

1 Perplexity about periodicity repeats perpetually: 2 A response to Brookshire

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10 **Abstract**

11 Brookshire (2022) claims that previous analyses of periodicity in detection performance after a reset
12 event suffer from extreme false-positive rates. Here we show that this conclusion is based on an
13 incorrect implementation of a null-hypothesis of aperiodicity, and that a correct implementation
14 confirms low false-positive rates. Furthermore, we clarify that the previously used method of
15 shuffling-in-time, and thereby shuffling-in-phase, cleanly implements the null hypothesis of no
16 temporal structure after the reset, and thereby of no phase locking to the reset. Moving from a
17 corresponding phase-locking spectrum to an inference on the periodicity of the underlying process
18 can be accomplished by parameterizing the spectrum. This can separate periodic from non-periodic
19 components, and quantify the strength of periodicity.

20 **Introduction**

21 Brookshire (2022) revisited reports of rhythmicity in detection performance (e.g., Landau and Fries,
22 2012; Fiebelkorn et al., 2013), and concluded that formerly employed methods lead to excessive
23 false-positive rates. Previous studies had presented, per trial, one reset event (a flash), followed by
24 one randomly timed probe, and had recorded the behavioral response (hit or miss). Across many
25 trials, the reset-aligned accuracy time course (ATC) was calculated. The ATC was then Fourier
26 transformed, and the resulting spectrum compared to spectra obtained after randomly pairing, across
27 trials, behavioral reports and probe time points, i.e., after “shuffling-in-time”. This procedure tests for
28 temporal structure. Brookshire makes the valuable point that rejecting the null hypothesis of no
29 temporal structure does not unequivocally demonstrate the presence of periodic structure, and
30 therefore argues that the null hypothesis should consist of a temporal structure that is aperiodic.

31 **The calculation of false positives - a single noisy time course is not noisy enough**

32 Brookshire’s implementation of the aperiodic null hypothesis is based on different types of noise
33 processes, primarily the first-order autoregressive (AR(1)) process and its special case, the random
34 walk. In an AR(1) process, the signal at time t is the sum of a specified fraction of the signal at time

35 t-1 plus a random step (Figure 1A). When many realizations of an AR(1) process are Fourier
36 transformed, their average spectrum decays monotonically with frequency according to $1/f^n$, without
37 peaks indicative of periodicity (Figure 1B, left). However, single realizations of an AR(1) process
38 often yield spectra that do not decline monotonically with frequency and thus have spectral peaks
39 (Figure 1B, right). Despite this fact, Brookshire simulates the ATC on the basis of a single AR(1)
40 realization; this realization is taken as a probability time course, and probabilistic draws from it
41 generate the hits and misses of all trials (and *all subjects*; Figure 1C, D); the resulting ATC is then
42 analyzed with the shuffling-in-time statistics, often yielding significant results for some frequency bins
43 (Figure 1E-G). Brookshire argues that these results should be considered false positives, because
44 the AR(1) process is aperiodic. However, as explained above, this does not hold for single AR(1)
45 realizations. When we use the code provided with Brookshire (2022) and modify it to implement
46 separate AR(1) realizations for each trial of each subject (Figure 2A), or even just for each subject
47 (Figure 2B), false positives are substantially diminished.

48 The use of a single time course to generate many simulated trials (and many subjects) trivially leads
49 to phase-locked modulation of simulated behavior. The hits and misses generated in single trials are
50 just (very) noisy replications of the single time course. If this time course is not entirely flat, then it
51 has some temporal structure, and the noisy replications of this temporal structure across trials are
52 equivalent to phase locking of the trials to the reset event. Thus, phase-locking metrics as used in
53 Landau and Fries (2012) should and do actually provide significant results in this case. The
54 significance increases when more trials are simulated (Figure 1G), demonstrating that many draws
55 of a single time course are not an implementation of “structured noise” as claimed by Brookshire.

56 Note that several previous studies modeled e.g. evidence accumulation as AR(1) process (drift
57 diffusion), but they consistently implemented separate AR(1) realizations per trial (Ratcliff and
58 McKoon, 2008; Shadlen and Kiani, 2013)). Other studies did use one function to model trends in
59 trial-averaged behavioral time courses, but they used deterministic processes, such as Gaussian or
60 exponential functions (Grabenhorst et al., 2019; Grabenhorst et al., 2021), and not stochastic ones,
61 such as AR(1) or random walk.

62 **From spectra to interpretation**

63 It is important to clarify what shuffling-in-time actually tests. Shuffling-in-time followed by Fourier
64 transformation is equivalent to shuffling-in-phase. If statistical tests based on shuffling-in-phase are
65 significant for a given frequency bin, this means that there is significant phase locking (to the reset
66 event) at that frequency bin. An isolated significant frequency bin in a phase-locking spectrum is
67 consistent with a periodicity in a frequency band including this frequency, i.e. with a spectrum
68 containing a distinct peak. Yet, it is also consistent with a different spectral pattern that is not
69 indicative of periodicity. To move from a phase-locking spectrum to the inference on a likely
70 underlying, periodic or non-periodic, process, one needs to consider the shape of the entire spectrum
71 or at least of a substantial part of the spectrum (Tosato et al., 2022). This interpretation of the
72 spectrum can be achieved by, e.g., parameterizing the spectrum (Donoghue et al., 2020). Such
73 parameterization can objectively separate periodic from non-periodic components, and quantify the
74 strength of the observed periodicity.

75 When periodicity has been established, the evidence can be further strengthened by replication, e.g.
76 across different conditions within one study (for a similar approach, see Vinck et al., 2022). Indeed,
77 several studies have found that different experimental conditions produce phase locking to the reset
78 event at very similar frequencies (Landau and Fries, 2012; Zhang et al., 2019).

79 **Methods proposed by Brookshire – and their problems**

80 Brookshire (2022) proposes two methods for analyzing behavioral time courses, namely “AR
81 surrogate” and “robust estimation”, which are presented as having better detection ratio (the ratio of
82 true positives to false positives). The AR surrogate method models the empirical ATC with an AR(1)
83 process, and then uses this AR(1) process to generate surrogate ATCs, which form the basis for
84 statistical testing. This method does generate multiple realizations of the AR(1) process. However,
85 the surrogate ATCs are scaled using the standard deviation of the noise, which unfortunately causes
86 their values to exceed the range of possible detection rates, i.e., 0 to 1. This leads to an inflation of
87 the power of the surrogate data compared to realistically simulated and empirical data. As a result,
88 Brookshire (2022) reports a very low false-positive rate with this method, which leads to falsely high
89 detection ratios. When the scaling is corrected, false-positive rate is higher (Figure 2, arrow; 0.08
90 instead of 0.03 in Brookshire (2022)), and detection ratio is slightly lower.

91 The second method proposed, the robust estimation method, is presented as having an acceptable
92 detection ratio. However, the true-positive rate of this methodology is unacceptably low (Figure 3; as
93 pointed out by several commentaries on this work, e.g., Fiebelkorn, 2022; Vinck et al., 2022). This
94 fact is masked in the detection ratios by false-positive rates approaching zero. Figure 3 shows the
95 false- and the true-positive rate for our method as well as the two methods proposed by Brookshire.
96 We simulated a periodic modulation with a frequency of 4 Hz and with modulation depths (defined
97 as in Brookshire (2022)) of 0.3, 0.2, and 0.1, similar to empirically observed modulation depths
98 (Busch et al., 2009; Landau et al., 2015; Benedetto and Morrone, 2017; Tomassini et al., 2017; Re
99 et al., 2019). On these simulated data, all methods were tested, and the methods proposed by
100 Brookshire (2022) suffer from very low true-positive rates (Figure 3).

101 **Conclusion**

102 Brookshire’s main claim of extreme false-positive rates in previous analyses is unfounded. Previous
103 analyses correctly tested for phase locking per frequency. Moving from a phase-locking spectrum to
104 an inference on (the periodicity of) the underlying process can proceed by parameterizing the phase-
105 locking spectrum - a fruitful endeavor for future work.

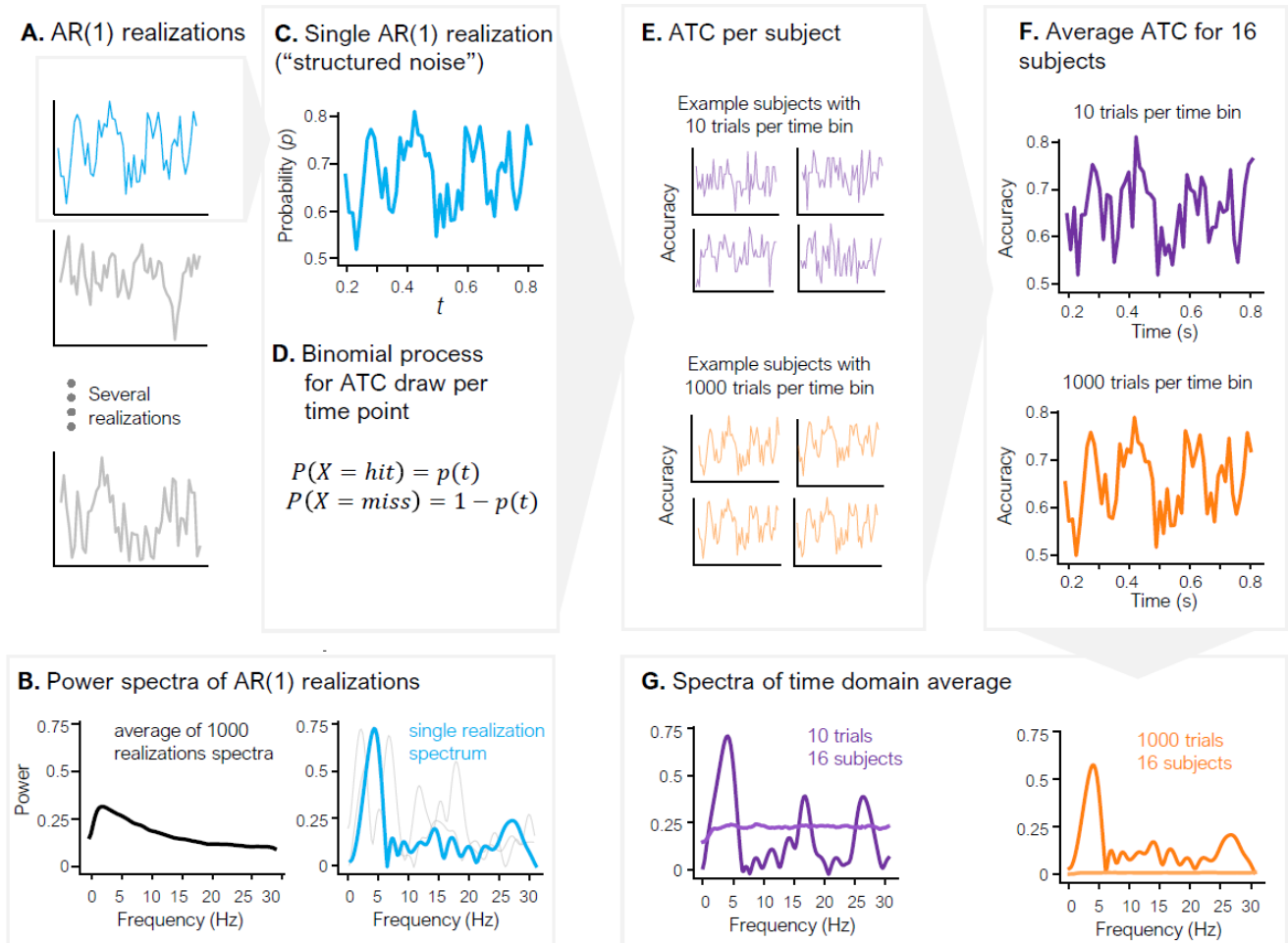
106 **Competing interests statement**

107 P.F. has a patent on thin-film electrodes and is beneficiary of a respective license contract with
108 Blackrock Microsystems LLC (Salt Lake City, UT, USA). P.F. is a member of the Advisory Board of
109 CorTec GmbH (Freiburg, Germany) and is managing director of Brain Science GmbH (Frankfurt am
110 Main, Germany).

111 **Author contributions statement**

112 **Daniele Re:** Conceptualization, Methodology, Software, Formal analysis, Visualization, Writing –
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114 Software, Formal analysis, Visualization, Writing – Original Draft, Writing – Review & Editing. **Ayelet**
115 **Landau:** Conceptualization, Methodology, Formal analysis, Visualization, Writing – Original Draft,
116 Writing – Review & Editing, Supervision, Project administration, Funding acquisition. **Pascal Fries:**
117 Conceptualization, Methodology, Formal analysis, Writing – Original Draft, Writing – Review &
118 Editing, Supervision, Project administration, Funding acquisition.

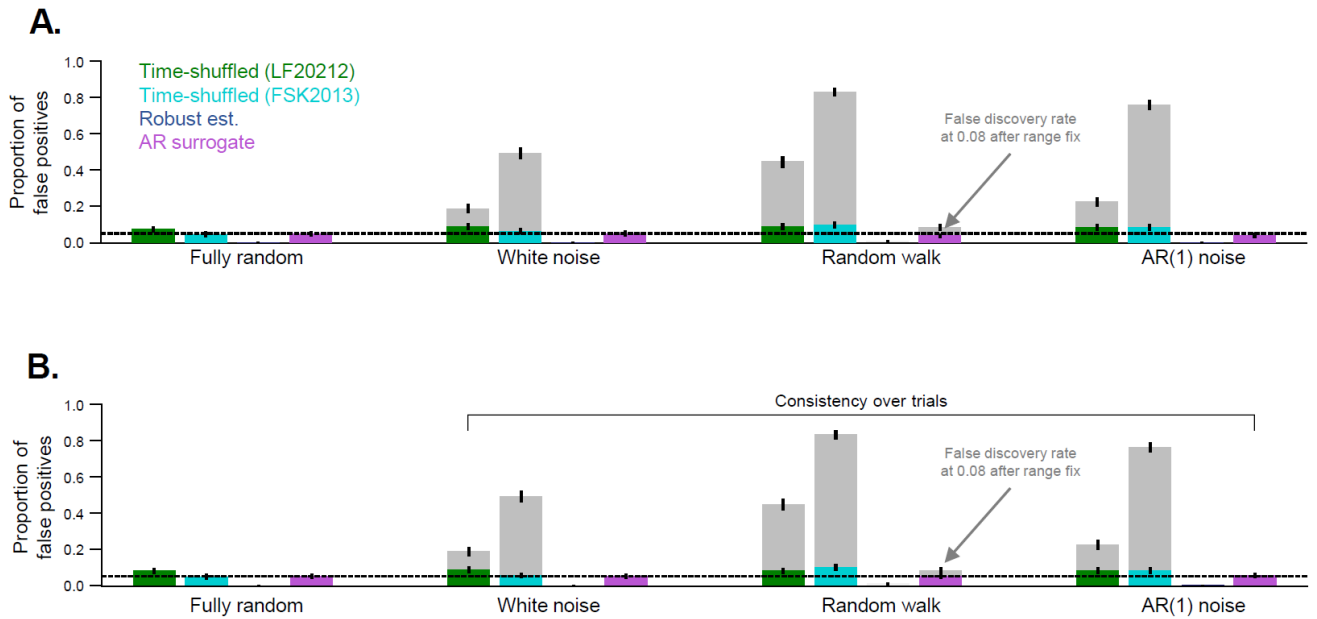
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121 **Figure 1:** Illustration of the null-hypothesis implementation proposed by Brookshire (2022).
 122 (A) Several realizations of an AR(1) process. (B) Left: Power spectrum averaged over 1000
 123 realizations. This average spectrum declines monotonically, except at its low-frequency end where
 124 it shows the effect of linear detrending. Right: Power spectra of single realizations, showing clear
 125 peaks. (C) The single realization used as a probability function for hit and misses underlying the
 126 ATC. Brookshire refers to this probability function as “structured noise”. (D) A binomial process is
 127 used to draw the single-trial outcome from the probability distribution in C for each time bin t . (E) The
 128 outcomes are averaged over trials to obtain the ATC for each subject. (F) The ATCs averaged over
 129 subjects are shown for different numbers of trials per time point (10 and 1000, respectively). Average
 130 ATCs are similar to the single AR(1) realization shown in C, more so, the more trials are included.
 131 (G) ATC power spectra, and corresponding significance thresholds (dashed), for 10 (purple) and
 132 1000 (orange) trials per time bin. Increasing trial numbers lead to increasing significance, contrary
 133 to what is expected from a noise process.

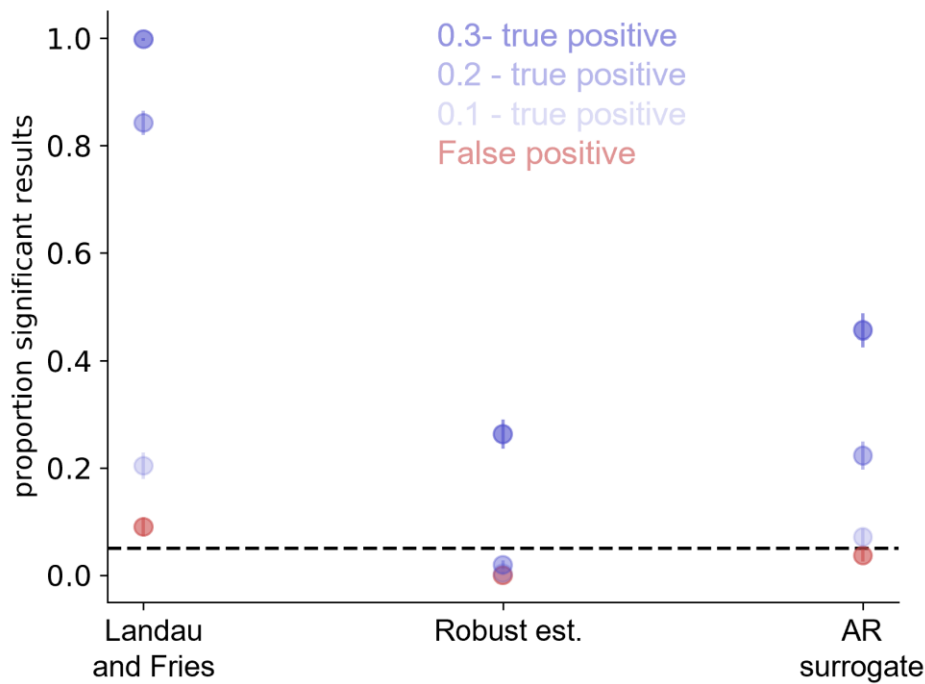
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136 **Figure 2:** Replotting Figure 3a from Brookshire (2022) using provided analysis code. The gray bars
137 show false-positive rates reported in Brookshire (2022), where trials and subjects were drawn from
138 a single realization of the chosen noise process (except for the Fully random condition). The colored
139 bars are based on the same analysis, only implementing separate realizations per trial (A) or per
140 subject (B), which resulted in false-positive rates close to 0.05, with negligible differences between
141 methods. Although separate realizations should be used per trial (panel A), even the use of separate
142 realizations merely at the level of each subject (panel B) is sufficient to have low false-positive rates
143 in all analysis methods. Note that the gray bars for the AR surrogate include a normalization step,
144 which leads to higher false-positive rates, and which was missing in Brookshire’s implementation
145 (see section on “Methods proposed by Brookshire – and their problems”).

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148 **Figure 3:** True-positive and false-positive rates for different analysis methods. We simulated a
149 probability time course characterized for all trials by a 4 Hz sinusoidal modulation, and additionally
150 added different random-walk noise, per trial. We simulated 3 conditions with sinusoidal modulation
151 depths (defined as in Brookshire (2022)) of 0.3, 0.2 or 0.1, and one condition without modulation.
152 For the conditions with modulation, the y-axis reflects the true-positive rate, and for the condition
153 without modulation the false-positive rate. As in figure 2, a previously used method (Landau and
154 Fries, 2012) results in low false-positive rates and reasonable true-positive rates. Robust estimation
155 and AR surrogate on the other hand show a true-positive rate below 0.5 for all conditions.

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