, such that

(i) For $l \geq l^{min*}$, the unique equilibrium is characterized by Proposition 3 A.). The allocation is first-best.

(ii) For $l < l^{min}$, the unique equilibrium is characterized by Proposition 4 A.). There is inefficient early liquidation and an inefficient scale of long-term investment.

(iii) For $l^{min} \leq l < l^{min*}$, an equilibrium without bank default as characterized by Proposition 3 A.) and an equilibrium with bank default as characterized by Proposition 4 A.) coexist.

The intuition for these findings is as follows. First, for small macroeconomic shocks, i.e., when l is larger than l^{min*} (and h is correspondingly lower), the amount of equity banks attract is sufficient to buffer such shocks. Banks invest the socially optimal amount in LT. They invest in an amount of liquidity that enables them to refinance in $t = 2$.

Second, for large macroeconomic shocks, i.e., when l is lower than l^{min} (and h is correspondingly higher), banks default in case of $\eta = l$. This implies the possibility of inefficient early liquidation of the LT investment. Moreover, banks must pay a risk premium to first-period debt holders, as these debt holders anticipate the possibility of default. Banks have to take this risk premium into account when considering the amount of liquidity required to repay the first depositors in case of $\eta = h$, i.e., banks have to invest more in ST. The fact that debt holders receive higher interest rates in the good state also has a negative effect on the attractiveness of investing in bank equity. This requires larger good-state marginal returns from LT investment, if investing in bank equity is to stay attractive compared to ST investment. As a consequence, the equilibrium amount of LT investment in case of bank default is below its efficient scale $\bar{K}$.

Last, for moderate macroeconomic shocks, i.e., for $l \in [l^{min}, l^{min*})$, the two types of equilibria just described coexist. Bank default occurs as a self-fulfilling prophecy. If agents hold a positive outlook on banks' solvency and thus depositors do not demand a risk premium and banks do not expect to default, then we obtain the equilibrium without bank default. If, however, there are doubts about banks' solvency and agents act accordingly, then there is indeed equilibrium bank default.

6 Banking Regulation

We will now discuss whether and how regulatory intervention can alleviate the inefficiencies we have identified so far. In particular, we investigate whether liquidity and/or capital regulation can prevent bank default and implement the first-best.

6.1 Liquidity requirements

We start with liquidity requirements. Let ζ $(0 < \zeta < 1)$ denote a regulatory liquidity ratio in the spirit of Basel III's LCR, i.e., ζ denotes the amount of liquidity relative to deposits that bank i is obliged to hold in $t = 1$:

$$\frac{l_B^i}{d_1^i} \geq \zeta.$$

As banks are initially identical, i.e., $l_B^i = L_B/n$ and $d_1^i = D_1/n$, regulation need not differentiate across banks. Let us suppose for the moment that ζ is at a level where banks do not default.

Banks' optimization problem. The maximization problem of bank i, given regulatory policy ζ, is

$$\max_{k^i} \left\{ \mathbb{E}\left(e_3^i\right) \right\} = \max_{k^i} \left\{ (1 + r_L^e)k^i + \left(\left(\frac{E_1 + D_1}{n} - k^i \right) (1 + r_s) \right. \right.$$
$$\left. \left. - (1 + r_s)\frac{D_1}{n} + d_2^i \right) (1 + r_s) - (1 + r_s)d_2^i \right\}$$

$$= \max_{k^i} \left\{ (1 + r_L^e)k^i + \left(\frac{E_1}{n} - k^i \right) (1 + r_s)^2 \right\}, \tag{28}$$

subject to $\frac{l_B^i}{d_1^i} = \frac{E_1 + D_1}{D_1} - \frac{k^i}{d_1^i} \geq \zeta$.[13]

Mandatory liquidity. The regulator has to stipulate ζ, which prevents bankruptcy of bank i in state $\eta = l$. Then, according to Lemma 3, the regulatory liquidity ratio ζ has to satisfy the following condition:

$$(1 + r_L^l)k^i + \left(\zeta d_1^i (1 + r_s) - (1 + r_s) \frac{D_1}{n} \right) (1 + r_s) \geq 0,$$

which yields

$$\zeta \geq 1 - \frac{(1 + r_L^l)k^i}{(1 + r_s)^2 \frac{D_1}{n}}.$$

As we have observed in Section 5.1, an equilibrium in which banks do not default implies an amount of LT investment $\bar{K}$. The same argument applies here. We obtain

Proposition 5

Let $l < l^{min}$, so that in the absence of banking regulation a no-bank-default equilibrium does not exist. There exists a critical level for l, given by

$$l^{crit} = \frac{(\bar{K} - W)(1 + r_s)^2}{f'(\bar{K})\bar{K}} = m \left(1 - \frac{W}{\bar{K}} \right), \tag{29}$$

such that

A.) If $l \geq l^{crit}$, a regulatory liquidity ratio ζ^{reg} given by

$$\zeta^{reg} = 1 - \frac{l f'(\bar{K})\bar{K}}{(1 + r_s)^2 D_1} = 1 - \frac{l}{m} \frac{\bar{K}}{\mu W} \tag{30}$$

ensures that there is a no-bank-default equilibrium with

(i) $K = \bar{K}$,

(ii) $L_B \in \left[\zeta^{reg} \mu W, W + \mu W - \bar{K} \right]$,

(iii) $E_1 = \bar{K} + L_B - \mu W$,

[13] We note that the maximization problem under liquidity requirements, assuming that no default occurs, is identical to the maximization problem in the no-bank-default case without governmental intervention (cf. Section 5.1), except that l_B^i has to be larger than ζd_1^i.

(iv) $L_I = W - E_1$,

(v) $r_{d2} = r_s$,

(vi) $r_{d1} = r_s$.

B.) If $l < l^{crit}$, there exists no regulatory liquidity ratio such that banks do not default and aggregate efficient investment $\bar{K}$ occurs in LT.

The proof is in Appendix A. By Assumption 1, $\zeta^{reg} < 1$ for $l \geq l^{crit}$. We note that l_B^i has not only to comply with liquidity regulation ζ^{reg}, but must also ensure that there is enough liquidity to repay the first depositors in $t = 2$. Thus, L_B must be at least equal to $\max\{\underline{L}_B, \zeta^{reg}\mu W\}$, which, however, comes down to just $\zeta^{reg}\mu W$ for $l < l^{min}$.

By Equation (13), $l^{min} = (m\mu W)/((1 + r_s)\bar{K})$. We obtain $l^{crit} < l^{min}$, since Equation (29) and Assumption 1 yield

$$1 - \frac{W}{\bar{K}} < \frac{\mu W}{(1 + r_s)\bar{K}};$$
$$W + \frac{\mu W}{1 + r_s} > \bar{K}.$$

For $l < l^{crit}$, there are not enough resources available at $t = 1$ to simultaneously achieve the efficient scale of LT investment $\bar{K}$ and hold enough liquidity to avoid default in $t = 2$ if $\eta = l$. Hence, liquidity regulation can implement the first-best when $l \in [l^{crit}, l^{min})$, but not when $l < l^{crit}$.

Uniqueness of equilibrium. We note that Proposition 5 *A.)* does not establish a no-bank-default equilibrium as the unique equilibrium. For uniqueness, i.e., to rule out the existence of an equilibrium with bank default for $l < l^{min*}$, making use of Lemma 3 with $K = K^*$ and r_{d1} according to Equation (17) we require stronger liquidity regulation:

$$\zeta \geq 1 - \frac{plf'(K^*)K^*}{(1 + r_s)^2\mu W} = 1 - \frac{l}{h}\frac{K^*}{\mu W} := \zeta^{reg*} \ (> \zeta^{reg})\,. \tag{31}$$

Implementability of the first-best as the unique equilibrium. With liquidity regulation ζ^{reg*}, it is possible to achieve the efficient scale of LT investment $\bar{K}$ only for values of l for which $W + \mu W \geq \zeta^{reg*}\mu W + \bar{K}$, which we can write as

$$s_2(l) \coloneqq lf'(K^*(l))K^*(l) - \frac{(\bar{K} - W)(1 + r_s)^2}{p} \geq 0. \tag{32}$$

Clearly, $s_2(0) < 0$. Furthermore, $s_2(l) < 0$ for all $l < l^{crit}$, since $K^*(l) < \bar{K}$ by Lemma 6 *(ii)* and since $\left|\frac{f''(K)K}{f'(K)}\right| < 1$. Comparing Expression (32) to Expression (24) immediately shows $s_2'(l) = s'(l)$. Hence, Lemma 7 B.) applies, and again we focus on the case where a unique l within (l^{crit}, m) solves Condition (32) with equality, which as in Lemma 7 *C.)* is the case if and only if

$$K^*(m) - (\bar{K} - W) > 0. \tag{33}$$

Denote this unique solution to Equation (32) by l^{crit*}. We note that

$$l^{crit*} = \frac{(\bar{K} - W)(1 + r_s)^2}{pf'(K^*(l^{crit*}))K^*(l^{crit*})} \; (> l^{crit}). \tag{34}$$

It follows that, for $l^{crit*} \leq l < l^{min*}$, liquidity regulation ζ^{reg*} implements a unique no-bank-default equilibrium with L_B at least equal to $\max\{\underline{L}_B^*, \zeta^{reg*}\mu W\} = \zeta^{reg*}\mu W$ and LT investment K at its efficient scale $\bar{K}$. We note that $\zeta^{reg*} < 1$, for $l \geq l^{crit*}$.

Feasibility of the first-best as the unique equilibrium. We note that $l^{crit*} < l^{min*}$. By comparing Equations (27) and (34), this follows from Assumption 1. Hence, there is indeed a range of parameter values $l \in [l^{crit*}, l^{min*})$ for which liquidity regulation ζ^{reg*} is feasible and effective in implementing the first-best as the unique equilibrium.

6.2 Capital requirements

We next consider a regulatory capital ratio ρ ($0 < \rho < 1$) which stipulates that the equity financing of a bank i has to satisfy

$$\frac{e_1^i}{k^i} \geq \rho.$$

Let us suppose for the moment that ρ is at a level where banks do not default. Then owners will supply bank equity capital to the amount of at least $E_1 = \rho\bar{K}$, and we obtain $r_{d1} = r_s$, $K = \bar{K}$. It follows that banks will indeed not default if

$$(1 + r_L^l)\bar{K} + [L_B(1 + r_s) - D_1(1 + r_s)]\,(1 + r_s) \geq 0.$$

With the balance sheet $E_1 + D_1 = \bar{K} + L_B$ and $E_1 = \rho\bar{K}$, this yields

$$lf'(\bar{K})\bar{K} + \left(\rho\bar{K} - \bar{K}\right)(1 + r_s)^2 \geq 0$$

and thus

$$\begin{aligned}\rho &\geq 1 - \frac{lf'(\bar{K})}{(1 + r_s)^2} \\ &= 1 - \frac{l}{m}.\end{aligned}$$

We obtain the following proposition:

Proposition 6

Let $l < l^{min}$, so that in the absence of banking regulation a no-bank-default equilibrium does not exist. Then

A.) If $l \geq l^{crit}$, the capital requirement ρ^{reg} given by

$$\rho^{reg} = 1 - \frac{lf'(\bar{K})}{(1 + r_s)^2} = 1 - \frac{l}{m} \tag{35}$$

ensures that there is a no-bank-default equilibrium with

(i) $K = \bar{K}$,

(ii) $E_1 \in [\rho^{reg}\bar{K}, W]$,

(iii) $L_B = E_1 + \mu W - \bar{K}$,

(iv) $L_I = W - E_1$,

(v) $r_{d2} = r_s$,

(vi) $r_{d1} = r_s$.

B.) If $l < l^{crit}$, there exists no capital requirement ensuring that banks do not default and aggregate efficient investment occurs in LT.

The proof is in Appendix A. The proposition is analogous to Proposition 5 in the previous subsection. We note that E_1 has not only to comply with capital regulation ρ^{reg}, but must also be large enough to allow for enough liquidity holdings to repay the first depositors in $t = 2$. Thus, E_1 has to equal at least $\max\left\{\rho^{reg}\bar{K}, \bar{K} + \underline{L}_B - \mu W\right\}$, which, however, comes down to just $\rho^{reg}\bar{K}$ for $l < l^{min}$.

Uniqueness of equilibrium. We note that Proposition 6 *A.)* does not establish a no-bank-default equilibrium as the unique equilibrium. Ruling out the existence of an equilibrium with bank default for $l < l^{min*}$ requires stronger capital regulation, as we will show next. If agents expect banks to default under a regulatory capital ratio ρ, owners will supply bank equity capital to the amount $E_1 = \rho K^*$, and we obtain $K = K^*$ and r_{d1} according to Equation (17). Making use of Lemma 3, ruling out the existence of such an equilibrium with bank default requires

$$lf'(K^*)K^* + \rho K^* \frac{(1+r_s)^2}{p} - K^* \frac{(1+r_s)^2}{p} \geq 0,$$

that is, a regulatory capital ratio that satisfies

$$\begin{aligned}\rho &\geq 1 - \frac{plf'(K^*)}{(1+r_s)^2} \\ &= 1 - \frac{l}{h} =: \rho^{reg*}.\end{aligned} \tag{36}$$

Comparing Equation (36) to Equation (35) shows that $\rho^{reg*} > \rho^{reg}$, i.e., the regulatory capital ratio required to implement the first-best as the *unique* equilibrium is stronger than the one required to implement it as *an* equilibrium.

Implementability of the first-best as the unique equilibrium. If LT investment is to achieve its efficient scale $\bar{K}$, banks' capital ratios will only satisfy $\rho^{reg*}\,(> \rho^{reg})$ if $E_1/\bar{K} \geq \rho^{reg*}$, which in turn is only possible if

$$\begin{aligned}\frac{W}{\bar{K}} &\geq \rho^{reg*}; \\ l &\geq h\left(1 - \frac{W}{\bar{K}}\right).\end{aligned} \tag{37}$$

Obviously, there is a unique value of $l\,(< m)$ that solves Condition (37) with equality. We denote this value by $l^{crit'}$ and note that

$$l^{crit'} = \frac{(\bar{K} - W)(1+r_s)^2}{pf'(K^*(l^{crit'}))\bar{K}}\,(> l^{crit}). \tag{38}$$

For $l \geq l^{crit'}$, capital regulation ρ^{reg*} rules out an equilibrium with bank default and establishes a no-bank-default equilibrium with E_1 equal to at least $\max\{\bar{K}+\underline{L}_B-\mu W, \rho^{reg*}\bar{K}\}$ and LT investment K at its efficient scale $\bar{K}$.

Feasibility of the first-best as the unique equilibrium. We note that since $K^*(l) < \bar{K}$ for all $l\,(< m)$, comparing Expression (38) to Expression (34) yields $l^{crit'} < l^{crit*}\,(< l^{min*})$. It follows that there is indeed a range of parameter values $l \in [l^{crit'}, l^{min*})$ for which capital requirements ρ^{reg*} are feasible and effective in implementing the first-best as the unique equilibrium.

6.3 Comparing liquidity and capital regulation

Comparing the results from Section 6.1 to those in Section 6.2, we obtain

Theorem 2

Let $l < l^{min*}$. Then

(i) For $l \geq l^{crit}$, both liquidity regulation ζ^{reg} and capital regulation ρ^{reg} can implement the first-best as an equilibrium outcome.

(ii) For $l \geq l^{crit*}\,(> l^{crit'} > l^{crit})$, both liquidity regulation ζ^{reg*} and capital regulation ρ^{reg*} can implement the first-best as the unique equilibrium outcome.

(iii) For $(l^{crit} <)\, l^{crit'} \leq l < l^{crit*}$, only capital regulation ρ^{reg*} can implement the first-best as the unique equilibrium outcome.

Overall, Theorem 2 states that to guarantee a Pareto-optimal outcome, capital regulation is necessary in Case *(iii)*. In Cases *(i)*-*(ii)*, either form of regulation will suffice.

Both capital and liquidity regulation have to establish an equilibrium without bank default that still leaves banks with enough funds available to achieve the efficient scale of long-term investment $\bar{K}$, if they are to implement the first-best. Liquidity regulation, on the one hand, achieves this by ensuring that banks hold an amount of safe liquid assets (relative to deposits) large enough to make up for potential losses from risky long-term investment, thereby ruling out bank default. Without the risk of default, LT investment and bank equity investment dominate ST investment as long as $K < \bar{K}$. Hence, owners will provide banks with enough equity capital to achieve $K = \bar{K}$. On the other hand, capital regulation requires banks to have a certain amount of equity E_1 relative to LT investment. Banks can only attract equity capital if they can offer a return on equity that dominates the return on ST investment. As the regulatory capital ratio constrains the banks' amount of LT investment for any given amount of equity, they will voluntarily invest in an amount of liquid assets high enough to rule out default, which in turn allows for excess returns on bank equity until an amount of LT investment $\bar{K}$ has been reached.

Combining liquidity and capital regulation. Although liquidity and capital requirements work in different ways, they can achieve similar outcomes. This becomes clear if we note that Proposition 6 entails that the optimal capital regulation ρ^{reg} to implement a no-bank-default equilibrium already implies liquidity holdings L_B that match those implied by the corresponding optimal liquidity regulation ζ^{reg} in Proposition 5. Vice versa, liquidity regulation ζ^{reg} already implies an amount of equity capital E_1 that matches that of the corresponding optimal capital regulation ρ^{reg}. Since ρ^{reg} and ζ^{reg} also feature the same threshold value for l, i.e., they can both implement the first-best only for $l \geq l^{crit}$, we can immediately state the following corollary:

Corollary 1

Capital and liquidity requirements are perfect substitutes in implementing the first-best as an equilibrium outcome.

In particular, this means that with regard to implementing the first-best as *an* equilibrium outcome, combining capital regulation with liquidity regulation cannot yield benefits over using only one of the two.

Capital regulation is more powerful. The reason why capital regulation is superior to liquidity regulation in implementing the first-best as the *unique* equilibrium outcome, as established in Theorem 2, can be set out as follows: Ruling out equilibria with bank default through liquidity regulation requires stricter liquidity requirements than the ones required for implementing the first-best as just *an* equilibrium (i.e., $\zeta^{reg*} > \zeta^{reg}$). The additional regulatory liquidity, however, does not have to be (fully) matched by more equity. This is because the amount of LT investment is smaller in the former kind of equilibria than in the latter one (i.e., $K^* < \bar{K}$), which implies that for any given amount of equity capital there are more resources left for investment in liquid assets. It follows that the strengthening of regulation needed to rule out the existence of equilibria with default as opposed to implementing the first-best as just *an* equilibrium, is more significant when relying on liquidity requirements rather than capital regulation. In other words, a regulatory capital ratio that rules out the existence of an equilibrium with default leaves banks more room than the corresponding liquidity regulation to achieve the efficient scale of LT investment in the remaining equilibrium without default. We illustrate this finding in the numerical example below. The following corollary restates the result:

Corollary 2

Capital regulation is more powerful than liquidity regulation in implementing the first-best as the unique equilibrium outcome.

In combination, Corollaries 1 and 2 imply that optimal banking regulation can be achieved through capital regulation alone. Adding liquidity requirements to optimal capital regulation is redundant.

Finally, we note that banking regulation harms neither owners nor debt holders. In equilibrium, they earn an expected return $1 + r_s$ in each period both with and without regulation. Since banking regulation restores the first-best, the entrepreneurs will then benefit.

6.4 Numerical example

We conclude the section with a simple example. Suppose $f(K) = \ln(1 + K)$. The production function satisfies the Inada condition at infinity, but not at 0. The latter is not a problem as long as $ph > (1 + r_s)^2$ is assumed. We note that $\left|\frac{f''(K)K}{f'(K)}\right| = \frac{K}{1+K} < 1$ and Condition (25) in Lemma 7 holds: $\frac{-f''(K)K}{f'(K)} - \frac{f'''(K)K}{-f''(K)} = -\frac{K}{1+K} \geq -1$.

Suppose further that $r_s = 0.02$, $p = 0.7$, $m = 15$, $W = 10$ and $\mu = 1.25$. Since we keep m fixed, $ph = m - (1 - p)l$. We obtain $\bar{K} = 13.42$, and Assumption 1 holds. Since Conditions (26) and (33) hold, we obtain unique values for l^{min*}, l^{crit*} and $l^{crit'}$. Figure 3 illustrates the necessity of banking regulation and the capabilities of capital and liquidity regulation, depending on the magnitude of macro-risk.

Figure 3: Implementability of a no-bank-default equilibrium as a function of l

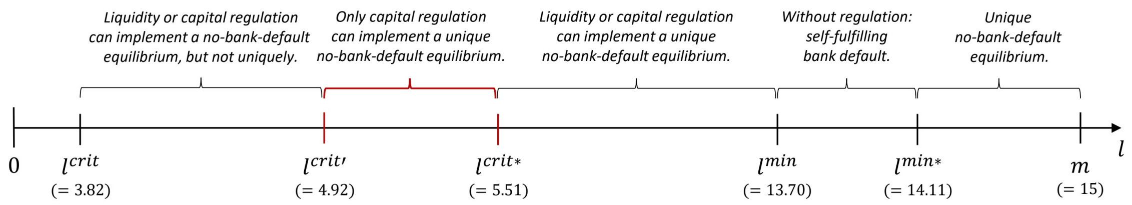
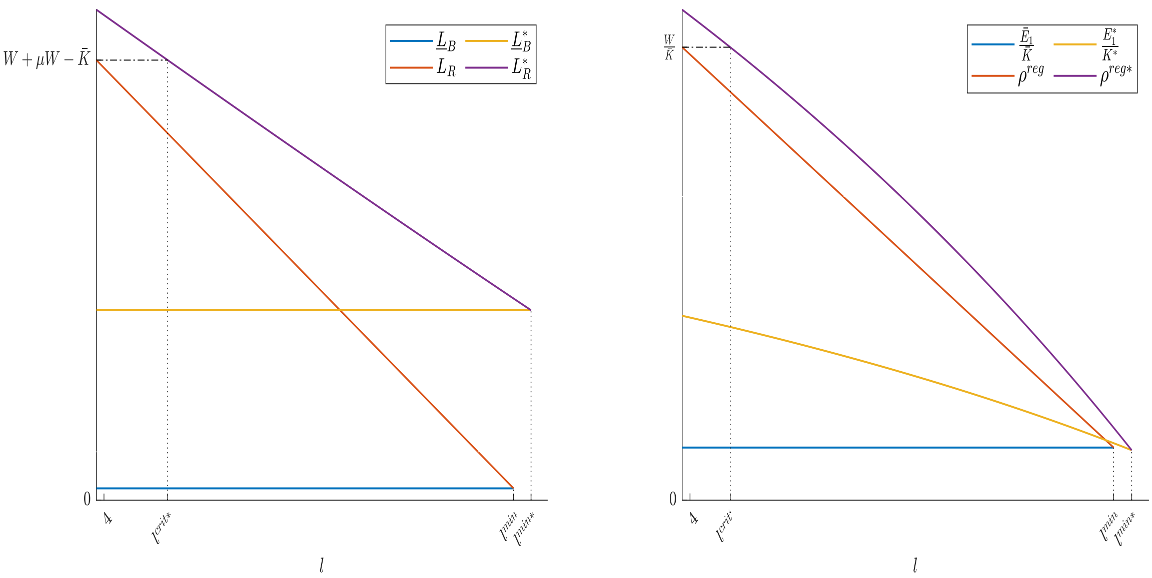


Figure 4 illustrates the liquidity and capital requirements that rule out bank default and implement the first-best as functions of l. Intuitively, Figure 4a shows that the liquidity holdings $L_R := \zeta^{reg}\mu W$ implied by the regulatory liquidity ratio ζ^{reg}, which implements the no-bank-default equilibrium as an equilibrium outcome, exceed $\underline{L}_B$ for $l < l^{min}$. Furthermore, the liquidity holdings $L_R^* := \zeta^{reg*}\mu W$ implied by ζ^{reg*}, which implements the no-bank-default equilibrium as the *unique* equilibrium, exceed $\underline{L}_B^*$ for $l < l^{min*}$. Analogously, Figure 4b shows that the capital ratio ρ^{reg} required to implement the no-bank-default equilibrium as an equilibrium exceeds $\frac{\bar{E}_1}{\bar{K}}$ for $l < l^{min}$. The capital ratio ρ^{reg*} that implements the no-bank-default equilibrium as the *unique* equilibrium exceeds $\frac{E_1^*}{K^*}$ for $l < l^{min*}$. We can further show that $\zeta_\Delta^{reg*}(l) := \frac{\zeta^{reg*}-\zeta^{reg}}{\zeta^{reg}} > \frac{\rho^{reg*}-\rho^{reg}}{\rho^{reg}} =: \rho_\Delta^{reg*}(l)$ for $l^{crit} < l < l^{min*}$. This is in line with our argument that in order to rule out the existence of an equilibrium with default, regulation has to be strengthened by more (in relative terms) if one relies on liquidity requirements rather than on a regulatory capital ratio. For instance, $\zeta_\Delta^{reg*}(5) = 0.17 > 0.11 = \rho_\Delta^{reg*}(5)$.

Figures 5 and 6 shed further light on why capital regulation is more powerful than liquidity regulation. Figure 5 illustrates the required minimum liquidity holdings under liquidity regulation as well as the implied minimum liquidity holdings under capital regulation, both as a function of l. Figure 5a considers regulation aimed at implementing the first-best as *an* equilibrium outcome. As we can see, the implied liquidity holdings under capital regulation, given by $\rho^{reg}\bar{K} + \mu W - \bar{K}$ via the banks' balance sheet, are equal to the liquidity holdings $\zeta^{reg}\mu W$ under liquidity regulation for all l. Implementing the first-best as an equilibrium outcome is not possible for $l < l^{crit}$, since the (implied) liquidity requirements ruling out bank default do not leave enough funds available to achieve the efficient scale of long-term investment: $\zeta^{reg}(l)\mu W > W + \mu W - \bar{K} = 9.08$

Figure 4: Banking regulation as a function of l



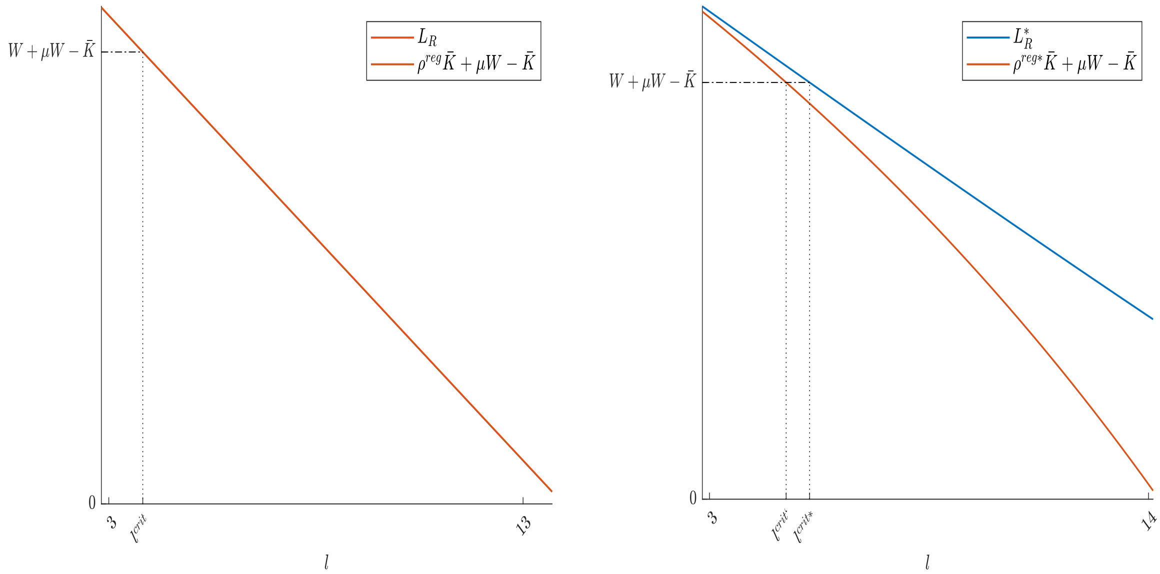
(a) Liquidity regulation

(b) Capital regulation

for $l < l^{crit}$. Figure 5b considers regulation aimed at implementing the first-best as the *unique* equilibrium outcome. Strikingly, the implied liquidity holdings under capital regulation, given by $\rho^{reg*}\bar{K} + \mu W - \bar{K}$, are smaller than the minimum liquidity holdings $\zeta^{reg*}\mu W$ under liquidity regulation for any given l. This means that for any given l, capital regulation ρ^{reg*} leaves banks with more free funds than liquidity regulation ζ^{reg*} to achieve the efficient scale of long-term investment $\bar{K}$ in the equilibrium without bank default. As a consequence, only capital regulation can implement the first-best as the unique equilibrium for $l^{crit'} \leq l < l^{crit*}$.

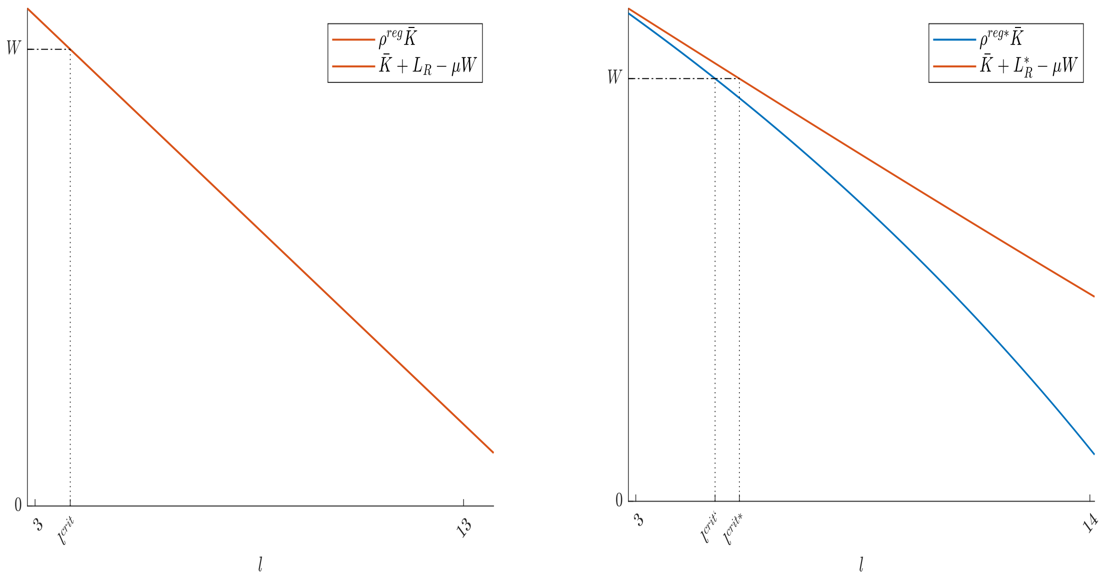
Figure 6 is analogous to Figure 5, except that it compares the banks' minimum amount of equity capital required under capital regulation to the banks' implicitly required minimum amount of equity capital under liquidity regulation, both as a function of l. Figure 6a considers regulation aimed at implementing the first-best as *an* equilibrium outcome. As we can see, the amount of equity capital under capital regulation, given by $\rho^{reg}\bar{K}$, is equal to the implied amount of equity capital $\bar{K} + \zeta^{reg}\mu W - \mu W$ under liquidity regulation for all l. Implementing the first-best as an equilibrium outcome is not possible for $l < l^{crit}$, since the (implied) capital holdings required to rule out bank default and

Figure 5: (Implied) regulatory liquidity holdings



(a) Equilibrium without bank default exists (b) Equilibrium without bank default is unique

Figure 6: (Implied) regulatory equity capital



(a) Equilibrium without bank default exists (b) Equilibrium without bank default is unique implement LT investment at its efficient scale $\bar{K}$ exceed the maximum available amount of equity capital that owners can provide: $\rho^{reg}(l)\bar{K} > W = 10$, for $l < l^{crit}$. Figure 6b considers regulation aimed at implementing the first-best as the *unique* equilibrium outcome. Strikingly, the implicitly required amount of equity capital under liquidity regulation, given by $\bar{K} + \zeta^{reg*}\mu W - \mu W$, is larger than the required equity capital $\rho^{reg*}\bar{K}$ under capital regulation, for any given l. This means that if the first-best amount of LT investment $\bar{K}$ is to be achieved, capital regulation ρ^{reg*} allows banks to operate with less equity capital than the implicitly required amount of equity capital under liquidity regulation ζ^{reg*}, for any given l. As a consequence, only capital regulation can implement the first-best as the unique equilibrium for $l^{crit'} \leq l < l^{crit*}$.

7 Conclusion

We propose a simple model to study the interplay of capital and liquidity regulation by focusing on future funding risks of banks. Since bank equity is protected by limited liability, banks may hold insufficient amounts of capital or liquidity to survive in all states of the world. Once it becomes clear that a bank may encounter solvency issues in the future, it is likely to experience refinancing problems and may have to default straightaway. With regard to regulatory and political discussions on whether a bank is insolvent or illiquid, this means that although a bank may not yet be technically insolvent, it may still experience liquidity problems related to doubts about solvency in the future.

If macroeconomic shocks are not too large, banking regulation can help to avoid bank default and implement the efficient allocations. Capital requirements turn out to be more powerful than liquidity requirements in implementing the first-best as the unique equilibrium. Capital regulation also solves the liquidity problem, since well-capitalized banks are not at risk of defaulting in the future and thus can always refinance their liquidity needs. To some extent, liquidity requirements can also address solvency risk, since they require banks to hold larger amounts of safe short-term assets, which can make up for losses from risky long-term investments. From this perspective, capital and liquidity requirements are substitutes rather than complements.

In terms of policy, equilibrium multiplicity in the sense of self-fulfilling expectations highlights the fact that (re)building investors' and depositors' trust in the resilience of the banking system is complementary to actual banking regulation. In the presence of potentially large macro-shocks policy faces a trade-off, since banking regulation then cannot rule out bank default in all future states of the world while at the same time leaving banks with sufficient funds to achieve their efficient scale of long-term investment. Regulation then has to either accept the possibility of bank default in very bad states of the world or strongly limit banks' scale of long-term investment in all states of the world. Lastly, adding liquidity requirements to optimal capital regulation is redundant, since under optimal capital regulation banks will already hold sufficient amounts of voluntary liquidity.

A Proofs

Proof of Proposition 1

Proposition 1 follows from the following considerations: First, at equilibrium prices, buying long-term assets by owners and debt holders in $t = 1$ and $t = 2$ must be as profitable as (repeatedly) investing in ST in order to maximize their expected consumption. Debt holders who buy long-term assets in $t = 2$ achieve a return

$$\frac{r^\eta}{q_2^\eta} = \frac{\frac{\eta(1+r_s)^2}{m}}{q_2^\eta},$$

which equals the one-period return from ST investment if and only if $q_2^\eta = \hat{q}_2^\eta$, for $\eta \in \{l, h\}$. Table 1 shows asset demand and supply in $t = 2$. For $q_2^\eta > \hat{q}_2^\eta$, ST investment dominates the returns from buying the asset, and thus asset supply exceeds demand. For $q_2^\eta < \hat{q}_2^\eta$, holding the asset is more profitable than selling it and investing in ST instead. Thus, owners do not supply the asset to the market. Late debt holders, however, would like to acquire assets. Since $\bar{K} - W < \mu W$ by Assumption 1, asset demand exceeds supply. It follows that market clearing is possible only for $q_2^\eta = \hat{q}_2^\eta$, $\eta \in \{l, h\}$.[14]

Table 1: *Supply and demand for long-term assets in $t = 2$*

Price	**Supply**	**Demand**
$q_2^\eta > \hat{q}_2^\eta$	$\bar{K}$	0
$q_2^\eta = \hat{q}_2^\eta$	$\bar{K} - W$	any value in $[0, \mu W]$
$q_2^\eta < \hat{q}_2^\eta$	$\bar{K} - W$	μW

To determine q_1, we note that, in period $t = 1$, the expected payoff in $t = 3$ of one unit of the long-term asset is $pr^h + (1-p)r^l = (1 + r_s)^2$. Anticipating equilibrium prices in $t = 2$, the payment that first-period debt holders can expect from selling one long-term asset in $t = 2$ is

$$p\hat{q}_2^h + (1-p)\hat{q}_2^l = 1 + r_s.$$

[14] At $q_2^\eta = \hat{q}_2^\eta$, owners are indifferent between selling their long-term assets, investing in ST, and holding the assets. Without loss of generality, we assume they continue to hold their assets for $q_2^\eta = \hat{q}_2^\eta$. However, any distribution of long-term asset holdings in $t = 2$ among owners and late debt holders constitutes an equilibrium if it satisfies the wealth constraint.

Considering further that ST investment yields a return $(1 + r_s)$ per period, asset market clearing in $t = 1$ implies $q_1 = \hat{q}_1 = 1$. Profit maximization by entrepreneurs then yields $a_L = \hat{a}_L = \bar{K}$.

Finally, we observe that the consumption plans given by Equation (7) are feasible. Since $\hat{q}_2^h > \hat{q}_2^l$, we know that $\hat{C}_2^D(\eta = h) > \hat{C}_2^D(\eta = l)$. With Assumption 1 we obtain $\hat{C}_2^D(\eta = l) > 0$. Furthermore, Assumption (1) together with $m > ph$ implies that $(1 + r_s)\frac{h}{m}(\bar{K} - W) < \mu W$ and thus

$$\begin{aligned}\hat{C}_2^D(\eta = h) &= (\bar{K} - W)\hat{q}_2^h + (\mu W + W - \bar{K})(1 + r_s) \\ &= \frac{h(1 + r_s)}{m}(\bar{K} - W) + (\mu W + W - \bar{K})(1 + r_s) \\ &< \mu W + (1 + r_s)(\mu W + W - \bar{K}),\end{aligned}$$

where $\mu W + (1 + r_s)(\mu W + W - \bar{K})$ is the total amount of resources available at $t = 2$. This completes the proof. □

Proof of Proposition 2

We prove the Proposition in two steps. In Step 1, we show that no pair except $(\gamma_2, \bar{K})$ with $\gamma_2 \in [0, 1)$ and $(1, K)$ with $K \leq \bar{K}$ can be Pareto-optimal. In Step 2, we will show that the pairs referred to here are Pareto-optimal.

Step 1

We can immediately state that no pair $(\gamma_2, \bar{K})$ with $K > \bar{K}$ can ever be Pareto-optimal, since in that case ST investment yields higher expected returns than LT investment. A marginal decrease in K, i.e., a shift in resources from LT investment to ST investment, accompanied by an adequate decrease in γ_2, would then obviously allow for a Pareto improvement. Thus, for the rest of Step 1, let $K \leq \bar{K}$. We consider three cases: $\gamma_2 = 0$, $\gamma_2 \in (0, 1)$, and $\gamma_2 = 1$.

Case (i): $\gamma_2 = 0$

For $\gamma_2 = 0$, expected period-2 consumption is zero. Then, no K that does not maximize expected period-3 consumption can be Pareto-optimal. Hence, the only candidate for a Pareto-optimum is $(0, K)$ with LT investment at its efficient scale $K = \bar{K}$.

Case (ii): $\gamma_2 \in (0,1)$

From Equations (8)-(9), we obtain

$$\frac{\partial C_{S2}}{\partial K} = -\gamma_2(1+r_s) < 0,$$
$$\frac{\partial Y}{\partial K} = mf'(K) - (1-\gamma_2)(1+r_s)^2 > 0.$$

Clearly, C_{S2} is decreasing in K. For $K \leq \bar{K}$, period-3 consumption Y is certainly increasing in K. Furthermore,

$$\frac{\partial C_{S2}}{\partial \gamma_2} = (1+r_s)(W+\mu W-K)+\mu W > 0,$$
$$\frac{\partial Y}{\partial \gamma_2} = -(1+r_s)[(1+r_s)(W+\mu W-K)+\mu W] < 0.$$

If the C_{S2}-indifference curve and the Y-indifference curve in (γ_2, K)-space are not tangent at (γ_2, K), then (γ_2, K) cannot be Pareto-optimal. Hence, let us turn to the tangency condition, which is

$$\frac{(1+r_s)(W+\mu W-K)+\mu W}{\gamma_2(1+r_s)} = \frac{(1+r_s)[(1+r_s)(W+\mu W-K)+\mu W]}{mf'(K)-(1-\gamma_2)(1+r_s)^2}.$$

This implies $\gamma_2(1+r_s)^2 = mf'(K) - (1-\gamma_2)(1+r_s)^2$ or $mf'(K) = (1+r_s)^2$ or $K = \bar{K}$. Hence, for $\gamma_2 \in (0,1)$, only pairs (γ_2, K) with $K = \bar{K}$ are candidates for Pareto-optima.

Case (iii): $\gamma_2 = 1$

Third-period consumption in case of $\gamma_2 = 1$ is simply given by $Y = mf(K)$, which is increasing in K. As already set out, pairs with $K > \bar{K}$ cannot be Pareto-optimal. Hence only pairs $(1, K)$ with $K \leq \bar{K}$ qualify as candidates for Pareto-optima.

Step 2

It remains to be shown that all pairs $(\gamma_2, \bar{K})$ with $\gamma_2 \in [0,1)$ and all pairs $(1, K)$ with $K \leq \bar{K}$ are Pareto-optimal. A pair (γ_2, K) is Pareto-optimal if it is not (weakly) Pareto-dominated by some other pair (γ_2', K').

First, consider the pair $(\gamma_2, K) = (0, \bar{K})$, which implies $C_{S2} = 0$ and $Y = mf(\bar{K}) + (1+r_s)\left[(1+r_s)(W+\mu W-\bar{K})+\mu W\right]$. Any pair (γ_2', K') with $\gamma_2' > 0$ or $K' \neq \bar{K}$ would yield $Y' < Y$. It follows that no other pair Pareto-dominates the pair $(0, \bar{K})$, which accordingly is Pareto-optimal.

Next, consider a pair $(\gamma_2, \bar{K})$ with $\gamma_2 \in (0,1)$. Suppose this pair is not Pareto-optimal, i.e., there exists a pair (γ_2', K') that Pareto-dominates it. In Step 1 we already established that this would require $K' = \bar{K}$. But any $(\gamma_2', \bar{K})$ with $\gamma_2' < \gamma_2$ decreases C_{S2}, while any $(\gamma_2', \bar{K})$ with $\gamma_2' > \gamma_2$ decreases Y. It follows that no other pair Pareto-dominates the pair $(\gamma_2, \bar{K})$ with $\gamma_2 \in (0,1)$. Hence, such a pair is Pareto-optimal.

Last, consider a pair $(1, K)$ with $K \leq \bar{K}$, which gives rise to $C_{S2} = (1+r_s)(W + \mu W - K) + \mu W$ and $Y = mf(K)$. Any pair (γ_2', K') with $\gamma_2' \leq 1$ and $K' \geq K$ yields $C_{S2}' \leq C_{S2}$. Any pair $(1, K')$ with $K' < K$ yields $Y' < Y$. Finally, any pair (γ_2', K') with $\gamma_2' < 1$ and $K' < K$ necessarily yields $C_{S2}' < C_{S2}$ or $Y' < Y$, since $K' < K$ implies moving (further) away from the efficient scale of LT investment $\bar{K}$. We have thus shown that no other pair Pareto-dominates the pair $(1, K)$ with $K \leq \bar{K}$, which accordingly is Pareto-optimal. □

Proof of Lemma 4

We prove $1+r_L^e = (1+r_s)^2$ by showing that neither $1+r_L^e > (1+r_s)^2$ nor $1+r_L^e < (1+r_s)^2$ can hold in equilibrium.

Step 1

First, suppose that $1 + r_L^e > (1 + r_s)^2$. This implies that the aggregate amount of LT investment K satisfies $K < (f')^{-1}\left(\frac{(1+r_s)^2}{m}\right) = \bar{K}$. Then,

$$\begin{aligned}\mathbb{E}(e_3^i) &= (1 + r_L^e)k^i + \left(\frac{E_1}{n} - k^i\right)(1 + r_s)^2 \\ &> (1 + r_s)^2 k^i + \left(\frac{E_1}{n} - k^i\right)(1 + r_s)^2 = \frac{E_1}{n}(1 + r_s)^2,\end{aligned}$$

i.e., the return on equity is higher than the return on investment in ST. This cannot be optimal, since by Assumption 1 and $r_{d1} = r_s$, $D_1 = D_2 = \mu W$, there are enough resources in the economy for a minimum ST investment $\underline{L}_B$ to ensure the banks' survival in $t = 2$ and an LT investment at its efficient scale $\bar{K}$:

$$\begin{aligned}W + D_1 &> D_1 - \frac{D_2}{1 + r_s} + \bar{K} \\ &= \underline{L}_B + \bar{K}.\end{aligned}$$

Hence, as long as $1 + r_L^e > (1 + r_s)^2$, owners would increase their investment in bank equity, and banks would channel these additional funds into LT investment. This yields a contradiction to $1 + r_L^e > (1 + r_s)^2$.

Step 2

Second, suppose that $1 + r_L^e < (1 + r_s)^2$. Then,

$$\mathbb{E}(e_3^i) < (1 + r_s)^2 k^i + \left(\frac{E_1}{n} - k^i\right)(1 + r_s)^2 = \frac{E_1}{n}(1 + r_s)^2.$$

This means that the owners would not invest in bank equity since the return on investment in ST is higher. Therefore we obtain a contradiction to $1 + r_L^e < (1 + r_s)^2$.

Step 3

Combining our results from Step 1 and Step 2, we find that in any equilibrium

$$1 + r_L^e = (1 + r_s)^2.$$

Together with $1 + r_L^e = mf'(K)$ it immediately follows that the equilibrium aggregate investment in LT is $\bar{K}$. □

Proof of Lemma 5

We first prove that $p(1+r_L^h) = (1+r_s)^2$ by showing that neither $p(1+r_L^h) > (1+r_s)^2$ nor $p(1 + r_L^h) < (1 + r_s)^2$ can hold in equilibrium. Then we argue that this implies $l_B^i = \underline{l}_B^*$.

Step 1

First, suppose that $p(1 + r_L^h) > (1 + r_s)^2$. This implies that aggregate LT investment K satisfies $K < (f')^{-1}\left(\frac{(1+r_s)^2}{ph}\right) < (f')^{-1}\left(\frac{(1+r_s)^2}{m}\right) = \bar{K}$. Using Equations (17) and (18) yields

$$\begin{aligned}
\mathbb{E}(e_3^i) &= p(1 + r_L^h)\frac{K}{n} + p\left(\frac{L_B}{n}(1 + r_s) - (1 + r_{d1})\frac{D_1}{n} + \frac{D_2}{n}\right)(1 + r_s) - p(1 + r_s)\frac{D_2}{n} \\
&= p(1 + r_L^h)\frac{K}{n} + p\frac{L_B}{n}(1 + r_s)^2 - (1 + r_s)^2\frac{D_1}{n} + (1 - p)(1 + r_s)^2\frac{L_B}{n} \\
&= p(1 + r_L^h)\frac{K}{n} + \frac{L_B}{n}(1 + r_s)^2 - (1 + r_s)^2\frac{D_1}{n} \\
&> (1 + r_s)^2\left(\frac{K}{n} + \frac{L_B}{n} - \frac{D_1}{n}\right) = (1 + r_s)^2\frac{E_1}{n}.
\end{aligned}$$

The return on equity is higher than the return on investment in ST. Assumption 1 implies

$$\begin{aligned} W + \mu W &> \mu W - \frac{p\mu W}{1+r_s} + \bar{K} \\ &> \mu W \left(1 - \frac{p}{1+r_s}\right) + (f')^{-1}\left(\frac{(1+r_s)^2}{ph}\right) \\ &= \underline{L}_B^* + (f')^{-1}\left(\frac{(1+r_s)^2}{ph}\right). \end{aligned}$$

This means that either (a) the banks' voluntary liquidity must exceed minimum liquidity $\underline{L}_B^* \coloneqq n\underline{l}_B^*$, which cannot be optimal as $p(1+r_L^h) > (1+r_s)^2$ implies that LT investment yields a higher return, or (b) owners must be investing some of their resources in ST, which cannot be optimal as bank equity yields a higher return. Hence we obtain a contradiction to $p(1+r_L^h) > (1+r_s)^2$.

Step 2

Second, suppose that $p(1+r_L^h) < (1+r_s)^2$. Then, we obtain

$$\begin{aligned} \mathbb{E}\left(e_3^i\right) &= p(1+r_L^h)\frac{K}{n} + \frac{L_B}{n}(1+r_s)^2 - (1+r_s)^2\frac{D_1}{n} \\ &< (1+r_s)^2\left(\frac{K}{n} + \frac{L_B}{n} - \frac{D_1}{n}\right) = (1+r_s)^2\frac{E_1}{n}, \end{aligned}$$

which implies that owners would not invest in bank equity since the return on investment in ST is higher. Hence, we obtain a contradiction to $p(1+r_L^h) < (1+r_s)^2$.

Step 3

Combining our results from Step 1 and Step 2, we find that in any equilibrium

$$p(1+r_L^h) = (1+r_s)^2. \tag{A.1}$$

From $p(1+r_L^h) = phf'(K)$ and Equation (A.1) it immediately follows that $K^* = (f')^{-1}\left(\frac{(1+r_s)^2}{ph}\right)$.

Step 4

Finally, we argue that $p(1+r_L^h) = (1+r_s)^2$ implies $l_B^i = \underline{l}_B^*$. Because of limited liability, the expected marginal return of an individual bank from investing in LT is $p(1+r_L^h)$, while the bank's expected marginal return from repeatedly investing in liquid assets is $p(1+r_s)^2$. Since $p(1+r_L^h) = (1+r_s)^2 > p(1+r_s)^2$, the bank holds only the minimum amount of liquidity required to survive in $t = 2$, $\eta = h$. Equation (21) follows from substituting $L_B = n\underline{l}_B = \frac{\mu W}{1+r_s} r_{d1}$ into Equation (17). With $r_{d1} = r^*_{d1}$ follows $l^i_B = \underline{l}^*_B$. $\square$

Proof of Lemma 6

Step 1

We first establish existence and uniqueness. From the assumption of concavity of $f(K)$, there exists a uniquely determined K for every $f'(K)$, i.e., there exists a uniquely determined K^* for all $l\,(< m)$.

Step 2

From Equation (20) we obtain

$$K^* = (f')^{-1}\left(\frac{(1+r_s)^2}{m-(1-p)l}\right) < (f')^{-1}\left(\frac{(1+r_s)^2}{m}\right) = \bar{K}.$$

Therefore $K^* < \bar{K}$ has to hold for all $l\,(< m)$.

Step 3

We rewrite Equation (20) as

$$f'(K^*) = \frac{(1+r_s)^2}{m-(1-p)l}.$$

We differentiate both sides with respect to K^* and l and obtain

$$\frac{\mathrm{d}K^*}{\mathrm{d}l} = \frac{(1-p)(1+r_s)^2}{[m-(1-p)l]^2 f''(K^*)} < 0. \tag{A.2}$$

$\square$

Proof of Lemma 7

Part A.)

For Part A.) of the lemma, we note that for $l < l^{min}$,

$$\begin{aligned} lf'(K^*(l))K^*(l) &< l^{min} f'(K^*(l))K^*(l) \\ &< l^{min} f'(\bar{K})\bar{K} \\ &= D_2(1+r_s), \end{aligned}$$

where the second inequality follows from Lemma 6 *(ii)* and the assumption that $\left|\frac{f''(K)K}{f'(K)}\right| <$ 1.

Part B.)

We next prove Part B.). Obviously, $s(0) = -D_2(1 + r_s) < 0$. Taking the derivative of $s(l)$ yields

$$s'(l) = f'(K^*)K^* + l\left[f''(K^*)K^* + f'(K^*)\right]\frac{\partial K^*}{\partial l}.$$

With $\partial K^*/\partial l$ according to Equation (A.2) and $(m - (1 - p)l)f'(K^*) = (1 + r_s)^2$, we obtain

$$\begin{aligned}
s'(l) &= \frac{(1+r_s)^2}{m-(1-p)l}K^* + l\left[f''(K^*)K^* + f'(K^*)\right]\frac{(1-p)(1+r_s)^2}{f''(K^*)(m-(1-p)l)^2} \\
&= \frac{(1+r_s)^2K^*}{m-(1-p)l}\left[1 + \frac{(1-p)l}{(m-(1-p)l)}\left(1 + \frac{f'(K^*)}{f''(K^*)K^*}\right)\right] \\
&= \frac{(1+r_s)^2K^*}{m-(1-p)l}\left[\frac{(m-(1-p)l)+(1-p)l}{(m-(1-p)l)} + \frac{(1-p)l}{(m-(1-p)l)}\frac{f'(K^*)}{f''(K^*)K^*}\right] \\
&= \underbrace{\frac{(1+r_s)^2K^*}{(m-(1-p)l)^2}}_{=:A}\underbrace{\left[m + (1-p)l\frac{f'(K^*)}{f''(K^*)K^*}\right]}_{=:B}.
\end{aligned}$$

We immediately see that $s'(0) = Am > 0$. Since $A > 0$, the FOC is given by $B = 0$. We show that B is strictly decreasing, which implies that if a solution to the FOC exists, then it is unique and yields a maximum. Taking the derivative:

$$\begin{aligned}
\frac{\partial B}{\partial l} = (1-p)\Bigg[&\frac{f'(K^*)}{f''(K^*)K^*} \\
&+ l\left[\frac{f''(K^*)}{f''(K^*)K^*} - \frac{f'(K^*)}{\left[f''(K^*)K^*\right]^2}\left[f'''(K^*)K^* + f''(K^*)\right]\right]\frac{\partial K^*}{\partial l}\Bigg].
\end{aligned}$$

This derivative is smaller than zero if

$$\begin{aligned}
1 + \frac{l}{K^*}\frac{\partial K^*}{\partial l}\left[\frac{f''(K^*)K^*}{f'(K^*)} - \frac{1}{f''(K^*)}\left[f'''(K^*)K^* + f''(K^*)\right]\right] &> 0; \\
1 + \frac{l}{K^*}\frac{\partial K^*}{\partial l}\left[\frac{f''(K^*)K^*}{f'(K^*)} - \frac{f'''(K^*)K^*}{f''(K^*)} - 1\right] &> 0; \\
1 - \frac{l}{K^*}\frac{\partial K^*}{\partial l} - \frac{l}{K^*}\frac{\partial K^*}{\partial l}\left[\frac{-f''(K^*)K^*}{f'(K^*)} - \frac{f'''(K^*)K^*}{-f''(K^*)}\right] &> 0.
\end{aligned}$$

The fact that $\partial K^*/\partial l < 0$ and that by assumption the term in brackets is greater than -1 completes the proof.

Part C.)

Last, we prove Part C.). From Part B.) we know that a unique solution $l \in (l^{min}, m)$ to the problem $s(l) = 0$ exists if and only if $s(m) > 0$, i.e., if and only if

$$\begin{aligned} mf'(K^*(m))K^*(m) - D_2(1+r_s) &> 0; \\ m\frac{(1+r_s)^2}{m-(1-p)m}K^*(m) - D_2(1+r_s) &> 0; \\ \frac{1+r_s}{p}K^*(m) - D_2 &> 0; \\ K^*(m) - \frac{D_2 p}{1+r_s} &> 0. \end{aligned}$$

□

Proof of Proposition 4

Part A.)

We show that for $l < l^{min*}$, there is an equilibrium with bank default in state $\eta = l$. To see this, suppose that banks default in equilibrium in state $\eta = l$ and thus $K = K^*$, $r_{d1} = r_s + 1 - p$ and $phf'(K^*) = (1+r_s)^2$. Then, by the definition of l^{min*}, a bank with $l_B^i = \underline{l}_B^*$ will default. Now suppose that bank i deviates by holding sufficient voluntary liquidity $l_B^i = \hat{l}_B^{i*}$ ($\hat{l}_B^{i*} > \underline{l}_B^*$) to just avoid default in $\eta = l$. Then its expected profits would be given by

$$\mathbb{E}(e_3^i) = p\left[hf'(K^*)k^i + \left(\hat{l}_B^{i*} - \underline{l}_B^*\right)(1+r_s)^2 - \frac{D_2}{n}(1+r_s)\right] + (1-p)\cdot 0, \tag{A.3}$$

where $e_3^i = 0$ for $\eta = l$, since the bank holds voluntary liquidity to only just survive in the bad state. If instead the bank holds only voluntary liquidity $\underline{l}_B^*$ and defaults in state $\eta = l$, its expected profits are given by

$$\mathbb{E}(e_3^i) = p\left[hf'(K^*)\left(k^i + \hat{l}_B^{i*} - \underline{l}_B^*\right) - \frac{D_2}{n}(1+r_s)\right] + (1-p)\cdot 0, \tag{A.4}$$

where $e_3^i = 0$ for $\eta = l$, as in case of default the bank is protected by limited liability.

Since $hf'(K^*) > phf'(K^*) = (1+r_s)^2$, Expression (A.4) is greater than Expression (A.3), hence the bank prefers to hold minimum liquidity $\underline{l}_B^*$ and default in state $\eta = l$. Accordingly, for $l < l^{min*}$ there is an equilibrium with bank default.

We further observe that the amount of resources owners invest in ST is given by $L_I = W - E_1$ and the equilibrium values of r_{d1}, l_b^i, K and E_1 are given according to Lemma 5 *(ii)* and Equations (20) and (22).

Part B.)

An equilibrium with bank default does not exist for $l > l^{min*}$. To see this, suppose there was an equilibrium with bank default and thus $K = K^*$, $r_{d1} = r_s + 1 - p$ and $phf'(K^*) = (1+r_s)^2$. Then Condition (23) is violated for a bank that holds minimum liquidity $\underline{l}_B^*$ and thus the bank will not default—a contradiction.

□

Proof of Proposition 5

Step 1

It is instructive to look first at the second part of the proposition. Suppose that $l = 0$. Then $\zeta^{reg} = 1$. The maximum amount of resources of the banking system in $t = 1$ is $W + \mu W$. Since $\zeta^{reg} = 1$ implies $L_B = \mu W$, the banking system can invest at most W in LT. As $W < \bar{K}$, it is impossible to achieve the efficient scale $\bar{K}$ in LT.

Step 2

We note that ζ^{reg} is monotonically decreasing in l (holding m constant). For $l = m$, $\zeta^{reg} = 1 - \frac{\bar{K}}{\mu W}$ and thus $\zeta^{reg}\mu W + \bar{K} = \mu W < \mu W + W$. Together with Step 1, it follows that there exists a unique critical value $l^{crit} \in (0, m)$ such that $\zeta^{reg}\mu W + \bar{K} = W + \mu W$. For all $l < l^{crit}$, it is impossible to achieve the efficient scale $\bar{K}$ in LT.

Step 3

We finally show that there exists an equilibrium in which no bank will default if $l \in [l^{crit}, l^{min})$. By construction, all banks survive if $\eta = l$ occurs. Since banks never default, debt holders always receive $1 + r_s$ per unit of invested resources, and we obtain $1 + r_L^e = (1+r_s)^2$ and $K = \bar{K}$. The expected equity of all banks in $t = 3$ is given by

$$\mathbb{E}[E_3] = \bar{K}(1 + r_L^e) + [L_B(1+r_s) - D_1(1+r_s)]\,(1+r_s)$$

$$= (\bar{K} + L_B - D_1)(1 + r_s)^2$$
$$= E_1(1 + r_s)^2.$$

Hence, the portfolio choice of owners maximizes their expected utility as they earn the return $(1 + r_s)^2$ over two periods on both types of investment (bank equity and ST). Finally, since return expectations on LT and ST are the same, the investment choices of banks in LT and ST are optimal. □

Proof of Proposition 6

The proof of the proposition is analogous to the proof of Proposition 5.

Step 1

It is instructive to look at the second part of the proposition first. Suppose that $l = 0$. Then $\rho^{reg} = 1$. The maximum amount of equity financing is W. Since $W < \bar{K}$ by Assumption 1, it is impossible to achieve $\rho^{reg} = 1$ if investment in LT is to be at the efficient scale $\bar{K}$.

Step 2

As ρ^{reg} is monotonically decreasing in l and becomes zero for $l = m$, there exists a critical value for l such that $W/\bar{K} = 1 - l/m$. It is given by l^{crit} according to Equation (29). For all $l < l^{crit}$, the same observation applies as for $l = 0$.

Step 3

We finally show that there exists an equilibrium in which no bank will default if $l \in [l^{crit}, l^{min})$. By construction, all banks survive for $\eta = l$. With $l \geq l^{crit}$, there are enough resources to achieve $\bar{K}$ in LT:

$$W + \mu W - L_B = W + \frac{l}{m}\bar{K} \geq W + \bar{K} - W = \bar{K}.$$

Since banks never default, $1 + r_L^e = (1 + r_s)^2$ and $K = \bar{K}$. The expected equity of all banks in $t = 3$ is given by

$$\mathrm{E}(E_3) = (1 + r_L^e)\bar{K} + [L_B(1 + r_s) - D_1(1 + r_s)]\,(1 + r_s)$$
$$= (\bar{K} + L_B - D_1)(1 + r_s)^2$$

$$= E_1(1 + r_s^2).$$

Hence, owners maximize expected utility as they earn the same expected return on both types of investment (bank equity and ST). □

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