

Nucleus	N	State	Binding energy in MeV	
			experimental	calculated
Ca ³⁹	20-1	1 d 3/2	15.63	16.9
Ca ⁴¹	20+1	1 f 7/2	8.36	10.5
Ca ⁴⁷	28-1	1 f 7/2	10.07	11.1
Ca ⁴⁹	28+1	2 p 3/2	5.00	5.8
Ti ⁵¹			6.36	7.2
Cr ⁵³			7.94	8.6
Fe ⁵⁵			9.30	9.9
Kr ⁸⁵	50-1	1 g 9/2	9.76	11.4
Sr ⁸⁷			11.12	12.4
Mo ⁹¹			13.13	14.5
Sr ⁸⁹	50+1	2 d 5/2	6.77	6.1
Zr ⁹¹			7.21	6.9
Mo ⁹³			7.85	7.8
Ba ¹³⁹	82+1	2 f 7/2	4.72	4.3
Ce ¹⁴¹			5.53	4.8
Nd ¹⁴³			5.46	5.4
Sm ¹⁴⁵			6.82	6.0
Pb ²⁰⁷	126-1	3 p 1/2	7.38	8.0
Po ²⁰⁹			7.69	8.4
Rn ²¹¹			7.96	8.8
Pb ²⁰⁹	126+1	2 g 9/2	3.93	3.6
Po ²¹¹			4.55	4.0

Table 1. Ground states of some nuclei with a closed neutron shell plus or minus 1 neutron¹⁰.

well compatible with scattering data. Further details about this investigation will be published elsewhere.

⁷ U. AMALDI, G. CAMPOS VENUTI, G. CORTELESSA, C. FRONTEROTTA, E. REALE, P. SALVADORI, and P. HILLMAN, Phys. Rev. Letters **8**, 171 [1964], and private communication by Dr. G. CORTELESSA on experiments with Ca⁴⁰.

⁸ D. M. BRINK and N. SHERMAN, Phys. Rev. Letters **14**, 393 [1965].

On the Relation between Experimentally Observed Single Particle Energies and Levels of the Extreme Single Particle Model

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Comparison of experimental levels with the distances between eigenvalues in realistic single particle potentials shows the latter to be usually somewhat larger. Especially in nonlocal potentials¹ the spacings are wider by a factor of about 1.5. As a

Nucleus	Z	State	Binding energy in MeV	
			experimental	calculated
K ³⁹	20-1	1 d 3/2	8.34	9.9
Sc ⁴¹	20+1	1 f 7/2	1.08	3.5
Sc ⁴³			4.93	5.5
Sc ⁴⁵			6.89	7.3
Sc ⁴⁷			8.49	9.1
Sc ⁴⁹			9.47	10.8
Cu ⁵⁹	28+1	2 p 3/2	3.42	3.2
Cu ⁶¹			4.81	4.5
Cu ⁶³			6.13	5.7
Cu ⁶⁵			7.45	6.9
Cu ⁶⁷			8.52	8.1
Sb ¹²¹	50+1	2 d 5/2	5.78	4.5
Tl ²⁰³	82-1	3 s 1/2	6.60	8.3
Tl ²⁰⁵			7.29	8.8
Tl ²⁰⁷			8.04	9.2
Tl ²⁰⁹			8.33	9.7
Bi ²⁰⁷	82+1	1 h 9/2	3.55	2.8
Bi ²⁰⁹			3.77	3.3
Bi ²¹¹			4.38	3.9

Table 2. Ground states of some nuclei with a closed proton shell plus or minus 1 proton¹⁰.

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⁹ H. MELDNER and G. SÜSSMANN, Z. Naturforsch. **20 a**, 1219 [1965].

¹⁰ The experimental data on the binding energies are taken from L. A. KÖNIG, J. H. E. MATTAUCH, and A. H. WAPSTRA, Nucl. Phys. **31**, 18 [1962].

matter of fact, one was forced in the last few years by other experimental evidence^{2,3} to increase the potential depths while the radius constants were decreased from about 1.4 to 1.2 fm.

The purpose of this note is to sketch some possible reasons with which deviations of order 1 MeV

¹ H. MELDNER, G. SÜSSMANN, and W. ULRICI, Z. Naturforsch. **20 a**, 1217 [1965].

² F. PEREY and B. BUCK, Nucl. Phys. **32**, 353 [1962]. — U. AMALDI, G. CAMPOS VENUTI, G. CORTELESSA, C. FRONTEROTTA, A. REALE, P. SALVADORI, and P. HILLMAN, Phys. Rev. Letters **8**, 171 [1964]. — In addition see the results of various (p,2p) experiments.

³ For radius constants see the results of scattering experiments by HOFSTADTER et al.

and compressions of observed levels may be explained without using the assumption of an anomalous effective mass near the FERMI level, as was proposed in ⁴.

A well known compression of levels near the FERMI energy results, if one introduces attractive residual two-body interactions of short range with the pairing model. There the FERMI surface is smoothed by an amount Δ . The energies q_k of quasiparticle excitations in an odd system are approximately given by

$$|q_k| = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2} - \Delta$$

where λ is the chemical potential or effective FERMI energy (see e. g. ⁵, p. 106). This yields the following ratio of extreme single particle to observed spacings

$$\frac{\epsilon_k - \lambda}{|q_k|} = \left(1 + \frac{2\Delta}{|q_k|}\right)^{1/2} = \left(\frac{1+2U_k V_k}{1-2U_k V_k}\right)^{1/2}. \quad (1)$$

From (1) it can be seen that the increase of that ratio in the vicinity of the chemical potential (FERMI level) may be understood as a consequence of pairing interactions, as shown in Fig. 1.

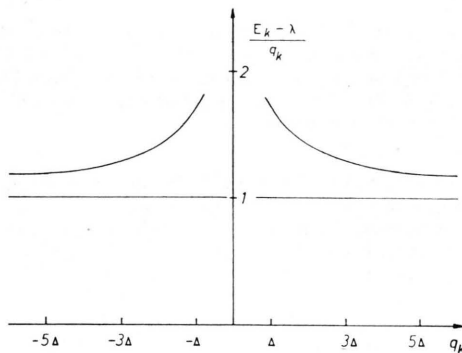


Fig. 1. Ratio of extreme single particle to observed spacings near the FERMI energy.

Fig. 1 in the present paper is related to Fig. 1 of ⁴ and a similar comparison with the observed levels as was carried out in ⁴ supports in the FERMI energy region also nonlocal potentials of a nonlocality (relative effective mass around 0.7) which is rather well established for $|\epsilon - \lambda| > \lambda$ (l. c. ²). Even at energies 5 Δ away from the FERMI surface

⁴ G. E. BROWN, J. H. GUNN, and P. GOULD, Nucl. Phys. **46**, 598 [1963].

⁵ G. E. BROWN, Unified Theory of Nuclear Models, North Holland Publishing Co., Amsterdam 1964.

⁶ S. G. NILSSON and O. PRIOR, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, 16 [1961].

the depression is still about 20 percent. A more quantitative example is shown in Fig. 2, where the lead 209 levels of a nonlocal calculation ¹ are approximately corrected for two-body residual interactions due to (1). Contrary to the ratio, the absolute shifts of the levels are not strongly dependent upon the choice of the chemical potential. Thus, for simplicity, it was taken to be equal to the energy of the $g\ 9/2$ level. A FERMI surface parameter Δ between 0.6 and 0.9 gives satisfactory agreement with experiments. Comparison with even-odd mass differences $P_n(N, Z)$ from which according to NILSSON and PRIOR ⁶ the best available information on the Δ parameters is expected, leads to values between 0.5 and 1.2 MeV in the Pb^{209} region. Theory also predicts non-zero matrix elements in the region of closed shells, since the FBCS method ⁷ gives even for a weak residual force strength an appreciable configurational admixture ⁸.

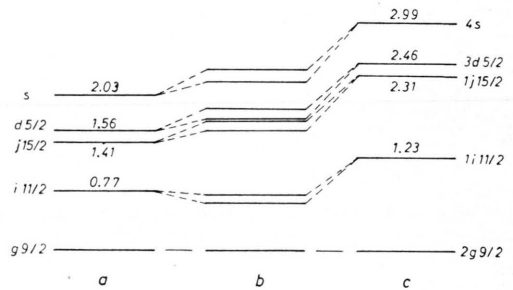


Fig. 2. a) Levels in Pb^{209} observed in $Pb^{208}(d,p)$ ⁹. The numbers are the corresponding energies in MeV. b) Single particle levels corrected with (1) and $\Delta=0.6$ MeV (upper), $\Delta=0.9$ MeV (lower levels). c) Extreme single particle levels ¹.

Another reason for deviations may be the influence of some rearrangement. Using the notation of BROWN ⁵ and abbreviating

$$T_i = \langle i|T|i\rangle, \quad V_{ji} = \langle j i|V|j i\rangle - \langle j i|V|i j\rangle,$$

one has for energy differences measured in stripping reactions:

$$\begin{aligned} E_{A'} - E_A &= \sum_{i=1}^{A-1} T_i' + \frac{1}{2} \sum_{j,i=1}^{A-1} V_{ji}' - \sum_{i=1}^{A-1} T_i - \frac{1}{2} \sum_{j,i=1}^{A-1} V_{ji} \\ &\quad + T_{A'} + \sum_{i=1}^{A-1} V_{iA'} - T_A - \sum_{i=1}^{A-1} V_{iA} \\ &\lesssim \epsilon_{A'} - \epsilon_A. \end{aligned}$$

⁷ H. J. MANG, J. K. POGGENBURG, and J. O. RASMUSSEN, Nucl. Phys. **64**, 353 [1965].

⁸ We thank Dr. K. DIETRICH for enlightening remarks on this point.

⁹ P. MUKKERJEE and B. L. COHEN, Phys. Rev. **127**, 1284 [1962].

The primes meaning that such matrix elements are calculated with single particle functions of the rearranged selfconsistent potential where one particle is in the state $A' \neq A$. This is expected to lower the quasiparticle energies compared to the eigenvalue differences for a constant potential. That correction of order A^{-1} will probably go in the same direction in each matrix element and therefore the summation may lead to an appreciable effect. Calculations of r^2 from $\sum |\psi|^2$ while putting the last particle in a level that was not occupied in the ground state, gave mean square radii which sometimes were increased by about one percent. Hence one should consider the influence of very slight fluctuations of the nuclear density, especially near closed shells. This may be justified to some extent also in analogy to the periodic system of elements where large fluctuations of the atomic radii occur at closed shells. In Fig. 3 some lead isotopes are considered for a quantitative example. Since the eigenvalues are very sensitive to variations of r_0 while the other parameters are kept fixed, a one half percent deviation of the selfconsistent radius from the $R = r_0 A^{1/3}$ line gives a remarkable (1 MeV) effect. An assumed density dependence near the neutron number 126 as given in the upper curve in Fig. 3 leads to a smoothed dependence of the last filled level on the particle numbers as is shown in the lower part of that figure. Here a constant potential depth U was assumed, since a reasonable δU for a simultaneous r_0 variation gives only a smaller correction. After all, the rearrangement is expected to be of order 1 MeV. Hence it would be question-

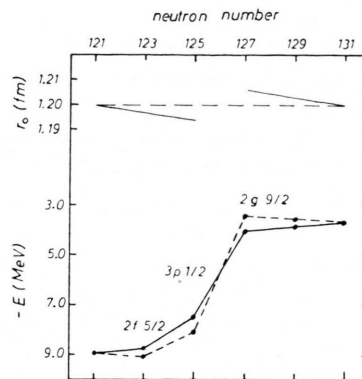


Fig. 3. Last filled levels in Pb isotopes.

able to draw quantitative conclusions on potential depths or nonlocalities from a comparison with single particle excitation energies which are about 1 to 3 MeV.

In conclusion, one should keep in mind that only rather sophisticated investigations of residual effects can enable a satisfactory quantitative comparison of the experimental levels near the FERMI surface with single particle energy differences. Such investigations, however, are reasonably done with eigenfunctions of nonlocal single particle potentials that give account of the data also at energies further removed from the FERMI level.

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Beugung langsamer Elektronen an kugelförmigen Kupfer-Kristallen mit unberührter Oberfläche

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Die Beugung langsamer Elektronen wurde durch die schon von EHRENBERG¹ benutzte und von GERMER und Mitarbeitern² vervollkommnete Leuchtschirm-

Beobachtung zu einer vielfach verwendeten Methode bei der Untersuchung von Oberflächen. Silicium und Germanium haben dabei Oberflächenstrukturen gezeigt, die aus ihrem Raumgitter nicht unmittelbar zu verstehen sind^{3, 4}. Metalle dagegen liefern Beugungsbilder, die ihren Raumgittern entsprechen⁵. Adsorption von Gasen auf reinen Oberflächen führt zu Beu-

¹ W. EHRENBERG, Phil. Mag. 18, 878 [1934].

² L. H. GERMER u. C. D. HARTMAN, Rev. Sci. Instr. 31, 784 [1960].

³ R. E. SCHLIER u. H. E. FARNSWORTH, J. Chem. Phys. 30, 917 [1959].

⁴ J. J. LANDER u. L. MORRISON, J. Chem. Phys. 33, 729 [1962].

⁵ siehe z. B. L. H. GERMER, A. U. MACRAE u. C. D. HARTMAN, J. Appl. Phys. 32, 2432 [1961].