CENTER FOR FINANCIAL STUDIES

CFS Working Paper Series

No. 717

Hugo van Buggenum, Hans Gersbach,
and Sebastian Zelzner

Contagious Stablecoins?

The CFS Working Paper Series

presents ongoing research on selected topics in the fields of money, banking and finance. The papers are circulated to encourage discussion and comment. Any opinions expressed in CFS Working Papers are those of the author(s) and not of the CFS.

The Center for Financial Studies, located in Goethe University Frankfurt's House of Finance, conducts independent and internationally oriented research in important areas of Finance. It serves as a forum for dialogue between academia, policy-making institutions and the financial industry. It offers a platform for top-level fundamental research as well as applied research relevant for the financial sector in Europe. CFS is funded by the non-profit-organization Gesellschaft für Kapitalmarktforschung e.V. (GfK). Established in 1967 and closely affiliated with the University of Frankfurt, it provides a strong link between the financial community and academia. GfK members comprise major players in Germany's financial industry. The funding institutions do not give prior review to CFS publications, nor do they necessarily share the views expressed therein.

Center for Financial Studies | Goethe University | Theodor-W.-Adorno-Platz 3 | 60323 Frankfurt am Main | Germany | Phone +49 69 798 30050 | info-cfs@hof.uni-frankfurt.de | www.gfk-cfs.de

Contagious Stablecoins?*

Hugo van Buggenum† Hans Gersbach‡ Sebastian Zelzner§

KOF Swiss Economic Institute at ETH Zurich,
Leonhardstrasse 21, 8092 Zürich, Switzerland.

This Version: January 31, 2024

Abstract

Can competing stablecoins produce efficient and stable outcomes? We study competition among stablecoins pegged to a stable currency. They are backed by interest-bearing safe assets and can be redeemed with the issuer or traded in a secondary market. If an issuer sticks to an appropriate investment and redemption rule, its stablecoin is invulnerable to runs. Since an issuer must pay interest on its stablecoin if other issuers also pay interest, competing interest-bearing stablecoins, however, are contagious and can render the economy inefficient and unstable. The efficient allocation is uniquely implemented when regulation prevents interest payments on stablecoins.

Keywords: Stablecoins, currency competition, free banking, private money, digital money.

JEL Classification: E4, E5, G1, G2

*We thank Aleksander Berentsen, Jean-Charles Rochet, and Lukas Voellmy, as well as participants at the SSES Annual Congress at the University of Neuchâtel 2023, the EEA-ESEM 2023 Annual Congress at Pompeu Fabra University, the VfS 2023 Annual Congress of the German Economic Association, and the Davos Hub during the World Economic Forum 2024 for useful comments and suggestions.

†hvanbuggenum@ethz.ch

‡hgersbach@ethz.ch

§szelzner@ethz.ch

1 Introduction

The latest attempts to construct a private money in the form of stablecoins are yet another test of whether competing private monies can serve as effective payment and saving instruments.[1] Stablecoins are a digital form of privately-created money. Unlike cryptocurrencies like Bitcoin and Ethereum, they aim to maintain a stable value against a national currency or a basket of national currencies. Well-known stablecoins like Tether and USDC are pegged one-to-one to the US Dollar and invest in liquid, dollar-denominated assets to maintain their peg. In that sense, they closely resemble traditional forms of banking and, maybe even more, money-market mutual funds (MMFs). Yet, stablecoins remain largely unregulated and in contrast to bank deposits and MMFs, which typically operate as open-end investment funds, most stablecoins can be traded in secondary markets.

While some believe that stablecoins are a promising form of private money, the history of free banking is full of examples of competing, privately-produced monies that were pegged to national currencies or commodities but still encountered severe difficulties in serving as a stable store of value and medium of exchange. During the recent great financial crisis, even MMFs were prone to runs. In light of the recent emergence of stablecoins, three questions are thus particularly relevant. First, how should an individual stablecoin be designed so that it is truly stable, i.e., always trades at the peg, and do we require any regulation to achieve this? Second, does competition between issuers make it more or less likely for stablecoins to be truly stable? Third, how should a system of competing stablecoins be designed, so that poorly designed stablecoins do not harm well-designed ones, i.e., how can contagion from poorly designed to well-designed stablecoins be avoided?

To answer these questions, we develop a continuous-time model to study the issuance and competition of stablecoins in a dynamic framework. Stablecoins are backed by safe, interest-bearing assets and are pegged to a stable currency. They serve to insure investors against an idiosyncratic investment horizon, much like traditional bank deposits that provide liquidity insurance in models following Bryant (1980) and Diamond and Dybvig (1983). Unlike deposits, however, stablecoins can be traded in a secondary market, much like current stablecoins can be traded on platforms like Binance and Coinbase.[2] A

[1]For an overview, see, e.g., Bains, Ismail, Melo and Sugimoto (2022), Catalini, de Gortari and Shah (2022) and Kosse, Glowka, Mattei and Rice (2023).

[2]We do not consider algorithmic stablecoins or stablecoins that are backed by other cryptoassets. See

particular feature of our model is that stablecoins look like a blend between open- and closed-end MMFs since issuers act as market makers in the secondary market by standing ready to buy coins at a quoted peg, i.e., by providing liquidity.[3] Our core contribution is to study how such an asset should be designed and regulated to obtain efficient outcomes in a competitive environment. We derive four main insights.

First, an individual stablecoin can be made both truly stable and attractive relative to an existing currency by means of an investment rule. This rule stipulates that at each point in time, a fraction of the issuer's disposable resources is invested in safe assets that dominate currency. Resources earmarked for redemption thus have to be limited to respect the investment rule.[4] Since stablecoins are traded in a secondary market, there is no reason for a run, unlike in the model of Engineer (1989), where a run can be triggered by the fear of not being able to consume in the future due to a redemption limit. And even if a run occurred accidentally, for which there is some evidence from lab experiments (Kiss, Rodriguez-Lara and Rosa-Garcia, 2018), there are no losses for investors selling stablecoins in the secondary market. We call stablecoins adhering to the investment rule *micro-well-designed*, as they are run-proof if the stablecoin is the only stablecoin in the market.

Second, a monopolistic issuer of a micro-well-designed stablecoin implements the efficient allocation by issuing a zero-interest stablecoin offered in an ICO (initial coin offering) at a discount. This arrangement provides insurance against investors' idiosyncratic investment horizon while simultaneously allowing investors to enjoy, on average, the return on safe assets. We call a stablecoin that is micro-well-designed, pays zero interest, and is issued at a discount, *macro-well-designed*, as it implements the efficient allocation.

Third, even if all stablecoins are micro-well-designed, competition between issuers entails a coordination problem. The reason is that an issuer has to match the returns of other stablecoins to satisfy its investors' dynamic incentive constraint—a stablecoin that is micro-well-designed but pays a positive interest rate is contagious for other stablecoins

Cao, Dai, Kou, Li and Yang (2021) for a model on stablecoin design based on option pricing and smart contracts implemented on the Ethereum platform. Mayer (2022) develops a model that rationalizes dual-token structures such as Terra/Luna.

[3]ETFs are likewise traded in a secondary market, but they are not directly created or redeemed by the issuer. Instead, this is done by authorized market participants (big institutional investors that act as market makers) who can trade the basket of securities that the ETF tracks with the ETF issuer in exchange for ETF shares.

[4]The rule may be implemented by using blockchain technology (Cong, Li and Wang, 2022).

by forcing them to offer the same returns. The intuition relates to work of Jacklin (1987, 1993), Haubrich and King (1990), and von Thadden (1997, 1998, 2002), who show that when investors can access other assets, this imposes tighter dynamic incentive constraints on liquidity-insurance contracts. The distinguishing feature of our model is that investors cannot access safe, interest-bearing assets themselves, but can trade a variety of stablecoins, which are backed by a safe interest-bearing asset, in a secondary market. Competition among issuers of micro-well-designed stablecoins in the secondary market may thus fail to produce efficient outcomes. In fact, a continuum of inefficient allocations featuring interest-bearing stablecoins exist as equilibria besides the efficient equilibrium.

Fourth, ruling out interest payments on competing stablecoins implements the efficient allocation as the unique equilibrium. This provides a rationale for policies preventing interest-bearing stablecoins. In the US, for instance, interest-bearing stablecoins would subject the issuer to banking or securities-market regulation, which is probably one of the reasons for the zero-interest design of stablecoins like Tether and USDC. The EU is, in fact, planning to prohibit interest-bearing stablecoins (Read and Diefenbach, 2022).

Given the above, the contribution of our paper can be summarized as follows. We study the design of stablecoins, as well as competition between them, through the lens of a model in which stablecoins resemble a blend between open- and closed-end MMFs—stablecoins can be traded in a secondary market, while the issuers simultaneously act as market makers that provide liquidity by standing ready to purchase their respective stablecoins at a quoted peg. This blending allows us to derive new results relative to traditional models of banking, which abstract from secondary markets for liquidity-insurance contracts. Moreover, by casting our model in continuous time, we study the interplay between two types of delicate dynamic incentives. First, when it is attempted to prevent runs by limiting redemption, runs can arise due to the fear of a future redemption limit. Second, in the presence of a secondary market and competition between stablecoins, investors have an incentive to sell efficient, zero-interest stablecoins and to buy inefficient, interest-bearing stablecoins, thus threatening efficient risk sharing. Our results show that investors' fear for a future redemption limit unravels since stablecoins can always be sold in the secondary market, and that inefficient risk sharing is avoided by prohibiting interest payments on stablecoins.

Model summary and detailed results. We derive our insights in a continuous-time, infinite-horizon model with overlapping cohorts of agents representing stablecoin investors. As argued, among others, by von Thadden (1997, 1998, 2002), continuous time allows for sufficient tractability while preserving the dynamic nature of liquidity-insurance contracts such as deposits, MMFs, and, in our case, stablecoins. Stablecoin investors exert effort to produce a homogeneous good at birth (representing market entry and initial investment) and consume at a random time of death (representing disinvestment and market exit). The random time of death entails an idiosyncratic investment horizon and thus a role for liquidity insurance. Goods are perfectly storable (we interpret them as stable currency) and can also be invested in scalable, safe investment opportunities that we call *trees*.

A tree gestates a deterministic amount of goods (*fruit*) at a stochastic date drawn from a Poisson distribution, entailing an idiosyncratic maturity. Trees are destroyed after producing fruit and fruit can be consumed, stored, or used to *plant* new trees. A well-diversified portfolio of trees eliminates the maturity risk and resembles a perpetual bond with exponentially decaying coupons similar as in Woodford (2001). We assume that the expected return on a tree equals the investors' rate of time preference, reflecting that our model captures a small sector of a macro economy in steady state. We further assume that the expected maturity of a tree is shorter than investors' expected investment horizon, reflecting that stablecoin issuers mostly invest in assets with short maturities.

We find no storage in the efficient Arrow-Debreu allocation, i.e., there is only investment in trees, and investors' consumption levels are independent from the point in time at which they exit, i.e., the optimal contingent-consumption schedule is smoothed to the extent that it becomes flat. In this sense, there is perfect insurance against investors' random investment horizon, and investors earn, on average, the rate of time preference on their initial investment. More specifically, those who exit early earn a high annualized return, whereas those who exit late earn a low annualized return.

In a decentralized economy where investors can trade trees, perfect insurance breaks down. The reason is that the market for trees allows investors to earn the rate of time preference at each point in time. Those who exit early therefore earn the same annualized return on their initial investment as those who exit late, so that consumption on exit increases with the realized investment horizon. This lack of insurance, manifesting itself as an increasing rather than a flat contingent-consumption schedule, is inefficient and well known: if investors can access interest-bearing assets, this can unravel efficient insurance (Jacklin, 1987, 1993; Haubrich and King, 1990; von Thadden, 1997, 1998, 2002).

The stablecoin economy allows investors to undertake storage on their own, but they can neither plant trees nor trade trees themselves. Instead, investors of a particular cohort—defined by time of entry—coalesce to form *stablecoin issuers*, which can plant trees, store goods, and issue redeemable stablecoins to their members in an ICO. Our view of the stablecoin economy is reminiscent to Wallace's (1988, 1990) view on banking, i.e., stablecoins are an alternative to market-based exchange of interest-bearing assets when such markets are unavailable or difficult to access for (unsophisticated) investors. We do, however, allow investors to trade existing stablecoins in a competitive secondary market, i.e., they cannot trade trees, but they can trade stablecoins.

We first consider a version of the stablecoin economy with a single cohort of investors that coalesce as an issuer, representing a platform with a single tradable stablecoin. By issuing stablecoins at a discount in the ICO, investing the proceeds in planting trees, maintaining a time-invariant peg with goods after the ICO, and paying zero interest, the issuer replicates the Arrow-Debreu insurance arrangement. The single issuer earns exactly enough fruit from its trees to meet redemption by all exiting investors and to keep the ratio of trees to stablecoins in circulation constant. If only exiting investors redeem their stablecoins, non-exiting investors have no incentive to redeem stablecoins or to sell in the secondary market. They know that the peg can be maintained, so that their only outside option, storage, earns the same return as the stablecoin.

If all investors ask for redemption, however, the fruit from the trees is insufficient to service all redemption requests at the peg. If the issuer then redeems as much as possible at the peg, maturing trees cannot be replaced, so that the stock of trees per stablecoin shrinks. This reduces the resources available for future redemption and thus forces the issuer to reduce the future peg—the run becomes self-fulfilling. In other words, investors know that the peg cannot be maintained, and in the secondary market, the stablecoin starts to trade at a discount relative to the current peg. Our results on the dynamic behavior of the peg are in line with empirical evidence on open-end investment funds. For instance, Feroli, Kashyap, Schoenholtz and Shin (2015) and Wang (2015) find that fund outflows predict future declines in a fund's net asset value (NAV), i.e., in the price at which the fund buys shares from its shareholders.

A run can be prevented by limiting the funds earmarked for redemption, which is reminiscent of the suspension of convertibility advocated by Diamond and Dybvig (1983) and others. More precisely, the issuer should ensure that it always has enough resources left to plant new trees and keep the ratio of trees to circulating stablecoins constant. Adhering to this *investment rule* is in the issuer's own interest since it avoids a deviation from the efficient allocation, even when a run happens by accident, as exiting investors can also trade the stablecoin at the peg in the secondary market. In this sense, stablecoins are micro-well-designed as long as limiting redemption is in line with regulation. In fact, regulation that allows MMFs to limit redemption has been introduced by US and EU authorities in response to some MMFs experiencing runs during the great financial crisis (Voellmy, 2021). We stress again the role of the secondary market in ruling out runs. In a dynamic environment with a redemption-limit policy, investors could still have an incentive to run for fear of not being able to consume due to a future redemption limit (Engineer, 1989). In our model, this fear unravels because if a redemption limit is set, investors are insured as they can consume by selling stablecoins in the secondary market.

We proceed by studying stablecoin issuance with overlapping cohorts of investors. Since each cohort's members can trade other issuers' stablecoins, this represents a platform with multiple tradable stablecoins. Each cohort has an incentive to issue a micro-well-designed stablecoin since this avoids runs. Whether a cohort can issue a macro-well-designed stablecoin depends on the return that investors can earn by trading other stablecoins. More precisely, when an issuer expects others to issue an interest-bearing micro-well-designed stablecoin, it has to pay at least that interest on its own stablecoin, as investors prefer to hold the stablecoin which offers the highest return. This means that the dynamic incentive constraint on liquidity-insurance contracts becomes tighter when investors can access other stablecoins—they have a stronger incentive to claim a need for liquidity to redeem their funds and reinvest them in a stablecoin with better returns.

A distinguishing feature of our model is that the return on investors' outside option is endogenous, as it depends on the returns of *all* other stablecoins. This feature implies that micro-well-designed stablecoins that pay interest are *contagious*, i.e., they force all other stablecoins to pay interest as well, thereby driving the economy away from the efficient equilibrium. Issuers thus face a coordination problem and there is a continuum of stationary equilibria, among which only one implements the efficient allocation. We also find a multitude of perfect-foresight equilibria with cyclical and non-stationary dynamics, demonstrating the instability of micro-well-designed but unregulated stablecoins. Regulation that prohibits interest payments on stablecoins implements the efficient allocation as the unique equilibrium.

Relation to the literature. Our paper relates to three strands of literature. First, in terms of methods, we relate closely to the literature on liquidity insurance following Bryant (1980) and Diamond and Dybvig (1983). We contribute to this literature by studying a dynamic framework in which liquidity-insurance contracts, stablecoins in our framework, can be redeemed with the issuer *and* traded on a secondary market. Further, competition between stablecoins in the secondary market generates dynamic incentive constraints similar to those studied by von Thadden (1997, 1998, 2002). To this literature, we provide a new angle by having investors' outside option depend on how other stablecoins are designed. The similarities between stablecoins and MMFs imply that we also relate to the theoretical literature on the design and stability of MMFs. For instance, Zheng (2017) shows how even floating-NAV MMFs, i.e., those that continuously adjust their redemption price to the value of underlying assets, can be subject to runs. Voellmy (2021) studies how recent changes in the regulation of MMFs can help to prevent runs of both fixed- and floating-NAV MMFs. We contribute by analyzing matters with an asset that looks like a blend between open- and closed-end MMFs.

Second, we relate to a growing literature on the risks, design, and regulation of cryptoassets. Stablecoins promise to be a stable store of value and means of payment. Our contribution focuses on how the stablecoin market should be designed to attain efficient allocations. Related but within a different framework, Fernández-Villaverde and Sanches (2019) analyze whether competition in private digital currencies can be compatible with price stability and efficient money supply. Li and Mayer (2021) develop a dynamic model of stablecoin management and argue that sufficient reserves are key for stability. D'Avernas, Maurin and Vandeweyer (2022) show that while sufficient collateral is essential to maintain a peg, decentralization can substitute for commitment in issuing stablecoins. Brunnermeier, James and Landau (2021), Guennewig (2022), and Rogoff and You (2023) study the issuance of digital money by retailers, notably by large online platforms such as Amazon or Alibaba. In this context, issuers have an incentive to limit interoperability between monies to bind consumers to their platform and to exploit intra-platform transaction data. Goldstein, Gupta and Sverchkov (2023) show that the tradablity of general-purpose tokens is essential for competitive pricing. We add to this literature by studying competition through the lens of dynamic incentives that can undermine efficient liquidity insurance.

Third, we relate to the free-banking literature. Gorton and Zhang (2021) assess that the realm of stablecoins resembles earlier free-banking episodes. They stress that experience from the latest US free-banking episode—the 1837–1862 *wildcat-banking* era—suggests that unregulated competition among stablecoins can result in fluctuating prices, as stablecoins may not always be accepted at par, and it is likely to produce runs. The idea of free banking has a long history and has been the subject of enduring analyses and debates, as recently reviewed by White (2014). Many theoretical studies on free banking are also relevant for competition between stablecoins. Gersbach (1998), for instance, shows that free banking leads to over-issuance of private monies. Other contributions on modeling private-money competition include Cavalcanti and Wallace (1999a,1999b), Cavalcanti, Erosa and Temzelides (1999, 2005), Williamson (1999), Aghion, Bolton and Dewatripont (2000), Berentsen (2006), and Martin and Schreft (2006). Their findings on whether free banking can be efficient vary, and they show that the way in which private monies are structured and compete matters. Our contribution is to show how unregulated competition between stablecoins, sharing features with traditional deposits, entails coordination problems that endanger efficient liquidity insurance and that generate instability.

Outline. Our paper proceeds as follows: Section 2 introduces the model environment. Section 3 studies an Arrow-Debreu setting and characterizes the efficient allocation. Section 4 considers a decentralized economy with tradable assets. Section 5 introduces the stablecoin economy, where investors coalesce to issue stablecoins backed by safe, interest-bearing assets, and trade the stablecoins in a secondary market. To start, we study a single cohort represented by a single issuer in Section 6. Section 7 considers overlapping cohorts and competition among issuers. Section 8 concludes. Lengthy proofs are relegated to the Appendix.

2 The Model

Time $t \in \mathbb{R}_+$ is continuous and lasts forever. For any stock variable X, we interpret X_t as a state at time t. For any variable Y, we let Y_t^+ be the right hand limit of Y_t and $\dot{Y}_t$ the right-hand derivative of Y_t w.r.t. to t.

The economy is inhabited by investors with a finite but idiosyncratic investment horizon. A unit mass of investors enter at time $t = 0$. Incumbent investors exit at rate δ (in the aggregate) and are replaced by new investors, so that there is a unit mass of investors alive at any point in time. There is a single, perfectly divisible and storable good available in the economy. The good can be produced by investors on entry. The aggregate stock of goods stored is S_t. One interpretation of this storage good in a broader sense is a stable national currency such as the US Dollar. Stablecoin issuers, which we introduce later, then redeem their coins into dollars.

There are also scalable, safe investment opportunities that we label as *trees* in the spirit of Lucas (1978). Goods can be used to plant trees at a one-to-one rate and trees have an idiosyncratic maturity: a tree generates a *fruit* of y goods with Poisson arrival rate ϕ and the tree is destroyed after generating fruit. The stock of trees A_t develops as

$$\dot{A}_t = I_t - \phi A_t, \tag{1}$$

where I_t is the investment made to plant trees. The expected real return r from planting a tree is $r = \phi(y - 1)$, which follows from solving

$$1 = \int_t^\infty \phi y e^{-(r+\phi)(\tau-t)} \mathrm{d}\tau. \tag{2}$$

Equation (2) elucidates that a well-diversified portfolio of trees resembles a perpetual bond with exponentially decaying coupons à la Woodford (2001)—when a unit mass of trees is planted at time t, it diminished to mass $e^{-\phi(\tau-t)}$ at time τ, thus generating flow income $\phi y e^{-\phi(\tau-t)}$ at time τ. Our specification of trees allows to model safe, interest-bearing assets with some average duration $1/\phi$ in an algebraically convenient way.

Preferences. The preferences of an investor entering at time s are represented by the function

$$\mathcal{U}_s = -h_s + \mathbb{E}\left\{e^{-\rho(T-s)} u(c_{T,s})\right\}, \tag{3}$$

where $h_s \geq 0$ is the investor's labor effort at time s, T is the time period at which the investor exits, $c_{T,s}$ is the investor's consumption on exit, ρ is the rate of time preference, and $u : \mathbb{R}_+ \mapsto \mathbb{R}$ is concave with the standard Inada properties. We interpret labor effort on entry as initial investment and consumption on exit as disinvestment. Disinvestment represents a liquidity-demand shock like in much of the banking literature since the investor wants to consume the storage good (dollars) on exit. The realized investment horizon $T - s$ thus constitutes idiosyncratic liquidity risk. An individual investor exits with Poisson arrival rate δ.

Assumptions on parameters. In order to obtain an efficient allocation that is stationary, we assume that y is such that $r = \rho$. This simplifies the analysis, as otherwise we would have to introduce growth effects. The assumption also views our stablecoin economy as being part of a bigger macro economy that is in steady state. We furthermore assume that $\phi > \delta$, i.e., the expected maturity of a tree is shorter than the investor's expected investment horizon. This ensures that the first welfare theorem holds in our model, which is not obvious with a double infinity of traders and dated goods (Shell, 1971). The assumption also implies that the efficient allocation requires no transfers between investors entering at different dates, allowing us to study in isolation a cohort of investors entering at a particular date.

Resource constraints. When the economy starts, there is a unit mass of investors providing labor effort (initial investment) h_0. There is no exit yet, so h_0 is used for planting trees or storage. Thus,

$$h_0 = A_0^+ + S_0^+. \tag{4}$$

Consumption by exiting investors (disinvestment), labor supply (initial investment) by new investors, and income from maturing trees become flows at time $t > 0$. Aggregate flow consumption is

$$c_t = \delta e^{-\delta t} c_{t,0} + \int_0^t \delta^2 e^{-\delta(t-s)} c_{t,s} \mathrm{d}s \tag{5}$$

and the aggregate flow labor supply is δh_t. To obtain Expression (5), we treat the cohort of investors present when the economy starts separately, which yields the first term. The other cohorts entering later yield the second term. Of the first cohort, which has initial mass 1, a mass $e^{-\delta t}$ is still alive at time t and in the aggregate, they exit at rate δ, leading to flow consumption $\delta e^{-\delta t} c_{t,0}$. New investors enter at rate δ and likewise exit at rate δ, so the contribution to time-t flow consumption by investors entering at time s is $\delta^2 e^{-\delta(t-s)} c_{t,s}$, which we integrate from $s = 0$ to $s = t$.

The aggregate resource constraint reads as

$$(\rho + \phi)A_t + \delta h_t = c_t + I_t + \dot{S}_t. \tag{6}$$

We note that from $r = \phi(y-1)$ and the assumption $r = \rho$ it follows that $\phi y = \rho + \phi$. The first term on the left-hand side of Equation (6) is therefore equal to $\phi y A_t$ and represents the resource income from maturing trees. The second term is resource income from labor by new investors. The right-hand side is the summand of spending through consumption by exiting investors, investment in trees, and changes in storage.

3 Arrow-Debreu Allocation

We study first an Arrow-Debreu setting in which the allocation is determined by a frictionless market for dated goods before the economy actually starts. The purpose is to derive an efficient benchmark allocation that a (system of competing) stablecoin(s) should ideally implement. Let p_t denote the Arrow-Debreu price of the time-t good.[5] The availability of storage implies $\dot{p}_t \leq 0\ \forall t$, i.e., prices cannot increase over time. We also have $S_t^+ > 0 \Rightarrow \dot{p}_t = 0$, i.e., storage is undertaken only when prices remain constant. Investors' ability to plant trees implies

$$p_t \geq V_t \equiv \int_t^\infty (\rho + \phi) e^{-\phi(\tau - t)} p_\tau \mathrm{d}\tau\ \forall t, \tag{7}$$

where V_t denotes the present value of the tree's fruit. Again, note for this formula that $\rho + \phi = \phi y$. More particularly, a well-diversified portfolio of trees with mass one at time t diminishes to mass $e^{-\phi(\tau-t)}$ at time $\tau > t$, as the trees mature at rate ϕ. This leads to a flow income $\phi y e^{-\phi(\tau-t)} = (\rho + \phi) e^{-\phi(\tau-t)}$ of goods at time τ, valued at price p_τ. Furthermore, $I_t > 0 \Rightarrow p_t = V_t$, i.e., a tree is planted only when the present value of its fruit equals the cost of planting. We note that $\dot{V}_t = -(\rho + \phi)p_t + \phi V_t$. With $p_t \geq V_t$, this

[5] The level of p_t does not matter. Only the relative prices matter, and p_0 can be normalized to 1.

implies

$$-\rho V_t \geq \dot{V}_t, \quad \text{with “=” if } I_t > 0. \tag{8}$$

An investor that enters at time s chooses h_s and $(c_{t,s})_{t=s}^{\infty}$ to maximize

$$-h_s + \int_s^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s}) \mathrm{d}t \tag{9}$$

subject to the budget constraint

$$\int_s^{\infty} \delta e^{-\delta(t-s)} p_t c_{t,s} \mathrm{d}t \leq p_s h_s. \tag{10}$$

Condition (10) states that the *expected* consumption expenditure of the investor, i.e., taking into account the probability distribution over the possible investment horizons $t-s$, cannot exceed the value of the investor's labor effort on entry. It is in this sense that the Arrow-Debreu environment allows the investor to take out actuarially fair (liquidity) insurance against its idiosyncratic investment horizon. Substituting Condition (10) into Expression (9), we find that the investor chooses $(c_{t,s})_{t=s}^{\infty}$ to maximize

$$\int_s^{\infty} \delta e^{-(\rho+\delta)(t-s)} \left[u(c_{t,s}) - e^{\rho(t-s)} \frac{p_t}{p_s} c_{t,s} \right] \mathrm{d}t, \tag{11}$$

so that the optimal consumption and labor choices satisfy

$$c_{t,s} = u'^{-1}\left(\frac{e^{\rho(t-s)} p_t}{p_s}\right) \quad \text{and} \quad h_s = \int_s^{\infty} \delta e^{-\delta(t-s)} \frac{p_t}{p_s} u'^{-1}\left(\frac{e^{\rho(t-s)} p_t}{p_s}\right) \mathrm{d}t. \tag{12}$$

Aggregate consumption at time $t > 0$ is thus given by

$$c_t = \delta e^{-\delta t} u'^{-1}\left(\frac{e^{\rho t} p_t}{p_0}\right) + \int_0^t \delta^2 e^{-\delta(t-s)} u'^{-1}\left(\frac{e^{\rho(t-s)} p_t}{p_s}\right) \mathrm{d}s. \tag{13}$$

We now characterize the allocation in which there is planting of trees at all times but no storage, which turns out to be the unique Arrow-Debreu equilibrium. With $I_t > 0 \; \forall t$, we obtain $\dot{p}_t = -\rho p_t \; \forall t$, i.e., prices decline at rate $r = \rho$, which also implies $S_t = 0 \; \forall t$. We define $c^\star \equiv u'^{-1}(1)$ for notational convenience. Since $p_t = p_s e^{-\rho(t-s)} \; \forall t \geq s$, the expressions for individual consumption, individual labor supply, and aggregate consumption simplify to

$$c_{t,s} = c^\star, \quad h_s = \frac{\delta c^\star}{\rho + \delta}, \quad \text{and} \quad c_t = \delta c^\star. \tag{14}$$

From the aggregate resource constraints we find that the stock of trees develops according to

$$\dot{A}_t = \rho A_t - \delta c^\star + \frac{\delta^2 c^\star}{\rho + \delta}, \quad \text{with } A_0^+ = \frac{\delta c^\star}{\rho + \delta} \quad \Rightarrow \quad \frac{\dot{A}_t}{A_t} = \rho \left(1 - \frac{A_0^+}{A_t}\right). \tag{15}$$

Clearly, this implies $A_t = A_0^+ \ \forall t > 0$, which in turn requires $I_t > 0 \ \forall t$. In other words, we found an Arrow-Debreu equilibrium, which happens to be the unique equilibrium.

Proposition 1. *The Arrow-Debreu equilibrium characterized above is the unique Arrow-Debreu equilibrium, and it is also Pareto efficient, i.e., the first welfare theorem holds.*

Uniqueness and Pareto efficiency of the Arrow-Debreu equilibrium arise naturally under the assumption $\phi > \delta$, as this allows a cohort of investors entering at a particular date to enjoy the fundamental return on trees without the need to trade with investors entering at other dates.

To see this, suppose, for the sake of illustration, that every cohort—defined by time of entry—has its own stock of trees. Let $A_{t,s}$ denote the stock of trees owned by cohort s at time $t \geq s$, where we measure the stock of trees relative to the initial size of the cohort so that $A_t = A_{t,0} + \int_0^t \delta A_{t,s} \mathrm{d}s$. The law of motion for $A_{t,s}$ if there is no storage is

$$\dot{A}_{t,s} = \rho A_{t,s} - \delta e^{-\delta(t-s)} c_{t,s}, \quad \text{where } A_{s,s}^+ = h_s, \tag{16}$$

i.e., the change in the stock of trees is the fundamental return on trees, which already accounts for maturing trees, minus the consumption by the cohort's exiting investors. The allocation requires trade between cohorts if and only if there exists a time $t > s$ for which $\dot{A}_{t,s} < -\phi A_{t,s}$, as the absence of such trades imposes $\dot{A}_{t,s} \geq -\phi A_{t,s} \ \forall t, s, \ t \geq s$. In words, if $\dot{A}_{t,s} < -\phi A_{t,s}$ at time t, cohort s wants to reduce its stock of trees at a rate greater than natural gestation, which is only feasible if the implicit ownership of trees is transferred to a different cohort.

Using Equation (14) in Equation (16), we obtain

$$\dot{A}_{t,s} = \rho A_{t,s} - (\rho + \delta) A_{s,s}^+ e^{-\delta(t-s)} \quad \Rightarrow \quad A_{t,s} = e^{-\delta(t-s)} A_{s,s}^+, \tag{17}$$

i.e., the stock of trees owned by cohort s diminishes at a rate δ. Implementing the Arrow-Debreu allocation therefore requires no trade between cohorts since we assumed $\delta < \phi$. This also implies that in a hypothetical, different Arrow-Debreu equilibrium, each cohort of investors should obtain at least the level of utility generated by the allocation characterized above. The resource constraints, however, imply that given the allocation above, one cohort cannot be made better off without making another cohort worse off. Thus, the equilibrium is unique *and* Pareto efficient.

4 A Decentralized Economy with Tradable Trees

Consider next a decentralized economy in which investors can undertake storage and plant trees by themselves. There are no markets for time-t goods where investors can trade in advance, but trees, which are identical once planted, can be sold in a continuously-open secondary market for v_t goods. We rule out short-selling. The purpose of this section is to show that a simple asset market implements an inefficient equilibrium, indicating that there is a role for more sophisticated trading arrangements, e.g., stablecoins, in order to implement a Pareto-efficient allocation.

We have $v_t \leq 1$ in any possible equilibrium, as $v_t > 1$ allows for arbitrage by planting trees and immediately selling them. Trees are only planted if they cannot be acquired more cheaply in the secondary market, so $I_t > 0 \Rightarrow v_t = 1$. The real return $r^+_{v,t}$ from holding a tree from time t to time $t + \varepsilon$, with $\varepsilon > 0$ but infinitesimal, satisfies

$$r^+_{v,t} v_t = \phi(y - v_t) + \dot{v}_t \quad \Rightarrow \quad r^+_{v,t} = \rho/v_t + \phi(1/v_t - 1) + \dot{v}_t/v_t. \tag{18}$$

Equation (18) is a standard asset-pricing equation—the return on trees is composed of three components: the fundamental return on trees ρ/v_t; the discount $1/v_t - 1$ of reacquiring a tree in the secondary market instead of replanting it after maturity; and price changes in the secondary market. All trees would be offered for sale if $r^+_{v,t} < 0$ because storage is available as an alternative investment, so we must have $A_t > 0 \Rightarrow r^+_{v,t} \geq 0$. Likewise, trees cannot dominate storage whenever storage is undertaken, so $S^+_t > 0 \Rightarrow r^+_{v,t} \leq 0$.

Due to the possibility of storing goods and trading trees, all incumbent investors can save at an effective real return of $\max\{r_{v,t}, 0\}$. Assuming that the process $(\max\{r_{v,t}, 0\})_{t=0}^{\infty}$

is integrable, an investor entering at time s consumes

$$c_{T,s} = e^{\int_s^T \max\{r_{v,t},0\}\mathrm{d}t} h_s \tag{19}$$

on exit at time T. It follows that h_s is chosen to maximize

$$\int_s^\infty \delta e^{-(\rho+\delta)(T-s)} u\left(e^{\int_s^T \max\{r_{v,t},0\}\mathrm{d}t} h_s\right) \mathrm{d}T - h_s. \tag{20}$$

The optimal choice for h_s thus follows uniquely from the first-order condition

$$0 = \int_s^\infty \delta e^{\int_s^T (\max\{r_{v,t},0\}-\rho-\delta)\mathrm{d}t} u'\left(e^{\int_s^T \max\{r_{v,t},0\}\mathrm{d}t} h_s\right) \mathrm{d}T - 1, \tag{21}$$

and aggregate consumption at time t satisfies

$$c_t = \delta e^{\int_0^t (\max\{r_{v,\tau},0\}-\delta)\mathrm{d}\tau} h_0 + \int_0^t \delta^2 e^{\int_s^t (\max\{r_{v,\tau},0\}-\delta)\mathrm{d}\tau} h_s \mathrm{d}s. \tag{22}$$

We now show that there is a decentralized equilibrium in which $I_t > 0 \ \forall t$, as is the case for the unique Arrow-Debreu equilibrium, but that this equilibrium is inefficient. We have $v_t = 1 \ \forall t$ when $I_t > 0 \ \forall t$, so that $r_{v,t} = \rho > 0 \ \forall t > 0$ and $S_t = 0 \ \forall t > 0$. From the aggregate resource constraint (6) we find that the stock of trees develops according to

$$\dot{A}_t = \rho A_t - \delta e^{(\rho-\delta)t} h_0 - \int_0^t \delta^2 e^{(\rho-\delta)(t-s)} h_s \mathrm{d}s + \delta h_t, \quad \text{with } A_0^+ = h_0. \tag{23}$$

Since $r_{v,t}$ $(= \rho)$ is time-invariant, we obtain $h_s = h_0 = A_0^+ \ \forall s$. Thus,

$$\dot{A}_t = \rho A_t + A_0^+ \left(\delta - \delta e^{(\rho-\delta)t} - \int_0^t \delta^2 e^{(\rho-\delta)(t-s)} \mathrm{d}s\right), \tag{24}$$

which in turn implies

$$A_t = A_0^+ \left[e^{(\rho-\delta)t} + \delta \int_0^t e^{(\rho-\delta)(t-s)} \mathrm{d}s\right] = A_0^+ \frac{\rho e^{(\rho-\delta)t} - \delta}{\rho - \delta} \quad \Rightarrow \quad \frac{\dot{A}_t}{A_t} = \frac{\rho(\rho-\delta)e^{(\rho-\delta)t}}{\rho e^{(\rho-\delta)t} - \delta}. \tag{25}$$

We have $\dot{A}_t/A_t \geq 0$ with $\lim_{t\to\infty}[\dot{A}_t/A_t] = \rho - \delta$ if $\rho \geq \delta$, and $\dot{A}_t/A_t > 0$ with $\lim_{t\to\infty} A_t = \frac{\delta}{\delta-\rho} A_0^+$ and $\lim_{t\to\infty}[\dot{A}_t/A_t] = 0$ if $\rho < \delta$. The stock of trees therefore always expands, so that $I_t > 0 \ \forall t$. In other words, we have found a decentralized equilibrium, for which the growth rate of the stock of trees converges to $\max\{0, \rho - \delta\}$.

The decentralized equilibrium above is Pareto inefficient—every investor is ex-ante worse off compared to the unique Pareto-efficient Arrow-Debreu equilibrium. This follows from the fact that individual consumption $c_{T,s}$ increases at a rate ρ with the realized investment horizon $T - s$, while prices in the Arrow-Debreu allocation decline at rate ρ. The decentralized allocation $(h_s, (c_{T,s})_{T=s}^{\infty})_{s=0}^{\infty}$ therefore respects the budget constraint for any investor in the Arrow-Debreu equilibrium, but is never chosen by an investor in the Arrow-Debreu equilibrium. Proposition 2 generalizes the inefficiency by additionally proving uniqueness:

Proposition 2. *The decentralized equilibrium is unique and Pareto inefficient.*

The inefficiency relates to the liquidity insurance that investors can buy in the Arrow-Debreu economy to insure against their idiosyncratic investment horizon. More particularly, investors choose a contract giving them a consumption level independent of the realized investment horizon—the optimal contingent-consumption schedule is flat. Offering such a schedule with an insurance contract is, however, infeasible in the decentralized economy. Investors would then claim exit immediately after entry and invest the proceeds in the secondary market to earn the return ρ and increase future consumption. This finding is in line with Jacklin (1987, 1993), Haubrich and King (1990), and von Thadden (1997, 1998, 2002), among others.

5 Preliminaries for a Stablecoin Economy

In what follows, we construct a trading arrangement with stablecoins to implement the Arrow-Debreu allocation. We assume that investors can claim entry only at the time of actual entry and that trees cannot be traded, whereas exit is unobservable and stablecoins are tradable in a secondary market. The cohort of investors entering at time s coalesce to issue a stablecoin. They form an *issuer* that plants trees and issues a perfectly divisible stablecoin in an initial coin offering (ICO) to the member investors and *only* to the member investors. Our setup aligns with Wallace's (1988, 1990) view of banking in the sense that stablecoins provide investors with an alternative to market-based exchange of trees when such markets are unavailable or difficult to access. The distinguishing feature here is that investors do have access to a secondary market for their own cohort's stablecoin, and potentially also to secondary markets for stablecoins issued by other cohorts.

We interpret the stablecoin version of our model as supported by a digital platform, for instance a mobile-phone app or an online portal. Investors can use the platform to hold a national currency (goods in our model), to create and hold stablecoins, to trade a stablecoin for other stablecoins or currency, and to pay for consumption. Consumption could represent the actual purchase of consumption goods, for instance with online retailers, or the purchase of cryptoassets like Bitcoin. We implicitly assume that those who sell consumption goods or cryptoassets prefer to hold national currency. When sellers accept the stablecoin in payment, they would then immediately convert it into currency and move this currency off the platform.

The observability of entry and the unobservability of exit imply a form of limited record-keeping by the platform. More particularly, investors can claim a need to consume at any time—a common assumption in the liquidity-insurance literature—, which would imply that currency is moved off the platform and stored by the investor until actual exit arises. Moving currency off the platform is, however, observed, so that the investor can be excluded from using this currency to participate in an ICO again.[6]

6 Monopolistic Issuance of Stablecoins

In this section we consider a setup with investors that hold and trade their own stablecoin only, i.e., we consider a single cohort that operates in isolation. This represents a platform with a single stablecoin. The purpose is to show that a well-designed stablecoin can uniquely implement the Pareto-efficient Arrow-Debreu allocation in a monopolistic environment.

A unit mass of investors enter at time $t = 0$, after which the pool of investors gradually declines at a rate δ. No new investors enter. The Arrow-Debreu allocation we seek to implement is

$$c_{t,0} = c^\star, \quad c_t = \delta e^{-\delta t} c^\star, \quad h_0 = \frac{\delta c^\star}{\rho + \delta}, \quad A_t = \frac{e^{-\delta t}\delta c^\star}{\rho + \delta}, \quad \text{and} \quad I_t = (\phi - \delta) A_t. \tag{26}$$

[6] One reason could be know-your-client requirements. In case of pseudo-anonymity, one can argue that investors find it too costly to move fiat currency off the platform and then on the platform again under a different identity.

D_0^+ units of the stablecoin are minted in an ICO by the issuer, who seeks to maximize its members' utility. The ICO's proceeds are used to plant A_0^+ trees. We set $A_0^+ = D_0^+$ so that the ICO price is normalized to one. The issuer allows investors to redeem a stablecoin for x_t goods at times $t > 0$, subject to a resource constraint—with A_t trees owned by the issuer, the flow payment to cover redemption is at most $\phi y A_t$ $(= (\phi + \rho) A_t)$. We refer to x_t as the *peg*, and the resource constraint, in essence, limits the amount of liquidity that the issuer can provide at the peg. The stablecoin can be traded among investors at price q_t in a secondary market at all times $t > 0$.

In order to characterize the peg and the ICO quantity, respectively, we distinguish between the issuer's choices at times $t > 0$ and time $t = 0$. Consider that the issuer has $A_{t'} > 0$ trees, $D_{t'}$ circulating coins, and $S_{t'} = 0$ goods in storage at time $t' > 0$, where we assume zero storage because we know that storage is inefficient, and we assume that the amount of trees is positive to have a meaningful constellation. The issuer chooses $(c_{t,0}, A_t, S_t)_{t=t'}^{\infty}$ to maximize

$$\int_{t'}^{\infty} \delta e^{-(\rho+\delta)t} u(c_{t,0}), \tag{27}$$

i.e., utility of the investors, subject to: the law of motion

$$\dot{A}_t + \dot{S}_t = \rho A_t - \delta e^{-\delta t} c_{t,0}; \tag{28}$$

the constraints $\dot{A}_t \geq -\phi A_t$ and $S_t \geq 0$; the starting values $A_{t'}$ and $S_{t'} = 0$; and the dynamic incentive-feasibility constraint $\dot{c}_{t,0} \geq 0$. The latter reflects the fact that consumption cannot decline because investors can trade the stablecoin and use storage as an alternative investment. More particularly, if the targeted consumption path would decline between time t_1 and t_2, investors could avoid this decline by redeeming or selling stablecoins at t_1, storing the proceeds, and then implicitly acquiring the consumption path as of t_2 by buying stablecoins at t_2. This strategy is feasible because the market value of an investor's stablecoin holdings at time t must equal the targeted consumption on exit at time t, as otherwise this consumption cannot be provided by the issuer on the equilibrium path. In this sense, the secondary market entails a tighter dynamic incentive constraint compared to a constellation without a secondary market, where an investor must store forever after redeeming.[7]

[7] When storage earns a return $r_{S,t}$, then without a secondary market, the dynamic incentive constraint

The targeted path for $(c_{t,0}, A_t, S_t)_{t=t'}^{\infty}$ follows directly from the Arrow-Debreu allocation. There is no storage, the consumption level on exit satisfies $c_{t,0} = \frac{(\rho+\delta)}{\delta} e^{\delta t'} A_{t'}\ \forall t \geq t'$, and the path for trees is $A_t = e^{-\delta(t-t')} A_{t'}\ \forall t \geq t'$. The issuer therefore sets the time-t' peg to

$$x_{t'} = \frac{\rho + \delta}{\delta} \frac{A_{t'}}{D_{t'}}, \tag{29}$$

as investors exiting at time t' hold $e^{\delta t'} D_{t'}$ stablecoins by construction, so that they consume $c_{t',0} = x_{t'} e^{\delta t'} D_{t'} = \frac{(\rho+\delta)}{\delta} e^{\delta t'} A_{t'}$. The argument can then be forward iterated to conclude that the peg is set according to (29) at all dates $t > 0$, i.e.,

$$x_t = \frac{\rho + \delta}{\delta} \frac{A_t}{D_t}. \tag{30}$$

Provided that the targeted path for trees is indeed implemented as of time $t' = 0$, i.e., $A_t = e^{-\delta t} A_0^+$, an exiting investor can then consume $c_{t,0} = e^{\delta t} x_t D_t = \frac{(\rho+\delta) e^{\delta t}}{\delta} A_t = \frac{\rho+\delta}{\delta} A_0^+$ by redeeming at the peg. The peg thus remains constant over time when the targeted path is implemented since this will imply $\dot{A}_t / A_t = \dot{D}_t / D_t = -\delta$, i.e., only exiting investors redeem so both the stock of stablecoins and the stock of trees contract at rate δ on the targeted path. Note that the way in which the issuer sets the peg according to Equation (30) resembles the design of floating-NAV MMFs. Such MMFs continuously adjust the NAV, i.e., the price at which the fund stands ready to redeem its shares, to reflect the value of the fund's assets per outstanding share. In our setup, the time-t peg (30) is likewise a function of the amount of trees per stablecoin in circulation at time t.

Turning to the ICO quantity, with the ICO price normalized to one so that we have $A_0^+ = D_0^+$, the issuer chooses D_0^+ to maximize

$$\int_0^{\infty} \delta e^{-(\delta+\rho)t} u\left(\frac{\delta + \rho}{\delta} D_0^+\right) \mathrm{d}t - D_0^+, \tag{31}$$

where we use that $c_{t,0} = \frac{\rho+\delta}{\delta} A_0^+$ on the targeted path. We therefore obtain $A_0^+ = D_0^+ = h_0 = \frac{\delta c^\star}{\rho+\delta}$ and $c_{t,0} = x_t D_0^+ = c^\star\ \forall t > 0$, i.e., the Arrow-Debreu allocation. Whether this allocation is implemented as an equilibrium depends on investors' incentives to participate in the ICO and to redeem coins only when they indeed exit the market. These incentives

would read as $c_{t,0} \int_t^{\infty} e^{\int_t^{\tau} (r_{S,s} - \rho - \delta) \mathrm{d}s} \mathrm{d}\tau \leq \int_t^{\infty} \delta c_{\tau,0} e^{-(\rho+\delta)(\tau - t)} \mathrm{d}\tau$, whilst with a secondary market it would read as $\dot{c}_{t,0} / c_{t,0} = r_{S,t}$. With storage earning a zero return, this does not make a difference. However, it would make a difference if storage earns some time-varying return $r_{S,t}$.

hinge on the issuer's redemption policy, i.e., how many stablecoins can be redeemed at date t against the peg x_t as specified in Equation (30), as well as the secondary market price q_t, as we show next.

Unlimited redemption. Suppose the issuer commits to use the full amount of income $(\phi+\rho)A_t$ from maturing trees for redemption when necessary. When a fraction χ_t of the stablecoins D_t in circulation is offered for redemption—thus assuming that the ICO indeed succeeded, as otherwise there would be no stablecoins in circulation—, the arrival rate of a successful redemption from the perspective of the investor is

$$\alpha_t = \frac{(\phi+\rho)A_t}{x_t \chi_t D_t}, \tag{32}$$

i.e., a flow α_t of the stablecoins offered for redemption is indeed redeemed.[8]

We focus on equilibria in which $x_t \geq q_t$, as otherwise not even the exiting investors would redeem their stablecoins with the issuer, i.e., there would be no redemption at all. If $x_t = q_t$, exiting investors are indifferent between selling their stablecoins or redeeming them, in which case we assume that all exiting investors choose to redeem, whilst non-exiting investors choose not to redeem. In that case, exiting investors will redeem all their stablecoins successfully as they have infinitesimal mass, i.e., $\chi_t \to 0$ so $\alpha_t \to \infty$. If $x_t > q_t$, all investors, both exiting and non-exiting, attempt to have their stablecoins redeemed, i.e., a run takes place. The reason is that a profit can be made by redeeming, i.e., if a non-exiting investor successfully redeems a stablecoin against the peg x_t, it can immediately buy back the same stablecoin at a lower price q_t. A run features $\chi_t = 1$ by definition and together with the peg given by (30) it follows that during a run

$$\alpha_t = \delta\frac{\phi+\rho}{\delta+\rho}. \tag{33}$$

From the above, we find that the return r_t^+ on stablecoins satisfies

$$r_t^+ = \delta\frac{\phi+\rho}{\delta+\rho}\max\left\{0, \frac{x_t}{q_t} - 1\right\} + \frac{\dot{q}_t}{q_t}. \tag{34}$$

According to Equation (34), the return is determined by: the flow profit that arises from

[8] An individual investor can also have a flow α_t of its coins redeemed because coins are perfectly divisible.

the strategy of redeeming stablecoins at x_t and buying them back at q_t in case $q_t < x_t$; and changes in the market price. The law of motion for the stock of stablecoins reads as

$$\dot{D}_t = \begin{cases} -\delta\frac{\phi+\rho}{\delta+\rho} D_t & \text{if } q_t < x_t, \\ -\delta D_t & \text{if } q_t = x_t. \end{cases} \tag{35}$$

The issuer cannot plant new trees during a run since all fruit is used for redemption. Thus,

$$\dot{A}_t = \begin{cases} -\phi A_t & \text{if } q_t < x_t, \\ -\delta A_t & \text{if } q_t = x_t. \end{cases} \tag{36}$$

Note that the stock of trees diminishes faster during a run since we assume $\phi > \delta$.

No runs. When a run does not take place, so that $q_t = x_t \; \forall t > 0$, we obtain

$$r_t^+ = \frac{\dot{q}_t}{q_t} = \frac{\dot{x}_t}{x_t} = \frac{\dot{A}_t}{A_t} - \frac{\dot{D}_t}{D_t} = 0, \tag{37}$$

where the second equality uses $q_t = x_t$, the third equality follows from the specification of x_t, and the fourth equality uses $\dot{D}_t/D_t = -\delta$. We thus have $x_0^+ = x_t = \frac{\delta+\rho}{\delta}$ since we have normalized $A_0^+ = D_0^+$. It follows that the Arrow-Debreu allocation is implemented and although investors can store goods, they have no incentive to do so. First, the return on the stablecoin at all times $t > 0$ is zero, i.e., it is not dominated by storage. Second, the investor strictly prefers to participate in the ICO over investing in storage at time $t = 0$ because $x_0^+ > 1$, i.e., the return from holding the stablecoin from time $t = 0$ to time $t = \varepsilon$, where $\varepsilon > 0$ is infinitesimal, approaches infinity. In other words, the issuer can implement the efficient Arrow-Debreu equilibrium by issuing stablecoins at a discount (relative to the peg at time $t = \varepsilon$) in the ICO. Once issued, provided there is no run, the peg then remains constant and the stablecoin effectively earns zero interest.

Runs. If the issuer does not limit redemption, there can also be unexpected runs at any time $t_R > 0$. To see this, suppose that $x_t = q_t \; \forall t < t_R$. Then consider what happens when the market price drops discontinuously and unexpectedly at time t_R, so that we have $q_{t_R} < x_{t_R}$. The aggregate resource constraint implies that during a run

$$\dot{S}_t = (\alpha_t x_t - \delta q_t)\, D_t - \delta S_t, \tag{38}$$

where S_t is the aggregate amount of storage that investors undertake on their own, δD_t is the flow of stablecoins offered for sale by exiting investors, δS_t is the flow of storage consumed by exiting investors, and $\alpha_t x_t D_t$ is the redemption flow paid by the issuer. Using the characterization of α_t, we can write

$$\dot{S}_t = \delta \left(\frac{\phi + \rho}{\delta + \rho} \frac{x_t}{q_t} - 1 \right) q_t D_t - \delta S_t. \tag{39}$$

It follows that during a run $S_t^+ > 0$: we have $x_t > q_t$ and we assume $\phi > \rho$, so that $\lim_{S_t \to 0} \dot{S}_t > 0$. The low price intuitively implies that, being forced to sell at a discount, exiting investors consume less than the resources paid out by the issuer. The difference is then stored by the non-exiting investors, as they cannot plant trees on their own. Investors are unwilling to undertake storage when $r_t^+ > 0$, and all investors want to sell stablecoins when $r_t^+ < 0$. Market clearance therefore requires $r_t^+ = 0$ during a run. Setting $q_t < x_t$ and $r_t^+ = 0$ in Equation (34), we thus find that the market price during a run develops as

$$0 = \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{x_t}{q_t} - 1 \right) + \frac{\dot{q}_t}{q_t}. \tag{40}$$

At this point, it is convenient to define $\theta_t \equiv q_t / x_t$, i.e., the discount at which the stablecoin trades. During a run, (40) implies that the discount develops as

$$\begin{aligned} 0 &= \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{1}{\theta_t} - 1 \right) + \frac{\dot{\theta}_t}{\theta_t} + \frac{\dot{x}_t}{x_t} \\ &= \delta \frac{\phi + \rho}{\delta + \rho} \left(\frac{1}{\theta_t} - 1 \right) + \frac{\dot{\theta}_t}{\theta_t} + \frac{\dot{A}_t}{A_t} - \frac{\dot{D}_t}{D_t} \\ &= \delta \frac{\phi + \rho}{\delta + \rho} \frac{1}{\theta_t} + \frac{\dot{\theta}_t}{\theta_t} - \phi, \end{aligned} \tag{41}$$

where the second line uses $\dot{x}_t / x_t = \dot{A}_t / A_t - \dot{D}_t / D_t \; \forall t$ and the third line uses the fact that $\dot{A}_t / A_t = -\phi$ and $\dot{D}_t / D_t = -\alpha_t$ during a run. In short, as long as $\theta_t < 1$, it must be that

$$\dot{\theta}_t = \phi \theta_t - \delta \frac{\phi + \rho}{\delta + \rho}. \tag{42}$$

Because q_t cannot drop discontinuously with positive probability (if it does, then at time $t-\varepsilon$ investors either already start to run or start to sell in market), it follows that starting from time t_R we have

$$\theta_t = \min\left\{\theta_{t_R} e^{\phi(t-t_R)} + \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}\left(1-e^{\phi(t-t_R)}\right), 1\right\}. \tag{43}$$

There is a steady state at $\theta = \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho} \equiv \underline{\theta} < 1$ and at $\theta = 1$. For $\theta_{t_R} < \underline{\theta}$, θ_t contracts at a rate that increases over time, so that at some time $t' > t_R$, we obtain $q_{t'} = 0$, which cannot be an equilibrium. The reason is that stablecoins must have a positive value because $x_t > 0$ even if the run continues forever. For $\theta_{t_R} \in (\underline{\theta}, 1)$, θ_t grows at an increasing rate until at some time $t' > t_R$, we obtain $\theta_{t'} = 1$. In short, at any point in time $t_R > 0$, a run can start unexpectedly if θ_{t_R} jumps from 1 into the interval $[\underline{\theta}, 1)$. The run either continues forever if $\theta_{t_R} = \underline{\theta}$, or it lasts for a finite period of time if $\theta_{t_R} \in (\underline{\theta}, 1)$. Once the run has stopped, storage develops as $\dot{S}_t = -\delta S_t$, i.e., the accumulated storage during a run gradually declines since the exiting investors consume their storage. The presence of storage is consistent with equilibrium because the stablecoin continues to earn zero return once the run has stopped.

A run implies that all investors are worse off, as the stock of trees contracts at an inefficiently high rate and part of the income from maturing trees is stored rather than used to plant new trees. In the proof of Proposition 3, where we fully characterize the allocations in case of a run, we show that consumption on exit drops permanently after the run has started, i.e., $c_{t,0} = \theta_{t_R} c^\star \ \forall t \geq t_R$.

Proposition 3. *Unexpected runs can occur when an issuer does not limit redemption and the resulting allocation in a run is Pareto inefficient.*

A run is self-fulfilling because the issuer cannot plant new trees during a run, as the income from maturing trees is used to cover redemption. This implies that x_t declines during the run, which explains why investors attempt to run—anticipating that x_t is going to decline, investors attempt to redeem stablecoins because they can store the redemption proceeds. This suggests that to prevent runs, the issuer should restrict redemption in such a way that, even in case of a run, investment is maintained at $I_t = (\phi - \delta)A_t$. We call stablecoins that satisfy this investment rule *micro-well-designed* and further analyze their properties below.

Example of a run. Figure 1 illustrates a run that starts at $t_R = 1$. The run starts with a sudden drop in the price q_t and continues until the peg x_t and the price q_t equalize again (Figure 1a). All investors attempt to redeem during the run and the amount of trees decreases faster than the amount of stablecoins in circulation (Figure 1b). Investors that do not exit use the proceeds from successful redemption to buy stablecoins from exiting investors and store the remainder, which leads to a positive stock of storage (Figure 1c). At some point, however, the stock of storage starts to decrease as exiting investors consume their storage. Individual consumption on exit drops discontinuously at the onset of the run, which reflects the sudden price drop (Figure 1d). Subsequently, consumption remains constant due to two forces that cancel each other out: on the one hand, the aggregate market value of stablecoins in circulation drops as the run continues; on the other, investors' stock of storage grows due to successful redemption.

Limited redemption. In a hypothetical run in which redemption is limited such that $I_t = (\phi - \delta)A_t$, i.e., the issuer adheres to the investment rule so that the stablecoin is micro-well-designed, we obtain $\alpha_t = \delta$. From the perspective of the issuer, the rate at which stablecoins are redeemed therefore equals δ, independently of whether a run takes place or not. The law of motion for stablecoins in circulation then reads

$$\dot{D}_t = -\delta D_t. \tag{44}$$

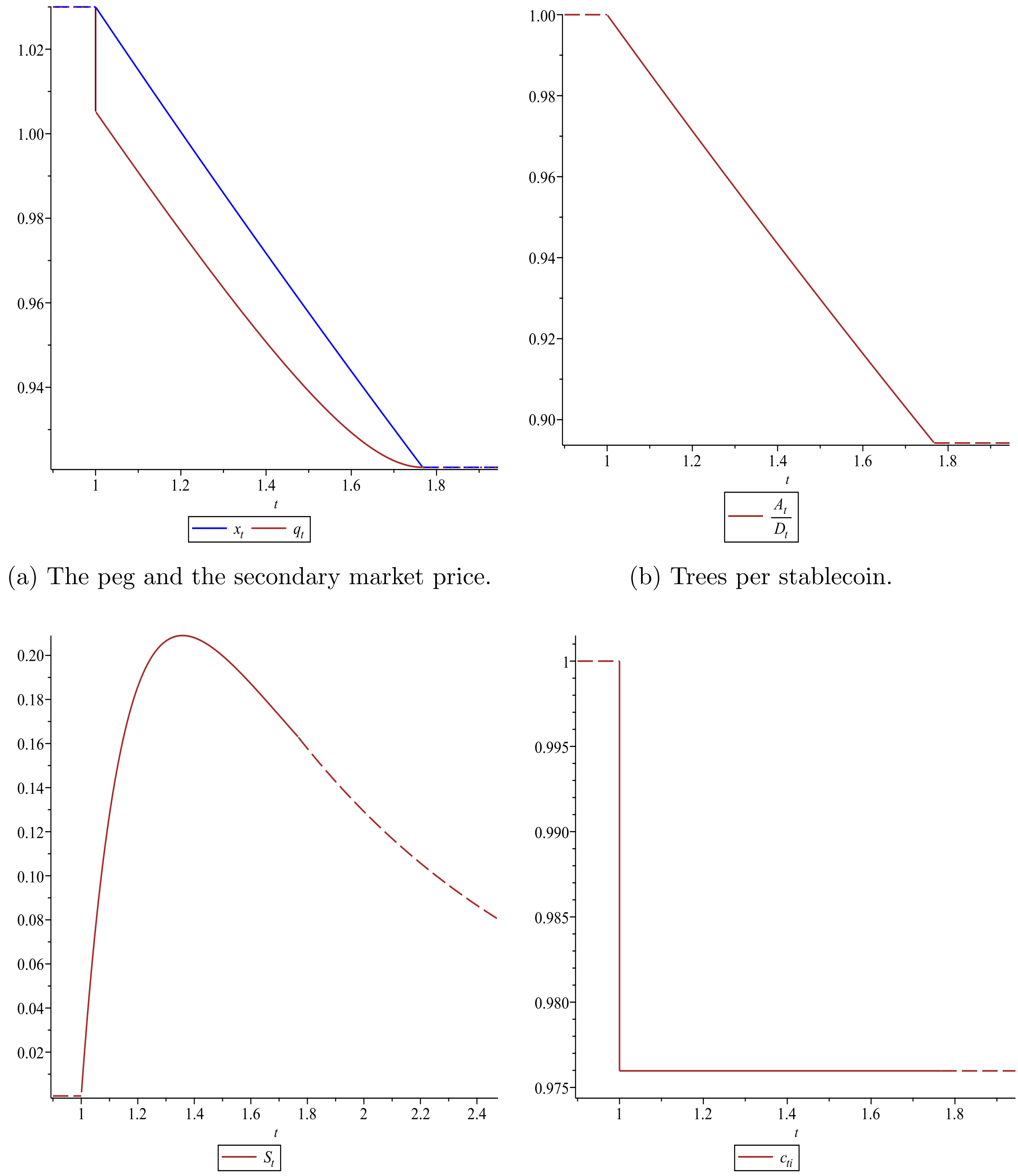
From Equation (38), based on the aggregate resource constraint, we obtain

$$\dot{S}_t = \delta(x_t - q_t)D_t - \delta S_t \tag{45}$$

during a run, so that, as before, $S_t^+ > 0$ when there is a run. This implies that the return on stablecoins should be zero, and during a run, this return reads as

$$r_t^+ = \delta(x_t/q_t - 1) + \dot{q}_t/q_t \tag{46}$$

Figure 1: A finite run



(a) The peg and the secondary market price.

(b) Trees per stablecoin.

(c) Aggregate savings.

(d) Individual consumption upon exit.

Note: The figure assumes a utility function $u = \ln(\cdot)$, which implies $c^\star = 1$ and thus $A_0 = D_0 = \frac{\delta}{\rho+\delta}$, $x_0 = \frac{\delta+\rho}{\delta}$. We let $\rho = 0.03, \delta = 1,\ \phi = 6$ and $\theta_{t_R} = \underline{\theta} + 0.01(1 - \underline{\theta})$.

because investors can now redeem at arrival rate δ instead of $\delta\frac{\phi+\rho}{\delta+\rho}$. Imposing $r_t^+ = 0$ and defining θ_t as before, we obtain

$$\begin{aligned}
0 &= \delta\left(\frac{1}{\theta_t} - 1\right) + \frac{\dot{\theta}_t}{\theta_t} + \frac{\dot{x}_t}{x_t} \\
&= \delta\left(\frac{1}{\theta_t} - 1\right) + \frac{\dot{\theta}_t}{\theta_t} + \frac{\dot{A}_t}{A_t} - \frac{\dot{D}_t}{D_t} \\
&= \delta\left(\frac{1}{\theta_t} - 1\right) + \frac{\dot{\theta}_t}{\theta_t},
\end{aligned} \tag{47}$$

where we used $\dot{A}_t/A_t = -\delta$ and $\dot{D}_t/D_t = -\delta$. Starting from time t_R, we thus have

$$\theta_t = \min\left\{(\theta_{t_R} - 1)e^{\delta(t-t_R)} + 1, 1\right\}. \tag{48}$$

There is now only one steady state at $\theta = 1$, i.e., a steady state in which there is no run. If a run unexpectedly occurred at time t_R, we would have $\theta_{t_R} < 1$. Then θ_t would diminish at an increasing rate until at some time $t' > t_R$ we would obtain $q_{t'} = 0$, which cannot be an equilibrium for reasons explained before.

Investors now have no strict incentive to run because a run does not change the dynamic development of the peg. Although a run implies that an exiting investor cannot redeem stablecoins (only an infinitesimal fraction of stablecoin holdings, i.e., a flow, can be redeemed), exiting investors can sell their coins at a price $q_t = x_t$ in the secondary market. The reason is that in case of a run, the issuer restricts redemption so that the dynamic development of the stock of trees is left unaffected.

We stress the role of the secondary market for this result. In dynamic models from the banking literature that add one or more (or, in our case, a continuum of) periods to the benchmark two-period model of Bryant (1980) and Diamond and Dybvig (1983), a redemption-limit policy typically fails to prevent runs, as investors have an incentive to run pre-emptively if they fear a future run. A future run namely implies the risk of not being able to obtain funds in case of a future need for liquidity, exactly because a redemption limit would then be put in place, as argued by Engineer (1989). In our model, however, investors with a liquidity need can always sell stablecoins in the secondary market, albeit at a potentially lower price than the peg, when they cannot or fail to redeem. The competitive nature of the secondary market implies that the market price remains unaffected by the run since, due to the investment rule, the run leaves the dynamic development of the stock of trees, and therefore also the dynamic development of the peg, unchanged.

We thus find that a monopolistic stablecoin issuer can uniquely implement the Arrow-Debreu allocation by minting a micro-well-designed stablecoin that is issued at a discount in the ICO. The investment rule, which, in essence, limits the amount of liquidity that the issuer provides at the peg over and above the income from maturing trees, guarantees that the peg remains constant over time. Since a constant peg represents zero interest payments, the stablecoin issued by a monopolist is also macro-well-designed.

Theorem 1 (Monopolistic stablecoins)**.** *A micro-well-designed stablecoin is invulnerable to runs since the issuer adheres to an investment rule. A macro-well-designed stablecoin uniquely implements the Pareto-efficient Arrow-Debreu allocation as an equilibrium.*

7 Stablecoin Competition

We now return to the baseline model with new investors entering at rate δ. The stablecoin economy then represents a platform with multiple tradable stablecoins. The purpose is to show that a system of competing, micro-well-designed stablecoins can but need not implement the Pareto-efficient Arrow-Debreu allocation.

7.1 Incumbent issuers

Consider first the choices of incumbent issuers at some point in time t', as we did for the monopolistic issuer. Incumbent issuer s is the coalition of investors entering at date $s \in [0, t')$. Let issuer s have $D_{t',s}$ stablecoins in circulation and let it own $A_{t',s} > 0$ trees, where these quantities are expressed relative to the associated cohort's initial size. Assume also that $D_{t',s}$ is fully held by the cohort of investors s and let $Z_{t,s}$ denote cohort s's holdings of other coins and/or storage at time $t \geq t'$. We suppose that $Z_{t',s} = 0$ to represent a situation in which the incumbent investors initially only hold their own stablecoin backed by the positive stock of trees $A_{t',s}$, much like we assumed $S_{t',0} = 0$ in Section 6.

The incumbent issuer's choices. Incumbent issuer s maximizes the utility of cohort s but has to take as given the return process $(r_t)_{t=t'}^{\infty}$—a key equilibrium variable that depends on the design of all other stablecoins. Here, r_t is the return that investors can earn by storing goods or trading other stablecoins, where the option of storage implies $r_t \geq 0 \; \forall t > t'$. More particularly, $(c_{t,s}, A_{t,s}, Z_{t,s})_{t=t'}^{\infty}$ is now targeted to maximize

$$\int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s}) \mathrm{d}t, \tag{49}$$

subject to: the law of motion

$$\dot{A}_{t,s} + \dot{Z}_{t,s} = \rho A_{t,s} + r_t Z_{t,s} - \delta e^{-\delta(t-s)} c_{t,s}; \tag{50}$$

the constraints $\dot{A}_{t,s} \geq -\phi A_{t,s}$ and $Z_{t,s} \geq 0$; the starting values $A_{t',s}$ and $Z_{t',s} = 0$; and the dynamic incentive-feasibility constraint $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$. The law of motion simply states that the change in the asset stock, which consists of trees, storage, and holdings of other stablecoins, is equal to the return on the asset stock minus the consumption by the cohort's exiting investors. The intuition for the incentive-feasibility constraint is similar as before. I.e., stablecoin s is designed to provide a consumption process $(c_{t,s})_{t=t'}^{\infty}$ for cohort s. The effective return on stablecoin s is therefore $r_{t,s}^+ = \dot{c}_{t,s}/c_{t,s}$. But with the outside option of trading other coins and/or storage, investors can earn an effective real return $\max\{r_t^+, \dot{c}_{t,s}/c_{t,s}\}$. The outside option would thus dominate stablecoin s when $r_t^+ > \dot{c}_{t,s}/c_{t,s}$, so that investors would deviate from the targeted consumption process. The dynamic incentive-feasibility constraint prevents such a deviation.

Regarding issuer s's desire to hold other stablecoins and/or storage, we find

Lemma 1. *Targeting $Z_{t,s} = 0 \; \forall t > t'$ is optimal only if $r_t \leq \rho \; \forall t > t'$.*

The intuition is that when the return on other assets—storage and other stablecoins—exceeds the fundamental return on trees $r = \rho$, the issuer prefers other assets over trees.

In general equilibrium we must have $S_t = Z_{t,0} + \delta \int_0^t Z_{t,s} \mathrm{d}s$, i.e., the aggregate holding of other assets must equal storage. With $r_t > \rho$ we must therefore have $S_t > 0$ on the one hand, since Lemma 1 applies to all incumbent issuers. On the other hand, $S_t > 0$ directly implies $r_t = 0$, a contradiction. In what follows, we can therefore focus on return processes $(r_t)_{t=t'}^{\infty}$ that satisfy $r_t \in [0, \rho] \; \forall t > t'$. It then turns out that the dynamic incentive-feasibility constraint is always binding and the constraint $\dot{A}_{t,s} \geq -\phi A_{t,s}$ is always slack:

Lemma 2. *Suppose* $r_t \in [0, \rho]$ $\forall t > t'$*. Then targeting* $Z_{t,s} = 0$ $\forall t > t'$ *is optimal. The optimal targeted consumption process* $(c_{t,s})_{t=t'}^{\infty}$ *furthermore satisfies* $\dot{c}_{t,s}/c_{t,s} = r_t^+$ $\forall t \geq t'$*, and the constraint* $\dot{A}_{t,s} \geq -\phi A_{t,s}$ *is always slack.*

From the proof of Lemma 2 we also obtain

Corollary 1. *The optimal targeted process* $(c_{t,s}, A_{t,s})_{t=t'}^{\infty}$ *is time consistent and implies that consumption on exit at time* t *is a multiple of the trees per investor at time* t*:*

$$c_{t,s} = a_{t,s} \left[\delta \int_t^{\infty} e^{\int_t^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau \right]^{-1}, \quad \text{where} \quad a_{t,s} \equiv e^{\delta(t-s)} A_{t,s}. \tag{51}$$

We note that the multiple in Equation (51), i.e., the term in square brackets, accounts for two factors: the fundamental return that trees will earn; and the process $(r_\tau)_{\tau=t}^{\infty}$ that future consumption growth has to match in order to satisfy the dynamic incentive-feasibility constraint. In essence, a higher required return r reduces current consumption given the current amount of trees. The reason is that a higher return requires stronger future consumption growth, which can only be financed when more trees are planted, thus requiring a lower level of current consumption.

Optimal coin design. Can issuer s design its stablecoin to implement the allocation described above? The answer is yes, as becomes evident by supposing that at any time $t \geq t'$, issuer s stands ready to redeem its stablecoin for $x_{t,s} > 0$ goods, subject to a resource constraint. With $A_{t,s}$ trees owned by issuer s at time t, the time-t flow payment to cover redemption is at most $\phi y A_{t,s}$ $(= (\phi + \rho) A_{t,s})$. The optimal allocation above, however, suggests that $\dot{A}_{t,s} = \rho A_{t,s} - \delta c_{t,s} e^{-\delta(t-s)}$. To maintain this path, the issuer should devote at most

$$\begin{aligned} \delta c_{t,s} e^{-\delta(t-s)} &= e^{-\delta(t-s)} \delta c_{t,s} \\ &= \left[\int_t^{\infty} e^{\int_t^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau \right]^{-1} A_{t,s} \end{aligned} \tag{52}$$

goods to cover redemption, where the second equality follows from Equation (51). Stablecoin s is micro-well-designed, i.e., it is invulnerable to runs, when its issuer adheres to the investment rule above. This is shown next.

From the perspective of an investor, the arrival rate of a successful withdrawal is

$$\alpha_{t,s} = \frac{\left[\int_t^\infty e^{\int_t^\tau [r_T - \rho - \delta]\mathrm{d}T} \mathrm{d}\tau\right]^{-1} A_{t,s}}{x_{t,s}\chi_{t,s}D_{t,s}}, \tag{53}$$

when a fraction $\chi_{t,s}$ of the stablecoins $D_{t,s}$ is offered for redemption. Similar as in Equation (30), issuer s sets the peg

$$x_{t,s} = \frac{1}{\delta \int_t^\infty e^{\int_t^\tau [r_T - \rho - \delta]\mathrm{d}T} \mathrm{d}\tau} \frac{A_{t,s}}{D_{t,s}} \tag{54}$$

to implement the optimal consumption path, as an exiting investor holds the amount $D_{t,s}e^{\delta(t-s)}$ of stablecoins by construction, so that it can consume $e^{\delta(t-s)}x_{t,s}D_{t,s}$ by redeeming. In the special case that $r_\tau = 0 \ \forall \tau \geq t$, i.e., the other stablecoins do not outperform storage, we get the exact same peg as set by the monopolistic issuer (see Equation (30)). When the other stablecoins earn a positive return, however, the peg is set at a lower level. The reason is that the issuer, when the targeted path is indeed implemented, must let the peg increase over time to match the return on other stablecoins. To pay for a peg that increases over time, the current peg has to be lower so that more resources can be devoted to planting trees.

Stablecoin s can be traded in a secondary market at price $q_{t,s}$. As before, we focus on equilibria in which $x_{t,s} \geq q_{t,s}$. When $x_{t,s} = q_{t,s}$, only exiting investors from cohort s redeem stablecoin s and when $x_{t,s} > q_{t,s}$, all investors, exiting or not, attempt to have stablecoin s redeemed, i.e., a run takes place. In case of a run, which features $\chi_{t,s} = 1$ by definition, the characterization of $x_{t,s}$ in Equation (54) implies that $\alpha_{t,s} = \delta$. The return earned by stablecoin s therefore satisfies

$$r_{t,s}^+ = \delta \max\{0, x_{t,s}/q_{t,s} - 1\} + \dot{q}_{t,s}/q_{t,s}, \tag{55}$$

which is the same equation as for the micro-well-designed monopolistic stablecoin (46), and, no matter whether a run takes place or not, we have

$$\dot{D}_{t,s} = -\delta D_{t,s} \quad \text{and} \quad \dot{A}_{t,s} = \left(\rho - \left[\int_t^\infty e^{\int_t^\tau [r_T - \rho - \delta]\mathrm{d}T} \mathrm{d}\tau\right]^{-1}\right) A_{t,s}. \tag{56}$$

The equilibrium return on stablecoin s satisfies $r_{t,s}^+ = r_t^+$ if there is no run at time t. This follows from

$$r_{t,s}^{+}=\frac{\dot{q}_{t,s}}{q_{t,s}}=\frac{\dot{x}_{t,s}}{x_{t,s}}=\left[\int_{t}^{\infty}e^{\int_{t}^{\tau}[r_{T}-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau\right]^{-1}+r_{t}^{+}-\rho-\delta+\frac{\dot{A}_{t,s}}{A_{t,s}}-\frac{\dot{D}_{t,s}}{D_{t,s}}=r_{t}^{+}, \tag{57}$$

where the second equality uses $q_{t,s}=x_{t,s}$, the third equality follows from Equation (54), and the fourth equality uses $\dot{D}_{t,s}/D_{t,s}=-\delta$ and $\dot{A}_{t,s}/A_{t,s}=\rho-\left[\int_{t}^{\infty}e^{\int_{t}^{\tau}[r_{T}-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau\right]^{-1}$. We note that Equation (57) implies that $x_{t,s}$ is only constant over time if $r_{t,s}^{+}=r_{t}^{+}=0$.

No runs under optimal coin design. There are no runs on stablecoin s when it is micro-well-designed, i.e., when issuer s adheres to the investment rule described above. To see this, suppose first that $x_{t,s}=q_{t,s}\ \forall t\in[t',t_{R})$. Then consider what happens if at time t_R the market price drops unexpectedly so that we have $q_{t_R,s}<x_{t_R,s}$. In the spirit of Equation (45), we obtain

$$\dot{Z}_{t,s}=\delta\left(x_{t,s}-q_{t,s}\right)D_{t,s}-\delta Z_{t,s}, \tag{58}$$

where $Z_{t,s}$ are cohort s's holdings of other coins and/or storage. The return on $Z_{t,s}$ is, by definition, r_t. It follows that $Z_{t,s}^{+}>0$ during a run because $\lim_{Z_{t,s}\to 0}\dot{Z}_{t,s}=\delta\left(x_{t,s}-q_{t,s}\right)D_{t,s}$, which is strictly positive because $x_{t,s}>q_{t,s}$ during a run. Cohort s thus needs to invest in other stablecoins and/or storage during the run. With $r_{t,s}^{+}>r_{t}^{+}$, all incumbent investors prefer to hold stablecoin s and with $r_{t,s}^{+}<r_{t}^{+}$ all investors want to sell stablecoin s. Market clearance therefore requires $r_{t,s}^{+}=r_{t}^{+}$ in case of a run. Thus, setting $r_{t,s}^{+}=r_{t}^{+}$ and $x_{t,s}>q_{t,s}$ in Equation (57), we find that the market price during a run develops as

$$r_{t}^{+}=\delta\left(x_{t,s}/q_{t,s}-1\right)+\dot{q}_{t,s}/q_{t,s}. \tag{59}$$

Defining $\theta_{t,s}\equiv q_{t,s}/x_{t,s}$, with $\theta_{t_R,s}<1$, we obtain

$$\begin{aligned}
r_{t}^{+}&=\delta\left(\frac{1}{\theta_{t,s}}-1\right)+\frac{\dot{\theta}_{t,s}}{\theta_{t,s}}+\frac{\dot{x}_{t,s}}{x_{t,s}}\\
&=\delta\left(\frac{1}{\theta_{t,s}}-1\right)+\frac{\dot{\theta}_{t,s}}{\theta_{t,s}}+\left[\int_{t}^{\infty}e^{\int_{t}^{\tau}[r_{T}-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau\right]^{-1}+r_{t}^{+}-\rho-\delta+\frac{\dot{A}_{t,s}}{A_{t,s}}-\frac{\dot{D}_{t,s}}{D_{t,s}}\\
&=\delta\left(\frac{1}{\theta_{t,s}}-1\right)+\frac{\dot{\theta}_{t,s}}{\theta_{t,s}}+r_{t}^{+}.
\end{aligned} \tag{60}$$

Starting from time t_R, we find

$$\theta_{t,s} = \min\left\{(\theta_{t_R,s} - 1)e^{\delta(t-t_R)} + 1, 1\right\}. \tag{61}$$

There is a unique steady state at $\theta = 1$. If $\theta_{t_R,s} < 1$, $\theta_{t,s}$ would decline at a rate that increases over time, so that at some time $t'' > t_R$ we would have $q_{t'',s} = 0$, which cannot be an equilibrium for reasons discussed earlier. Thus, there is no run on stablecoin s, exactly because it is micro-well-designed. However, as we will show below, there is a continuum of processes $(r_t)_{t=t'}^{\infty}$ that are consistent with an equilibrium.

7.2 New stablecoin issuers

We next derive the time-t' choices of the new issuer representing the cohort of investors entering at time t'. The new issuer becomes incumbent at time $t' + \varepsilon$, so we can build on the choices of incumbent issuers. More particularly, issuer t' mints a micro-well-designed stablecoin and we need to solve for the ICO quantity $D^+_{t',t'}$. We again use the normalization $D^+_{t',t'} = A^+_{t',t'}$ so that the ICO price is one. It is clear from the choices of an incumbent issuer that the consumption path targeted and, in fact, implemented by the new issuer, satisfies

$$c_{t,t'} = c_{t',t'} e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau}, \quad \text{with} \quad c_{t',t'} = D^+_{t',t'} \left[\delta \int_{t'}^{\infty} e^{\int_{t'}^{\tau}[r_T - \rho - \delta]\mathrm{d}T} \mathrm{d}\tau\right]^{-1}. \tag{62}$$

Once issued, the return $r_{t,t'}$ earned by stablecoin t' thus mimics the return process $(r_t)_{t=t'}^{\infty}$, and since $x_{t,t'} = q_{t,t'}$ $\forall t > t'$ and $q^+_{t',t'} D^+_{t',t'} = c^+_{t',t'}$, the cohort of investors t' are willing to participate in the ICO if and only if[9]

$$\delta \int_{t'}^{\infty} e^{\int_{t'}^{\tau}[r_T - \rho - \delta]\mathrm{d}T} \mathrm{d}\tau \leq 1, \tag{63}$$

which holds true since the return process $(r_t)_{t=t'}^{\infty}$ satisfies $r_t \in [0, \rho]$ $\forall t > t'$. More particularly, there is an ICO discount unless $r_t = \rho$ $\forall t$. The discount is the higher, the lower the return on outside options implied by $(r_t)_{t=t'}^{\infty}$. If $r_t = 0$ $\forall t' > t$, we obtain an ICO discount $\frac{\delta}{\rho+\delta}$ and a constant peg and consumption path thereafter, i.e., exactly as with the monopolistic issuer described in Section 6.

[9]To be precise, participation requires $\lim_{\varepsilon\to 0} q_{t'+\varepsilon,t'} \geq \lim_{\varepsilon\to 0} e^{\int_{t'}^{t'+\varepsilon} r_\tau \mathrm{d}\tau}$, which reduces to (63).

With the ICO taking place and generating the consumption process as specified by Equation (62), it follows that the new issuer t' chooses the ICO quantity $D^+_{t',t'}$ to maximize

$$\int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-t')} u \left(\left[\delta \int_{t'}^{\infty} e^{\int_{t'}^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau \right]^{-1} e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau} D^+_{t',t'} \right) \mathrm{d}t - D^+_{t',t'}. \tag{64}$$

The resulting ICO quantity is therefore uniquely determined by the first-order condition

$$0 = \int_{t'}^{\infty} e^{\int_{t'}^{t} [r_\tau - \rho - \delta] \mathrm{d}\tau} u' \left(\left[\delta \int_{t'}^{\infty} e^{\int_{t'}^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau \right]^{-1} e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau} D^+_{t',t'} \right) \mathrm{d}t - \int_{t'}^{\infty} e^{\int_{t'}^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau. \tag{65}$$

7.3 General equilibrium

We now characterize the equilibrium return processes $(r_t)_{t=0}^{\infty}$ and the associated allocations. It is useful to explicitly introduce the equilibrium concept for an economy with competing stablecoins.

Sequential markets equilibrium. We focus on equilibria with micro-well-designed stablecoins, as this allows the issuers to prevent runs. Recall that we normalize ICO prices to one and that the equilibrium return process $(r_t)_{t=0}^{\infty}$ is taken as given by the stablecoin issuers and the investors. As we explain in Section 7.1, the process has to satisfy $r_t \in [0, \rho]\ \forall t$, which, by Lemma 2, allows us to focus on an equilibrium with each cohort holding only its own stablecoin, i.e., $(Z_{t,s})_{t=s}^{\infty} = 0\ \forall s$. Lemma 2 also tells us that with $r_t \in [0, \rho]$, the constraint $\dot{A}_{t,s} \geq -\phi A_{t,s}$ is always slack and the dynamic incentive-feasibility constraint always binds. We therefore obtain the following characterization:

Definition 1 (Sequential markets equilibrium)**.** *A competitive equilibrium with competing, micro-well-designed stablecoins, is a sequence for: the investors' labor effort and contingent-consumption schedules, i.e.,* $(h_s, (c_{t,s})_{t=s}^{\infty})_{s=0}^{\infty}$*; the stock of trees and amount of stablecoins in circulation for each issuer s, i.e.,* $((A_{t,s}, D_{t,s})_{t=s}^{\infty})_{s=0}^{\infty}$*; the peg for each stablecoin s, i.e.,* $((x_{t,s})_{t=s}^{\infty})_{s=0}^{\infty}$*; the secondary market price for each stablecoin s, i.e.,* $((q_{t,s})_{t=s}^{\infty})_{s=0}^{\infty}$*; the return earned by each stablecoin s, i.e.,* $((r_{t,s})_{t=s}^{\infty})_{s=0}^{\infty}$*; and the return process* $(r_t)_{t=0}^{\infty} \in [0, \rho]$ *on the investors' outside option, such that:*

(i) any incumbent issuer s maximizes at any time $t' > s \geq 0$ *the utility of cohort s given* $(r_t)_{t=t'}^{\infty}$ *and* $(A_{t',s}, D_{t',s})$*, i.e.,* $(c_{t,s}, A_{t,s})_{t=t'}^{\infty}$ *maximizes (49) s.t.: the law of*

motion (50) *with* $Z_{t,s} = 0\ \forall t \geq t'$*; the constraint* $\dot{A}_{t,s} \geq -\phi A_{t,s}\ \forall t \geq t'$*; the starting values* $A_{t',s}$ *and* $D_{t',s}$*; the incentive-feasibility constraint* $\dot{c}_{t,s}/c_{t,s} = r_t\ \forall t \geq t'$*; and the peg* $(x_{t,s})_{t=t'}^{\infty}$ *satisfies* (54)*;*

(ii) *any incumbent issuer* $s \geq 0$ *adheres to the investment rule* $\dot{A}_{t,s} = \rho A_{t,s} - \delta c_{t,s} e^{-\delta(t-s)}$ *so that it limits the funds earmarked for redemption by RHS of* (52) *and its stock of trees and circulating stablecoins develop according to* (56)*;*

(iii) *any new issuer* $s \geq 0$ *maximizes the expected utility of cohort* s *given* $(r_t)_{t=s}^{\infty}$ *and the anticipated optimal consumption plan* $(c_{t,s})_{t=s}^{\infty}$ *by issuing the optimal amount of stablecoins, i.e.,* $D_{s,s}^{+}$ *maximizes* (64) *and the issuer's ICO succeeds so that* $h_s = A_{s,s}^{+} = D_{s,s}^{+}$*;*

(iv) *the secondary market for stablecoins clears, i.e.,* $q_{t,s} = x_{t,s}$*, so that* $r_{t,s}^{+} = \dot{x}_{t,s}/x_{t,s}$*;*

(v) *the return on investors' outside option equals the highest attainable return given the availability of storage and the tradability of stablecoins, i.e., it is* $(r_t^{+})_{t=0}^{\infty} = (\max\{\max_{s\in[0,t)}\{r_{t,s}^{+}\}, 0\})_{t=0}^{\infty}$.

We are now ready to state the first main result on stablecoin competition:

Theorem 2 (Instability of a system of competing stablecoins)**.** *For every piecewise continuous return process* $r_t \in [0, \rho]\ \forall t > 0$*, there exists a sequential markets equilibrium in which each cohort issues a micro-well-designed stablecoin that generates the return* r_t *at each point in time once the stablecoin has been issued.*

This result follows directly from the fact that with $\dot{A}_{t,s}$ and $\dot{D}_{t,s}$ given by Equation (56), we have $\dot{x}_{t,s}/x_{t,s} = r_t^{+}$ according to Equation (54). In other words, if the return on the outside option is $r_t \in [0, \rho]$, every stablecoin in circulation at time t will also earn r_t as follows from an incumbent issuer's choices. The incumbent issuer ideally offers a zero return on its stablecoin to provide the corresponding cohort of investors with the flat contingent-consumption schedule that characterizes the Arrow-Debreu allocation, but consumption needs to grow at least at the rate r_t earned by investors' outside option. Accordingly, also offering a return r_t on the stablecoin is the second-best thing to do. This makes any piecewise continuous process $r_t \in [0, \rho]\ \forall t > 0$ self-fulfilling.[10]

[10]We assume piecewise continuity so that we have well-defined integrals over r_t.

The self-fulfilling nature of equilibria highlighted above suggests that even if all issuers mint a micro-well-designed stablecoin, competition between stablecoin issuers is a source of instability. An issuer's potential to implement the efficient allocation by means of a micro-well-designed stablecoin requires, on the one hand, that the stablecoin pays zero interest. Thus, it must be *macro-well-designed*, i.e., provide the investors with a flat contingent-consumption schedule so that consumption is independent of the realized investment horizon. On the other hand, the issuer's ability to mint a macro-well-designed stablecoin depends on the actions of all other coin issuers, i.e., issuers face a coordination game.

More particularly, without regulation, incumbent issuers anticipate competition from other incumbent issuers as well as from future issuers. If all incumbent and future issuers mint macro-well-designed stablecoins, it is in the individual interest of a single issuer to mint a macro-well-designed stablecoin, too. However, if one issuer mints an interest-bearing, micro-well-designed stablecoin, all the incumbent issuers have to change the design of their stablecoins. This implies that the equilibrium allocation in the stablecoin economy moves away from the Pareto-efficient Arrow-Debreu allocation, i.e., a single interest-bearing, micro-well-designed stablecoin is *contagious*.

Our findings highlight how a secondary market for stablecoins can but need not lead to a breakdown of efficient liquidity insurance. It is well-known from the banking literature that efficient liquidity insurance unravels if investors can directly trade interest-bearing assets—trees in our model. In the stablecoin economy, although investors cannot trade trees directly, a related but novel issue arises since investors can invest in other stablecoins via the secondary market. The novelty is that the stablecoins earn an endogenous return, i.e., the return that a stablecoin earns depends, through the dynamic incentive-feasibility constraint, on the return earned by other coins. Although the secondary-market helps to prevent runs on individual coins, as argued in Section 6, it does open the door to a coordination problem that renders the stablecoin economy unstable and inefficient.

The discussion above suggests that regulatory intervention can rule out coordination problems by preventing interest payments on stablecoins. This is our second and last finding with regard to stablecoin competition.

Theorem 3 (Regulating interest payments on stablecoins)**.** *If regulation ensures* $r_t = 0\ \forall t$*, the efficient Arrow-Debreu allocation is implemented as the unique equilibrium.*

8 Conclusion

We have developed a model to study how stablecoins should be designed, whether there is contagion when multiple stablecoins compete via a secondary market, and how such contagion can be prevented. The optimal design is straightforward at the micro-level: an investment rule with limited redemption avoids runs and guarantees the stability of the coin in the presence of a secondary market. Such a micro-well-designed stablecoin implements the Pareto-efficient allocation by paying zero interest when it faces no competition from other stablecoins via the secondary market. Micro-well-designed coins, however, are not sufficient at the macro-level because a single, micro-well-designed stablecoin that pays interest is contagious for other stablecoins. This provides a rationale for prohibiting stablecoin issuers from paying interest.

There are several extensions that can be pursued with the current model. One could, for instance, add a convenience yield to the holding of stablecoins if they can be used with lower transaction costs to make payments compared to national currencies. Moreover, one might introduce agency conflicts when a private company issues and operates stablecoins on behalf of investors. Agency conflicts could also necessitate standardized requirements that a stablecoin issuer has to fulfill in order to mint stablecoins, similar to the licenses that commercial banks need to be able to operate in the current monetary system. Finally, one could study how repeated issuance of stablecoins affects the result, i.e., we could allow a cohort of investors to issue further coins according to some predetermined plan once they have started issuance.

One can certainly agree with Gorton and Zhang (2021) that the new world of stablecoins posits similar problems to those encountered in earlier free-banking eras. But the knowledge on how to address these problems, the technical and financial infrastructure, and our entire monetary system have evolved considerably since then. The search for stablecoins that compete and produce favorable outcomes therefore continues. The current paper is a step in that direction.

A Proofs

We provide all our proofs in continuous time. Our continuous-time model can also be cast as the limit of a discrete-time model, for which we find that all our results hold true. The associated proofs in discrete time are available on request.

A.1 Proof of Proposition 1

The first step is to derive a lower bound on the utility $\mathcal{U}_s$ that an investor entering at time s can attain in an Arrow-Debreu equilibrium. Let $I_{t,s}$ denote the time-t investment made to plant trees by an investor entering at time s. We show first that regardless of the process for Arrow-Debreu prices $(p_t)_{t=0}^{\infty}$, the investor can finance

$$h_s = \frac{\delta}{\rho + \delta} c^\star \quad \text{and} \quad c_{t,s} = c^\star \ \forall t \geq s \tag{66}$$

by planting the following amount of trees:

$$I_{t,s} = \begin{cases} 0 & \text{if } t < s, \\ \frac{\delta}{\rho+\delta} c^\star & \text{if } t = s, \\ e^{-\delta(t-s)} \frac{\delta(\phi-\delta)}{\delta+\rho} c^\star & \text{if } t > s. \end{cases} \tag{67}$$

By assumption $\phi > \delta$, so these quantities are non-negative (and therefore feasible). Time-t net expenditure by the investor, i.e., expected consumption expenditure and investment expenditure minus labor income, then reads as

$$E_{t,s} = \begin{cases} 0 & \text{if } t < s, \\ I_{s,s} p_s - h_s p_s = 0 & \text{if } t = s, \\ \delta e^{-\delta(t-s)} c_{t,s} p_t + I_{t,s} p_t = e^{-\delta(t-s)} \frac{\delta(\phi+\rho)}{\delta+\rho} c^\star p_t & \text{if } t > s, \end{cases} \tag{68}$$

where $c_{t,s} = c^\star$ by Equation (66), and the income from planted trees gestating at time t reads as

$$\begin{aligned} Y_{t,s} &= \phi y p_t \int_0^t e^{-\phi(t-\tau)} I_{\tau,s} \mathrm{d}\tau \\ &= (\phi+\rho) p_t \int_0^t e^{-\phi(t-\tau)} I_{\tau,s} \mathrm{d}\tau \\ &= \mathbf{1}_{\{t>s\}} \frac{\delta(\phi+\rho)}{\delta+\rho} c^\star p_t \left[e^{-\phi(t-s)} + (\phi-\delta) \int_s^t e^{-\phi(t-\tau)} e^{-\delta(\tau-s)} \mathrm{d}\tau \right] \\ &= \mathbf{1}_{\{t>s\}} e^{-\delta(t-s)} \frac{\delta(\phi+\rho)}{\delta+\rho} c^\star p_t. \end{aligned} \tag{69}$$

The net income $(Y_{t,s})_{t=0}^\infty$ generated by the planting schedule $(I_{t,s})_{t=0}^\infty$ therefore exactly covers the net expenditure $(E_{t,s})_{t=0}^\infty$, and it is a feasible planting schedule since $\phi > \delta$ implies that $(I_{t,s})_{t=0}^\infty$ is never negative. The utility obtained by the investor in an Arrow-Debreu equilibrium is therefore bounded from below by

$$\begin{aligned} \mathcal{U}_s \geq \underline{\mathcal{U}} &\equiv -\frac{\delta}{\rho+\delta} c^\star + \int_s^\infty \delta e^{-(\rho+\delta)(t-s)} u(c^\star) \mathrm{d}t \\ &= \frac{\delta \left[u(c^\star) - c^\star \right]}{\rho+\delta}. \end{aligned} \tag{70}$$

The next step is to find an upper bound on $\mathcal{U}_s$ from the resource constraints and the fact that $\mathcal{U}_s \geq \underline{\mathcal{U}}$ $\forall s$ must hold in an Arrow-Debreu equilibrium. Define the welfare measure

$$\mathcal{W} = \mathcal{U}_0 + \delta \int_0^\infty e^{-\rho s} \mathcal{U}_s \mathrm{d}s. \tag{71}$$

The utility of investors entering at time 0 thus receives weight one, and the utility of investors entering at time $s > 0$ is treated as a flow that accounts for the rate of entry δ as well as discounting according to the rate of time preference ρ. From the analysis above, we can find a lower bound on the realized welfare $\mathcal{W}$ in an Arrow-Debreu equilibrium:

$$\begin{aligned} \mathcal{W} \geq \underline{\mathcal{W}} &\equiv \underline{\mathcal{U}} \left[1 + \delta \int_0^\infty e^{-\rho s} \mathrm{d}s \right] \\ &= \frac{\rho+\delta}{\rho} \underline{\mathcal{U}} \\ &= \frac{\delta \left[u(c^\star) - c^\star \right]}{\rho}. \end{aligned} \tag{72}$$

The aggregate resource constraints, on the other hand, imply an upper bound

$$
\overline{\mathcal{W}} \equiv \max_{\left((I_t,h_t,(c_{t,s})_{s=0}^{s=t})\right)_{t=0}^{\infty}} \left\{ \begin{array}{l} \int_0^\infty e^{-\rho t}\left(\delta e^{-\delta t}u(c_{t,0}) + \int_0^t \delta^2 e^{-\delta(t-s)}u(c_{t,s})\mathrm{d}s - \delta h_t\right)\mathrm{d}t \\ \quad - h_0 \end{array} \right\}
$$

$$
\begin{aligned}
\text{s.t.} \quad & \dot{A}_t && = I_t - \phi A_t && \forall t>0, \quad (\lambda_{A,t}) \\
& \dot{S}_t && = \delta h_t + (\phi+\rho)A_t - I_t - \delta e^{-\delta t}c_{t,0} - \textstyle\int_0^t \delta^2 e^{-\delta(t-s)}c_{t,s}\mathrm{d}s && \forall t>0, \quad (\lambda_{S,t}) \\
& h_t && \geq 0, && (\mu_{h,t}) \\
& S_t && \geq 0, && (\mu_{S,t}) \\
& h_0 && \geq A_0^+ + S_0^+. && (\chi_0)
\end{aligned} \tag{73}
$$

We ignore the constraint $I_t \geq 0$ in $\overline{\mathcal{W}}$, which is without loss because adding constraints will only lower attainable welfare—we are going to show that $\underline{\mathcal{W}} = \overline{\mathcal{W}}$, i.e., $\overline{\mathcal{W}}$ is in fact attainable. The solution to maximizing $\overline{\mathcal{W}}$ follows by defining the Hamiltonian

$$
\begin{aligned}
\mathcal{H}_t = {} & \delta e^{-\delta t}u(c_{t,0}) + \int_0^t \delta^2 e^{-\delta(t-s)}u(c_{t,s})\mathrm{d}s - h_t(\delta - \mu_{h,t}) + \mu_{S,t}S_t + \lambda_{A,t}\left(I_t - \phi A_t\right) \\
& + \lambda_{S,t}\left(\delta h_t + (\phi+\rho)A_t - I_t - \delta e^{-\delta t}c_{t,0} - \int_0^t \delta^2 e^{-\delta(t-s)}c_{t,s}\mathrm{d}s\right),
\end{aligned} \tag{74}
$$

where $\lambda_{A,t}$ and $\lambda_{S,t}$ are the co-states and $\mu_{h,t}$ and $\mu_{S,t}$ the Lagrange multipliers for the non-negativity constraints. Optimality conditions for h_0, A_0^+, and S_0^+ imply

$$
h_0: \quad 0 = -1 + \mu_{h,0} + \chi_0, \tag{75}
$$

$$
A_0^+: \quad 0 = -\chi_0 + \lambda_{A,0}, \tag{76}
$$

$$
S_0^+: \quad 0 = -\chi_0 + \lambda_{S,0}, \tag{77}
$$

where χ_0 is the Lagrange multiplier for the constraint $h_0 = A_0^+ + S_0^+$; the first-order conditions for the controls $I_t, h_t, (c_{t,s})_{s=0}^{s=t}$ imply

$$
I_t: \quad 0 = \lambda_{A,t} - \lambda_{S,t}, \tag{78}
$$

$$
h_t: \quad 0 = -\delta + \mu_{h,t} + \delta\lambda_{S,t}, \tag{79}
$$

$$
c_{t,s}: \quad 0 = u'(c_{t,s}) - \lambda_{S,t}; \tag{80}
$$

and the co-states develop according to

$$A_t: \quad 0 = -\rho\lambda_{A,t} - \phi\lambda_{A,t} + (\phi+\rho)\lambda_{S,t} + \dot{\lambda}_{A,t}, \tag{81}$$

$$S_t: \quad 0 = -\rho\lambda_{S,t} + \mu_{S,t} + \dot{\lambda}_{S,t}. \tag{82}$$

It follows from the condition for I_t that we have $\lambda_{A,t} = \lambda_{S,t}\ \forall t$. The law of motion for the costate $\lambda_{A,t}$ then implies $\dot{\lambda}_{A,t} = 0\ \forall t$ i.e., λ_A, and therefore also λ_S, are time-invariant. The condition for S_t then implies that μ_S is also time-invariant, with $\mu_S = \rho\lambda_S = \rho\lambda_A$. Because zero consumption is clearly sub-optimal, there must be t for which $h_t > 0$, which in turn implies $\lambda_S = 1$ from the condition for h_t. Accordingly, $\lambda_S = \lambda_A = 1$ and $\mu_S = \rho$; there is no storage and consumption by exiting investors is given by $c_{t,s} = c^\star$, as follows from the condition for $c_{t,s}$. We obtain

$$\overline{\mathcal{W}} = \overline{\mathcal{U}}_0 + \delta\int_0^\infty e^{-\rho s}\overline{\mathcal{U}}_s \mathrm{d}s, \quad \overline{\mathcal{U}}_s \equiv -h_s + \frac{\delta}{\rho+\delta}c^\star + \underline{\mathcal{U}}, \tag{83}$$

where $\overline{\mathcal{U}}_s$ is the utility attained by investors entering at time s. We find that

$$\overline{\mathcal{W}} - \underline{\mathcal{W}} = \frac{\delta c^\star}{\rho} - h_0 - \delta\int_0^\infty e^{-\rho s}h_s \mathrm{d}s, \tag{84}$$

where it remains to determine the value of $h_0 + \delta\int_0^\infty e^{-\rho s}h_s\mathrm{d}s$ in the solution for $\overline{\mathcal{W}}$. With $S_t = 0\ \forall t$, we can forward iterate on $\dot{A}_t = I_t - \phi A_t$ by using $I_t = \delta h_t + (\phi+\rho)A_t - \delta e^{-\delta t}c_{t,0} - \int_0^t \delta^2 e^{-\delta(t-s)}c_{t,s}\mathrm{d}s$ and $A_0^+ = h_0$ to find

$$\begin{aligned} h_0 + \delta\int_0^\infty e^{-\rho s}h_s\mathrm{d}s &= \delta\int_0^\infty e^{-(\rho+\delta)t}c_{t,0}\mathrm{d}t + \delta^2\int_0^\infty e^{-\rho s}\int_s^\infty e^{-(\rho+\delta)(t-s)}c_{t,s}\mathrm{d}t\mathrm{d}s \\ &\quad + \lim_{T\to\infty}\left[e^{-\rho T}A_T\right] \\ &= \frac{\delta c^\star}{\rho}, \end{aligned} \tag{85}$$

where we have used the fact that $\lim_{T\to\infty}\left[e^{-\rho T}A_T\right] = 0$ by the transversality condition. We thus obtain $\overline{\mathcal{W}} = \underline{\mathcal{W}}$, i.e., the social welfare in an Arrow-Debreu equilibrium must be $\overline{\mathcal{W}} = \underline{\mathcal{W}}$. Combined with the fact that $\mathcal{U}_s \geq \underline{\mathcal{U}}\ \forall s$, it follows that $\mathcal{U}_s = \underline{\mathcal{U}}\ \forall s$ in an Arrow-Debreu equilibrium. An immediate corollary is that the Arrow-Debreu allocation is Pareto efficient, as it otherwise cannot attain $\overline{\mathcal{W}}$.

Finally, because

$$c_{t,s} = c^\star \quad \forall t \geq 0,\ s \in [0,t]; \quad \text{and} \quad S_t = 0 \quad \forall t \tag{86}$$

are necessary to attain $\overline{\mathcal{W}}$, and because $\mathcal{U}_s = \underline{\mathcal{U}}\ \forall s$ implies $h_s = \frac{\delta}{\rho+\delta}c^\star\ \forall s$, it follows that A_t develops according to $A_0^+ = h_0$ and

$$\dot{A}_t = \rho A_t + \delta h_t - \left(\delta e^{-\delta t} + \delta^2 \int_0^t e^{-\delta(t-s)}\mathrm{d}s\right) c^\star \quad \Rightarrow \quad A_t = \frac{\delta}{\rho+\delta}c^\star \quad \forall t > 0, \tag{87}$$

which logically satisfies $\dot{A}_t > -\phi A_t$ so that $I_t > 0$. Thus, the Arrow-Debreu allocation is unique and Pareto efficient. □

A.2 Proof of Proposition 2

Suppose first that for some $t_S \geq 0$ we have $S_t = 0$ for all $t \leq t_S$ but $S_{t_S}^+ > 0$. We set out to show that this cannot be part of a decentralized equilibrium. The supposition $S_{t_S}^+ > 0$ implies $r_{v,t_S}^+ = 0$, so that Equation (18) implies $\dot{v}_{t_S} < 0$. This, in turn, implies $v_{t_S+\varepsilon} < 1$ for $\varepsilon > 0$ but sufficiently small since $v_{t_S} \leq 1$. We must therefore have $I_{t_S}^+ = 0$, as no trees are planted when the price of trees falls short of the cost of planting. This finding can then be generalized for all $t > t_S$ by considering matters at time $t_S + \varepsilon$.

The income for maturing trees at that time $t_S + \varepsilon$ is $\phi y A_{t_S+\varepsilon} = (\phi+\rho)A_{t_S+\varepsilon}$ and incumbent investors' aggregate wealth (storage plus the market value of trees) is given by $S_{t_S+\varepsilon} + v_{t_S+\varepsilon}A_{t_S+\varepsilon}$. Incumbent investors consume their wealth on exit by construction, so the aggregate flow consumption at $t_S + \varepsilon$ satisfies $c_{t_S+\varepsilon} = \delta\left(S_{t_S+\varepsilon} + v_{t+\varepsilon}A_{t+\varepsilon}\right)$. The aggregate resource constraint (6) therefore implies

$$\begin{aligned}
\dot{S}_{t_S+\varepsilon} &= \delta h_{t_S+\varepsilon} + (\phi+\rho)A_{t_S+\varepsilon} - c_{t_S+\varepsilon} - I_{t_S+\varepsilon} \\
&= \delta h_{t_S+\varepsilon} + (\phi+\rho)A_{t_S+\varepsilon} - \delta\left(S_{t_S+\varepsilon} + v_{t+\varepsilon}A_{t+\varepsilon}\right) - I_{t_S+\varepsilon} \\
&\geq \delta h_{t_S+\varepsilon} + (\phi+\rho-\delta)A_{t_S+\varepsilon} - \delta S_{t_S+\varepsilon} - I_{t_S+\varepsilon},
\end{aligned} \tag{88}$$

where we have used that $v_{t_S+\varepsilon} \leq 1$ to arrive at the third line and where it has to be noted that $h_{t_S+\varepsilon} > 0$. The latter property follows from the fact that zero labor supply is sub-optimal for the investor entering at $t_S + \varepsilon$ as u satisfies Inada conditions.

Taking the limit $\varepsilon \to 0$ in (88), we obtain

$$\begin{aligned} \dot{S}^+_{t_S} &\geq \delta h^+_{t_S} + (\phi + \rho - \delta)A^+_{t_S} - \delta S^+_{t_S} - I^+_{t_S} \\ &= \delta h^+_{t_S} + (\phi + \rho - \delta)A^+_{t_S} - \delta S^+_{t_S}, \end{aligned} \tag{89}$$

where we have used the fact that the supposition $S^+_{t_S} > 0$ implies $I^+_{t_S} = 0$, as we have demonstrated above, to arrive at the second line. From (89) it follows that $\lim_{S^+_{t_S} \to 0} \dot{S}^+_{t_S} > 0$ when $\phi > \delta$. We can thus forward iterate the argument above to conclude that the supposition $S^+_{t_S} > 0$ implies $I^+_{t_S+\varepsilon}, r^+_{v,t_S+\varepsilon} = 0\ \forall \varepsilon \geq 0$ and $S^+_{t_S+\varepsilon} > 0\ \forall \varepsilon \geq 0$.

The above, however, cannot be an equilibrium. Equation (18) implies that with $r^+_{v,t_S+\varepsilon} = 0\ \forall \varepsilon \geq 0$, we have $\dot{v}_{t_S+\varepsilon} = -(\rho + \phi) + \phi v_{t_S+\varepsilon}\ \forall \varepsilon \geq 0$. Since this differential equation is linear and $v_{t_S} \leq 1$, we will have $v_T = 0$ for some $T \geq t_S$, i.e., the price of trees hits zero within a finite amount of time. This contradicts the notion that trees must have a strictly positive fundamental value. I.e., the probability that the tree matures within an investor's investment horizon is strictly positive, and the investor can then store the resulting income until consumption on exit.

Suppose next that $S_t = 0\ \forall t$. We show that this must imply $I_t > 0\ \forall t$, i.e., the case we solve explicitly in Section 4. From the supposition and the resource constraint (4) for $t = 0$ we find $I_0 = h_0$, where $h_0 > 0$ is positive for the reasons explained earlier. Building on Equation (88), we have for $t > 0$:

$$\begin{aligned} 0 = \dot{S}_t &= \delta h_t + (\phi + \rho)A_t - \delta(S_t + v_t A_t) - I_t \\ &= \delta h_t + (\phi + \rho)A_t - \delta v_t A_t - I_t \\ &\geq \delta h_t + (\phi + \rho - \delta)A_t - I_t \\ &> -I_t, \end{aligned} \tag{90}$$

where we use $S_t = \dot{S}_t = 0$, $v_t \leq 1$, $h_t > 0$, and $\phi > \delta$. Concluding, $I_t > 0\ \forall t$ in a decentralized equilibrium, which yields the uniquely determined allocation derived in Section 4. We have argued in Section 4 that this allocation is Pareto inefficient. □

A.3 Proof of Proposition 3

We derive the full-fledged allocations in case of a run. It holds true that

$$x_t = \frac{\delta + \rho}{\delta} \frac{A_t}{D_t}. \tag{91}$$

We furthermore have

$$\alpha_t = \delta \frac{\phi + \rho}{\delta + \rho} \tag{92}$$

and

$$\dot{A}_t = -\phi A_t, \quad \dot{D}_t = -\delta \frac{\phi + \rho}{\delta + \rho} D_t \tag{93}$$

during a run. An exiting investor consumes

$$c_{t,0} = q_t d_t + s_t \tag{94}$$

during a run, where d_t and s_t denote individual stablecoin holdings and storage. We only need to account for the secondary market price q_t because the amount of stablecoins successfully redeemed by the investor is a flow during a run in continuous time. By construction, we have $d_t = D_t e^{\delta t}$ and $s_t = e^{\delta t} S_t$. Aggregate flow consumption during a run therefore reads as

$$c_t = \delta e^{-\delta t} c_{t,0} = \delta \left(q_t D_t + S_t \right). \tag{95}$$

No trees are planted during a run, and the aggregate flow income from gestating trees is $y\phi A_t = (\phi + \rho) A_t$. The change in aggregate storage is therefore

$$\begin{aligned} \dot{S}_t &= (\phi + \rho) A_t - \delta \left(q_t D_t + S_t \right) \\ &= (\phi + \rho) \frac{A_t}{D_t} D_t - \delta \left(q_t D_t + S_t \right) \\ &= \delta \frac{\phi + \rho}{\delta + \rho} x_t D_t - \delta \left(q_t D_t + S_t \right) \\ &= \delta \left(\frac{\phi + \rho}{\delta + \rho} x_t - q_t \right) D_t - \delta S_t, \end{aligned} \tag{96}$$

where we use (91) to arrive at the third line.

Note that because $s_t = S_t e^{\delta t}$ we have

$$\frac{\dot{s}_t}{s_t} = \frac{\dot{S}_t}{S_t} + \delta \quad \Rightarrow \quad \dot{s}_t = \left(\dot{S}_t + \delta S_t \right) \frac{s_t}{S_t}. \tag{97}$$

Combining this with (96) and using $s_t = S_t e^{\delta t}$, we therefore find for individual storage that

$$\begin{aligned}\dot{s}_t &= \delta\left(\frac{\phi+\rho}{\delta+\rho}x_t - q_t\right)D_t e^{\delta_t} \\ &= (\alpha_t x_t - \delta q_t)d_t,\end{aligned} \tag{98}$$

i.e., the change in individual storage equals the flow revenue from redeeming stablecoins minus the purchasing of stablecoins on the secondary market, where the latter equals the sale of stablecoins by the flow of investors exiting.

With $x_t > q_t$ during a run and $\phi > \delta$ by assumption, it also follows from (96) that $S_t^+ > 0$ during a run because $S_t = 0$ implies

$$\dot{S}_t = \delta\left(\frac{\phi+\rho}{\delta+\rho}x_t - q_t\right)D_t > 0. \tag{99}$$

We therefore know that for $t \geq t_R$, where t_R is the time at which the run starts, the discount develops according to

$$\frac{q_t}{x_t} = \theta_t = \min\left\{\theta_{t_R}e^{\phi(t-t_R)} + \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}\left(1-e^{\phi(t-t_R)}\right), 1\right\}, \tag{100}$$

with the run continuing forever when $\theta_{t_R} = \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho} = \underline{\theta}$, in which case $\theta_t = \theta_{t_R}\ \forall t \geq t_R$, and lasting for a finite amount of time when $\theta_{t_R} \in (\underline{\theta}, 1)$.

To characterize the development of storage during the run, we can use (96):

$$\begin{aligned}\dot{S}_t &= \delta\left(\frac{\phi+\rho}{\delta+\rho}x_t - q_t\right)D_t - \delta S_t \\ &= \delta\left(\frac{\phi+\rho}{\delta+\rho} - \theta_t\right)x_t D_t - \delta S_t \\ &= \delta\left(\frac{\phi+\rho}{\delta+\rho} - \theta_t\right)\frac{\delta+\rho}{\delta}A_t - \delta S_t \\ &= \left(\frac{\phi+\rho}{\delta+\rho} - \theta_{t_R}e^{\phi(t-t_R)} - \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}\left(1-e^{\phi(t-t_R)}\right)\right)(\delta+\rho)e^{-\phi(t-t_R)}A_{t_R} - \delta S_t \\ &= \left(\frac{\phi-\delta}{\delta}\frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho} - \left(\theta_{t_R} - \frac{\delta}{\phi}\frac{\phi+\rho}{\delta+\rho}\right)e^{\phi(t-t_R)}\right)(\delta+\rho)e^{-\phi(t-t_R)}A_{t_R} - \delta S_t \\ &= \left(\frac{\phi-\delta}{\delta}\underline{\theta} - (\theta_{t_R} - \underline{\theta})e^{\phi(t-t_R)}\right)(\delta+\rho)e^{-\phi(t-t_R)}A_{t_R} - \delta S_t.\end{aligned} \tag{101}$$

Equation (101) can be rearranged and multiplied by an integrating factor $\mathcal{I}_t$ (not to be confused with investment I_t) to find

$$\dot{S}_t\mathcal{I}_t + \delta S_t\mathcal{I}_t = \left(\frac{\phi-\delta}{\delta}\underline{\theta} - (\theta_{t_R} - \underline{\theta})e^{\phi(t-t_R)}\right)(\delta+\rho)e^{-\phi(t-t_R)}\mathcal{I}_t A_{t_R}. \tag{102}$$

Choosing $\mathcal{I}_t$ such that $\dot{\mathcal{I}}_t = \delta\mathcal{I}_t$, i.e., $\mathcal{I}_t = \mathcal{I}_{t_R}e^{\delta(t-t_R)}$ implies by the product rule that

$$\frac{\partial(S_t\mathcal{I}_t)}{\partial t} = \left(\frac{\phi-\delta}{\delta}\underline{\theta} - (\theta_{t_R} - \underline{\theta})e^{\phi(t-t_R)}\right)(\delta+\rho)e^{(\delta-\phi)(t-t_R)}\mathcal{I}_{t_R}A_{t_R}. \tag{103}$$

Therefore,

$$\begin{aligned}
S_t\mathcal{I}_t &= S_{t_R}\mathcal{I}_{t_R} + \int_{t_R}^{t}\left(\frac{\phi-\delta}{\delta}\underline{\theta} - (\theta_{t_R} - \underline{\theta})e^{\phi(\tau-t_R)}\right)(\delta+\rho)e^{(\delta-\phi)(\tau-t_R)}\mathcal{I}_{t_R}A_{t_R}\mathrm{d}\tau \\
&= S_{t_R}\mathcal{I}_{t_R} + (\delta+\rho)\mathcal{I}_{t_R}A_{t_R}\int_{t_R}^{t}\left(\frac{\phi-\delta}{\delta}\underline{\theta}e^{(\delta-\phi)(\tau-t_R)} - (\theta_{t_R} - \underline{\theta})e^{\delta(\tau-t_R)}\right)\mathrm{d}\tau \\
&= S_{t_R}\mathcal{I}_{t_R} - \frac{\delta+\rho}{\delta}\mathcal{I}_{t_R}A_{t_R}\int_{t_R}^{t}\left(\underline{\theta}\frac{\partial e^{(\delta-\phi)(\tau-t_R)}}{\partial\tau} + (\theta_{t_R} - \underline{\theta})\frac{\partial e^{\delta(\tau-t_R)}}{\partial\tau}\right)\mathrm{d}\tau \\
&= S_{t_R}\mathcal{I}_{t_R} - \frac{\delta+\rho}{\delta}\mathcal{I}_{t_R}A_{t_R}\left[\underline{\theta}e^{(\delta-\phi)(\tau-t_R)} + (\theta_{t_R} - \underline{\theta})e^{\delta(\tau-t_R)}\right]_{\tau=t_R}^{\tau=t} \\
&= S_{t_R}\mathcal{I}_{t_R} + \frac{\delta+\rho}{\delta}\mathcal{I}_{t_R}A_{t_R}\left(\underline{\theta}e^{\delta(t-t_R)}(1-e^{-\phi(t-t_R)}) - \theta_{t_R}(e^{\delta(t-t_R)}-1)\right).
\end{aligned} \tag{104}$$

With $S_{t_R} = 0$ and $\mathcal{I}_t = \mathcal{I}_{t_R}e^{\delta(t-t_R)}$, it follows that

$$S_t = \frac{\delta+\rho}{\delta}A_{t_R}\left(\underline{\theta}(1-e^{-\phi(t-t_R)}) - \theta_{t_R}(1-e^{-\delta(t-t_R)})\right). \tag{105}$$

When the run ends, it is clear that $\dot{S}_t = -\delta S_t$, i.e., investors exiting consume their storage. Having storage after the run has stopped is consistent with equilibrium because the return on the stablecoin is zero.

Individual consumption by investors exiting during a run can be characterized as follows:

$$
\begin{aligned}
c_{t,0} &= \left(q_t D_t + S_t\right) e^{\delta t} \\
&= \left(\theta_t x_t D_t + S_t\right) e^{\delta t} \\
&= \left(\theta_t \frac{\delta+\rho}{\delta} A_t + S_t\right) e^{\delta t} \\
&= \frac{\delta+\rho}{\delta} A_{t_R} \left(\theta_t e^{-\phi(t-t_R)} + \underline{\theta}(1-e^{-\phi(t-t_R)}) - \theta_{t_R}(1-e^{-\delta(t-t_R)})\right) e^{\delta t} \\
&= \frac{\delta+\rho}{\delta} A_{t_R} e^{\delta t_R} \theta_{t_R} \\
&= \theta_{t_R} c^{\star},
\end{aligned}
\tag{106}
$$

where the last line uses $A_{t_R} = D_{t_R} = e^{-\delta t_R} D_0^+$, $D_0^+ = h_0$, and Equation (26). When the run starts, individual consumption therefore drops permanently by exactly a fraction θ_{t_R}, and remains constant thereafter, even when the run has stopped. This proves that the run is Pareto inefficient. □

A.4 Proof of Lemma 1

We first show that $r_t^+ > \rho$ implies that $I_{t,s} = 0$, i.e., the issuer targets zero planting of trees at time t when $r_t^+ > \rho$. Consider a proof by contradiction: Suppose that $I_{t,s} > 0$. The mass of trees remaining from this investment at time $t+\varepsilon$ is $f_{t+\varepsilon,s} = I_{t,s} e^{-\phi\varepsilon}$ and the flow of fruit generated by it at time $\tau \in (t, t+\varepsilon]$ is $g_{\tau,s} = y\phi I_{t,s} e^{-\phi(\tau-t)} = (\phi+\rho) I_{t,s} e^{-\phi(\tau-t)}$. Suppose that instead we now plant $f_{t+\varepsilon,s}$ trees at time $t+\varepsilon$, financed by investing $f_{t+\varepsilon,s} e^{-\int_t^{t+\varepsilon} r_\tau \mathrm{d}\tau}$ in the alternative asset Z at time t, where we note that the alternative asset Z, i.e., storage and other stablecoins, is fully liquid. We can likewise replace the sequence of fruit $(g_{\tau,s})_{\tau=t}^{\tau=t+\varepsilon}$ by additionally investing $\int_t^{t+\varepsilon} g_{\tau,s} e^{-\int_t^\tau r_T \mathrm{d}T} \mathrm{d}\tau$ in the alternative asset at time t. The proposed alternative strategy thus leaves everything after time t unaffected, while leading to a net reduction in resources used for investment activity at time t given by

$$
\begin{aligned}
d_{t,s} &= I_{t,s} - f_{t+\varepsilon,s} e^{-\int_t^{t+\varepsilon} r_\tau \mathrm{d}\tau} - \int_t^{t+\varepsilon} g_{\tau,s} e^{-\int_t^\tau r_T \mathrm{d}T} \mathrm{d}\tau \\
&= I_{t,s} \left[1 - e^{-\int_t^{t+\varepsilon}(\phi+r_\tau)\mathrm{d}\tau} - (\phi+\rho) \int_t^{t+\varepsilon} e^{-\int_t^\tau (\phi+r_T)\mathrm{d}T} \mathrm{d}\tau\right].
\end{aligned}
\tag{107}
$$

Dividing $d_{t,s}$ by the length of time ε over which we deploy the alternative investment strategy and taking the limit $\varepsilon \to 0$, we obtain

$$\begin{aligned}\lim_{\varepsilon\to 0}\left[\frac{d_{t,s}}{\varepsilon}\right] &= I_{t,s}\lim_{\varepsilon\to 0}\left[\frac{1-e^{-\int_t^{t+\varepsilon}(\phi+r_\tau)\mathrm{d}\tau}-(\phi+\rho)\int_t^{t+\varepsilon}e^{-\int_t^{\tau}(\phi+r_T)\mathrm{d}T}\mathrm{d}\tau}{\varepsilon}\right]\\ &= I_{t,s}\lim_{\varepsilon\to 0}\left[(\phi+r_{t+\varepsilon})e^{-\int_t^{t+\varepsilon}(\phi+r_\tau)\mathrm{d}\tau}-(\phi+\rho)e^{-\int_t^{t+\varepsilon}(\phi+r_T)\mathrm{d}T}\right]\\ &= I_{t,s}\left[r_t^+-\rho\right]\\ &> 0,\end{aligned} \tag{108}$$

where we apply L'Hôpital's rule to arrive at the second line, the fact that $r_t^+ \equiv \lim_{\varepsilon\to 0} r_{t+\varepsilon}$ to arrive at the third line, and finally the supposition that $I_{t,s} > 0$ together with $r_t^+ > \rho$ to arrive at the fourth line. The alternative investment strategy thus frees up resources at time t when it is deployed over a sufficiently short time horizon ε, implying that targeting $I_{t,s} > 0$ cannot be optimal when $r_t^+ > \rho$.

The next step is to show that the issuer targets $Z_{t,s}^+ > 0$ when $r_t^+ > \rho$. We established in the first step that $r_t^+ > \rho \Rightarrow I_{t,s} = 0$, so that $\dot{A}_{t,s} = -\phi A_{t,s}$. Now suppose that $Z_{t,s}^+ = 0$ to establish a contradiction. The issuer's resource constraints at time t and $t+\varepsilon$ with $\varepsilon \to 0$ would then imply

$$c_{t,s} \geq \frac{(\rho+\phi)A_{t,s}e^{\delta(t-s)}}{\delta} \quad \text{and} \quad \lim_{\varepsilon\to 0} c_{t+\varepsilon,s} \leq \lim_{\varepsilon\to 0}\frac{(\rho+\phi)A_{t,s}e^{(\delta-\phi)\varepsilon+\delta(t-s)}}{\delta}. \tag{109}$$

It follows that

$$\begin{aligned}\frac{\dot{c}_{t,s}}{c_{t,s}} &\equiv \frac{1}{c_{t,s}}\lim_{\varepsilon\to 0}\left[\frac{c_{t+\varepsilon,s}-c_{t,s}}{\varepsilon}\right]\\ &= \lim_{\varepsilon\to 0}\left[\frac{c_{t+\varepsilon,s}/c_{t,s}-1}{\varepsilon}\right]\\ &\leq \lim_{\varepsilon\to 0}\frac{e^{(\delta-\phi)\varepsilon}-1}{\varepsilon}\\ &= \delta-\phi\\ &< 0,\end{aligned} \tag{110}$$

where we use (109) to arrive at the third line, L'Hôpital's rule to arrive at the fourth line, and the fact that $\delta < \phi$ to arrive at the fifth line. Thus, $Z_{t,s}^+ = 0 \wedge I_{t,s} = 0 \Rightarrow \dot{c}_{t,s}/c_{t,s} \leq 0$. This however implies that the constraint $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$ is violated because $r_t^+ > \rho$ by supposition. Concluding, $r_t^+ > \rho \Rightarrow Z_{t,s}^+ > 0$ or, equivalently, $Z_{t,s}^+ = 0 \Rightarrow r_t \in [0,\rho]$, where we know that $r_t \geq 0$ because storage is available as an alternative investment. $\square$

A.5 Proof of Lemma 2

We first define $a_{t,s} = A_{t,s}e^{\delta(t-s)}$ and $z_{t,s} = Z_{t,s}e^{\delta(t-s)}$, i.e., the amount of trees and the market value of other investments (storage and/or other stablecoins) owned by cohort s in per-investor terms. This allows us to redefine the optimization problem as

$$\begin{aligned} \max_{(c_{t,s},a_{t,s},z_{t,s})_{t=t'}^{\infty}} & \int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s})\mathrm{d}t \\ \text{s.t.} \quad & \dot{a}_{t,s} + \dot{z}_{t,s} = (\rho+\delta)a_{t,s} + (r_t+\delta)z_{t,s} - \delta c_{t,s}, \\ & \dot{a}_{t,s} \geq (\delta-\phi)a_{t,s}, \quad z_{t,s} \geq 0, \quad \text{and} \quad \dot{c}_{t,s}/c_{t,s} \geq r_t, \end{aligned} \tag{111}$$

and subject to the starting values $a_{t',s} > 0$ and $z_{t',s} = 0$.

We next verify that when the constraint $\dot{a}_{t,s} \geq (\delta-\phi)a_{t,s}$ is slack, this must imply $\dot{c}_{t,s}/c_{t,s} = r_t^+$ when $r_t \in [0,\rho]$ $\forall t > t'$. We thus consider

$$\begin{aligned} \max_{(c_{t,s},a_{t,s},z_{t,s})_{t=t'}^{\infty}} & \int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s})\mathrm{d}t \\ \text{s.t.} \quad & \dot{a}_{t,s} + \dot{z}_{t,s} = (\rho+\delta)a_{t,s} + (r_t+\delta)z_{t,s} - \delta c_{t,s}, \\ & z_t \geq 0, \quad \dot{c}_{t,s}/c_{t,s} \geq r_t^+, \quad \text{and} \quad \lim_{T\to\infty}\left[e^{-(\rho+\delta)(T-t')}a_{T,s}\right] \geq 0, \end{aligned} \tag{112}$$

where the last condition constitutes a no-ponzi-scheme condition for $a_{t,s}$, which we need to include here because $a_{t,s} < 0$, i.e., borrowing, is now feasible. We consider the same starting values as before. It is immediately clear that we can set $z_{t,s} = 0$ $\forall t \geq t'$ because we have $r_t \leq \rho$ $\forall t \geq t'$ by supposition, i.e., the return on trees weakly dominates the return on alternative investments. The problem therefore reduces to

$$\begin{aligned} \max_{(c_{t,s},a_{t,s})_{t=t'}^{\infty}} & \int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-s)} u(c_{t,s})\mathrm{d}t \\ \text{s.t.} \quad & \dot{a}_{t,s} = (\rho+\delta)a_{t,s} - \delta c_{t,s}, \quad c_{t,s}/c_{t,s} \geq r_t^+, \quad \lim_{T\to\infty}\left[e^{-(\rho+\delta)(T-t')}a_{T,s}\right] \geq 0. \end{aligned} \tag{113}$$

Suppose now that the solution $(c_{t,s}, a_{t,s})_{t=t'}^{\infty}$ to (113) implies the existence of a $T \geq t'$ for which $\dot{c}_{T,s}/c_{T,s} > r_T^+$. We show that this leads to a contradiction, so that we must in fact have $\dot{c}_{t,s}/c_{t,s} = r_t^+$ $\forall t \geq t'$. Assuming $\dot{c}_{T,s}/c_{T,s} > r_T^+$ for some $T \geq t'$ implies the existence of a $t_1 \geq t'$ and a $t_3 > t_1$ (we will use a point in time $t_2 \in (t_1, t_3)$ later on) such that

$$c_{t_3,s} > c_{t_1,s} e^{\int_{t_1}^{t_3} r_\tau \mathrm{d}_\tau}. \tag{114}$$

To see why, note that the opposite must imply

$$c_{T+\varepsilon,s} \leq c_{T,s} e^{\int_T^{T+\varepsilon} r_\tau \mathrm{d}_\tau} \ \forall T \geq t', \ \forall \varepsilon > 0, \tag{115}$$

which must in turn imply

$$\frac{1}{c_{T,s}} \frac{c_{T+\varepsilon,s} - c_{T,s}}{\varepsilon} \leq \frac{e^{\int_T^{T+\varepsilon} r_\tau \mathrm{d}_\tau} - 1}{\varepsilon} \ \forall T \geq t', \ \forall \varepsilon > 0. \tag{116}$$

One particular instance of (116) is the case in which $\varepsilon \to 0$, i.e., (116) must imply that $\forall T \geq t'$, we have

$$\begin{aligned} \frac{\dot{c}_{T,s}}{c_T} \equiv \frac{1}{c_{T,s}} \lim_{\varepsilon \to 0} \left[\frac{c_{T+\varepsilon,s} - c_{T,s}}{\varepsilon} \right] &\leq \lim_{\varepsilon \to 0} \left[\frac{e^{\int_T^{T+\varepsilon} r_\tau \mathrm{d}_\tau} - 1}{\varepsilon} \right] \\ &= \lim_{\varepsilon \to 0} \left[r_{T+\varepsilon} e^{\int_T^{T+\varepsilon} r_\tau \mathrm{d}_\tau} \right] \\ &= r_T^+, \end{aligned} \tag{117}$$

where we apply L'Hôpital's rule to arrive at the second line and use $r_T^+ \equiv \lim_{\varepsilon \to 0} r_{T+\varepsilon}$ to arrive at the third line. Equation (117) clearly contradicts the existence of a $T \geq t'$ for which $\dot{c}_{T,s}/c_{T,s} > r_T^+$.

The existence of a $t_1 \geq t'$ and $t_3 > t_1$ for which (114) holds allows us to construct an alternative, feasible solution $(c'_{t,s}, a'_{t,s})_{t=t'}^\infty$ to (113) that is strictly better than $(c_{t,s}, a_{t,s})_{t=t'}^\infty$. We illustrate this graphically by means of Figure 2 before delving into the details.

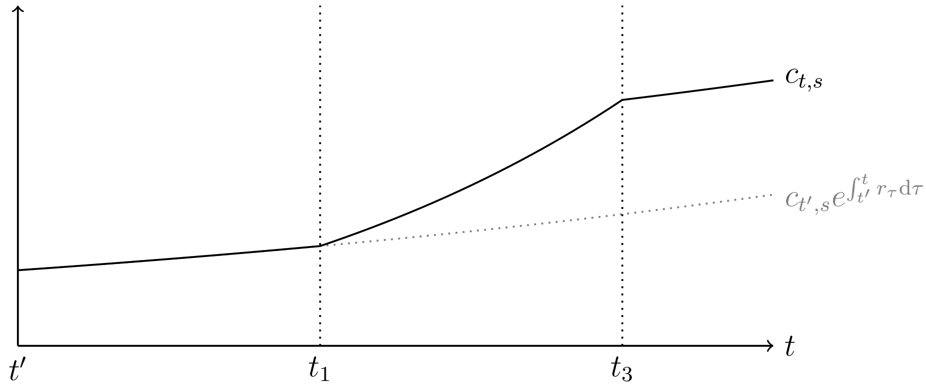
The existence of a $t_1 \geq t'$ and $t_3 > t_1$ for which (114) holds basically implies that the consumption process $(c_{t,s})_{t=t'}^\infty$ grows faster than r_t^+ over the time interval $[t_1, t_3]$, which we illustrate in Figure 2a. This consumption process never declines because $r_t^+ \geq 0$ due to the availability of storage and the fact that feasibility implies $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$. We focus on improving on this consumption process over the time interval $[t', t_3]$, i.e., we keep consumption after time t_3 fixed. The asset a that we use to finance the consumption processes earns the return $\rho + \delta$, so that an alternative consumption process $(c'_{t,s})_{t=t'}^{t=t_3}$ is feasible when it has the same present value as the process $(c_{t,s})_{t=t'}^{t=t_3}$, where the present value is determined by applying $\rho + \delta$ as the discount rate. We then observe that the discount rate applied to utility u in (113) is also $\rho+\delta$. The strict concavity of u therefore implies that an alternative consumption process $(c'_{t,s})_{t=t'}^{t=t_3}$ that has the same present value as $(c_{t,s})_{t=t'}^{t=t_3}$ but is flatter, i.e., that does not decline but that grows at a weakly lower rate on the interval $[t', t_3]$ and at a strictly lower rate on some sub-interval with positive mass, will be strictly better. The fact that $(c_{t,s})_{t=t'}^{t=t_3}$ grows faster than r_t^+ on the interval $[t_1, t_3]$ allows us to construct such an alternative process while respecting the constraint $\dot{c}'_{t,s}/c'_{t,s} \geq r_t^+$ everywhere.

We proceed to do this in the following way. First, we leave consumption on the interval $[t', t_1]$ unaffected and reduce the growth rate of consumption on the interval $[t_1, t_3]$ to exactly r_t^+. The adjusted consumption process $(\tilde{c}_{t,s})_{t=t'}^{t=t_3}$ is depicted in Figure 2b and it clearly has a lower present value than the original process $(c_{t,s})_{t=t'}^{t=t_3}$, i.e., resources have been freed up. We subsequently use these slack resources to increase consumption on the entire interval $[t', t_3]$ by a constant. This leads to the consumption process $(c'_{t,s})_{t=t'}^{t=t_3}$, depicted in Figure 2c, that: has exactly the same present value as $(c_{t,s})_{t=t'}^{t=t_3}$; and grows at rate r_t^+ over the interval $[t_1, t_3)$. It furthermore features $c'_{t_3,s} < c_{t_3,s}$, as otherwise $c'_{t,s} > c_{t,s}$ $\forall t \in [t', t_3)$, which would violate the notion that $(c'_{t,s})_{t=t'}^{t=t_3}$ and $(c_{t,s})_{t=t'}^{t=t_3}$ have the same present value. In turn, $c'_{t_3,s} < c_{t_3,s}$, implies that we also have $\dot{c}'_{t_3,s}/c'_{t_3,s} > \dot{c}_{t_3,s}/c_{t_3,s} \geq r_{t_3}^+$, as we left consumption after t_3 unaffected. Our alternative consumption process is therefore feasible because it has the same present value as the original and because it grows at at least the rate r_t^+. From Figure 2c we see that it is clearly flatter than the original process in the interval $[t', t_3]$—it intersects the original process from above at some unique point $t_2 \in (t_1, t_3)$ and it never declines. In other words, the alternative consumption process is not only feasible, but also strictly better than the original one.

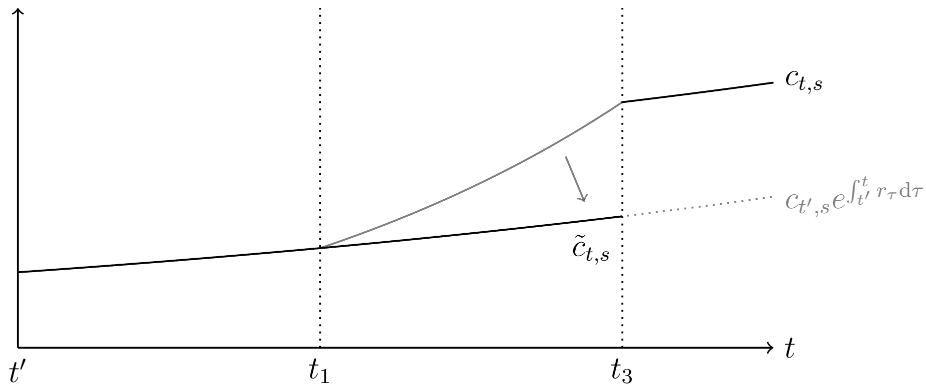
We now specify explicitly the alternative solution $(c'_{t,s}, a'_{t,s})_{t=t'}^{\infty}$ to formalize the verbal reasoning above:

$$c'_{t,s} = \begin{cases} c_{t,s} + f & \text{if } t' \leq t < t_1, \\ c_{t_1,s} e^{\int_{t_1}^{t} r_T \mathrm{d}T} + f & \text{if } t_1 \leq t \leq t_3, \\ c_{t,s} & \text{if } t > t_3; \end{cases}, \quad f = \frac{\int_{t_1}^{t_3} e^{-(\rho+\delta)(\tau - t')} \left[c_{\tau,s} - c_{t_1,s} e^{\int_{t_1}^{\tau} r_T \mathrm{d}T} \right] \mathrm{d}\tau}{\int_{t'}^{t_3} e^{-(\rho+\delta)(\tau-t')} \mathrm{d}\tau},$$

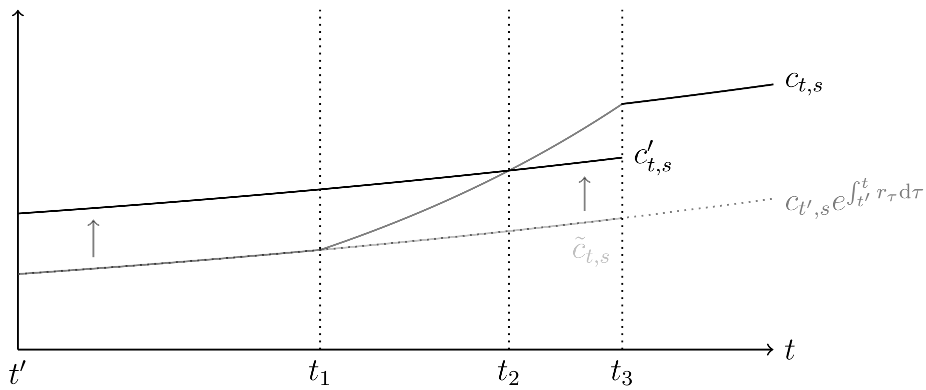
$$a'_{t,s} = a_{t',s} e^{(\rho+\delta)(t-t')} - \delta \int_{t'}^{t} c'_{\tau,s} e^{(\rho+\delta)(t-\tau)} \mathrm{d}\tau, \tag{118}$$



(a) The consumption process $(c_{t,s})_{t=t'}^{\infty}$, which grows at a rate greater that r_t on the interval $[t_1, t_2]$ and at a rate equal to r_t otherwise.



(b) Constructing $(\tilde{c}_{t,s})_{t=t'}^{t=t_3}$ from $(c_{t,s})_{t=t'}^{t=t_3}$ by setting $\tilde{c}_{t,s} = c_{t',s}e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau}$.



(c) Constructing $(c'_{t,s})_{t=t'}^{t=t_3}$ from $(\tilde{c}_{t,s})_{t=t'}^{t=t_3}$ by adding a constant f to $\tilde{c}_{t,s}$ $\forall t \in [t', t_3]$ so that the present value of $(c'_{t,s})_{t=t'}^{t=t_3}$ equals that of $(c'_{t,s})_{t=t'}^{t=t_3}$, i.e., $\int_{t'}^{t_3} e^{-(\rho+\delta)(t-t')}[c'_{t,s} - c_{t,s}]\mathrm{d}t = 0$.

Figure 2: Construction of the alternative consumption process $(c'_{t,s})_{t=t'}^{t=t_3}$ from $(c_{t,s})_{t=t'}^{t=t_3}$.

where we note that

$$\int_{t'}^{t_3} e^{-(\rho+\delta)(t-t')} \left(c'_{t,s} - c_{t,s}\right) \mathrm{d}t = 0 \tag{119}$$

by the construction of f. Note furthermore that a discontinuous process $(c_{t,s})_{t=t'}^{\infty}$ is trivially sub-optimal due to the strict concavity of u, i.e., there should be no jumps in the consumption process. This, in turn, implies that the existence of a $t_1 \geq t'$ and $t_3 > t_1$ for which (114) holds implies that we also have

$$c_{t,s} > c_{t_1,s} e^{\int_{t_1}^{t} r_\tau \mathrm{d}\tau} \ \forall t \in (t_3 - \mu, t_3], \quad \text{where} \quad \mu \in (0, t_3 - t_1). \tag{120}$$

By (120), we therefore have $f > 0$. We also have $c'_{t_3} < c_{t_3}$ because

$$\begin{aligned} f &= \frac{\int_{t_1}^{t_3} e^{-(\rho+\delta)(\tau-t')} \left[c_{\tau,s} - c_{t_1,s} e^{\int_{t_1}^{\tau} r_T \mathrm{d}T}\right] \mathrm{d}\tau}{\int_{t'}^{t_3} e^{-(\rho+\delta)(\tau-t')} \mathrm{d}\tau} \\ &< \frac{\int_{t_1}^{t_3} e^{-(\rho+\delta)(\tau-t')} \left[c_{t_3,s} - c_{t_1,s} e^{\int_{t_1}^{t_3} r_T \mathrm{d}T}\right] \mathrm{d}\tau}{\int_{t'}^{t_3} e^{-(\rho+\delta)(\tau-t')} \mathrm{d}\tau} \\ &\leq \frac{\int_{t_1}^{t_3} e^{-(\rho+\delta)(\tau-t')} \left[c_{t_3,s} - c_{t_1,s} e^{\int_{t_1}^{t_3} r_T \mathrm{d}T}\right] \mathrm{d}\tau}{\int_{t_1}^{t_3} e^{-(\rho+\delta)(\tau-t')} \mathrm{d}\tau} \\ &= c_{t_3,s} - c_{t_1,s} e^{\int_{t_1}^{t_3} r_T \mathrm{d}T}, \end{aligned} \tag{121}$$

where we use $\dot{c}_{t,s}/c_{t,s} \geq r_t^+ \ \forall t \geq t'$ and (120) to arrive at the second line. We furthermore have the existence of a $t_2 \in (t_1, t_3)$ for which

$$c'_{t,s} \begin{cases} > c_{t,s} & \text{if } t < t_2, \\ = c_{t,s} & \text{if } t = t_2, \\ < c_{t,s} & \text{if } t > t_2. \end{cases} \tag{122}$$

It follows from the fact that $\dot{c}_{t,s}/c_{t,s} \geq \dot{c}'_{t,s}/c'_{t,s} \geq r_t^+ \geq 0 \ \forall t \in [t', t_3)$ that $(c'_{t,s})_{t=t'}^{t=t_3}$ and $(c_{t,s})_{t=t'}^{t=t_3}$ are both increasing and intersect exactly once, i.e. at t_2, where $c_{t,s}$ intersects $c'_{t,s}$ from below.

We can now prove that $(c'_{t,s})_{t=t'}^{\infty}$ is strictly better than $(c_{t,s})_{t=t'}^{\infty}$. More particularly, we have

$$
\begin{aligned}
\int_{t'}^{\infty} \delta e^{-(\rho+\delta)(t-s)} \left[u(c'_{t,s}) - u(c_{t,s})\right] \mathrm{d}t &= \int_{t'}^{t_2} e^{-(\rho+\delta)(t-s)} \int_{c_{t,s}}^{c'_{t,s}} u'(c)\mathrm{d}c\mathrm{d}t \\
&\quad - \int_{t_2}^{t_3} e^{-(\rho+\delta)(t-s)} \int_{c'_{t,s}}^{c_{t,s}} u'(c)\mathrm{d}c\mathrm{d}t \\
&> \int_{t'}^{t_2} e^{-(\rho+\delta)(t-s)} \int_{c_{t,s}}^{c'_{t,s}} u'(c'_{t,s})\mathrm{d}c\mathrm{d}t \\
&\quad - \int_{t_2}^{t_3} e^{-(\rho+\delta)(t-s)} \int_{c'_{t,s}}^{c_{t,s}} u'(c'_{t,s})\mathrm{d}c\mathrm{d}t \\
&= \int_{t'}^{t_2} e^{-(\rho+\delta)(t-s)} u'(c'_{t,s})[c'_{t,s} - c_{t,s}]\mathrm{d}t \\
&\quad - \int_{t_2}^{t_3} e^{-(\rho+\delta)(t-s)} u'(c'_{t,s})[c_{t,s} - c'_{t,s}]\mathrm{d}t \\
&\geq u'(c'_{t_2,s}) \int_{t'}^{t_2} e^{-(\rho+\delta)(t-s)} \left[c'_{t,s} - c_{t,s}\right] \mathrm{d}t \\
&\quad - u'(c'_{t_2,s}) \int_{t_2}^{t_3} e^{-(\rho+\delta)(t-s)} \left[c_{t,s} - c'_{t,s}\right] \mathrm{d}t \\
&= 0,
\end{aligned}
\tag{123}
$$

where we use that $c'_{t,s} > c_{t,s}$ $\forall t \in [t', t_2)$ and $c'_{t,s} < c_{t,s}$ $\forall t \in (t_2, t_3]$ together with the fact that $u'' < 0$ to arrive at the "$>$", the fact that $c'_{t,s} \leq c'_{t_2,s} \forall t \in [t', t_2)$ and $c'_{t,s} \geq c'_{t_2,s} \forall t \in (t_2, t_3]$ together with the fact that $u'' < 0$ to arrive at the "$\geq$", and finally Equation (119) to arrive at the "$= 0$". We can conclude from (123) that the alternative consumption process $(c'_{t,s})_{t=t'}^{\infty}$ generates strictly higher utility than $(c_{t,s})_{t=t'}^{\infty}$.

We finally verify the feasibility of $(c'_{t,s}, a'_{t,s})_{t=t'}^{\infty}$ to arrive at the desired contradiction. From the specification of the consumption process, in particular that $c'_{t_3,s} < c_{t_3,s}$, it follows that $\dot{c}'_{t,s}/c'_{t,s} \geq r_t^+$ $\forall t \geq t'$, as otherwise $(c_{t,s})_{t=t'}^{\infty}$ would have violated $\dot{c}_{t,s}/c_{t,s} \geq r_t^+$ $\forall t \geq t'$ as well. The specification of $(a'_{t,s})_{t=t'}^{\infty}$ furthermore implies directly that

$$
\dot{a}'_{t,s} = (\rho + \delta)a'_{t,s} - \delta c'_{t,s} \tag{124}
$$

and the starting condition is satisfied because $a'_{t',s} = a_{t',s}$. Forward iterating on (124) yields

$$
\begin{aligned}
a_{t',s} = a'_{t',s} &= \delta \int_{t'}^{\infty} e^{-(\rho+\delta)(t-t')} c'_{t,s}\mathrm{d}t + \lim_{T\to\infty} \left[e^{-(\rho+\delta)(T-t')} a'_{T,s}\right] \\
&= \delta \int_{t'}^{\infty} e^{-(\rho+\delta)(t-t')} c_{t,s}\mathrm{d}t + \lim_{T\to\infty} \left[e^{-(\rho+\delta)(T-t')} a'_{T,s}\right],
\end{aligned}
\tag{125}
$$

where we use (119) to arrive at the second line. Because forward iterating on the law of motion in (113) likewise yields

$$a_{t',s} = \delta \int_{t'}^{\infty} e^{-(\rho+\delta)(t-t')} c_{t,s} \mathrm{d}t + \lim_{T\to\infty} \left[e^{-(\rho+\delta)(T-t')} a_{T,s} \right], \tag{126}$$

it follows from (125) and (126) that $\lim_{T\to\infty} \left[e^{-(\rho+\delta)(T-t')} a'_{T,s} \right] = \lim_{T\to\infty} \left[e^{-(\rho+\delta)(T-t')} a_{T,s} \right]$. In other words, $(a'_{t,s})_{t=t'}^{\infty}$ satisfies the no-ponzi-scheme condition since $(a_{t,s})_{t=t'}^{\infty}$ satisfies it as well. We have now arrived at a contradiction: $(c'_{t,s}, a'_{t,s})_{t=t'}^{\infty}$ is feasible *and* is strictly better than $(c_{t,s}, a_{t,s})_{t=t'}^{\infty}$. We have therefore proved that $\dot{c}_{t,s}/c_{t,s} = r_t^+$ must hold for all $t \geq t'$ when the constraint $\dot{a}_{t,s} \geq (\delta - \phi) a_{t,s}$ is slack and $r_t^+ \in [0, \rho]$ $\forall t \geq t'$.

The last step is to show that the constraint $\dot{a}_{t,s} \geq (\delta - \phi) a_{t,s}$ $\forall t' \geq t$ is indeed not binding when we have $\dot{c}_{t,s}/c_{t,s} = r_t^+ \in [0, \rho]$ $\forall t \geq t'$. The latter implies $c_{t,s} = c_{t',s} e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau}$, and the dynamics of $a_{t,s}$ according to the law of motion in (113) can then be written as

$$\dot{a}_{t,s} = (\rho + \delta) a_{t,s} - \delta c_{t',s} e^{\int_{t'}^{t} r_\tau d\tau}. \tag{127}$$

Forward iterating on Equation (127) and then imposing $\lim_{T\to 0} \left[e^{-(\rho+\delta)(T-t')} a_{T,s} \right] = 0$, i.e., the transversality condition, implies that

$$c_{t',s} = a_{t',s} \left[\delta \int_{t'}^{\infty} e^{\int_{t'}^{\tau} [r_T - \rho - \delta] \mathrm{d}T} \mathrm{d}\tau \right]^{-1}. \tag{128}$$

Again using the fact that $c_{t,s} = c_{t',s} e^{\int_{t'}^{t} r_\tau \mathrm{d}\tau}$ and that the law of motion $\dot{a}_{t,s} = (\rho + \delta) a_{t,s} - \delta c_{t,s}$ implies

$$a_{t,s} = a_{t',s} e^{(\rho+\delta)(t-t')} - \int_{t'}^{t} e^{(\rho+\delta)(t-\tau)} c_{\tau,s} \mathrm{d}\tau. \tag{129}$$

We therefore also find

$$
\begin{aligned}
\frac{a_{t,s}}{\delta c_{t,s}} &= e^{(\rho+\delta)(t-t')}\frac{a_{t',s}}{\delta c_{t',s}}\frac{c_{t',s}}{c_{t,s}} - \frac{c_{t',s}}{c_{t,s}}\int_{t'}^{t} e^{(\rho+\delta)(t-\tau)+\int_{t'}^{\tau} r_T \mathrm{d}T}\mathrm{d}\tau \\
&= e^{-\int_{t'}^{t} r_\tau \mathrm{d}\tau} e^{(\rho+\delta)(t-t')}\frac{a_{t',s}}{\delta c_{t',s}} - e^{-\int_{t'}^{t} r_\tau \mathrm{d}\tau}\int_{t'}^{t} e^{(\rho+\delta)(t-\tilde{\tau})+\int_{t'}^{\tau} r_T \mathrm{d}T}\mathrm{d}\tau \\
&= e^{-\int_{t'}^{t}[r_\tau-\rho-\delta]\mathrm{d}\tau}\frac{a_{t',s}}{\delta c_{t',s}} - \int_{t'}^{t} e^{-\int_{\tau}^{t}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau \\
&= e^{-\int_{t'}^{t}[r_\tau-\rho-\delta]\mathrm{d}\tau}\int_{t'}^{\infty} e^{\int_{t'}^{\tau}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau - \int_{t'}^{t} e^{-\int_{\tau}^{t}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau \\
&= \int_{t'}^{\infty} e^{\int_{t}^{\tau}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau - \int_{t'}^{t} e^{-\int_{\tau}^{t}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau \\
&= \int_{t}^{\infty} e^{\int_{t}^{\tau}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau,
\end{aligned}
\tag{130}
$$

which implies that (128) generalizes for all $t \geq t'$:

$$
c_{t,s} = a_{t,s}\left[\delta\int_{t}^{\infty} e^{\int_{t}^{\tau}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau\right]^{-1},
\tag{131}
$$

i.e., the solution is fully time-consistent.

Substituting (131) into the law of motion in (113) yields

$$
\begin{aligned}
\dot{a}_{t,s} &= \left[\rho+\delta-\left[\int_{t}^{\infty} e^{\int_{t}^{\tau}[r_T-\rho-\delta]\mathrm{d}T}\mathrm{d}\tau\right]^{-1}\right]a_{t,s} \\
&\geq \left[\rho+\delta-\left[\int_{t}^{\infty} e^{-(\rho+\delta)(\tau-t)}\mathrm{d}\tau\right]^{-1}\right]a_{t,s} \\
&= 0,
\end{aligned}
\tag{132}
$$

where we use $r_t \geq 0\ \forall t > t'$ to arrive at the second line. It follows that $\dot{a}_{t,s} \geq (\delta-\phi)a_{t,s}$ because we have $\delta < \phi$. This concludes the proof. □

References

Aghion, P., Bolton, P. and Dewatripont, M. (2000). Contagious bank failures in a free banking system. *European Economic Review*, **44** (4), 713–718.

Bains, P., Ismail, A., Melo, F. and Sugimoto, N. (2022). *Regulating the crypto ecosystem: The case of stablecoins and arrangements*. Fintech Note 2022/008, International Monetary Fund.

Berentsen, A. (2006). On the private provision of fiat currency. *European Economic Review*, **50** (7), 1683–1698.

Brunnermeier, M., James, H. and Landau, J.-P. (2021). *The digitalization of money.* BIS Working Papers 941, Bank for International Settlements.

Bryant, J. (1980). A model of reserves, bank runs, and deposit insurance. *Journal of Banking Finance*, **4** (4), 335–344.

Cao, Y., Dai, M., Kou, S., Li, L. and Yang, C. (2021). *Designing stable coins.* NUS RMI Working Paper Series 2021-05, National University of Singapore.

Catalini, C., de Gortari, A. and Shah, N. (2022). Some simple economics of stablecoins. *Annual Review of Financial Economics*, **14** (1), 117–135.

Cavalcanti, R., Erosa, A. and Temzelides, T. (1999). Private money and reserve management in a random-matching model. *Journal of Political Economy*, **107** (5), 929–945.

—, — and — (2005). Liquidity, money creation and destruction, and the returns to banking. *International Economic Review*, **46** (2), 675–706.

— and Wallace, N. (1999a). Inside and outside money as alternative media of exchange. *Journal of Money, Credit and Banking*, **31** (3), 443–457.

— and — (1999b). A model of private bank-note issue. *Review of Economic Dynamics*, **2** (1), 104–136.

Cong, L. W., Li, Y. and Wang, N. (2022). Token-based platform finance. *Journal of Financial Economics*, **144** (3), 972–991.

D'Avernas, A., Maurin, V. and Vandeweyer, Q. (2022). *Can Stablecoins be Stable?* Working Paper 2022-131, Becker Friedman Institute.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, **91** (3), 401–419.

Engineer, M. (1989). Bank runs and the suspension of deposit convertibility. *Journal of Monetary Economics*, **24** (3), 443–454.

Fernández-Villaverde, J. and Sanches, D. (2019). Can currency competition work? *Journal of Monetary Economics*, **106** (C), 1–15.

Feroli, M., Kashyap, A. K., Schoenholtz, K. L. and Shin, H. S. (2015). *Market tantrums and monetary policy.* Chicago Booth Resarch Paper 14-09, University of Chicago.

GERSBACH, H. (1998). Liquidity creation, efficiency, and free banking. *Journal of Financial Intermediation*, **7** (1), 91–118.

GOLDSTEIN, I., GUPTA, D. and SVERCHKOV, R. (2023). Utility tokens as a commitment to competition. Mimeo, Available at SSRN: https://ssrn.com/abstract=3484627.

GORTON, G. and ZHANG, J. (2021). *Taming wildcat stablecoins*. Law Review 90, University of Chicago.

GUENNEWIG, M. (2022). Currency competition with firms. Mimeo, Available at SSRN: https://ssrn.com/abstract=3940408.

HAUBRICH, J. G. and KING, R. G. (1990). Banking and insurance. *Journal of Monetary Economics*, **26** (3), 361–386.

JACKLIN, C. (1987). Demand deposits, trading restrictions, and risk sharing. In E. C. Prescott and N. Wallace (eds.), *Contractual arrangements for intertemporal trade*, *2*, Minneapolis, MN: University of Minnesota Press, pp. 26–47.

JACKLIN, C. J. (1993). Market rate versus fixed rate demand deposits. *Journal of Monetary Economics*, **32** (2), 237–258.

KISS, H. J., RODRIGUEZ-LARA, I. and ROSA-GARCIA, A. (2018). Panic bank runs. *Economics Letters*, **162**, 146–149.

KOSSE, A., GLOWKA, M., MATTEI, I. and RICE, T. (2023). *Will the real stablecoin please stand up?* BIS Working Papers 141, Bank for International Settlements.

LI, Y. and MAYER, S. (2021). *Money Creation in Decentralized Finance: A Dynamic Model of Stablecoin and Crypto Shadow Banking*. CESifo Working Paper Series 9260, CESifo.

LUCAS, R. E. (1978). Asset prices in an exchange economy. *Econometrica*, **46** (6), 1429–1445.

MARTIN, A. and SCHREFT, S. L. (2006). Currency competition: A partial vindication of Hayek. *Journal of Monetary Economics*, **53** (8), 2085–2111.

MAYER, S. (2022). Token-based platforms and speculators. Mimeo, Available at SSRN: https://ssrn.com/abstract=3471977.

READ, O. and DIEFENBACH, C. (2022). *The Path to the EU Regulation Markets in Crypto-assets (MiCA)*. wifin Working Paper Series 13/2022, RheinMain University of Applied Sciences, Wiesbaden Institute of Finance and Insurance (wifin).

ROGOFF, K. and YOU, Y. (2023). Redeemable platform currencies. *Review of Economic Studies*, **90** (2), 975–1008.

SHELL, K. (1971). Notes on the economics of infinity. *Journal of Political Economy*, **79** (5), 1002–1011.

VOELLMY, L. (2021). Preventing runs with fees and gates. *Journal of Banking & Finance*, **125**, 106065.

VON THADDEN, E.-L. (1997). The term-structure of investment and the banks' insurance function. *European Economic Review*, **41** (7), 1355–1374.

— (1998). Intermediated versus direct investment: Optimal liquidity provision and dynamic incentive compatibility. *Journal of Financial Intermediation*, **7** (2), 177–197.

— (2002). An incentive problem in the dynamic theory of banking. *Journal of Mathematical Economics*, **38** (1), 271–292.

WALLACE, N. (1988). Another attempt to explain an illiquid banking system: the Diamond and Dybvig model with sequential service taken seriously. *Quarterly Review*, **12** (Fall), 3–16.

— (1990). A banking model in which partial suspension is best. *Quarterly Review*, **14** (Fall), 11–23.

WANG, J. J. (2015). *Asset Managers and Financial Instability: Evidence of Run Behavior and Run Incentives in Corporate Bond Funds*. M-RCBG Associate Working Paper 46, Haravard Kennedy School.

WHITE, L. (2014). Free banking in history and theory. In H. W. White, V. Vanberg and E. A. Kohler (eds.), *Renewing the search for a monetary constitution*, *9*, Washington, DC: Cato Institute, pp. 187–211.

WILLIAMSON, S. D. (1999). Private money. *Journal of Money, Credit and Banking*, **31** (3), 469–491.

WOODFORD, M. (2001). Fiscal requirements for price stability. *Journal of Money, Credit and Banking*, **33** (3), 669–728.

ZHENG, Y. (2017). *A dynamic theory of mutual fund runs and liquidity management*. Working Paper Series 42, European Systemic Risk Board.

CFS Working Paper Series

Recent Issues

All CFS Working Papers are available at *www.ifk-cfs.de*.

No.	Authors	Title
716	Cathy Yi-Hsuan Chen, Roman Kräussl, and Patrick Verwijmeren	The green sin: how exchange rate volatility and financial openness affect green premia
715	Alessandro Moro, Andrea Zaghini	The green sin: how exchange rate volatility and financial openness affect green premia
714	Hans Gersbach, Hans H. Haller, And Sebastian Zelzner	Enough Liquidity with Enough Capital—and Vice Versa?
713	Volker Brühl	Generative Artificial Intelligence (GAI) – foundations, use cases and economic potential
712	Isabelle Cathérine Hinsche and Rainer Klump	Mirror, Mirror on the Wall, Who Is Transitioning Amongst Them All?
711	Volker Brühl	Big Tech, the platform economy and the European digital markets
710	Andrea Zaghini	Unconventional green
709	Gilbert Fridgen, Roman Kräussl, Orestis Papageorgiou, and Alessandro Tugnetti	The Fundamental Value of Art NFTs
708	Roman Kräussl, Tobi Oladiran, and Denitsa Stefanova	ESG as Protection Against Downside Risk
707	Roman Kräussl, Joshua Pollet, and Denitsa Stefanova	Closed-end Funds and Discount Control Mechanisms
706	Roman Kräussl, Zsofia Kräussl, Joshua Pollet, and Kalle Rinne	The Performance of Marketplace Lenders