NOTIZEN

The Influence of the f_0 Meson in the Two-Photon Exchange on the Relative Ratio of $\sigma_{e^- p}$ to $\sigma_{e^+ p}$ and the Polarization of the Recoil Protons in Elastic Electron-Proton Scattering

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In a former paper [1] we have calculated the two-photon exchange contributions in elastic (e,p)-scattering processes. With this result we have calculated the correction term in α^3 to the ROSENBLUTH formula (interference term of the one- and two-photon exchange amplitude) and that to the proton-antiproton annihilation in an electron-positron pair. Using the procedure given by HEARN and LEADER [2], an expansion of the photon-nucleon vertex was given in terms of the COMPTON scattering amplitude for the proton. Because one can neglect the cut in the s-plane [3] we have considered dispersion relations in the t-channel only. The corrections to the ROSENBLUTH formula and those to the $p\bar{p}$ annihilation into an $e^+ e^-$-pair are smaller by a factor α than the contributions of the one-photon exchange diagram; so we got a deviation of $\sim 1\%$ [4, 5] for a point nucleon. One can have only a possible influence in the resonances of the COMPTON scattering amplitude. Let A_B be the contribution of the one-photon exchange amplitude and A_C that for the two-photon case. Then near the resonance energy $\omega_R \approx 300$ MeV for COMPTON scattering A_C is predominantly imaginary, being just the shadow of the channel for the photo meson production in the resonant 3-3 channel, as DRELL, RUDERMANN [6] and FUBINI [3] have shown. Therefore A_C is approximately $\pi/2$ out of phase with A_B, a real potential scattering term in BORN approximation, and the interference term in

$$\frac{d\sigma_{e^- p}}{d\Omega} \sim |\alpha A_B + \alpha^2 A_C|^2 \approx \alpha^2 |A_B|^2 + 2\alpha^3 \operatorname{Re} A_B^* A_C + \dots \quad (1)$$

is actually very small [7, 8].

FLAMM and KUMMER [9] have calculated the influence of A_C by introducing a tensor resonance model for COMPTON scattering. The result is, that the maximal deviation from the straight line behaviour as a function of $\operatorname{tg}^2(\vartheta/2)$ [10, 11]

$$[A(t) + B(t) \operatorname{tg}^2(\vartheta/2)] \quad (2)$$

appears only at very small scattering angles ϑ.

The deviation from the ROSENBLUTH formula comes out to be $\sim 10\%$ for an impulse transfer of $t = -30\, f^{-2}$ and small ϑ and the assumption, that the coupling constants used in this model are of the order of 1. We find [8], that the deviations from (2) give a correction of 7% for $t = -30\, f^{-2}$ and $\vartheta = 5^\circ$ and the f_0 meson in the intermediate state, using dispersion relation methods.

But if we introduce the POMMERANCHUCK trajectory [12] for the f_0 meson, the correction amounts to be 12%. For the coupling constants we have assumed $\mathfrak{f}_{\gamma f_0} = \mathfrak{f}_{\gamma\pi}$ and $\mathfrak{f}_{f_0 \mathcal{N}} = \mathfrak{f}_{\pi \mathcal{N}}$ in both cases.

In order to get quantities for a better discussion in experiments and to have a direct measure for the contribution of A_C, we have calculated from (1) the relative ratio

$$\Delta = \frac{\sigma_{e^- p} - \sigma_{e^+ p}}{\sigma_{e^- p} + \sigma_{e^+ p}} = 2\alpha \frac{\operatorname{Re} A_B^* A_C}{|A_B|^2} \quad (3)$$

and the polarization P of the recoil protons in the elastic (e,p)-scattering processes, using unpolarized electrons [8, 13]

$$P = \frac{\operatorname{Tr.}\{M^+ \boldsymbol{\sigma}\, \boldsymbol{s}\, M\}}{\operatorname{Tr.}\{M^+ M\}}. \quad (4)$$

M denotes the transition matrix, $\boldsymbol{\sigma}$ the PAULI matrices and $\boldsymbol{s}$ the spin direction of the recoil proton ($\boldsymbol{s}^2 = 1$).

Expanding the S-matrix in powers of α, we get

$$S = 1 + i M_2 + i M_4 + \dots, \quad (5)$$

where M_2 corresponds to the one-photon exchange term A_B (hermitian!) and M_4 to the A_C. To lowest order in α, the polarization is then given by

$$P = 2i \frac{\operatorname{Tr.}\{M_2 (\boldsymbol{\sigma s}) \operatorname{Im} M_4\}}{\operatorname{Tr.}\{M_2^+ M_2\}}. \quad (6)$$

In order to calculate the M_4 resp. A_C, the unitarity condition, calculated for the channels given in Fig. 1, was used.

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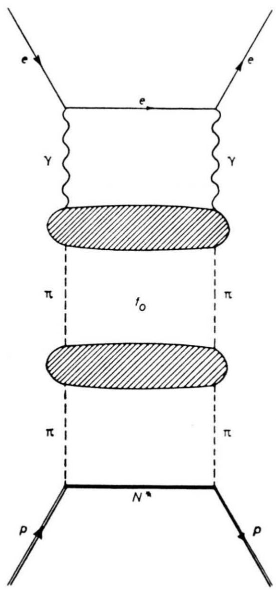

Fig. 1.

To calculate the contribution of this diagram, we have used for the $(\pi \mathfrak{N} \mathfrak{N}^*)$-vertex the isobaric model of GOURDIN and SALIN[14, 15], who calculated the photo production processes.

Further, to calculate $\operatorname{Im} M_4$ for the channels given in Fig. 1, a technique similar to that given in [13] was used. To obtain the form factor in the $(\gamma\,\pi)$-vertex, the f_0 resonance was introduced and for the pion form factor a subtracted dispersion relation[7] was used, viz.:

$$F_\pi(q'^2) = \exp - \left[\frac{q'^2}{\pi} \int_{4\mu^2}^{\infty} \frac{\delta_1(\sigma^2)\,d\sigma^2}{\sigma^2(\sigma^2 - q'^2 - i\,\varepsilon)} \right], \tag{7}$$

where $\delta_1(\sigma^2)$ is the d-wave $\pi\,\pi$-scattering phase shift. At the resonance there holds

$$\delta_1(q_r^2) = \pi/2 \quad (q_r^2 = m f_0^2).$$

With the assumptions used in [13–15] we find that

$$\Delta_{\text{theoret.}} \sim -0{,}079 \quad \text{for} \quad t = q^2 = -19{,}5\,f^{-2}.$$

The experiments[16, 17] give

$$\Delta_{\text{exp.}} = -0{,}094 \pm 0{,}046 \quad \text{for} \quad q^2 = -19{,}5\,f^{-2}.$$

For the polarization (projection on a plane perpendicular to the coplanar reaction plane) we find a maximum value of $-1{,}2\%$ for electron energies $E_1 = 1{,}2$ GeV and $\cos \vartheta_{\text{CMS}} = \pi/2$. The experimental results[18] are of the same order.

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Electron Paramagnetic Resonance of Eu^{2+} in CdF_2

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The paramagnetic resonance spectrum of Eu^{2+} in CdF_2 (about 0,1 mole % Eu) shows the same characteristics as in other alkaline earth fluorides with a CaF_2-type lattice[1–4].

The angular variation shows that the Eu^{2+}-ions in CdF_2 are in an electric field of cubic symmetry occupying Cd sites. The ground state of the Eu^{2+}-ions is $^8S_{7/2}$. For this case BAKER, BLEANEY and HAYES[2] give a spin HAMILTONIAN of the following form:

$$H = g\,\beta\,\boldsymbol{H}\cdot\boldsymbol{S} + A\,\boldsymbol{I}\cdot\boldsymbol{S} + B_4(O_4^0 + 5\,O_4^4) + B_6(O_6^0 - 21\,O_6^4).$$

Taking this spin HAMILTONIAN, the splitting of the $^8S_{7/2}$ ground state in a cubic crystal field (8-fold coordination) and with hyperfine interaction in the magnetic field H was calculated[2, 4, 5]. The magnetic dipole transitions $\Delta M = \pm 1$ (without hyperfine interaction) are given by

$$\begin{aligned}
&M = \pm 7/2 \to \pm 5/2 \quad [\text{abbreviated: } \pm 7/2]: \quad h\,\nu = \Delta E_{\pm 7/2}(H) = g\,\beta\,H_{\pm 7/2} + Q_7\,,\\
&\qquad Q_7 = \pm 20\,p\,b_4 \pm 6\,q\,b_6 + (-1 + 114\,\varphi - 345\,\varphi^2 + 84\,\psi)\;10\,b_4^2/(g\,\beta\,H_{\pm 7/2})\,;\\
&M = \pm 5/2 \to \pm 3/2 \quad [\pm 5/2]: \quad h\,\nu = \Delta E_{\pm 5/2}(H) = g\,\beta\,H_{\pm 5/2} + Q_5\,,\\
&\qquad Q_5 = \mp 10\,p\,b_4 \mp 14\,q\,b_6 + (-92\,\varphi + 455\,\varphi^2 - 441\,\psi)\,20\,b_4^2/(3\,g\,\beta\,H_{\pm 5/2})\,;
\end{aligned} \tag{1}$$

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