Interactional niche in the development of geometrical and spatial thinking in the familial context

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KURZFASSUNG DER ARBEIT

Interaktionale Nische der Entwicklung mathematischer Raumvorstellung bei Vorschulkindern im familialen Kontext

Während die Auswirkung der Familie auf die Lernprozesse im Mathematikunterricht in der mathematikdidaktischen Diskussion relativ breit diskutiert wird, kommen die Forschungen über die Auswirkung der Familie auf die frühkindlichen Lernprozesse in der Mathematik erst in Gang. Internationale Forschungsprojekte zeigen auf, dass familiale Aktivitäten wichtige Bestandteile für die mathematische Denkentwicklung eines Kindes sind. Die familialen Situationen werden dabei nicht nur im Sinne einer spezifischen, kulturell geprägten Mathematik erlebt, sondern auch als eine kulturell geprägte „Praxis des Mathematiklernens“ gesehen. Insbesondere weisen sie auch auf die herausragende Bedeutung ethnischer und kultureller Hintergründe der Familien für mathematische Lernprozesse hin. „Early Steps in Mathematics Learning - Family Study“ (erStMaL-FaSt) ist eine der ersten Studien in Deutschland, in der Kinder im Kindergartenalter mit ihrer Familie in mathematischen und raumgeometrischen Situationen beobachtet werden und bei der zudem der ethnische familiale Hintergrund mitbedacht wird (vgl. Kap. 1). In Deutschland lebt eine große multi-ethnische Anzahl von Bürgern (ca. 20% der Bevölkerung), bei der Kinder aus einem multilingualen Umfeld, abhängig von den kulturellen Wurzeln, im deutschen Bildungssystem häufig als Risikokinder eingestuft werden. Die Studie ist ein Teil des Projekts „early Steps in Mathematics Learning“ (erStMaL) und longitudinal angelegt, deren Gegenstand frühe mathematische Lernprozesse in den Inhaltsbereichen Geometrie und Messen sind (Acar Bayraktar & Krummheuer, 2011) (vgl. Kap.1.4).

Das Analyseinteresse in erStMaL-FaSt richtet sich auf den Einfluss familialer Interaktionen für die Entwicklung der mathematischen Raumvorstellung bei Kindern türkischer Einwanderfamilien im Vorschulalter. Die Arbeit ist ebenfalls longitudinal angelegt und es wird eine interaktionistische, sozial-konstruktivistische Perspektive eingenommen. Es interessiert, wie in

diesem familialen Interaktionssystem ein „Mathematics Learning Support System“ (MLSS) für die Entwicklung der Raumvorstellung beim Kind emergiert. Es ist weitgehend ungeklärt, wie solche MLSS im Einzelnen funktionieren und sie sich im Zuge der kindlichen Entwicklung verändern. Für die Analyse wird theoretisch auf das Konzept der „interaktionalen Nische mathematischer Denkentwicklung“ (NMD) zurückgegriffen (vgl. Kap.2.2). Eine NMD besteht aus den kulturspezifischen, von einer Gruppe oder Gesellschaft bereitgestellten Lernangeboten (Allokationsaspekt), realen Interaktionsituationen (Situationsaspekt) und dem individuellen Beitrag des interessierenden Kindes (Aktionsaspekt) (Krummheuer & Schütte, 2014; Krummheuer, 2011). Mit Hilfe dieses theoretischen Konstrukts werden raumgeometrisch fundierte Spielsituationen zwischen Eltern und ihren Kindern rekonstruiert (vgl. Kap. 3). Dadurch wird der Blick auf eine neu entwickelte mathematikdidaktische Theorie gerichtet, die „Interaktionale Nische mathematischer Denkentwicklung im familien Kontext“ genannt wird (Acar Bayraktar & Krummheuer, 2011,2014; Acar Bayraktar, 2016,2014,a,b,c,d, 2012a,b). In diesen empirisch hergeleiteten Begriff fließt die Mathematik in die Theorieentwicklung nicht nur als Anwendungsfall sondern auch als konstitutive Dimension für eine umfassender verstandene Entwicklungstheorie mathematischen Denkens ein (vgl. Kap.2.2.4). Darüber hinaus werden die verschiedenen familialen Support Systeme beschreibbar (vgl. Kap.2.3). Hierbei sind die ethnisch kulturellen Hintergründe der Familien mitberücksichtigt worden (vgl. Kap.1.5).

Anhand des vielfältigen Datenmaterials werden zunächst drei unterschiedliche einander ergänzende Perspektiven auf die Unterstützung in mathematischen familialen Kontexten entwickelt.

1. **Perspektive: Interaktionstheorie mathematischen Lernens**

Mit der ersten Perspektive wird der Blick auf einen interaktionistischen Ansatz zum Mathematiklernen gerichtet, welcher für mathematischen Lehr-Lern- Prozesse in der Grundschule entwickelt wurde (Krummheuer & Brandt, 2001; s. a. Brandt 2004, Krummheuer 2011a). Diese basiert auf drei grundlegenden Annahmen (vgl. Kap.2.2.1).:

„(1) Sowohl der zu lernende „Stoff“ als auch die dazu (mehr oder weniger funktional) emergierenden Lernbedingungen werden lokal im interaktiven Austausch zwischen den Beteiligten hervorgebracht. (2) Die konstitutive soziale Bedingung der Möglichkeit des Lernens eines mathematischen Inhalts, Begriffs und/oder Verfahrens ist die Partizipation an einer kollektiven Argumentation. (3) Ausdruck eines erfolgreichen Lernprozesses eines Kindes oder Schülers ist die zunehmend autonomere Partizipation in Fortgang der Interaktion oder in folgenden Interaktionen, die thematisch auf die aktuelle Situation Bezug nehmen. Der Erfolg ist abhängig von den ‚Partizipationsspielräumen’ (Brandt 2004, S. 58), welcher dem lernenden Kind in der Interaktion eröffnet, wird“ (Krummheuer, 2011d, S.495).

Auf der Basis dieses Ansatzes habe ich geometrische Spiel- Situationen in familialen Kontext untersucht und das mathematische Lernen als einen dualen Prozess konzipiert, in dem der Aufbau von Wissensbausteinen und die zunehmend autonomere Partizipation des Kindes in sozialen Situationen gegenübergestellt werden. Dabei werden in einzelnen Situationen des mathematischen Lernens die intentionalen und sozialverträglichen Verhaltensweisen der Teilnehmer in den interaktiven Prozessen dargestellt.

2. **Perspektive: Familiensystemtheorie (Systemische Familientherapie)**

Unter der Berücksichtigung interner und externer Faktoren in den Familien (vgl. Kap.2.2.2), wird die Familie unter dieser Perspektive als ein soziales System aufgefasst. Jede Beziehung zwischen dem Kind, den Eltern, Großeltern und Geschwistern wird als ein Teil dieses Systems verstanden und als ein Teilsystem betrachtet (Bornstein, 1989; 2002a,b, c, d, e; Bornstein & Sawyer, 2008; Collins et al., 2002; Crawford, 2012; Goodfellow & Laverty, 2003; Howe, Brody & Recchia, 2006; Laakso, 1995; Morgaine, 2001; Paquette, 1994, 2004; Parke, 2004; Salonen et al., 2007; Smith, 2005; Smith & Drew, 2002; Tamis-Lemonda, 2004). Von daher ist die erste

Fragestellung, wie die Gewohnheitsmuster der jeweiligen Familiendynamiken die mathematischen Entwicklungsprozesse des Kindes beeinflussen und wie diese Muster lokal im interaktiven Austausch zwischen den Beteiligten zum Ausdruck gebracht werden. Darauf aufbauend lautet die zweite Fragestellung, wie solche Gewohnheitsmuster und Familiendynamiken auf die Partizipationsspielräume des Kindes während des Spiels Einfluss nehmen. Im interaktionistischen Sinne (1.Perspektive) geht es keinesfalls um die Konzepte, die dem Kind und den Familienmitgliedern als Interaktionspartner bewusst sein müssen und von ihnen intentional zur Anwendung gebracht werden.

3. **Perspektive: Selbstreguliertes (selbstgesteuertes) und fremdreguliertes (fremdgesteuertes) Lernen**

In der dritten Perspektive wird die Frage gestellt, wie das Kind in seinem Lernverhalten während einer Spielsituation mit den kognitiven und psychosozialen Bedingungen umgeht (vgl. Kap.2.2.3). Um dieses Verhalten zu beschreiben, habe ich mich auf die externen und internen Steuerungen des Kindes konzentriert. In Anlehnung an die Bereiche der pädagogischen Psychologie, Schulpädagogik, differenzial und sozial-kognitiven Entwicklungspsychologie, Erwachsenenbildung sowie der Konzepte des Selbstbestimmten Lernens (Nader-Grosbois et. al., 2008) habe ich die Eigenbeteiligung und Leistung des Kindes in Spielsituationen untersucht. Hierbei wird das selbstgesteuerte Lernen als ist eine Form des Lernens verstanden, bei der das Kind in Abhängigkeit von der Art der Lernmotivation selbstbestimmt eine oder mehrere Selbststeuerungsmaßnahmen kognitiver, metakognitiver, volitionaler und verhaltensmäßiger Art ergreift und den Fortgang des Lernprozesses selbst überwacht[1]. Im entgegengesetzten Fall sprechen wir von fremdreguliertem Lernen. Insofern werden beim fremdgesteuerten Lernen die wesentlichen Entscheidungen über die Zielsetzung, Inhalte und Methoden nicht vom Lernenden sondern von der

[1] https://portal.hogrefe.com/dorsch/lernen-selbstgesteuertes/

Lehrkraft (hier besonders von anderen Familienmitgliedern) getroffen[2].
Das fremdregulierte Lernen kann durch direkte und explizite Förderungsmaßnahmen gesteuert werden. Aus diesem Grund erfordert das Lernen mit fremder Hilfe eine gute Steuerung und Kontrolle. In diesem Zusammenhang können Lehren und Trainieren als „abnehmende Unterstützungsstrategien" angesehen werden, deren Verwendung in verschieden Bereich möglich ist (Bannert, 2007, S.106)[3]. Diese können in Form vom Geben eines „intellektuellen Schubs" (Kniffka, 2010, S. 4)[4] erfolgen. Hier spricht man von „scaffolding" (engl. "Baugerüst"; vgl. Wood, Bruner & Ross 1976), welches Unterstützungshandlungen bezeichnet, die bei Erwachsenen in der Interaktion mit einem Kind eingesetzt werden (Kniffka, 2010, S. 1). Sinngemäß ist hierbei der Begriff des Scaffolding als „eine Metapher für maßgeschneiderte und befristete Hilfen" zu sehen, die ein kompetenterer Partner (z. B. Erwachsene, Lehrkraft, Lehrende, Mutter, Erzieherin) dem weniger kompetenten Partner (der Lernenden, der Lehrling, das Kind, den Schülerinnen und Schülern) anbietet, „um eine Aufgabe zu lösen, die sie andernfalls nicht hätten bearbeiten können" (Kaiser et al., 2015, S.377)[5]. Dadurch gibt der Erwachsene dem Kind „eine Orientierungsgrundlage, für die Ausführung der Tätigkeit" (Schnotz, 2011, p.134)[6] und auch eine Gelegenheit, von der das Kind profitieren und seine kognitiven Fähigkeiten ausbauen kann.
Auf der Basis dieses Ansatzes sollten die Lernbedingungen im

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[2] https://prezi.com/nszkyjap-hhv/fremdgesteuertes-vs-selbstgesteuertes-lernen/

[3] Bannert, M. (2007). Metakognition beim Lernen mit Hypermedien. Erfassung, Beschreibung und Vermittlung wirksamer metakognitiver Strategien und regulationsaktivitäten. Münster: Waxmann.

[4] Kniffka, G. (2010). Scaffolding. In proDaZ: Deutsch als Zweitsprache in allen Fächern. Retrieved from http://www.unidue.de/prodaz/konzept.php

[5] Kaiser, G., Blum, W., Ferri, R. B., & Greefrath, G. (2015). Anwendungen und Modellieren. In R. Bruder, L. Hefendehl-Hebeker, B. Schmidt-Thieme,H.-G. Weigand (Eds.), Handbuch der Mathematikdidaktik (pp. 357-384). Berlin: Springer.

[6] Schnotz, W. (2011). Pädagogische Psychologie Kompakt. 2nd Edition. Weinheim: Julius Beltz GmbH & Co. KG.

interaktiven Austausch zwischen dem Kind und den Familienmitgliedern definiert werden.

In Anbetracht der oben genannten drei Perspektiven, werden zwei Fragestellungen in dieser Dissertation untersucht:

1- Wie zeichnet sich eine interaktionale Nische raumgeometrischer Denkentwicklung in einem familialen Kontext für ein Vorschulkind ab?

2- Wie sollte ein Mathematics Learning Support System aufgebaut sein und im Fortgang der Interaktion zwischen das Kind und Familienmitgliedern realisiert werden?

Bezugnehmend auf das Konzept der Entwicklungsnische im familialen Kontext wird ein Blockspiel aus dem Inhaltsbereich der mathematischen Raumvorstellung eingesetzt (vgl. Kap.3). Dafür werden drei bilinguale, deutsch-türkische Familien (Familie Gül, Familie Ak und Familie Kil) ausgewählt und eine theoretische Reflexion des Begriffs MLSS durch eine konzeptionelle Einbindung der NMD-Familie angestrebt (vgl. Kap. 4).

Tabellarisch lassen sich die gewonnenen Ergebnisse zur NMD-Familie und MLSS in der folgenden Weise zusammenfassen:

NMD-Familien	**Inhaltskomponente**	**Kooperationskomponente**	**Vermittlungskomponente**
Aspekt: Allocation **Familie Kil** **Familie Ak** **Familie Gül**	„Raumgeometrie“ Spiel: Bauherr 01,02	Spiel von den Familienmitgliedern und dem Kind	Theorien zur Entwicklung räumlicher Fähigkeiten
Aspekt: Situation Familie Kil	• explorativ und disputational interaktive Aushandlungen von Teilen der räumlichen Körper • Deutungen unter	• verschiedene Partizipationsspielräume: expandiert und beschränkt • Eltern: Tutor • Ayse (Kind): Tutee	• Verschiedene Scaffolding Prozesse der Mutter & Vater • Ermöglichung der Ausführung und Leistung der verschiedenen räumlichen Besonderheiten • Positive Auswirkung auf Ayses kognitive Entwicklung

	geometrischen Rahmungen • Arbeitskonsensus		durch emotionale Rückversicherung ihrer Eltern
Aspekt: Situation Familie Ak	• kumulativ und disputational interaktive Aushandlungen von Teilen der räumlichen Körper; • Arbeitsinterim	• verschiedene beschränkte Partizipationsspielräume • Eltern: Tutor • Aleyna (Kind): Tutee	• Fokus auf die Arithmetik • arithmetisch-analytische Rahmung von den Eltern • Folk Psychologie und Folk Pädagogie • räumgeometrische Besonderheit der räumlichen Körper • Balance zwischen Mutter und Vater
Aspekt: Situation Familie Gül	• explorativ und disputational interaktive Aushandlungen von Teilen der räumlichen Körper • Deutungen unter geometrischen und arithmetischen Rahmungen • Arbeitskonsensus und Arbeitsinterim	• verschiedene Partizipationsspielräume: legitimate peripheral Partizipation und autonome Partizipation • Erwachsene: Tutee • Älter Bruder: Tutor • Berk (Kind): Tutee	• Modellierung durch Grußmutter und älteren Bruder. • Ermöglichung der Erkennung die räumlichen Körper, die falsch oder richtig nachgebaut wurden. • situationelles Lernen für Berk
Aspekt: Aktion **Ayse Familie Kil**	• Erkundung verschiedener Möglichkeiten des Nachbauens in vertikaler und horizontaler Ebene • explizite Erfahrungen über spezifische geometrische Besonderheiten	• Verschiedene Typen der Regulierung: Selbst- und Fremd- Regulierung	• gut entwickelte Lernchancen in der Raumgeometrie • implizites und explizites Erfahren von raumgeometrischen Eigenschaften • Entdeckung vertikaler und horizontaler Ebenen

Aspekt: Aktion **Aleyna Familie Ak**	• Erkundung verschiedener Möglichkeiten des Nachbauens • explizite Erfahrungen über arithmetisch-analytische Besonderheiten	• Verschiedene Typen der Selbstregulierung: zentral und umweltbedingt Selbstregulierungen	• Arithmetische Aktivitäten • gut entwickelteLernchancen in der Arithmetik • schwache Lernchancen in der Raumgeometrie
Aspekt: Aktion **Berk Familie Gül**	• Erkundung verschiedener Möglichkeiten des Nachbauens • explizite Erfahrungen über geometrische Besonderheiten	• differenzierte Rezipientenrollen	• Lernchanen in der Raumgeometrie • schwache Lernchancen in der Arithmetik • implizites Erfahren von raumgeometrischen Eigenschaften • Beobachtung jeder gelegenen Bauaktivitäten den Familienmitgliedern

Beim Vergleich der NMD-Familie in den Tabellen bietet sich eine Betrachtung der Zellen „Situationsaspekt" und „Aktionsaspekte" an (grau unterlegte Zellen). Hier zeigt sich in jeder Familie ein Supportsystem welches als je spezifische Ausformung in eine übergreifende interaktionale Nische mathematischer Denkentwicklung integriert ist.

Mithilfe der dargelegten Perspektiven und Begrifflichkeiten zeigt sich, dass MLSS, Familiensysteme und die Selbst- u. Fremdregulierung im Rahmen des Interaktionsprozesses in einer direkten Abhängigkeit zu einander stehen. Hinzu kommt, dass die ethnisch kulturellen Hintergründe der Familien keine bedeutsamen Parameter darstellen: Die Familienmitglieder führen interaktive Aushandlungsprozesse mit den Kindern, welche in räumlich geometrischen oder arithmetisch-analytischen Rahmungen hervorgebracht werden. Dadurch ermöglichen die Familienmitglieder den Kindern, eine verbesserte Perspektive auf die Mathematische Problemstellung. Im Blockspiel können die Kinder verschiedene Figuren aus den gewählten Karten nachbauen, erkunden und hierbei mehr oder weniger raumgeometrische Erfahrungen machen kann. Die Rekonstruktion der Bearbeitungsprozesse in den Blockspielen erbrachte drei unterschiedlich strukturierte

Aushandlungsprozesse (disputational, kumulativ, explorativ) und die unter den drei Perspektiven vorgenommenen interpretativen Analysen zeichnen ein Bild davon, wie die Kinder die verschiedenen Partizipationsspielräume ausgestalten, obwohl die Aushandlungsprozesse ähnliche Strukturen aufweisen. Darüber hinaus deuten die Analyseergebnisse darauf hin, dass die Familienmitglieder gleiche unterstützende Aktivitäten in Form von Fremdregulierung kreieren und verschiedene Scaffolding-Prozesse realisieren. Es lässt sich zwar keine standardisierte NMD-Familie definieren, jedoch lassen sich Bedingungen für differenzierte Formen der NMD-Familie beschreiben. Anhand der Analyseergebnisse komme ich zu dem Ergebnis, dass in den raumgeometrisch konzipierten Blockspielen in Familien äußerst unterschiedliche Lernunterstützungen und Förderungen für die Kinder auftreten.

Nach der zusammenfassenden Darstellung der Ergebnisse (vgl. Kap. 5.1) werden Fragestellungen aufgeworfen, die sich auf der Grundlage der gefundenen Einsichten für Folgeprojekte anbieten (vgl. Kap.5.2): Die Einflüsse der ethnischen Hintergründe von Mono- und Bilingualität in sowohl türkischen als auch deutschen Gesellschaften, sollten in einer empirisch-kulturübergreifenden Forschung, detailliert berücksichtigt werden. Darüber hinaus könnten folgende Fragestellungen untersucht werden: Wie können Familienmitglieder und Lehrkräfte mathematische Denkentwicklungen von Deutsch-Türkischen Kindern gemeinsam fördern? In welcher Frequenz und Gewichtung sollten die Familienmitglieder, Eltern, Erzieher(innen) und Lehrkräfte, den Kindern zu Hause, im Kindergarten oder in der Schule mathematische Erfahrungen anbieten.

1. INTRODUCTION

1.1. About This Work

In the analysis of mathematics education in early childhood it is necessary to consider the familial context, which has a significant influence on development in early childhood. Many reputable international research studies emphasize that the more children experience mathematical situations in their families, the more different emerging forms of participation occur for the children that enable them to learn mathematics in the early years. In this sense mathematical activities in the familial context are cornerstones of children's mathematical development, which is also affected by the ethnic, cultural, educational and linguistic features of their families. Germany has a population of approximately 82 million, about 7.2 million of whom are immigrants (Statisches Bundesamt 2009, pp.28-32). Children in immigrant families grow up with multiculturalism and multilingualism, therefore these children are categorized as a risk group in Germany. "Early Steps in Mathematics Learning – Family Study" (erStMaL-FaSt) is the one of the first familial studies in Germany to deal with the impact of familial socialization on mathematics learning. The study enables us to observe children from different ethnic groups with their family members in different mathematical play situations. The family study (erStMaL-FaSt) is empirically performed within the framework of the erStMaL (Early Steps in Mathematics Learning) project, which relates to the investigation of longitudinal mathematical cognitive development in preschool and early primary-school ages from a socio-constructivist perspective. This study uses two selected mathematical domains, Geometry and Measurement, and four play situations within these two mathematical domains.

My PhD study is situated in erStMaL-FaSt. Therefore, in the beginning of this first chapter, I briefly touch upon IDeA Centre and the erStMaL project and then elaborate on erStMaL-FaSt. As parts of my research concepts, I specify two themes of erStMaL-FaSt: family and play. Thereafter I elaborate upon my research interest. The aim of my study is the research and development of theoretical insights in the functioning of familial interactions for the formation of geometrical (spatial) thinking and learning of children of Turkish ethnic background. Therefore, still in Chapter 1, I present some background on the Turkish people who live in Germany and the spatial development of the children.

This study is designed as a longitudinal study and constructed from interactionist and socio-constructivist perspectives. From a socio-constructivist perspective the cognitive development of an individual is constitutively bound to the participation of this individual in a variety of social interactions. In this regard the presence of each family member provides the child with some "learning opportunities" that are embedded in the interactive process of negotiation of meaning about mathematical play. During the interaction of such various mathematical learning situations, there occur different emerging forms of participation and support. For the purpose of analysing the spatial development of a child in interaction processes in play situations with family members, various statuses of participation are constructed and theoretically described in terms of the concept of the "interactional niche in the development of mathematical thinking in the familial context" (NMT-Family) (Acar & Krummheuer, 2011), which is adapted to the special needs of familial interaction processes. The concept of the "interactional niche in the development of mathematical thinking" (NMT) consists of the "learning offerings" provided by a group or society, which are specific to their culture and are categorized as aspects of "allocation", and of the situationally emerging performance occurring in the process of meaning negotiation, both of which are subsumed under the aspect of the "situation", and of the individual contribution of the particular child, which constitutes the aspect of "child's contribution" (Krummheuer 2011a, 2011b, 2012, 2014; Krummheuer & Schütte 2014). Thereby NMT-Family is constructed as a *subconcept* of NMT, which offers the advantage of closer analyses and comparisons between familial mathematical learning occasions in early childhood and primary school ages.

Within the scope of NMT-Family, a "mathematics learning support system" (MLSS) is an interactional system which may emerge between the child and the family members in the course of the interaction process of concrete situations in play (Krummheuer & Acar Bayraktar, 2011). All these topics are addressed in

Chapter 2 as theoretical approaches and in Chapter 3 as the research method of this study. In Chapter 4 the data collection and analysis is clarified in respect of these approaches. Returning to the idea of MLSS within the familial context, this part especially sheds light on the topic of how family members can be supportive or helpful for the geometrical thinking and learning process of a child. Chapter 4 also discusses the emergence of support systems with respect to the observed family dynamics and interaction systems. Thereupon these support systems are typified with respect to NMT-Family and the results are presented in Chapter 5.

1.2. Mathematics in the Early Years

Pure mathematics is, in its way, the poetry of logical ideas.

Albert Einstein

Mathematics does not have a generally accepted definition, but it is seen as a science or group of related sciences dealing with the logic of quantity, shape and arrangement[7]. Barrow defines mathematics as the catalogue of all possible patterns and orders, and as the description of the world in which inevitably there exist patterns and orders of life (2010, p.371). Similarly Devlin identifies mathematics as the science of patterns (2000, p.7) and emphasizes that these patterns can be either real or imagined, visual or mental, static or dynamic, qualitative or quantitative, utilitarian or recreational (2000, p.8). Mathematics is regarded as comprehensive, abstract and also a concrete fact in the universe (Ginsburg, 2006, pp.17-19; see also Devlin, 2000).

Mathematics, as “the eyes of mind” (Devlin, 2000, p.9), occurs in each activity in the everyday life. To play with blocks, to bake a cake, to see or count house numbers on the street or car licence plates, to measure the weight of fruit or flour,

[7] Retrieved from http://www.thefreedictionary.com/Mathematics and http://en.wikipedia.org/wiki/Definitions_of_mathematics on 17 October 2014.

to cook a meal or to set the table, etc. and many more actions all consist of mathematics. Therefore, early in life, children often engage in mathematics more genuine than that taught in school (Ginsburg & Ertle 2008, p.55) and this real-life practice of mathematics outside the classroom is a core component of education from very early ages to the higher grades (Lee & Ginsburg 2009, p.39).

Most of the research sustains the idea that true mathematics involves broad strands of big areas (Lee & Ginsburg, 2009, p.39). Just as mathematics is the science of patterns, there exist different kinds of patterns which give rise to different branches of mathematics (Devlin, 2000, p.7). For example, number theory studies (and arithmetic uses) the patterns of number and counting; geometry studies the patterns of shape; calculus (or algebra) studies the patterns of motion; logic studies the patterns of reasoning; probability theory studies the patterns of chance; topology studies the patterns of closeness and position (Devlin, 2000, p.8). These areas are also determined by the NCTM (the USA National Council of Teachers of Mathematics) as content standards, which are categorized as “number and operations”, “geometry” (shape and space), “measurement”, “algebra” (particularly pattern) and “data analysis and probability” (NCTM, 2000, 2006; Clements, Sarama & DiBlase, 2004; Ginsburg, 2006; Clements & Sarama, 2007, 2014; Cross et al., 2009; Ginsburg & Ertle, 2008; Saracho & Spodek, 2008; Science Education Resource Center, 2008; Lee & Ginsburg, 2009; National Research Council, 2009; Van Nes, 2009; Copley, 2010; Brandt, Krummheuer & Vogel, 2011; Sperry Smith, 2012). Each of these five areas is regarded as one of mathematical content and defined as follows (Clements, Sarama & DiBiase, 2004):

1. *Number and Operations:* Numbers are used to tell how many objects can be described and ordered and they involve numerous relations. Operations with numbers are used to model a variety of real word situations and to solve problems. Number and operations can be represented and carried out in various ways.

2. *Geometry:* Geometry is used to understand and represent the object directions, locations in the world, and their relations with each other. Geometric shapes are described, analysed, transformed, and composed and decomposed into other shapes.
3. *Measurement:* Measures are determined by repeating a unit or by using a tool. Comparing and Measuring are used to specify "how much" of an attribute (e.g. length) objects possess.
4. *Algebra and Patterns:* Patterns are used and extended to recognize relationships and to make generalizations.
5. *Data Analysis and Probability:* Data analysis is used to classify, represent and use information to ask and answer questions.

Considering each of these five mathematical domains[8], early experiences in mathematics have major importance on children's learning in the first six years of life (NCTM, 2013, p.1). During this period of their lives, children have an ability to learn each mathematical domain and develop their interests in them (Clements & Sarama, 2014, p.1). It is thus crucial in which way and how often children are exposed to mathematics in its whole range of subdomains in the early years of

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[8] Clements and colleagues (2004) emphasize that each mathematical domain has a mutual effect with each other. They describe each mutual affect as follows: (1) Number can be used to quantify properties of geometric objects (e.g. number of sides or angles). Geometric objects provide models for number and operations (e.g. number line or arrays for multiplication). (2) Number and operations are essential elements of measurement. The measurement process subdivides continuous quantities such as length to make them countable. Measurement provides a model and an application for both number and arithmetic operations. (3) Geometry provides the major context for learning and teaching measurement. Measurement quantifies the attributes of geometric figures, such as side length and angle measure. (4) In geometric measurement, the measurement process usually synthesizes the domains of number and operations, on the one hand, and geometry on the other. (5) Algebra can be used to identify, describe and extend number patterns. (6) Number concepts are essential in analysing data. (7) Measures are often used and analysed as data. (8) Data analysis can be used to organize information to uncover patterns.

their lives. However, most adults think that mathematics consists of “numbers”, or rather “numeracy”[9] and they emphasize numbers or use counting as an integral part of the interaction with their children (Pound, 2006, p.51; 2008; Clements & Sarama, 2014; Casey et al., 2008; see also Blevins-Knabe, 2008; Devlin, 2014; Acar, 2011a,b; Tiedemann, 2012; Schuler, 2013; Newcombe, 2010, 2013). Thus “number sense” appears as an abstract cognitive competence, which is more operant than “color, shape, or appearance” (Devlin 2000, p.37; s. a. Newcombe, Huttenlocher, & Learmonth, 1999). But mathematics is more than this; in fact, it is about life (Devlin, 2000, p.76). As a powerful tool for communication, it includes the knowledge and understanding of the world (Pound, 2006), which means that mathematical thinking and learning is influenced by a complex system of different factors. Therefore, to understand the functioning of mathematical thinking and learning it is necessary to describe a theory including different factors of children’s lives. In Frankfurt, Germany, a mathematical research project is underway in which it is intended to develop such a theory in mathematics education with different factors.

1.3. IDeA Centre and Project erStMaL

Young children love to think mathematically.

(Clements & Sarama, 2014, p.1)

IDeA (the Centre for Research on Individual Development and Adaptive Education of Children at Risk) is a research centre which investigates extensively the development of children at risk and the processes of individual learning in preschool, kindergarten, and primary school age by including and combining cognitive, neuro-cognitive and socio-emotional factors (see also Krummheuer, 2013). In 2008, IDeA was constituted by the German Institute for International Educational Research (DIPF), the Frankfurt Goethe University and the Sigmund

[9] Numeracy is defined as proficiency, which involves confidence and competence with numbers and measures (Pound, 2006, p.2). Moreover, it includes understanding of the number system, computational skills and inclination and ability to solve number problems (ibid.).

Freud Institute (SFI), which are all located in Frankfurt am Main in Germany. Financial support provided by the Ministry of Higher Education, Research, and the Arts of the state of Hessen[10]. Within IDeA, experts from the fields of psychology, educational science, psycholinguistics, neuroscience, sociology, and psychoanalysis cooperate and exchange work on associated issues of child development.

In this research centre, mathematics learning and the development of mathematical thinking in the early years are investigated in the project, **E**a**r**ly **St**eps in **Ma**thematics **L**earning (erStMaL). It investigates the mathematical development of children, as a longitudinal study, from the age of three until the third year of primary school. The project is based on different research perspectives[11] on the development of mathematical thinking:

1. the development of mathematical concepts, the classification of the emerging mathematical activities in mathematical situations of play and exploration
2. the reconstruction of development of mathematical thinking
3. interactional support systems for the acquisition of mathematical concepts and operations/processes and participation in mathematical discourses
4. multi-modal aspects of children's mathematical concepts in discourses within preschool and primary mathematics classes (e.g., speech, gestures, actions, inscriptions)
5. mathematical, pedagogical, and psychological aspects of the activities of adults in the interaction processes.

[10] Retrieved from http://www.idea-frankfurt.eu/en/about-idea/about-idea?set_language=en on 5 April 2014

[11] Each numeral point takes place in the scope of different research perspectives (e.g. conceptual change, multimodality, inclusion, sociolinguistics, professionalization), which will not be enlarged upon in this work. The only perspective used is "socio-constructivism", which is discussed in the text chapter.

The first survey period of erStMaL covers kindergarten[12] children, whereas in the second survey period the same children are observed at primary school age (see also Acar Bayraktar et al. 2011, Acar Bayraktar 2014a,b,c). The aim of the project is to reconstruct the development of mathematical thinking, which is concerned with the handling of numbers, dates, and probability, with spatial thinking and measurement, with geometrical forms and bodies as well as patterns and structures (see Vogel, 2013, 2014).

The socio-constructive elements in the development of mathematical thinking are investigated in different settings, for which specific situations of play and exploration have been developed (see Vogel, 2014a,b, 2013, 2012; Vogel & Jung, 2014; Vogel & Wippermann, 2005). These situations are performed in groups of four or two children. Additionally, some mathematical play situations take place that are instructed by caregivers at the kindergartens participating in the erStMaL project. Moreover, mathematical situations of play and exploration are observed in the family environment as a sub-project of erStMaL, which is called the Family Study. Eventually, all these mathematical situations of play and exploration are video-recorded and analysed with the help of different qualitative methods (e.g., methods of conversation analysis, qualitative content analysis, argumentation analysis etc.). Standardized tests are used for measuring the development of mathematical concepts and basic cognitive skills. Thereby we acquire a huge data collection within the scope of the erStMaL project.

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[12] In the English-speaking world, the word "kindergarten" usually means an institution in which children spend just one year before entering primary school from ages 5 to 6 (Krummheuer, 2013a, p.250). In Germany, the word "*Kindergarten*" designates the institution which children attend between the ages of 3 and 6 before they start primary school (ibid.). In the USA one might use the abbreviation "pre-k" (ibid.). In its organization and administration, the German *Kindergarten* is separate from primary schools and "early *Kindergarten* age" refers to the age group between age 3 and 4 (ibid.). "In Germany, kindergarten is an institution for children from ages 3 to 6 and kindergarten is not a compulsory institution of education and is separated from primary school" (Brandt, 2014, p.228). Please note that the word "kindergarten" is used here in the German sense.

Provisional results of the erStMaL project are:

1. Children establish relationships between various mathematical domains to solve mathematical problems.
2. The expression of children's mathematical concepts is located on a continuum between mathematical language and everyday speech.
3. The argumentative use of diagrams and narrative-structured arguments appear successively in the mathematical thinking of child development.
4. In particular, children with risk factors seem to introduce their mathematical ideas through a gestural leeway of participation.
5. The reconstruction of code-switching processes between formal and informal language suggests that there are different supporting forms of implicit pedagogy.

(for more see Brandt et al., 2011; Brandt, 2014, 2013; Krummheuer, 2014, 2013a,b; Vogel, 2014a,b,2013). Currently the whole dataset collected by the project is still in the ongoing process of analysis, from the longitudinal perspective through qualitative and quantitative evaluation methods[13].

As mentioned above, a family study is performed within the erStMaL project, which is also designed as a longitudinal study. This study is the core area of my dissertation and in the next part I focus on its structure and set up a basis on my research interest.

1.4. A Familial Study in the erStMaL Project: erStMaL-FaSt

The family study is a sub-project of the erStMaL project and it deals with the impact of familial socialization on mathematics learning. The idea of a family study occurred from the need to investigate the importance of families in the mathematical development of children. For this family study, three criteria were

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[13] For further information about the erStMaL project see: Benz et al., 2014; Brandt 2014, 2013; Brandt et al., 2010; Brandt et al., 2011; Krummheuer 2014, 2013a, 2013b, 2012, 2011, 2009; Krummheuer & Schütte 2014; Vogel 2014a, 2014b, 2013.

determined: ethnic background (German or Turkish), duration of the formal education of the parents (more or less than 10 years) and the sibling situation within the family. Before going into further detail about the family study, first I will state the reasons for choosing the family as an observation group and the need for the three criteria specified in this study. In the following discussion of some of the theoretical aspects, I touch upon the theme "family" first and then try to understand the purpose of the three criteria ("Ethnic background of the family", "Duration of formal education of parents/family members" and "Sibling situation in the family") for the family study.

1.4.1. Main Issue of erStMaL-FaSt: Family

The family is a micro unit of the social system, which is a group of people affiliated by consanguinity, affinity or co-residence. The most common form of family includes only the husband, the wife, and their unmarried children, which in sociology is also called the nuclear family[14]. This principal institution is one of the critical social settings and thus the cornerstone for the socialization of children, in which children develop and learn (National Research Council, 2009). Similarly Hawighorst (2000) defines "family" as a social system, which in everyday family practices provides the resources with which the basis for individual education processes are constituted and more or less good pre-conditions are offered for success in school:

> ...die Familie wird als soziales System verstanden, in dem im Kontext der familiären Alltagspraxis die Ressourcen vermittelt werden, die die Grundlage für Gestaltungsmöglichkeiten individueller Bildungsverläufe darstellen und die mehr oder weniger gute Voraussetzungen für schulischen Erfolg bieten (Hawighorst, 2000, p.32)[15].

[14] Retrieved from http://oregonstate.edu/instruct/anth370/gloss.html on 10 July 2014.

[15] Family is percieved as a social system and this system procure some resources in the context of everyday practices. These resources constitute various designs of individual educational paths and thus offer the more or less favorable conditions for academic success of children.

This expression emphasizes that the education process first begins in the family then continues in school. This micro unit contributes to children's social growth during the time that they experience school or academic life and go on their own way. The family is the first institution for children as they prepare themselves for the real world. It influences children's development in many ways, including parenting practices, provision of resources, interactions with school, and involvement in the community (National Research Council, 2009, pp.101-102). Hughes states that the family makes up children's community and lets them grow up within a closely linked network of people (Hughes 1986, p.36). Similarly, in Germany, the Federal Ministry of Family Affairs indicates that children are testing, discovering and coming to know their enigmatic world with its forms, colours, and sounds; or with its surprises and regulations, first in their family (BMFuS, 2002, p.18).

Parents form the representative part of the community for their own children, who learn language, culture, rules of common life and daily routines in the family. The US National Research Council emphasizes that each parent has different attitudes, values and beliefs in raising young children, which result in different emphases on educational activities in the home (National Research Council, 2009, pp.101-102). Parents are the primary caregivers of children (Connecticut State Board of Education, 2007; National Research Council, 2009). They prepare their children for life with a repertoire of, how to act, react and take on responsibilities. Mills defines parents as childrens' first and continuing "educators" (Mills, 2002, p.1).

Mostly parents perceive caring for their children as "a type of employment" (Kim & Fram 2009, p.78), and do not recognize the importance of childcare for children's ongoing lives. Moreover, parents may not always be aware of, or be able to identify, their own contribution to the development of their child (Bottle 1999, p.56). Therefore, childcare is a kind of "strategy" (Kim & Fram 2009, p.78), in order to let children to live efficiently. In this familial strategy, while parental social status and educational level appear to have a significant influence on childcare, cultural factors also have a significant influence on formative values and beliefs about child development (Kim & Fram 2009, p.79).

The process of engagement with parents fosters refinement of children's thoughts and makes their performances more effective, in which lingual development and the articulation of ideas are the central facts to learn and develop. The activities, toys, materials and social events introduced to children in their home environments shape their thought processes and performances. Thereby home culture supports children's thinking and play in the emergence of their skills and abilities in each developmental domain (Connecticut State Board of Education, 2007, p.viii).

In the light of all these definitions and emphases on the importance of "the family" I want to go into the representative function of the family, as mentioned above, which enables children to learn language, culture and rules of common life. In the next part I discuss the meanings and functions of the three criteria of the family study ("Ethnic background", "Duration of the formal education of parents/family members", "Sibling situation in the family"), and their importance in the development of children, which constitute the representative function of the family. Regarding all these facts I give thought to the mathematical thinking and learning of children and muse on the purpose of specifying three criteria for the family study.

1.4.1.1. 1. Criterion: Ethnic background of the families

The first criterion in the family study is the ethnic background of the families, whether they are Turkish or German. Because ethnic background is a mediator to characterize the cultural identity of particular communities within German society in Germany, this criterion is quite important for the family study to answer the question how the ethnic background of the child affects mathematical development in the familial context.

Germany as a multiethnic society

The Federal Republic of Germany is a country that offers plenty of opportunities for immigrants to gain better living conditions and educational opportunities.

Thus, the societal reality in the Federal Republic of Germany has been characterized for more than 40 years by the continuing process of natives and immigrants living side-by-side (BMAS, 2006, p.18). Currently, in Germany there are about 82 million people, of whom 20.5 per cent are immigrants (Statisches Bundesamt (DESTATIS), 2009, 2014, 2015). Since the 1950s, the Federal Republic of Germany has been subject to immigration and has allowed immigrants and moreover their descendants to enter and settle in the country. The majority of immigrants, about 8.6 million people, have a German passport, while there are approximately 7.1 million foreign nationals. As a result of the first official permission of immigration, Germany has embarked on being a multicultural society. But with the birth of children and the maturing of migrant youth from the second and third generations, Germany has developed into a multiethnic society (BMAS, 2006, p.18; see also DESTATIS, 2009, 2014, 2015). An ethnic society may be defined as an involuntary group of people who share a heritage, kinship ties, a sense of identification, political and economic interests, and cultural and linguistic characteristics (Banks 1979, p.239). Accordingly, a multiethnic society is an ethnically heterogeneous society whose members belong to more than one ethnic group. Thereby the necessity for a comprehensive social integration policy and encouragement of intercultural competence as important creative resources for natives and migrants living and growing (up) together is recognized and multiethnicity is constituted in Germany (BMAS, 2006, p.18; see also DESTATIS, 2009, 2014, 2015). The cultural and lingual situatedness of a family represents the main structure of its ethnicity:

Culture as a "mixed salad"

Children are born and socialized in a family environment to become productive citizens in their society (Saracho, 2010, p.119). In this regard, the development of the children is not only a cognitive but also a social process. In the socially interactive processes of children take place the creation of shared activities, which are named "cultural practices" (Greenfield,1997, p.303), shared meanings, which are named "cultural interpretations" (Greenfield, 1997, p.303).

Culture is a broad and comprehensive concept that includes all the ways of being (Terry & Irving, 2010, p.110), and the cluster of learned and shared beliefs, values (achievement, individualism, collectivism etc.), practices (rituals and ceremonies), behaviours (roles, customs, traditions etc.), symbols (institutions, language, ideas, objects, artifacts etc.) and attitudes (moral, political, religious etc.) that are characteristic of a particular group of people that are communicated from one generation to another (Gardiner & Kosmitzki, 2008, p.5). Similarly UNESCO defines the term "culture" as a set of distinctive spiritual, material, intellectual and emotional futures of society or a social group, which encompasses, in addition to art and literature, lifestyles, ways of living together, value systems, traditions and beliefs (UNESCO, 2002, p.1). Thereby parents, who prepare children to function competently in the physical, economic and psychosocial situations that are characteristic of their culture (Bornstein & Bohr, 2011, p.1), are seen as adaptively a central node in the nexus between culture and child development.

In the course of daily life, parents enable their children to experience social and physical settings, while the children familiarize themselves with the culturally constructed ideas and cultural belief systems of their parents (Harkness et al., 2007, p.35). As noted above, culture has several components, including values and behavioural styles; language and dialects; nonverbal communications; perspectives, worldviews, and frames of reference (Banks, 2006). Each family is a "nuclear social system" and belongs to a culture. The children's hereditary environment makes each child a unique individual within this nuclear system of the heritage culture (Saracho, 2010, pp.119-120). In each different ecological setting, every child interacts with cultural and historical factors in the balance of demands with the requirements of culture (Gardiner & Kosmitzki, 2008, p.36; see also Bronfenbrenner, 1970; Vygotsky, 1978). As a member of society, the child performs and acquires different knowledge, beliefs, arts, morals, laws, customs and any other capabilities and habits as a complex whole. Furthermore, in contrast to culturally and ethnically homogenous societies, in multicultural and multiethnic societies, children develop into members of more than one ethnic group, which have different cultures. Thus, social, physical and ecological settings, as "culture-specific constructs" or "universal typologies" (Harkness & Super, 2010, p.1), inevitably play a role to identify the children's cultural identity and to shape it through daily experiences. In this context, instead of a single and homogeneous culture as a monoculture, a child can acquire and combine two or more cultures, in other words, to be "bi- or multi-cultural". Thereby their "cultural maintenance" (of customs from the old culture) is perpetuated, while they pursue the public domain as a "cultural adaptation" (Bornstein & Bohr, 2011, p.2). In accordance with public and individual habits, children furnish their acts with the way of thinking, perceiving, associating and interpreting that constitutes the characterization of multiculturalism. In multiculturalism, many cultures live together with more or less tolerance and all the societies, with their different value systems, behaviour patterns, traditions, institutions, beliefs, arts and literatures, dwell in one place.

As a "mixed salad", each individual in the society belongs to many different cultures but has an ability to function in at least two different groups in the society (Fries, 2008, p.6). Here the metaphor of a mixed salad delineates also "a macro culture with wide boundaries", in which micro cultures are retained (Banks, 1979, p.238). Through interaction, the micro cultures as "fluids" (Holloway et al., 1995, p.453) merge with each other and construct models, creating a shared, intersubjective understanding of key beliefs, values and strategies. For attaining valued goals, this "co-construction" process is dynamic; cultural knowledge is produced – not just reproduced – through ongoing interaction (Holloway et al., 1995, p.453). In this regard, the cultural identity of children depends on the social and physical locations of where their parents live and where they come from. The term "culture" should thus be linked with the term "immigration", which directly affects the cultural identity of children.

Immigration and children

Immigration is the physical relocation of a person and is typically thought of as relocation to another country (Bornstein & Cote, 2009, p.2). In 2013, the

International Organization for Migration stated that about 192 million people were living outside their place of birth, which is about 3 per cent of the world's population[16]. Due to political, social and economic conditions, many people leave their own country and voluntarily demand a permanent residence in a new country. To provide their children with better economic, welfare and health standards, as well as educational and employment opportunities, parents will make sacrifices and move to a new continent and culture at great economic, physical, and psychological cost. Although they wish to promote their children's positive development, they cannot abandon their own culture and customs, and they inculcate their traditional beliefs in their offspring. Through their displacement, each family member negotiates and navigates through a new culture, different belief systems and new social networks with "significant effects" on their family life (Bornstein & Bohr, 2011, p.2). During the whole social and cultural shifts they need to adapt themselves culturally, psychologically and socially to the resettlement as they straddle two cultures, producing tension and conflict in the family. Whatever the reason for immigration might be, immigrants have a particularly difficult ti3me adjusting to the "new" culture (Cote, 2011, p.3).

When parents migrate to a new culture, they carry their culture with them, which is the foundation of child-rearing, and goals for the development of their offspring, but they "encounter new implicit cognitions and explicit practices concerning childrearing in their culture of destination" (Bornstein & Bohr, 2011, pp.3-4). In the adaptation process it is difficult for immigrant parents to decide between the cognitions or practices implied by retaining their indigenous culture and modifying to the new conventions. Moreover, their children, as members of immigrant communities or as members who live with the society of another culture and assimilate aspects of its behaviour, become bi- or multi-cultural through interaction with other children or caregivers from different cultures (Fries, 2008, p.6). These cultural differences in the environment of the children of the children precipitate the formation of their cultural identity.

[16] Retrieved from http://www.iom.int/jahia/Jahia/about-migration/lang/en on 8 December.2013.

Whereas immigrant children have few opportunities to participate in and learn about their “heritage” culture (Birman & Poff, 2011, p.1), they can be involved in the “new culture” more quickly and easily by attending kindergarten and school. Meanwhile, their parents and grandparents cannot so easily become comfortable enough with the new culture and language as to become socially integrated into their “new country”. Thereby immigration brings adaptation difficulties and language barriers between parents and their children, which are destabilizing of parent-child relationships and intimacies between parents and child. As a consequence, there occurs a gap between immigrant parents and their children in their cultural worlds.

Immigrant parents maintain their own traditions and language at home, whereas they encourage their children to develop the competencies necessary to function in the “new culture” (Suárez-Orozco & Suárez-Orozco, 2001, pp. 88-89). Thus the social relationships of immigrant families come to be more complex than particular relationships as a product of larger social problems in that they exist “in the margins of two cultures” and walk on a “tightrope” (Civil, Planas & Quintos 2005, p.81). The immigrant children have plenty of opportunities to acquire the new culture but in their everyday lives they concurrently confront two or moral culture types: their heritage culture and the culture in their new country. Thereby they become acutely aware of nuances of behaviours. While they are simultaneously adopting cognitions and practising the new culture by retaining their old one, they turn out to be “bicultural” or “multi-cultural” (Bornstein & Bohr, 2011, p.2). Never truly belonging either “here” (the new culture) or “there” (the old culture) (Suárez-Orozco & Suárez-Orozco, 2001, p.92), they acquire two different cultures, constitute a bicultural society in public, and move back and forth on a continuum between two feelings. Neverthless they are still regarded as “strange” and “foreign” in public, whereas they are esteemed “normal” at home (ibid.).

Language of immigrants

Language is a "key tool of communication" (Kaufmann, 2010, p.79) and a symbolic system consisting of the structural relations between "signifiers" and "signifieds" (Aksu-Koc, 1988, pp.3-4). Furthermore, as a symbolic vehicle for the representation of physical, social or logical knowledge through concrete and formal operations, it shifts to coincide with sociocultural change in "multifaceted and dynamic ways" (Baquedano-López & Kattan, 2007). Thereby language takes its place in centre of the culture (Reich et al., 2002) as a type of action which assimilates objects of this life history into a "synchronic system" (McNeill, 1985, p.258). A synchronic system includes the relationship between social structures in society, as educational systems, and individual language development, including its relations to eventual academic success (Jørgensen & Quist, 2007a,b).

For the matter of immigration and, correspondingly, multiculturalism, language metamorphoses into the structure of multilingualism. In societies in Europe – particularly in Germany – which were previously monolingual, immigration has made language diversity more salient (ICMI, 2009, p.5). That is to say, the consequence of some kind of disturbance in the "language order", such as migration or conquest, brings language systems into some kind of unexpected and "unnatural" contact with one another, and often leads to structural simplification (Auer & Wie, 2007, p.2). In particular the generation of bilinguals and multi-linguals has an immigration background and they adapt or are forced to adapt to the monolingual majority's language norms, to communicate with foreigners, not with people of their own (bilingual) community (ibid.). For that matter language looms large in the socialization of immigrants as a cultural artifact. In multi- or bilingualism, the norms are enforced by effective institutions, particularly the school or work system, and their stability is guaranteed by the fact that they are backed up by a large corpus of written documents which are easily accessible to everybody since the respective societies are literate to a very high degree (ibid.). Thereby bi- or multi-linguals come to have a unique and specific linguistic configuration, which is actually not the sum of two complete or incomplete monolinguals (Paradis, 2007, p.34; see also Grosjean, 1995). Being bi- or multilingual enables them to express their emotions with different languages instead of one language and hence they struggle to choose between different ways of feeling and differing cultural norms of expression, and hence transcending a particular emotional world (Besemeres, 2004). Consequently, their purely personal feelings become partly dependent on cultural forms of the society and reciprocally the presentation of the self and the perception of the world switch together with the bi- or multilingual's language switch (Dewaele, 2007, p.112). Thereby many of these children "share" and experience more than one culture and language with different groups that they interact with (Fries, 2008, p.6).

For child development, it is quite important at what age immigration takes place (Dewaele, 2007). The language patterns of children living in a multilingual context are complex in that children learn the "heritage" language from immediate family members whereas the second language gradually enters their lives via television, contact with peers and day-care in school or kindergarten (Verhoeven & Strömqvist, 2001, p.1). Furthermore, culture and language influence a wide array of family functions including roles, decision-making patterns, and cognitions and practices related to child-rearing and child development (Bornstein & Bohr, 2011). Thus, most bi- or multilingual children form a minority group who learn two or more languages in a successive manner. As these children mature, it becomes increasingly difficult for them to discuss abstract concepts and complex issues with their parents, from whom they receive no formal instruction in their heritage language (Birman & Poff, 2011). Personal beliefs and attitudes influence the parents' own linguistic practices and interaction strategies with their children, and this in turn has an impact on the children's language development (Lanza, 2007, p.53) and especially mathematical learning (Barwell, 2009).

Effect of ethnic background of families on mathematical thinking and learning in the early years

Very few cross-national studies have examined the influences of family and ethnicity on early mathematics learning (Hunstsinger et al., 1997, p.372). Considering the cultural, migrational and lingual situatedness of the family, as mentioned above, the ethnicity of parents and/or grandparents seems to have a crucial effect on the development – and here particularly mathematical development – of children. Mathematical thinking and learning cannot be irrespective of culture and language (Novotná & Moraová, 2005; also Halliday & Hasan, 1985). The language, culture, beliefs, and practices of each family member are interwoven in the daily routines at home (Miller et al., 2005, p.176). Each activity of everyday life can be mathematical for the child, while home culture supports children's thinking and playing in the emergence in the mathematical thinking and learning of the child (see Bottle, 1999). The home environment and the attitudes of parents to mathematics have an essential effect on mathematical thinking and learning (Eccles 1993; Hunstsinger et al., 1997; Bottle, 1999), which also requires the situatedness of culture and language (Perkkilä & Aarnos, 2007, p.1271). In this regard, ethnicity strongly influences methods of negotiation in familial situations at home which may directly influence children's performance in mathematics learning (Hunstsinger et al., 1997, p.378).

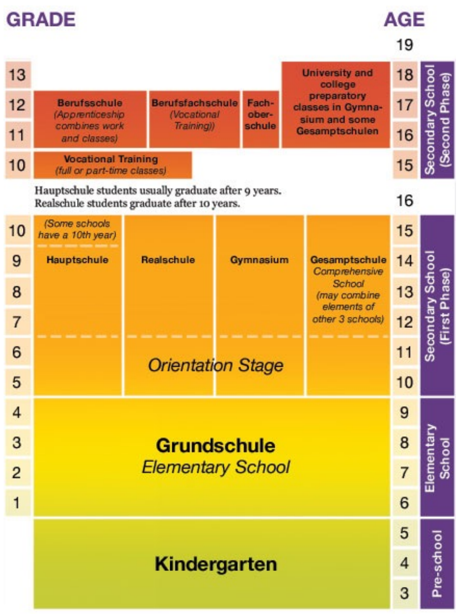

Fig. 1.1 The German school system

(source: http://www.howtoGermany.com/pages/Germanschools.html)

The aim of erStMaL-FaSt is to observe and find out whether there is any difference between the mathematical thinking and learning of children whose families have different ethnicity and, if there is, how this difference and its influences on children's mathematical development can be shaped.

1.4.1.2. 2. Criterion: Duration of formal education of parents

The second criterion in the family study is the duration of the formal education of the parents. This is categorized into two groups: higher educational level, represented by more than 10 years of formal education, and lower educational level, less than 10 years of formal education. In the formal education system in Germany, nursery and preschool education begins with *Kindergarten*, which children may attend between the ages of two and six years[17]. Primary school education takes four years, from the age of six to ten (Engel, 2008). After fourth year, according to their abilities and the wishes of their families, children progress to different types of secondary school: *Hauptschule* (general secondary school), *Realschule* (intermediate secondary school), *Gymnasium* (upper secondary school) and *Gesamtschule* (comprehensive secondary school) (ibid.).

Between the ages of 16 and 19, students obtain their secondary education diploma, which is awarded after more than 10 years of formal education, i.e., 10th school grade. Therefore, in the family study parents are categorized according to their school grades. If they attended school only from grade 1 to 10, then their education level is deemed lower educational level. When parents have at least a secondary education diploma, which means that they at least attained 12th grade, for the purposes of this study, they are deemed to have higher educational level.

According to the USA National Institutes of Health, the education level of a parent is a significant predictor of a child's educational achievements and behavioural

[17] See footnote 6 above.

outcomes[18]. As an information and referral agency for their child (Peyton et al., 2001), a parent's education level plays a crucial role in the child's development and stature, adjustment and success (Bornstein, 2002a,b,c,d). It relates to a number of parenting behaviours which are associated with the child's behavioural and socio-emotional performance and competence (Smith et al., 2002, p.398).

Parents with lower levels of education may not feel capable of assisting their child or playing a role in their lives as they may not understand school educational material or feel comfortable with their abilities (Shapiro, 2009, p.5). On the contrary, parents with higher levels of education may have more information about child development and more opportunity to become more involved in the formal education of their children (Hill et al., 2004; Shapiro, 2009). Amidst the many influences on child development, parental education level affects parental involvement in the child's development and their perspectives on preparing their children for the physical, psychosocial and economic conditions of their lives. Moreover, education level affects the child's experiences of growth and maturation, parenting behaviours, quality of parent-child interactions in everyday life and educational activities with parents during early childhood development (Zimmerman & Pons, 1986; Peyton et al., 2001; Bornstein, 2002a,b,c,d; Smith et al., 2002; Bornstein & Sawyer, 2008; Engel 2008; Shapiro, 2009).

Put succinctly, education level plays a significant role in the amount of parental involvement in the child's education and development and here particularly the child's mathematical thinking and learning. In the course of daily routines, a child engages in mathematics with his(her) parents and the level of parental education determines the quality of such mathematical events. Numerous studies emphasize that the level of parental education is strongly associated with the frequency, quality and intensiveness of mathematical interactions between parents and their children (Anderson, 1997; Benigno & Ellis, 2008; Blevins-Knabe, 2008; Coates & Thompson, 1999; Hannula et al., 2007; Pound, 2006,

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[18]Retrieved from http://everydaylife.globalpost.com/education-level-parent-affect-childs-achievement-school-6869.html on 1 March 2015.

2008; Tiedemann, 2012; Silinskas et al., 2010; Street et al., 2005; Vandermaas-Peeler, 2008).

Therefore erStMaL-FaSt aims to observe and reveal whether there is any difference between mathematical thinking and learning of children whose families have different educational levels and, if there is, how this difference and its influences on mathematical development of children can be determined.

1.4.1.3. 3. Criterion: Sibling situation within the family

The third criterion in the family study is the sibling situation within the family which represents the status of the children in the family. When a child is an only child in the family, (s)he is called and categorized as a single child in this study. When there is more then one child in the family, then the child is categorized as a sibling, which means that (s)he has one or more brothers or one or more sisters or both.

Developmental research on the family process traditionally focuses more on the parent-child relationship and less on sibling relationships (Segrin & Flora, 2011; Doron, 2009; Bornstein & Sawyer, 2008; Howe & Recchia, 2006; Bornstein 2002,a,b,c,d). However, the sibling relationship is an integral component of the family system (Howe & Recchia, 2006, p.1) and the longest relationship in human life that begins in childhood, continues throughout life, and only ends with the death of one of the siblings (Doron, 2009, p.23). In the family dynamics each child's birth induces many changes, joys, responsibilities, and transformations (Furman & Lanthier, 2002) that have significant influences on the child's developmental processes (Doron, 2009). Moreover, relationships with siblings provide a context in which children can practise learning and social skills and interaction styles that have been learned from parents or others (Parke, 2004, p.374). The sibling relationship, as a natural laboratory, enables young children to learn how to interact with others who are interesting and engaging playmates, to manage disagreements in constructive ways, and to regulate both positive and negative emotions in socially acceptable ways (Howe & Recchia, 2006, p.1; see also Parke, 2004; Segrin & Flora, 2011). All siblings differ from each other, but through their interactions they develop specific interaction patterns and social understanding skills which generalize, shape, and regulate their behavioural patterns.

In the preschool years, siblings play an important role in one another's social lives (Pepler et al.,1981, p.1347). While they are growing up, they can join in activities together and take role-playing from each other (Doron, 2009). In this regard the birth order shapes the sibling relationships in the family dynamics. Older siblings function as competent socializing agents of younger children and as tutors, managers or supervisors of their younger brother or sister's behaviour during social interactions, while they may also function as monitors, gatekeepers who extend or limit opportunities to interact with other children outside of the family (Parke, 2004, p.374; Zukow-Goldring, 2002, p.254). Thereby older siblings are thought of as "the responsible ones", whereas youngest siblings are "the free spirits or the funny ones" of their family (Crawford, 2012, p.1). In this regard the different positions of the siblings establish that none of the children experience the family in the same way (ibid.).

When the first born grows up without any sibling, then (s)he is considered as an only child. The only child can live out his(her) own identity and spend more time with the parents and does not need to fight to get their attention (Harmut, 2007, p.15). Conversely, the only child may have less social experience and more specific interaction patterns than the child with siblings. However, the only child gets more attention from the father and mother than siblings, because the parents do not need to allocate time for each child but rather for only one child (Harmut, 2007). Thereby the only child has a fruitful and intensive relationship with his or her parents. In this regard, both the personal growth of each child and the arrival of new siblings change family dynamics perpetually. There emerge different relationships between first borns, latter borns and parents in that the relative amount of attention from parents to the first born may decline and be reallocated to the new arrivals (Furman & Lanthier, 2002; Segrin & Flora, 2011). Each child thus experiences reciprocally different family dynamics.

In their daily routines a child would most likely engage in mathematics with his(her) siblings, and the quality of sibling relationships might specify the quality of such mathematical events. However there exist no studies emphasizing whether the sibling relationships or being an only child are strongly associated with the frequency, quality and intensiveness of mathematical interactions in the familial context. Therefore, erStMaL-FaSt aims to observe and find out whether there is any effect from having siblings in the mathematical play situations of children, and how being an only child or a child with siblings influences mathematical development in the familial context.

1.4.1.4. Mathematical thinking and learning in the early years in the family

Young children have an ability to learn mathematics (Copley, 2010, p.4), but mathematical activities are not usually a dominant feature in young children's lives (Blevins-Knabe, 2008, p.2). Mathematical activities should be made relevant to children's real-life experiences of the world and/or to their concerns and interests (Pound, 2006, p.83). The home provides a rich learning environment where they can ask questions, reflect, and argue, and therefore construct their mathematical knowledge (Carruthers & Worthington, 2006, p.219). Moreover, the child's cultural and linguistic endowment is seen as the fundamental medium of learning (Pound, 2006, p.viii).

While the child is constructing and linking together the worlds inside and outside, (s)he involuntarily benefits from linguistic or cultural representations including mathematical ideas in everyday life. Moreover, Hustinger and colleagues emphasize that ethnicity directly influences parental teaching methods and therefore ethnicity, parents' beliefs and parents' practices have relevant influences on the early mathematics performance and development of children (Hustinger et al., 1997, p.386). In the model of Eccles (1993), characteristics of the context and family members (parents, siblings, grandparents) shape and influence the parents' beliefs and practices (parent-child beliefs and behaviours), which in turn affect their children's performance and educational level in mathematics thinking and learning (ibid.). A cluster of variables – parent, family and neighborhood characteristics – includes ethnic status, education, income, occupation and family composition, indirect as well as direct, has an impact on the mathematical thinking and learning of children. Each family member, who personifies their heritage, culture and language (see Chapter 1.2.1.1), and the familial environment, thus plays a crucial role in a child's mathematical development.

As parents support their children's mathematical learning, siblings and grandparents can also provide children with different mathematical learning situations in the everyday environment. Through everyday experiences the child learns a wealth of informal knowledge and strategies to deal with situations that have a mathematical dimension (Copley, 2010, p.4). By cooking, shopping, walking, eating, reading, listening, or playing, the child can be exposed to mathematical situations and learn mathematics informally (Pound, 2006; Ginsburg & Ertle, 2008). In this way, the child continually constructs his(her) own meanings of mathematics and mathematical ideas based on his(her) experiences with the home environment, interactions with adults and other children, and daily observations (Copley, 2010, p.5; see also Carruthers & Worthington, 2006). In this regard language, culture, beliefs, and practices are interwoven and incorporated into the core of the mathematical thinking and learning process in the early years (ICMI, 2009, p.7; see also Anghileri,1995; Cao et al., 2005; Miller et al., 2005; Pound, 2006, 2088; Civil et al. 2008, 2006, 2005).

As a matter of course family dynamics are inextricably intertwined with the child's mathematical experiences in the familial environment. Through these family dynamics, the child can reflect, explore, and link everyday experiences to mathematics. Thereby mathematical thinking and learning come to be a "jigsaw" (Pound, 2006, p.23) in which the child can make connections between things that are known and new information and experiences. In this regard the child can learn and construct the complex meanings of mathematics and at the same time go on building on this knowledge, which can be used in his or her future school life (Carruthers & Worthington, 2006). Pound emphasizes that children themselves set the pace and the sequence of their own learning, within a supportive structure provided by adults (2006, p.23). Considering this idea, interaction processes with family members seem to allow children to imitate and respond to the mathematical thinking of others (Pound, 2006, p.45). Pound (2008) points out that children profit from discussing mathematical ideas with adults, through which they can learn mathematics effectively. In this way each family member makes the uses of mathematics apparent so that the child can benefit from them and learn complex mathematical meanings and understandings. On entry to school, the child's informal and home-based mathematical knowledge enables him or her to cope easily with mathematical schemes, features, and different mathematical understandings (Carruthers & Worthington, 2006; Pound, 2006, 2008; Copley, 2008, 2010).

1.4.1.5. Summary

The three criteria of erStMaL-FaSt – the level of parental education, the ethnic background of parents and the sibling situation in the family – have crucial roles in the child's mathematical developmental process. Parents' education level and ethnic background affect the child's development directly. Moreover, they both indirectly affect the sibling situation in the family. The more parents engage their children in the everyday situations of daily life, the more mathematical development the child can experience. Young children in every setting experience mathematics through these familial practices (NCTM, 2013, p.1) and the family is the best medium for children to learn mathematics effectively and enjoyably. The familial environment gives children various different opportunities to experience mathematical activities, which are potentially significant for learning mathematics in different cultural, ethnic and linguistic contexts.

1.4.2. Main concept of erStMaL-FaSt: play

As a social event in early childhood education, play is a dominant, prominent, and important instrument for the child and adult alike, as a way of using mind, or better yet, an attitude towards the use of mind (Bruner, 1983, p.69). Play is the first and most important defining behaviour of a child and cannot be replaced by any other activity (Bodrova & Leong, 1996). Through play children construct their understanding of the world, re-create their knowledge, employ their own rules, make ideas part of their reality, and discover solutions to complex problems (Connecticut State Board of Education, 2007, p.10). Thus, play contributes to and enhances all areas of early childhood development. Bruner remarks that play under the control of the player gives the child his(her) first and most crucial opportunity to have the courage to think, to talk, and perhaps even to be himself or herself (Bruner, 1983, p.69). During play, children perform and learn cooperation, problem solving, language and mathematical and scientific concepts (Connecticut State Board of Education, 2007, p.10). Furthermore, while children are taking the opportunity to negotiate their thoughts during play, they become able to express and control their emotions and to make inquires about the information given by parents or older siblings. In this sense, play enables dyadic exchanges, such as face-to-face interactions, and extra-dyadic exchanges, such as object-oriented interactions (Tamis-LeMonda et al., 2002). In addition, play enables the child to use language and culture to shape his(her) interactions cooperatively with his(her) family members, who can be role models for the child (Coates & Thompson, 1999, p.205; see also Bornstein, 2002a,b,c,d, 2008). Through these interactions, children also gain confidence and learn to trust others. Thus, Erikson (1963, pp.222-223) suggests that play is of prime importance in the mastery of emotional needs (see also Bruner, 1972). During play activities, parents or siblings influence not only the child's cognitive learning, but also the child's social-emotional competence by negotiations and relations in the family system. While children learn to give, receive and share, and to convey ideas and feelings, they come to see the perspectives of others and make choices. Play is in one sense a social act but it is also the typical activity of a child. Thereby emotional, cognitive, communicative, social and cultural functions are intensely embedded in the nature of play (Tamis-LeMonda et al., 2002). Families provide plenty of opportunities for children to play and explore and thus make positive contributions to the lives of children (Connecticut State Board of Education, 2007; Bernstein, 2002a,b,c,d). Play furnishes children with models and techniques for how to operate, both cooperatively and on their own.

As a result of all these, play is the crucial factor in child development. The question then arises how and in which ways play affects the mathematical development of the child in the familial context. Therefore, we chose “play” as the main concept of erStMaL-FaSt.

1.4.3. Data Collection of erStMaL-FaSt

For the family study, we chose children from main erStMaL study according to the following three criteria: the ethnic background (German or Turkish), the duration of the formal education of the parents (more or less than 10 years) and the sibling situation within the families (see below, 1.4.3.1).

The sample of the erStMaL project consists of 178 participant children from 12 day-care centres in Frankfurt am Main, Germany (Vogel & Jung, 2014, p.9). From these participants 120 children were selected, who were aged around four in 2009 (Acar Bayraktar et al., 2011, p.14). Of these 120 children, 49 matched the three criteria of erStMaL-FaSt. Therefore, these children and their families were contacted and asked if they would like to participate in erStMaL-FaSt. At the beginning of the family study, 12 families agreed to participate. However, in 2012, these numbers had decreased and only eight of them went on to participate in erStMaL-FaSt. The detailed research design is shown in Table 1.1.

Table 1.1 Research design of erStMaL-FaSt

Eight Families		with sibling	without sibling
Higher Educational Level	German Turk	1	2
	German	2	1
Lower Educational Level	German Turk	1	-
	German	1	-

The data consists of videorecording and transcripts of the recordings. Once a year, an appointment was arranged with each family. This led to the accumulation of a collection of data on each child. In these appointments, the erStMaL child was video recorded together with members of the family while they played a game. For each child, the following table was designed (Table 1.2).

Table 1.2 Observation design of erStMaL-FaSt

Observation design	erStMaL child as a single child	erStMaL child as a sibling
erStMaL child is playing with	mother or one member of family (e.g. father)	mother or one member of family (e.g. father) and sibling
	mother and one member of family (e.g. father)	mother, sibling and one member of family (e.g. father)

1.4.4. Play situations and observations of erStMaL-FaSt

For erStMaL-FaSt, four play situations are conceived, which refer to two mathematical domains: geometry and measurement (Acar Bayraktar & Krummheuer, 2011, p.143; see also Acar Bayraktar et al., 2011). Each mathematical play situation is one of several mathematical situations of play and exploration, designed as special empirical research instruments for the erStMaL-FaSt and constructed according to specific didactic design patterns, or "design patterns of mathematical situations" (see Vogel, 2014a,b, 2013, 2012; Vogel & Jung, 2014; Vogel & Wippermann, 2005).

These design patterns are used to document and relay expert knowledge about the arrangement of mathematical teaching-learning environments (Vogel, 2014, p.234; see also Vogel & Wippermann, 2005). They are also used to describe the diagnosis of situations, to categorize the situations of observations and to stay in a mathematical portfolio (Vogel, 2012, p.9). There are three aspects of the design patterns of the mathematical situations of play and exploration: organizational, realization-related and mathematical aspects. Whereas the organizational aspect is decisive in the organization of research, the realization-related aspect is relevant to "the actual implementation of the play situations and the mathematical aspects support the guiding adult in their decision-making during the situation" (Vogel, 2014, p.234), In this sense Vogel constructed the structure for the "design pattern of mathematical situations (of play and exploration)" in the erStMaL project as follows (Vogel, 2012, p.6).

Each aspect of structure is related to the others and they can be selected for specific situations (Vogel, 2014a,b, 2013, 2012). Within the scope of "realization-related aspects", possible activities, verbal and gestural impulses, spoken instructions and intended mathematical and pedagogical ideas of adults, which emerge during the play situation, are written and detailed in these design patterns of the erStMaL project (Acar Bayraktar et al., 2011). These "mathematical stagings (the mathematical situations of play and exploration)" might vary by mathematical context as well (Vogel, 2012, p.6; Vogel, 2014, p.234).

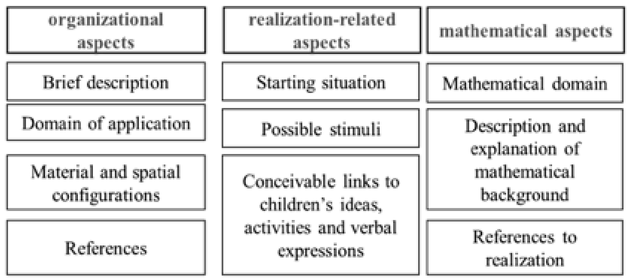

Fig. 1.2 Structure of the "design pattern of mathematical situations" in the erStMaL project

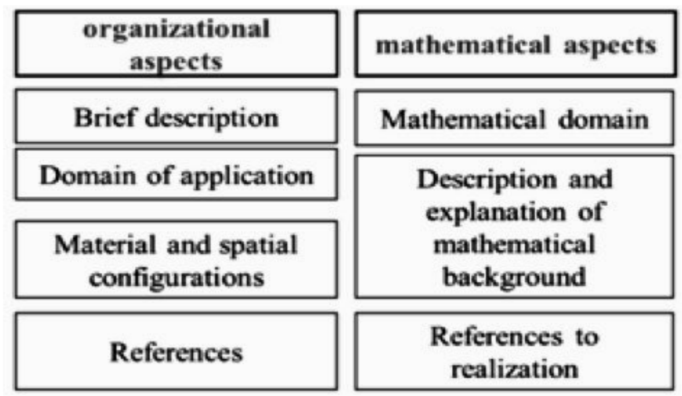

Fig. 1.3 Structure of the "design pattern of mathematical situations" in the erStMaL-FaSt

For the erStMaL-FaSt, these subgroups in the design patterns of stagings consist of just the organizational and mathematical aspects, through which a didactic free play situation can be provided for the child, while (s)he is playing with his(her) family. In this sense each situation of play and exploration focuses on one mathematical task or problem, which is presented in a playful or exploratory context according to the age of the child and represents the starting point of a common process of dispute (Vogel, 2014, p.225). One particular mathematical domain should be addressed (see section 1.2) and compatible materials, arrangement of space and mathematical task should be chosen.

Realization-related aspects are not needed for the erStMaL-FaSt since the aim is to let children and their families play and negotiate freely during the mathematical situations of play and exploration. Moreover, for the family study, only two mathematical domains – geometry and measurement – are used to conceive these mathematical stagings. The aim is to observe negotiation processes in mathematical stagings, which are either about geometry or measurement, within the familial context. Therefore, only brief descriptions of each play situation and instruction manuals for parents are composed for the families in erStMaL-FaSt. There are no intended activities, no spoken instructions and no pedagogical ideas in the design patterns of the mathematical situations in order to keep the play situation open and to let parents play artlessly with their children. In each brief description of a specific design pattern: (1) definition of play situation, (2) application field, (3) intended mathematical domain, (4) mathematical context, (5) materials and playroom, (6) instruction manual are introduced below. The outline drawings of specific design patterns are shown both in English and in German in Fig. 1.4:

English Version:

GOETHE UNIVERSITÄT FRANKFURT AM MAIN — LOEWE Exzellente Forschung für Hessens Zukunft — IDeA Individual Development & Adaptive Education

Mathematical situation of play and exploration

erStMaL - Familystudy
(Date: xx.xx.xxxx)

FaSt – Play X

Definition of play situation (1)	How can the play situation be described?
Application field (2)	Which fields are scheduled at the application of the play situation?
Intended mathematical Domain (3)	Which mathematical domain is chosen for the play situation?
Mathematical context (4)	How can the chosen mathematical content de described?
Materials and play room (5)	Which materials and structures are needed?
Instruction manual (6)	Which impulses and information are needed for the participation of the play situation?

German Version:

GOETHE UNIVERSITÄT FRANKFURT AM MAIN — LOEWE Exzellente Forschung für Hessens Zukunft — IDeA Individual Development & Adaptive Education

Spiel- und Erkundungssituation

erStMaL-Familienstudie
(Stand: xx.xx.xxxx)

FaSt- Spiel X

Kurzbeschreibung der Spiel- und Erkundungssituation (1)	Wie kann die Spiel- und Erkundungssituation beschrieben werden?
Einsatzbereich (2)	Welcher Einsatzbereich ist für die Spiel- und Erkundungssituation vorgesehen? (Familienstudie, Erstbesuch/Zweitbesuch, Altersbereich)
Mathematischer Bereich (3)	Welchem mathematischen Bereich kann die Spiel- und Erkundungssituation zugeordnet werden?
Mathematischer Gehalt (4)	Wie kann der mathematische Gehalt der Spiel- und Erkundungssituation beschrieben werden?
Material und Raum (5)	Welches Material wird für die Durchführung der Spiel- und Erkundungssituation benötigt? Wie muss der Raum für die Spiel- und Erkundungssituation vorbereitet sein?
Impulse und Hinweise für die Familien (6)	Welche Impulse und Hinweise benötigen die teilnehmenden Familien, um an der Spiel- und Erkundungssituation teilzunehmen?

Fig. 1.4. Template design pattern for a mathematical play situation

In the family study, there were four different play situations, which were expanded and upgraded each year before the observations (see Acar & Brandt, 2010). According to the recording schedules each year, each play situation and uniform didactic design patterns would be categorized. For data collection, an appointment was arranged with the family giving them flexibility to choose the place and time. Before they began to play in the meeting, all the games were explained in the language family members preferred: either German or Turkish. At the same time, all the play materials were shown to family members. In addition, they were told that they were free to play in any language they want. Instruction manuals for each game were offered in both languages as well. These introduction manuals and game materials were provided by myself and put at the disposal of the family in the recording room.

Original Instruction Manual of erStMaL-FaSt:

In German Language:

Spielanleitung „Fühlsäckchen"

Ziel des Spiels ist es, die im grünen Säckchen versteckte Figur auf dem Tisch wiederzuerkennen.

Zu Beginn wird von jedem vorhandenen Figuren-Paar jeweils eine Figur auf den Tisch gelegt. Die restlichen Figuren werden im roten Säckchen versteckt.
Es startet ein Spieler und steckt – ohne dass dies von den Mitspielern gesehen werden kann - **eine** im roten Säckchen liegende Figur in das grüne Säckchen. Der links neben ihm sitzende Spieler versucht jetzt, die Figur zu ertasten und die gleiche Figur auf dem Tisch zu erkennen. Er darf die Figur dafür nicht aus dem Säckchen nehmen.
Hat er die richtige Figur erkannt, erhält er einen Punkt und legt die Figur aus dem grünen Säckchen zurück in das rote Säckchen. Jetzt ist er selber an der Reihe, eine Figur auszuwählen und im grünen Säckchen zu verstecken.
Insgesamt werden drei Runden gespielt.

Wer am Schluss die meisten Punkte hat, hat gewonnen.

In Turkish Language:

Oyun klavuzu „Kesedekini Hisset"

Bu oyunun amacı yeşil kesenin içinde bulunan geometrik şekli bakmadan sadece elleyerek ne olduğunu anlamak ve masanın üzerindeki şekiller arasından seçip göstermek.

Oyun malzemesi olarak yeşil/ kırmızı kese ve geometrik şekillerle dolu 2 adet şeffaf kutu göreceksiniz. Kutular içerik olarak aynı olup, içinde her biri farklı 7 tane geometrik şekil bulunmaktadır. **Oyuna başlamadan önce** bir kutuyu masanın üzerine diğer kutuyu da kırmızı kesenin içine boşaltın.

Oyuna başlayan oyuncu kırmızı kesenin içinde bulunan geometrik şekillerden birini seçerek **kimse görmeyecek şekilde** yeşil kesenin içine koyar. Soldaki oyuncu da yeşil kesenin içindeki geometrik şekilin ne olduğunu **sadece dokunarak; keseden çıkarmadan ve bakmadan;** masadaki şekiller arasından hangisi olduğunu tahmin etmeye çalışır. Daha sonra yeşil kesenin içindeki şekil çıkartılır ve oyuncunun tahmini ile aynı olup olmadığına bakılır. Eğer oyuncunun tahmini doğru çıkarsa 1 puan; yanlış çıkarsa 0 puan alır ve şekil kırmızı kesenin içine geri bırakılır.

Aynı işlemi sırayla diğer oyuncular da yaparlar. Bu şekilde 3 tur oynanır ve en çok puanı alan oyuncu oyunu kazanmış olur.

Fig. 1.4 Template from erSTMaL-FaSt game instruction manual

The family was then left alone to make themselves comfortable while the video-recorders are turned on. The members of the family were supposed to choose at least two games out of four and to play them.

1.5. Research Interest

In this section, I present the main points of research interest and the formation of my prevailing research questions. First, I explain the purpose of picking "German Turks" as the group of interest in this study. In this regard I discuss their Turkish background and their current status in Germany, and the effect of being "German Turks" on the mathematical development of the children. I specify the mathematical topic at issue and its constituents. In this study, "geometry" is chosen as the mathematical domain. Moreover, its relation and connections with spatial abilities are discussed. Finally, block play and its features, and the purpose of preferring it as a focal medium to observe the German Turk families and their children are thematized and explained. In conclusion, I amalgamate all these elements to identify the specific research question of this study.

1.5.1. Group in this Study: "German Turks"

"German Turks" is a term which signifies people living in Germany who originate from Turkey[19]. In the literature of cross-cultural studies, different terms are used for this group, e.g. Turkish Germans (Kilinc, 2014), German-Turkish (Kaya, 2007), Germany-born Turks (King & Kilinc, 2013), or Euro–Turks (Kaya & Kentel, 2005; Østergaard-Nielsen, 2003). In the present work, the term "Turk" denotes the ethnic identity of these people, while the term "German" denotes the country, in which these people live. In this regard I adopt the term "German Turks" to refer to the people who have Turkish ethnicity and live in Germany.

German Turks constitute the largest ethnic minority in the Federal Republic of Germany (Durgel et al., 2009, p.837). The Federal Statistical Office of Germany (Statistisches Bundesamt) reports that about 1.6 million Turks were living in

[19] Retrieved from http://en.wikipedia.org/wiki/Turks_in_Germany on 29 April 2015.

Germany in 2008, whereas their population had decreased to about 1.5 million at the end of 2014 (see Table 1.3)[20].

Table 1.3 Ethnic minority populations in the Federal Republic of Germany

Naturalization of foreigners in Germany per (selected) country and year				
Country of origin	**31.12.2008**	**31.12.2010**	**31.12.2012**	**31.12.2014**
Turkey	1 688 370	1 629 480	1 575 717	1 527 118
Poland	393 848	419 435	532 375	674 152
Italy	523 162	517 546	529 417	574 530
Greece	287 187	276 685	298 254	328 564
Croatia	223 056	220 199	224 971	263 347
Russia	188 253	191 270	202 090	221 413
Bosnia-Herzegovina	156 804	152 444	155 308	163 519

Source: Statistisches Bundesamt, 2009, 2014, 2015a, b; translated by Acar Bayraktar.

After the Second World War, Germany hired cheap foreign labour to fuel the German economic miracle (Mueller, 2006). In the 1960s and 1970s the Turks who immigrated to Germany as "guest workers" were clearly labelled as "aliens" (Schmidt, 2011, p.82) and/or "outsiders" (Müller, 2006, p.420) by German society. The initial stipulation allowed guest workers to stay in Germany for up two years and then to return to their homeland (Karcher, 2010: Kilinc, 2014; King & Kilinc, 2013; Schmidt, 2011). Some of them did return to Turkey, but most took up residence and started a family or reunified their family in Germany (see Heinrich Böll Stiftung, 2010; Schmidt, 2011; Mueller, 2006; Kilinc, 2014; King & Kilinc, 2013; Statistisches Bundesamt, 2009, 2014, 2015a, b). In this way the first generation of German Turks was formed, who came from Turkey as "guest workers", and then stayed and led the rest of their lives in Germany. Subsequently they brought their families from Turkey – especially their spouses and children (King & Kilinc, 2013, p.4) – pursuant to the Family Reunification Act under the German Basic Law and the European Convention on Human Rights (Karcher, 2010, p.6). Some of the Turkish immigrants started a family in Germany, which thus constituted the second generation of German Turks, who were born in Germany but whose affiliation to their Turkish roots was strongly

[20] Retrieved from https://www.destatis.de/DE/ZahlenFakten/GesellschaftStaat/Bevoelkerung/MigrationIntegration/AuslaendischeBevolkerung/Tabellen/StaatsangehoerigkeitJahre.html on 29 April 2015.

influenced by their parents (Schmidt, 2011, p.83). Later on, this second generation of German Turks gave birth to the third generation, who were born in Germany and are naturalized German citizens (Schmidt, 2011, p.84). The second generation is thus perceived as the German Turks whose parents immigrated to Germany, and accordingly the third generation is those whose grandparents immigrated to Germany (for more, see Statistisches Bundesamt, 2015b, p.583). The German federal statistical office defines and classifies generations of people with non-German citizenship as in Table 1.4.

Table 1.4 Classification of people with non-German citizenship by country of birth in the Federal Republic of Germany

	country of birth	
	foreign country (people with own migration experience)	**national country (people without own migration experience)**
citizenship: non-german	**I. immigrated aliens** a) first-generation aliens	**II. non-immigrated aliens** a) second-generation aliens (parents are parts of I.) b) third-generation aliens (parents are parts of II.)
Source: Statistisches Bundesamt, 2015b, p.581; translated by Acar Bayraktar.		

"German Turks" is that the majority of the first generation of Turkish immigrants to Germany came from rural and undeveloped areas of Turkey, and on average their education level was low (Abadan-Unat, 2002). Durgel and colleagues substantiated that Turkish immigrants assimilated to the German culture significantly and related positively to the goals of close warm relationships and personal and economic potential (2009, p.843). German Turks acquire the personal independence typical of German culture, but they cannot disregard the interdependence of Turkish culture (Durgel et al., 2009, pp.843). The majority of first- and second-generation German Turks feel themselves close to Turkish culture, maintain close ties with Turkey (Kaya & Kentel, 2005) and value lifelong close ties with family members, including the expectation of a financial contribution to the family from their offspring. As the independence of German culture and the interdependence of Turkish culture coincide and confront each other, German Turks experience the conflict of never truly belonging either “here” (the new culture) or “there” (the heritage culture) (Suárez-Orozco & Suárez-Orozco, 2001, p.92).

As a consequence, the purely personal feelings and experiences of German Turks become reciprocally dependent on the cultural and lingual forms of German and Turkish societies in Germany. Their children (and/or grandchildren) share and experience both Turkish and German cultures and languages. Moreover they gather together all the social, cultural and lingual norms that produce both mixed culture and language (Dirim & Auer, 2004; Heinrich Böll Stiftung, 2010; Jørgensen 2003a,b; Kilinc, 2014; King & Kilinc, 2013; Mueller, 2006; Otyakmaz, 2008; Röhrborn, 2002; Reich et al., 2002; Schmidt, 2011; Pfaff, 2001; Yada, 2005). In this sense they have their own personal histories of dual identity and their own strategies for balancing Turkish traditions and German influences on their own lives (Horrocks & Kolinsky, 1996, p.xxi). Consequently, both their language and cultural patterns are relatively complex, which influences a wide array of family functions including roles, decision-making patterns, and cognitions and practices related to child-rearing and child development (for more, see 1.2.1.1).

Considering the three criteria of erStMaL-FaSt – the level of parental education, the ethnic background of the parents and the sibling situation in the family – the process of mathematical developmental of Turkish children becomes an issue to explore. The majority of first-generation Turkish immigrants have a low educational level, they earn(ed) their living in Germany by doing manual labour. In general, they do not feel capable of assisting their children in their school work, but they do especially hope that the educational achievement of their children (i.e., the second generation of German Turks) will enable them to „have a better life” than their parents (i.e. the first generation of German Turks) (Hawighorst, 2000). However, their parents’ educational level and their ethnic background affect these children’s development directly, so that they mostly have relatively low school success and come to have a low educational level like their parents by German education standards (see also Lohse, 2001; Hawighorst, 2000; Schmidt, 2011)[21].

The development of language and the articulation of ideas is central to the learning and development of children (Atherton, 2010), and their language, culture, beliefs and practices are so interwoven with each other that it greatly affects their learning and developmental process, and here particularly mathematical learning and development. All these factors are incorporated into the core of the mathematical thinking and learning process in the early years (ICMI, 2009, p.7; see also Anghileri, 1995; Benigno & Ellis, 2008, 2004; Blevins-Knabe, 2008; Blevins-Knabe et al., 2000; Cao et al., 2005; Civil et al. 2008, 2006, 2005; Hawighorst, 2000; Miller et al., 2005; Pound, 2006, 2008). The cultural, migrational and lingual situatedness of these families affect children's mathematical thinking and learning, which construct their social and individual identities as well (Zevenbergen, 2003).

Most of the first- and second-generation Turkish immigrant parents comprehend the importance of formal education in time and that mathematics has a key position in their children's school life in that it initiates their academic success (Hawighorst, 2000). Therefore, some German Turk parents uphold the idea that their children must be good at mathematics in order to succeed in their school lives and eventually graduate from university (Hawighorst, 2000). This approach of parents might be highly associated with their children's behavioural and socio-emotional performance and competence in their school lives in that the majority of second- and third-generation German Turks do come to have higher academic and especially mathematical success than their parents (Statistisches Bundesamt (DESTATIS), 2009, 2014, 2015; BMBF, 2006). They can be better qualified and thus more specialized to work in different occupations.

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[21] Türken ohne Schulabschluss. Türkische Einwanderer und ihrer Kinder haben schlechte Aussichten, sich in Deutschland zu integrieren. Was sind die Ursachen? Was muss in Zukunft geschehen? *Die Zeit*. (27 January 2009). Retrieved from http://www.zeit.de/online/2009/05/integration-tuerken-schulabschluss

While the fourth-generation Turkish immigrants are continuously exposed to German language in their daily lives via television, contact with peers, and day-care in school or kindergarten, they learn Turkish as a heritage language from immediate family members. In this sense, these children as German Turks are exposed to Turkish cultural traditions through their parents and/or grandparents but they become educated in German, through which they may eventually attain higher education and professional occupations. The beliefs and attitudes of the family members – as German Turks – influence the linguistic practice and interaction strategies of the children, which directly affects their mathematical thinking and learning. In this sense, children's mathematical experiences in the familial environment are inextricably intertwined with the family dynamics. While these children (and/or grandchildren) mature as the third and later generations of German Turks, it may become increasingly difficult for them to discuss abstract concepts and complex issues with their parents and/or grandparents (i.e. the first or second generations of Turkish immigrants) or to receive formal instruction in the Turkish language. On the other hand, as the second and third generations receive their formal education in Germany, that they can pass their competences on to their children and assist them in the German language during their formal education. By and by these generations of German Turks become more integrated into using the German language and proficient in it, so that they can discuss abstract concepts and complex issues with their children in German.

Through such family dynamics, children can reflect on, explore and link everyday experiences to mathematics. In this sense the process of engagement with an adult person in the family enables children to refine their thinking or performance to make it more effective (Acar, 2011b). Moreover, parenting behaviours and quality of parent-child interactions in everyday life seem to affect children's experiences including their ability to achieve mathematical and academic success in the course of their growth and maturation. The familial environment provides children with different opportunities to experience mathematical activities, which are potentially significant for learning mathematics in their cultural and lingual contexts (see 1.2.1). In this regard different learning possibilities emerge for these children through language proficiency of their family members in German.

Whereas children can take advantage of both Turkish and German norms facultatively in the early years, the use of German language and culture becomes formal and compulsory during their preschool or school education in Germany (Meyer & Prediger, 2011a,b; Prediger & Wessel 2011; Otyakmaz, 2008; Yada, 2005; Dirim & Auer, 2004; Jørgensen 2003a,b; Röhrborn, 2002; Reich et al., 2002; Pfaff, 2001). Their mathematical thinking and learning process comes to be a "jigsaw" (Pound, 2006, p.23), in which they make dual connections – German and Turkish – between things that are known and new information and experiences. The children can learn and construct the complex meanings of mathematics and concomitantly go on building on this knowledge, which can be used in their future school life (Carruthers & Worthington, 2006). In this regard the more children are exposed to the assistance of their family members in the German language during mathematical activities, the better they can be prepared for school and reach higher levels of formal educational than the early generations of German Turks. So indeed, statistical comparison reveals a discernible increase in educational levels between the first and later generations of Turkish immigrants in Germany (BMBF, 2006, p.132).

Home environment, Turkish ethnicity and the attitudes of Turkish parents to mathematics have essential effects on children's mathematical thinking and learning (Bottle, 1999; Hunstsinger et al., 1997; Eccles 1993), which also requires the situatedness of mixed (German and Turkish) culture and language (Perkkilä & Aarnos, 2007). However there exist no studies emphasizing either the sibling relationships or being an only child, which are strongly associated with the frequency, quality and the intensiveness of mathematical interactions in the familial context in Germany. As a matter of fact, a child most likely engages in mathematics with his(her) siblings and the quality of sibling relationships specifies the quality of mathematical events in the daily routines (see section 1.2.1).

1.5.2. Mathematical Topic: Spatial abilities and Geometry

From birth to death human beings are constantly confronted with space. In the surrounding space they gain their first experience through activities based on looking, hearing and listening. By and by they expand their experiences by acting, reaching, playing with shapes and figures in the world. While a baby is trying to reach a ball, a child in the kindergarten might try to build a train station or a mountaineer try to reach the highest level of Mount Everest. An infant sees its mother's face in one view from below, in another when cuddled in her arms, and yet another view from an infant seat (Sperry Smith, 2012, p.196). An adult can orient himself and go on moving in space, but the young child reaches for a rattle on the tray or crawls to the coffee table and pulls herself up by the rim (Sperry Smith, 2012, p.196). While an adult is able to picture a building as a static object, children perceive the word differently. Overall, children confront these activities with different geometric phenomena and geometry problems.

We need to understand how a child explores and learns about space; how a child can think and get specific ideas about this "living" area; and what activities and teaching approaches can help the child to develop such spatial abilities and to learn geometry (Clements, 1999). Considering the idea that geometry in early childhood is inextricably intertwined with children's spatial abilities, in the present study the mathematical domain of geometry is at issue.

1.5.3. Summary

Children naturally love to explore geometric and spatial aspects of the world around them (Copley, 2000, p.110). For understanding not only the spatial world but also other topics in mathematics and in art, science, and social studies, it is crucial that children should be frequently exposed to block play and activities involving geometrical issues (NCTM 2000 p.97).

Children experience mathematics first in the family and then in preschool, kindergarten and (primary) school (Tiedemann, 2012a,b, 2010,a,b). In this sense family members are children's first educators in learning geometry, and familial activities in the mathematical context are cornerstones of children's mathematical abilities (Acar, 2011b, p.1861). Moreover, the social and individual construction of identity has a great influence on the geometry learning performance of children (see also Zevenbergen, 2003).

International studies indicate the weakness in children's geometric achievements and the lack of thought about geometry in formal education at primary level (Mullis et al., 1997, 2008). In this regard there exist no studies about the geometrical and spatial development of children with Turkish backround (see section 1.5.1.) who live in Germany. Moreover, there is little research on how block play affects the geometrical and spatial developmental processes of these children. Bounding all these issues, my research interest is the geometrical and spatial development of German Turk children during block play within a familial context.

2. THEORETICAL APPROACHES

With respect to the concept of erStMaL-FaSt (see 1.4 above), the present study is designed as a longitudinal study and based on empirically grounded theory. Considering the research interest (see 1.5 above), interactionist and socio-constructivist perspectives are used in order to examine the interaction processes between children and family members during block play. Mathematics is regarded as “a social construction” (Cobb & Bauersfeld, 1995, p.4; see also Tiedemann 2013) and mathematics learning is a “social process, which takes place in the interaction between human beings” (Krummheuer, 1999b, p.332; see also Bruner, 1990, 1996; Bauersfeld,1995; Erikson, 1982; Krummheuer, 1992). In this sense children’s idiosyncratic activities during block play, the dynamic interactions between individual, social and cultural factors and mathematics learning, which emerge through such interactions, are taken into consideration. Concerning the process of negotiation of meaning in the family during block play, the learning opportunities provided, and the emerging forms of participation and support, are analysed in depth.

In the following sections, first, the mathematical domain of geometry is scrutinized. Second, the social constructivist approach and its basic theoretical tenets are discussed. Subsequently the concept of the “interactional niche in the development of geometrical and spatial thinking in the familial context” (NMT-Family) and “mathematics learning support system” (MLSS) are furnished with the purpose of identifying the special needs of familial interaction processes.

2.1. Geometry and Spatial Abilities

Geometry offers an important way to interpret and reflect on the physical environment (Clements, 1999, p.77). In this sense, spatial abilities enable to understand, interpret and appreciate all geometric phenomena of the world (NCTM, 1989). This intervention takes place first initially and then gradually through concrete experiences with eye and hand (Van den Heuvel-Panhuizen & Buys, 2005, p.115). Geometry pertains to spatial abilities (Bishop, 1983, p.176) and the development of spatial ability is an essential tool for mathematical thinking using geometry (Sperry Smith, 2012, p.196). Therefore geometry and spatial abilities are interwoven and incorporated into the core of mathematics education and developmental psychology (see also Bishop, 1983). However, geometry and spatial abilities are mostly ignored or minimized in early childhood education and in the professional development of early childhood teachers as well (Clements & Sarama, 2011a, p.133; see also Ginsburg et al., 2006; Sarama & Clements, 2009a).

In the next section, first “spatial abilities”, which are the *sine qua non* of geometry, and then geometry as one of the mathematical domains, are introduced. Their meanings and functions and their importance in the development of children are discussed. Thereupon all these factors are reflected in the introduction of block play and the purpose of choosing block play in this study. Consequently, I discuss how familial context affects geometrical thinking and learning in block play with regard to the development of children’s spatial abilities.

2.1.1. Spatial abilities

Fundamentally spatial ability is the ability to perceive the environment through the senses, to learn about the environment and the relationship between objects (Karaman & Yontar Toğrol, 2010, p.3). In this sense it can be also defined as the awareness of things and our ability to use this awareness to solve spatial problems (ibid.). Moreover, it refers to the capacity to understand and remember the spatial relations among geometrical objects (Spatial Test Battery, n.d.).

Spatial abilities generally relate to skills in representing, transforming, generating and recalling symbolic, nonlinguistic information (Linn & Petersen, 1985, p.1482). Moreover, such abilities enable us to formulate, generate, retain, retrieve and transform well-structured visual and mental images, and to manipulate these images in the mind (Lean & Clements, 1981, pp. 267; Lohmann, 1996, p.99; Maier, 1994, 1999; Merschmeyer-Brüwer, 2001; Obersteiner, 2012; Brandl, 2011; Wölpert, 1983).

Spatial ability is “not unitary” (Lohman, 1996, p.99), “not monolithic” and “not static” (Spatial Test Battery, n.d., p.1), but made up of numerous sub-skills, which are interrelated as pivotal constructs of all models of human abilities and develop throughout the human being's life (see Spatial Test Battery, n.d.; Lohman, 1996). In this sense spatial ability is a unique type of intelligence distinguishable from other forms of intelligence, such as verbal ability, reasoning ability and memory skills (Spatial Test Battery, n.d., p.1; see also Karaman & Yontar Toğrol, 2010). Therefore, it is seen as a measurable trait that a person has, and as a way of characterizing a person's ability to perform mentally such operations as rotation, perspective change and so forth (National Research Council, 2006, p.26). In this regard Newcombe emphasizes that spatial abilities can be typically measured through tests, in which subjects are asked to form accurate mental images of spatial relationships and to change them in some way (Newcombe, 2013, p.3).

Spatial abilities can be tested and observed from different perspectives. Linn and Petersen designated these perspectives as (1) psychometric, (2) differential, (3) cognitive and (4) strategic (1985, p.1480; see also Lüthje, 2010; Grüßing, 2012; Glück et al., 2005; Büchter, 2011; Rost, 1977).

1. **The psychometric perspective** involves the comparison of correlations between different spatial tasks in order to define "factors" in the spatial abilities (Linn & Petersen, 1985, p.1480). In this sense spatial ability is treated as one of the separate sub-capacities of intelligence. To identify the components of intelligence and spatial abilities, different aspects known as "factors" are constituted in pursuance of each test method (Büchter, 2011; Carroll, 1993; Gardner, 1983; Grüßing, 2012; Linn & Petersen, 1985; Lohman, 1979, 1988, 1996; Maier, 1994, 1996, 1999; McGee, 1979; Michael et al., 1957; Obersteiner, 2012; Pinkernell, 2003; Rost, 1977; Souvignier, 2000; Thurstone, 1938; Wachs, 1993; Yilmaz, 2009). These test methods can be performance tests, paper-and-pencil tests, verbal tests, and film or dynamic computer-based tests etc.

   Definitions of all the spatial factors from different researchers are aggregated and categorized as follows (Carroll, 1993; Linn & Petersen, 1985; Lohman, 1979, 1988, 1996; Michael et al., 1957; Thurstone, 1938):

Visualization: is the ability

- to manipulate visual patterns, as indicated by level of difficulty and complexity in visual stimulus material that can be handled successfully,
- to apprehend and identify a visual pattern, knowing in advance what is to be apprehended, when the pattern is disguised or obscured in some way,
- to explore visually a wide and complicated spatial field,
- to detect or manipulate figural stimuli embedded in a “noisy” visual context,
- to match and combine incompleted, disconnected, vague, visual stimuli into a meaningful whole,
- to identify a common object from a series of incomplete pictures presented successively.

Spatial Relations: is the ability

- to recognize the identity of an object when it is seen from different angles,
- to visualize a rigid configuration when it is moved into different position,
- comprehend the nature of the arrangement of elements within a visual stimulus pattern primarily with respect to the examinee’s body as the frame of reference.

Spatial Orientation: is the ability

- to think about those spatial relations in which the bodily orientation of the observer is an essential part of problem,
- to determine spatial relationships with respect to the orientation of their own bodies, in spite of distracting information.

Mental Rotation: is the ability

- to rotate a two- or three-dimensional figure rapidly and accurately,
- to visualize movement or placement among the internal parts of configuration,
- to require mental manipulation of visual objects involving a specified sequence of movements,
- mentally to rotate, turn, twist, or invert one or more objects, or parts, of a configuration (constituting a test item) according to relatively explicit directions as to what the nature and order of manipulations should be,
- to recognize the new position, location or changed appearance of objects that have been moved or modified, within a more or less complex configuration,
- to manipulate relatively simple visual patterns, by whatever means (mental rotation, transformation, or otherwise).

Kinaesthetic Imagery: is the ability

- to make right–left discriminations with respect to the location of the human body, so that the left and right hands seem to use vicariously,
- tentatively to move in response to a simple visual stimulus.

In the present study neither factor test nor factor analysis is applied or needed. Pinkernell (2003) is critical that in psychometric models empirical evidence is missing, although such evidence comprises a large part of mathematics lessons. In this sense, in the present study factors are used only to diagnose the spatial abilities of children during the observation of the interaction processes within the family during play and it does not pursue any further particulars of psychometric perspective.

2. **The strategic perspective** involves the identification of the qualitatively different strategies used to solve a given spatial ability task by different

individuals (Linn & Petersen, 1985, p.1480). Mostly it is based upon age differences and gender perspectives (e.g. Alyman & Peters, 1993; Battista, 1990; Büchter, 2011; Casey et al., 1997; Linn & Petersen, 1985; Lohaus et al., 1999; Lüthje, 2010; Nigl, 1981; Souvignier, 2000).

With respect to the socio-constructivist frame of the present study, the solution strategies of children are considered and discussed within their emergent situational context. These are free play situations and the analytical focus on the interactive process of the negotiation of meaning is the aim rather than "testing" children through a task.

3. **The differential perspective** involves the comparison of spatial abilities for different populations (Linn & Petersen, 1985, p.1480): e.g., the difference between males and females; cultural differences; age differences etc. Regarding test performances in the psychometric tests, many researchers have observed gender differences in their works (Battista, 1990; 1997; Büchter, 2011; Casey et al., 2008; Ellis et al., 2013; Farell, 1957; Grüßing, 2012; Levine et al., 1999; Linn & Petersen, 1985; Lüthje, 2010; Maier, 1999; McGuinness & Morley, 1991; Souvignier, 2000; Terlecki et al., 2008; 1995).

   While gender difference is conspicuous in studies of adolescents and school-age children (Battista, 1990; Büchter 2011; Casey et al., 1995; Ellis et al. 2013; Grüßing 2012; Feng et al., 2007; Linn & Petersen, 1985; Maier 1999,1994; McGuinness & Morley, 1991), for preschool-age children the gender difference is not anticipated (Büchter, 2011; Grüßing, 2012; Levine et al., 1999; Lüthje, 2010). Therefore, in the present study the sex of the children is considered to be undistinguishable in the children of preschool age.

4. **The cognitive perspective** involves the identification of the processes used universally to solve a particular spatial ability task, albeit with quantitatively different efficiency (Linn & Petersen, 1985, p.1480). It revolves around the notation to understand the internal processes of our mind (McLeod 2007, p.1). In this sense it is another process-oriented approach for a theoretical description of spatial abilities, in which mental structure theories are in the

foreground (Lüthje, 2010, p.59). Therefore differential, strategic and psychometric perspectives furnish the basis for the identification of mental processes of children's minds (for more, see Barratt, 1953; Paivio, 1971, 1976; Cooper & Shephard, 1973; Kosslyn & Pomerantz, 1977; Shephard & Cooper, 1982; Carter et al., 1983; Just & Carpenter, 1985; Baddeley, 1986; Lohman et al., 1987; Lohaus et al., 1999; Lohman,1999; Schultz, 1991; Souvignier, 2000; Newcombe & Huttenlocher, 2003; Anderson, 1995; Lüthje, 2010; Newcombe et al., 2013; Newcombe, 2013). So indeed Newcombe, Uttal and Sauter emphasize that human spatial cognition plays a central role in the evolution of our species, in adaptation, and current everyday functioning (Newcombe et al., 2013, p.5). In this sense spatial skills are a key component of human intellect, and hence need to be incorporated in any successful model of the architecture of the human mind (ibid.).

In the present study the social constructivist perspective is embraced. Hence it focuses particularly on children's learning during interactions in the familial context. In this regard the interdependence of social and individual processes in the co-construction of knowledge are issued and discussed. Therefore "the influence of social and cultural factors on the child's cognition" is acknowledged and "the impetus for understanding this influence is reviewed" (Palincsar, 1998, p.345), which avoids going into the detail of the cognitive perspective in this part.

Regarding these four perspectives, the term *spatial* is brought together with different terms by many researchers. Instead of the word "abilities", a review of the literature in this field indicates that a variety of terminology, like spatial sense, spatial insight, spatial imagery, spatial thinking, spatial reasoning, spatial representation are used, which focus on gaining interaction with space (see Bennie & Smit, 1999; Battista, 2007; Büchter, 2011; Clements, 1998, 1999; Clements & Battista, 1992; Clements & Sarama, 2007, 2011; Copley, 2000; Lüthje, 2010; Maier, 1994, 1999; Ministry of Education, 2005; National Council for Curriculum and Assessment, 2014; National Research Council, 2006, 2009; Newcombe, 2013; Newcombe & Huttenlocher, 2003; Newcombe & Stiff, 2012;

Pinkernell, 2003; Rost, 1977; Sperry Smith, 2012; Science Education Resource Centre, 2008; Van den Heuvel-Panhuizen & Buys, 2005; Van der Merwe, 2009). The definition of each term used is clarified as follows:

- **Spatial sense** means “an intuitive feel for one’s surroundings and the objects in them” (NCTM, 1989, p.49). Similarly, Copley defines children’s spatial sense as an awareness of themselves in relation to the people and objects around them (Copley, 2000, p.105). Sperry Smith defines spatial sense differently, as a person’s way of using cues from the environment to orient themself in relation to the world (2012, p.196).

- **Spatial insight** is an important aspect of the intellectual capability (Van den Heuvel-Panhuizen & Buys, 2005, p.117), which refers to the capacity to gain an accurate and deep understanding of spatial elements.

- The US National Research Council defines **spatial thinking** as one of the thinking forms, which is a collection of cognitive skills that consist of declarative and perceptual forms of knowledge and some cognitive operations that can be used to transform, combine, or otherwise operate on this knowledge (2006, p.12). The National Research Council defines spatial thinking as a constructive amalgam of three elements: concepts of space, tools of representation, and processes of reasoning (2006, p.12). Therefore, it is asserted that it uses representations to help people remember, understand, reason, and communicate about the properties of and relations between objects represented in space, whether or not those objects themselves are inherently spatial (National Research Council, 2006, p.26). Similarly, Newcombe claims that spatial thinking concerns the locations of objects, their shapes, their relations to each other, and the paths they take as they move (2013, p.28).

  Clements and Sarama (2014) consider the hierarchical development of children’s spatial thinking and categorize it into two major abilities: “spatial orientation” and “spatial visualization and imagery”. They scrutinize all the spatial abilities and development of spatial thinking of children age by age. Their approach can be regarded as a conjunction of psychometric and differential perspectives in the spatial research field. Moreover, their approach touches upon the cognitive perspective as well. In this regard they literally define “spatial orientation” as follows (Clements & Sarama, 2014, p.124):

> Spatial orientation is knowing where you are and how to get around in the world; that is, understanding relationships between different positions in space, at first with respect to your own position and your movement through it, and eventually from a more abstract perspective including maps and coordinates. This essential competence is not only linked to mathematics knowledge but also how we *remember* things.

Children’s developmental progression about spatial orientation is defined and clarified by age in Table 2.1, which is based on Clements and Sarama (2014, pp.137-139).

Like spatial orientation, Clements and Sarama apply psychometric, differential and cognitive perspectives to define “spatial visualization and imagery”, which they define as follows (Clements & Sarama, 2014, p.127):

> Spatial images are internal representations of objects that appear to be similar to real-world objects. Spatial visualization abilities are processes involved in generating and manipulating mental images of two- and three-dimensional objects, including moving, matching, and combining them. Such visualization might guide the drawing of figures or diagrams on paper or computer screens.

Table 2.1 Developmental progression of children’s spatial orientation

Age (Years)	Developmental Progression – Spatial Orientation
0–2	**Landmark and Path User:** Uses a distant landmark to find an object or location near it, if they have not personally moved relative to the landmark.
	Understands initial vocabulary of spatial relations and location.
2–3	**Local–Self Framework User:** Uses distant landmarks to find objects or locations near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time.

	Orients a horizontal or vertical line in space.
4	**Small Local Framework User:** Locates objects after movement even if target is not specified ahead of time. Searches a small area comprehensively, often using a circular search pattern.
	Extrapolates lines from positions on both areas and determines where they intersect in meaningful contexts.
5	**Local Framework User:** Locates objects after movement (relates several locations separately from own position), maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g. "about halfway" between two landmarks) and keeps track of own location in open areas or mazes. Some use coordinate labels in simple situations.
6	**Map User:** Locates objects using maps with pictorial cues.
	Can extrapolate two coordinates, understanding the integration of them to one position, as well as use coordinate labels in simple situations.
7	**Coordinate Plotter:** Reads and plots coordinates on maps.
8	**Route Map Follower:** Follows a simple route map with more accurate direction and distances.
	Framework User: Uses general frameworks that include the observer and landmarks. May not use precise measurement even when that would be helpful, unless guided to do so.
	Can follow and create maps, even if spatial relations are transformed.

In this regard, the children's development progression about spatial visualization and imagery are defined and illustrated by ages in Table 2.2 which is also based on Clements and Sarama (2014,pp. 139-140).

Table 2.2 Developmental progression of children's spatial visualization and imagery

Age (Years)	Developmental Progression – Spatial Visualization and Imagery
0–3	Understands initial vocabulary of spatial relations and location.
4	**Simple Turner:** Mentally turns object in easy tasks.
	Given a shape with the top marked with colour, correctly identifies which of three shapes it would look like if it were turned "like this" (90-degree turn is demonstrated) before physically moving the shape.

5	**Beginning Slider, Flipper, Turner:** Uses the correct motions but not always accurate in direction and amount.
	Knows a shape has to be flipped to match another shape but flips in the wrong direction.
6	**Slider, Flipper, Turner:** Performs slides and flips, often only horizontal and vertical using manipulative. Performs turns of 45, 90 and 180 degrees.
	Knows a shape must be turned 90 degrees to the right to fit into a puzzle.
7	**Diagonal Mover:** Performs diagonal slides and flips.
	Knows a shape has to be turned flipped over an oblique line (45-degree orientation) to fit into puzzle.
8	Mental Mover: Predicts results of moving shapes using mental images.
	"If you turned these 120 degrees, it would be just like this one."

- **Spatial reasoning** offers ways to interpret and describe physical environments (NCTM, 2000, p.41). According to the NCTM Principles and Standards for Teaching Mathematics, spatial reasoning is constituted of three parts: (1) examining spatial patterns and noting of spatial regularities; (2) supporting spatial statements by showing that they apply in other spatial cases or rejecting spatial statements by providing counterexamples; and (3) explaining spatial reasons by answering the question "why" (NCTM, 2000; see also National Council for Curriculum and Assessment, 2014).

- **Spatial representation** is the way in which space is represented in the brain[22]. Schwartz and Heiser emphasize spatial representation that people construct and transform in their mind's eye (2006, p.283). In this sense it partakes of perceptual processes and experiences (ibid.). The process of working with mental spatial representations is called "imagery" (ibid.).

In respect of all the definitions and information above, spatial ability can be assumed to be a skill which is based on spatial thinking, spatial reasoning and

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[22] Retrieved from
http://www.bcp.psych.ualberta.ca/~mike/Pearl_Street/Dictionary/contents/S/spatial_representation.html

spatial representation and visualization. Spatial sense and spatial insight can be perceived as baselines of spatial ability. Through spatial sense a child is able to notice the surroundings and the objects, and to perceive and learn the environment. Furthermore learning to think spatially develops in an evolutionary process, as step-by-step children experience and explore maps, position words, and opportunities to manipulate shapes into various positions from early childhood years up to the end of school years (Copley, 2000, p.113). Thereby spatial thinking becomes an invaluable lifelong habit of mind (National Research Council, 2006, p.3) and shapes children's school and social lives. In that regard, through spatial abilities a child can think spatially, confront geometric phenomena and solve geometry problems which enable him or her to learn to grasp the surrounding world better (Van den Heuvel-Panhuizen & Buys, 2005, p.115).

Clements and Sarama claim that spatial ability is an essential human ability which contributes to mathematical ability (Clements & Sarama 2007, p.489). Moreover mathematics achievement is related to spatial ability and hence spatial ability plays a major role in learning many topics of mathematics (ibid.). Spatial ability, especially, is the *sine qua non* of being able to understand the mathematics of space, which refers to the mathematical domain of geometry. Van den Heuvel-Panhuizen and Buys indicate that geometry needs to be given a full-fledged position in primary school education, if only to give the child's natural development of spatial insight the greatest opportunity (2005, p.117). In the next section, the mathematical domain of "geometry" in early childhood and primary school education is discussed to show more clearly the relation of spatial abilities with our physical world.

2.1.2. Geometry

Geometry originated in the ancient practice of earth measurement used in agriculture, the building of pyramids, and the observation of the patterns in the movement of the stars used in navigation (Kemeny, 2002; Education Development Center, 2000). Bishop defines geometry as "the mathematics of space" (1983, p.175). Similarly the Education Ministry of Ontario states that geometry is the science of shapes and space (2008, p.14). The word “geometry” comes from two Greek words: *geo* meaning “earth”, and *metron* “measure” (Ministry of Education 2005, p.188; see also Devlin, 2000). Geometry serves as a tool for the study of other topics in mathematics and science (Clements, 1999, p.77; see also Clements & Sarama, 2011a). It refers to human activities, acting and thinking, which are performed by even the youngest of children (Van den Heuvel-Panhuizen & Buys, 2005, p.117). In the literature, geometry is defined as “grasping space ... that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it.” (Freudenthal in NCTM, 1989, p.48; see also Sarama & Clements, 2008, 2009; Clements & Sarama, 2007, 2011).

For most people throughout human history the geometry of the world they experienced was planar Euclidean geometry, which accords extremely well with our everyday experiences (Devlin, 2014). So indeed, in the learning–teaching trajectory description, geometry is intended to be understood as *grasping* the physical world (Van den Heuvel-Panhuizen & Buys, 2005, p.145). Furthermore, Kemeny emphasizes that geometry is the subject area where the development of abstract reasoning began, culminating in the first systematic organization of mathematical knowledge by Euclid around 350 BC (Devlin, 2000; Kemeny, 2002). Contemporary mathematics education includes Euclidean geometry and utilizes Euclid’s deductive system to build definitions, postulates, theorems, and proofs, which have served as the blueprint for representing mathematical knowledge since its inception (Kemeny, 2002).

Euclidean geometry is usually described as a set of objects of three kinds, namely, “points”, “lines” and “planes”; the relations between them are incidence, order (“lying between”), congruence (or the concept of motion), and continuity[23]. In this regard Euclidean geometry comprehensively includes two-dimensional, three-dimensional and high-dimensional spaces (for more, see Devlin, 2000;

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23 Euclidean geometry. Encyclopedia of Mathematics. URL: http://www.encyclopediaof-math.org/index.php?title=Euclidean_geometry&oldid=34034

Müller, 2000; Strehl, 2003; Müller-Philipp & Gorski, 2005; Franke, 2007; Krauter & Bescherer, 2013).

I. **Plane geometry (Euclidean plane geometry)** is the study of geometry based on definitions, undefined terms (point, line and plane)[24] and shapes like circles and triangles, which can be called *elements* (Devlin, 2000, p.79) and drawn on a flat surface, a plane[25] (see Fig. 2.1).

Term	*Graphic*	*Symbol*	*Dimension*
Point	•	• A	Zero
Line		$\overline{AB}$	One
Plane		Plane C	Two

Fig. 2.1. Basic definitions of plane geometry

Devlin defines these elements as a regular polygon, which is a figure made up of equal straight-line edges, each adjacent pair of which meets at exactly the same angle (2000, p.79). Moreover he indicates that the simplest such figure is an equilateral triangle, where the sides are all equal and the angle of each vertex is 60 degrees, and then a square; followed by a regular pentagon (a 108-degree angle between touching edges); a regular hexagon (angles of 120 degrees), etc. (ibid.) (see Fig. 2.2). A regular polygon may have any number of sides (ibid.).

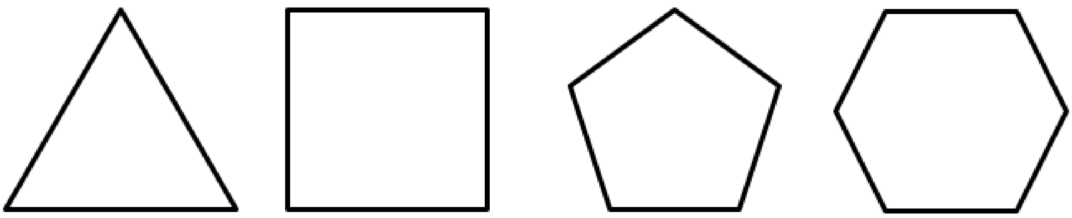


Fig. 2.2 Regular polygons: Some elements of plane geometry (Devlin, 2000, p.79).

II. **Solid geometry (three-dimensional Euclidean geometry)** is the ordinary space of three dimensions, the kind of space we live in or the kind of space

[24] Retrieved from http://www.regentsprep.org/regents/math/geometry/gg1/Euclidean.htm

[25] Retrieved from http://wiki.answers.com/Q/What_is_Euclidean_geometry and http://en.wikipedia.org/wiki/List_of_geometry_topics

which can be imagined in the coordinate system. It is related to stereometry which deals with the measurements of volumes of various solid figures (three-dimensional figures) including pyramids, cylinders, cones, spheres and prisms etc.[26] (see Fig. 2.3).

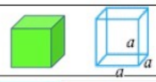
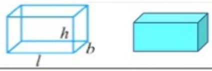
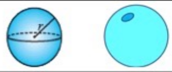
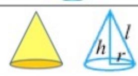
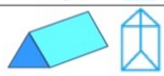
Name of Solid	*Graphic*	*Dimension*
Cube		Three
Cuboid		Three
Cylinder		Three
Sphere		Three
Cone	**(Right circular cone)** 	Three
Pyramid	**(Square based Pyramid)** 	Three
Prism	**(Triangular prism)** 	Three

Fig. 2.3 Various solid figures.

III. n-dimensional Euclidean geometry (high-dimensional Euclidean geometry) is the geometric space with n dimensions. The term “n” identifies the total amount of dimensions in the present geometric space. For example, four-dimensional Euclidean geometry represents four dimensions in Euclidean space, which is much more complex than three-dimensional space.

Euclidean space is metric and norm whose dimensions are treated as regular, same. Higher dimensional euclidean spaces are indistinguishable from others (see Müller, 2000; Devlin, 2000; Strehl, 2003; Müller-Philipp & Gorski,2005; Franke, 2007; Krauter & Bescherer, 2013).

In contemporary primary mathematics education, geometry involves activities which have properties of Euclidean geometry, such as connecting points with line segments and recognizing figures such as triangles, squares and rectangles (Karaman & Yontar Toğrol , 2010, p.1). The geometry of Euclidean space is based on ordinary space in the common elementary geometry, which is taught

[26] Retrieved from https://en.wikipedia.org/wiki/Solid_geometry

in early childhood and primary mathematics education. The core goal of teaching geometry is to develop children's spatial abilities to perceive the environment through their senses, and to learn about the environment and the relationship between objects. In that regard geometry is an attempt to understand space, shape and dimension and is one of the mathematical domains, which also deals with the spatial relationships, properties, movement and location of two-dimensional figures and three-dimensional shapes (Education Development Center, Inc., 2000, p.1; Ministry of Education, 2005, p.188; see also Clements, 2004, 2001, 1998; Clements & Sarama, 2014, 2011a,b, 2009, 2007, 2000, 1999; Copley, 2010, 2000, 1999; Sperry Smith, 2012; Sarama & Clements, 2009a, b, 2008, 2004, 2003, 2002, 2000; Saracho & Spodek, 2008; Van den Heuvel-Panhuizen & Buys, 2005).

Having introduced the basics of geometry, I will next go into the details of learning and teaching geometry in early childhood and primary education. First the trajectory of geometry education and then the learning processes of children in early childhood and primary-school age are discussed in the next section. Section 2.1.4 presents block play, which requires geometrical knowledge and spatial abilities, to explain the purpose of selecting block play as a focal medium for the observation in this study.

2.1.3. Learning and teaching geometry: The connection with spatial abilities

Geometry in early childhood and primary school education involves shape, size, position, direction and movement, and describes and classifies the physical world we live in (Copley, 2000, p.105), which includes two-dimensional (2D) and three-dimensional (3D) space (Cross et al., 2009, p.35). Geometry has different aspects, such as orienting, constructing and operating with shapes and figures, and these aspects involve activities in space and on a plane (Van den Heuvel-Panhuizen & Buys, 2005, p.145). A great deal of geometry knowledge can be imparted to children in the real-world situations of everyday life. They will not be immersed in geometrical understanding automatically, however. During everyday activities in children's lives, their capabilities should be engaged with instructions and experiences fruitful for meeting the challenge of geometry (see also Kemeny, 2002). Therefore, first, a child should be able to recognize and represent geometric shapes and structures in the environment (see 2.1.2), and then effective instructions should be provided during specific activities at home and in educational settings. Moreover, such opportunities provide a context to develop number and other mathematical concepts. Learning numbers and arithmetic benefits from geometrical knowledge and spatial abilities, as reflected in the high correlation between geometrical, arithmetical and mathematical skills in the primary school ages (see Clements & Sarama, 2007; Obersteiner, 2012; Newcombe, 2010, 2013).

The US National Council of Teachers of Mathematics (NCTM) determined principles and standards in school mathematics as a guide for enhancing children's mathematical learning from pre-kindergarten through to grade 8 mathematics (NCTM, 1989, 2000, 2006, 2013). As mentioned in section 1.2, the standards describe different mathematical contents and processes that children should learn, while the principles describe particular features of high-quality mathematics education (NCTM, 2000, p.11).

The NCTM and other research councils and associations, like the NCCA[27], NAEYC[28], NRC[29], PSB[30], CSBE[31]; EDU[32] etc., regard geometry as one of the contents of mathematics, and thematize spatial skills as the baselines of geometry education in preschool years (see NCTM, 1989, 2000, 2006; NCCA, 2014; NRC, 2006, 2000; PSB, 2005; CSBE, 2007; EDU, 2005, 2008). Therefore, in many educational resources it is difficult to find any pure geometrical notions

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[27] National Council for Curriculum and Assessment (United States).

[28] National Association for the Education of Young Children (United States).

[29] National Research Council (United States).

[30] Ontario Ministry of Education.

[31] Connecticut State Board of Education .

[32] Ministry of Education (Ontario).

rather than spatial ones (cf. Battista, 1990, 2007; Bennie & Smit, 1999; Casey et al., 2008; Clements, 2001, 2004; Clements & Battista, 1992; Clements & Sarama, 2000, 2007, 2009, 2011a, 2011b, 2014; Copley, 1999, 2000, 2010; Cross et al., 2009; Van Nes, 2009; Newcombe, 2010, 2013; Newcombe & Huttenlocher, 2003; Newcombe & Stieff, 2012; Newcombe et al., 2013; Nigl, 1981; Oberdorf & Taylor-Cox, 1999; Obersteiner, 2012; Pinkernell, 2003; Rosenstein et al., 1996; Schwartz & Heiser, 2006; Sarama & Clements, 2000, 2002, 2003, 2004, 2008, 2009a,b; Schultz, 1991; Souvignier, 2000; Sperry Smith, 2012; Van den Heuvel-Panhuizen & Buys, 2005; Van der Merwe, 2009). In this sense spatial abilities are regarded as "malleable" (Newcombe, 2010, p.31) in that they can be improved by geometrical activities at home, and in pre-kindergarten, kindergarten and school settings, when appropriately supportive conditions are provided. Geometry in primary education is of use in organizing and ordering all kinds of spatial situations, such as making models, graphs and diagrams (Heuvel-Panhuizen & Buys, 2005, p.119).

The NCTM generates and identifies expectations and standards for geometry from pre-kindergarten through to grade 2 (see Fig. 2.4).

Geometry Standard for Grades Pre-K–2

Instructional programs from prekindergarten through grade 12 should enable all students to—	**Expectations** In prekindergarten through grade 2 all students should–
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	• recognize, name, build, draw, compare, and sort two- and three-dimensional shapes; • describe attributes and parts of two- and three-dimensional shapes; • investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	• describe, name, and interpret relative positions in space and apply ideas about relative position; • describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance; • find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.
Apply transformations and use symmetry to analyze mathematical situations	• recognize and apply slides, flips, and turns; • recognize and create shapes that have symmetry.
Use visualization, spatial reasoning, and geometric modeling to solve problems	• create mental images of geometric shapes using spatial memory and spatial visualization; • recognize and represent shapes from different perspectives; • relate ideas in geometry to ideas in number and measurement; • recognize geometric shapes and structures in the environment and specify their location.

Fig. 2.4 Geometry standards and expectations for pre-kindergarten through to grade 2 (NCTM, 2000, p.96).

Pedagogical and learning strategies are specified for challenging and supporting children to understand geometry well. In this sense the continual improvement of geometry education is attained when children can experience geometry effectively in every setting. Through such standards it is expected that children will actively learn and understand geometric phenomena in order to build on their prior knowledge and spatial abilities (see NCTM, 1989, 2000).

Regarding these standards and principles; in 2006 the NCTM determined improved curriculum focal points for pre-kindergarten through to grade 8 mathematics. Curriculum focal points describe connections which consist of related content, including contexts and material, to ensure continuing development from previous grade levels (Cross et al., 2009, p.123). Similar to the Principles and Standards for School Mathematics (PSSM), it provides guidance on the basis for the descriptions of foundational and achievable mathematics contents from early childhood through to grade 8. Regarding the focus of interest of this study, in the following figure only the mathematical domain "geometry" for pre-kindergarten through to grade 3 is thematized (see Fig. 2.5).

Prekindergarten curriculum the focal points (NCTM, 2006, p.11)
Geometry Identifying shapes and describing spatial relationships Children develop spatial reasoning by working from two perspectives on space as they examine the shapes of objects and inspect their relative positions. They find shapes in their environments and describe them in their own words. They build pictures and designs by combining two- and three-dimensional shapes, and they solve such problems as deciding which piece will fit into a space in a puzzle. They discuss the relative positions of objects with vocabulary such as "above," "below," and "next to."
Kindergarten curriculum the focal points (NCTM, 2006, p.12)
Geometry: Describing shapes and space Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.
Grade 1 curriculum the focal points (NCTM, 2006, p.13)
Geometry: Composing and decomposing geometric shapes Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry.
Grade 3 curriculum the focal points (NCTM, 2006, p.15)
Geometry Describing and analyzing properties of two-dimensional shapes Children describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Children investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.

Fig. 2.5 US NCTM Curriculum focal points for prekindergarten through grade 3 geometry.

The curriculum focal points are clear areas of emphasis, calling for instruction that helps children to learn geometry, giving them a foundation for increasing their spatial understanding as they encounter richer and more challenging mathematics (NCTM, 2006, p.5). With the purpose of specifying the understanding, knowledge and skills of children, the following topics are included in the curriculum focal points: recognizing shape, creating mental images of shapes, discovery of the properties of shape, topological geometry of closed/open curves, motion geometry, early perspective and points of view, lines of symmetry, mapping, using a grid and early coordinate geometry, logo-computer software, measurement, early concepts of angles, area and volume (Sperry Smith, 2012, p.206). Moreover, the primary goal of the early geometry curriculum is to begin the process of developing spatial sense, which helps the child to experience space and shape in appropriate ways (Sperry Smith, 2012, p.210). All the possible contexts for geometry learning are broadly determined in order to allow children to investigate initial spatial understandings, identify and develop relevant supporting spatial skills, and gain experience with varied and interesting applications of the new knowledge (Ontario Ministry of Education, 2005, p.25).

The NCTM standard describes and discusses the mathematics to be learned in detail at each grade band and defines influential PSSM (NCTM, 1989, 2000) and curriculum focal points (NCTM, 2006). However, they do not specify what is to be learned at individual grade levels (Cross et al., 2009, p.122). Therefore Cross et al. determine teaching-learning paths which are based on research that shows that young children generally follow particular paths when learning number-relations operations and geometric measurement (Cross et al., 2009, p.121). These paths consist of significant steps in mathematics learning and each new step in the learning path builds on the earlier steps (ibid.). Regarding the focus of interest of the present study, teaching-learning paths for spatial and geometric thinking in 2D and 3D contexts are determined in Figures 2.5 and 2.6. Therefore, children are grouped by age and the activities in which children should be engaged are outlined (Cross et al., 2008, pp.184-185).

Space and Shape in Two Dimensions

Steps/Ages (Level of Thinking)	**a. Perceive, Say, Describe/ Discuss, and Construct Objects in 2-D Space**	**b. Perceive, Say, Describe/ Discuss, and Construct Spatial Relations in 2-D Space**	**c. Perceive, Say, Describe/ Discuss, and Construct Compositions and Decompositions in 2-D Space**
Step 1 (Ages 2 and 3)			
1. Level: Thinking visually/ holistically	Recognition and informal description (including at least circles, squares, then triangles, rectangles).	Recognize shapes in many different orientations and sizes. Trial-and-error geometric movements (informal, not quantified). • Use relational language, including vertical directionality terms as "up" and "down", referring to a 2-D environment. • Informally recognizes area as filling 2-D space (e.g., "I need more papers to cover this table").	Solve simple puzzles involving things in the world. Create pictures by representing single objects, each with a different shape.
2. Level: Thinking about parts	Shapes by number of sides (starting with restricted cases, e.g., prototypical equilateral triangle, square).		
Step 2 (Age 4)			
1. Level: Thinking visually/ holistically	Recognition and informal description at multiple orientations, sizes and shapes (including circles and half/quarter circles, squares and rectangles, triangles and others <the pattern block rhombus, trapezoids, hexagons regular>)	Recognize shapes (to the left) in many different orientations, sizes and shapes (e.g., "long" and "skinny" rectangles and triangles). • Math shapes by using geometric motions to superimpose them. • Use relational words of proximity, such as "beside," "next to," and "between", referring to a 2-D environment.	
2. Level: Thinking about parts	Describe and name shapes by number of sides (up to the number they can count). Describe and name shapes by number of corners (vertices).	Move shapes using slides, flips, and turns. • Use relational language involving frames of reference, such as "to this side of," "above." • Compare areas by superimposition. For rectangular spaces • Tile a rectangular space with physical tiles (squares, right triangles, and rectangles with unit lengths) and guidance.	Move shapes using slides, flips, and turns to combine shapes to build pictures. For rectangular spaces • Copy a design shown a grid, placing squares onto squared-grid paper.
3. Level: Relating parts and wholes	Sides of same/ different length. • Right vs. nonright angles.	Predict effects of rigid geometric motions.	Combine shapes with intentionality, recognizing them as new shapes. In an "equilateral triangle world," create pattern block blue rhombus, trapezoid, and hexagons from triangles.
Step 3 (Age 5)			
1. Level: Thinking visually/ holistically	Recognition and informal description, varying orientation, sizes, shapes (includes all above, as well as octagons, parallelograms, convex/concave figures).		
2. Level: Thinking about parts	Shapes by number of sides and corners (including new shapes).	Create and record original compositions made using squares, right triangles, and rectangles on grid paper. Extend to equilateral grids and pattern blocks (those with multiples of 60° and 120° angles). • Begin to use relational language of "right" and "left." • Draw a complete covering of a rectangle area. Count squares in rectangular arrays correctly and (increasingly) systematically.	
3. Level: Relating parts and wholes	Measure of sides (simple units), gross comparison of angle sizes.	Compare area using superimposition. • For rectangular regions, draw and count by rows (initially may only count some rows as rows). • Identify and create symmetric figures using motions (e.g., paper folding; also mirrors as reflections).	Composition on grids and in puzzles with systematicity and anticipation, using a variety of shape sets (e.g., pattern blocks; rectangular grids with squares, right triangles, and rectangles, tangrams).

NOTE: Most of the time should be spent on 2-D, about 85 percent (there are many beneficial overlapping activities).

Fig. 2.5 Steps/ages (level of thinking) in two-dimensional space (Cross et al., 2009, pp. 177-179).

Moreover these activities cover a range of difficulty, including *perceive*, *say*, *describe/discuss* and *construct* in order to illustrate how children's engagement with mathematics should build and develop over the prekindergarten years (ibid.).

Space and Shape in Three Dimensions

Steps/Ages (Level of Thinking)	**a. Perceive, Say, Describe/ Discuss, and Construct Objects in 3-D Space**	**b. Perceive, Say, Describe/ Discuss, and Construct Spatial Relations in 3-D Space**	**c. Perceive, Say, Describe/ Discuss, and Construct Compositions and Decompositions in 3-D Space**
Step 1 (Ages 2 and 3)			
1. Level: Thinking visually/ holistically	See and describe pictures of objects of all sorts. (3-D to 2-D). *Research indicates that very young children mainly use shape for object identification. Research says children with lower socioeconomic status have difficulty with describing objectsand need to learn the vocabulary to do so.	Understand and use relational language, including "in," "out," "on," "off," and "under," along with such vertical directionality terms as "up" and "down".	Represent real-world objects with blocks that have a similar shape. • Combine unit blocks by stacking.
2. Level: Thinking about parts	Discriminate between 2-D and 3-D shapes intuitively, marked by accurate matching or naming.		
Step 2 (Age 4)			
1. Level: Thinking visually/ holistically	Describe the difference between 2-D and 3-D shapes, and names common 3-D shapes informally and with mathematical names ("ball"/sphere; "box" or rectangular prism, "rectangular block," or "triangular block"; "can"/cylinder).	Match 3-D shapes. • Uses relational words of proximity, such as "beside," "next to" and "between," "above," "below," "over," and "under."	
2. Level: Thinking about parts	Identify faces of 3-D objects as 2-D shapes and name those shapes. • Uses relational language involving frames of reference such as "in front of," "in back of," "behind," "before."	Identify (matches) the faces of 3-D shapes to (congruent) 2-D shapes, and match faces of congruent 2-D shapes, naming the 2-D shapes. • Represent 2-D and 3-D relationships with objects.	Combine building blocks, using multiple spatial relations.
3. Level: Relating parts and wholes	Informally describe why some blocks "stack well" and others do not.		Compose building blocks to produce composite shapes. Produce arches, enclosures, corners and crosses systematically.
Step 3 (Age 5)			
1. Level: Thinking visually/ holistically	Name common 3-D shapes with mathematical terms (spheres, cylinder, rectangle, prism, pyramid).		
2. Level: Thinking about parts	Begin to use relational language of "right" and "left".	Fill rectangular containers with cubes, filling one layer at a time.	
3. Level: Relating parts and wholes	Describe congruent faces and, in context (e.g., block building), parallel faces of blocks.	Understand and can replicate the perspective of a different viewer.	Substitution of shapes. Build complex structures. Build structures from pictured models.

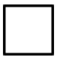
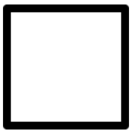
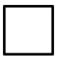
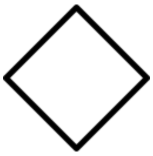
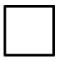
NOTE: Less time on 3-D than on 2-D, about 10 percent of the time on 3-D.

Fig. 2.6 Steps/ages (level of thinking) in three-dimensional space (Cross et al.,2009, pp. 186-187).

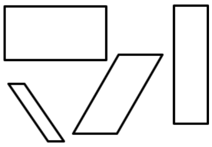
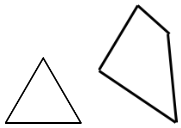
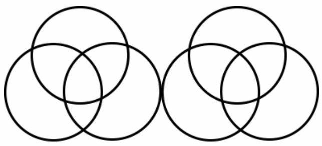
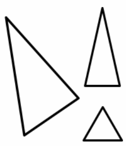
Regarding the NCTM Standards and Principles and other studies, Clements and Sarama (2014) stated the contemporary learning trajectories for childrens' mathematical development; they refer to spatial thinking, shape and how to compose and decompose shapes (2014, pp.124-185). They discuss learning trajectories of geometry at various ages and 2D and 3D shapes.

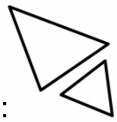
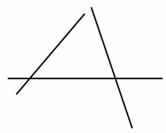
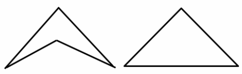
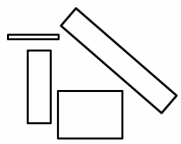
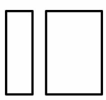
They point out that hierarchical development in children's spatial thinking evolves integrally, and they classify all developmental progress in terms of geometrical phenomena. They illustrate the comprehension of 2D shapes (Clements & Sarama, 2014, pp.157-169) and 3D shapes (Clements & Sarama, 2014, pp.174-175) by the age[33] and developmental progress of the child (Tables 2.3 and 2.4).

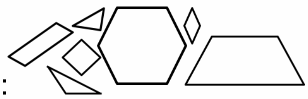
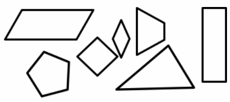
Table 2.3 Child development process on two-dimensional space

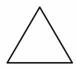
Age (Years)	Developmental Progression – 2D Shapes
0–2	**"Same Thing" Comparer:** *Comparing* Compares real-world objects.
	Says two pictures of houses are the same or different.
	Shape Matcher – Sizes: *Comparing* Matches familiar shapes with *different sizes.*
	Matches  to 
	Shape Matcher – Sizes: *Comparing* Matches familiar shapes with *different sizes.*
	Matches  to 
3	**Shape Recognizer – Typical:** *Classifying* Recognizes and names a typical circle, square, and, less often, triangle. May physically rotate shapes in atypical orientations to mentally match them to a prototype.
	Names this a "square": 
	Some children correctly name different sizes,shapes and orientations of rectangles that look rectangular but not rectangles.

[33] It is important to note that the ages in the first column of Table 2.3 and Table 2.4 are approximate.

	Names the shapes “rectangles” (including the non-rectangular parallelogram): 
	“Similar” Comparer: *Comparing* Judges two shapes the same if they are visually more similar than different.
	“These are the same: They are pointy at the top.” 
3–4	**Shape Matcher – More Shapes:** *Comparing* Matches a wider variety of shapes with *same size* and *orientation*.
	Shape Matcher – More Shapes and Orientations: *Comparing* Matches a wider variety of shapes with different sizes and orientations.
	Matches these shapes: 
	Shape Matcher – Combinations: *Comparing* Matches *combinations* of shapes to each other.
	Matches these shapes: 
4	**Shape Recognizer – Circles, Squares, and Triangles:** *Classifying* Recognizes some less typical squares and triangles and may recognize some rectangles, but usually not rhombuses (diamonds). Often doesn’t differentiate sides/corners.
	Names these as triangles: 
	Part Comparer: *Comparing* Says two shapes are the same after matching one side on each.

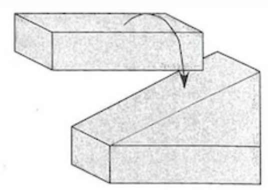
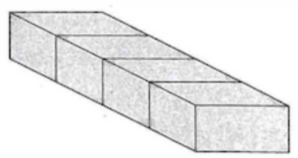
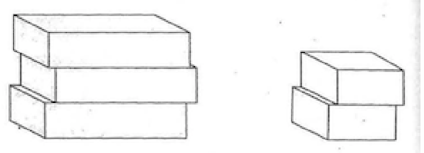
	“These are the same” (matching the two sides): 
	Constructor of Shapes from parts – Looks Like: *Parts* Uses manipulations representing parts of shapes, such as sides, to make a shape that “looks like” a goal shape. May think of angles as a corner (which is “pointy”).
	Asked to make a triangle with sticks, create this: 
	Some Attributes Comparer: *Comparing* Looks for differences in attributes, but may examine only part of shape.
	“These are the same” (indicating the top halves of the shapes are similar): 
4–5	**Shape Recognizer – All Rectangles:** *Classifying* Recognizes more rectangle size, shapes, and orientations of rectangles.
	Correctly names these shapes “rectangles”: 
	Side Recognizer: *Parts* Identifies sides as distinct geometric objects.
	Asked what this shape is , says it is a “quadrilateral” (or has four sides) after counting each, running finger along the length of each side.
	Most Attributes Comparer: *Comparing* Looks for differences in attributes, examining full shapes, but may ignore some spatial relationships.
	“These are the same.” 

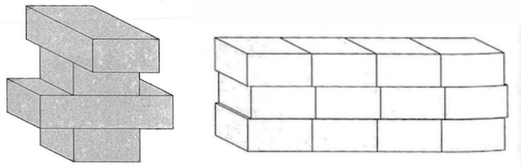
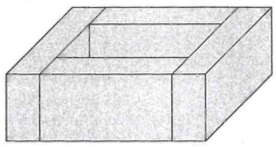
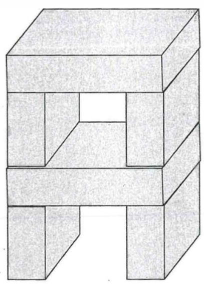
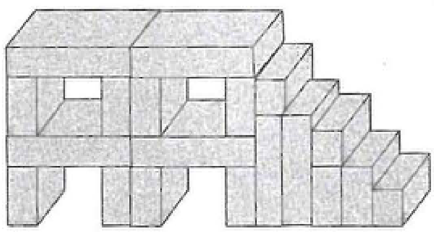
	Corner (Angle) Recognizer: *Parts* Recognizes angles as separate geometric objects, at least in the limited context of “corners.”
	Asked why is this a triangle, says “It has three angles” and counts them, pointing clearly to each vertex (point at the corner).
5	**Shape Recognizer – More Shapes:** *Classifying* Recognizes most familiar shapes and typical examples of other shapes and typical examples of other shapes, such as hexagon, rhombus (diamond) and trapezoid.
	Correctly identifies and names all of these shapes: 
6	**Shape Identifier:** *Classifying* Names most common shapes, including rhombus, without making mistakes such as calling ovals “circles”. Recognizes (at least) right angles,so distinguishes between a rectangle and a parallelogram without right angles.
	Correctly names all of the following shapes: 
7	**Angle Recognizer – More Contexts:** *Parts* Can recognize and describe contexts in which angle knowledge is relevant, including corners (can discuss “sharper” angles), crossings (e.g., a pair of scissors), and, later, bent objects and bends (sometimes bends in paths and slopes). Only later can explicitly understand how angle concepts relate to these contexts (e.g., initially may not think of bends in roads as angles; may not be able to add horizontal or vertical to complete the angle in slope contexts; may even see corners as more or less “sharp” without representing lines that constitute them). Often does not relate these contexts and may represent only some features of angles in each (e.g., oblique line for a ramp in a slope context).
	Parts of Shapes Identifier: *Classifying* Identifies shapes in terms of their components.
	“No matter how skinny it looks, that’s triangle because it has three sides and three angles.” 
	Congruence Determiner: *Comparing* Determines congruence by comparing all attributes and all spatial relationships.

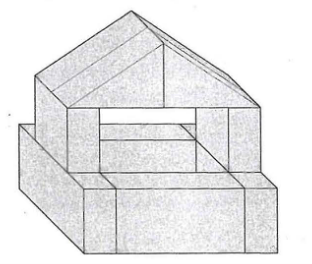
	Says that two shapes are the same shape and the same size after comparing every one of their sides and angles.
	Congruence Superposer: *Comparing* Moves and places objects on top of each other to determine congruence.
	Says that two shapes are the same shape and the same size because they can be laid on top of each other.
	Constructor of Shapes from Parts – Exact: *Representing* Uses manipulative representing parts of shapes, such as sides and angle "connectors", to make a shape that is completely correct, based on knowledge of components and relations.
	Asked to make a triangle with sticks, creates this: 
8+	**Angle Representer:** *Parts* Represents various angle contexts as two lines, explicitly including the reference line (horizontal or vertical for slope; a "line of sight" for turn contexts) and, at least implicitly, the size of the angle as the rotation between lines (may still maintain misconceptions about angle measure, such as relating angle size to length of side's distance between end points, and may not apply these understandings to multiple contexts).
	Congruence Representer: *Comparing* Refers to geometric properties and explains with transformations.
	"These must be 'congruent', because they have equal sides, all square corners, and I can move them on top of each other exactly."
	Parts of Shapes Identifier: *Classifying* Uses class membership (e.g., to sort), not explicitly based on properties.
	"I put the triangles over here, and the quadrilaterals, including squares, rectangles, rhombuses, and trapezoids, over there."
	Shape Property Identifier: *Classifying* Uses properties explicitly. Can see the variants in the changes of state or shape, but maintaining the shapes' properties.
	"I put the shapes with opposite sides parallel over here, and those with four sides but not both pairs of sides parallel over there."
	Property Class Identifier: *Classifying* Uses class membership for shapes (e.g.,to sort or consider shapes "similar") explicitly based on properties, including angle measure. Is aware of restrictions of transformations and also of the definitions and can integrate the two. Sorts hierarchically, based on properties.

	"I put 'equilateral triangles' over here, and 'scalene triangles' over here. The 'isosceles triangles' are all these… they included the equilaterals."
	Angle Synthesizer: *Parts* Combines various meanings of angle (turn, corner, slant), including angle measure.
	"This ramp is at a 45 degree angle to the ground."

Table 2.4 Child development process on three-dimensional space.

Age (Years)	Developmental Progression – 3D Shapes
0–1	**Pre-Composer (3D):** Either places blocks randomly or manipulates shapes as individuals, but does not combine them to compose a larger shape. May pound, clap together, or use slide blocks or single blocks to represent an objects, such as a house or truck.
1	**Stacker:** Shows use of the spatial relationship of "on" to stack blocks, but choice of blocks is unsystematic.
	
1½	**Line Maker:** Shows use of relationships of "next to" to make a line of blocks, which are one dimensional.
	
2	**Same Shape Stacker** (previously, Congruency Stacker): Shows use of relationship of "on" to stack congruent blocks, or those that show a similarly helpful relationship to make stacks or lines.
	
	Piece Assembler (3D): Builds vertical and horizontal components within a building, but within a limited range, such as building a "floor" or a simple "wall." These, then, are two-dimensional structures.

	
3–4	**Picture Maker (3D):** Uses multiple spatial relations, extending in multiple directions and with multiple points of contact among components, showing flexibility in integrating parts of the structure. Produces arches, enclosures, corners and crosses, but may use unsystematic trial and error and simple addition of pieces.
	
4–5	**Shape Composer (3D):** Composes shapes with anticipation, understanding what 3D shape will be produced with a composition of two or more other (simple, familiar) 3D shapes. Can produce arches (with vertical interior space), enclosures (with internal horizontal space), corners, and nosses systematically. Builds enclosures and arches several blocks high. Later in this level, children add depth to make 3D structures, and they add roofs across structures multiple blocks high (but they may have no internal spaces).
	
5–6	**Substitution Composer and Shape Composite Repeater (3D):** Substitutes a composite for a congruent whole. Builds complex bridges with multiple arches, with ramps and stairs at the ends. Structures are 3D, often including roofs and multiple internal spaces.
	

6–8+	**Shape Composer – Units of Units (3D):** Makes complex towers or other structures, involving multiple levels with ceilings (fitting the ceilings), and adult-like structures with blocks, including arches and other substructures.
	

Clements points out that children in the age range of three to eight years are a special group who act with their whole beings while engaging in mathematics (Clements, 2001, p.272). They should be offered sustained and frequent times in which they themselves enact the core of mathematical content and talk about what they are doing and why they are doing it (Cross et al., 2009, p.125). Moreover they possess informal mathematical abilities and enjoy using them, while they are developing their informal mathematical knowledge in everyday life (ibid., p.271). Through activities such as block building, paper folding and the use of geoboards the children are supplied with effective and active geometry practices (see Sperry Smith, 2012). Many relationships between objects in the home, pre-kindergarten and kindergarten settings naturally enable children to engage in informal learning and foster a good beginning for school life (see Sperry Smith, 2012). In mathematics learning, effort creates ability (Cross et al., 2009, p.125) and playing with blocks is one of the ways to enhance the scientific and mathematical abilities and achievements of children. In the next section I explain the importance and benefits of block play and then its relation to spatial skills and learning geometry.

2.1.4. Block play

Block play refers to any activity performed with unadorned wooden forms[34] in space (see Hewitt, 2001). It indicates possible building ideas and performing them with wooden forms, i.e., blocks. Blocks are the most favoured and useful equipment for children in order to make their own repeatable forms, in which they focus on a building action by constructing shapes, forms or any world that they can imagine. Moreover it enables children to learn a diverse range of valuable competencies and knowledge, from social skills to the foundations for later mathematics achievement (Kersh et al., 2008, p.237; Hewitt, 2001). In this sense block play, i.e., block building activity, can be seen as a valuable activity for children to express themselves and the world in which they live, while they are building many wildly imaginative structures (see also Sperry Smith, 2012).

Block play allows children to interact with both science and mathematics content in authentic, meaningful and hands-on ways (Lindeman & Anderson, 2015, p.42). As "constructive workers" (Hewitt, 2001, p.8) children engage in designing, engineering and the arts in the context of the scientific process and logical/mathematical problems while they are playing with blocks (Lindeman & Anderson, 2015, p.42). Through these "hands-on materials" (Anderson, 2010, p.56) children utilize their intuitive and informal capabilities in the block building activity. Hewitt emphasizes that children build three basic forms with blocks: "forms of life" (representing objects from the world: houses, furniture, trees), "forms of knowledge" (giving physical substance to abstract ideas: number and geometry); and "forms of beauty" (creating imaginative designs, mainly based on symmetry, for aesthetic appreciation) (Hewitt, 2001, p.9).

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[34] A wide range of educational block games is available on the market. Whereas some of them elicit free play situations for children, some require children to follow instructions step-by-step (Ferrara et al., 2011). In the current chapter, considering all types of block play, I bring together whole featured thoughts about block play. Therefore I standardize the type of blocks and regard them as unadorned wooden forms.

Most researchers attach great importance to block play, through which children can understand many important concepts in geometry, data analysis, numeracy, patterns, measurement, sorting and sequencing (Tepylo, Moss, & Stephenson, 2015; Lindeman & Anderson, 2015; Acar Bayraktar 2014a,b,c; National Council for Curriculum and Assessment, 2014; Clements & Sarama, 2014, 2011a; Sperry Smith, 2012; Ferrara et al., 2011; Newcombe, 2010; Anderson, 2010; Cross et al., 2009; Van Nes, 2009; Tunks, 2009; Kersh et al., 2008; Casey et al., 2008; Ministry of Education, 2008, 2005; Ginsburg, 2006; Sarama & Clements, 2003; Hewitt, 2001; Copley, 2000; Clements, 1999; Battista, 1998; Leeb-Lundberg, 1996; Rosenstein et al., 1996; Bullock, 1992; Cartwright, 1988; Child Action, n.d.). While children are playing with blocks, they count, classify, sort and match the blocks. Block play enables children to experience and learn mathematical concepts such as "bigger than", "smaller than" or "need more or need less" (Bullock,1992). Rosenstein Caldwell, and Crown emphasize that through block play children become able to

1. investigate and predict the results of combining, subdividing and changing shapes,
2. use tessellations to explore properties of geometric shapes and their relationships to the concepts of area and perimeter,
3. explore geometric transformations such as rotations (turns), reflections (flips) and translations (slides),
4. understand the variety of ways in which geometric shapes and objects can be measured,
5. explore, understand and apply the concepts of symmetry, similarity and congruence,
6. develop, understand and apply a variety of strategies for determining perimeter, area, surface area, angle measurement and volume,

7. investigate, explore and describe the geometry in nature and real-world applications, using models, manipulations, and appropriate technology (Rosenstein et al., 1996, pp. 215-223).

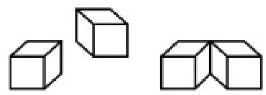
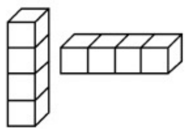
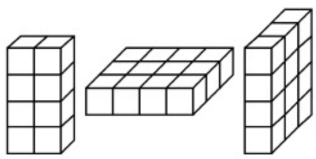
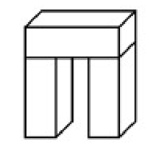
Block play also contributes to children's physical growth in addition to their cognitive growth (Bullock, 1992; Tunks, 2009). They gain a wide variety of learning opportunities to improve their spatial skills and motor skills, to understand the world and to think mathematically, while they are discovering and establishing equivalencies in length, height, weight, area and volume during block play (see Ferrara et al., 2011; Sarama & Clements, 2003; Casey et al., 2008; Hewitt, 2001; Bullock, 1992). While combining blocks to compose/decompose a structure or a building, children rotate, orient and classify blocks and explore their spatial relationships. By moving, lifting, carrying, bending, reaching, pushing and pulling they foster motor skills and coordination of muscles (Bullock, 1992, p.16).

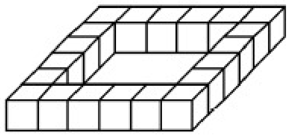
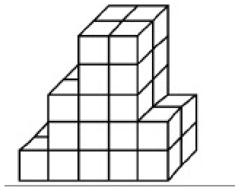
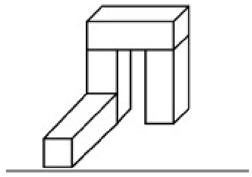
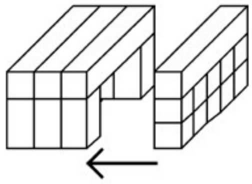
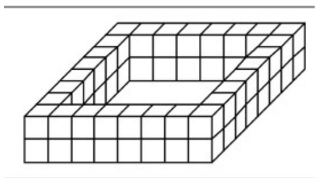
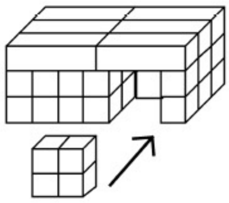
Through reaching, grasping, balancing, stacking and moving blocks children learn hand–eye coordination and the sense of balance and symmetry (Bullock, 1992, p.16). Block play can serve as a resource for developing spatial visualization and imagination as well (Casey et al. 2008, p.304; see also Tepylo, Moss, & Stephenson, 2015; Ferrara et al., 2011). Moreover, it can increase use of "spatial language" (Newcombe, 2010, p.34; see also Tepylo, Moss, & Stephenson, 2015; Ferrara et al., 2011; Tunks, 2009) when children are exposed to appropriate instructions about spatial and geometrical issues. They learn the names of geometric block shapes, debate verbally with their peers, and explain how their structures are built (Tunks, 2009, p.3; see also Stroud, 1995). Similarly, Ferrara et al. point out that, especially in the context of guided play, interaction with blocks naturally elicits elevated levels of spatial language (2011, p.143).

Children learn to master the physical environment and their own bodies, integratively and constructively, while they explore the structural properties of blocks during play (Casey et al., 2008). They use informal skills and ideas relating to number, shape and pattern as they play with blocks (Ginsburg, 2006, p.145).

While they incorporate the spatial dimensionality of blocks and structures, children learn part-whole relationships and understand basic concepts of block building (ibid.). They engage in ideas of shape, space, pattern, spatial and object-space relationships, gravity, size, distance, proportion, force and physical properties (see Lindeman & Anderson, 2015; Anderson, 2010; Leeb-Lundberg, 1996; Ginsburg, 2006). Furthermore, they become able to combine several basic block forms into one structure so that they can exhibit increasing hierarchical integration (Casey et al., 2008, p.275). Namely children begin to understand part-whole relationships and build on this understanding so that they become able to create more complex structures by using one or more blocks and integrating, or tying, or combining them with other blocks (ibid.). In this sense they begin to produce hierarchically integrated structures of greater complexity (ibid.) and come to play with mathematics directly (Ginsburg, 2006, p.155). So indeed, the previous chapter in the present study (see 2.1.3.) fortifies this idea and shows each step of children's hierarchical integration into block building activities (see Table 2.4). Similar to Clements and Sarama (2014), Casey et al. illustrate the hierarchical integration levels of block building as follows (Table 2.5).

Table 2.5 Levels of block building (Casey et al., 2008, pp.287-288)

Level	Description	Example
0	Random block placement	
1	1D structures – row of single blocks, or stack of single blocks	
2	2D structures (no internal space) – structure with no width (a wall), no height (a floor), or no length (a two-block-wide tower)	
3	2D structures with vertical internal space – arches	

4	2D structures with horizontal internal space - enclosure only one block high (no height)	
5	3D structures – 3D piles with no internal space	
6	3D structures – 2D vertical or horizontal internal space plus depth to make a 3D structure: arch +1 or more blocks placed in front or behind, or 2 separate walls, 2 blocks high + 1 or more blocks connecting the 2 walls	
7	3D horizontal enclosure: one-block-high enclosure (or partial enclosure)+ layer of roof blocks – adds height to make a 3D structure. irregular one-block-high enclosure with roof-gaps/sloops	
8	3D horizontal enclosure: two blocks high irregular two-block high enclosure-gaps/sloppy	
9	3D horizontal enclosure: two blocks high+ roof+ divided internal space	

Through block play children can connect their informal knowledge to more formal school mathematics and reach “solid content knowledge and develop higher-order thinking”, which leads to mathematical achievement in formal education (Sarama & Clements, 2003, p.484, see also Acar Bayraktar 2014a,b,c; Anderson, 2010; Battista, 1998; Casey et al., 2008; Clements, 1999; Clements & Sarama, 2014, 2011a; Copley, 2000; Cross et al., 2009; Ferrara et al., 2011; Ginsburg, 2006; Hewitt, 2001; Kersh et al., 2008; Lindeman & Anderson, 2015; Ministry of Education, 2008, 2005; National Council for Curriculum and

Assessment, 2014; Newcombe, 2010; Sarama & Clements, 2003; Sperry Smith, 2012; Tepylo, Moss, & Stephenson, 2015; Tunks, 2009; Van Nes, 2009).

While children are building, rebuilding, balancing, linking, rotating, sorting, orienting and knocking down any structure, they experience problem solving, logical thinking, cooperating and utilizing many different strategies with their peers reliably and authentically (Lindeman & Anderson, 2015; Christenson & James, 2015; National Council for Curriculum and Assessment, 2014; Tunks, 2009; Child Action, n.d.; Ministry of Education, 2005). They experience taking turns, sharing and respecting the rights of others, have knowledge of various roles and skills, and learn to cooperate and play together, while exploring, matching and classifying the sizes, shapes, distances and proportions in block play (Bullock, 1992; Bayraktar, 2014b). Namely, in addition to physical activity, they become engaged in another's world and thinking while shaping up their own thinking (Battista, 1998). Moreover, block play enables children to learn patience, and contributes to a sense of accomplishment (Bullock, 1992, p.16). Children perceive block play as "experientially real" (Van Nes, 2009, p.29) in that they can stimulate their personal strategies and motivate each other during block building activities. In this sense block play facilitates increasing independence, confidence and self-esteem, from experimenting with a variety of roles and skills and feeling a sense of success (Bullock, 1992, p.16). It stimulates the imagination, creativity and joy of children so that they can also experience interpersonal relationships, cooperating and playing together (ibid.). Block play thus enhances children's social skills and contributes to children's social and emotional growth in addition to physical and cognitive growth (Tepylo, Moss, & Stephenson, 2015; Tunks, 2009; Bullock, 1992).

In conclusion, block play eventuates in perfect learning situations, which have a rich potential and a full range of mathematical activities (Sarama & Clements, 2003, p.484). It fuses playful learning and geometry education and is "an excellent instrumental in promoting learning in a content-rich, developmentally appropriate preschool" (Christenson & James, 2015, p.28). However, the potential of block play is generally disregarded, and blocks are rarely seen in classrooms beyond kindergarten (Hewitt, 2001, p.12; see also Sperry Smith, 2012, Ferrara et al., 2011). Teachers, principals, parent organizations, and school boards should view block building as an essential part of the curriculum (Sperry Smith, 2012, p.201). Tepylo and colleagues highlight that the more children play with blocks, the more sophisticated they become as builders (Tepylo, Moss, & Stephenson, 2015, p.19). Regarding this, Lindeman and Anderson point out that during block play children need to come back to their block structures and creations again and again to improve on and expand their skills (2015, p.39). Children's experiences with their families during block building activities are of crucial importance in the children's growth. Moreover, naturalistic interactions between parents and children can build a foundation for important spatial concepts (Ferrara et al., 2011, p.150) and cultivate children's spatial thinking and geometry learning. Considering this idea, in the current study I observe children's block play with their parents and other relatives in order to examine the cultivation of parental and familial interactions during block building activities with children. Thus, in the next section the issue of "learning through interaction processes" is discussed in detail.

2.2. Interactional Niche in the Development of Mathematical Thinking

The central research purpose of this work is to examine the relationship between the participation of children and family members in play situations and to find out how they interact with each other and how individual content-related learning occurs. In this regard, the concept of "interactional niche in the development of mathematical thinking in the familial context" (NMT-Family) (Acar & Krummheuer, 2011) is used.

The interactional niche in the development of mathematical thinking is particularly based on symbolic interactionism (Blumer, 1969), the cultural historical approach of Vygotksky and Leont'ev, (see Wertsch & Tulviste, 1992; Ernest, 2010; Bruner, 1996) and the "phenomenological sociology" of Alfred Schütz (Schütz &

Luckmann, 1979) and its expansion into ethnomethodology (Garfinkel, 1972) (Krummheuer, 2012, p.321; see also Krummheuer, 2013a, 2011a,c).

Before I go into further detail about the present study, first I would like to clarify briefly each of these approaches:

- **Symbolic interactionism** (Blumer, 1969; 1973, 1975, 1986) goes through the concepts of joint action and acting unit to describe the interactions that extend from dyads to complex institutions (Denzin, 2008, p.5; see also Charon, 2004). Its central tenet is that meaning develops out of interaction and interpretation (Yackel, 1995, p.132). Human beings act towards things on the basis of the meanings that the things have for them (Blumer, 1986, p.2). People should be accepted as social and thinking beings whose actions cannot be understood without focusing on social interaction (Charon, 2004, p.31). The meaning of a thing for each human being arises out of the social interaction and each meaning grows out of the ways in which other human beings act towards the human being with regard to the thing (Blumer, 1969, p.4). In this sense, meanings are social products which are constructed and modified through an interpretative process of human beings when they encounter and deal with things (Blumer, 1986, p.2). Additionally the meanings are regarded as creations in that they contain the cause of human actions which are the result of what is occurring in the present situation of human beings (Blumer, 1969, p.4; Charon, 2004, p.31). Thus human beings develop their understanding while they are interacting with each other and learning occurs through negotiating with meanings. Symbolic interactionism tends to focus on the micro level, which can also vary from researcher to researcher. As Voigt (1996) points out, the symbolic interactionist approach is very useful, it emphasizes the individual's sense-making processes as well as social processes, to examine and to deduce individuals' learning in detail.
- **The phenomenological sociology of Alfred Schütz (Schütz & Luckmann, 1979) and its expansion into ethnomethodology (Garfinkel, 1972)**

Phenomenological sociology deals with different fields and types of experiences and realizations in everyday life or in the everyday "life-world" (Lebenswelt) which is an intersubjective and social world (Schütz & Luckmann, 1974, p.16). The everyday life-world refers to the individual's subjectively experienced world and represents "the dynamic result of constitutive acts of interpreting experiences and worlds on the part of individuals" (Witte, 2014, p.212). In this matter phenomenological social science examines how people make sense of the social world (Packer, 2011, p.155). Moreover its aim is to explain "the thought-objects constructed by common sense" in terms of "the mental constructs or thought-objects of science" (Schütz, 1970, p.272; in Packer, 2011, p.157). Therefore, it deals with implicit, taken-for-granted forms of knowledge that are widely accepted as everyday "common sense" (Johnson, 2008, p.137). In this approach each shared knowledge is unique, special and personal for each individual, but the knowledge of an individual can exist in another individual too, when these individuals are exposed to the same facts. Thus, some human beings know some things, whereas some human beings know other things. But each piece of knowledge can be transmitted from one human being to another through social interactions, which makes knowledge socially distributive. Therefore, the knowledge of the human being is structural, distributive and social in genesis (Schütz, 1962, p.11; see also Heap & Roth, 1973). From that point of view, each human being is at the centre of his/her own environment, which is structured in "strata" or "layers" around the individual (Overgaard & Zahavi, 2009, p.9).

Ethnomethodology is literally defined as the study of the practical activities or methods of an *ethnos* (folk) (Loseke, 1999, p.189; see also Packer, 2011). It concerns how individuals construct meaning or "definitions of the situation", which emerge from how individuals announce and impart sense-making perceptions and perspectives to one another (Maynard & Clayman, 1991, p.386). The theoretical proposal of ethnomethodology is "that there is a self-generating order in concrete activities, an order whose scientific appreciation depends upon neither prior description, nor empirical generalization, nor formal specification of variable elements and their analytic relations" (Maynard & Kardash, 2006, p.1483). It focuses attention on the importance of implicit practical knowledge which enables individuals to make sense of one another's actions without extensive verbal explanation (Johnson, 2008, p.138). Moreover this perspective makes it possible to investigate objectively individuals' achievements, practices and concerted behaviours, which are embedded in their everyday lives (Maynard & Kardash, 1991). Researchers can concretize individuals' "definitions", bring them together and interpret repeated patterns of behaviour in relation to the context (Hung et al., 2012, p.8). In this sense ethnomethodology is "individualistic" and unstable (Maynard & Clayman, 1991, p.411).

Garfinkel inverted the phenomenological primacy accorded to subjective experience in favour of studying public activities and common practices through which members achieve the apparent reality of those objects (Maynard, 1986, p.348; in Packer, 2011, p.191). In this manner, the ethnomethodology of Garfinkel (1967) is concerned with how social agents "construct meaning or definitions of the situation" (Maynard & Clayman, 1991, p.386) and how they "structure their social environment in a meaningful way" (Overgaard & Zahavi, 2009, p.17). Therefore, it is "the process itself as a phenomenon for investigation" (Wilson, 1970, p.78) and is strongly influenced by phenomenological sociology. It documents methods and practices through which individuals make sense of their world (Yakkaldevi, 2013). From this point of view it deals with sociological and linguistic courses. Garfinkel defines it as "doing, recognizing and using ethnographies" (Garfinkel, 1967, p.10). Thus it examines the "everyday methods that people use for the production of social order" (Garfinkel, 2002; in Yakkaldevi, 2013, p.46). In this manner it thematizes social structures and how individuals set up interpretations and opinions that they have. Here social structures are products of social interaction, through which individuals give out and pass on sense-making perceptions to one another. According to Garfinkel, all reactions – from confusion to anger – are illustrations of the fragility of the social order, which is an order that individuals themselves help to produce, but which they nevertheless tend to take for granted (Garfinkel, 1967, pp. 42-43). In this sense, ethnomethodology has special techniques to divulge the everyday practices that human beings engage in while setting up a social order. Johnson labels this perspective the "social construction of reality", which deals more explicitly with how the larger institutional structures of society are grounded in the routine practices and interaction patterns[35] through which individuals' subjective consciousness is formed (2008, p.138). Ethnomethodology discusses "the interpretive process itself as a phenomenon for investigation" (Wilson, 1970, p.78), whereas phenomenological sociology searches for "interpretation for social action" (Heap & Roth, 1991, p.358). Ethnomethodology tries to understand how individuals, as social agents, manage meanings, which are described and explained in the social reality of their lives, whereas phenomenological sociology examines how individuals experience their own social reality in their common lives (Overgaard & Zahavi, 2009). In like manner, Heap and Roth describe the relationship between phenomenological sociology and ethnomethodology successively as follows:

> For phenomenology objects in the "real" world are reduced to objects of immediate consciousness and are seen as constituted in and through intentional acts of consciousness. For ethnomethodology, the "objective" features of the social world are reduced to the interpretative procedures by which that world is assembled and accomplished in concrete, ongoing, social situations. For phenomenology, the foundational nexus of meaning in the world is immediate consciousness; for ethnomethodology, the foundational nexus of meaning in the social world is the immediately

[35] Interaction patterns emerge in interactions and are a kind of routines or structures which can be reconstructed by dint of a detailed analysis of the interaction processes. These routines contain implicit interlocutors, rather unconscious rules which determine the process of interaction. The benefit of those patterns is to stabilize the progress of the mathematical interaction and to guarantee the functionality. Mutual coordination of the interlocutors can be realized. Interaction patterns emerge in these interactions between interlocutors (Huth, 2014, p.150)

present, directly observed social situation … The domain of phenomenological inquiry consists solely of the recognizable structures of immediate consciousness; while the domain of ethnomethodological inquiry consists solely of members' situated practices which produce for themselves and for observers the sense of objective social structures. (Heap & Roth, 1973, pp.363-365).

- **Cultural historical approach** (see Wertsch & Tulviste, 1992; Ernest, 2010). Krummheuer defines this approach as that "which takes culture as a given that the child adapts to by its development; an important issue hereby is the notion of language that stores and transmits the cultural accomplishments in a symbolic form allowing the child to enter into this culture, step by step, finally becoming a full participant" (2012, pp.321-322). According to Krummheuer (2012), culture is a course of action which emerges continuously and locally during mutual exchanges of meanings in the interaction between human beings. In this sense the child is affected by his or her integration in the culture (Brandt, 2013, p.233), while his or her individual developmental progression is fulfilled. Observing and imitating the actions of other individuals in the course of interaction, children take an active role as they explore their cultural environment and co-construct it. Regarding the interactional perspective, Krummheuer defines this as "leeway of participation" (org. "Partizipationsspielraum"; Brandt, 2004) (see section 2.2.4.), which means room for freedom of action (Krummheuer, 2012, p.322). Moreover, Krummheuer demonstrates that the child individually utilizes the leeway of participation which is interactively accomplished and to be understood as a result of the culture the participants share (ibid.).

Regarding these three theoretical approaches, Krummheuer developed the concept of "interactional niche in the development of mathematical thinking" (Krummheuer, 2011), using the theoretical framework "developmental niche" of Super and Harkness (1986). The term "developmental niche" is a "theoretical framework of studying cultural regulation of the micro-environment of the child, and it attempts to describe the environment from the point of view of the child in order to understand processes of development and acquisition of culture" (Super & Harkness, 1986, p.552). It is generated as a juncture of cultural anthropology and developmental psychology[36]. The authors introduce three major subsystems of a developmental niche, which operate together and share the common function of mediating the individual's developmental experience within the larger culture: (1) the physical and social settings in which the child lives, (2) culturally regulated customs of childcare and rearing, and (3) the psychology of the caretakers (Super and Harkness 1986, p.552) (see Fig. 2.6). In this model the child is established as the central object of "particular set of inherited dispositions" (Harkness et al., 2007, p.34).

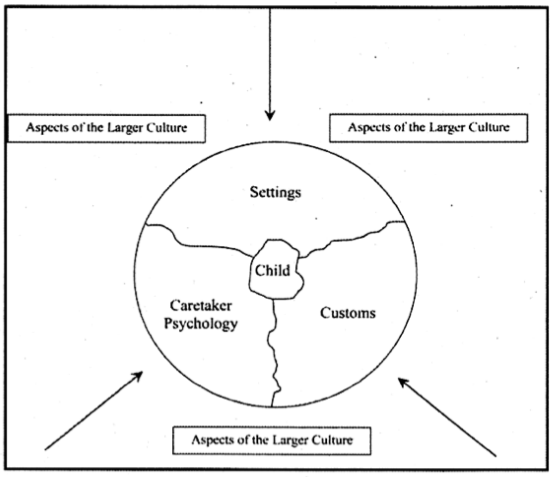


Fig. 2.6. The "developmental niche" (Super & Harkness, 1986, p.552).

Nevertheless, these three components of the developmental niche lack focus on the situational aspects of social interaction processes. Although they form the cultural context of child development (Super & Harkness 1986, p.552), local productions of social interaction processes of mathematics education in children's development are not taken into account. Krummheuer (2012, 2011c,d, 2013) worked on this theoretical concept, modified it to mathematical development, and named it the "interactional niche in the development of mathematical thinking" (NMT). He merged the categories "customs" and "caretaker psychology" to the component "pedagogy and education", redefined

[36] For more, see Super & Harkness, 2002, 1994, 1986.

the category “settings” as “cooperation” and added the new component “content”. Hence NMT consists of “the provided *learning offerings* of a group or society, which are specific to their culture and will be categorized as aspects of *allocation*, and of situationally emerging performance occurring in the process of meaning negotiation, which will be subsumed under the aspect of the situation” (Krummheuer, 2012, p.323; see also Krummheuer, 2011c,d, Acar Bayraktar, in press-c). Through the allocational aspect, the activeness and the emergence of interaction during play can be examined in a chosen mathematical domain. Through the situational aspect, it how players participate and what they perform during interactive negotiations in the play situation can also be examined. So that this concept enables us to combine these three novel components with both of the mentioned aspects (Krummheuer, 2012, 2011c,d; see also Acar Bayraktar, in press-c).

Krummheuer later worked with Markus Schütte (2014) on the situational and allocation aspects of NMT and associated them with a third aspect – the child’s contribution (Krummheuer, 2014; Krummheuer & Schütte, in press, 2014; see also Acar Bayraktar, in press-c). In this improved version of NMT, the aspects of allocation and situation are kept constant and defined the same as before: allocation refers to the learning offerings provided by a group or a society, which specifically highlight cultural representations; and situation consists of the emerging performance occurring within the process of negotiating meaning. In addition to these, the aspect of the child’s contribution is defined as the situational and individual contribution of the particular child. In this regard, NMT is structured as follows (Fig. 2.7):

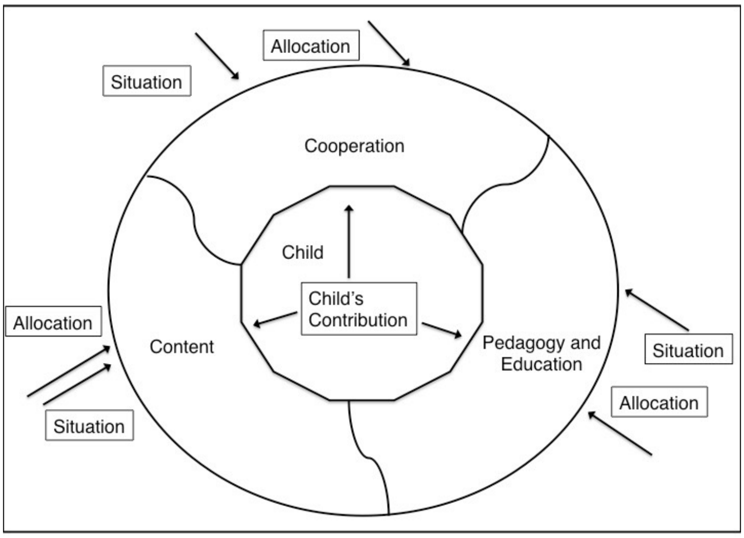


Fig. 2.7. The "interactional niche in the development of mathematical thinking" (NMT).

In terms of the development of mathematical thinking, one should also consider a time axis for the progress of interactional niche in the development of mathematical thinking. The temporal assumption shows the specific characteristics of NMT, which constituent parts are relevant to that time, and over time these characteristics can change with development of the child. However, some points about the way and manner of this development of the child still remain for further research. In this regard the following diagram (Fig. 2.8) shows the relationship between NMT and the time axis, which provides evidence for the child's further development.

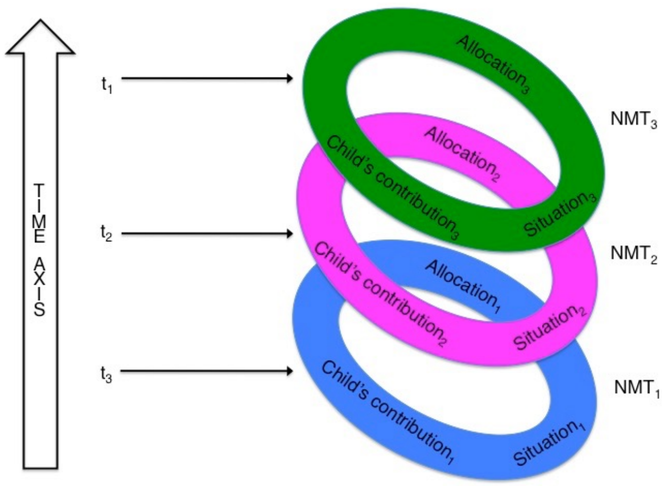


Fig. 2.8. Time axis of NMT

This modification allows a combination of each of these novel components with the three different aspects. Moreover it includes the advantage of a closer analysis of the relationship between mathematical learning occasions with those which take place in preschool, kindergarten and/or primary mathematics classes. Moreover it makes it possible to analyse the learning offerings, while analysing the mathematical situations according to the emerging performances of participants and the child's contribution. Thus, this framework enables us to observe and examine child development clearly through social interaction processes in mathematical situations. Moreover, this concept answers the question, "How can the situationally emerging form of participation of a child in a social encounter be conceptualized as a moment in the child's development in mathematical thinking?" (Krummheuer, 2014, p.72). In this sense the interactional perspective on mathematics learning features in the concept of NMT.

In the next section I discuss the interactional perspective on mathematics learning in detail. Thereupon, I explain the family systems theory and regulation theories. Thereafter I come to the theoretical concept of NMT-Family, which constitutes the main structure of the present study.

2.2.1. Interaction theory in mathematics teaching and learning

> Interaktion ist nicht Medium, sondern Konstituente des Lernens
> (Krummheuer, 1992, p.34).

The interactional theory of mathematics teaching and learning is "an approach of mathematics education to understand the learning of mathematics as highly socially constituted in interaction" (Huth, 2014, p.148) and aims "at understanding these individual meanings, as well as their interactive generation- and clarification-processes" (Krummheuer, 2002, p.341). It emanates from socio-constructivist principles and is based on ethnomethodology (Garfinkel, 1967), symbolic interactionism (Blumer, 1969) and cultural psychology (Bruner, 1996). The theoretical principles of ethnomethodology (Garfinkel, 1967) and symbolic interactionism (Blumer, 1969) were defined in the introduction to this section

(2.2), but cultural psychology (Bruner, 1996) was not discussed. Therefore before I go into any further details about the interactionist perspective, I want to describe briefly how it functions.

2.2.1.1. Cultural psychology (Bruner, 1990, 1996)

Cultural psychology is a term coined by Jerome Bruner in order to define social components of cognitive processes from the psychological perspective (1990, 1996). Bruner defines the cultural approach as "that the child only gradually comes to appreciate that she is acting not directly on the world but on beliefs she holds about that world" (1996, p.49). Each individual development must be expressible in the particular symbolic system of a given culture, and members of a culture should employ specific culturally accomplished ways to interpret the psychological disposition of individuals (Krummheuer, 1999, p.333, citing Bruner, 1986, p.35). In this regard Bruner's cultural psychology involves two perspectives together: folk psychology and folk pedagogy.

Folk psychology is "a theory about mental phenomena that common folk allegedly hold, a theory in terms of which mental concepts are understood" (Goldman, 1993, p.16). Bruner defines it as a reflection of "certain, 'wired-in' human tendencies (like seeing people normally as operating under their own control)" and concomitantly as a reflection of "some deeply ingrained cultural beliefs about 'the mind'" (Bruner 1996, pp.35-45). It denotes "the interpretations of human beings' experiences, the constructions of their understandings and anticipations one an others" (Voigt, 1999, pp.358).

Folk pedagogy is explicitly learning process of human beings, which is crucial for teaching and learning (Voigt, 1999, pp.358). It is based on the idea that it is possible to help human beings to learn (Voigt, 1999, p.360). Bruner indicates:

> One way of presenting the general matter of folk psychology and folk pedagogy most starkly is by contrasting our own human species with non-human primates. In our species, children show an astonishingly strong "predisposition to culture" they are sensitive to and eager to adopt the folkways they see around them. They show a striking interest in the activity

> of their parents and peers and with no prompting at all try to imitate what they observe … Folk pedagogies, for example, reflect a variety of assumptions about children: they may be seen as willful and needing correction; as innocent and to be protected from a vulgar society; as needing skills to be developed only through practice; as empty vessels to be filled with knowledge that only adults can provide; as egocentric and in need of socialization. (Bruner, 1996, p.47-49).

Strauss exemplifies cultural psychology with classroom mathematics: “teachers’ folk psychology and folk pedagogy can be viewed as a baseline understanding of the mind and its workings, and teachers’ new knowledge that goes beyond their folk psychology and pedagogy can be seen as professional knowledge that is not shared by non-teachers” (2001, p.242).

2.2.1.2. Interactionist perspective

The interactionist perspective on teaching and learning mathematics refers to a specific domain of sociology which is epistemologically compatible with the constructivist psychological perspective (Voigt, 1994, p.276). Thus, it focuses empirically on mathematics teaching and learning situations in typical elementary classes and specifically uses micro sociology as a basis. For researchers of mathematics education, this approach enables social aspects to be considered, while avoiding the danger of overemphasizing the cultural aspects (Voigt, 1995, p.166).

Bauersfeld, Krummheuer and Voigt modified the sociological concepts in order to deal with the specifics of teaching and learning mathematics (Bauersfeld, 1980; Bauersfeld, Krummheuer, & Voigt, 1988; Krummheuer, 1983; Voigt, 1984) and focused their attention on the negotiation of mathematical meanings in the local events of classroom life (Voigt, 1995, p.166). By and by their working groups also focused on classroom situations comprehensively from the interactionist perspective (Krummheuer, 1992, 1995, 1997, 2000a,b,c, 2002, 2007a,b, 2011c, 2012; Brandt, 2002, 2004, 2007; Brandt, Fetzer & Schütte, 2010; Jungwirth & Krummheuer, 2006; Fetzer, 2007; Krummheuer & Brandt, 2001; Krummheuer &

Voigt, 1991; Schreiber; 2010; Schütte, 2009; Voigt 1994, 1995 etc.). They drew attention to the “fundamental learning steps” which are facilitated by individuals during social interaction processes in primary school mathematics classrooms (Schütte & Krummheuer, 2012, pp.358-359). Moreover, they exhaustively studied “the cognitive aspect of content-related learning processes from a sociological perspective” (Krummheuer, 2000c, p.22). For this, they focused on individuals’ sense-making processes and on the ways in which they interactively constitute and stabilize mathematical meanings (Voigt, 1995, p.166). Therefore, their approach makes it possible to scrutinize all the features, occurrences and steps of individuals in the interaction processes of mathematics in the classroom. From this point of view, this approach illuminates my research methodology in order to observe and to examine the relationship between the participation of children and family members in play interactions and individual content-related learning.

In the interactionist approach, mathematical teaching and learning situations in a classroom culture are regarded as a “process of interaction” (Krummheuer, 2000c, p.22). One appreciates that mathematical discourses are highly socially structured by perceiving the local classroom as a micro-culture and establishing the mathematical society as the primary point of reference of negotiation (Voigt, 1994). The interactionist approach considers that students and teachers constitute social norms and mathematical practices in the matter of interaction processes in mathematics classrooms (Cobb & Bauersfeld, 1995). From this point of view, teachers and students bring forth mathematical culture by joining the process of meanings, which mediates between cognition and culture (Bauersfeld, Krummheuer & Voigt, 1988). Bauersfeld (1980) calls this process the “everyday mathematics classroom situation”, which refers to “mutually referring actions of the participants of a social event in mathematics classroom” (Krummheuer, 2007b, p.61). In such situations students and teachers come up with their senses and purposes. They interpret the ongoing classroom situation in the course of “sensible and tenable” events and “develop their content-related understandings”, while they are participating in the joint creation of the interactions (Krummheuer, 2000c, p.22). Such situations include high and regular potential for change, which enable different learning opportunities for each individual.

From the interactionist approach, in usual classroom lessons each student and teacher forms an idea about the situation and then takes action in a way appropriate to their assessment. They define the *situation* subjectively and individually through their individual interpretation activities. In the interaction process they structure the components of *meanings*, which can be perceived as an ongoing process of redefinition of the situation in view of the interactionist approach. In this regard the term “situation” refers to a permanent and individual flux of *meanings* during the process of interaction. Goffman defines the term “situational” as “a concern for what one individual can be alive to at a particular moment, this often involving a few other particular individuals and not necessarily restricted to the mutually monitored arena of a face-to-face gathering” (1974, p.8; see Krummheuer 2007b, p.61 and see also Krummheuer, 1992, 1995, 2011c). Each student is a “member of a mathematical community” and “the individual subjectivity” of students is an important consideration (Cobb & Bauersfeld, 1995, p.11). In this sense teachers and as well as students “are seen to mutually influence each other’s activity in classroom situations” (ibid.). Moreover, each move and decision of students and teacher becomes part of the dynamics of the everyday classroom situation (Krummheuer, 2007b, p.61) and each mathematical *situation* becomes accounting practice with the status of mathematical coherence.

In the interactionist approach *mathematical meanings* emerge between individuals and are accomplished in the course of social interaction. They are perceived as products of social processes i.e. social interactions but not as independent existences from the acting individuals and from their interaction (Cobb, & Bauersfeld, 1995; Voigt,1994). They cannot be constructed inside of individuals but rather are coordinated by means of interactive negotiations between the persons involved (Krummheuer,2007,2002,2000b; Voigt, 1994; Cobb, & Bauersfeld, 1995).

In usual classroom settings, mathematical meanings “are ultimately subject to criteria of consistency with the tenets of mathematical correctness as endorsed by the mathematics community” (Clarke, 2001, p.22). Specifically they arise from the definitions of the situation and from the subjectively intentionally shaped attributions of sense by the persons involved (Krummheuer,2007b,p.70; see also Krummheuer 2000b,2002; Voigt, 1994). From this point of view, the “negotiation of meaning” leads individuals to the jointly shared and together exercised interpretations of mathematical meanings which are permanently subject to the process of change and brought about imputable dynamics of interaction. The “negotiation of meaning” refers to the interactive accomplishment of intersubjectivity[37]. Specifically, it is about the theoretical diffusion of social situations, in which mathematical learning processes emerge and are established and sustained (Krummheuer 1992, 1995, 1997, 2011c). In this manner the “negotiation of meaning” focuses principally on the here-and-now interaction of persons and less on the personal performance of these individuals who fulfil the reconstruction of the mathematical situation and the locally created mathematical productions. Specifically, this process mediates between cognition, which is the set of mental abilities and processes, and culture, which is brought forth jointly by human beings (Cobb & Bauersfeld, 1995, p.1; see also Bauersfeld, Krummheuer, & Voigt, 1988). Therefore “learning is characterized by the subjective reconstruction of societal means and models through negotiation of meaning in social interaction” (Bauersfeld, 1988, p.39).

From the interactionist viewpoint, the typical teaching-learning situation in a classroom is regarded as the teacher constructing meanings for objects which differ from those constructed by the students (Cobb, & Bauersfeld, 1995, pp.295-296). Here the students have to negotiate meaning in order to arrive at a taken-to-be-shared (or taken-as-shared) meaning and thereby to constitute mathematical meanings interactively (Voigt, 1994; Cobb, & Bauersfeld, 1995).

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[37] Intersubjectivity refers to a mutual or taken-as-shared understanding of an object or event (Cobb & Bauersfeld, 1995, p.295).

The term “taken as shared” refers to “the participants’ conviction that meanings are shared, or the participants’ willingness to neglect doubts in view of inevitable ambiguities, or the presumption that the meanings will be shared if the others will *read between lines*” (Voigt, 1995, pp.172-173). In this sense “taken-as-shared meanings emerge in the interaction process through negotiation, as do the forms of interaction” (Brandt, 2002, p.378). Cobb and Bauersfeld emphasize that the participant individuals constitute “taken as shared” through negotiation of meaning, although they do not necessarily share knowledge (1995, p.297). The social process of negotiation enables one to achieve a mutual or taken-as-shared understanding of an object or event (ibid., p.295). Voigt exemplifies this process as follows:

> The participant of the interaction monitors his action in accordance with what he assumes to be the other participants’ background understandings, expectations, etc. At the same time, the other participants make sense of this action by adopting what they believe to be the actor’s background understandings, intentions, etc. The subsequent actions of the other participants are interpreted by the former actor with regard to his expectations and can prompt a reconsideration, and so on… (1994, pp.280-281)

During this interaction process students and teacher create a “network of mathematical meanings taken-to-be-shared”, which is called by Voigt a “mathematical theme” (1994, p.283). It is generated interactively between participants and altered in terms of negotiation through taken-as-shared mathematical meanings. A mathematical theme can be perceived as the “topic of discourse” but not as a “fixed body of knowledge” (Voigt, 1994) so that participants experience both agreements and disagreements externally and with constraint during the division of labour. Thereby they learn to argue when they can design mathematics instructions in the everyday classroom situations and argue about them on a sophisticated mathematical level (Krummheuer, 2007b). For this they use “accounting practice” (Garfinkel, 1967, p.1), which refers to “techniques and methods that help to demonstrate the rationality of the action while acting and trying to make his or her actions accountable" (Krummheuer, 2000b, p.237; see also Cobb, & Bauersfeld, 1995). In this sense participants construct an *account* in order to provide an intersubjective mathematical object and to make it observable and understandable to someone else or themselves (Leiter, 1980; see also Voigt, 1994, 1995). Furthermore, there occurs a "reflexive" relationship between interaction and learning, which are mutually dependent on each other (Voigt, 1995, pp.177-192). "Reflexivity" takes place as a property of the relationship between the context of a classroom culture, a so-called micro-culture, and particular meanings that are interactively constituted in the situation (ibid.; see also Leiter, 1980; Yackel, 1995; Krummheuer, 1995). In this sense, "reflexivity" becomes a "conjecture of doing something and intimating its accountability" (Krummheuer,1995, p.239). Reflexive relationships can be between "the students' mathematical activity and the social relationships they established", or "the quality of a student's explanation and the social situation in which it is developed", or "mathematical themes and individual contributions" (Cobb & Bauersfeld, 1995, p.296). In such cases, individuals interpret mathematical meanings according to their prior experiences and design mathematical meanings on a cognitive level during the interaction process. Each participant avails himself/herself of his/her background understanding, which is necessarily extant for each individual in order to design new mathematical concepts that are brought in. Each participant experiences both agreements and disagreements with others externally and with constraint. Thus, they depend on each other while having different goals which they are developing through coordination (Krummheuer, 1995, p.254).

In usual classroom teaching and learning, each participant can think differently while using the same words or gestures (Krummheuer, 2011c, 1992, 1995; Schreiber, 2010; Voigt, 1984). Such disparity between individuals occurs because of different background understandings that individuals already have. This event is called "framing" (Krummheuer, 1983, 1984, 1992, 1995, 2011c; see also Gellert, 2010; Schreiber, 2010; Schütte, 2009), which is based on Goffmann's work on "frame analysis" (1974, 1980). Krummheuer defines “framing” as a schematized interpretation of a social event, which emerges by processes of cognitive routinization and interactive standardization (Krummheuer, 1995, p.249; see also Krummheuer, 1983, 1984, 1992, 2011c). Moreover it “refers to the cognitive constitution of meaning whereby an individual tries to create a meaning for a situation” (Cobb & Bauersfeld, 1995, p.294). Framing takes place as “an individual routinization of sense making and social standardization of these individuals processes by which a taken-as-commonly-shared reality emerges” (Krummheuer, 1995, p.250). In this regard, the development and restructuring of framings refer to “the purely individual achievements of sense making” (Krummheuer, 1995, p.255).

In the mathematics classroom teacher and students try to understand each other and they mostly “interact as if they interpret the mathematical topic of their discourse as the same, although they cannot be actually certain that their subjective background understandings are consistent with those of the other participants” (Voigt, 1995, p.172). They show indications that they interpret and furnish the mathematical statement through a process of negotiation. Whereas the teacher can frame the situation in the sense of curriculum focal points for primary mathematics classes, the students can frame the situation as far as their “emotional rejections, cognitive oversophistications, and/or cognitive oversimplifications” allow (Krummheuer, 1995, p.252). Therefore, the core of the argument may be viewed in the light of different framings whereby each participating individual can fulfil either nearly same or totally different framings. These framings of individual participants lead the process of negotiation of taken-as-shared meanings either to the “working consensus” or to the “working interim”, with reference to the work of Goffman (Krummheuer, 1992, 1995).

Goffman defines “working consensus” as “an interactional *modus vivendi*” (Goffman, 1959, p.9) and states:

> Together the participants contribute to a single over-all definition of the situation which involves not so much a real agreement as to what exists but

rather a real agreement as to whose claims concerning what issues will be temporarily honored. (Goffman, 1959, p.9).

In a working consensus, the participants do not have any sensible understanding about the argument or any achievement of their expectations, but rather they construct the argument in such a way that it becomes rational and acceptable to all participants. In this way, shared actions and utterances of individuals are perceived as familiar representamen in an interaction situation and the meanings are adjusted to their background understandings. Such framings bring forth familiarized representamen (see Schreiber, 2010). The framing process occurs in "consonance[38] with the constructing activities of other individuals by the social means of negotiation" (Krummheuer, 1995, p.250). In this sense, the working consensus is a "transient convergence on a locally viable interpretation" of participants through negotiation of meaning in the social interaction (Clarke, 2001, p.22).

The usual primary school mathematics lesson paves the way for different framings between teacher and students and also between students themselves (Krummheuer, 1992, 1995). During classic whole class teaching, between the teacher and the students "misunderstandings" occur (Voigt, 1994, p.278; see also Krummheuer, 1992,1995). The newly presented mathematical concepts necessitate a reconstruction in the students' background understandings and, especially while the teacher is introducing such new mathematical concepts, a disparity between the teacher's and the students' background knowledge necessarily emerges (Voigt, 1994, p.278). In this sense, the mathematical concept or idea that the teacher intends to convey to the students can be "ambiguous" or "unambiguous" by virtue of the characterization of mathematical discourses in the classroom (Voigt, 1994, pp.277-278). The necessary condition for learning is the negotiation of taken-as-shared meaning between the teacher and students. If such ambiguities are allowed to subsist in the agreements about

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[38] Krummheuer defines this term as "a pleasing combination of sounds simultaneously produced" (Krummheuer, 1995, p.250).

mathematical meanings of participants, different framings occur and they lead the teaching-learning situation to the "working interim". With reference to Goffman, Krummheuer defines "working interim" as a working consensus under the condition of a framing difference in mathematics classrooms (Krummheuer, 1995, p.252; see also Krummheuer, 1983, 1984, 1992, 1995; Voigt, 1985, 1994, 1995; Gellert, 2010; Schreiber, 2010; Schütte, 2009). It has a "fragile and provisional meaning" in the teaching-learning situation in the mathematics classroom (Cobb & Bauersfeld,1995, p.297), causing specific structural asymmetry in the definition of the situation by teacher and students.

In this regard the framings of individuals have to match up to the ongoing mathematical framing processes regadless of whether they are generalized coherence or incoherence. Thus, such framings result the argument in the essential and different characteristics of a consensus emerged through interaction. Moreover, such situations raise difficulties, especially for multicultural and multilingual students, in understanding, learning and interpreting mathematical meanings during the negotiation of taken-as-shared meaning (Nazarkiewicz, 2010; Schütte, 2010; Acar Bayraktar & Krummheuer, 2011). On the other hand, mathematical meanings can be negotiated even if the participants do not explicitly argue from different points of view (Nazarkiewicz, 2010; Schütte, 2010). Krummheuer proposed that the obvious way to express and to reason specific meanings is the use of language (Krummheuer, 1995, 2000b, 2007b), however, in later works he demonstrates that the use of language is not the only way to argue and express interpretations (Krummheuer, 2014, 2013a, 2012, 2011c; see also Huth 2014, 2011). In such cases the individual conceptions have become compatible so that the individuals interact as if they ascribe the same meanings to objects, even if the observer can reconstruct different subjective meanings (Cobb & Bauersfeld, 1995, p.296).

2.2.1.3. Collective argumentation

From interactionist point of view, learning processes emerge from participants' collective argumentation through negotiation of meaning (Brandt, 2002, p.378).

According to Krummheuer, in order for students to be in the fundamental condition for mathematics learning, one should encourage them to participate in the process of “collective argumentation” in classroom settings (Miller, 1986, p.294). In this process, individuals can conjointly involve and engage themselves in the interaction process and generate individual learning processes collectively (Miller, 1986; Cobb & Bauersfeld, 1995; see also Brandt, 2004; Schütte, 2009). Thus “no single participant could produce all foundations or reasoning in the emerging connection and combination of statements”, while “the individually uttered arguments are reflecting the dynamic of the interaction process” (Brandt, 2002,p. 379). Thus each “collective argumentation” process is explorative and rhetorical, referring to the “isolated metacommunicative activity that emerges [in] everyday classroom activities and follows an ordinary action when the validity of the claimed argumentation is doubted or challenged” (Krummheuer, 1995, p.232; see also 1992, 1997; Brandt, 2002, 2004; Brandt & Krummheuer, 2001; Cobb & Bauersfeld, 1995).

In the work of Krummheuer, *argumentation*[39] is seen primarily as a social phenomenon, in which individuals cooperate, try to adjust their intentions and interpretations, and verbally present the rationale of their actions (Krummheuer,2000b,p.155). Students reflect, set up and review hypotheses and make rational decisions while they are accomplishing common features, namely, participating in the joint creation of interactions (Krummheuer, 2000b). In this sense, Cobb and Bauersfeld describe argumentation in two ways:

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[39] Study of argumentation is one of the specific issues of the interactionist approach and aims to analyse argumentation process in classroom interaction (Brandt, 2002; Krummheuer, 2007b; Brandt & Krummheuer, 2001). For this two types of analysis are used, namely “analysis of argumentation” (Krummheuer, 2007b, p.62; see also 1995, 1997) and “analysis of participation” (Brandt, 2002, 1997; Krummheuer, 2007b; Brandt & Krummheuer, 2001). In the current study the “analysis of argumentation” is not used as a research methodology. Therefore, I touch upon it briefly and avoid going into the methodology of the argumentation approach. “analysis of participation” is featured under “leeway of participation” in section 2.2.2.

(a) A primarily social process in which cooperating individuals try to adjust their interpretations and interactions by verbally presenting rationales for their actions.

(b) The techniques or methods used to establish the validity or claim of a statement. During argumentation, if one explains a solution, the implicit message is that the claim is valid. A successful argumentation refurbishes a challenged claim into a consensurable or acceptable one for all participants. (Cobb & Bauersfeld, 1995, p.293)

The aim of argumentation is “to convince oneself as well as the other participants of the property of one’s own reasoning and to win over the other participants to this special kind of rational enterprise” (Krummerer, 1995, p.247). Argument can be defined as “the final sequence of statements accepted by all participants, which are more or less completely reconstructable by the participants or by an observer” (ibid.). In this sense, argumentation can be described as an attempt “to transfer the collectively doubted into the collectively accepted by collectively shared means” (Krummheuer, 1995, p.239). Subsequently the impact of argumentation occurs predominantly in the framing processes, which lead the argumentation process to either a working consensus or a working interim. In this regard *argument* is deemed to be “framing-dependent” (Krummheuer, 1995, p.250), it takes place unsystematically and simply through fragile interaction processes in the context of professional subject-related learning (for more, see Gellert, 2010; Krummheuer, 1992).

To conceptualize learning processes within argumentation, Krummheuer makes use of Bruner’s term “format”, meaning a “standardized, initially microcosmic interaction pattern between an adult and an infant that contains demarcated roles that eventually become reversible” (Bruner, 1983, p.120). In this regard, the participation of the learner in the process of interaction is the constitutive social condition for learning (Krummheuer, 2000c, p.23; see also Krummheuer, 1995, 1992; Krummheuer & Brandt, 2001; Schreiber; 2010; Schütte, 2009). Arguments and the replies to arguments change given formats, which provides the possibility of the individual's cognitive construction. In this regard individuals' participation in such argumentations makes them learn mathematics more explicitly and in a more sophisticated way, and each statement of argumentation improve students' interactional effectiveness during negotaiation of taken-as-shared meanings (Krummheuer, 2007b). Through such a process each individual's thinking about their formatted experiences constructs more general types of arguments cognitively. From this point of view, children come to learn mathematics through their contribution to such arguments and activities (Krummheuer, 2014, 2013a,b, 2011,a,b,c,d, 2009, 2007a,b, 2003, 2002; Krummheuer & Schütte, in press). In this sense, learning mathematics can be perceived as argumentative learning, which means that "the participation in argumentations is a pre-condition for the possibility to learn and not only the desired outcome" (Brandt, 2007, p.1172; see also Krummheuer, 2007b). Moreover, "the individual constitution of meaning is strongly related to the participants' collective constitution of meaning in the interaction" (Brandt, 2002, p.378). Therefore, the cognitive development of an individual is constitutively bound to his or her participation in a variety of social interactions. As "interlocutors" individuals establish interpretations, expectations of others' activities, and obligations for their own activity which enable mutual adaptation (Brandt, 2002, 2004; Bauersfeld, 1980, 1995; Bauersfeld, Krummheuer & Voigt, 1988; Krummheuer, 2002; Jungwirth & Krummheuer, 2006; Krummheuer & Brandt, 2001; Krummheuer & Voigt, 1991; Schreiber; 2010; Schütte, 2009). Each individual "takes part in" and/or becomes "part of"[40] the interaction processes in mathematics classrooms (Fetzer, 2007, p.1212; see also Fetzer, 2006) and can change her or his status of participation by taking action (ibid.). The learning process is the increasing autonomy of individuals in such stable interactional structures (Brandt, 2002, p.379). It occurs during the co-creation of interaction, both directly and indirectly, by the participation of individuals in an appropriately developed collective argumentation process

[40] "To be a part of" means that learners orient themselves on the behaviour of others. "To take part" rneans that one's own behaviour is used as orientation for others (Huth, 2014, p.149).

(Krummheuer, 1995, 1992, 1997, 2000b, 2007b; Brandt, 2002, 2004; Brandt & Krummheuer, 2001).

In this regard the participants coordinate their individual goals and intentions in the course of adjusting their actions and negotiating their definitions of the situation. Through such actions, participants become able to demonstrate the seriousness and accountability of their participation. Fetzer defines taking part as an active form of participation, whereas being part of is receptive (ibid.). Furthermore, she points outs that from being a receptive participant of the classroom interaction an individual can change his or her status of participation and take action (ibid.).

Relating to this, I now turn to Sfard's works about "learning-as-participation" (Sfard, 1998, 2001, 2008; Sfard & Lavi, 2005), which enables me to view the term „participation" in an extended way.

2.2.1.4. Learning-as-Participation (Sfard, 2008)

Sfard emphasizes that "learning a subject is now conceived of as a process of becoming a member of a certain community" (1998, p.6). For that she uses a research discourse which is grounded in the metaphor of leading as improving participation in historically established forms of activity. She calls it "participationism", which "implies that the identity of an individual, like an identity of a living organ, is a function of his or her being (or becoming) a part of a greater entity" (1998, p.6). The participationist tradition grew out of the criticism of acquisitionism (2008, p.301). According to Sfard, the "acquisition metaphor" emphasizes the inward movement of the object known as knowledge and stresses the way in which possession determines the identity of the possessor (1998, p.6). Additionally, she points out that the "participation metaphor" "gives prominence to the aspect of mutuality characteristic of the part-whole relation" and "makes salient the dialectic nature of the learning interaction: The whole and the parts affect and inform each other." (ibid.). In this regard she emphasizes that "learning is nothing else than a special kind of social interaction aimed at modification of other social interactions" (2001, p.4). Moreover, Sfard believes that “becoming a participant in mathematical discourse is tantamount to learning to think in a mathematical way” (ibid., p.5). There the “participationist” perspective conceptualizes learning as becoming a participant in certain activity and it differs from the “acquisitionist” perspective, grounded in the metaphor of learning-as-acquisition (of mental scheme, concept, skill, and so forth) (Sfard & Lavi, 2005, p.238). In this sense, Sfard replaces “the metaphor of learning-as-acquisition” with “the metaphor of learning-as-participation” (2008, p.92). Moreover, Sfard amalgamates two terms “communication” and “cognition” that she creates the term the term “commognition” (2008). She defines thinking as a “logical entailment” (Sfard, 2008, p.292) and “the individualized version of interpersonal communication as a communicative interaction in which one person plays the roles of all interlocutors” (2008, p.xvii). For Sfard, cognitive processes are “individualized forms of interpersonal communication”, whereas communication is “collectively performed rule-driven activity that mediates and coordinates other activities of actors” (2008, p.92). She defines “commognition” as a unity of the thinking (individual cognition) and the (interpersonal) communicating, which encompasses these different (intrapersonal and interpersonal) manifestations (2008). The commognitive vision of human development grew out of the participationist assumption that collective implementations of historically established forms of activity are the primary source of individual growth (Sfard, 2008, p.292). Thereupon, she considers participationism “as the best option, so far, for those who want to know what makes human learning special” (2015, p.130). Sfard also claims that participationism “provides answers to the question of how it happens that human activities evolve and grow in complexity from one generation to another” (ibid.). Moreover, she declares that this can be assumed as a specific form of a discourse, in which mathematics and mathematics learning occur by means of “participation in communicational activities of any collective that practices this discourse” (Sfard, 2008, p.91)

By the participation of individuals in an appropriately developed collective argumentation process, different types of participation of individuals emerge in the interaction process. Considering the idea of Sfard (2008), in the next part I come to the point about different types of participation of individuals. Brandt and Krummheuer describe them according to the individuals' use of language (Krummheuer & Brandt, 2001). These differences enable researchers to analyse the types of participation of individuals in their ongoing interaction processes in great detail ("analysis of participation") (Krummheuer, 2007b, p.62; see also Krummheuer, 1995, 1997; Brandt, 1997, 1999, 2004).

2.2.1.5. Analysis of participation

Brandt and Krummheuer (2001) illustrate the conception of learning-as-participation in primary everyday mathematics classroom situations and the implications of a participationist view on teaching and learning mathematics at school. They observe complex structures of conversation in everyday classroom activities and apply Goffman's (1981) idea of the dissolution of the speaker–hearer dyad and decomposition of their roles. For this, they amalgamate Goffman's approach with Levinson's (1988) idea about utterances in the course of collective argumentation. They take two terms from Goffmann – "speaker" and "hearer" (Goffman,1981) – which are simple participation statuses in the dyadic interaction process, but realize that these two terms are insufficient to determine the roles of participants in the process of collective argumentation. Owing to the fact that more amendments and modifications of participation profiles are needed, Brandt and Krummheuer categorize speaking and non-speaking persons in the collective argumentation process according to the constitution of the participants. They consider polyadic interaction processes or "multi-party interaction" (Krummheuer & Brandt, 2001), and generate the production and recipient profiles of contributors. For this, they categorize speaking and non-speaking persons in the collective argumentation process according to

(1) gestical/acoustical appearance: the argumentative expression: "sounding box" or "*Lautsprecherfunktion*",

(2) syntactical construction with certain words and expressions: the argumentative use of an utterance: the "formulation" or "formulation function"

(3) thematic/semantic contribution to the negotiation of taken-as-shared meaning: the argumentative function of an utterance: “content” or “content function” (Brandt, 2002, pp.380-381; Krummheuer,2011a, p.85; see also Brandt & Krummheuer, 2001, p.42).

Regarding these three features of speaking and non-speaking persons, Brandt and Krummheuer (2001) generate the responsibilities of each contributor to the collective argumentation process. Whereas they adopt the basic ideas of Goffmann’s work, in some further works of Brandt she does not use the original terms of Goffmann, which were originally in English, but rather she translates the German terms of their works into English by keeping the German terms constant (creator, traducer, formulator, paraphaser, initiator, imitator, inventor). Unlike Brandt, Krummheuer uses the original terms of Goffma n(1981) at his analysis in his English-language publications (e.g. Krummheuer, 2007b, 2011a). Moreover he groups contributors according to their responsibilities in the interaction processes (Krummheuer, 2011, p.85). In order to emphasize this use, the contributors responsible for the content of utterances are labelled in blue whereas contributors responsible for the formulation of utterances are labelled in green in the table (see Table 2.6). In the current study, I use Goffman’s terms, like Krummheuer, in my analysis to avoid the any confusion about different word meanings.

Table 2.6 Responsibilities of speaking person and non-speaking person (Krummheuer 2011a, p.85; Brandt, 2002, pp.380-381)

speaking person				**nonspeaking person with responsibility**		
	sounding box	formulation	content		formulation	content
author	+	+	+	/		
ghostee	+	-	+	ghoster	+	-
spokesman	+	+	-	sponser	-	+
relayer	+	-	-	deviser	+	+

In the interactional theory of mathematics learning, Krummheuer and Brandt call this categorization “production design” (Krummheuer, 2007b, p.67; 2011a; see also Krummheuer & Brandt, 2001) and they define each mentioned term as follows (Krummheuer 2011a, p.85; Krummheuer & Brandt, 2001, p.41; see also Goffmann, 1981):

- If a speaker is responsible in both components (syntactically and semantically) for his or her utterance, then the speaker is called an “author“ (Krummheuer, 2011a, p.85).

- If a speaker takes the identical formulation of the parts of a preceding utterance and with them attempts to express his own new idea, the speaker is called a “ghostee’’ (Krummheuer, 2011a, p.85). When person A verbally expresses his/her own idea (content) in the same words as person B (formulation) with the same acoustical appearance (sounding box). Then the person A becomes the ghostee, whereas person B takes the role of ghoster (Krummheuer & Brandt, 2001, p.41; translated by Acar Bayraktar).

- If a speaker takes the idea of a preceding utterance and then tries to express this idea with his/her own new formulation, this speaker is called a “spokesman’’ (Krummheuer, 2011a, p.85). When person A verbally expresses the idea of person B (content) in his/her own words (formulation) with his/her own acoustical appearance (sounding box). Then person A becomes the spokesman whereas person B takes the role of sponser (Krummheuer & Brandt, 2001, p.41; translated by Acar Bayraktar).

- A speaker who neither takes responsibility for nor has originality in the semantic content of their utterances, is call a “relayer’’ of an utterance (Krummheuer, 2011a, p.85). When person A verbally expresses the idea of person B (content) in the same words as person B (formulation) with the same acoustical appearance (sounding box). Then person A becomes the “ relayer”, whereas person B takes the role of “deviser” (Krummheuer & Brandt, 2001, p.41; translated by Acar Bayraktar).

Regarding production design (see Table 2.6), Krummheuer and Brandt aim to describe learning processes as the product of participation in collective argumentation (Brandt & Krummheuer, 2001). They perceive the argumentative content of an utterance as an “explanative idea” in order to describe the interactive genesis of a collective argumentation and the learning process by participating in formats of argumentation (Brandt, 2002, pp. 381-384; see also Brandt & Krummheuer, 2001).

Regarding the features above, Brandt and Krummheuer (2001) divide the accessibility of each utterance in the polyadic interaction process of the collective argumentation into two groups: “*direct participation* of the recipient in the utterance” and “*not direct participation* of the recipient in the utterance” (Brandt & Krummheuer, 2001, p.51). Each of these is separated into two subgroups according to their equal and unequal involvement in the developing conversation, depending on direct participation or indirect participation (see Goffman, 1981; Krummheuer, 2011a). Considering the interactional theory of mathematics learning, Krummheuer and Brandt call this categorization “recipient design” (Table 2.7) (Krummheuer, 2007b, pp. 83-84; see also Krummheuer & Brandt, 2001).

Table 2.7 Recipient design (Krummheuer, 2011a, p.84)

Accessibility of an utterance			
direct participation of the recipient in the utterance		**not direct participation** of the recipient in the utterance	
addressed by the speaker	**not addressed** by the speaker	**tolerated** by the speaker	**excluded** by the speaker
conversation partner	co-hearer	over-hearer	eavesdropper

In the “*direct participation* of the recipient in the utterance” the utterance of the speaker addresses the recipient directly (see Table 2.7; Krummheuer & Brandt, 2001, p.51; translated by Acar Bayraktar). In such participation the speaker has a conversation partner, “to whom the speaker allocates the right to take the next turn” (Krummheuer, 2011a, p.83). When the recipient “is directly addressed, but not assumed to be the next speaker but may be later involved" in the current situation, then such recipient is called a 'co-hearer'" (ibid.); "this recipient status is associated with the obligation of a high level of attentiveness" (ibid.). In "*not direct participation* of the recipient in the utterance" the speaker does not address the recipient directly (see Table 2.7; Krummheuer & Brandt, 2001, p.51; translated by Acar Bayraktar). In such participation when the recipient is "tolerated by the speaker, but not considered in the same manner to take part in the conversation", then such recipient is called an "over-hearer" (Krummheuer, 2011a, p.83). Lastly, the "eavesdropper" can be defined as a listener who is deliberately excluded from the utterance (Krummheuer, 2011a, pp.83-84).

In this chapter the method "analysis of participation" is discussed only in the school environment. In the current study I consider and use each term for the family environment.

Regarding above-mentioned theoretical approaches of Sfard (2008) and Brandt & Krummheuer (2001), one should also consider another theoretical approach called as "situated learning" from Lave and Wenger (1991). This perspective is formed parallel to the idea of interactionist approach and argues the idea that the learning can emerge not only "situational" (see section 2.2.1.2.) but also situated.

2.2.1.6. Situated Learning (Lave & Wenger, 1991)

Jean Lave and Etienne Wenger (1991) define "learning" as a reifiable independent process of a situated activity, which just occurs in any unspecified place by putting on display different forms of participations. Learning occurs in *certain forms of social engagements* in which newcomers come to be experienced members and, in time, old-timers of a community of practice[41]. Therefore, Lave and Wenger determine the nature of learning as *situated* and try

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[41] Lave and Wenger define the concept of "community of practice" as an intuitive notion, which serves a purpose and requires a more rigorous treatment (1991, p.42). Moreover, they point out that community of practice does imply participation in an activity system about which participants share understandings concerning what they are doing and what that means in their lives and for their communities (1991, p.98).

to answer the question: “What kinds of social engagements provide the proper context for learning to take place?” (Hanks, 1991, p.14). Their starting point is the Vygotskian approach[42] of the socialization of newcomers into knowledge by a form of apprenticeship (Cox, 2005, p.2). From the idea of apprenticeship, they move to *situated learning* and for this use the concepts of “the relational character of knowledge and learning”, “the negotiated character of meaning” and “the concerned (engaged, dilemma-driven) nature of learning activity for the people involved” (Lave & Wenger, 1991, p.33). They stress “comprehensive understanding involving the whole person on activity in and with the world and on the view that agent, activity and the world mutually constitute each other” (ibid.). Considering these two perspectives, they thematize the term “legitimate peripheral participation”.

In the work of Lave and Wenger cognitive and sociological perspectives are drawn together. In this sense their work can be defined as a progression of the key ideas of Lave (1988) about situated cognition (Graven & Lerman, 2003). They categorize “learning” as the *real* occurrence, when it is generated socially but not internally, whereas they consider “knowledge” as internal and cerebral (Fincher, 2003). In this sense, they propose a new model of a learning theory that can “move away from psychological and cognitive explanations of learning to a more social and situated view of learning and a shift from a focus on the individual as learner to learning as participation in the social world” (Graven & Lerman, 2003, p.186). Lave and Wenger see the notion of situated learning as “a transitory concept, a bridge, between a view according to which cognitive processes (and thus learning) are primary and a view according to which social practice is the primary, generative phenomenon, and learning is one of its characteristics” (Lave & Wenger, 1991, p.34). They regard *learning* not only as situated in practice, which is “some independently reifiable process that just

[42] According to Vygotsky, knowledge is socially embedded, and learning occurs from socially mediated collaborative processes, which enable the zone of proximal development for each individual (1978, see also Clancey,1995; for more see footnote 32 above).

happened to be located somewhere", but also as "an integral part of generative social practice in the lived-world" (Lave & Wenger, 1991, p.35). Considering this, their aim is to bring a specific analytic approach to the understanding of "learning" and to develop a view of learning that should stand on its own, reserving the analysis of schooling or other specific educational forms for the future (Lave & Wenger, 1991, pp.35-43). Therefore, they propose the term "legitimate peripheral participation" with the aim of determining "the engagement in social practice, which entails learning as an integral constituent" (ibid.). This does not mean that legitimate peripheral participation is an educational form, but rather it is much less a pedagogical strategy or a teaching technique, which is a way of understanding learning.

According to Lave and Wenger, "there are multiple, varied, more- or less-engaged and -inclusive ways of being located in the fields of participation defined by a community" (1991, p.36). Thus they describe different forms of participation: "central participation", "complete participation", "legitimate peripheral participation" and "full participation" (1991, pp.29-43). While defining peripheral participation as a being located in the social world, they perceive legitimate peripherality as a position at the articulation of related communities and also a source of power or powerlessness, in affording or preventing articulation and interchange among communities of practice. They see peripheral participation as a positive term, a dynamic concept and a kind of partial participation that the individual is not "disconnected" from the practice of interest (ibid.).

Lave and Wenger define the term *central participation* as a kind of participation, in which the individual's occurrence takes place in the centre (physical, political, or metaphorical) of a community, whereas *complete participation* is a closed domain of knowledge or collective practice for which there might be measurable degrees of "acquisition" by newcomers (1991, p.36). They emphasize that "the mastery of knowledge and skill requires newcomers to move toward the full participation in the sociocultural practices of a community" (1991, p.29). *Legitimate peripheral participation* enables one to speak about the relations between newcomers and old-timers and about their activities, identities, artifacts and communities of knowledge and practice (ibid.). In this sense it is more than being on the "observational" lookout for the newcomers (1991, p.95). It crucially takes in *participation as a characteristic of ways of belonging* and *a way of learning the culture of practice*.

Sfard thus verifies the work of Lave and Wenger and emphasizes that one can participate in mathematical discourses either "actually" or "peripherally" (Sfard, 2015, p.136). Similar to Sfard (2008), Krummheuer also underlines that learning occurs only through participation not acquisition (2011a, 2013a,2015) and he points out the importance of the rules of arguing in these mathematical discourses. Regarding the work of Lave and Wenger (1991), Krummheuer defines legitimate peripheral participation and full participation as sensitizing concepts[43], which put forward "a perspective and a framework in which way this theory approaches the empirical domains of interest" (Krummheuer, 2011a, p.82). Furthermore, he considers participation profiles of production and recipient designs (Krummheuer & Brandt, 2001) and categorize them as definitive concepts[44], which takes in "empirical elements, which clearly help to describe what a certain phenomenon looks like" (ibid.). Considering both concepts he defines the relationship between the concept of "legitimate peripheral participation" (Lave & Wenger, 1991, p.37) and "relayer, over-hearer, co-hearer, eavesdropper" (Krummheuer & Brandt, 2001) as follows:

> Students participating in a production design in the role of a relayer can be taken to be at the very beginning of a cooperative learning process. They try to imitate what they hear and see and possibly this initiates reflections inside their cognitive system that might lead to a restructuring of their definitions of

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43 For this Krummheuer benefits from the idea of Blumer (1954): "Whereas definitive concepts provide prescriptions of what to see, sensitizing concepts merely suggest certain directions along which to look" (Blumer, 1954, p.7).

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> the situation. In a definitive sense, this role in a production design could be associated with Lave & Wenger's concept of the "legitimate peripheral participation" … Students participating as recipients of an utterance might also be in an attitude of learning. Generally, one has to consider that a student in any status of an emerging recipient design might be able to pick up some ideas that change his/her interpretation of the problem situation. Considering the dynamics of the interactional turn-taking process, the commitment of a recipient to listen and pursue the ongoing actions increases when the speaking person addresses him/her directly. In this case, the obligation arises for the recipient to take the next turn. That implies a certain degree of attentiveness, which can be seen as a positive condition of initiating a learning process. Considering the term of "legitimate peripheral participation", one would possibly only see it represented in the indirectly addressed statuses of the over-hearer and the eavesdropper and would not identify this concept with the relayer in the production design. (Krummheuer 2011a, pp.87-88).

Krummheuer emphasizes that the participation forms of *ghostee* and *spokesman* in a production design (Krummheuer & Brandt, 2001) can be perceived as "novel representations of the sensitizing concept of learning-as-participation in a definitive form that suggest stages in a learning process developing from imitation to autonomy" (Krummheuer, 2011a, p.88). Regarding the participation in communicational activities of collective mathematical practices (Sfard, 2008), Krummheuer observes that "becoming a member of a mathematical discourse" requires "the learning of the rules and routines of these mathematical discourses" (2011a, p.82; also in 2013a, p.252). Therefore, Krummheuer states that "teaching has a systemic effect on situations that enable certain members to participate as learners" (2011a, p.8). Referring to this, the conceptual shift from acquisition to participation is "an integral feature of the process of social interaction" (ibid.).

Regarding the theoretical basis above, mathematics learning is perceived as a social and active process, in which learners interact with each other and actively construct meaning as they participate in increasingly substantial ways in the re-enactment of established mathematical practices (Cobb, Yackel, & McClain, 2000, p.21; in Anghileri, 2006, p.33; see also Bauersfeld,1995; Bruner, 1990, 1996; Erickson, 1982; Jungwirth & Krummheuer, 2008; Krummheuer, 1992, 1999; Voigt, 1995). Specifically, Krummheuer defines mathematics learning as "a dual process, as the individual's cognitive construction of knowledge and as his increasingly autonomous participation in social situations" (2014, p.72). In this sense, each learning situation is constituted socially through interactive processes, which are vital parts of the "nature" of learning (Krummheuer, 1999, p.331). In other words, "mathematics learning takes place in the *social* and the medium of this social world is the interaction" (Huth, 2014, p.150).

With all these theoretical backgrounds in mind, how family dynamics shape this social and active process, is the next point that I will discuss in the coming section.

2.2.2. Family Systems Theory

Family system theory lays emphasis on the internal and external factors of a family and regards the family as a social system (see also section 1.4.1.). In the current study it helps me to understand how family members deal with each other and how these relationships affect the child's development.

This approach considers "the interdependence among the roles and functions of all family members" (Parke, 2004, p.366) and helps me "to understand fully the nature of family relationships". Moreover, this theory acknowledges "how family lives are intertwined and how the ability of families responds and adapts to challenging and changing circumstances" (Silverstein & Ruiz, 2006; in Goodfellow, 2010, p.7). Regarding these, in the current study the family is perceived as a social system, which has "subsystems, coalitions, or alliances" that are bounded with "interrelated elements or assemblage of objects related to each other by some regular interaction or interdependence" (Morgaine, 2001).

Subsystems of a family are composed of individuals and their relationships with in and of each other (Bornstein & Sawyer, 2008, p.382; see also Morgaine, 2001). In this regard the family system is a dynamic system, which "is simply a more or less self-contained set of elements that interact in complex, often nonlinear ways to form coherent patterns" (Vallacher & Nowak, 1994, p.xv; in Bornstein & Sawyer, 2008, p.382). Each subsystem interacts with others, while they exert influence on each other reciprocally and multi-directly. The subsystems have boundaries, but they are permeable and vague so that they can influence each other internally and externally.

The subsystems of family consist of the parent-child, the grandparent-child, and the sibling subsystems. Each subsystem has its own dynamics, while they affect each other reciprocally and multi-directly. Principally these interrelations also have impact on child development in direct and/or indirect ways (Parke, 2004). The main thing is to consider "mutual influences among family subsystems, such as the marital relationship, the parent-child relationship, the sibling relationship etc.," in the course of "viewing individuals within the context of their larger family systems" (Cox & Paley, 2003, p.193). These models are called multiple models which exist "from the individual up through the whole family in the context of different levels" (ibid., p.195). In this regard the processes of child development are affected by the whole and partial formations of alliances within the family (ibid.). Dombeck and Wells-Moran (2006) sum up the key insights of family system theory as follows:

- the problems people have frequently reflect problems experienced by the families and groups those people are a part of,
- it is necessary to address family or group problems at the family or group level (the "system" level) if they are to be resolved, and
- the way to identify what is going wrong within a family or group is to pay attention to how the boundaries governing the family or group members are functioning. Frequently, role transgressions (e.g., abuse situations, failures to carry out particular responsibilities, etc.) serve as good indicators that boundaries are not functioning properly (p.2).

The current study focuses on the mathematical development of children between the ages of five and eight and conceives family systems specifically for this age group. This age group comes up to the middle childhood, which refers to children between the ages of five and 12 years (e.g. Bornstein, 2002a,b,c,d,e). In the following paragraphs I present the subsystems of the family system in detail.

2.2.2.1. The Parent-Child Subsystem

Parents are the first and the most continuous provider of services and care for their children. They inform their children about any issue, from birth to death, while they also satisfy the emotional, physical and motivational needs of their children. In this regard parents have a crucial role in the development of children in mathematics as well as in any other realities of life. Bornstein defines parents as the “final common pathway” to children’s development, adjustment and success (Bornstein, 2002a, p.xi). He points out that parents meet the biological, physical and health requirements of children and are thus most consistent and caring people in the lives of children (ibid.).

While parents are interacting socially with their children, they make provisions, organizations and arrangements for children’s home, local and social environments that can encourage children to understand and engage in the world. Thereby such support enables children “to enter the word of learning” (Bornstein, 2002a, p.ix). Parental awareness of community services and their participation in shaping the institutions of the community promote the maintenance of values and norms that influence their children (Parke, 2004, p.385). Hence parents’ cultural background, their ethnicity, the quality of their relationship, their education and economic level have direct or indirect influence on the quality of parents’ relationships with their children and the children’s development (Bornstein & Sawyer, 2008; Bornstein 2002a,b,c,d,e; Parke, 2004).

Parke (2004) emphasizes that in research with families, it is necessary to take into account the acculturation level of parents, which affects the family processes and child outcomes (see also section 1.4.1.1). He indicates that “intergenerational differences in acculturation can create role strains between parents and children that have implications for child-rearing styles, disciplinary practices, and overall parent-child relations" (Parke, 2004, p.385).

The marital quality also affects the relationship between family members and their children. Marital discord is linked to the family functioning and the quality of parenting, which can give rise to difficulties in children's externalizations and internalizations (Parke, 2004, p.372). Parents who are distressed and negative in their interactions as a couple expose their children to their cold, unresponsive and angry behaviours. Children who have experienced such parental behaviours display "more anger and noncompliance" in their own lives (ibid., p.373).

Collins et al. (2002) point out that the education level of parents has an influence on the communication and parents' styles of interaction with their children (see also section 1.4.1.2.). Parents should "arrange for and interact with out-of-home childcare personnel and with adults who provide instruction and supervision in out-of-school learning and recreational settings" (Collins et al., 2002, p.77). In this sense, the level of parental education enters into their communication styles with their children, the children's social environment and "daily informal and formal activities, which promote or discourage children's peer relationships" (Parke, 2004, p.371). Parents with lower levels of education have less frequent interactions with their children in middle childhood and when these children start school, the frequency of their interactions become less than half (Collins et al., 2002, p.79). Fundamentally parents use their "interpersonal-linguistic skill in naming, categorizing, directing shared attention, and explaining aspects of the world" to their children (Laakso, 1995, p.445). In this sense "the quality of parental advice", "parental guidance about social relationships" and their "monitoring" actions affect children's social competence with peers (Parke, 2004, p.371). Lower academic skills of parents give rise to poor monitoring of children, especially once they are attending primary school. In contrast, high parental guidance, often with advice-giving strategies, efforts to keep children from being influenced by peers and talking to them about the future consequences of their behaviour, lead children to low levels of antisocial behaviour and higher levels of academic achievement (ibid.). At the age of five children enter a wider social world and begin to “determine their own experiences including their contacts with particular others” (Collins et al., 2002, p.73). Both fathers and mothers increase their attention to their children’s school achievement and homework during middle childhood (Collins et al., 2002, p.80).

Whereas some studies indicate that fathers’ and mothers’ communication styles do not differ, some have found “different types of communication deviances” between fathers and mothers when they are conveying information to their child (Collins et al., 2002; Herzog, 1998; Laakso, 1995; Mullis & Mullis, 1986; Paquette, 1994, 2004; Parke, 2002, Tamis-LeMonda, 2004; Tomasello et al., 1990; Bornstein, 2002a,b,c,d,e). According to some family system approaches, fathers tend to make more requests for information, give more exact and elaborative descriptions in play situations, and use a greater proportion of verbalizations describing form, shape and direction relations than mothers in course of interacting with their children (Laakso, 1995; Bornstein, 2002a,b,c,d,e; McGillicuddy-DeLisi, 1988). Moreover, fathers evoke the “activation function” during play interactions with their children, which involves an exploratory system whereby children experience novel issues in physical and social environments (Tamis-Lemonda, 2004, p.222; Paquette, 1994, 2004; Bowlby, 1969).

Beyond the cultural models, the characterization of fathers’ role in families is a bit limited. According to traditional models of society, fathers are “financial providers” (Tamis-Lemonda, 2004, p.220) and thus in western industrialized nations they spend less time than mothers with their children (Bornstein & Sawyer, 2008). Whereas mothers spend about 65–80 per cent of their free time with their children, fathers spend less free time in direct one-to-one interaction with their children (Bornstein & Sawyer, 2008, p.386). Therefore, usually they take less responsibility than mothers for child caring. While mothers mostly attend to „the child’s calm and comfort“, fathers foster children’s “openness to the world” (Tamis-LeMonda, 2004, p.220). Fathers tend to encourage risk taking while simultaneously protecting their young from danger. During play activities with their fathers, children experience standing up for their own beliefs, while their fathers encourage them to face up to unfamiliar occurrences and their own mistakes, hence justifying themselves and taking risks in new sets of circumstances (ibid.). Such occasions lead children's social competences and functions to develop; they open children up to the outside world.

Beside these, fathers encourage their children to complete tasks in the shortest amount of time, which is the primary goal in problem solving (Laakso, 1995, p. 447). Laakso points out that in the parent-child conversation children experience more communicative breakdowns with their fathers than with their mothers and thus there occur different communication styles between mother-child and father-child dyads (Laakso, 1995, p.446; see also Tomasello et al., 1990). Fathers ask questions more than mothers, offer their children more information, use more elaborative labels, and come up with more imperative and short utterances in the interaction process with their children (Mullis & Mullis, 1986). Furthermore, fathers give more responsibility to their children in completing their given tasks, while they pose more questions and vary the instructions given to their children more flexibly. Therefore, fathers' and mothers' teaching styles differ from each other due to complex factors of interplay as well (Laakso, 1995, p.446). In recent years the level of fathers' involvement with children has increased so that fathers too can become competent caregivers and playmates and thus affect and manage aspects of family life equally with mothers (Parke, 2002, p.62). Although fathering is multidetermined with individual, family, institutional and cultural factors all influencing this role, father's role is brought up it is issued less often than mothers' role (ibid.).

Mothers are the "major caregivers" of their children (Barnard & Solchany, 2002). They use more directivity than fathers and minimal-effort strategies (Mullis & Mullis, 1986; Russell & Russell, 1987; Leonard, 1993). The roles in the mother-child interaction are somewhat different in that the mother takes on more responsibility for the social exchange. Depending on the child's capacity or developmental level, the mother provides feedback of an instructional nature during the negotiation process with her child (Barnard & Solchany 2002, p.16). Bornstein (1989) labels this type of instructional feedback as didactic exchange, in which interactive content is rich in terms of positive affect and verbal stimulation (see also Barnard & Solchany, 2002). In the emerging negotiation process mothers express their agreements, disagreements, individual decisions and short counter-assertions, through which children become attached to their mothers and experience their consistently sensitive care. In this regard the mother-child relationship aims to calm and comfort rather than to arouse in childrens' activations (Lamb, 1981; Paquette, 2004; cf. Tamis-LeMonda, 2004; Bornstein 2002 a,b,c,d,e; Salonen et al., 2007). Maybe therefore mothers endure with forbearance and rely on low-level strategies during problem solving (cf. Laakso 1995, p.446). In this sense they focus on mostly having and meeting children's emotional needs in a balanced manner and thus do not criticize their children. Thus, mothers mostly take the role of *nurturer*, one who provides emotional support, creates safety, is available to others, and can be a mediator (Telos Residential Treatment, n.d., p.5). While mothers interact more frequently in connection with caregiving and household tasks, fathers typically are involved relatively more in physical/outdoor play interactions with their children (Collins et al., 2002, p.80). Mothers are more sensitive than fathers in that "they evaluate the strategies chosen by the children and verbalize relevant and critical aspects of the construction task so as to promote the children's learning of the proper method for solving them" (Laakso, 1995, p.447). Similarly, Parke (2002) indicates that, from birth to the end of middle childhood, mothers are more likely to shoulder "the managerial role" for children than fathers (p. 30). Both positive and negative emotional expressions and conflictual interactions are more likely in mother-child than in father-child interactions (Collins et al., 2002, p.80). Moreover, in the interaction process of the mother-child dyad, mothers provide more feedback of an instructional nature during the exchange, in which a mutually adaptive "waltz" occurs between mother and child (Barnard et al., 1989; in Barnard & Solchany, 2002, p.16). This "waltz" reflects the emotional availability, reciprocity and mutuality in the child-mother relationship (ibid., p.17). Furthermore, Barnard and Solchany (2002) point out that the mother must remain both consistent and contingent in responding to her child. They found mothers with low levels of education to be more contingent in responding to the child (Barnard et al., 1989;

in Barnard & Solchany, 2002, p.16). In this regard Barnard and Solchany (2002) indicate that interactive content must be rich in terms of positive affect, verbal stimulation and range of play materials provided, while the adaptive patterns between mother and child should change over time relative to the emerging developmental capacities of the child (2002, p.16).

2.2.2.2. The Grandparent-Child Subsystem

Grandparents are the senior educators and the most experienced care providers for their children's children. While they are still informing their own children about any issue from birth to death, they become the superior reference care-giving issues.

Grandparents create the "parenting pattern" in a nuclear family (Bornstein & Sawyer 2008, pp.388-389). While they exist as parents of their own children, they also affect the way their children act as parents (Smith, 2005). Thus, grandparents can become "custodial parents (or one who has care and control of the grandchildren)" for their grandchildren, at the same time as satisfying the emotional, physical and motivational needs of their children, who are also parents at that time (Smith & Drew, 2002, p.154; see also Goodfellow & Laverty, 2003). With continuing "social change, including changes in family composition and the rise in maternal employment", it is possible for grandparents to play a significant role in the lives of children (Goodfellow & Laverty, 2003, p.19). In this regard their influence can be direct, as "resulting from contact and face-to-face interaction", or indirect, as "being mediated by other means such as parental behavior" (Smith, 2005, p.685). Hence the prominent issue in caring for grandchildren is the quality of the relationship between grandparents and parents. If they have a "harmonious" relationship, then grandparents provide stability, support and a "responsive, intensive form of nurturance" to the grandchildren and family while providing financial assistance and childcare as well (Smith & Drew, 2002; Goodfellow & Laverty, 2003). Financial assistance or support can be seen as indirect influence of grandparents, while the care-giving issues about grandchildren can be seen as direct influence of grandparents (see Smith, 2005).

As grandparents achieve the right “balance” with their own children in their lives, they foster their grandchildren’s development in the critical early years in which the grandchildren learn to make personal choices and to use the capacity of retaining a degree of autonomy and sense of self (Goodfellow & Laverty, 2003, p.19). Hence grandparents can be assumed as “significant attachment figures during the early years of their grandchildren’s lives” and also “contributors to the childcare system in supporting economic growth” (Goodfellow & Laverty, 2003, p.19).

Similar to the parent-child subsystem, in the grandparent-grandchild subsystem grandmothers are more involved with their grandchildren than grandfathers, as a consequence of mothers having greater involvement with their children than fathers (Smith, 2005; Dench & Ogg, 2002). Specifically, the grandparents on the mother’s side become more involved with their grandchildren than those on the father’s side (ibid.). As a “substitute figure” for the parent (Smith & Drew 2002, p.145) the grandmother takes the role of significant attachment figure during her grandchildren’s lives and subsists as “an integral part of the family unit” (Bornstein &Sawyer 2008, p.389), by which she transmits her believes and behaviours across the generations (ibid.). In this informal caring of the grandmother, she directly influences the children’s acts resulting from contact and face-to-face interaction (Bornstein & Sawyer 2008; Smith 2005). Smith and Drew (2002) categorize the grandparent’s role in the family into five different styles:

- formal role: following prescribed roles with a clear demarcation between parenting and grandparenting responsibilities,
- fun seeker role: seeing grandchildren as fun and a source of self-indulgence or mutuality of satisfaction,
- role as surrogate parent: taking actual care-giving responsibility,
- role as reservoir of family wisdom: dispensing special skills or resources, with authority,

- distant role: only infrequent contacts with grandchildren on ritual occasions (p.152).

Beside this, they point out that the fun seeker role of grandparents mostly takes in place while grandchildren are younger, whereas grandparents take on formal roles when grandchildren are older (ibid.). Furthermore, they emphasize that immigration and cultural differences affect grandparent-grandchild relationships. Because of immigration the culture of grandchildren in a new country differs from the heritage culture of the grandparents (see section 1.4.1.1.). Since they have generational differences, their adaptation of a "new" culture progresses divergently, which leads to intergenerational difficulties and disagreements between the three generations (grandparents-parents-children) (Coll & Pachter, 2002). Grandparents and grandchildren are in possession of a sufficient repertoire of behaviours so that interlocking sequences are possible, and a smooth-flowing interactive system develops (Perner, Ruffman & Leekam 1994). As creators of the "parenting pattern" in the nuclear family (Bornstein & Sawyer 2008, pp.388-389) and *"sufficient"* persons in childrenrearing, grandparents can scaffold their grandchildren. Grandparents thus procure such a subsystem in the negotiation process and take the "transitional object role" towards their grandchildren unwittingly (Smith & Drew 2002, p.145; Perner, Ruffman & Leekam, 1994), so that they can erect and strengthen standards, serve as models for emulation and give advice (Doron, 2009, p.24).

2.2.2.3. The Sibling Subsystem

In section 1.4.1.3. above, I have already discussed the importance of sibling's existence, so I refer to it here briefly.

In a family with a newborn, the family system changes significantly. While parents have a new baby, their children get a new companion in their own lives. Each child can affect the family system in different ways. During the preschool years, siblings play an important role in one another's social lives (Pepler et al., 1981). While they are interacting with one another, they influence their social and emotional growth and development as well. For example, children "who perceive that they are treated less positively than their sibling" are "more likely than their sibling to show negative personality adjustment in adolescence" (Daniels et al., 1985, in Collins et al., 2002, p.86). Such interaction patterns between parents and children serve as an "important context in which children deal with differential treatment and complex social emotions such as rivalry and jealousy" (Parke, 2004, p.374).

Smith and Drew point out that the younger sibling elicits many more explanations from older sibling than from the adults and enjoys a "privileged" teaching status (2002, p.274). Older siblings often act and are treated as experts, whereas younger siblings embody the novice role more often (ibid.). Similarly, Parke highlights that "older siblings function as tutors, managers, or supervisors of their younger brother or sister's behavior during social interactions" and also "any function as gatekeepers who extend or limit opportunities to interact with other children outside of the family" (Parke, 2004, p.374). In this regard, in the family system siblings have different positions: the oldest sibling is "the responsible one" and the youngest sibling is "the funny one" (Crawford, 2012; Doron, 2009; Sulloway, 2001) (see also section 1.4.1.3). Furthermore, younger siblings accept such help or direction from older siblings (Abramovitch et al., 2014, p.64). Such situations allow the younger one to imitate and to "organize information in the mind by relating concepts together" (Brahier, 2009, p.53) that enable the child to be a learner and follower in any everyday situation (Crawford, 2012; Howe, Brody & Recchia, 2006; Abramovitch et al., 1986; Azmitia & Hesser, 1993). Specifically, playing with siblings allows the younger one to be cultivated, to "furnish ideas with those models and techniques for how to operate on his own" (ibid.), so that having older siblings leads to precocious development for the younger ones (Perner, Ruffman & Leekam 1994).

The main reason for taking "family systems" into consideration in this study is the habitual emergent interaction patterns, which are "repetitive cycles" that enable "maintain[ing] the family's equilibrium" (Morgaine, 2001; Bornstein & Sawyer, 2008). During interaction processes such a system and its subsystems can play a crucial role when family members are negotiating taken-as-shared meanings (Laakso, 1995, p.447).

2.2.3. Self-Regulation and Family Members' Regulation

The further social cognitive research to consider are about regulation theories. Regulation types arise from the theoretical basis of the interactionist approach. I will not go into detail about cognitive, neurocognitive and individualist approaches, but rather I argue regulation theories from the social perspective by means of the *participation metaphor* of Sfard (1998, 2008) and the interactionist theory of mathematics teaching and learning (Brandt & Krummheuer, 2001).

2.2.3.1. Self-Regulation

Self-regulation concepts deal with "how individuals manage their own problem-solving or learning behaviour, by incorporating cognitive, metacognitive, motivational, social and emotional aspects of their functioning" (Nader-Grosbois et al., 2008, p.97). Zimmerman (2000) defines self-regulation as "self-generated thoughts, feelings, and actions that are planned and cyclically adapted to the attainment of personal goals" (p.14). Feedbacks from prior performance allow one to adjust current efforts while they are emerging at the same time. Zimmerman focuses on the self-regulation concept from the social cognitive perspective. According to him, self-regulation is an interaction of personal, behavioural and environmental triadic processes (Zimmerman, 2000, p.13). Moreover, self-regulation is contingent on "self-beliefs and affective reactions, such as doubts and fears, about specific performance contexts" (ibid., p.14). Zimmerman defines three different types of self-regulation, which are constantly changing during the course of learning and performance (Zimmerman, 2000, p.14; see also Nader-Grosbois et al., 2008, p.97):

- Behavioural self-regulation: Self-observation, self-adjustment of the method of learning, depending on the performance towards the goal
- Covert self-regulation: Management of one's cognitive, affective states;

- Environmental self-regulation: Adjustment to learning conditions: social and material environment. Here the learner adjusts the “material means and local conditions of his or her learning environment” by mobilizing his/her social environment through communicative solicitations e.g. asking for help or any clue, searching for any approval or disapproval for his/her reactions or initiating referential joint attention on the task with the partner (Nader-Grosbois et al., 2008, pp.97-98).

Nader-Grosbois et al. (2008) argue that the exploration of the means at one’s disposal involves both behavioural and covert self-regulation. These are “the individual’s identification of an objective orienting the task and the planning of a sequence of actions”, whereas “environmental self-regulation is sustained by exploration of the means at one’s disposal” (p.97). More specifically, self-regulation depends on a context and “social resources on a self-selective basis” (Zimmerman, 2000, p.31). The regulation types described above include “affective and behavioral processes, and a resilient sense of self-efficacy to control them” (Zimmerman, 2000, p.34). Therefore, this approach emphasizes “the role of socializing agents in the development of self-regulation, such as parents, teachers and peers” (ibid.). The current study examines how a child contributes to interaction processes in the familial discourses. For this, self-regulation theories are taken into consideration as well as interactional participation profiles. Zimmerman’s work helps me to approach this section from a social cognitive perspective alongside the basic perspective (socio-constructivisim) of this study.

Self-regulation pertains to “the extent in which one is able to be one’s own teacher” (Simons & Beukhof, 1987, p.4). Moreover, it implies that “individual behavior is predominantly regulated by internal forces, therefore negating the impact of external factors” (Jackson, Mackenzie & Hobfoll, 2000, p.276). The term “self-regulation” is about having “intrinsically or fully internalized and integrated extrinsic motivation” (Deci et al.,1996, p.179). Because of my interactionist research approach, I avoid going into any details about the cognitive and metacognitive aspects of self-regulation theories. Especially I take into consideration the study of Jackson, Mackenzie and Hobfoll (2000), in which they draw attention to the idea that “autonomy and relatedness to others are dual human needs and self-regulation theories should regard the interaction” (p.276). Similarly, Nader-Grosbois and colleagues (2008) define self-regulation as follows, which I consider especially pertinent to my research:

> to attain an identified objective, the person plans, explores the means at his or her disposal, maintains his or her attention and motivation during the problem solving, evaluates and adjusts his or her actions, and when necessary, solicits the social environment by asking for help, joint attention or approval. (Nader-Grosbois et al., 2008, p.98)

Certain groups of individuals socially mediate and control their lives independently, while being “autonomous, self-directing, unique, and assertive” individuals, who also value privacy and freedom of choice (Jackson et al., 2000, p.275). In this way self-regulation from the social cognitive perspective keeps individuals in the community and does not isolate them from each other (ibid.). Similarly, Simons and Beukhof emphasize that external and internal components interact strongly (1987, p.8).

Deci et al. (1996) argue that “autonomous self-regulation is related to positive educational outcomes” (p.171). They form the concept of self-regulation as a continuous sequence in which adjacent elements are not perceptibly different from each other, but the extremes are quite distinct. Deci et al. speak about different types of self-regulation [45] and how “different interpersonal and

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[45] Deci et al. classify self-regulation into four degrees, from very low to very high (1996). The four different types of self-regulation are: external, introjected, identified and integrated (Deci et al.,1996, p.168). In this study, only external regulation is taken into consideration because of the high cognitive and meta-cognitive features of other types of regulation, whereas external regulation has the feature of social cognition and is thematized in the main text above. The remaining three types can be defined as follows. Introjected-regulation is a moderately low degree of self-regulation and represents a behaviour controlled by demands or contingencies

developmental contexts lead individuals to use different types of regulation" (Deci et al., 1996, p.166). With respect to these contexts and particular feedbacks or behaviours in the interaction process, individuals can be more or less self-regulated. In the current study only one of them is taken into consideration: external regulation, which refers to the lowest degree of self-regulation. According to Deci et al., external regulation is the behaviour controlled by demands or contingencies overtly external to the individual (1996, pp.168-170). This type of regulation implies being offered a reward or other incentive, or a means to avoid punishment. It involves doing an activity only because the person feels forced by some external agent (ibid.). External regulation depends on external demands, which are also intentional. Deci et al. (1996) relate it to parents, which makes it relevant to the analysis in the current study. They emphasize that "external reasons include behaving because of rewards, punishments, or demands imposed by" parents (Deci et al.,1996, p.170). External regulation is perceived as an "autonomy-supportive parenting style", which offers choices and considers the child's perspective willingly and evidently when making decisions (Deci et al., 1996, p.176). External regulation in this field refers to "the sequence of examples presented, and the kinds of examples given" by parents (Simons & Beukhof, 1987, p.8). Such types of parental reactions are noticed as autonomy support, which relates positively to children's school achievement and educational outcomes (Deci et al., 1996, pp.171-176).

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inside the person, such as self-esteem contingencies (Deci et al., 1996, p.168). Identified-regulation is a moderately high degree of self-regulation, which refers to the behaviour chosen when person identifies with the importance of the activity (ibid.). Integrated regulation is the highest degree of self-regulation, which is a behaviour experienced as "wholly free" because the regulation has been integrated with the person's sense of self (ibid.).

2.2.3.2. Family Members' Regulation [46] : External Regulation & Scaffolding[47,48]

Parental regulation, or parents' regulation, is one of the socio-constructivist approaches in family education and refers to the parental adaptations according to the reactions of children in the interaction processes. The term parental regulation (or parents' regulation) is a distinct term in the literature which includes scaffolding and external regulation strategies together in the familial studies. In the current study I not only focus on parent-child interactions but also grandmother-child and sibling-child interactions. Therefore I am interested not only in the regulation of parents but also in the regulation of "family members" more broadly. Therefore, please note that in presenting the theoretical perspectives in this section I adapt any terms about parents to refer of family members. Thus, I call this section family members' regulation instead of parents' regulation.

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[46] Hammond and Müller (2012) emphasize that parental scaffolding differs from scaffolding. Whereas they consider parental scaffolding as unique among potential forms of parental influence on children, they assume scaffolding as intentionally directed at attempting to improve a child's problem solving (Hammond & Müller, 2012, p.280). In the current study I refer to scaffolding as one aspect of parental regulation, regarding six scaffolding functions. Therefore, parental regulation includes two points external regulation and scaffolding. From Hammond and Müller's point of view, external regulation represents parental scaffolding in this study.

[47] Although all work about scaffolding emanates from the acquisition metaphor (Sfard, 2008), this perspective helps me to interpret parents' reactions during the interaction process. Therefore I take the theoretical term "scaffolding" into consideration in this study, whilst I believe that learning is something which occurs only through participation of the child and others and not through acquisition (Sfard, 2008). In this regard, I want to emphasize here that, in my view, scaffolding occurs through participation instead of adults teaching while the child's autonomy increases during the interaction process.

[48] In most studies the metaphor of scaffolding is considered as "supporting learners to engage [with] a content" (Renninger & Granott, 2005, p.111; see also Fernández et al., 2001; Anghileri, 2006; Salonen, Lepola & Vauras, 2007; Nader-Grosbois et al., 2008; Bibok, Carbondale & Müller, 2009; Hammond & Müller, 2012).

Nader-Grosbois et al. (2008) point out that parental regulation consists of scaffolding and external regulation. This perspective includes developmental and cognitive psychology, family system theories and epistemology. I prefer to pack them all into the interaction theory of thinking and learning.

Bruner (1983) highlights that parents elicit interactive play settings, which promotes child development to sophisticated levels. In this sense Bruner assumes scaffolding and external regulation as two genesis as parents' initiatives for supporting children's learning. Parents adapt their regulation strategies: they decrease executive, directive scaffolding by adjusting their verbal and nonverbal interventions, in order to enable the child to develop (Nader-Grosbois et al., 2008, p.96).

As already discussed in section 2.2.3.1, external regulation refers to the lowest degree of self-regulation (see Nader-Grosbois et al., 2008; Simons & Beukhof, 1987; Deci et al., 1996). Parents reflect on the child's perspective voluntarily and obviously, which enables the child an increasing or decreasing autonomy. Boekaerts (1997) defined external regulation as a form of aid that "leaves the learner little autonomy and hardly any responsibility for the learning process", while "scaffolding" refers to a metaphor which "captures the idea of an adaptable and temporary support that helps an individual during the initial period of gaining expertise" (pp.171-172). Specifically, all together functions of scaffolding leads the child to self-regulation, whereas the usage of some functions of scaffolding, namely sensitivity and mind-mindedness, convey the child to external regulation (Bernier, Carlson & Whipple, 2010). Scaffolding enables a shift from external regulation to self-regulation, "where the child is assisted in attaining a higher level of independent functioning in terms of cognitive self-regulation and motivational autonomy" (Salonen et al., 2007, p.79). Therefore, in scaffolding the children's cognitive and emotional development can be facilitated, whereas external regulation encourages only one of these. Both scaffolding and external regulation involve the negotiation of meaning, but only scaffolding renders possible the "transfer of responsibility for learning within a social context" (Salonen et al., 2007, p.79).

In scaffolding, the “tutor” or “expert” should “strike a balance between working with children at their current level of competency and at the same time challenging them”, while “responding contingently to children’s ongoing activity and expanding on that activity and directing it in more challenging directions” (Bibok et al., 2009, p.18). In external regulation, the “tutor” or “expert” only responds contingently to children’s ongoing activity (Bibok et al., 2009; Nader et al., 2008). In this sense, scaffolding is a process that “simultaneously aims to regulate both children’s motivation (recruitment, frustration control) and cognition (reduction in degree of freedom, marking critical features, demonstration)”, whereas external regulation aims to control preferably children’s motivation or cognition (Bibok et al., 2009, p.18). In the current study, external regulation is assumed as a realization of one scaffolding function.

The term “scaffolding” firstly appeared in the work of Wood, Bruner and Ross (1976) about the role of tutoring in problem solving. They define scaffolding as an “adult controlling those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence” (Wood et al., 1976, p.90).

In their work, Wood and colleagues refer to the adult person as an “expert”, who “tutors” children during 3D-structure building (1976), and the “novice” or “tutee” as a person who is less adult or less expert and thus gets help from an “expert” (Bruner, 1986; Wood et al., 1976; Hammond & Müller, 2012). Their wor, aimed at examining “some of the major implications of [the] interactive, instructional relationship between the developing child and his elders for the study of skill acquisition and problem solving” (Wood et al., 1976, p.89). They define the usual type of tutoring as an “actual pattern of instruction” “in which one member *knows the answer* and the other does not, rather like a *practical* in which only the instructor *knows how.*” (Wood et al., 1976, pp.89-97). Moreover Wood et al. (1976) speak about “blind” actions, which refers to “(...hardly random) trying-out behaviour” (p.90). They emphasize also that *blind* actions “are [a] necessary condition for the children to discover the nature of the final objective and some of the means for achieving it" (Wood et al., 1976, p.91). While children get a "sense of possible outcomes" and achieve the final objective, the *tutor* takes the role of "activator", whose support is required "in search of the structure of the problem" (Wood et al., 1976, pp.90-91). The *tutor* enables children to learn a subject through his or her instructions in the interaction process. This process is called scaffolding, which is an "interactive system of exchange that tutors operate with an impilicit theory of the learner's acts" (ibid., p.99). How the tutor operates them is defined under six functions:

1. **Recruitment:**

   The tutor's first and obvious task is to enlist the problem solver's interest in and adherence to the requirements of the task.

2. **Reduction in degrees of freedom:** This involves simplifying the task by reducing the number of constituent acts required to reach [a] solution … The *scaffolding* tutor fills in the rest and lets the learner perfect the component sub-routines that he can manage.

3. **Direction maintenance:** Learners lag and regress to other aims, given limits in their interests and capacities. The tutor has the role of keeping them in pursuit of a particular objective. Partly it involves keeping the child *in the field* and partly a deployment of enthusiasm and sympathy to keep him motivated.

4. **Marking critical features:** A tutor by a variety of means marks or accentuates certain features of the task that are relevant. His marking provides information about the discrepancy between what the child has produced and what he would recognize as a correct production. His task is to interpret discrepancies.

5. **Frustration control:** There should be some such maxim as *problem solving should be less dangerous or stressful with a tutor than without*. Whether this is accomplished *by face saving* for errors or by exploiting the learner's wish to please or by other means, is of only minor importance. The major risk is in creating too much dependency on the tutor.

6. **Demonstration:** Demonstrating or *modelling*[49] solutions to a task, when closely observed, involves considerably more than simply performing in the presence of the tutee. It often involves an *idealization* of the act to be performed and it may involve completion or even explication of a solution already partially executed by the tutee himself. In this sense, the tutor is *imitating* in idealized form an attempted solution tried (or assumed to be tried) by the *tutee* in the expectation that the learner will then *imitate* it back in a more appropriate form (Wood et al., 1976, p.98).

Referring to these functions, *scaffolding* enables "a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted efforts" (Wood et al., 1976, p.90). Bruner (1978, 1986) combined the scaffolding concept with a Vygotskyian approach (1981) and termed it "vicarious consciousness". Consciousness is determined as a "reflection", which is a way of keeping the mind from reacting without careful consideration of individuals' words or actions (Bruner, 1978, p.72). Referring to vicarious consciousness, Bruner talks about the act of an adult person who "serves a vicarious form of consciousness until such a time as the learner is able to master his own action through his own consciousness and control" (Bruner, 1986, p.123). Vicarious consciousness refers to "temporary intellectual support that a teacher offers in order to draw the learner up towards a higher level of understanding" (Fernández et al., 2001, p.41).

Anghileri (2006) highlights scaffolding is not a teaching process. According to her, "what is needed for a metaphor of classroom practice is the notion of a flexible and dynamic scaffold in which teachers are responsive to individuals even within the classroom setting" while students learn independently and autonomously (Anghileri, 2006, pp.49-50). I adopt this idea and carry it out in a familial context, which allows me to think about the notion of a flexible and

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[49] Wood et al. (1976) define *modelling* as "an act that is to be imitated" and emphasize that "observed instances of imitation were all of a kind as to suggest that the only acts that children imitate are those they can already do fairly well" (p.99).

dynamic scaffolding in which family members are responsive to each other within the setting of mathematical play situations.

Regarding the "scaffolding" concept, Fernández and colleagues examined if "the concept of scaffolding is made useful for understanding learning in symmetrical groups" (Fernández et al., 2001, p.41). In their work, symmetrical groups refers to peer groups of children/students, whereas asymmetrical groups consist of teacher and children/students. Their results indicate that children in symmetrical talk are following implicit ground rules and they have "same effect as scaffolding without needing any conscious intention" (Fernández et al., 2001, p.53). They also found that in

> asymmetrical interaction a teacher might explicitly plan how to show children an idealized version of a problem to help them understand it, in symmetrical talk the idealized version often emerges in an unplanned way through attempts by children to share understandings and to explain solutions as they work together. (Fernández et al., 2001, p.53).

With the aim of discussing such symmetrical and asymmetrical interaction processes they consider three "social ways of thinking", which are three educationally significant ways of talking and arguing (Fernández et al., 2001, p.42). They define these three types of talk as follows:

a) **Disputational talk:** "Characterised by disagreements and individualised decision-making, and short assertions and counter-assertions … The orientation of disputational talk is more individualised and competitive. Each participant aims to 'win', and so there are no attempts to construct joint understanding or to reason together."

b) **Cumulative talk:** "Speakers build positively but uncritically on what the other has said; it is characterised by repetitions, confirmations and elaborations … The orientation of cumulative talk is to solidarity; it achieves agreement without critiques or reasons being voiced."

c) **Exploratory talk:** "Participants engage critically but constructively with each other's ideas, offering justifications and alternative hypotheses. Knowledge is

> made publicly accountable and reasoning is more visible in the talk, and progress results from the eventual agreements reached … Exploratory talk is dedicated to the common pursuit of the best solutions; it is orientated to critical, co-operative, situated reasoning. All relevant information is shared. Participants strive to reach an agreement and take joint responsibility for decisions. It is expected that participants give reasons for opinions. Challenges are acceptable and alternatives are discussed before a decision is taken. All the members of a group are encouraged to talk by the other members." (Fernández et al., 2001, pp.42-43).

Regarding these types of talk, the results indicate that through the exploratory talk of peers working together on a common task "they all support each other and so travel further, in an intellectual sense, than they would have if using other types of talk or when working alone" (Fernández et al., 2001, p.53). Furthermore Fernández et al. emphasize that disputational talking in symmetrical groups restricts the zone of proximal development of children, whereas exploratory talk in symmetrical groups expands it. Thereupon they define the usage of six scaffolding functions as follows:

> Where, as an "asymmetrical" interaction, a teacher might explicitly plan how to show children an idealised version of a problem to help them understand it, in symmetrical talk the idealised version often emerges in an unplanned way through attempts by children to share understandings and to explain solutions as they work together. The children in symmetrical talk may not be consciously trying to scaffold the development of each other's understanding (as might a tutor), but the implicit ground rules that they are following have this effect anyway (without needing any conscious intention) (Fernández et al., 2001, p.53).

Moreover Fernández et al. highlight that children do not scaffold each other's learning but rather "they achieve this simply by using effective communicative strategies for solving a problem together" (ibid.). In this sense I regard such talk types as different communication styles while combining them with the notion of a flexible and dynamic scaffolding (Anghileri, 2006). Thus, I aim to interpret, define and understand either *situational or situated* interaction processes between family members and children in detail.

Tiedemann (2012a,b,2013) defines different support tasks which are realized by the child-adult dyad during the interaction process in playing games and reading picture books (2013, 2012a,b, 2010). She uses the term “exploration” as one of the types of support which is focused on the child’s exploration in the game, which is rarely restricted in terms of time or method (Tiedemann, 2013, p.2226). The child is “free to explore its ideas, questions and interests and it is the support system that ensures that there is enough room for that purpose and help if necessary” (ibid.). Similarly, Salonen et al. (2007), discuss established interaction patterns between parents and children which “lead cumulatively toward developmental continuities” of children (p.78). Their work deals with the question of how “novel interaction patterns emerge, become amplified, and stabilize over time through a system’s internal regulatory processes” (ibid.). Referring to the concept of scaffolding, Salonen et al. (2007) articulate “the socio-cognitive, emotional and motivational aspects of optimal and non-optimal parent-child guidance patterns in terms of dynamic scaffolding match vs. mismatch” (p.77). In their work, like Anghileri (2006) and Fernández et al. (2001), they regard parental scaffolding as a “dynamic system” and “the interpersonal relational views of dyadic interaction”, which also match the interactionist approach of this study as set out in section 2.2.1 (Salonen et al., 2007, p.90). Consequently, their research about parent-child interaction patterns underscores the developmental importance of coordination or dynamic “match”, i.e., the reciprocity, mutuality and synchrony of parents’ and their children’s behaviours (Salonen et al., 2007, p.79). Moreover Salonen et al. (2007) indicate that adults realize contingent shifting, minimal-sufficiency principles, and motivational and emotional regulation. By means of contingent shifting the adult person “calibrates the task demands to meet the child’s skill levels and varies the quality and amount of assistance according to the changing level of the child’s independent functioning”, while “the minimal-sufficiency principle states that only a minimum of external incentives or rewards, or adult directiveness" should be offered (Salonen et al., 2007, p.79). These realizations are used to maximize the child's autonomous task-related efforts or intrinsic motivation (ibid.). In the minimal-sufficiency principle, "the child is being pulled by the moderate discrepancy or constructive friction toward new levels of independent activity, since the parent's assistance is minimized, and indirect cueing is used" (ibid.). Emotional regulation requires "the parent's ability to detect the child's affective signals and modulate them appropriately", while motivational regulation necessitates "the parent's ability to perceive and respond in a supportive manner to the child's motivational signs such as verbalized expectations and causal attributions" (ibid., p.80).

The ideas of Tiedemann (2013, 2012a,b, 2010), Anghileri (2006), Fernández et al. (2001) and Salonen et al. (2007), lead me to the need to define the features of different interaction dynamics between family members and child. Therefore, in the current study, I prefer to use the three communication styles of Fernández et al. (2001) in order to define features of interaction flux. Hence, I introduce the terms "disputational", "cumulative" and "exploratory" and their definitions into the interaction theory of mathematics learning, which helps me to understand and determine the interaction flux between family members and child. From this point of view I concern myself with the question: "In which ways do family members achieve thinking and learning about mathematical meanings jointly, while they negotiate taken-as-shared meanings within the setting of mathematical play situations?".

Bjorklund, Huber and Reubens (2004, p. 355) "investigate parents' interactions with their young children while playing a board game and examine the relationship between parental behaviours and children's strategy use in a microgenetic study". Their study focused on arithmetical issues, and they perceived playing a board game at home as an "ideal venue for informal instruction and opportunity for parents to teach children important technological skills in a highly motivating" environment (Bjorklund, Hubert & Reubens, 2004, p.348). Regarding sociocultural and cognitive theories, they carried out a microgenetic study of strategy development in an informal social context which aimed to find the correlations between parental verbal behaviours and children's strategic performance. For this, they categorized parental verbal behaviours into six groups: prompt, prompt after error, affirmation, disaffirmation, provide answer and instruction, modelling, re-representation under the group “cognitive directives” (see Fig. 2.9).

Their results indicate that parents are “sensitive to the cognitive and social demands of their children in the different contexts” and “engage their children more in the math context than in the game context and vary their behavior accordingly” (Bjorklund et al., 2004, p.355). Furthermore their results show that “most parents provided appropriate support to their children, giving more prompts and cognitive directives to children who needed them and making fewer such comments to children who did not” (ibid.,p.356). Furthermore, “parents adjust their behavior to the abilities of their children, which in turn is associated with changes in children’s behavior” (ibid., pp.356-357).

Behaviour	*Description*
Prompt	Suggestion by parent as to the generation of an answer without mention of the use of any specific strategy (e.g. "How many do you have?").
Prompt after error	Child makes error in calculation and parent prompts for a recalculation (e.g. "Are you sure?").
Affirmation	The parent demonstrates agreement to child's answer or response to a roll of the dice or a math problem (e.g. "That's right!").
Disaffirmation	This is a type of correction, a definitive negative response indicating an incorrect response; disaffirmation could be an explicit no (e.g. "No, that's not right.").
Provide answer	The parent provides the child with the correct answer or spontaneously produces the answer without the child saying anything.
Cognitive directives	Three types of behaviours that direct the child's thinking about the problem: modelling, instruction, and re-representation.
(a) Modelling	The demonstration of a strategy independent of instruction, that is, the parent models a behaviour for the child to observe and imitate (e.g., the parent counts the dice).
(b) Instruction	When the parent suggests the use of any specific strategy (e.g., "If you have 6 on that one and 2 on that one, you don't have to count the 6—you know it's 6, what's after 6?").
(c) Re-representation	When the parent re-presents the problem in a way that is more familiar to the child (e.g., holding up fingers to represent the dice or addend).

Fig. 2.9. Parental behaviours coded by Bjorklund, Hubert and Reubens (2004, p.351)

Bjorklund and colleagues concluded that “each parent–child pair needs to be examined in greater detail in order to increase the accuracy of predictions” about specific patterns of parent or child behaviour in a play context (ibid., p.357). Regarding this discussion of Bjorklund et al. (2004), I consider only the coded parental behaviours (2004, p.351) in order to examine dynamic interaction processes between family members and the focus child in detail. Therefore, I prefer to use the term “activities of family members” instead of “parental behaviours” on account of my research interest. In this sense when observing the activity of family members during the interaction process, I consider these coded behaviours from Bjorklund et al. (2004).

In relation to self-regulation theories, Vermunt (1998) works on students’ learning models and their qualities. To gain some initial understandings about the “regulation of high-quality learning”, Vermut researched the learning activities, strategies, conceptions and interpretations of students (Vermunt, 1996). He found that, in usual classroom situations, external regulations by the teacher are less productive than self-regulation by students and concluded that to improve the quality of student learning the classical instructional designs of agents should exert less control over the learning process (Vermut, 1998). Moreover, he points out that “learning is not a passive, knowledge-consuming and externally directed process, but an active, constructive and self-directed process in which the learner builds up internal knowledge representations that form a personal interpretation of his or her learning experiences” (Vermunt, 1998, p.150). Regarding this he states that external regulation is a large component of a reproduction-directed learning style, whereas self-regulation of learning processes is positively associated with the use of all processing strategies, which students employ analytically, thoroughly and in detail (Vermunt, 1998, p.167). In external regulation of learning processes learners should “be directed mainly by the didactic measures in the study materials or of the lecturers” gradually (ibid.).

This work of Vermunt (1998) leads me to the idea that external regulation and self-regulation approaches lead children to a learning process, but how far and in which way should be queried. In this regard my study aims to observe and examine the ways children learn regardless of which regulation and participation types emerge in the play situation with family members. Most self-regulation theories approach the learning process from the metacognitive and cognitive perspective, in which learning is perceived as an "acquisition" (Sfard, 2008). Here it is important to emphasize that I approach learning processes with a "participationist perspective" (Sfard, 2008)[50]. Regarding interaction theory of teaching and learning mathematics, the learning process is regarded as the occurrence of increasing autonomy (see Schütte, 2009; Miller, 1986). In this respect the self-regulation approach can help me only to interpret possible or potential learning situations by means of the child's contribution (see Schütte, 2009; Miller, 1986).

2.2.4. Interactional niche in the development of geometrical and spatial thinking in the mathematical familial context

Considering some of the results of the erStMaL-FaSt project, we know that learning mathematics in the early years occurs in different forms of participation as young children experience various mathematical situations within their families (Acar Bayraktar in press.-a,b; 2014a,b,c,d; 2012,1,b; 2011a,b; Acar & Krummheuer, 2011, 2014). In an attempt to describe the effect of the negotiation of taken-as-shared meaning in the process of children's mathematics learning in the familial context, we adapted the concept of NMT to the create the concept "interactional niche in the development of mathematical thinking in the familial context" (NMT-Family) (Acar & Krummheuer, 2011).

Like the concept of NMT, NTM-Family is particularly based on symbolic interactionism (Blumer, 1969), the cultural historical approach of Vygotksky and Leont'ev (see Wertsch & Tulviste, 1992; Ernest, 2010; Bruner, 1996), the phenomenological sociology of Alfred Schütz (Schütz & Luckmann, 1979) and its expansion into ethnomethodology (Garfinkel, 1972) (Krummheuer, 2012, p.321;

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[50] For more see section 2.2.2.2.2.

see also Acar Bayraktar in press.-a,b; 2014a,b,c,d; 2012,1,b; 2011a,b; Acar & Krummheuer, 2011, 2014)

The present study examines the learning of geometry, as one of the mathematical domains, in the early years in the familial context. Therefore, I am specifically interested in interaction processes between family members and children while they are playing with wooden blocks and negotiating family norms, including those that are specific to mathematics. In this regard symbolic interactionism helps to emphasize the individual's sense-making processes as well as the social processes, to examine and to deduce individuals' learning in detail. Considering this research interest (family interactions during block play) it is clear that the phenomenological sociology of Alfred Schütz (Schütz & Luckmann, 1979) and its expansion into ethnomethodology (Garfinkel, 1972) can help to seek and to find out how family members produce patterned courses of action; how they share internalized value systems and meanings of others, and through which methods and procedures they generate cognitive-interpretive solutions. Keeping these questions in mind, the present study aims to understand how family members "cope with the task of describing and explaining the order of the reality in which they live" (Overgaard & Zahavi, 2009, p.18). Addition to this, the cultural historical approach helps me to observe and interpret the negotiation of taken-as-shared meanings of family members during play with building blocks. While I am trying to understand how children and family members manage meanings, which are described and explained in the common social reality of their lives, I can also examine how they perform their own realities during block building activities. In sum, this concept offers the possibility for close analyses and comparisons between familial mathematical learning opportunities for children in early childhood and preschool ages.

As a consequence, the concept of the "interactional niche in the development of geometrical and spatial thinking in the mathematical familial context" (NMT-Family) (Acar & Krummheuer, 2011) is determined as the main concept of this study.

NMT-Family is constructed as a sub-concept of NMT. Similar to the concept of NMT, it consists of the aspects of allocation, situation and the child's contribution (Acar in press-a,b; 2014a,b,c,d; 2011a,b; Acar & Krummheuer, 2011, 2014). Allocation refers to the learning offerings provided by a group or a society which specifically highlight cultural representations. The aspect of situation consists of the emerging performance occurring within the process of negotiating meaning. Lastly, the aspect of the child's contribution is the situational and individual contribution of the particular child.

In Table 2.8, the three components of NMT-Family are described in relation to the components of content, cooperation and pedagogy and education.

Table 2.8 The structure of NMT-Family (Bayraktar, 2014c)

NMT-Family	**2.2.4.1** **component: content**	**2.2.4.2** **component: cooperation**	**2.2.4.3** **component: pedagogy and education**
aspect: allocation	2.2.4.1.1 mathematical domain: geometry	2.2.4.2.1 play as a familial arrangement for cooperation family system	2.2.4.3.1 developmental theories of mathematics education and proposals of active participation of family members on this theoretical basis
aspect: situation	2.2.4.1.2 interactive negotiation of the rules of play and the content framing	2.2.4.2.2 leeway of participation immigration	2.2.4.3.2 folk theories of mathematics education, everyday routines in mathematics education
aspect: child's contribution	2.2.4.1.3 individual actions	2.2.4.2.3 individual participation profile	2.2.4.3.3 competence theories

Each cell of Table 2.8 is described in the following sub-sections, while determining addtional theoretical backgrounds:

2.2.4.1. Content

2.2.4.1.1. Content × Allocation:

In the practice of the family study, children and their families are confronted with mathematical play situations. At this part the subject mathematical domain and its features are taken into consideration. For this study, the subject mathematical domain is geometry (for further theoretical insights see section 2.1).

2.2.4.1.2. Content × Situation:

The play situations are designed to offer families different opportunities to negotiate meanings interactively. The rules of the play situation and/or mathematical topics are the focus of the negotiation processes between family members. Therefore, interactional theory on mathematics teaching and learning is considered (see section 2.2.1). How the negotiation processes between family members are formed in the play situation, how the situation is reconstructed, if they achieve a mutual or taken-as-shared understanding of an object or event, if negotiation processes between family members lead the argumentation to a working consensus or working interim, are some of the questions that can be answered in this part.

2.2.4.1.3. Content × Contribution:

The child in focus might contribute more or less actively to the negotiation processes in the play situations. In such processes either different forms of efficient and original ideas can be expressed and realized, or the activities of other participants can be pursued by the child (see Krummheuer & Brandt, 2001). How the child frames and/or reframes the situation, in which way the child interprets the negotiation process and takes action, in which way he/she achieves a mutual or taken-as-shared understanding of an object or event, are some of the points that can be addressed in this part.

2.2.4.2. Cooperation

2.2.4.2.1. Cooperation × Allocation:

In play situations family members can cooperate with each other. The process of cooperation between family members and the child provides different opportunities to refine their thinking and make their performance more effective (see section 1.4.1.). How family members interact with each other and how their family systems take a role in the negotiation processes, should be taken into consideration. Therefore, family systems theory is described above in detail (see section 2.2.2.).

2.2.4.2.2. Cooperation × Situation:

Depending on the cooperation process, different leeways of participation of the family members and of the child can emerge. In this sense, different types of participation in the interaction encourage the child to explore, by co-constructs, aspects of the cultural environment during play (Brandt, 2004, pp.32-43).

The current part enables us to see clearly which types of participation of family members occur in the interaction process. While they negotiate about mathematical meanings between each other, how their "leeway of participation" is generated is the subject observed in this part.

With the exception of recipient and production designs of interaction theory (Krummheuer & Brandt, 2001), I have also considered another works in which the learning situations through participation in social engagements are studied (Lave & Wenger, 1991; Sfard, 1998, 2001, 2008) (see section 2.2.1.). My aim is to keep a broad scope of notations of "different leeways of participation" and to diagnose them openly during observation processes in this research.

The term "leeway of participation" is a translation of "*Partizipationspielraum*" (Brandt, 2004) and means "room for freedom of action" (Krummheuer, 2012, p.322). It focuses on individuals as unique learners, and the kinds of responsibilities they take in the joint creation of the interaction (Krummheuer & Brandt, 2001). In this regard leeway of participation is „a result of the culture the participants share" (Krummheuer, 2012, p.322; see also Krummheuer & Brandt, 2001; Krummheuer & Fetzer, 2005).

In the course interaction "the individual's reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings" (Yackel & Cobb,1996, p.460; in Brandt, 2002, p.378). Each individual makes use of leeway of participation which enables a "comprehensible" development of mathematical thinking (Krummheuer, 2012, p.322). By observing and imitating actions of other participants and taking active influence on the course of interaction manage the statuses of each individual's participation (Krummheuer, 2012). Each individual explores and co-constructs their cultural environment which enables the increasing autonomy of the individual in its specific type of participation in accomplishing mathematical situations (Krummheuer, 2013a). Each individual takes part and/or becomes part of the interaction processes of the mathematics classroom (Fetzer, 2007, p.1212; see also Fetzer, 2006).

Based on the three different interactionist approaches in interaction theory (Brandt & Krummheuer, 2001; Lave & Wenger, 1991; Sfard, 2008), different forms of participation are taken into consideration in the current study. Regarding all these facts, the participation types of family members should be observed and analysed in this part of NMT.

2.2.4.2.3. Cooperation × Contribution:

Brandt (2004) explains that the participants interactively accomplish different versions of leeways of participation that are conducive or restrictive to the mathematical development of a child (see also Krummheuer, 2011c, 2012). The children are involved in social settings in the play situations, which are variously structured as in child-parent interactions and/or child-sibling interactions. These social settings need to be fulfilled in the process of interaction. In this way, different "leeways of participation" for the child emerge (see Brandt, 2004, 2006), in which individually different participation profiles of the child are generated in the joint interaction.

The current part of NMT enables us to see clearly which types of participation the child in focus should display in the interaction process. While he/she negotiates with family members about mathematical meanings, how his/her "leeway of participation" is generated is the subject observed in this part.

With the aim of interpreting and observing clearly the learning situation of the child in focus in the current study, I now refer to some social cognitive research in which different regulation types are studied (Nader-Grosbois et al., 2008; Jackson, Mackenzie & Hobfoll, 2000; Zimmerman, 2000; Boekaerts,1999; Vermunt, 1998; Deci, Ryan & Williams,1996). In this way "sensible and tenable" occurrences for the child "develop his/her content-related understandings" (Krummheuer, 2000c, p.22) and background knowledge during negotiation of meanings can be diagnosed and interpreted. As a matter of course the concepts of learning-as-participation, *situated* learning (Lave & Wenger, 1991) and *situational* learning (Krummheuer & Brandt, 2001) should be considered in this part too.

Referring to all theoretical background about different types of self-regulation (see section 2.2.3.) and forms of participation (Brandt & Krummheuer, 2001; Lave & Wenger, 1991; Sfard, 2008), one can speak about different types of participation of children. In this sense, the role-taking process is assumed to be continually changing towards increasing autonomy for the learner (Krummheuer, 2000c). Moreover, through the manifestation of self-regulation and external regulation during the negotiation of meaning, the participation profiles of the child during interaction processes can be named "self-regulatory participant" for the self-regulated child and "external-regulatory participant" for the externally regulated child. In this regard self-regulation types and different forms of participation (Brandt & Krummheuer, 2001; Lave & Wenger, 1991; Sfard, 2008) are taken into consideration in this part of NMT, while observing and analysing childrens' contribution in my data in the current study.

2.2.4.3. Pedagogy and Education

2.2.4.3.1. Pedagogy and Education × Allocation:

Developmental theories and theories of mathematics education describe and delineate learning paths in the familial context for children's mathematical growth (Bayraktar & Krummheuer, 2012). For this part the theoretical features of teaching and learning geometry should be taken into consideration (see section 2.1.).

2.2.4.3.2. Pedagogy and Education × Situation:

Bruner emphasizes that folk pedagogy reflects a variety of assumptions about children: they may be seen as wilful and needing correction; as innocent and to be protected from a vulgar society; as needing skills to be developed only through practice; as empty vessels to be filled with knowledge that only adults can provide; as egocentric and in need of socialization (1996, p.49). Brandt highlights that the different concepts of folk pedagogy of Bruner are linked to different concepts or ideas of "mind" and "knowledge" in general (Bruner, 1996, p.50 f.), which offer the opportunity for content-related deliberations of different instruction practices (Brandt, 2014, p.57). With respect to these ideas of folk pedagogy (Bruner, 1996, see also Brandt, 2013, 2014), the participating adults and children become situationally active and operant in the concrete interaction process. The current part thus enables us to see clearly to what kind of learning opportunities the child is exposed. Moreover, during content-related deliberations the kind of instruction practices realized by parents is another subject observed in this part. For this, I refer to various theoretical works. Regarding folk psychology and folk pedagogy, how family members negotiate meanings with children is the main subject. Therefore, a broad scope of terms (see sections 2.2.1., 2.2.2., 2.2.3.) about the plane of interpersonal interaction should be considered in this part in order to observe situational or situated negotiation processes openly.

Folk psychology and folk pedagogy (Bruner, 1996)

Folk psychology refers to (in the case of this study) how family members' minds work while engaged mathematics teaching and learning and what kind of knowledge about each other the family members have. For example, while a child is playing with his mother, the mother provides her own folk theories for guiding her child's learning. By guiding him, she regards herself as teaching her child something that he does not know. Olson and Bruner (1996) define folk psychology and folk pedagogy as follows:

> Not only are we steered in ordinary interaction by our folk psychology, but we are steered in the activity of helping children learn about the world by a body of assumptions that make up what we may call "folk pedagogy". Folk pedagogy is visible in many contexts: Watch any mother, any teacher, even any baby-sitter with a child and you will be struck at how much of what they do is guided by notions of what children's minds are like and how one may help them learn, even though they may not be able to verbalize their pedagogical principles. (p.10)

Correspondingly, folk pedagogy refers to "the roles of instruction in fostering learning" (Strauss, 2001, p.242). For example, while a child is playing with her mother, she is guided by her mother (through her folk psychology); the meanings that the child comes to learn is the topic of folk pedagogy. In this sense, folk pedagogy works on the ways that children learn. Namely, folk pedagogy is "based on the idea that is possible to help somebody else to learn" (Krummheuer,1999, p.360). Similarly, Bruner demonstrates that children learn insights from stories they are told by family members, and the way of "learning from stories" is an independent mood (cf. Bruner, 1996; in Krummheuer, 1999, p.333). In that regard each child is responsible for his/her own learning and thinking, and each learning mechanism develops through the child's perspective, which is from the world that he/she lives in (ibid.). It is important to notice that folk psychology and folk pedagogy are concepts about the conceivability of learning and teaching. In this sense they do not necessarily have to be *intentional* and can be realized through interaction as a background of the interpretation of family members (see also Brandt & Tiedemann, 2011; Naujok, 1999).

Considering this idea, I would like to think about folk psychology and folk pedagogy from the perspective of family research. For example, in the work of Hawighorst (2005) there is an interview with an immigrant mother who was a participant in that study. Regarding the use of computers in the German mathematics lessons, the mother tells the difference between her children's and her own childhood experiences in school:

> Our teacher always gave us exercises that could be applied to real life. For example we have to calculate the size of our garden. Or we where to measure the size of a plot of land. Very tangible… At school they (her children) flit around with their heads in rose-colored clouds. They don't view life realistically. (Hawighorst, 2005, p.95).

Her comparison can be used to illustrate the two ideas:

- "folk psychology" refers to real-life exercises, which are very tangible, such as those the mother experienced in her childhood.
- "folk pedagogy" refers to the mother's teaching approach to her children, which is that the children don't view life realistically because of the use of computers in mathematics lessons and that they should apply tangible experiences to learn mathematics.

But how do family members share their background knowledge and how do they impart their folk theories to their children? To answer these questions, one should consider the theoretical background of parental regulation. For this I refer to Nader-Grosbois and colleagues (2008), in which they found that the "correlations between parents' regulation and children's self-regulation strategies indicated positive links concerning strategies relating to joint attention and to motivation; and negative links concerning strategies relating to the exploration of means and evaluation" (p.95). One should also consider that scaffolding emerges from the family member-child interaction and it determines the occurrence of six scaffolding functions. Hammond et al. (2012) emphasize that scaffolding "requires heavy involvement [by] parents" (p.274). Regarding this idea I assume for this study that scaffolding demands of heavy involvement by family members and child. Moreover, I conjecture that each individual's backing down and observing initiates the external regulation or scaffolding process, which is maintained and developed by the quality of interaction between individuals (see sections 2.2.1 and 2.2.3.).

2.2.4.3.3. Pedagogy and Education × Contribution:

In terms of the child's participation profile (see Brandt, 2004, 2006), the learning process of the child can be characterized. The child is intuitively able to describe the change and/or progress of his/her own participation in the development of mathematical thinking. In this sense appropriate theories like "self-regulation", "external regulation" (see Nader-Grosbois et al., 2008) or "situated learning" (see Lave & Wenger, 1991) can be grouped under the title "competence theories".

Beside all of the above, one must also consider and take into account the learning theories mentioned in the sections "Mathematical thinking and learning in the early years in the family" (see 1.4.1.4.), "Learning and teaching geometry" (2.1.3.),"Block play" (2.1.4.) and in the part "Cooperation × Situation" (see 2.2.4.2.2.) for this part as well.

Being enlightened by theoretical perspectives, the scaffolding processes and regulation and communication types mentioned above seem to enable children to experience a "cooperative learning" process through "collective argumentation" (Krummheuer, 2007a; see also section 2.2.1.). In such a process, symmetrical and asymmetrical groups can either decline or benefit from the *tutoring* or *scaffolding* of an *expert.* Furthermore, children can create for themselves a learning situation through relatively favourable cooperation with each other. Here the solution strategies of the game can also bring to undesired perfection, which is technically unintended by the *expert/tutor* (Krummheuer, 2007a).
Bruner highlights that "there is not one kind of learning and any learner has a host of learning strategies at [his/her] command" (1985a, p.8). For example, during a play situation children need "some periods of solitude to combine their ideas from their own head with the ideas that their partners have in theirs"

(Bruner,1983, p.69). Thereby children can "furnish ideas with those models and techniques for how to operate on [their] own" (ibid.) and "organize information in the mind by relating concepts together" (Brahier, 2009, p.53). Similarly Paradise and Rogoff point out that learning can be also "observational", while children participate with keen attention to ongoing events (2009, p.107). In this sense individuals equip each other's menus of learning possibilities in the course of their interactions. Collective and cooperative processes promote the range of these menus, when individuals profit from them wisely (see Bruner 1985a). Therefore children as individuals should be free to explore different ideas, questions and interests at any rate, technically intended or unintended, in the mathematical situation. Hereby collective and cooperative processes make it possible for children to learn and explore mathematics.

2.3. MLSS

During "the situationally emerging form of participation of a child in a social encounter", in this case a mathematics learning process, one should note the concept of support system (Krummheuer, 2014,p.72), along with Bruner's term "format"[51] (Bruner,1983). With this term, Bruner (1983) models how language acquisition of children occurs through telling stories in the family, while contingently an adult takes the role as "teacher" and children take the role of "learners" (ibid.). Moreover, these functions are the principal vehicle for the language acquisition support system (LASS) (Krummheuer, 1995, p.254). Bruner talks about LASS is a specific pattern of interaction in the child-adult discourse, through which children become engaged in social relationships with the use of language. In his later work, Bruner emphasizes that language learning takes place when children participate in these formats in the course of story-telling (1990). Moreover, according to Bruner, through negotiation of taken-as-shared meanings children accomplish an interaction process with the adult person

[51] Format means a "standardized, initially microcosmic interaction pattern between an adult and an infant that contains demarcated roles that eventually become reversible" (Bruner, 1983, p.120). For more see section 2.2.1.

whereby language acquisition of the child can emerge. The origin of patterns of interaction generates Bruner's LASS (1983, 1990) and Krummheuer and Schütte (in press) define it as follows:

> neither the mother nor the child applies these formats intentionally (…) they emerge spontaneously between mother and child in the course of interaction that is regulated by the interaction moves chained in the ongoing reiteration of the "adjacency pairs" of initiation and reply.

Therefore Bruner (1983, 1986, 1990) determines LASS as a concrete interaction form between child and adult (Bruner specifically makes mention of mother), which is not only the "genetically given capability of the human species to learn its mother tongue" but also "the cultural embeddedness of the child's development that provides a systematic backing (framing) for its development" (Krummheuer & Schütte, in press). For this, the acquisition of mother tongue is taken into consideration as "the prime example of a socio-constructivist conceptualization of any fundamental learning process in early childhood" (Krummheuer & Schütte, in press).

Similar to Bruner (1983, 1986), Rogoff points out that "the mutual roles played by children and their caregivers rely both upon the interest of the caregivers in fostering mature roles and skills and on children's own eagerness to participate in adult activities and to push their development" (Rogoff 1989, p.209; see also Rogoff, 1990). Tiedemann promotes Rogoff's idea and emphasizes that support for learning takes place within social activities, while the child supports his or her own learning as much as the mother does. In her work Tiedemann observes the engagement of child-mother dyads in playing games and reading picture books (2013,2012a,b, 2010). She defines support systems as "projects of cooperation" through which child-mother dyads establish different but certain kinds of support (Tiedemann, 2013, p.2220; see also Tiedemann, 2012a,b, 2010). With the purpose of defining such support systems in detail, Tiedemann adopts the concept of "discourse acquisition support system" (DASS) (Hausendorf and Quasthoff, 1996, p.301).

In coining the term DASS, Hausendorf and Quasthoff (1996) took the LASS concept of Bruner (1983, 1986) and regenerated it for the emerging support system in conversation, by which the development of children in the course of story-telling is influenced (1996). With respect to Bruner's concept of LASS and Hausendorf and Quasthoff's DASS (1996), Tiedemann proposes a similar concept for the learning of mathematics, which, by analogy, she called "mathematics acquisition support system" (MASS)[52] (2010a, 2012a,b, 2013). The two theoretical concepts (Bruner, 1983, 1986, 1990; Hausendorf & Quasthoff, 1996) thus led Tiedemann (2013) to ask two main questions: What is ensured by the MASS? and What does it focus on?

According to Tiedemann, support for learning early years mathematics represents helping young children become fluent in a mathematical discourse (Tiedemann, 2013, p.2219). Based on Bruner (1983, 1986) and Hausendorf & Quasthoff (1996), Tiedemann assumes that "families establish a support system for mathematical learning processes by means of their everyday discourses" (Tiedemann, 2010a, p.154; 2012a,b, 2013) and this system can be defined as follows:

> Mathematischer Support besteht in Interaktionsmustern und -routinen, die von Erwachsenem und Kind gemeinsam realisiert werden und das Kind darin unterstützen, an einem mathematischen Diskurs teilzuhaben. (Tiedemann, 2010, p.154).

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[52] In her works Tiedemann emphasizes that she mentions such support systems for mathematics learning instead of mathematics acquisition, although in her concept she uses the term mathematics *acquisition* support system (2012a,p.110). Here her aim is not to draw attention to the use of each word but rather she aims to keep the mode of expression of mathematics support system, namely, by analogy with the term used for a support system in language learning by Bruner (1983). In the current study, the theoretical background includes the participation metaphor of Sfard (2008) (for more see 2.2.2.2.2.). Therefore, principally, the use of each word has an important and critical value in the current study. With the aim of avoiding confusion, I keep Tiedemann's term MASS as she wrote it.

Tiedemann describes a mathematics support system as the realization of patterns and routines in an adult-child interaction, in which the child is supported to participate in a mathematical discourse. Like Bruner (1983,1986), Tiedemann points out that the support system is generalized in a concrete social encounter by two persons (at least) and it is a “certain kind of format that is established by the interlocutors” (Tiedemann, 2013, p.2220; see also Krummheuer & Brandt, 2001; Krummheuer, 2007a). Tiedemann also identifies support systems which are created by child-mother dyads relating to a *situational* context. Tiedemann concludes that support systems do not depend on any material or any point of time but rather they vary during interaction processes with reference to their focus (2010a, 2012a,b, 2013).

By analogy to Tiedemann’s concept of MASS (2010a, 2012a,b, 2013), in the current study, support system for mathematics learning is redefined by reference to Sfard’s idea of “learning-as-participation” (2008). In addition to this, one considers also the idea that mathematical discourses are “determined by their specific demands of proposing and justifying their statements subsumed under the notion of proof” (Krummheuer 2013a p.252) and that for learning mathematics a child should experience such discourses in terms of collective argumentation process (see section 2.2.1.).

Regarding this idea and the abovementioned interactionist approaches (sections 2.2.1 and 2.2.2), a “mathematics learning support system” (MLSS) (Acar Bayraktar & Krummheuer 2011a; Acar Bayraktar, 2011a,b,2012a,b,2014a,b,c,d; in press-a,b; Krummheuer, 2011b, c) is constituted and defined as follows:

> Wir verwenden den Begriff „Mathematiklernen“, wenn wir diesen integralen Prozess von Erwerb und Denkentwicklung meinen. Der Begriff des „Erwerbs“ beziehe sich auf mathematische Begriff e und Prozeduren und der Begriff der „Denkentwicklung“ auf die argumentativen Zusammenhänge, die zur „logischen“ Herleitung derartiger Begriffe und Verfahren führen und die damit verbundenen Praxen des Argumentierens. Mit Blick auf diese terminologischen Regelungen soll das für Mathematiklernen zu unterstellende doppelt

funktionale Supportsystem Mathematics Learning Support System (MLSS) genannt werden. (Krummheuer, 2011c, p.34)

By dint of "taking part" and/or "being part" (Fetzer, 2007b), in such systems learning occurs as an ongoing and dynamic activity through negotiation of taken-as-shared meanings. The learning of mathematics can be perceived as comprehensive only when it is assumed to be not only a social but also an active process. Therefore, different types of MLSSs can be reconstructed during such negotiation processes in a concrete social and mathematical encounters.

In terms of Bruner's format (1983,1986,1990), Krummheuer and Schütte (in press) reconstruct a two-fold adaptability in the "microcosm" of NMT. One aspect is the adaptation which "can result in a pattern of interaction that one could characterize as a format" and this leads to the MLSS (Krummheuer & Schütte, in press). In such interaction process, the process "adapts itself to the possibilities of participation of the involved children in that it generates a kind of conversation that enables at least some children to contribute actively to this interaction" (ibid.). In NMT, this fold is characterized as the interface of allocation and situation aspects. The second aspect mentioned by Krummheuer and Schütte is the adaptation of individual himself to the pattern of interaction by constructing "appropriate changes in his definition of the situation to a commonly shared interpretation" (ibid.). The individual uses "this patterned process of negotiation as his MLSS and the changes in his definition of the situation are an expression of his cognitive achievement of adaptation" (ibid.). Thus, the individual can act with increasing autonomy in the evolving format in the situation and namely learn mathematics. Regarding this idea of adaptation, this aspect is characterized as the interface of the situation and contribution aspects of NMT (Krummheuer & Schütte, in press).

Interestingly Sfard's (2006) idea supports the idea of Krummheuer and Schütte (in press) about learning mathematics. She perceives learning mathematics as "individualizing mathematical discourse, that is, as the process of becoming able to have mathematical communication not only with others, but also with onself"

(2006, p.162). Sfard's idea strengthens the idea of two-fold adaptability (Krummheuer and Schütte, in press) which leads me to think about the occurrence and functioning of MLSS in the familial context by virtue of NMT-Family.

For the familial contexts one can so far say that learning mathematics in the early years emerges in different forms of participation (Acar 2011a, p.1861; Acar Bayraktar, 2014a,b,c). Children experience mathematical situations in their families, while support occurs not only through "correct instructions" but also through "incorrect instructions" (Acar Bayraktar,2014a). In this sense children and family members construct new definitions of the situation through the negotiation of taken-as-shared meanings and this process reflects the mode of functioning of MLSS (Acar Bayraktar & Krummheuer, 2011; Acar Bayraktar, 2014a,b,c,d;in press-a,b). Therefore, in the current study, MLSS is perceived as part of a familial "micro cosmos" that emerges during such play situations.

To understand the functioning of MLSS in a familial context more deeply, I use the concept of NMT-Family (see section 2.2.2.), which enables me to observe the functioning of MLSS in great detail from a participational and developmental perspectives. Moreover, this concept sheds light on two questions regarding the functioning of MLSS:

- Which kinds of 'format' (Bruner, 1983) provide a learning situation to the child in a familial system? and,
- How do these 'formats' provide a learning situation in different observation phases?

Regarding these questions the current study aims to identify the ways of occurrence and functioning of MLSS.

2.4. Research Questions with Theoretical Aspects

Regarding all the abovementioned theoretical aspects and the structure of erStMaL-FaSt, my research questions consist of two main questions:

1. How does interactional niche occur in the development of geometrical and spatial thinking in the German-Turk familial context?
2. How does a mathematical learning support system occur and function in the familial context?

The family provides plenty of opportunity for children to play, explore and make positive contributions to each other's lives. Therefore how families combine their values and beliefs about how children should develop when they provide support for the learning of academic subject knowledge, such as mathematics, is not well understood (Acar Bayraktar, in press-a). In this regard in the current study the participation of family members in block play is examined in order to answer the research questions. Until now, my research results have indicated that during each play situation with the participation of a family member, whether father, mother, sibling or grandmother, the child explores something about the mathematical topic. The presence of family members provides children with different learning opportunities that arise from the interactive process in regard to the negotiation of taken-as-shared meanings about the mathematical block play. In this way, different forms of participation and support emerge during the interaction processes between family members. As already mentioned in theoretical approaches, the cognitive development of an individual is bound to their participation in a variety of social interactions. Moreover, individuals can support their own and others' development while they are negotiating with each other interactively.

In this regard empirical findings and results in the current study can shed light on the question in which way and how much family members should expose children to block play situations before primary school age.

3. Materials and Methods

Based on theoretical aspects presented in chapters 1 and 2, I regard NMT-Family and MLSS as phenomena which emerge between child and family members in the interaction. For this reason, my research materials consist of video-recordings in which family members engage in block play and transcripts of them. Such video-recordings enable me to observe play situations repeatedly and to interpret procedures and patterns of interaction between family members respectively. Moreover, the transcripts enable me to consult and analyse these practices of family members. They thus help me to reflect, to see and to reconstruct the factors that emerge in the process of negotiation of taken-as-shared meanings.

For this study, I chose three families from the eight participant families of the erStMaL-FaSt project based on their ethnic background, the duration of the formal education of the parents and the sibling situation within the families (see also section 1.4.). The three families are referred to as the Gül Family, the Ak Family and the Kil Family. From collected data of erStMaL-FaSt, I chose two play situations from the first and second observation phases of these families. These mathematical play situations are games called “Building 01” and “Building 02”, which are based on block play. Then I scrutinized and examined the initiated and recorded interactions of these families in these play situations regarding the theoretical backgrounds mentioned above (see sections 2.1 and 2.2).

In the next section, I describe first both play situations and then give further information about my observation material in more detail. Then I present my research methodology, which is based on the analysis of interaction and participation, and a comparative analysis (Krummheuer & Brandt, 2001).

3.1. Material: “Building 01” and “Building 02” Games

The example mathematical games selected from erStMaL-FaSt (see section 1.4.) are “Building 01” and “Building 02”, which refer to spatial thinking and are based upon the game “Make ‘n’ Break” (Lawson & Lawson, 2008). In the game, families are supposed to build a three-dimensional version of a geometrical shape depicted on a card with wooden blocks which fit easily in the children's hands. The play situation offers families different opportunities to perform the relations between 2D and 3D representations.

In each mathematical play situation, the playing cards and wooden blocks are provided by the research team of erStMaL-FaSt (see section 1.4.4.). "Believing that materials with less obvious purposes provide more possibilities for imaginative play" (Tepylo et al., 2015, p.23), both the playing cards and wooden blocks are designed with uniform size, colour and weight (see Fig. 3.1)

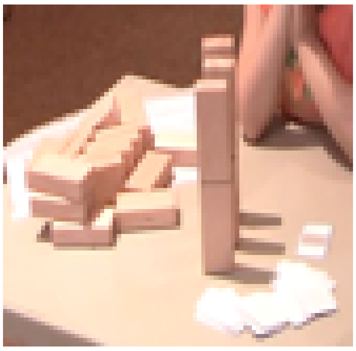


Fig. 3.1. The materials provided in the mathematical games "Building 01" and „"Building 02"

In each round, one player chooses a card from the deck and builds the figure depicted as an image, or in other words a 2D representation, on the card (see chapter 2.1.). The aim of play is to build a figure, as seen on the chosen card, properly with the wooden blocks and thereby to gain points; the player with the most points is the winner. To check the compatibility between the built figure and the figure seen on the card, the other players should examine if the built figure is correctly built or not and give feedback. If it is correct, then the player is awarded the number of points shown on the card.

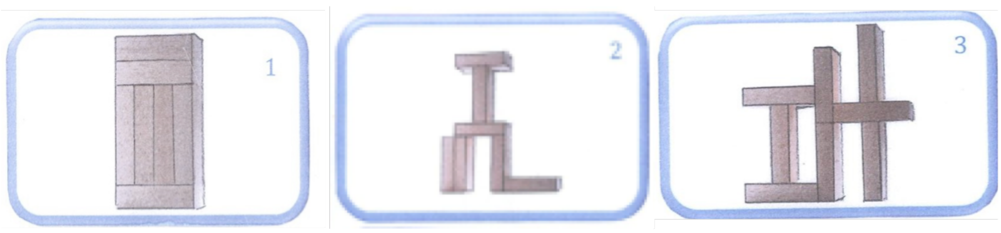


Fig. 3.2 The game cards in different levels of Building 01

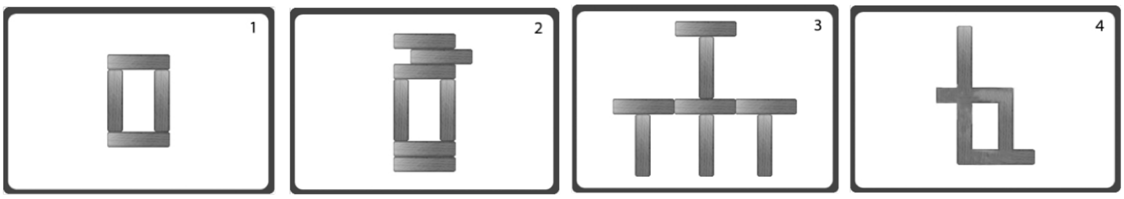


Fig. 3.3 The game cards in different levels of Building 02

In the game, cards are placed on the table face down. Each card has a difficulty level ranging from 1 to 3 in the game “Building 01” (see Fig. 3.2), whereas the difficulty levels range from 1 to 4 in the game “Building 02” (see Fig.3.3). Levels of cards are written on the cards and each card shows how many points a player will erceive if he/she builds the figure correctly. The cards with the number 1 are the easiest and the cards with the number 4 are the hardest. In total, players play five rounds by turns of each player in both games. Both “Building 01” and “Building 02” have the same play structure, they differ only in the difficulty levels of the figures.

the original specific design pattern of play „Building 01“ in German	**Translation of the specific design pattern of play „Building 01“ in English**

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

IDeA Individual Development & Adaptive Education

Spiel- und Erkundungssituation im IDeA-Projekt (Familienstudie)

Bauen01F (B01F)

erStMaL-Studie (Stand: 04.03.2010)

Kurzbeschreibung der Spiel- und Erkundungssituation	Wie kann die Spiel- und Erkundungssituation beschrieben werden? In der Spiel- und Erkundungssituation können die Kinder und ihre Familien Quadergebäude von zweidimensionalen Abbildungen mit entsprechenden Quadern nachbauen.
Einsatzbereich	Welcher Einsatzbereich ist für die Spiel- und Erkundungssituation vorgesehen? (Familienstudie, Erstbesuch/Zweitbesuch, Altersbereich) • Familienstudie • Erstbesuch • 4+ Jahre
Mathematischer Bereich	Welchem mathematischen Bereich kann die Spiel- und Erkundungssituation zugeordnet werden? Geometry and Spatial Thinking
Mathematischer Gehalt	Wie kann der mathematische Gehalt der Spiel- und Erkundungssituation beschrieben werden? Objektdarstellung durch einfache Mittel, Vereinfachung, Abstraktion; Zusammenhang zwischen zwei- und dreidimensionalen Darstellungen: Kavallierperspektive in ein dreidimensionales Modell übertragen
Material und Raum	Welches Material wird für die Durchführung der Spiel- und Erkundungssituation benötigt? Wie muss der Raum für die Spiel- und Erkundungssituation vorbereitet sein? • Holzquader in ausreichender Anzahl • Verschieden komplexe Abbildungen von Quadergebäuden auf Spielkarten (Komplexität wird durch Punktzahlen kenntlich gemacht)
Impulse und Hinweise für die Familien	Welche Impulse und Hinweise benötigen die teilnehmenden Familien, um an der Spiel- und Erkundungssituation teilzunehmen? **Spielregeln:** • Die Spielkarten werden gemischt und verdeckt auf einem Stapel in der Mitte des Tisches platziert. • Die Teilnehmenden ziehen nacheinander eine Karte und versuchen jeweils die darauf abgebildeten Gebäude nachzubauen. • Es werden fünf Runden gespielt. Danach vergleichen die Teilnehmenden ihre jeweils erspielte Anzahl an Punkten auf ihren Karten.

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

LOEWE Exzellente Forschung für Hessens Zukunft

IDeA Individual Development & Adaptive Education

Mathematical situations of play and exploration

FaSt – Building 01 (B01)

erStMaL-Familystudy (Stand: 04.03.2010)

Definition of play situation (1)	How can the play situation be described? In the play, families are supposed to build three-dimensional version of the picture with wooden blocks.
Application field (2)	Which fields are scheduled at the application of the play situation? • Family study • First meeting • age 4+
Intended mathematical domain (3)	Which mathematical domain is chosen for the play situation? Geometry and Spatial Thinking
Mathematical context (4)	How can the chosen mathematical content de described? Exploration of the two-dimensional illustrations from the three-dimensional buildings and performing the relations between two- and three-dimensional representations.
Materials and play room (5)	Which materials and structures are needed? • Wooden blocks in the same size and weight • The game cards in different levels (From1 to 3) • The given introduction manuals
Instruction manual (6)	Which impulses and informations are needed for the participation of the play situation? • Play cards should be mixed and placed on the table face down. • The youngest player begins to play. • Each player chooses one card from the deck and builds the figure, which is seen on the card as an image. To check the compatibility between the built figure and the seen figure on the card, other players should examine if the built figure is correctly built or not and give a feedback. If it is correct, then the player gets as many points as seen on the card. • In total, five rounds should be played by turns of each player.

Fig. 3.4 The specific design pattern of the game Building 01

Both games are constructed according to design patterns[53] specific to erStMaL-FaSt, namely, “design patterns of mathematical situations” (see also Vogel, 2012, 2013, 2014a,b; Vogel & Wippermann, 2005). In each brief description, a specific design pattern contains (1) definition of the play situation, (2) application field, (3) intended mathematical domain, (4) mathematical context, (5) materials and playroom, and (6) instruction manual. Specific design patterns of both “Building 01” and “Building 02”, and their translations are shown in Figs. 3.4 and 3.5.

the original specific design pattern of play „Building 02“ in German	**Translation of the specific design pattern of play „Building 02“ in English**

the original specific design pattern of play „Building 02“ in German

Spiel- und Erkundungssituation im IDeA-Projekt (erStMaL- Familienstudie)

FBauen02 (FB02) — **erStMaL-FaSt** (Stand: 01.05.2011)

Kurzbeschreibung der Spiel- und Erkundungssituation	Wie kann die Spiel- und Erkundungssituation beschrieben werden? In der Spiel- und Erkundungssituation werden von den Kindern und ihre Familienangehörigen Quadergebäude auf der Grundlage von zweidimensionalen **Darstellungen** mit entsprechenden Quadern nachgebaut.
Einsatzbereich	Welcher Einsatzbereich ist für die Spiel- und Erkundungssituation vorgesehen? (Familienstudie, Erstbesuch/Zweitbesuch, Altersbereich) • Familienstudie • Zweitbesuch • 4+ Jahre
Mathematischer Bereich	Welchem mathematischen Bereich kann die Spiel- und Erkundungssituation zugeordnet werden? Geometry and Spatial Thinking
Mathematischer Gehalt	Wie kann der mathematische Gehalt der Spiel- und Erkundungssituation beschrieben werden? Darstellung dreidimensionaler Objekte durch schwarz-weiß zweidimensionale Bilder: **Frontansicht**
Material und Raum	Welches Material wird für die Durchführung der Spiel- und Erkundungssituation benötigt? Wie muss der Raum für die Spiel- und Erkundungssituation vorbereitet sein? • Holzquader in ausreichender Anzahl • Verschiedene Abbildungen von Quadergebäuden auf Spielkarten (Schwierigkeitsgrad wird durch eine entsprechende Punktzahl auf den Karten kenntlich gemacht: 1-2-3-4) Insgesamt 30 Spielkarten. 
Impulse und Hinweise für die Familien	Welche Impulse und Hinweise benötigen die teilnehmenden Familien, um an der Spiel- und Erkundungssituation teilzunehmen? **Spielregeln:** • Die Spielkarten werden gemischt und verdeckt auf einem Stapel in der Mitte des Tisches platziert. • Der jüngste Spieler, die jüngste Spielerin fängt an. • Die Spielerinnen und Spieler ziehen nacheinander eine Karte und versuchen jeweils die darauf abgebildeten Gebäude nachzubauen. • Es werden fünf Runden gespielt. Danach vergleichen die Teilnehmenden ihre jeweils erspielte Anzahl an Punkten auf ihren Karten. • Die Punkte werden während des Spiels mit Stäbchen dokumentiert.

Translation of the specific design pattern of play „Building 02“ in English

Spiel- und Erkundungssituation im IDeA-Projekt (erStMaL- Familienstudie)

FaSt-Building02 (FB02) — **erStMaL-FaSt** (Stand: 01.05.2011)

Definition of play situation (1)	How can the play situation be described? In the play, families are supposed to build three-dimensional version of the picture with wooden blocks.
Application field (2)	Which fields are scheduled at the application of the play situation? • Family study • First meeting • age 4+
Intended mathematical domain (3)	Which mathematical domain is chosen for the play situation? Geometry and Spatial Thinking
Mathematical context (4)	How can the chosen mathematical content de described? Exploration of the two-dimensional illustrations from front elevation Exploration of the three-dimensional buildings and performing the relations between two- and three-dimensional representations.
Materials and play room (5)	Which materials and structures are needed? • Wooden blocks in the same size and weight • The game cards in different levels (From1 to 4) • The given introduction manuals
Instruction manual (6)	Which impulses and Informations are needed for the participation of the play situation? • Play cards should be mixed and placed on the table face down. • The youngest player begins to play. • Each player chooses one card from the deck and builds the figure, which is seen on the card as an image. To check the compatibility between the built figure and the seen figure on the card, other players should examine if the built figure is correctly built or not and give a feedback. If it is correct, then the player gets as many points as seen on the card. • In total, five rounds should be played by turns of each player.

Fig. 3.5 The specific design pattern of the game Building 02

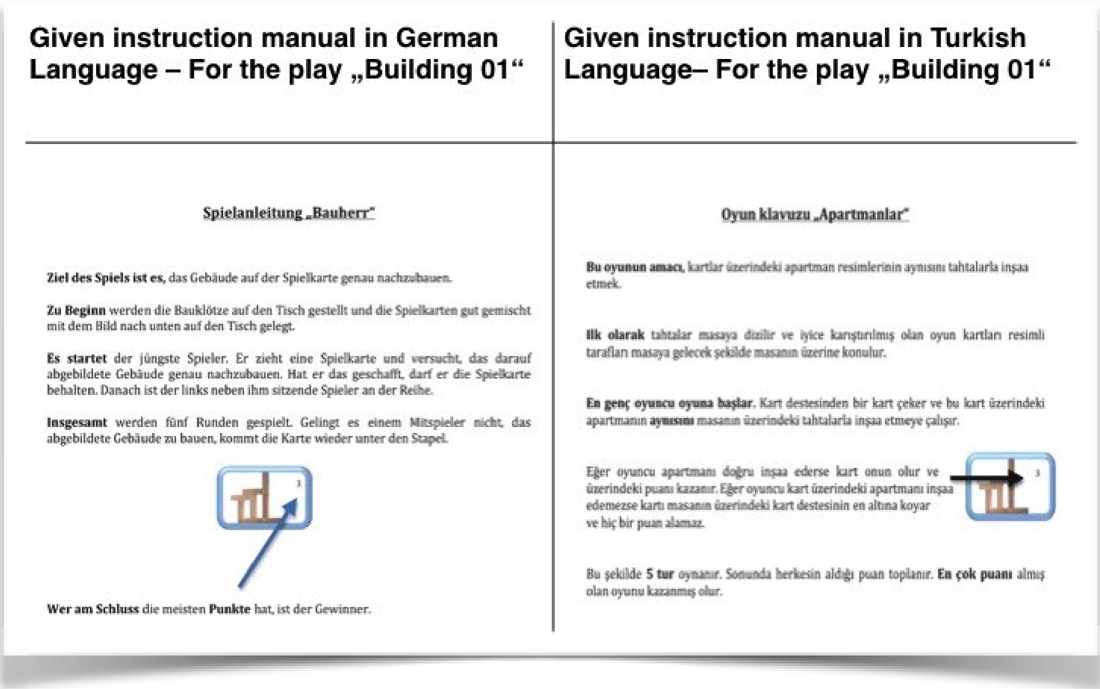
Data collection consists of video-recordings and transcripts of them. In the ongoing erStMaL-FaSt, once a year, an appointment is arranged with each family. This leads step by step to the collection of data on each child. In these

[53] For further details about design patterns, see section 1.4.4

appointments, the erStMaL child is video-recorded together with members of the family while they are playing. By setting up a design for the observation of each child, it is also possible to observe the game partners of the child. For data collection, an appointment is arranged with the family, giving them the flexibility to choose place and time.

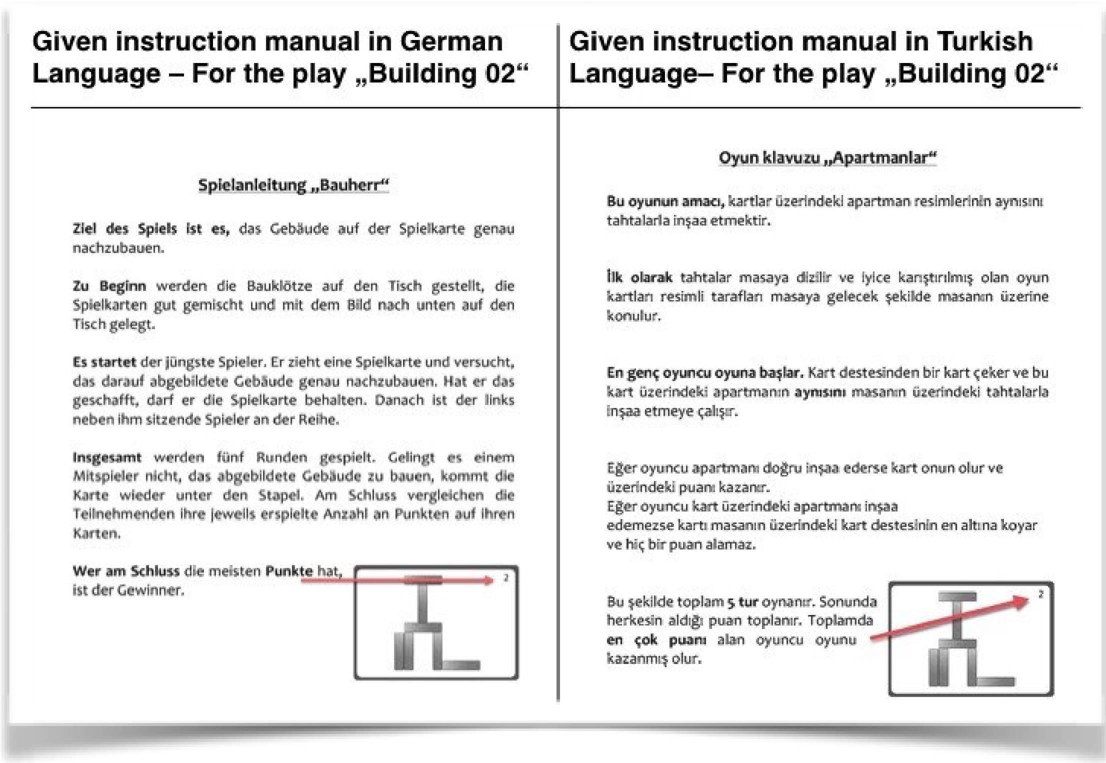
Before family members begin to play, the games are explained in the language family members prefer: either German or Turkish. At the same time, all play materials are shown to the family members. In addition, they are told that they are also free to play in any language they want. Instruction manuals for each game are composed and provided in both languages as well (see Figs. 3.6 and 3.7).

These instruction manuals and game materials are provided by me and put at the disposal of the family in the recording room. Afterwards, the family members are left by themselves to get comfortable while the video-recorders are turned on.



Given instruction manual in German Language – For the play „Building 01“	Given instruction manual in Turkish Language– For the play „Building 01“
Spielanleitung „Bauherr“ **Ziel des Spiels ist es,** das Gebäude auf der Spielkarte genau nachzubauen. **Zu Beginn** werden die Bauklötze auf den Tisch gestellt und die Spielkarten gut gemischt mit dem Bild nach unten auf den Tisch gelegt. **Es startet** der jüngste Spieler. Er zieht eine Spielkarte und versucht, das darauf abgebildete Gebäude genau nachzubauen. Hat er das geschafft, darf er die Spielkarte behalten. Danach ist der links neben ihm sitzende Spieler an der Reihe. **Insgesamt** werden fünf Runden gespielt. Gelingt es einem Mitspieler nicht, das abgebildete Gebäude zu bauen, kommt die Karte wieder unter den Stapel. **Wer am Schluss** die meisten **Punkte** hat, ist der Gewinner.	**Oyun klavuzu „Apartmanlar“** **Bu oyunun amacı,** kartlar üzerindeki apartman resimlerinin aynısını tahtalarla inşaa etmek. **Ilk olarak** tahtalar masaya dizilir ve iyice karıştırılmış olan oyun kartları resimli tarafları masaya gelecek şekilde masanın üzerine konulur. **En genç oyuncu oyuna başlar.** Kart destesinden bir kart çeker ve bu kart üzerindeki apartmanın **aynısını** masanın üzerindeki tahtalarla inşaa etmeye çalışır. Eğer oyuncu apartmanı doğru inşaa ederse kart onun olur ve üzerindeki puanı kazanır. Eğer oyuncu kart üzerindeki apartmanı inşaa edemezse kartı masanın üzerindeki kart destesinin en altına koyar ve hiç bir puan alamaz. Bu şekilde **5 tur** oynanır. Sonunda herkesin aldığı puan toplanır. **En çok puanı** almış olan oyunu kazanmış olur.

Fig. 3.6 Instruction manual of the game Building 01



Given instruction manual in German Language – For the play „Building 02“

Spielanleitung „Bauherr“

Ziel des Spiels ist es, das Gebäude auf der Spielkarte genau nachzubauen.

Zu Beginn werden die Bauklötze auf den Tisch gestellt, die Spielkarten gut gemischt und mit dem Bild nach unten auf den Tisch gelegt.

Es startet der jüngste Spieler. Er zieht eine Spielkarte und versucht, das darauf abgebildete Gebäude genau nachzubauen. Hat er das geschafft, darf er die Spielkarte behalten. Danach ist der links neben ihm sitzende Spieler an der Reihe.

Insgesamt werden fünf Runden gespielt. Gelingt es einem Mitspieler nicht, das abgebildete Gebäude zu bauen, kommt die Karte wieder unter den Stapel. Am Schluss vergleichen die Teilnehmenden ihre jeweils erspielte Anzahl an Punkten auf ihren Karten.

Wer am Schluss die meisten **Punkte** hat, ist der Gewinner.

Given instruction manual in Turkish Language– For the play „Building 02“

Oyun klavuzu „Apartmanlar“

Bu oyunun amacı, kartlar üzerindeki apartman resimlerinin aynısını tahtalarla inşaa etmektir.

İlk olarak tahtalar masaya dizilir ve iyice karıştırılmış olan oyun kartları resimli tarafları masaya gelecek şekilde masanın üzerine konulur.

En genç oyuncu oyuna başlar. Kart destesinden bir kart çeker ve bu kart üzerindeki apartmanın **aynısını** masanın üzerindeki tahtalarla inşaa etmeye çalışır.

Eğer oyuncu apartmanı doğru inşaa ederse kart onun olur ve üzerindeki puanı kazanır.
Eğer oyuncu kart üzerindeki apartmanı inşaa edemezse kartı masanın üzerindeki kart destesinin en altına koyar ve hiç bir puan alamaz.

Bu şekilde toplam **5 tur** oynanır. Sonunda herkesin aldığı puan toplanır. Toplamda **en çok puanı** alan oyuncu oyunu kazanmış olur.

Fig. 3.7 Instruction manual of the game Building 02

Before I go further details about my research data, first I want to describe my research methodology with the aim of avoiding any theoretical dilemma. Thereafter I will present the data collected in this study regarding this research methodology.

3.2. Method

This study is laid out as a comparative set of case studies. The aim of theoretical sampling, which evolves during the process, “is to maximize opportunities to compare events, incidents, or happenings to determine how a category varies in terms of its properties and dimensions” (Strauss & Corbin, 1998, p.202). Characteristically it features as “reconstructive social research” (Bohnsack, 1993)[54] and consists of two aspects: the observed family episodes and the steps

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[54] Bohnsack (1996) describes the applied research practice as “reconstructive” in two respects:
(1) The analysis of the observed classroom episodes is oriented towards the reconstruction

of analysis. The analysis of the observed episodes is oriented towards the reconstruction of the processes of interaction between family members and the focus child, which is geared to theories of interaction, argumentation and participation. The steps of analysis employed usually have been partly developed through concrete research work (Krummheuer, 2002, p.340). This means that while the processes of interaction between family members and child are reconstructed, interpretations of their actions within the processes of interaction are also reflected. Thereon these proceedings are inevitably reconstructed with the aim of adapting the research methods to the object of research. Similarly, Brandt (1999) defines this method as follows:

> First we construct the learning situations separately by interaction, argumentation analysis and participation analysis. The next step is the comparison of analysis of different interaction processes. The selection of episodes in the ongoing research process is guided by comparison, which causes modifications of our analytical methods and the construction of theoretical elements. (p.309)

In this regard the current study lays out the execution of the analysis of interaction, which was determined by the working group around Bauersfeld as already mentioned in section 2.2.1 (see Bauersfeld, 1980; Cobb & Bauersfeld, 1995; Bauersfeld, Krummheuer, & Voigt, 1988; Krummheuer, 1983; Voigt, 1984,1995). It is a basic and initial procedure; which "serves to reconstruct the process of negotiation of meaning and leads to reliable interpretations, which are grounded on the principals of conversation analysis" (Krummheuer, 2015,p.53).

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of the processes of interaction among the students and the teacher, which is geared to theories of interaction, argumentation and participation. (2) The steps of analysis employed in this action usually have been partly developed through concrete research work (for both aspects, see Bohnsack, 1993, p.8, cited in Krummheuer, 2002, p.340). According to Krummheuer and Brandt (2001), the reflection of this method of analysis and evaluation is based on the second use of the term "reconstructive social science". For more please see Krummheuer & Brandt, 2001; Krummheuer, 2002, 2009, 2011c, 2012, 2013a, 2014; Brandt, 2002, 2004; Tiedemann, 2012; Schütte, 2009; Schreiber, 2010.

In this regard the analysis of interaction is a method derived from ethnomethodological conversation analysis[55] (Eberle, 1997; Sacks, 1998; see also section 2.2.1.)[56]. It consists of five steps, which are described respectively as follows:

- Outlining of the interaction units: According to research criteria one should first outline the relevant units of the interaction process, in order to reflect the research process.
- General description: This is to classify immanent conjectured meanings of superficial descriptions. For this, one should consider the generality about the known facts of life and culture (for more, see Bohnsack, 1993, 1996; Kelle & Kluge, 1999).
- Detailed and sequential analysis of individual remarks: Here alternative interpretations should be (re-)constructed.
- Turn-by-Turn Analysis: Alternatively constructed interpretations should be reduced according to those interpretations which prove worth over the following joint interaction process of participants[57].

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[55] Ethnomethodological conversation analysis addresses “the purposefulness and intelligibility of social actions, and discern[s] the tacit understandings and assumptions that guide the accomplishment of social actions” (Arminen, 2006, p.28). Moreover this method opens up social actions as *situated* or/and *situational* activities “that emerge from their practical management within their realization” (ibid.) (see also Krummheuer, 2011a; Krummheuer & Naujok, 1999; Krummheuer & Brandt, 2001; Krummheuer & Fetzer, 2005; Schreiber, 2010; Schütte, 2009; Fetzer, 2007a).

[56] For detailed information about the analysis of interaction, see Brandt, 2004, 2006; Fetzer, 2007; Krummheuer & Brandt, 2001; Krummheuer, 2011b; Krummheuer & Naujok, 1999; see also Tiedemann, 2012; Schreiber, 2010; Schütte, 2009.

[57] Turn-by-turn analysis deals with the question “Wie reagieren andere Interaktanten auf eine Äußerung, wie scheinen sie die Äußerung zu interpretieren, was wird gemeinsam aus der Situation gemacht?” (Krummheuer, 2011b,p.236).

- Summarization: By counting the diversity of interpretations (abovementioned), the chosen sequence should be elaborated and summarized considering the regarded theories.

Considering these five steps, skipping or returning one to other can occur during the process of analysis. This also enriches the construction and reconstruction options of the cases. It is crucial to make sure of taking all adequate maxims into consideration in the course of elaboration (Krummheuer, 2011b, p.235). In this regard, for the analysis of interaction, “mathematical meanings are ascribed to the empirical phenomena and the focus of attention is on the processes of mathematization” through the transformation of empirical situations into mathematical statements (Voigt, 1994, p.276). On the basis of the results of the analysis of interaction, in the current study the analysis of participation and the comparative analysis are applied in a hierarchical order.

The analysis of participation is another method of analysis established by Krummheuer and Brandt (2001), which consists of recipient and production designs (for more see section 2.2.1.). These networks of terms allow us to describe and distinguish the requirements and structures of participations in respect of interaction possibilities in the family discourses (see Brandt, 2004; Krummheuer & Brandt, 2001).

After the analyses of interaction and participation, I “compare” the set of case studies through the comparative analysis, which leads me to the result of this study. In the process of comparison, four steps should be applied: (1) transcription, (2) selection of detailed analysis, (3) case-by-case assessment, and lastly (4) comparative analysis (Brandt, 2004, p.44). Regarding this, in the current study, data collection consists of video-recordings and transcripts of them.

In the method of comparative analysis[58] one should compare interpretations of different episodes according to “certain constructions of theory” (Krummheuer, 2002, p.343). In this method one takes advantage of the method of “abduction”, which is the type of inference by which most current solidly grounded theories are developed (ibid.). Basically, abduction[59] makes it possible to construct new theoretical elements through referencing the analysed sequences in different cases (Brandt, 2004). In other words, comparative analysis offers the empirical control and theoretical orientation that enable me to produce and create theoretical elements abductively (for more see Brandt, 2004; Krummheuer & Brandt, 2001). From a methodological point of view “the comparison of interpretations of different parts of reality represents a possibility to identify the specific characteristics of these particular cases in relation to each other” (Krummheuer, 2002, p.344). Comparative analysis takes place as a “local methodology of discovery”, which enables to control two things. Krummheuer explains them as follows:

> (1) the estimation of the ability to generalize the claim of the developed theory: “conceptual representativeness”: to find the representativeness of the developed theoretical concepts within the interpretations of the selected parts of reality, unlike quantitative research, which aims at representativeness on the level of the sampling.....(2) the documentation of the complexity of the reality, which could be made understandable through the corresponding development of a theory. : the fact that comparative procedures aim at

[58] Actually the “comparation” method contains the analysis of interaction, the analysis of argumentation, the analysis of participation and the comparative analysis. In the current study the analysis of argumentation is not utilized and thematized, hence I do not need to refer this analysis method here (for more, see Brandt, 2004; Krummheuer, 1995, 1997, 2007b, 2011b; Brandt & Krummheuer,2001, Krummheuer & Fetzer, 2005)

[59] According to Peirce (1978), the form of abduction can be defined as follows:

“The surprising fact, C, is observed —> But if A were true, C would be a matter of course. Hence, there is a reason to suspect that A is true.” (Peirce, 1978, 5.189).

grasping the selected part of reality by their specificity. (Krummheuer, 2002, p.340-344).

The chosen samples of families are thus presented based on the hierarchical dependency on the results of the analysis of interaction and an interpretation of the process of negotiation of taken-as-shared meaning.

With reference to the method of the current study, I return to the point about research materials, which I am synthesizing with my research method.

3.3. Synthesis of Research Materials and Methods

For this study, I selected three German Turk families, who are participants in the erStMaL project, and identified their video-recordings in two observation phases of erStMaL-FaSt. In these videos I searched for the moments which testify the possible potential for child's development through block play with family members obviously and clearly, and picked them out. Thereupon I transcribed these sequences in the original languages are spoken by family members, in German, in Turkish or in both languages as a mixed language (see section 1.4.1.1.), and then translated them into English. In terms of NMT-Family as a main concept of this study, I have analysed these sequences in detail by subjecting them to analysis of interaction and participation. After analysing them case-by-case, I compare these familial play situations according to my research insights by a comparative analysis (Krummheuer & Brandt, 2001). This study is presented as a comparative study and its methodology is characterized as "reconstructive and interpretative" (Krummheuer & Schütte, 2014, p.129).

The play situations "Building 01 and 02" require some specific determinations for clear understandings in the transcription process. In both play situations wooden blocks are used, which are all the same size and colour. I name the sides of the blocks as X, Y and Z sides (see Fig. 3.8) considering the theory of Euclidian geometry (see section 2.1.2.).

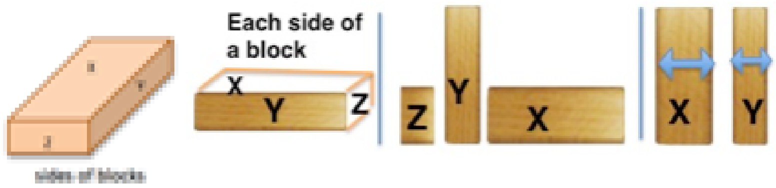

Fig. 3.8 Sides of blocks labelled X, Y and Z.

Furthermore, for the clear definition of building corpuses, I predicate the 3D representation of both built and building corpuses on architectural drawing, namely, “the third-angle orthographic projection” (see Fig. 3.9).

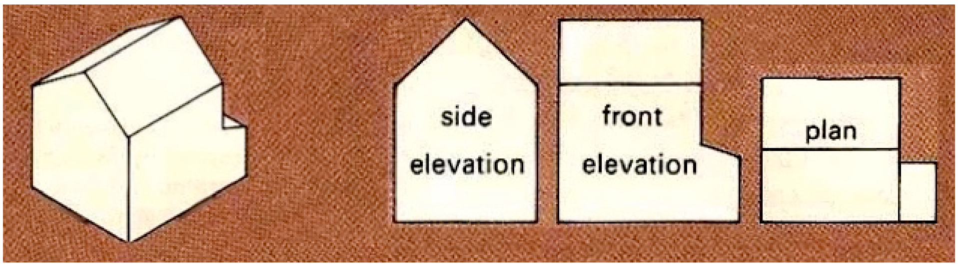

Fig. 3.9 Third-angle orthographic projection of a building (Darling, n.d.)

Regarding this, in the current study both built corpuses are shown with camera perspective and side-front elevations and also as a plan. In order to avoid misunderstanding, I prefer to use the term “top elevation” instead of using the term “plan” or “top view” or “bird’s-eye view”. The term “elavation” refers to a “drawing that shows one face of a building”[60] (McKendry, 1995). Referring to work of Darling (n.d.) I apply these terms for each building corpus in the current study as follows, while I consider the sides (X,Y,Z) of the blocks (Fig. 3.10):

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[60] The glossary of Architecture by Jennifer McKendry. Retrieved from http://www.mckendry.net/GLOSSARY/GLOSSARY.htm

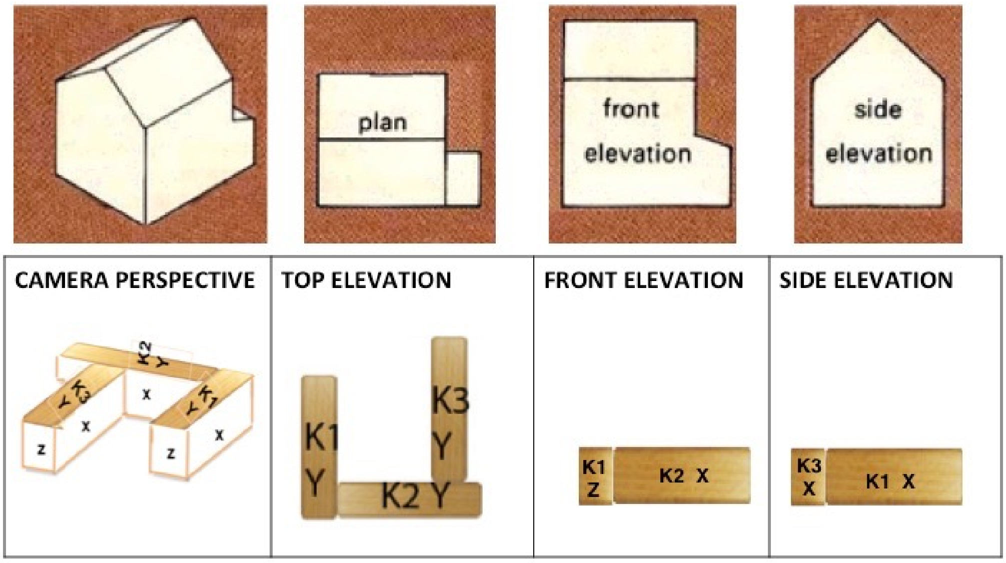


Fig. 3.10 An example of building corpus with third-angle orthographic projection

In the transcriptions I consider the language used by the participants, who are from German Turkish society in Germany, or for the purpose of this study, “German Turks” (see section 1.5.1). They can speak in German or Turkish, or use both languages “by switching” (Acar, 2011b; see also section 1.4.). In the transcriptions, utterances in German are written in regular font, while utterances in Turkish are <u>underlined</u>.

Each transcription consists of five columns (see Table 3.1):

Table 3.1 Structure of transcription

Column 1	Column 2	Column 3	Column 4	Column 5
Serially numbered lines	Time line	Marks for interaction process	Abbreviations of the names of the interacting people	Verbal (regular font) and nonverbal (*italic* font) actions

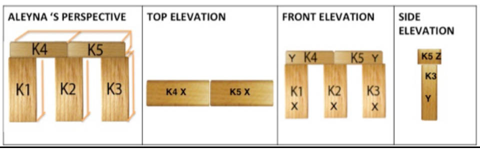
Additionally, the following rules are considered in the transcription process (see Table 3.2):

Table 3.2 Rules of transcriptions

@Column 3: Marks for interaction process	
. . ...	Breaks of. 1, 2 or. 3 seconds
(4 sec.)	Breaks of a specified time span
-	Even pitch
#	There is no break, the second speaker follows immediately from the first.
> P > K	The next block of simultaneous speech is indicated by a change in arrow direction.
< M < S	Indicates where people are talking at the same time.
@Column 5: Verbal (regular font) and nonverbal (italic font) actions	
standard speech	in German (regular font)
underlined speech	in Turkish
double-underlined speech	in English
bold	Accentuated word
s p a c e d	Spoken slowly
(word)	Unclear utterance

A sample transcription is presented in Table 3.3.

Table 3.3 An example of one of the transcriptions in the current study

21	01: 24		Mother	yes. exactly.
22			Aleyna	*puts K5 on its Y side horizontally bonded to K4 on K2 and K3* 

In analysis of the data, I use the term "turn" with two different meanings:

- In interactional theory of mathematics teaching and learning (see 2.2.1), the term, "turn" refers to "an opportunity or obligation to do something that comes successively to each of a number of people"[61] during the interaction process. In the analyses of the current study this kind of "turn" is named as "i-turn" (interaction-turn).

- By virtue of the game structures of "Building 01 and 02", participants should play five rounds by turns of each player in total (see 3.1). In this regard the word "turn" can also refer to the play routines, in the usual sense in which "turn" is used in everyday life. For this, I use the term "p-turn" (play turn), which refers to the turn of each player in the games provided.

Moreover, in the analysis, I use the terms of production design (author, ghostee, ghoster, spokesman, sponser, relayer, deviser) not only at the language level but also in motional (as in gesture and action) level in the sense of Krummheuer and Brandt (2001; see also section 2.2.1). In this regard I define

- *author* as a person, who expresses his/her own idea (content) either **verbally** or **motionally.**

- *ghostee* as a person who takes the identical formulation of the parts of a preceding either **utterance or motion** and with them attempts to then express his/her own new idea.

- *spokesman* as a person who takes the idea of a preceding utterance and then tries to express this idea with his/her own new formulation either **verbally** or **motionally**.

- *relayer* as a person, who neither takes responsibility for nor has originality in either the **semantic or motional** content of their utterances.

The synthesis of the method and material in this chapter allows me to undertake a deep analysis considering all the steps and factors in the process of negotiation between family members. Moreover, these methods divulge the way of negotiation, how the meanings are constituted and negotiated in the familial

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[61] Retrieved from http://www.oxforddictionaries.com/definition/english/turn

discourses, and what kind of differences they comprehend. Such deep analysis enables me to gain an in-depth understanding about the child's mathematical development while considering the child's history, motives, intentions, personal meanings, social discourses and interactions, cultural forms and artifacts. In this regard such deep analysis enables me to examine the dynamic interactions between individual, social and cultural factors. Moreover it makes it possible to describe the interaction patterns which provide mathematical development for the child. Here it is important to note that I benefit from the concept of NMT-Family; in which already exist content, cooperation, and pedagogy and education (see section 2.2.2). On the authority of this concept of NMT-Family and Bruner's theory about *formats* (1983, see also section 2.2.1.) I describe the emerging MLSSs in each family and compare them regarding allocation, situational and contribution.

4. DATA COLLECTION AND ANALYSIS

Three families are observed in the current study: Family Kil, Family Ak and Family Gül. This chapter consists of the presentation and analysis of the data collected on the three families and the comparison of these analyses.

4.1. The Kil Family

The Kil family are German Turks who live in a major city in German. Ayse is the focus child, and she is an only child. She can speak German and rudimentary Turkish. Her parents grew up in Germany, and they speak with their daughter mostly in German. Ayse's mother completed 10 years of formal education, she has a higher education and works as a laboratory assistant. Ayse's father completed 13 years of formal education, has a higher education qualification and works as a mechanical engineer. Ayse and her parents all speak fluent German and rudimentary Turkish. Owing to her parents' jobs, her grandparents (parents of her father) take care of Ayse after kindergarten and school. In the family they speak both in German and Turkish with each other.

In the first observation phase, Ayse is five years old and the meeting with Ayse's family takes place in the kindergarten that Ayse attends daily. Both mother and father attend the meeting and in total they play two different games from the erStMaL-FaSt. Ayse plays one game with her parents, which facilitates the emergence of polyadic interaction processes. First, she plays only with her father the chosen game, 'Building 01'. Although they are informed by the instruction manual that they should play only in total five rounds, they play in total 14 rounds by turns. In the course of the game they build all the given cards. They conduct the negotiation process only in German.

In the second observation phase Ayse is six years and six months of age and the meeting with Ayse's family takes place at their home. Her parents attend the meeting and in total they play four different games from the erStMaL-FaSt. Ayse plays two games with her parents as a triad, which facilitates the emergence of polyadic interaction processes. Then Ayse plays one game with her father and one another with her mother, both of which enable the emergence of dyadic interaction processes. The chosen game in the second observation phase of erStMaL-FaSt is ‘Building 02’ and Ayse’s game partner is her mother. In total, they play seven rounds by turns and conduct the negotiation process both in German and Turkish.

4.1.1. ‘Building 01’ from the first observation period

The chosen game starts with father’s p-turn. Up to the chosen and transcribed scene, Ayse and her father have played just two rounds by turns and both have built their two chosen cards correctly. The third round begins with father’s p-turn and proceeds with Ayse’s p-turn. The recording position of the chosen scene and cards chosen by Ayse and her father are shown as follows:

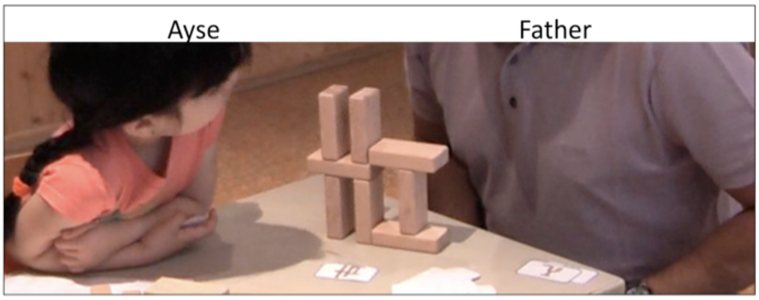


Fig. 4.1 Recording position in the first observation period of Ayse

For clarity, the cards chosen by father and by Ayse are named first and second chosen card, respectively.

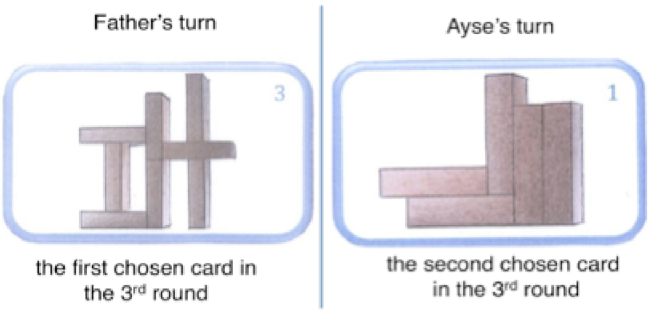


Fig. 4.2 The chosen cards in the third round

In the third round Ayse’s father begins, picks up a card and builds the corpus. He completes the assignment and asks whether the corpus he has built is correct.

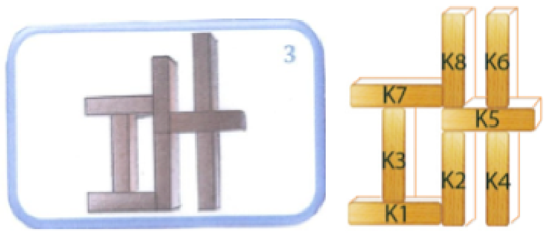
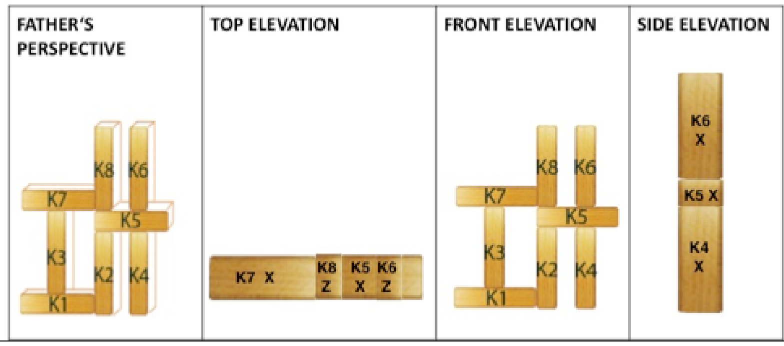
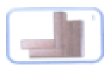
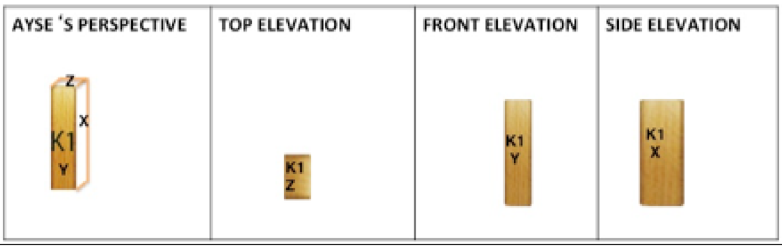
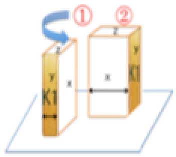
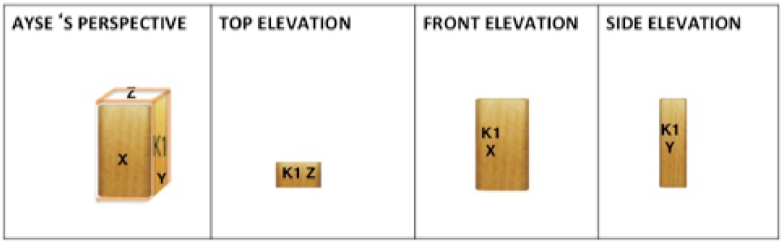
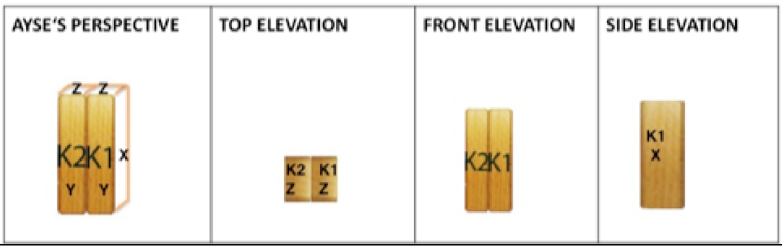
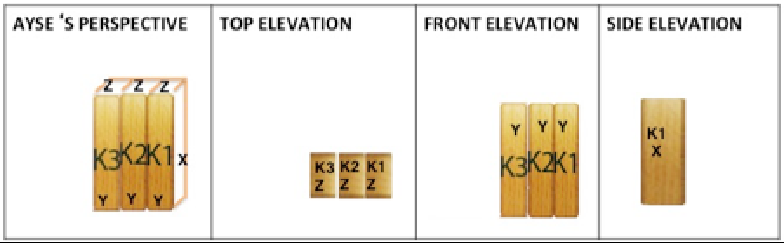
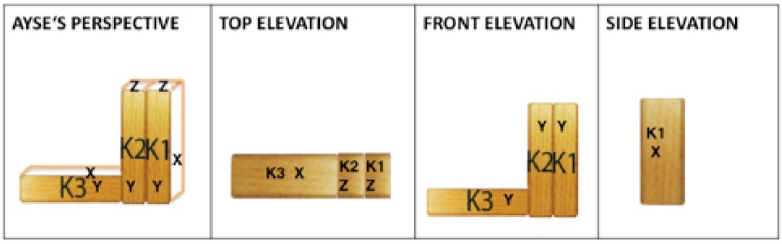
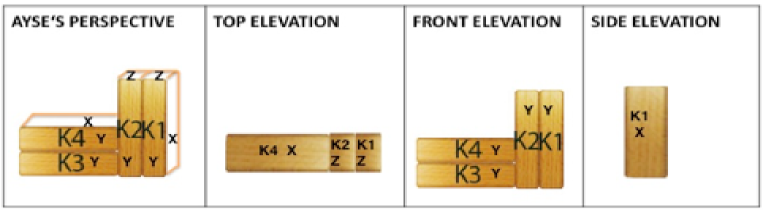


Fig. 4.3 The chosen card and the corpus built by father (the frontal elevation)

Transcription of third round

1			Father	right? 
2			Ayse	yes. *holds the deck of her previous cards*
3			Father	cool.
4			Ayse	but one long and one short. *looks at the card*
5			Father	yes I think the picture is wrong. *bends forward*
6				*and shows the card with his index finger*
7			Ayse	hıııım
8			Father	just look. this is a short block or a short
9				piece of wood.
10			Ayse	*looks at the residual pile of wooden blocks*
11	4.30		Father	building block.
12			Ayse	*leans back, still looking at the pile of blocks*
13			Father	*shows the pile of blocks* but there
14				are no short blocks. I think the picture is
15				wrong. I would say; it is all right. O.K.? *(intensively)*
16				*puts the card in his left hand and piles it on his*
17				*other cards* you too?
18			Ayse	*pushes her father's building with her right hand while*
19				*she is still holding the deck of her previous cards* hıı
20		>	Father	good. *bowls over his building and pushes the blocks*
21				*to the other pile*
22		>	Ayse	*picks a new card with her right hand* 
23				*turns the card face, looks at it, then looks at her*
24				*father and laughs*
25			Father	yes.
26			Ayse	nope(what)..

27			Father	not so difficult. *scratches his face with his left*
28				*hand*
29			Ayse	*picks up the card and lays it on the table so that*
30				*she can see the top elevation of the chosen card*
31				*separates the cards from the deck of her previous cards*
32				but then..one more.. a long one. heh?
33		<		*looks at the pile of blocks on the table*
34			Father	No no it looks like that. *takes the deck of her cards away*
35		<		*from her hand* let them away. *holds the deck of Ayse's*
36				*cards, shuffles it and looks at her* did you lose one?
37			Ayse	No. *looks at her father*
38			Father	O.K. *lays the card in front of Ayse on the table*
39			Ayse	I want another one.
40	05:00		Father	may- but it is easy. just get started. *shows the*
41				*chosen card, which lays in front of Ayse on*
42		>		*the table* just get started.
43		>	Ayse	*props her head on her left hand* hıııım *takes the first*
44				*block (K1) and sets it on its Z side, that Ayse can*
45				*see its Y side* 
46			Father	exactly.
47			Ayse	*turns the block (K1) counterclockwise, around its Y side* 
48				*that she can see its X side* like that? 
49			Father	no. *turns the block to the previous position (<45*>)*
50				it was already right. like that. the next block.
51			Ayse	*takes the second block (K2), sets its on its Z side*
52				*next to the K1 that their X sides touches each other* 
53				(not understandable) hey my- *takes the deck of her*
54		<		*previous cards, which lie on the table* hey my cards!

55				*puts them back on the table in front of her*
56		<	Father	*pulls the chosen card a bit to his side*
57			Ayse	*pulls the chosen card back to her side, that it lies*
58				*next to her previous cards, then she grasps two blocks*
59				*(K1 an K2)in her both hand and pulls them together a*
60				*bit to the right takes another block (K3) and sets it*
61				*on its Y side next to the K2, that its X side touches*
62				*the another X side of the K2* 
63			Father	hıııııııım ıh ıh!
64			Ayse	*takes the block(K3) back, lays it on its X side next*
65				*to the K2, that its Z side touches the another*
66				*X side of the K2* hıııım hı? 
67			Father	hım hııhıı
68			Ayse	*takes the block(K4) and sets it on the K3 in the same*
69				*way, so that their X sides touch each other and looks*
70				*at her father* hı? 
71	05:30		Father	hı? I don't know *holds his right hand*
72				*on his mouth and wobbles his finger*
73			Ayse	hı? *tends to take the card but lets it lie, looks at it*
74			Father	One is still missing *holds his right hand on his*
75				*mouth and touches his lips with his forefinger*
76		<	Ayse	hııh *pulls K4 a bit to the left*
77		<	Father	it's there *shows on the card the jut of the block f*
78		>		the long one. exactly. right.
79		>	Ayse	*takes the fifth block(K5) to her left hand and sets*
80				*it on its Z side between K4 and K2, but K1 and K2*
81				*fall down* hım hııh!
82			Father	*quietly* yes. they can be set upright again.

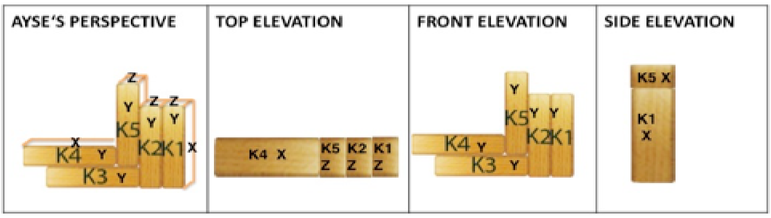
<table>
<tr><td>83</td><td></td><td></td><td>Ayse</td><td>hıı hıı sets first K2 in the upright position that</td></tr>
<tr><td>84</td><td></td><td></td><td></td><td>its X side touches the X side of K5, then sets K1 in</td></tr>
<tr><td>85</td><td></td><td></td><td></td><td>the upright position hıı hı
</td></tr>
<tr><td>86</td><td>05:47</td><td></td><td>Father</td><td>very good. pinches Ayse's cheek cool.</td></tr>
</table>

Fig. 4.4. Transcription of third round.

It is important to emphasize here that a part of the chosen transcript (<01-35>) and its analysis are published in Acar Bayraktar, 2014. In the transcription, "ıh ıh" refers to the typical Turkish expression of disapproving of something (see line <63>), while "hııhıı" is a typical Turkish expression of approving of something. (see line <67>). Additionally "hı?" refers to the typical Turkish expression, which is mostly used at the end of the sentence for linking alternatives like the use of the conjuction "or?". There is also another meaning of the word "hı?" in Turkish that is used occasionally for asking for information, similar to the use of the word "what?" in English.

Interaction analysis

After the father builds the corpus vertically on the table, he asks whether the built corpus is right <01>. The father's question can be interpreted in two different ways:

- Maybe he is not really sure about the rightness of the built corpus and thus he asks Ayse whether it is right or wrong. From a participatory point of view the father's reaction signalizes that he takes on the role of *novice*, reserving the role of *tutor* for Ayse (see section 2.2.3.). Namely he gives the impression of pushing Ayse to judge the rightness of the corpus and to make a decision on whether he built the corpus right.

- Another possibility is that he is just trying to get to know whether the built corpus is identical with the figure on the card or if Ayse finds his corpus identical to the figure on the chosen card. Maybe he tries to evoke Ayse's spatial abilities by getting her to think about the built corpus. In this regard his question can provide a stimulus for Ayse to perform and exercise her *spatial visualization* and *spatial orientation* (see section 2.1). Thereby he might also try to facilitate the development of Ayse's metacognitive and self-regulation skills by using direct elicitation in terms of *parental regulation* (see section 2.2.2.3). Therefore he might pose such a question to encourage her to generate and to evaluate alternatives in order to solve the spatial problem.

From the participatory point of view, by posing such a question the father might try to provide Ayse a specific leeway of participation so that she can take an opportunity for examining and musing on the built corpus as in the role of an *author* (see 2.2.2.2). Additionally, the father might have a role as a *tutor* (see 2.2.2.2.), which is the standard role for an adult person from the socio-constructivist perspective. He might take on the role of *expert* and try to scaffold Ayse's skills in such a way that is possible for Ayse to internalize her spatial knowledge and convert it into a tool for *conscious control* (see 2.2.2.3). Thus he might provide Ayse with a *vicarious* form so that she can master her own action through her own *consciousness* and *control* (see 2.2.2.3). From a supportive perspective, the father might try to engage Ayse in the play situation by posing such questions to let her feel free to express her own ideas and to become deeply involved in the play situation. Thus his reaction seems to be a type of supportive action, namely *motivation*.

Ayse says, "Yes". Her reaction might show a temporary agreement that her father built the corpus correctly in accordance with the chosen card <02>. Namely her father's question seems to work on Ayse to the effect that she can judge the rightness of the corpus and make a decision – that the built corpus is right. In this regard Ayse gives the impression of activating her *visualization* (see section 2.1) in which she operates her spatial visualization and spatial orientation. Moreover she seems to be able to take the opportunity to examine and muse on the built corpus. From a participatory point of view she looks like an *author* in that she expresses her own idea about the built corpus. Ayse's leeway seems quite open in that she participates in the situation actively. Concordantly she seems to assign her father the role of *activator* (see 2.2.2.3), who activates Ayse's *knowledge of the result* (see 2.2.2.3) in his own p-turn in the play situation. From a developmental perspective, Ayse should be able to represent blocks at the detailed level of shapes (see section 2.1). However, the built corpus does not exactly match the figure on the card (see Fig. 4.3.). By approving the correctness of the built corpus perhaps she ignores her father's mistake in the built corpus and she genuinely means that it is right. Another possibility is that she cannot really discriminate the wrong arrangement of blocks in the built corpus and cannot really represent each block at the detailed level.

Thereupon the father reacts by saying "cool" <03>. With regard to the meaning of the word "cool", which implies approval, his utterance looks like an exclamation and signalizes four possibilities:

- He finds his corpus excellent and thus says "cool" just for the built corpus, or
- He is glad of Ayse's agreement and reacts that he finds Ayse's reaction "excellent", or
- He congratulates himself "well done" on his success, or
- He understands, that Ayse has ignored his mistake in the built corpus and thus she means it is right. She recognizes his *blind action* but doesn't engage critically. Therefore he finds Ayse "cool".

From a participatory point of view, taking the first three possibilities into account the father appears to be a *novice*, who needs approval for building the right corpus. In this sense, he reserves the role of *tutor* for Ayse, as in line <1> where she decides the rightness of the built corpus. In the fourth possibility, the father gives the impression that he is acting as *tutor* as in line <1>, where he responds to Ayse's emotional state by means of *scaffolding* (see 2.2.2.3.).

While Ayse is looking at the chosen card, she adds "but one long and one short" <04>. Probably she is looking at the card with the aim of scrutinizing it. She might realize a disparity between the built corpus and the figure on the card and tries to indicate it to her father. In her comment she does not exactly explain what is long and what is short, but considering that there is a difference between the built corpus and the figure on the chosen card, her reaction can be interpreted as follows:

When the blocks on the chosen card and in the corpus are compared from the frontal elevation, the block K8 does not match block h on the card. Block h appears like shorter than K8. In this regard Ayse might mean that

- block h on the card is short but block K8 is long, or
- block h on the card is short but block K6 is long,
- block h is a short and block f is a long block on the card (see Fig. 4.4).

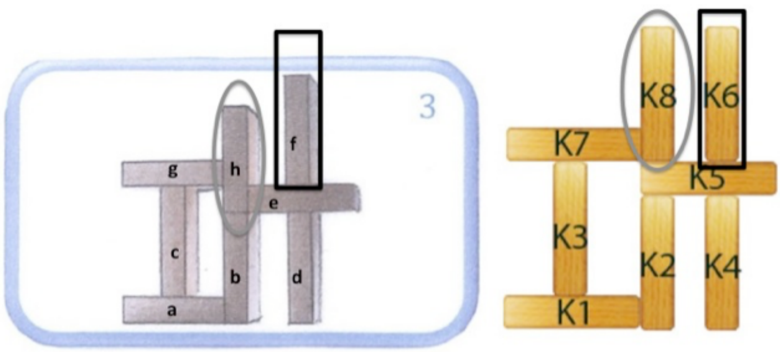


Fig. 4.4. Comparison from the frontal elevation between the built corpus and the figure on the chosen card

There is also another possibility, which is that Ayse might be comparing the length of the corpus and the figure. In this regard Ayse might mean that either

- the built corpus is long and the figure on the card is short, or inversely
- the figure on the card is long and the built corpus is short (see Fig. 4.4.).

In this context, Ayse might steer her assertion with geometrical argument and thus seems to be able to realize differences either between the figure on the card and the built corpus, or each block position on the card and on the corpus. In terms of the developmental perspective, her reaction indicates that she is a *congruence determiner* (see Table 2.4) by identifying the disparity between the figure on the card and the built corpus. By virtue of her visualization she may be able to represent blocks at the detailed level of shapes. Moreover, she gives the impression of being very capable of coordinating both structures *topologically* and realizing that either blocks or structures (built corpus and figure on the card) ostensibly are not the same length. In this sense, she gives the impression of *determining* the *congruence by comparing all attributes and all spatial relationships* (see Table 2.4).

From a participatory point of view, Ayse is coming up with a totally new idea and takes the role of *author*. By pointing out one long and one short “thing”, she thematizes a specific idea of her own. Moreover the father’s questioning in line <1> seems to activate Ayse’s *knowledge of the result* and evokes Ayse’s spatial abilities by means of scaffolding. In this regard Ayse’s reaction reinforces the idea that her father provides a *vicarious* form so that she can master her own action through her own *consciousness* and *control*. Ayse might assign the role of *activator* to her father, while she is taking the role of *tutee*. Moreover her reaction strengths the interpretation in the previous lines that she ignores her father’s mistake in the built corpus and she means it is right at first glance. Furthermore her utterance seems critical but constructive. She exposes her idea and rationalizes it somehow visible. In this regard her reaction might show that she broaches the feature of either built corpus or chosen card in an *exploratory way* (see 2.2.3), that she engages in the talk critically, but offers her justification through arguing the disparity of the built corpus and the figure. She points out the relevant characteristics of either the card or the corpus.

The father says, “yes, I think the picture is wrong”, while he is showing the figure on the card <05-06>. In this context, he expresses his agreement with a positive statement. Moreover, he gives the impression of taking in Ayse’s point and maybe thus comments on this matter by responding affirmatively. Therefore, he might show the figure on the card, which is “drawn wrongly”, and state it as a reason for the disparity between the built corpus and the figure on the card. His expression can be interpreted in five different ways:

- He builds the corpus vertically on the table and might put emphasis on the stability of the corpus. The figure on the card is less stable and would easily collapse, whereas the construction he builds seems to be quite robust; especially the cross consisting of K4, K5, K6 and K8 is statically better integrated into the entire of corpus than the cross consisting of d, e and f (see Fig. 4.4.). Therefore the “picture” on the chosen card is incorrectly drawn.

- He realizes the difference between the card and the built figure, but he does not care about the deficient jut of the block h (Ya-Yb) (see Fig. 4.5.) and focuses only on gestalt of the corpus and the figure. So indeed, regarding the deficient part of the figure (Ya-Yb), the gestalt of the built corpus is identical to the gestalt of the figure. Maybe, the difference between the built corpus and the figure is less important for him than for his daughter.

- He perceives two different blocks (h and f) on the card and probably comments as if he needs longer or shorter blocks, with which the corpus can be properly built up as seen on the card. He makes a statement that the figure on the chosen card is wrong, in spite of the fact that all the blocks are the same length.

- Maybe he “sees” Ayse’s point and tries not to take on the responsibility of building a corpus not identical with the figure on the card. As a *standard opinion*, the father *has to be* an *expert* and *has to achieve everything better* than a child (see 2.2.2.3). Therefore, he might try to act as a standard role model for his daughter, who is “expected” not to make a mistake. In this sense he might recognize the incongruence of the structures (figure on the card and the built corpus) but interpret it as a mistake of the “picturing”. So, he does not “want” to claim responsibility for this mistake, what he has done. For this reason, he imputes the fault of the corpus to the chosen card.

- From a socio-constructivist perspective the father acts as a tutor, who knows the answer in this manner. Maybe he tries to offer such a position that Ayse can recognize the essential fault in the building activity and solve it without any assistance from her father by means of self-regulation (see 2.2.2.2. and 2.2.2.3). Namely, he might pretend to offer a thought-provoking reason to Ayse to keep her deeply “in the spatial problem”,[62] in terms of scaffolding.

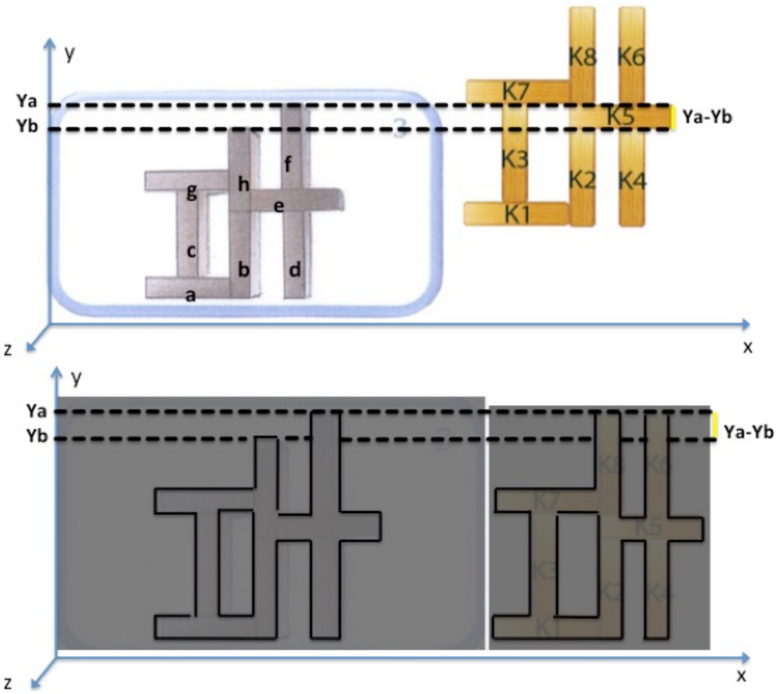


Fig. 4.5. The deficient jut of block h (Ya-Yb) and the *gestalt* of the chosen card and of the built corpus from the frontal elevation in coordinate axis

In any case the father comes up with a new argument. Hence from the participatory point of view he takes the role of *author*. Moreover by offering his justification and hypotheses he engages the talk critically but constructively with Ayse’s ideas. He gives the impression of catching Ayse’s idea and upon coming up with a new hypothesis that “the card (picture) is wrong”. Thus he is trying to be cooperative with Ayse. In this regard they seem to negotiate using *exploratory talk* which Ayse broached just before <4>. From a supportive perspective, his reaction can be called an *instruction*, by which he suggests the use of any specific strategy as a supportive activity. By imputing the fault of the corpus to the chosen

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[62] This refers to solving any emergent problem during block building activities in the current play situation. It does not have any cognitive, clinical or psychological meaning in terms of problem solving theory.

card and by diagnosing the chosen card as wrong he uses a specific strategy which Ayse can also use in her next p-turns.

Ayse says “hıııım” <07>. Considering that the utterance “hıııım” is a typical Turkish expression and denotes thinking about something, Ayse’s reaction can be interpreted as a *musing* process. Maybe her father’s reaction makes her think about the card, the built corpus and/or their relationship. While she is musing on her father’s comment, she might try to react simultaneously and thus makes such a sound. Her reaction gives the impression that her father’s scaffolding function works on Ayse, evoking her spatial skills.

Thereon the father says, “Look, this is a short block or a short piece of wood” <08-09>. His definition reinforces the interpretations in lines <05-06>. One possibility is that he might *really* perceive two different blocks (h and f) on the card and thus might comment as if he needs longer or shorter blocks, with which the corpus can be properly built up as seen on the card. Thus he might try to explain the fault on the card in detail, that one “block” – most probably block h – is drawn shorter than others on the card. Another possibility is that he might mean by the utterance “short piece of wood” the jut of the block f, which makes it longer than block h (see Fig. 4.5.). A further possibility is that he might *pretend* to perceive there exists a short block or a short piece of wood in the “picture” <05-06>. By reacting in such way he might try to simplify the matter so that Ayse can convert it and resolve the discrepancy by means of scaffolding. In this sense, he seems to *mark a critical feature* of a block on the card. In any event his reaction signalizes that he identifies each block on the card with a different length and seems to pursue Ayse’s argument in line <04>. From a participatory point of view, the father seems to take the role of *spokesman* while Ayse becomes a *sponsor* (see 2.2.2.2). He takes Ayse’s geometrical idea and points out the relevant characteristics of the blocks either on the card or on the corpus. From a supportive perspective his reaction might be interpreted as a supportive activity, namely *re-representation*, in which he reinforces Ayse’s hypothesis with different words. From a developmental perspective, the father seems to prevent any *framing* (see section 2.2.1) of the spatial ideas and to steer their discussion with a geometrical approach as Ayse did in line <04>. By conveying his thoughts, he seems to try to give Ayse a reason based on geometry for why he identifies the figure on the card as wrong. Thus his reaction reinforces the idea that they negotiate with *exploratory talk* in which they can collaborate and understand each other's points of view.

Ayse looks at the pile of blocks <10>. Her reaction might be interpreted as an examination whether all blocks are the same length or size. Moreover her reaction might manifest that she shows regard to her father's argument and muses on it.

The father adds: "Building block" <11>. Some family system theories (see 2.2.2.1.1.) promote the idea that fathers tend to make more requests for information and use a greater proportion of verbalizations describing form, shape and direction relations in the course of interacting with their children in structured tasks. Moreover, fathers differ from mothers in that they tend to give more exact and elaborate descriptions of the playing cards. In this regard, Ayse's father might try either to use *correct description* or to give *definite instruction* to Ayse about what he is speaking. Maybe he tries to let Ayse understand his point of view and thus tries to give an exact and elaborative description of the card.

Ayse leans back, while she is still looking at the pile of blocks <12>. Her ongoing action can be interpreted as Ayse examining the blocks in the pile in the course of her father's elaborative description of the card.

By pointing to the pile of the blocks, the father says "but there are no short blocks. I think the picture is wrong. I would say; it is all right. Okay?"<13-15>. Maybe he tries to underline his previous argument from just before. His reaction can be interpreted in two different ways:

- He *really* perceives two different blocks (h and f) on the card and claims that he needs shorter blocks, with which the corpus can be properly built up as seen on the card. But there are no short blocks in the pile. Thus the "picture" of the figure on the chosen card is drawn wrongly and he would say that he built the corpus "right".

- He *pretends* to perceive a block on the card as if it is short. Maybe he tries to highlight a critical feature of the matter and thus provide Ayse with a learning situation so that she can explore where the problem exactly lies. With regard to scaffolding perception, he might try to let Ayse review and muse on the built corpus.

From a supportive perspective, it seems that the father maintains his previous argument <08-09> and *re-represents* it. Therefore his reaction can be interpreted as a supportive activity r*e-representation* by which he reinforces Ayse's hypothesis <04>, that there exist "one short and one long" block, with different words. From a participatory point of view the father acts as a *spokesman*, while he seems to ascribe Ayse the role of *sponsor*. He gives the impression that he takes Ayse's geometrical idea of and points out the relevant characteristics of blocks both on the card and on the corpus. In this regard he tries to negotiate using *exploratory talk* with Ayse and comes up with the same hypothesis, "the card (picture) is wrong", again as in line <05-06>. Thereupon he asks Ayse for approval. By posing such a question, he might try to get feedback from Ayse as in line <01> and thus try to offer Ayse an expanded leeway, in which she can take an opportunity to interrogate his idea and come up with a new idea as an *author* after musing on his outcomes. Moreover his reaction signalizes one of the scaffolding functions, namely *marking critical features,* which enables Ayse to use her spatial knowledge and convert it into a tool for *conscious control*. The father seems to remark on a certain of feature of his chosen card and built corpus that "there are no short blocks" and "the picture is wrong" <13-15>.

The father might thus render Ayse such a *vicarious* form that she can master his action or declaration. From a supportive perspective, the father might try to *engage* Ayse in the play situation by posing such a question to let her feel free to express her own ideas and to become deeply involved in the play situation. Thus his reaction seems to be one type of supportive action, namely *motivation.*

Thereafter the father sets the chosen card on the other cards, which he chose at the previous p-turns, and he asks Ayse, "You too?" <16-17>. His action strengthens the interpretation <13-15> that he tries somehow to get feedback from Ayse. Taking the card away might show that he does not want any further discussion about the built corpus in terms of *reduction of degrees of freedom*, which is one of the functions of scaffolding. Another possibility of his reaction is that he tries for yet further discussion about the built corpus and thus implicitly tries to let Ayse open her own leeway and persistently to turn up. Thus, he might rather gently force Ayse to think again about his mistake, which might enable an expanded leeway of participation for her. In this sense, he might pretend to insist on the *rightness* of the built corpus to evoke Ayse's spatial ideas in terms of scaffolding. From the participatory point of view, the father appears to reserve the role of *tutee* for Ayse, whereas he seems to be an *activator,* who gives the impression of trying to activate Ayse's spatial knowledge by means of scaffolding. Moreover some family system theories reinforce this idea that fathers evoke the *activation function* during play interactions with their children, which involves an exploratory system in which children experience novel issues in physical and social environments. Thereby the father seems to maintain the negotiation with Ayse using *exploratory talk.* By asking Ayse for approval, he signalizes that he is cooperative and strives to reach an agreement with her. Moreover, by asking her for approval, he might sustain his offer, that Ayse can exchange her ideas and take the role of *author*. Thereby the interaction process again gives the impression of rendering an expanded leeway of participation to Ayse.

Thereafter Ayse pushes the built corpus with her right hand and voices "hıı" <18-19>. Her reaction is a typical Turkish expression of approving something. Therefore, her reaction can be interpreted as being in agreement with her father. By approving wordlessly, Ayse might demonstrate that she is not sure about her father's declaration and action – whether the card is wrong, or the corpus is right. Maybe her father's reaction made her think about the built corpus and her *self-regulation* can become mobilized. Maybe therefore she is uttering "hıı" rather than saying "yes". However, she seems not to make any clear commentary and not to go into any further discussion, which might also be seen as not rejecting her father's argument.

Bowling over the built corpus and pushing the blocks onto the pile, the father says, “Good.” <20-21>. His reaction might show that he assumes that his p-turn is successfully over. Thereby he behaves as though Ayse has not rejected his argument and he either pretends or really intends to turn a blind eye to the disparity between the built corpus and the figure on the card. Maybe he tries to provoke Ayse and to call her attention to the blocks, which are all the same length, by bowling over the corpus and pushing them. In this regard he appears to be realizing one of the scaffolding processes in that he *marks* one of the *critical features* – being the same length – of the blocks. From the participatory point of view, he can be regarded as an *activator*.

Ayse picks up a new card from the deck <22> (see Fig. 4.6.). The father’s p-turn in the third round ends and Ayse’s p-turn begins. It seems that the discussion about the rightness of the built corpus has ended unambiguously. A working consensus between the father and his daughter thus seems to emerge. This might show that they do not continue discussing the built corpus or the chosen card or the length of the blocks. From a supportive perspective his reaction can be seen as a supportive activity, namely, *conclusion*, as his p-turn is successfully over. He concludes the situation by bowling over the built corpus, pushing the blocks onto the pile and saying, “Good”.

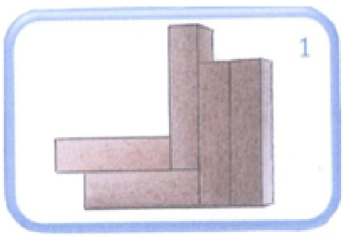


Fig. 4.6. The card chosen by Ayse in the third round

Ayse looks at the card then laughs at her father <23-24>. She might laugh at her father to signalize it is either easy or difficult for her to build as a corpus. In this sense, Ayse’s reaction can be interpreted in two different ways:

- When the card is difficult for Ayse, it might seem that she implicitly asks her father for help to build a corpus correctly. Thereby her reaction might also show that she accepts her father as a *tutor.*

- When the card is easy for Ayse, Ayse's reaction can be a kind of *gratification*, that she can achieve her p-turn successfully, by building such an easy figure.

The father says, "Yes" <25>. It is unclear what he is confirming with his "yes". One possibility could be that the father confirms that it is now Ayse's p-turn and the figure on the chosen card is easy to build, as Ayse laughs. Another possibility could be that the figure on the chosen card might be difficult and therefore he might approve Ayse's reaction with a supportive message that he can help her to build a corpus. From a supportive perspective, the father's reaction can be called *affirmation*, in that he demonstrates somehow his agreement with Ayse, whether she finds the card difficult or easy.

Ayse says, "Nope (what).." <26>. Originally she utters "ne..", which has a double-meaning in German-Turkish bilingual conversation. In German "ne" is a slang expression for "no". In Turkish "ne" means "what". Thus, Ayse's reaction in German language can be interpreted as a refusal of her father's intentions, which she ascribes to his utterance "yes" before. Beside this, in Turkish language she might ask her father in a short way either "what" she should do or "what" he means. One of the interpretations of lines <23-24> can be reinforced; that she finds the card difficult and therefore she asks her father "what" she should do. Thereon her father tells Ayse "not so difficult.", while he is scratching his face with his left hand <27-28>. His reaction can be interpreted in two different ways:

- His reaction can be interpreted as a critique or clue that the figure on the card is "not so difficult" to build and Ayse can do it easily. From a participatory point of view, Ayse is both his play partner and his competitor in the game. In this sense, he might compare his previous card with Ayse's current card and find it not as difficult as his previous card. Hence, he might point out that the figure on Ayse's card is not so difficult to build up as a corpus.

- Another possibility is that he tries to go on with the scaffolding process as in the previous p-turn. Therefore, he might try to *control* Ayse's *frustration* and ensure that she does not give up, in the terms of scaffolding. He might predict the consequence of Ayse's anticipated action – that she might give up –, and thus he tries to use a direct elicitation that the figure on the card is not so difficult and she can build it. He might try to facilitate Ayse's *self-regulation* by predicting the consequences of anticipated actions. From the participatory point of view he might take the role of *tutor*. From a supportive perspective, his expression can be interpreted as a supportive activitiy, namely *motivation*. He might try to *motivate* Ayse in the meaning of encouragement that the card easy for Ayse to build.

In both possibilities, the father and Ayse seem to interact in an *exploratory way* (see 2.2.2.3) in that the father engages in the negotiation process with Ayse critically and constructively. Furthermore, he gives the impression that he defines one of the relevant characteristics of the chosen card.

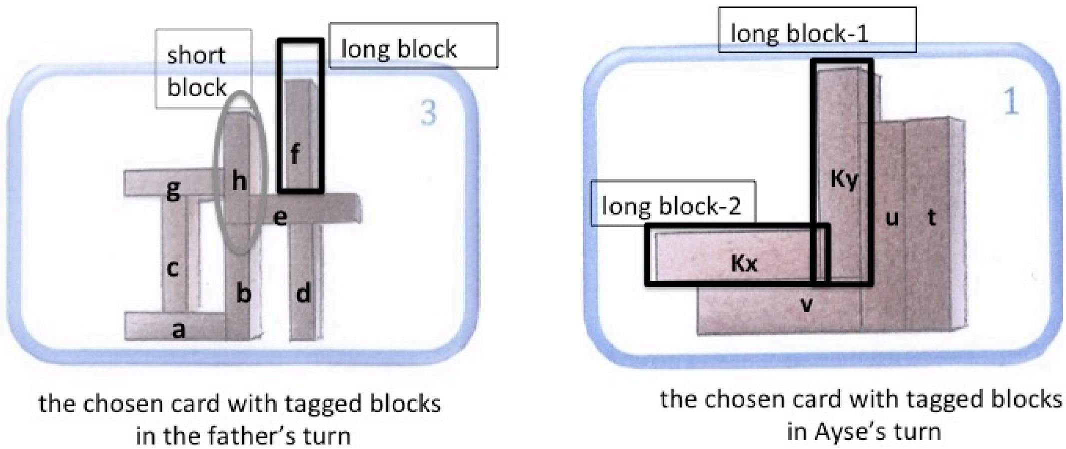


Fig. 4.7 Long blocks on Ayse's and her father's cards from the frontal elevation

Ayse lays the card on the table and separates it from the other cards, which she has done in her previous p-turns <29-32>. Then by looking at the pile of blocks on the table, she comments "but..one more.. a long one. heh?" <32-33>.

By putting the card on the table Ayse might try to check the card and focus on it. By looking at the pile of blocks she might attempt to check and find out whether there are long and short blocks in the pile or not. By using the adjectives “one more” and “a long one”, Ayse might mention two blocks:

- In her father's p-turn, which was just before her p-turn, she already saw “one long” block (see Fig. 4. 7). In the figure on the card she just sees two “long” blocks (see Fig. 4.7). This might evoke the idea that she needs just “one more” block, because in accordance with her father’s card there already exists “a long one”.

- She points out “one long block more” but not “three short blocks more”. At her father’s p-turn (see line <13-15>) he claims that there are no short blocks. By considering his claim, Ayse might muse on the existence of long blocks alone (see Fig. 4.7) and thus she might come up with the idea that she needs one long block more instead of three short blocks (see Fig. 4.7). From the participatory point of view, by coming up with this idea Ayse might take the role of *spokesman*, while she ascribes the role of *sponsor* to her father (see 2.2.2.2.). Thereby she broaches the issue of having either long or short blocks which they discussed in the previous p-turn. She might take the idea of her father, that there is no short block, and might formalize his idea in a different way, that she needs one long block more. Thereby she might keep on focusing on the same theme as in her father’s p-turn, however she remarks the focus by using antonym words (e.g. short and long).

- Ayse might repeat her argument, which she has already mentioned at her father’s p-turn (see line <04>), and might try to open a discussion about her father’s argument and cannot let go of her previous argument that she needs different lengths of blocks to build the figure on her card properly. In this way she might also try to spoil the non-ambiguity of their previous discussion in her father’s p-turn.

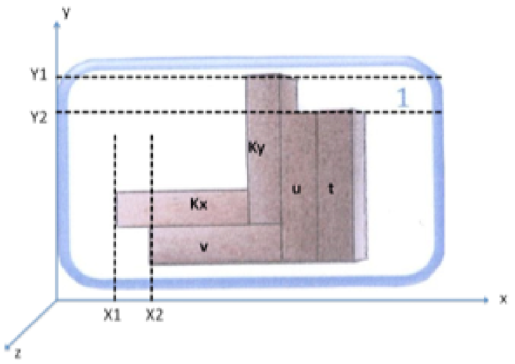


Fig. 4.8. Juts of Kx and Ky from the frontal elevation in coordinate axis: X2-X1 and Y1-Y2

From a developmental perspective, it can be said that Ayse is highly attentive, and that she can apprehend the jut of the block Kx (X2-X1) and the jut of the block Ky (Y1-Y2) (see Fig. 4.8). Her reaction reinforces the idea, as in the previous p-turn, that she is able to represent blocks at the detailed level of shapes and to coordinate the structure topologically. She seems to able to look for differences in attributes by examining full shapes.

The question “heh?” might mean that she needs some feedback or a clue from her father. Maybe she asks him for approval, if she is doing it right. Moreover, her reaction signalizes Ayse’s *environmental self-regulation* in that she mobilizes her environment through communicative solicitation by posing such a question (see 2.2.2.2). From the participatory point of view, her reaction shows she accepts her father as a *tutor*, who is an expert in introducing spatial tools in the building activity, while she takes the role of *tutee*. Besides this she seems to call for negotiating with her father using *exploratory talk* by offering him her hypotheses. She gives the impression of striving to discuss the alternatives and reach an agreement with her father.

The father says, “No no it looks like that.” and then he adds “let them away” while he is taking the deck of her previous cards away from Ayse’s hand <34-35>. His answer can be interpreted as that no long block is needed to build the figure on the card. Furthermore, he might mean that the figure on the card looks like two different lengths of blocks are needed to build the figure correctly. Thus, it seems that he creates a conflict between his arguments: two similar problems are exactly expressed as two different contrary positions. In his p-turn he rationalizes that the chosen card is drawn incorrectly, whereas in Ayse's p-turn he claims that the card makes it look as if two different lengths of blocks are needed. But now he gives the impression of emphasizing that actually two different lengths of blocks are not needed to build such a corpus. Namely he does not draw on his previous arguments <05, 08, 14-15> that the card is wrong and that short blocks are needed to build the corpus properly. From a supportive perspective his reaction can be interpreted as a supportive activity, namely *disaffirmation*. It is a type of correction and a definitive negative response indicating an incorrect reaction. From the participatory point of view, Ayse is his game partner, and he might try not to give any other explanation or clue to Ayse about her card by reason of their *competition* in the play situation. Another possibility is that he might take the role of tutor, while he is reserving the role of *tutee* for Ayse. In terms of scaffolding he might try to maintain *recruitment* by saying "no no it looks like that". Namely he gives the impression of preventing Ayse from abandoning her p-turn. Maybe he tries to *ensure that she does not give up*. In that respect he gives the impression of negotiating with Ayse using *exploratory talk* by sharing some relevant information ("the card looks like that") with her. Moreover, he shows sign of offering a claim that the card looks like that and striving to reach an agreement with Ayse. Besides, by taking the cards away from Ayse's hand and saying "let them away" <34-35>, in terms of scaffolding, he might try to keep Ayse *in the field* so that she focuses on the chosen card in spite of the other cards. In a similar manner he might also try to keep Ayse *in pursuit of a particular objective* so that she can *directly* maintain the building activity. Therefore, through his reaction, the father successively uses two *scaffolding functions*: *recruitment* and *direction maintenance.* He seems to take the role of *tutor* and to ascribe the role of *tutee* to Ayse.

On the heels of his reaction <34-35> the father resumes by questioning Ayse: "Did you lose one?", while he is holding and shuffling the deck of Ayse's cards <35-36>. Most probably he asks Ayse if she has lost one of her cards. His reaction can be interpreted in at least two different ways:

- Maybe he is more interested in taking care of Ayse's previous cards but not discussing either the chosen card or the length of the blocks. He might try to change the subject about which they are talking. Namely the father seems to prefer to avoid further discussion about the blocks. He might simultaneously try to control the game and to obstruct Ayse's endeavour to spoiling the non-ambiguity of their previous discussion <32-33> in his p-turn.

- Another possibility is that he tries to *keep* Ayse *in the field* so that she can focus on the current chosen card in spite of the other cards. Maybe therefore he tries to understand the reason why Ayse holds her previous cards, and thus might pose such a question.

From the participatory point of view, in both possibilities, the father seems to take the role of *tutor*, while ascribing the role of *tutee* to Ayse. He tries to assist Ayse somehow and keep her attention on the ongoing play activity; during this he gears her behaviour to her needs.

Thereon Ayse says "no." and looks at her father <37>. By giving a negative response Ayse might mean that she did not lose any card from her previous p-turns. Another possibility is that she might try to drop the changed subject, and thus give a negative, short and exact response to him.

The father says "Okay" and lays the card in front of Ayse on the table <38>. Although it is not clear whether he okays Ayse's reply or the non-lost card, it seems that he perpetuates not talking about either the cards or Ayse's hypotheses or the length of the blocks. Considering first interpretations in lines <27-28> and <34>, one possibility is that the father might try not to give any explanation to Ayse or go into any further discussion about the cards, by reason of being in *competition* with his daughter in the play situation. Another possibility is that he might try to go on with the scaffolding process as in the previous p-turn. Therefore he might try to keep Ayse *in the field* so that she can focus on the current chosen card in spite of the other cards. He gives the impression of striving to keep Ayse *in pursuit of a particular objective* – her current p-turn. Moreover he might try to enable her to *concentrate* on and *master* the building action in hand.

He might reserve the role of *tutee* for Ayse and himself take the role of *tutor* from the participatory point of view.

Ayse tells her father that she wants another card <39>. With her reaction Ayse might mean that she wants to get a card with a figure that can be built easily. Thereby her reaction reinforces the idea in line <22-24> that she might find the card difficult and ask her father for help to build the corpus correctly. In this regard her reaction can be interpreted in a way that Ayse takes her father's reaction – changing the subject – as a negative response and therefore tries to quit the chosen card. On the other hand, it could be seen as frustration problem of Ayse that she foresees not being able to build the corpus correctly. Moreover, her reaction seems to confirm the idea in lines <38, 34-36, 32-33, 27-28) that the father strives to ensure that she does not give up by using *scaffolding*. Neverthless, by posing such a type of implicit request, her reaction shows Ayse's *environmental self-regulation* in that she tries either to adapt other *material* or *local conditions* in the play situation or to initiate referential joint attention in the play situation with the partner, i.e., the father (see 2.2.2.3).

When shown the card, the father says "but it is easy. just get started. … just get started." <40-42>. His reaction can be interpreted in three different ways:

- His comment can mean that the card is easy to build and Ayse should just get started to experience that she is able to build it easily. In terms of *scaffolding* he might try to *control* Ayse's *frustration* and encourage her to *concentrate* on and *master* the current building action. Therefore, he might try to motivate Ayse in a positive way to make her believe that the card is easy and she can do it. Namely he might try to encourage Ayse to build the figure on the card. From a supportive perspective, his reaction can be interpreted as a supportive activitiy, namely *motivation.* He seems to strive for *motivating* Ayse by way of encouragement that the chosen card so easy that she can build it. In this regard, from the participatory point of view, he might ascribe the role of *tutee* to Ayse and take the role of *activator* himself. Moreover, he seems to negotiate with Ayse using *exploratory talk* by pointing out the relevant characteristic of the chosen card and striving to reach an agreement that she can build it. Through his reaction he provides Ayse with an expanded leeway so that she can explore, perform and examine any building possibilities in detail.

- Another possibility is that the father might not wish to waste time discussing the chosen card and tries instead to push Ayse eventually to get started and finish her p-turn. Some family system theories point out that fathers seem to convey to their children that completing a task in the shortest amount of time is the primary goal in problem solving (see 2.2.2.2.1.). In this respect, Ayse's father might try to complete the game in the shortest amount of time and to keep turns of the play situation as short as possible.

- Ayse is his play partner and also his *competitor* in the play situation. In this sense, he might compare his previous card with Ayse's current card. One possibility is that he finds it easier than his previous card. His reaction brings to mind his previous utterance at line <27> that the chosen card is not so difficult. Thus, he might keep his view and point out that Ayse's card is easy and she should start to build it. From a participatory point of view his reaction also brings to mind the previous interpretations that the father and his daughter are in *competition* in the play situation (see lines <38>, <34>, <27>). Therefore, another possibility is that he might find Ayse's card harder than his previous card. With a view to Ayse losing points, he might pretend that her card is easy and to let her start building it. Because Ayse is his competitor in the play situation he might try to win the game by pretending.

Whatever, from a supportive perspective, the father's reaction can be seen as a supportive activity, namely *motivation*, in that he motivates Ayse either in a positive or negative way by encouraging her to get started to build the corpus. Additionally, Ayse's *environmental self-regulation* seems to work on her father in that he seems to engage in joint attention on Ayse's card with Ayse.

Ayse props her head on her left hand and utters "hıııım" <43-44>. Her reaction is a typical Turkish expression that denotes thinking about something. In this regard she appears to be in a *consideration* process. Most probably her father's reaction makes her think either about her next step, whether she should go on with this card or not, or her father's fundamental aim, whether he is just trying to win the game or to let her win. Thereon she takes the first block (K1) and sets it on its Z side so that she can see its Y side <44-45>. Through this reaction Ayse might show that she *accepts being in the field* in spite of giving up the game, in terms of *scaffolding*. Thereby her reaction signalizes that her father's feedback or *scaffolding functions* work on Ayse; he makes her think. The father confirms Ayse: "exactly." <46>. His reaction can be seen in three different ways:

- He might find Ayse's card is just easier than his previous card. Therefore, he might insist on his claim that Ayse can build the figure on the card.

- In terms of competition he might find Ayse's card harder than his previous card. Intending for Ayse to lose points, he might pretend to motivate Ayse in a way that she can build the figure on the card.

- He might try to keep up the *scaffolding process* and thus tries to *control* Ayse's *frustration.* He might find Ayse's action "exactly right" and thus try to deploy Ayse's enthusiasm in order to keep her motivated and *in the field.* Thereby, from the participatory point of view, he is taking the role of *activator*, while ascribing the role of *tutee* to Ayse.

In any event, from a supportive perspective his reaction seems to be a supportive activity, namely *affirmation*, in that the father assents to Ayse's reaction.

Ayse turns the block (K1) counterclockwise around its Y side (see Fig. 4.9) so that she can see its X side.

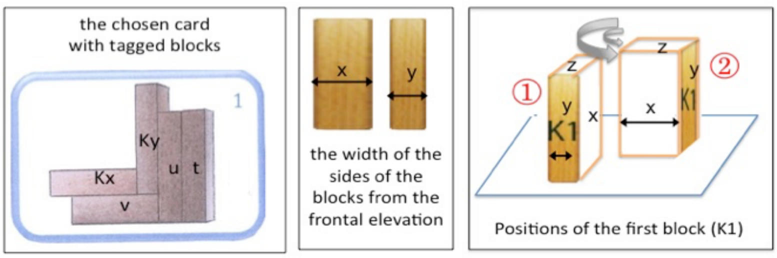


Fig. 4.9. The first and second position of the first block (K1), which is set by Ayse

Maybe Ayse interprets her father’s reaction in a negative way, that he might be trying to make her lose, and thus she changes the position of the block. She might be *motivated* in a positive way, in that she might try to reveal her spatial ability and with a view to doing her best she might change the position of the block. Another possibility is that she might change the position of the block (K1) in order to get some more reaction or clue from her father. Thereon she asks “like that?” <47-48>. Her reaction reinforces the idea in lines <23-24> and <32-33> that she either asks for her father’s help or seeks approval of her action. By posing such a question she might try to get a clue from her father, whether she sets the block right or the position of the block should be “like that”, i.e., as in second position. Her reaction shows *environmental self-regulation* in that she tries to mobilize her father through communicative solicitation by posing a question and seeking his approval of her action. From a developmental perspective, the changing position of the block can be a *width problem* in Ayse’s visualization (see Fig. 4.9). It seems that Ayse cannot distinguish the *width* of the block sides. Considering Ayse’s remark <4> in her father’s p-turn, her reaction can be also interpreted as that she can actually determine all attributes of the figure on the card, as she did in line <4>. Because of her father’s argument in the previous p-turn, in this p-turn she might consider her father’s criticisms at lines <8,14-15> and thus be confused about either the width or length of the wooden blocks. Another possibility is that she might visualize the Y side of the blocks as wide as the X sides. With regard to the participatory point of view, her reaction shows she is taking the role of *tutee* and *percieveing* her father as a *tutor*, who is an *expert* in introducing spatial tools into the building activity. This seems to be maintaining a negotiation with Ayse and her father. She gives the impression of striving to discuss the alternatives and reach an agreement with her father.

The father says “no” and then turns the block to the previous position. Then he adds: “it was already right. the next block.” <49-50>. Firstly, the father disaffirms Ayse. Most probably he just gives a negative response to her question ”like that?” in lines <47-48>. He might mean that she shouldn’t set the block “like that”. Turning the block to the previous position might be an exercise to show the correct position of the block. His reaction brings to mind his remark in the previous p-turn <8,14-15>.

By saying “the next block.” the father might mean that she should attend to the next block and leave the current one. This account of fathers’ behavior seems to be repetitious as it was presented just above that the father seems to convey his child to complete block building activity in the shortest amount of time. In this respect he might try to complete the game in the shortest amount of time and to keep turns of the play situation as short as possible. Moreover his reaction reinforces the second interpretation in lines <40-42>, that he might not wish to waste time building the corpus and thus tries to push Ayse eventually to get started and finish her turn. From a supportive perspective he seems to realize two different support activities consecutively: first *disaffirmation* and then *modelling.* First, he expresses his disagreement and then by changing the position of the block he *models* the correct position of the block – how it should be set.

Furthermore, his reaction brings to the mind another aspect of family system theory; that fathers tend to make more requests for information, give more exact and elaborative descriptions of the cards and use a greater proportion of verbalizations describing form, shape and direction relations than do mothers in the course of interacting with their children. So indeed Ayse’s father really directs the block by *modelling* and verbalizes his *disapproval* and gives an elaborative description about the position of the block and the coming step. He seems to realize some *scaffolding* functions too: *marking critical features* and *demonstration.* By means of *marking critical features* he provides information about Ayse’s act, that she shouldn’t set the block “like that”. Thereon he appears to use *demonstrate* how the block actually should be set. In the sense of *demonstration*, he might be *imitating* in idealized form the position of the block in the expectation that Ayse will then *imitate* it back in a more appropriate form. Regarding participation he seems to be a *tutor,* while reserving the role of *tutee* for Ayse. Additionally, through his disaffirmation and demonstration he gives the impression of maintaining the *exploratory* negotiation in that his approach is critical but cooperative and he offers his justification and hypotheses to Ayse.

Ayse takes the second block (K2) and sets it vertically on its Z side next to the K1 so that their X sides touch each other <51-52>. Her reaction might show that she *relays* her father's *demonstration.* In terms of scaffolding, her reaction signalizes that her father's reaction works on Ayse, that she executes the building activity in the same way as her father. Likewise, her reaction might manifest that her father's *idealization* made it clear in which way she should go on building the corpus. Her father's *scaffolding* seems to work on Ayse. From a developmental perspective her reaction shows she has quite strong spatial visualization in that she can realize or see the difference between the wide and narrow sides of blocks and she can exactly imitate her father's action in one go. From a participatory point of view, she gives the impression of taking the role of *relayer* while ascribing the role of *deviser* to her father.

Thereon she takes her previous cards, which lie on the table, and says "hey my cards!" while she is putting them back on the table in front of her <53-55>. This reaction might means she tries to have her previous cards within reach. Moreover, her reaction recalls the interpretations in lines <38> and <34-35> that the father tries to keep Ayse *in the field* so that she can focus on the current chosen card in spite of the other cards. In this sense, by setting the next block and then pointing out her previous cards, she might try to refute her father's aim that she can be in the field, concentrate on and master building action while focusing on her previous cards as well. From a participatory point of view, she appears to strive to expand her leeway and concentrate on more than one thing.

The father pulls the chosen card a bit towards himself <56>. Most probably he is trying to see the chosen card more clearly. Another possibility is that he might try to point out the chosen card to Ayse, in terms of scaffolding. Maybe he tries implicitly to show Ayse that she should concentrate only on the chosen card. Moreover, his reaction might show that he tries to demonstrate to Ayse the way of concentrating on the chosen card. By dint of Ayse's previous reaction <51-52>

he might have seen that she imitates his *demonstration* and maybe therefore he performs a new *idealized subject* in order to let Ayse *imitate* it back in a more appropriate form. From the participatory point of view, he gives the impression of being a *tutor,* while reserving the role of *tutee* for Ayse. Moreover from the supportive perspective his reaction can be interpreted as a supportive activity, namely *modelling*, in that he models behaviour which Ayse can imitate.

Ayse pulls the chosen card back to her side, so that it lies next to her previous cards, then she put two blocks (K1 and K2) in order and pulls them together a bit to the right. Thereafter she takes another block (K3) and sets it vertically on its Y side next to K2, so that its X side touches another X side of K2 <57-62> (see Fig. 4.10). Pulling and setting the chosen card back to her side and then regulating two blocks (K1 and K2) <57-60> might signalize that she tries either to control her own turn or to concentrate on the chosen card, as her father demonstrated. Bearing in mind the previous interpretation in line <56>, that the father demonstrates behaviour which Ayse can *imitate*, her reaction signalizes that the father's scaffolding works on Ayse. So indeed, she gives the impression of *imitating* of her father by pulling and setting the chosen card to her side.

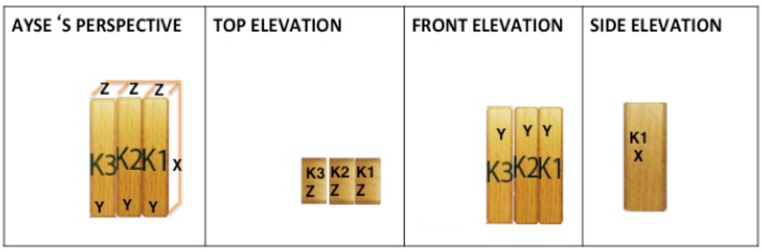


Fig. 4.10. The position of the third block (K3) which is set by Ayse

Thereon by setting the third block she indicates belief in herself, that she can further build the corpus as her father said at lines <40-42>. Moreover, from a developmental perspective, she appears to set the third block in the same way that her father demonstrated in lines <49-50>. She gives the impression of maintaining the *imitation* in idealized form of block building activity by her father. In this context the father's realization of *scaffolding* in lines <49-50> and <56> seem to work on Ayse that she *initiates* her father's *demonstrations.* From a participatory point of view, her reaction signalizes that she keeps on negotiating in an *exploratory* way with her father. She seeks the best solution, as her father offered, and approaches the task constructively and collectively. Moreover, her reaction indicates that the scaffolding process enables Ayse to turn up by imitating her father and to reach an eventual agreement. She gives the impression of taking the role of *relayer* while ascribing the role of *deviser* to her father.

The father says “hıııııııım”, which is a typical Turkish expression that denotes thinking about something <63>. In this sense, his reaction can be interpreted as a *consideration* process – that Ayse’s last step might make him think either about the position of the third block or her reaction. Thereon he adds “ıh ıh!” <63>, which is one of the Turkish expressions of disapproval of something. Namely, both of his utterances signalize that he thinks about Ayse’s last step, examines it and then tends to disapprove of it. Maybe he tries to disapprove of Ayse’s action or to strive for her to control her own p-turn. In both ways he shows disagreement with Ayse. By disapproving wordlessly, the father might indicate he is not sure whether she is building the corpus correctly Maybe therefore he is uttering “ıh ıh!” rather than saying “no” or “wrong”. So indeed, considering the built corpus and the figure on the card, it cannot be exactly diagnosed whether Ayse set the block in the corpus right or wrong. From the developmental perspective, it seems that Ayse has different opportunities to set the next block (K4) and to achieve an identical corpus to the figure on the card:

- The first possibility is that she continues the building activity by moving the third block (K3) a bit upwards and setting the fourth block (K4) horizontally underneath it (see Fig. 4.11.):

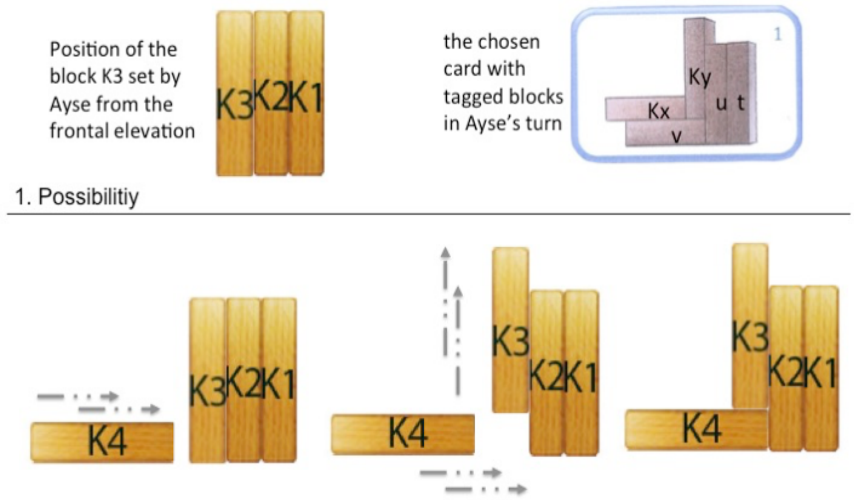


Fig. 4.11. One possible position of the third (K3) and the fourth block (K4) from the frontal elevation

- The second possibility is that she continues the building activity by knocking down the third block and places the fourth block (K4) horizontally upon it (see Fig. 4.12.).

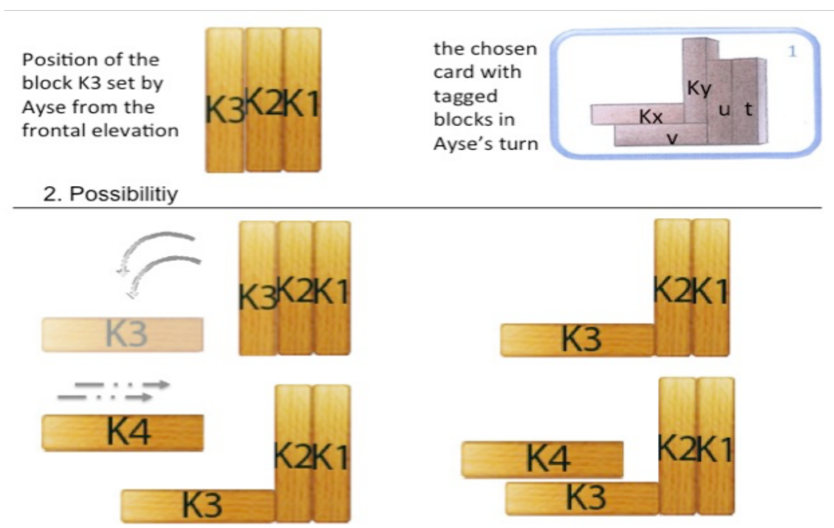


Fig. 4.12. Second possible position of the third (K3) and the fourth block (K4) from the frontal elevation

- The third possibility is that she continues the building activity by knocking down the third block and placing the fourth block (K4) perpendicularly to it (see Fig. 4.13).

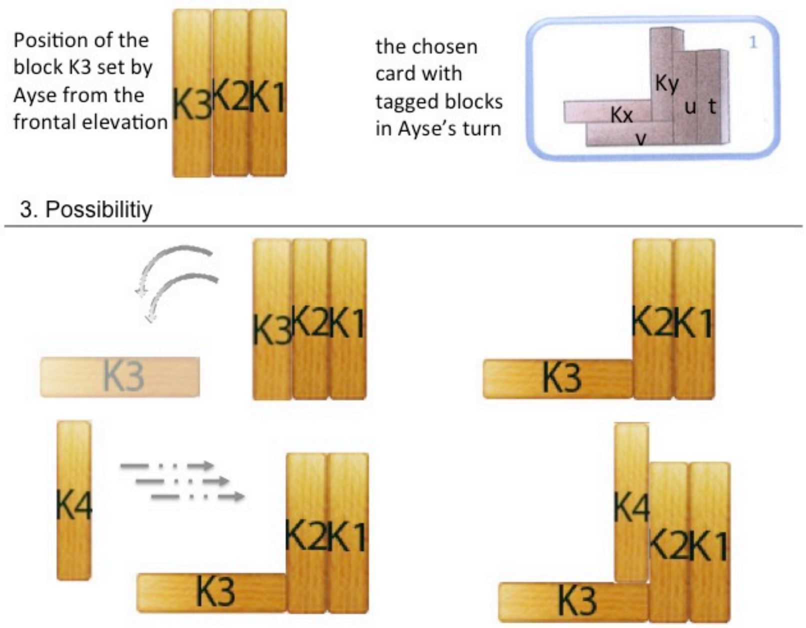


Fig. 4.13. The third possible position of the third (K3) and the fourth block (K4) from the frontal elevation

With reference to these three possibilities, it cannot be determined whether Ayse builds the corpus rightly or wrongly. As things stand, the interaction process between Ayse and her father seems to shape her further actions. Maybe therefore the father is careful to avoid making any certain commentary and does not enter into any further discussion about Ayse's building action. Moreover, he might try to direct Ayse's attention to a specific part of the chosen card or built corpus and enhance her self-regulation by using *referential nonverbal and verbal acts*. From a supportive perspective, his reaction demonstrates one of the supportive activities, namely *disaffirmation*. He realizes his disagreement by uttering "<u>ıh ıh</u>!". From a participatory point of view, he might try to direct Ayse implicitly and keep his role as a *tutor.* Maybe he tries to prevent any restriction of Ayse's leeway of participation of and thus he disapproves wordlessly and only by uttering "<u>ıh ıh</u>!" instead of saying "no" or "wrong". In this regard he somehow enables Ayse to move forward and continue building the corpus, which renders her an unrestricted leeway of participation. He and Ayse together maintain the *exploratory* negotiation process as they seek the best solution and approaches constructively and cooperatively.

Ayse takes the block (K3) back, lays it on its X side next to the K2, that its Z side touches the another X side of the K2 (see Fig. 4.13.), and then utters "hıııııııım hı?" <64-66>. Ayse's first utterance might be interpreted as her contribution to a *consideration* process, that she might take her father's reaction into account. She continues with "hı?" another typical Turkish expression, which is mostly used for linking alternatives in the sense of "or?", while she is setting the third block in the corpus. In terms of *environmental self-regulation*, her reaction reinforces the idea in lines <23-24, 32-33, 47-48> that she adapts local conditions in the play situation and *mobilizes her social environment through communicative solicitations.* Her utterance "hı?" might show that she asks her father for help or some feedback. Another possibility is that she might take in her father's disagreement and try to fix it by knocking the block down and setting it horizontally instead of vertically next to the second block (K2) (see Fig. 4.14.). From a developmental perspective her action signalizes that she has relatively strongly developed spatial abilities, through which she can relate, represent and identify 2D shapes to 3D shapes. Moreover she seems to compose and decompose building blocks using multiple spatial relations to produce composite shapes. Furthermore her reaction strengthens two interpretations in the previous line <63> that she chooses to knock down the third block and to superimpose the fourth block (K4) either perpendicularly or horizontally on it (K3).

1. Position of the block K3 set by Ayse
2. Position of the block K3 set by Ayse

the chosen card with tagged blocks in Ayse's turn

Fig. 4.14. Positions of the third block from the frontal elevation

From a participatory point of view, her reaction seems that she and her father keep on negotiating in an *exploratory way*. Moreover her reaction indicates that the father's *scaffolding* evokes her spatial abilities and she offers her hypothesis and turns up by expressing her own idea. In this sense she acts as an *author*. By posing the question "hı?" <64-66> she seems to ascribe the role of *tutor* to her father. Her reaction legitimatizes the interpretation in line <63> that the father tries to prevent any restriction of leeway of participation of Ayse, and so indeed his tutoring seems to work on Ayse.

The father utters "hım hııhıı"<67>. Firstly he utters "hım", which is a typical Turkish expression that denotes thinking about something for a while. In this sense, his first utterance can be interpreted as a *consideration* process and might show that Ayse's reaction made him think about either how she actually should set the block or whether she sets the third block right. Maybe therefore he appears to be taking a minute to think. Thereon he utters "hııhıı", which is a typical Turkish expression of approval. His reaction indicates the last position of block K3 as set by Ayse is correct. Briefly*,* from his reaction, he seems to think for a minute about the position of block K3, whether has Ayse set it right, and then he approves it. In this regard his reaction can be seen as a supportive activity, namely *affirmation*, through which he shows his agreement. Moreover he might try to direct Ayse's attention to a specific part of the chosen card or built corpus and enhance Ayse's *self-regulation* by using *referential nonverbal and verbal acts*. By approving her *wordlessly* the father might not exactly be sure whether she has built the corpus correctly. Maybe therefore he is uttering "hııhıı" rather than saying "yes" or "right". So indeed, when the built corpus and the figure on the card are compared, it cannot be exactly diagnosed whether Ayse has set the blocks correctly, because of the ongoing building process (see Fig. 4.12. and Fig. 4.13.). As in line <63>, the interaction process between Ayse and her father gives the impression of shaping her further actions. Moreover the father's reaction shows again his desire not to restrict Ayse's leeway of participation and thus he approves her wordlessly and only by uttering "hııhıı" instead of saying "yes" or "right". In this regard he somehow enables Ayse to turn up and continue to build the corpus, which renders her an unrestricted leeway of participation. Ayse and her father realize the negotiation process in an *exploratory* way in that their approaches are constructive and cooperative and they seek the best solution together.

Thereafter Ayse takes a block (K4), sets it on K3 in a same way, so that their X sides touch each other, looks at her father, and asks “hı?” <68-70>. Considering the meaning of “hı?” in the Turkish language, Ayse might be trying to ask her father, “should it be set like that, or?” as mentioned at lines <47-48>. The second interpretation about her reaction can be that she tries to pose a question such as “what should I do?”. Namely, her utterance might seem that either she asks her father obliquely for help to build a corpus correctly or tries to get aclue from her father if she is building the corpus right. By means of *environmental self-regulation*, her reaction reinforces the idea in lines <23-24, 32-33, 47-48, 64-66> that she *mobilizes* her *social environment through communicative solicitations* by posing such a question and seeking approval of her action. From a developmental perspective, Ayse’s geometrical action can be interpreted in the following way:

The father’s previous arguments about the chosen cards in lines <13-15> and <34> might effect Ayse in such a way that she yields that different lengths of blocks are needed to build the corpus identical to the figure on the card. Thereby she might tread in her father’s footsteps and thus might neglect – as her father in line <05-06> – the juts of the blocks ((X2-X1) and (Y1-Y2)) on the chosen card and focus only on the *gestalt configuration* of the figure without the juts of blocks (see Fig. 4.15.). So indeed, considering the figure on the chosen card without juts, the gestalt of the built corpus assimilates with the gestalt of the figure on the chosen card (see Fig. 4.15.). It seems that Ayse applies strong spatial thought and visualization as she constructs the figure without juts in her mind and then builds it.

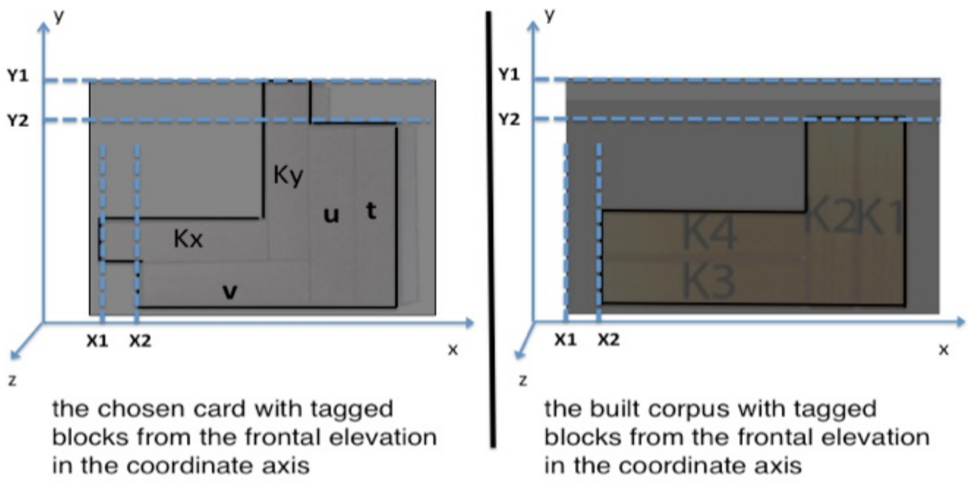


Fig. 4.15. Gestalt of the chosen card and the built corpus from the frontal elevation in coordinate axis

Furthermore, it is not obvious from Ayse's action whether she builds the corpus correctly. From a developmental perspective, the possibilities of Ayse's further actions to achieve a corpus identical to the figure on the card can be interpreted in two different ways:

- She set block K4 on K3 and then might push block K4 a bit to the opposite side of K2 – to the left – so that the fifth block (K5) perpendicularly fits in between (see Fig. 4.16.). She can thus achieve a corpus that accords with the figure on the card.

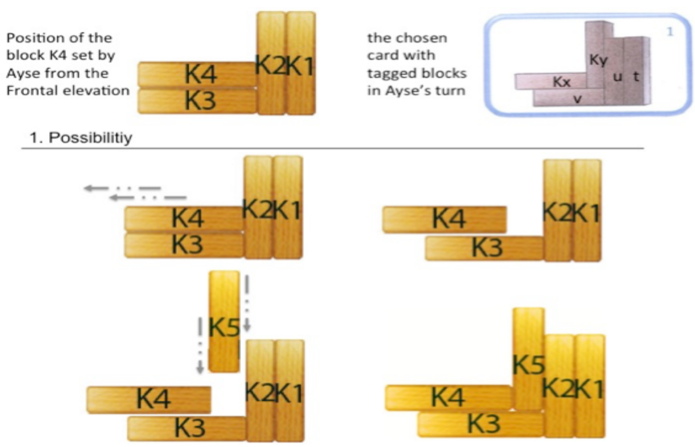


Fig. 4.16. First possible position of the fourth (K4) and the fifth block (K5) from the frontal elevation

- Ayse set block K4 on K3 and then might push block K4 in a right angle so that it stands perpendicularly on K3 and next to K2 (see Fig. 4.17.). Thereon she can take one more block (K5) and set it vertically on K3 and next to K4 so that it is adjacent to K3 and K4. She can thus achieve a corpus that accords with the figure on the card.

With reference to these two possibilities it cannot be determined whether Ayse builds the corpus right or wrong. As things stand, the negotation process between Ayse and her father seems to shape her further actions. From a developmental perspective, it remains open whether her action can be perceived as the action of a *picture maker* (see Table 2.4), *who can match shapes using gestalt configuration,* or not.

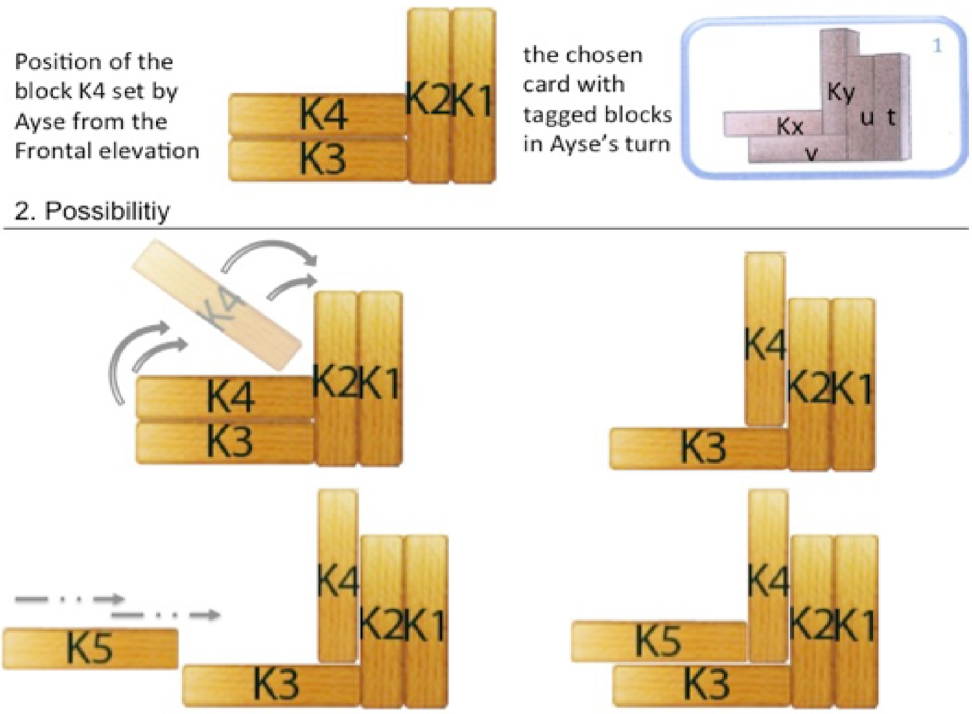


Fig. 4.17. Second possible position of the fourth (K4) and the fifth block (K5) from the frontal elevation

The father says "<u>hı?</u> I don't know" while holding his right hand to his mouth and wobbling his finger <71-72>. Regarding the meaning of "<u>hı?</u>" in Turkish language, his reaction might show that he tries to pose a question either "what has she actually done?" or "what has she actually set?". Then he says that he doesn't know. His reaction can be interpreted in three different ways:

- He might mean that he does not know, either what Ayse is actually trying to do or whether she sets the block right. He might literally mean that he does not really have any idea. From a participatory point of view, he takes the role of *novice*, while reserving the role of *tutor* for Ayse.
- He might pretend not to have any idea. By means of the *competition* in the play situation, he might try to win the game by pretending and giving no clue to Ayse.
- He might try to let Ayse think about her action in terms of *scaffolding* and therefore he appears not to be making any clear commentary and not going into any further discussion about Ayse's building action. From a supportive perspective, his reaction seems to be a type of supportive activity, namely *prompt after error.* He gives the impression of prompting Ayse implicitly to be careful and to think more about the corpus. Moreover he might try to direct Ayse's attention to a specific part of the chosen card and thus enhance Ayse's *self-regulation* by using *referential nonverbal and verbal acts*. He seems to negotiate with Ayse in *exploratory* talk that enables her to offer her hypothesis and to turn up by expressing her own idea. From the participatory point of view, he might take the role of *tutor* whereas he seems to offer Ayse a participation profile whereby she can muse on the built corpus and exchange her ideas by taking the role of *author*. Thereby the interaction process gives an impression of rendering an expanded leeway of participation to Ayse. In that regard he might ascribe the role of *tutee* to Ayse.

Ayse utters "<u>hı?</u>", tends to take the card but lets it lie and looks at it <73>. Considering the meaning of "<u>hı?</u>" in Turkish language, her utterance can be interpreted in two different ways:

- The use of "<u>hı</u>?" might be an exclamation indicating Ayse's astonishment, in the sense of "what?!". She might mean "what?! Don't you really know?!". To explain her astonishment, “standard” social constructivist approaches might be referred to, in which the adult is a standard role model for the children as an expert. As a “standard opinion” she might suppose that her father “has to know” “everything” better than a child. Therefore, she might be astonished or confused at how an adult person – her father – does not know what she should do or how the corpus should be properly built. Maybe therefore she lets the card lie instead of taking it from the table.

- By giving the same reaction as in the previous lines <64-66>, <68-70>, Ayse might maintain her questioning. Thus her utterance might mean that either she asks her father obscurely for help to build a corpus correctly or tries to get a clue from her father if she is building the corpus right. Furthermore her movements signalize that she starts to think about the built corpus and tries to check the card for guidance on whether she builds it right. In this context, the father’s reaction<71-72> seems to work on Ayse, causing her to muse on the rightness of the built corpus. Her reaction reinforces the idea in lines <23-24, 32-33, 47-48, 64-66, 68-70> that she poses such a question – like a communicative solicitation – in order to obtain approval for her action and mobilize her social environment by means of *environmental self-regulation.*

The father says, “One is still missing” while holding his right hand to his mouth and touching his lips with his forefinger <74-75>. “One is still missing” might mean that “one block” is missing or “the jut of one block” is missing in the built corpus. Regarding Ayse’s question in lines <32-33> “but..one more.. a long one. heh?”, the father’s word “one” might characterize “one more…a long one” in the terms used by Ayse, which is interpreted as a long block. In this context he might mean that “one long block” is missing. Bearing in mind two possibilities in line <68-70> (p.33), he might try to emphasize that when she changes the position of block K4, she will already have one long block and then she will need “one more long” – the fifth block (K5) – to build the corpus properly (see Fig.1.16 and Fig.1.17). Another interpretation is that, if she changes the position of K4 (see Fig.1.16 and Fig.1.17), then she might have one jut in the built corpus and will need one more jut to get the proper corpus. Thus the father comes up with the geometrical argument that emphasizes the existence of the missing block or jut in the built corpus. His reaction may be interpreted in a way that he *frames* the situation as a numeric one. Through his utterance, “One is still missing”, he seems to bring a numerical approach and hereby contends the amount of the “missing” blocks or juts, which is geometrical absence in the built corpus. However, he neither counts the “missing” blocks or juts nor gives a definitive target of the location where the “one” should be set. By emphasizing the missing block, he seems strongly to suggest a geometrical approach. From a developmental perspective his reaction might activate both Ayse’s numerical and geometrical skills so that she can use two approaches at the same time for the one situation.

Regarding the two previous interpretations of line <73>, the father’s reaction might also show that Ayse’s reaction works on him. Whatever she means at line <73>, he gives the impression of tagging either information or a clue about the built corpus. Thereby he might try to show that he is an expert and knows the way to build a proper corpus better than Ayse. Therefore, he might try to signalize his knowledge of building a corpus properly. From a participatory point of view, his reaction might be interpreted in two different ways:

- Regarding the element of *competition* in the play situation, he might try to win the game by pretending to give a clue to Ayse. Thereby he ascribes the role of *competitor* for both.

- In terms of *scaffolding* he might try to help Ayse enact a sequence of actions, which he knows to be expedient as a *tutor.* He simplifies the building activity so that his feedback is regulated to a level that Ayse can use to achieve the correctly built corpus. Thus he seems to realize one of the scaffolding functions, namely *reduction in degrees of freedom*. Moreover he engages in the negotiation process critically but constructively. He offers Ayse a hypothesis, which can be accounted for Ayse. In this regard it seems that the negotiation process is accomplished in an *exploratory way* so that she can turn up by finding the “missing” block or jut. Thereby he might also try to direct Ayse’s attention to a specific part of the chosen card or the built corpus and

> enhance Ayse’s *self-regulation* by *using referential verbal acts*. Moreover his reaction can be deemed a supportive activity, namely *instruction*, through which the father suggests the use of a specific strategy. Thereby Ayse can organize her further building actions pursuant to the clue or information given by her father. Thus he might provide Ayse with a “vicarious” form so that she can master her own action through her own *consciousness* and *control*. In that regard he seems to offer Ayse a participation profile such that she can muse on her further steps in the building activity and perform her ideas by taking the role of *author*. Furthermore the interaction process renders an expanded leeway of participation to Ayse. In that regard the father might ascribe the role of *tutee* to Ayse, whereas he is taking the role of tutor, who is an *expert* in introducing her to spatial tools in the building activity.

Ayse reacts with “hııh” while pulling block K4 a bit to the left <76>, while her father is saying “it’s there” by showing the jut of the block Ky on the chosen card <77>. Ayse’s utterance is a typical Turkish expression, which might be an exclamatory remark at picking up on her father’s point. Therefore her reaction can be interpreted as a *notice,* that there is “one thing missing” which could be a block or a jut. From a developmental perspective, her reaction shows strong spatial abilities in that she can get the point of her father instantly and perform it by pulling the block K4 a bit to the left <76>. From a participatory point of view she comes up with her own idea in that she pulls the fourth block a bit to the left. Thus, Ayse seems to have found out which block is missing and where it should be set. In that respect she gives the impression of participating as an *author*. Moreover, the father’s scaffolding, his *referential verbal acts* and her own endeavour for *environmental self-regulation* seem to work on her so that she comes to realize the “missing thing”, either block or jut, and tries to fix it. Thereby she demonstrates *self-regulation* in that she identifies the error in the built corpus *autonomously* and adjusts the building activity. In that regard Ayse gives the impression of activating her *visual discrimination*. She seems to participate as a relatively *autonomous* person with respect to spatial visualization and spatial orientation. From a developmental perspective, she seems to be a *shape*

composer, who can *build complex structures of units* (see Table 2.4.). Ayse's leeway seems quite open in that she participates actively in the situation. Moreover, she becomes an active participant in the building activity so that she can achieve a learning process. By means of *learning-as-participation*, her participation provides an answer to the question of how the built corpus can match the figure on the card (see 2.2.1.). Concordantly she seems to assign to her father the role of *activator,* who activates Ayse's *knowledge of the result* in the play situation so that she seems to use her father's instruction or clue.
Thereon the father states, "it's there", showing the jut of the block Ky on the chosen card <77>. His statement appears to be a justification for Ayse's action. Considering his statement at line <74-75>, he might mean that the "missing jut" or "missing block" is there. Maybe the object "it" symbolizes "block K4" in his phrase and represents the meaning "the fourth block is there". He seems to emphasize the correct position of K4, where it has to be, by pointing it out on the card. Another possibility is that he points out the place of jut on the card, to where Ayse should set the block K4 in order to achieve the presence of the one jut (see Fig. 4.18.). Thereby his reaction can be interpreted as a supportive activity, namely *instruction* that he suggests the use of any specific strategy. Moreover, he seems to render a scaffolding process for Ayse by pointing out the relevant characteristics of the block on the card. In this sense he seems to realize one of the scaffolding functions, namely *marking critical features*. He seems to try drawing Ayse's attention to features of the position of the block, which is important or relevant for its completion. From a participatory point of view, he gives the impression of acting as a *tutor,* while reserving the role of *tutee* for Ayse.

Thereon he adds, "the long one. Exactly. Right." <78>, when Ayse takes the fifth block (K5) in her left hand and sets it on its Z side between K4 and K2 <79-80>. Immediately afterwards, K1 and K2 fall down while Ayse utters "<u>hım</u> <u>hııhıı</u>" <80-81> (see Fig. 4.16. and Fig. 4.18.).

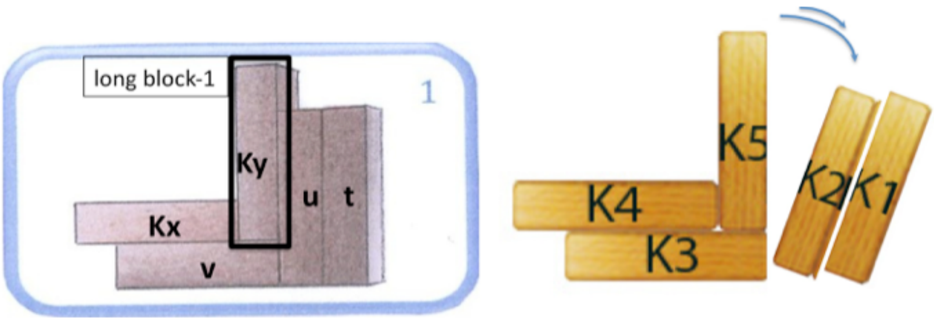


Fig. 4.18. The possible “long” block in compliance with father’s remark and the built corpus with two blocks falling down from the frontal elevation

The utterance “the long one” <78> of the father appears to be an iteration of the conflict in his arguments as in lines <05-06,08-09,13-15,34-35>. In his turn he rationalizes that short blocks are needed to build corpus properly but they don’t exist in the pile of blocks and thus the image on chosen card is wrongly drawn. However, during negotiation process in Ayse’s turn he determines two blocks (Kx and Ky) on the chosen card as long blocks and thereby does not draw on his previous arguments <05, 08, 14-15> that the card is incorrect. Moreover, he claims that the chosen card in Ayse’s turn gives the impression that long blocks are needed but actually they are not needed. He might try to emphasize that the block Ayse set is the “long one”, on which Ayse commented, “but..one more.. a long one. heh?” at lines <32-33>. Thus, he might try to clarify his argument at lines <34-35>, why the blocks on the card give the impression that long blocks are needed. By pushing block K4 to the left side, Ayse creates a jut in the built corpus and it gives the impression of being a long block. Maybe therefore the father says “the long one” <78> right after Ayse’s action.

In terms of scaffolding, he seems to try drawing Ayse’s attention to the feature of the blocks that is important or relevant for the completion of building the corpus. In this sense his reaction seems to be *marking critical features*. From a participatory point of view, the father seems to take the role of *relayer*, while ascribing to Ayse the role of *deviser.* Namely, he uses the issue of having/using a long block, which Ayse has stated in lines <32-33>. He gives the impression of taking his daughter’s idea and formalizing it in the same way as her. The father goes on “exactly. right.” <78>. His ongoing reaction reinforces the idea that he tries to clarify his argument at lines <34-35>, why the blocks on the card give the impression long blocks are needed, and Ayse does “exactly” what he means. Namely she constitutes a jut by setting block K4 to the left and thus the block gives the impression of being long. In this sense she set up the corpus “exactly. right.” <78>. From a supportive perspective, his reaction seems to be a supportive activity, namely *affirmation*, in assenting to Ayse’s reaction. Moreover, he seems to deploy Ayse’s enthusiasm in order to keep her motivated and *in the field* by the terms of scaffolding. Thus, he might also try to *control* Ayse’s *frustration*. From the participatory point of view, he is taking the role of *tutor*, while ascribing the role of *tutee* to Ayse.

Simultaneously Ayse takes the fifth block (K5) in her left hand and sets it on its Z side between K4 and K2 <79-80>. From a developmental perspective, Ayse appears to have strong spatial skills in that she is able to cope with the existence of the missing block. She seems to diagnose what is lacking and where it is lacking in the built corpus. This is most probably why she takes the fifth block and sets it between K4 and K2. Thereby the corpus becomes identical to the figure on the card. Moreover, her reaction seems to lend credence to the first possibility of Ayse’s further actions (see Fig. 4.18). From a participatory point of view, Ayse seems to take the idea of her father, that there is a missing block on the built corpus, and she might combine it with her own idea, that she works out which block is missing and sets it in a right place. Thereby she seems to take the role of *ghostee,* ascribing the role of *ghoster* to her father. The father’s scaffolding and her *environmental self-regulation* seem to work on Ayse in that she comes to realize and perform where the fifth block should be set. Thereby she signalizes *self-regulation* in that she identifies the deficiency in the built corpus and adjusts the building activity.

Immediately afterwards K1 and K2 fall down while Ayse is uttering “hım hııhıı” <80-81> (see Fig. 4.18.). Ayse’s first utterance can be interpreted as an expression of her *consideration* process that she might muse on further building activities. Ayse’s second utterance “hııhıı” is a typical Turkish expression of approval. Through her reaction she might show that she either approves her own action or settles for two blocks falling. Ayse might try to think about the falling of the blocks (K1, K2), thereon she might manage it her mind and thus might accept the “falling” of the blocks (see Fig.4.18).

The father says quietly “yes. they can be set upright again.” <82>. His reaction gives the impression of remarking that the collapsed blocks can be fixed and the corpus can be rearranged, in particular that the fallen blocks (”they”) can be brought to the upright position again. From the developmental perspective, the father seems to prevent any *framing* of the spatial ideas and to steer his remark with a geometrical approach. Bearing in mind the family system theories that fathers tend to make more requests for information and use a greater proportion of verbalizations describing form, shape and direction relations in the course of interacting with their children in structured tasks, his reaction gives an exact and elaborative description about setting blocks.

From a supportive perspective, the father’s reaction can be seen as a supportive activitiy to motivate Ayse to fix the collapsed blocks and thereby accomplish her turn. In this sense, he seems to *motivate* Ayse in a positive way by encouraging her that the falling blocks can be set up again. Thus, his reaction can be identified as a supportive activity, namely *motivation*, which might operate in a positive way for Ayse. Moreover, he lets Ayse continue her building activity so that she can strive to fix the problem and achieve her turn. In terms of scaffolding, his reaction can be interpreted either as *direction maintenance* or *marking critical features*. By means of *direction maintenance*, he emphasizes the next building action as a goal to keep Ayse *in the field*, whereas by means of *marking critical features* he accentuates features of the building activity in that the fallen blocks can be set up again. In both regards, from a participatory point of view, the father seems to take the role of *tutor*, while ascribing the role of *tutee* to Ayse.

Ayse utters “hıı hıı” while first setting K2 in the upright position so that its X side touches the X side of K5, then while setting K1 in the upright position she utters “hıı hıı” again <83-85>. Her reaction might mean that she gets her father’s point and expresses her agreement by saying "hıı hıı", which is a typical Turkish expression of aggrement, while she is returning both blocks to their previous position step by step. She builds the figure as seen on the card correctly and conducts her turn (see Fig. 4.19.). From a supportive perspective, father's *motivation* and *instruction* seem to work on Ayse, so that she can fix the problem and set both blocks in an upright position again. In other words her father's *scaffolding* and her *environmental self-regulation* seem to work on Ayse that she can set up the collapsed blocks again. Thereby she signalizes *self-regulation* as she identifies the imperfection in the built corpus and adjusts the building activity. From a developmental perspective, Ayse apparently takes note of her father's *elaborative descriptions, demonstrations* and *motivations* and therefore she tries to knuckle down to build the corpus properly to match the figure on the chosen card. Thereby Ayse seems active and strives to build the right corpus. From a participatory point of view, her reaction indicates that she implements her father's *scaffolding* and turns up by relaying her father's idea and seems to reach an eventual agreement. In this sense she gives the impression of taking the role of *relayer* while ascribing the role of *deviser* to her father. It seems Ayse and her father accomplish an *exploratory* negotiation process whereby they are collaborating, reaching agreement with each other and understanding each other's points of view.

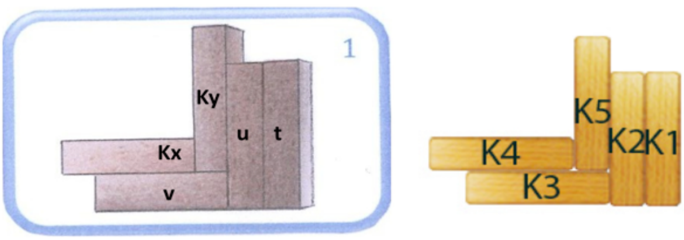


Fig. 4.19 The chosen card and the built corpus from frontal elevation

Father says, "very good", and then pinches Ayse's cheek while he is saying "cool." <86>. Most probably he admires Ayse's success. By pinching her cheek and saying "very good" and "cool" he might try to acknowledge her achievement, and consequently his satisfaction, and thus to motivate Ayse for further turns in the play situation. From the supportive perspective, his reaction can be interpreted as *motivation* and *conclusion*. Thus, Ayse's turn is completed.

Summing up from the perspective of interactional niche in the first observation period

In the chosen scene, one type of tutoring emerges. From an allocative perspective, the father is the official game partner of Ayse and her competitor in the play situation. But he – situationally – sets about the *scaffolding* process and comes up with a geometrical approach to argue about the built corpus. Thereby he gets a grip on the play situation and in a relatively resolute manner adopts scaffolding functions in the negotiation process with Ayse. It seems that the negotiation process between Ayse and her father is accomplished in an *exploratory way* in that they are collaborating, reaching agreement with each other, and understanding each other's points of view. Moreover they negotiate critically but collectively and constructively. Thereby the negotiation processes renders Ayse an expanded leeway of participation. Owing to her father's scaffolding, a leeway of participation emerges such that Ayse can act as a self-regulatory participant, whereas her father takes the role of tutor during the whole interaction process in the game. By referentially requesting help or a demonstration, seeking approval of her actions and initiating referential joint attention of her father, Ayse adapts the local conditions of her learning environment and mobilizes her social environment through such communicative solicitations. She comes to realize that she can build a corpus identical to the figure on the chosen card without needing different sized (short or long) blocks. Through her father's scaffolding, his *referential verbal and nonverbal acts* and her *self-regulation*, she explores and performs whole spatial consequences in the block building activity. In this sense, the learning process for Ayse emerges through her participation, in which she achieves the ability to build the corpus correctly as an autonomous person. Therefore, her father takes the role of *activator* who implicitly activates Ayse's self-regulation through his tutoring. In this way Ayse succeeds in her turn. Within this context there emerges a developmental niche for Ayse.

Table 4.1 The roles taken in the chosen scene of the first observation

	Father's turn & Ayse's turn
Ayse	Tutee as environmental self-regulatory participant
Father	Tutor as an activator

According to the whole analysis, the three components of an interactional developmental niche in Ayse's familial context can be structured as follows:

Component "content":

Allocation x Content: In the chosen scenes Ayse and her father are confronted with a mathematical and spatial play situation. The chosen game is structured in the mathematical domain of geometry and is based upon the game "Make 'n' Break" (Lawson & Lawson, 2008). The aim of the game is to build the 2D representations on the cards as 3D shapes properly with the wooden blocks provided, which are all of the same size and weight.

Situation x Content: The chosen play situation enables Ayse and her father to negotiate interactively about the correctness of the building corpuses. There emerges a dyadic interaction process, in which mostly geometrical and occasionally numerical features of chosen cards are thematized. During block building activity, Ayse and her father mostly put forward their justifications, alternative hypotheses and agreements. Moreover, they share all relevant information, strive to reach an agreement and dedicate themselves to pursuit of the best solution. Thus, they engage in the interaction process critically but constructively and collectively. In this respect the negotiation process between father and daughter emerges as an *exploratory* one. Reviewing the built corpus in the father's turn, the father argues that the built corpus is right and the figure on the chosen card is wrong. In the course of the negotiation process in the father's turn, a *working consensus* occurs between Ayse and her father. Reviewing the built corpus in Ayse's turn, the father engages in the negotiation with Ayse through *scaffolding* functions. In this regard the interaction process enables Ayse to use self-regulation so that she can build the corpus to match the figure on the card without needing any long or short blocks.

Contribution x Content: Ayse performs building a corpus identical to the figure on the card. Through her father's *scaffolding* she identifies that the blocks are all the same length and examines directly the feasibility of building the corpus without needing any long or short blocks. Considering the figure on the chosen card without juts, she experiences that the *gestalt* of the built corpus assimilates with the *gestalt* of the figure on the chosen card. Moreover, in the course of building activity, she comes to realize the last missing block in the built corpus and sets it right so that she accomplishes her turn. Thus she performs the spatial relations between 2D and 3D objects and relates some parts to the whole. She also identifies the faces of 3D blocks with 2D blocks so that she can correspondingly restore the collapsed blocks to the built corpus. Thereby she also earns praise from her father for showing that she can compose and decompose 2D and 3D structures and build 3D structures from 2D images.

<u>Component "cooperation":</u>

Allocation x Cooperation: In the play situation Ayse and her father are game partners. Ayse's father is the adult person and he mostly directs the play situation. Moreover, he is Ayse's "official" conversation partner, who allocates the right to take the next turn.

Situation x Cooperation: In this dyadic interaction process Ayse's father is her game partner. Thereby he takes the role of *tutor* and ascribes the role of *tutee* to Ayse. Furthermore, his tutoring and scaffolding implicitly activate Ayse's *self-regulation* so that she becomes able to mobilize her social environment through communicative solicitations. Therefore, Ayse latently ascribes the role of *activator* to her father. By dint of her father's scaffolding, Ayse has an expanded leeway of participation so that she can undertake almost all the *participation* profiles: *author, ghostee, spokesman, sponsor, relayer, deviser* (see section 2.2.1.). Mostly the father comes up with geometrical arguments whereas he rarely takes a numerical approach. Thereby he prevents any *framing* of the spatial ideas and steers the negotiation process with Ayse into the geometrical approach. Due to her father's *tutoring* Ayse actively explores, experiences and performs spatial features in the building activity and thus accomplishes her turn. She takes all four roles of *production design* in the course of negotiating with her father in both *disputational* and *exploratory ways*.

Contribution x Cooperation: Ayse argues collaboratively with her father in the course of block building activities in both p-turns. In the negotiation process, how to build the right corpus and the necessity of long and short blocks are at issue. Ayse apparently cares about her father's elaborative descriptions, demonstrations and motivations and this constitutes her *environmental self-regulation*. Thereby she becomes able to mobilize her social environment through communicative solicitations. She asks for help or a clue, seeks approval or disapproval for her reactions, and initiates referential joint attention on the block building activity with her father. In this regard, she latently ascribes the role of *activator* to her father, while she procures for herself self-regulatory participation in the whole interaction process so that she performs her own ideas originally and builds a corpus identical to the figure on the chosen card without needing any short or long blocks.

<u>Component "Pedagogy and Education":</u>

Allocation x Pedagogy and Education: Block building provides a view of children's initial abilities to compose 3D objects. In the chosen game, four goals are pursued: spatial structuring, operating shapes and figures, static balance between blocks, and matching the faces of 3D shapes with 2D shapes. According to the US National Research Council, five-year-old children can understand and replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National Research Council, 2009, p.191). The chosen scene includes references to exploring and examining spatial structuring, visualizing and kinaesthetic imagery.

Situation x Pedagogy and Education: In the chosen play situation the father strikes a balance between scaffolding and playing. While he is playing with Ayse, he challenges her at the same time. He doesn't respond contingently to Ayse's ongoing building activity but expands on the building activity and direct it in more challenging directions. In that regard he simultaneously realizes a scaffolding process in which he maintains both verbal and nonverbal behaviours, which offer Ayse choices (questions, suggestions or comments) directly relevant to her current or immediately prior building activities.

Considering the six functions of the scaffolding process, the father's reactions can be categorized as follows:

- **Recruitment:** The father involves Ayse's interest in the play situation and adherence to the requirements of the play situation. By saying, "no no it looks like that" he weans Ayse away from abandonment of her turn and ensures that she does not give up on building the corpus.

- **Direction maintenance:** The father ensures that Ayse's block building activity is directed towards achieving particular outcomes that contribute to completion of the built corpus. He ensures that Ayse's building activity is directed towards achieving particular outcomes that contribute to completion of the corpus. By taking Ayse's previous cards away from her hand and saying "let them away" <34-35>, he tries to keep Ayse *in pursuit of a particular objective* so that she can *directly* maintain the building activity and become involved only in her current turn in the play situation. Similarly, by saying, "yes. they can be set upright again." <82> he maintains Ayse's building activity so that she can strive to fix the matter and complete her turn. In this sense he emphasizes the next building action as a goal to keep Ayse *in the field.*

- **Frustration control:** The father manages and regulates Ayse's negative emotional reactions to difficulties in the building activity in order to maintain her commitment to finishing and achieving her turn. Thereby he responds to her *emotional state.* By telling Ayse, "not so difficult." <27-28>; "but it is easy. just get started. … just get started." <40-42>; "exactly." <46> ; "exactly. right." <78>, he deploys Ayse's enthusiasm in order to keep her motivated *in the field* and allow her *to concentrate on* and *master* the block building activity.

Therefore, he might try to motivate Ayse in a positive way to make her believe that the figure on the card is easy and she can achieve her turn.

- **Demonstration:** The father models the “idealized” position of the blocks to the requirements of the building activity so that they may be imitated by Ayse in the course of her turn. By changing the position of the block he uses *maternal planning* and models the correct position of the block and how it should be set <49-50>. thus he performs a new *idealized subject* in order to let Ayse *imitate* it in a more appropriate form.

- **Reduction in degrees of freedom:** The father simplifies the further steps of block building activity by reducing the number of constituent acts required to reach the correct built corpus. By stating, “One is still missing” <74-75>, he simplifies the building activity so that his feedback is regulated to a level that Ayse can use for correction to achieve the right built corpus. Thus he tries to help Ayse enact a sequence of actions, which he knows to be expedient as a *tutor*.

- **Marking critical features:** The father draws Ayse’s attention to features or aspects of the building activity that are important or relevant for its completion. By telling Ayse, “the long one” <78> and “it was already right. the next block.” <49-50>; and “yes. they can be set upright again.” <82>; and by stating “it’s there” while showing the jut of the block on the chosen card <77>, the father accentuates some aspects of the building activity in terms of confirming and checking.

    - In this sense he seems to realize all six scaffolding functions. In the course of the *scaffolding* process he applies the principle of contingent shifting, the minimal-sufficiency principle, and emotional and motivational regulation. In terms of contingent shifting, he calibrates the building activity and varies the quality and amount of assistance according to the changing level of the independent functioning of Ayse. By means of the minimal-sufficiency principle his assistance is minimized and indirect cueing is used in order to let Ayse be *pulled* by the moderate discrepancy or constructive friction towards new levels of independent activity. In terms of emotional and motivational regulation he perceives and responds in an auxiliary manner to Ayse's motivational and emotional signs. Therefore the father maintains a scaffolding process for Ayse. His adjustments in the course of the play situation are based on his interpretations of nonverbal and verbal behaviour from expressing changes with Ayse. Thus they lead to a gradual increase of Ayse's active participation and enhance her *self-regulation*. Therefore he seems to take the role of *activator* as he implicitly activates Ayse's self-regulation by realizing a *scaffolding* process. Moreover by using *referential verbal and nonverbal acts* the father enables Ayse to *internalize* her spatial knowledge, to convert it into a tool for *conscious control* and then to master her own actions through her own *consciousness* and *control*. This emotional and cognitive context of the scaffolding process enables Ayse to negotiate with her father in an *exploratory* way so that they reach an agreement in the building activity. The negotiation process between father and daughter thus corresponds to the transfer of responsibility for managing the block building activity and self-regulated learning for Ayse.

Contribution x Pedagogy and Education: In the chosen play situation Ayse seems to accept her father as a *tutor,* who is an *expert* in introducing spatial tools in the building activity, while she takes the role of *tutee*. She seems to have strong spatial abilities in that she can "see", "interrogate", "realize", "examine" and "perform" the block building activities during the interaction process. Through a *collective argumentation* process with her father, she fixes the missing block in the built corpus. The interaction process emerges in an *exploratory* way as they cooperate, collaborate, understand each other's points of view and reach an agreement. Ayse realizes an *environmental self-regulation* process, in which she chooses a strategy (building the corpus without using any long block) and adapts its features to the model (she builds a corpus identical to the figure on the card). Regarding geometrical developmental theories, Ayse acts as a picture maker, congurance determiner and shape composer of units of units (see Table 2.4). She proves that she can represent blocks at the detailed level of shapes and coordinate the structure topologically. Thus she adapts the *local conditions* of her learning environment and mobilizes her social environment through such communicative solicitations. Through her increasing autonomy Ayse performs her own idea and she gets learning opportunities for building the right corpus without needing any long or short block.

Through the father's *scaffolding* and Ayse's *environmental self-regulation* there emerges an exploratory negotiation process. In this way Ayse achieves her turn and there occurs an interactional niche in the development of her geometrical thinking. Due to these three components, the interactional developmental niche in Kil's familial context is structured as follows (see Table 4.2).

Table 4.2 NMT-Family Kil in the game "Building 01"

NMT-Family Kil **Building 01**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.	Playing with father	Development of spatial skills and transformational abilities in spatial thinking and learning
aspect of situation	Negotiation between father and Ayse Working consensus	Expanded leeway of participation Tutor and tutee	Scaffolding process by father Enabling the success of spatial abilities Activating Ayse's self-regulation
aspect of Contribution	Exploring feasibility of the building corpus without needing any long or short blocks	Environmental self-regulation of participant Father as activator	Environmental self-regulation by Ayse. Building an identical corpus to the figure on the chosen card without needing any short or long block Having quite strong spatial abilities

Bearing this NMT table in mind, I will now present another scene of Ayse from a subsequent observation session. In the second observation, the meeting takes place at Ayse's home and both her parents are present. Ayse plays one game with her father, then one with her mother. Afterwards they play one game together as a child-mother-father tetrad, which facilitates the emergence of a polyadic interaction process.

In the present analysis, a scene from the game "Building 02" is chosen, which is materialized in the second observation phase. Ayse's game partner is her mother.

4.1.2. The game "Building 02" from the second observation period

The chosen and transcribed scene is the third round of the play situation. It begins with Ayse's p-turn and ends with the mother's p-turn. In total they play seven rounds by turns in the recording position as shown in Fig. 4.20.

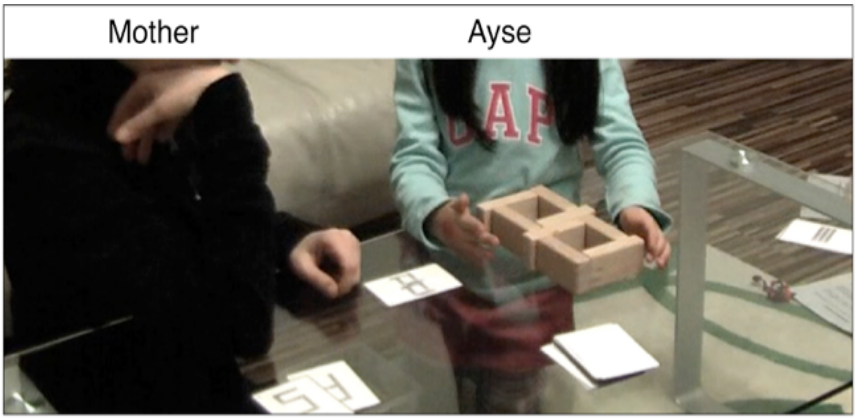


Fig. 4.20 Recording position of Ayse in the second observation period

At the beginning of the chosen scene, Ayse picks up a card from the pile (see Fig. 4. 21.) and looks at it while she is holding it with her right hand.

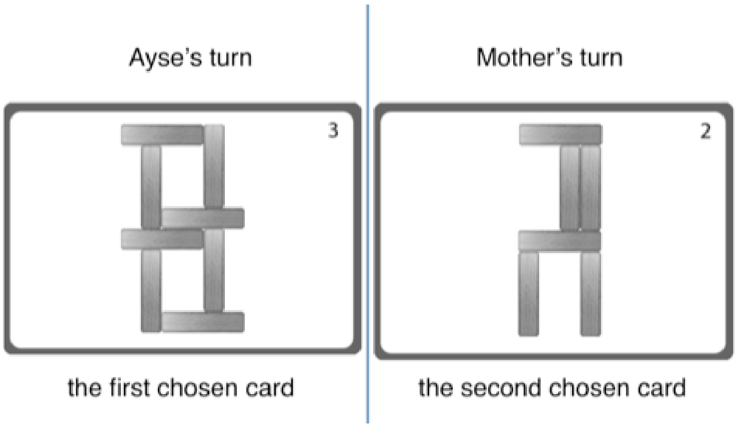
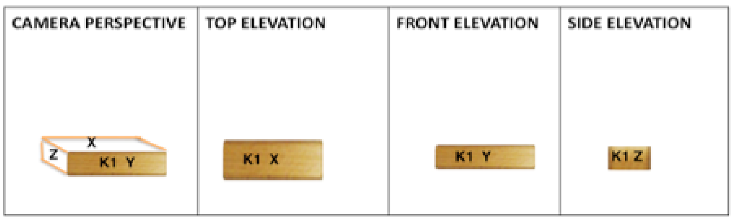


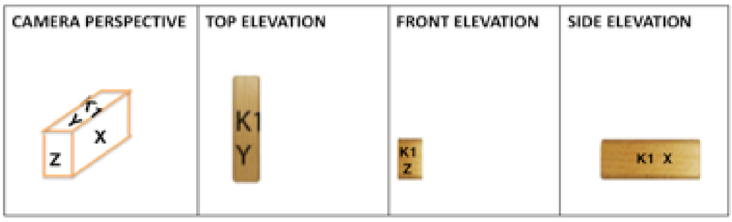
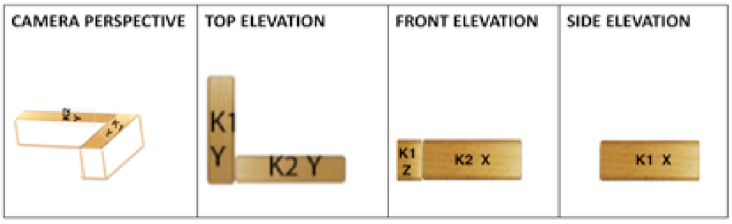
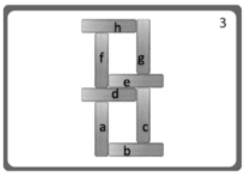
Fig. 4.21 The chosen cards in the third round

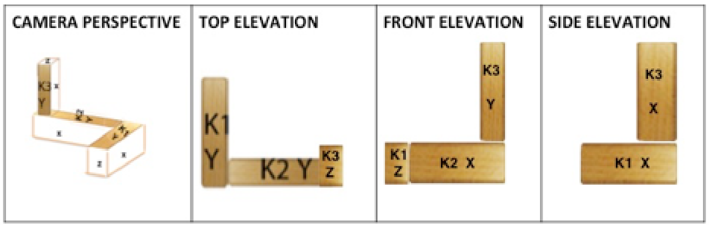
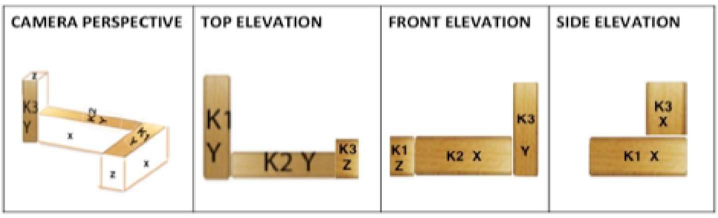
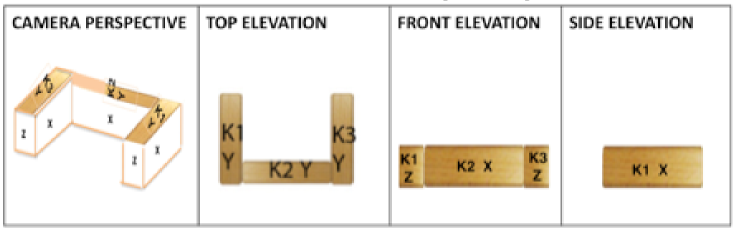
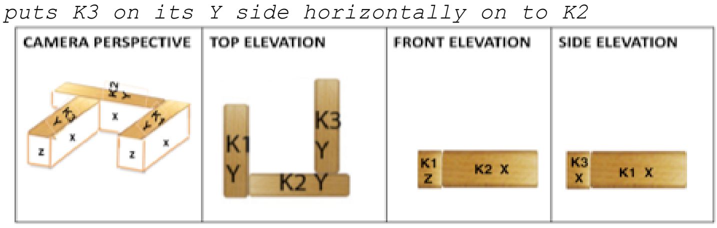
Her mother breaths deeply and says "oh yeah..". Then Ayse puts her left hand in front of her mouth and shakes her head side to side as if she wants to express that she is scared. Her mother takes the card from Ayse's hand, lays it in front of her, looks at it and says that they carry of it (see Fig. 4.22). Thereafter the chosen transcript begins. For clarity, the chosen cards of Ayse's turn and the mother's turn are named sequentially as the first chosen card and the second chosen card (see Fig. 4.22).

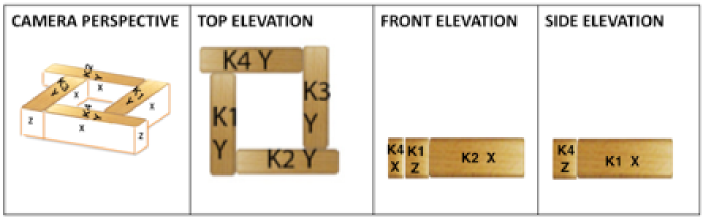
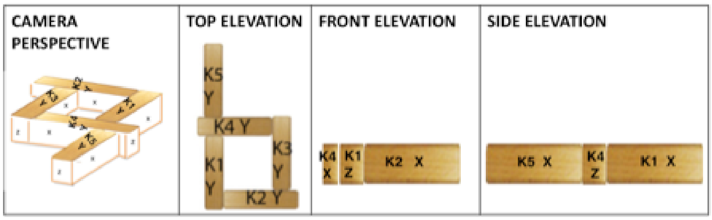
During the interaction process they use the German language and occasionally Turkish language by switching (see section 1.4.1.1.). In the transcript German speech is written in normal font and Turkish speech is underlined.

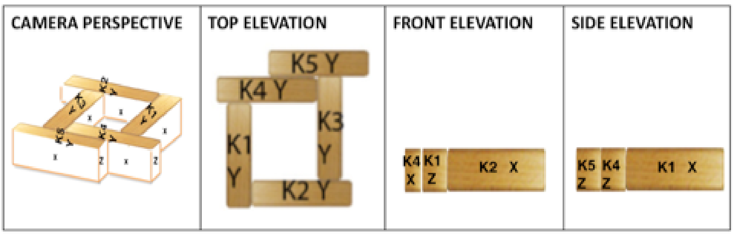
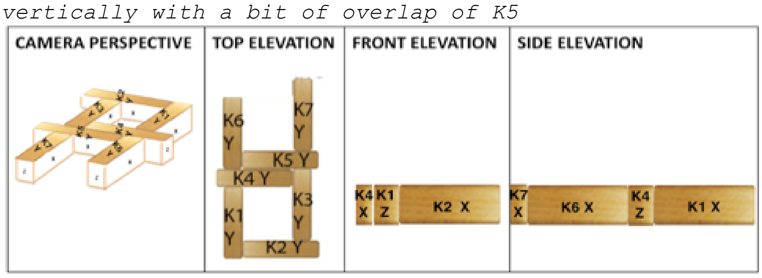
Transcript

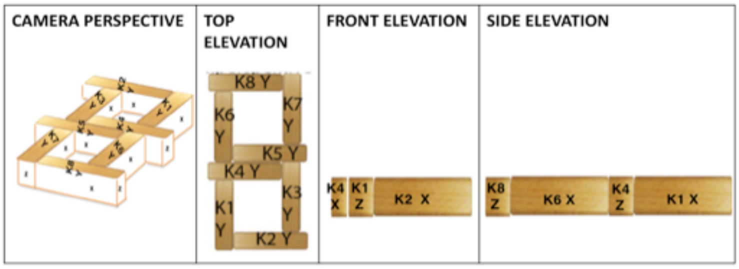
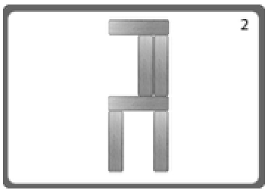
1	06.17		Ayse	O.K. *holds K1* then we always make alternately bricks.
2				*puts K1 on the table in front of her on X side* 
3			Mother	just look. not like that. *takes K1, turns it 90*

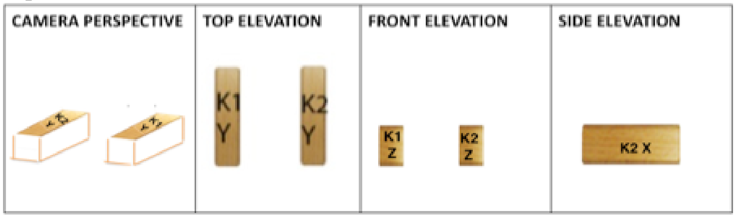
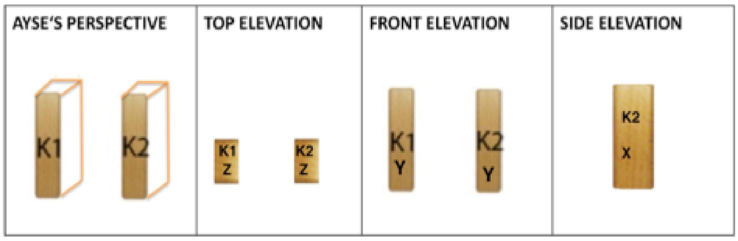
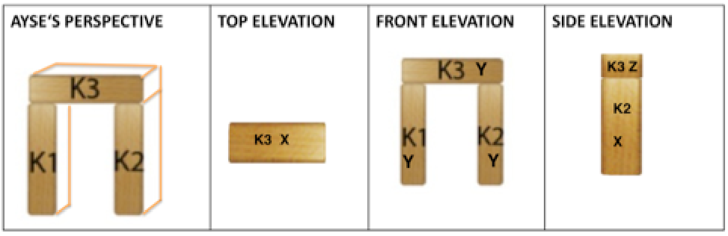
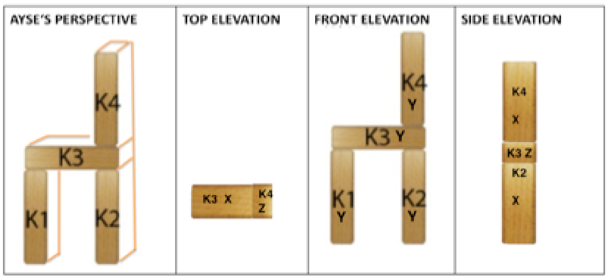
4				*degree and sets it on Y Side.* 
5				but like that. it is from the top. *rubs her*
6				*right index finger on the Y side on the block* O.K?
7			Ayse	Hııhıı
8			Mother	from the side- the side can be seen *crosses off her*
9				*right index finger on the Y side on the top*
10				*of the block*
11	06:30		Ayse	and now it is you. *puts her left hand on the Y*
12				*side of the block* you must here. *holds her left hand*
13		>		*next to K1,right hand a bit further to the left side*
14				*of the block*
15		>	Mother	yes. do we want to do so that we alternate?
16			Ayse	yes.
17			Mother	*puts K2 on its Y side with the right angle next to K1* 
18				so..and now we have this block. *shows block a on the*
19				*card.* and this one. *shows block b on the card* 
20				now you make this one. *shows block c on the card*
21			Ayse	*takes K3, puts it directly in the upright position*

22				*on K2.* 
23			Mother	no! First you should think about it! *shows block c*
24				*on the card* just look, how the block stands
25				that the side can be seen.
26			Ayse	*puts K3 on the upright position next to K2* 
27			Mother	lay down.
28			Ayse	*puts K3 on its Y side with the right angle next to K2* 
29			Mother	exactly. and then? I put this here. *shows the block b*
30				*on the card* now you make this block *shows block c*
31				*on the card then looks at Ayse*
32	07:00		Ayse	*looks at her mother* I have already.
33			Mother	noo.just look, where your block is. *Shows block b*
34				yours begins here. *shows edge of block b on the card*
35			Ayse	*puts K3 on its Y side horizontally on to K2* 
36			Mother	*still shows the edge of the block b on the card*
37				exactly O.K.! yes.right. *without changing the*

38				*position of K3, moves it a bit towards K1*
39			Ayse	*(not understandable)*
40			Mother	*looks at the figure* let me look. yes right. *takes K4*
41				so now I put one- now I make. now I put this block,
42				O.K.? *shows block d on the card*
43			Ayse	Hıııııııım *leans over the table*
44			Mother	but you have to move away from there. *puts K4*
45				*on its Y side on K1 intersecting with K3* 
46				then the camera cannot detect us.
47			Ayse	ehem *leans back*
48	48		Mother	so O.K.? did you understand it? so now it is your turn
49				which one would you like to put? this one or this
50				one? *shows blocks e and f on the card.*
51			Ayse	*shows block f* these-
52			Mother	one? *shows blocks e and f on the card.*
53	07:30		Ayse	*puts K5 in the same way as K1 on the K4 vertically* 
54			Mother	*points at column of block a and f on the card with*
55		<		*her right index finger* try to put it exactly at the
56				same straight like these.
57		<	Ayse	noo these. *points at the block*
58				*e on the card with her right index finger*
59			Mother	noo. now you have these. *shows block f on the card*
60				now you make-

61		>	Ayse	no!
62		>	Mother	Allright. Then do it.
63			Ayse	*puts K5 on its Y side horizontally overlapping on K4* 
64		<	Mother	hıhıı then I make this one. ok? *shows block f*
65				*on the card and puts K6 on its Y side vertically to*
66				*K4 at the same straight with K1, then moves K5 a bit*
67				*towards K6*
68		<	Ayse	and then I make this one. *shows block g on the card*
69				may I make then both of them? noo! you make the block
70				on the top. *takes K7 and puts it across K6 in the*
71		>		*same way onto K5 vertically.*
72		>	Mother	you can make all of them, when you would like to.
73		#	Ayse	no! the all-
74		#	Mother	you have understood it already.
75		<	Ayse	the all is mine. *takes K7 away and holds*
76	8:00	<	Mother	now just think about it- just think- just look-
77				it is same.. *shows on the card* it is at the same
78		>		length like here. *shows on the card*
79		>	Ayse	ahso *puts K7 across K6 in the same way onto K5*
80				*vertically with a bit of overlap of K5* 
81		<	Mother	you should pay attention. O.K.?
82				now just one more block.
83		<	Ayse	*sets K8 horizontally bonded to K7,K6 on its Y side*

84				*parallel to K2,K4,K5* the figure is a bit crooked
85				it has to -
86			Mother	hııhıı
87		>	Ayse	thereon. *put the figure to the right position, which*
88				*slopes*
89		>	Mother	*helps Ayse to regulate the set blocks* 
90			Ayse	*holds on the corpus with both her hands* it is done.
91			Mother	yes, it is done.
92			Ayse	but, cool. *pushes all blocks on the table with her*
93				*arms to the centre of the table*
94			Mother	yes *lays the chosen card on the pile of Ayse's cards*
95	08.30		Ayse	I can't. *takes pile of her previous cards* and who
96				has the most then that can.
97			Mother	we play 5 rounds. *chooses a card from the pile of*
98				*remaining cards*
99			Ayse	and when I have the most?
100			Mother	*sets the chosen card in front of her* then what do I
101				have to do now? *looks at the chosen card* 
102			Ayse	should I help you? *looks at her mother* please it
103				works easily. *takes two blocks from the box and*
104				*sets them first vertically on their Z sides but then*

105				*lays them on their Y sides on the table* 
106			Mother	you can put in the upright position.
107			Ayse	no then falls down
108			Mother	no we put it up *puts both blocks up so that Ayse can*
109				*see their Y sides in the upright position in front*
110				*of her* 
111	09:00		Ayse	but it falls down. *puts a new block, K3, horizontally*
112				*on K1 and K2 on its Y side* 
113			Mother	*wraps both her hands around the corpus*
114			Ayse	I do it for you-ok? *puts a block, K4,on its Z side*
115				*vertically to K3 at the same straight with K2* 
116			Mother	hııhıı
117			Ayse	give me more
118			Mother	*gives Ayse one Block(K5)from the pile*
119			Ayse	*sets K5 vertically on its Z side next to K4*

120				you always give me blocks then.
121			Mother	*brings K4 and K5 closer* hııhıı like that I think- or?
122			Ayse	*puts K6 on its x side on K4 and K5*
123			Mother	great *laughs at Ayse* great O.K.

Fig. 4.22 Transcription of third round.

Interaction analysis

Ayse takes the first block (K1) in her hand and adds “then we always make alternately brick” and sets it (K1) horizontally on its X side on the table <01-02>. By setting the block horizontally on the table, she gives the impression of starting to build a corpus vertically on the table, that she lays the base of the corpus by setting the first block as a base. In respect of the concept of the family study, the figures on the playing cards in “Building 02” are designed in such way that they can be built both vertically and horizontally on the table (see Fig.4.23). Thereby in each turn there are two possibilities: the player can build the corpus either vertically or horizontally on the table.

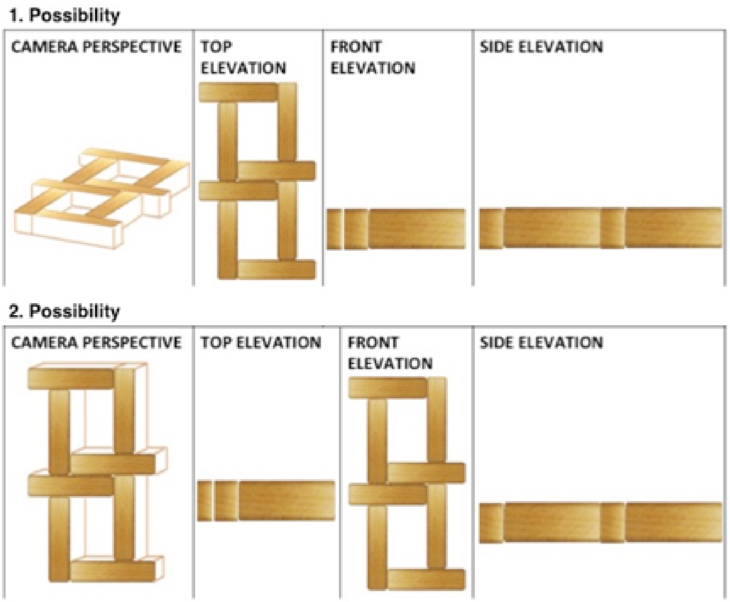


Fig. 4.23. The building possibilities of the chosen card and their elevations.

In the first observation period, with the game “Building 01”, Ayse played with her father and built different corpuses, which were always set vertically on the table (see 4.3.1.). Furthermore, in the previous turns of “Building 02”, until the chosen scene, both she and her mother built all corpuses vertically on the table. Bearing in mind, Ayse's first observation phase and the beginning of the chosen play situation, it does not seem surprising that she comes up with the idea to build a vertical corpus on the table. She might still focus on setting the block vertically on the table.

Ayse says "then we always make alternately brick" <01-02>. Through her utterance she might try either to pose an implicit question or to request, whether they can build the corpus together. Moreover, it might be a sign of trying to get some feedback or help from her mother. When her sentence is examined word-by-word, her reaction can be interpreted as follows:

- The word "alternately" means to do or perform something in turn repeatedly. In this sense she might mean that she tries to offer to build the figure on the card in turn with her mother repeatedly in her p-turn. Namely she seems to ask her mother for help to build the corpus together in sequence in her p-turn. Thereby she might mean that each player should attend the building corpus and setting a block on it alternately.

- The word "brick" might signify the word "wooden brick" and might be used to emphasize the blocks that remain at their disposal to build the corpus. In this sense she might mean that they sequentially take a block and set it on the building corpus. They can build the corpus together by taking turns one after the other.

From a participatory point of view, Ayse seems to take the role of *author*. By setting the first block on the building corpus and posing such a question, whether they can build the corpus together, she gives the impression of coming up with a totally new idea. Moreover, by posing such a question she might officially ascribe to her mother either the role *collaborative game partner*, with whom she can build the corpus collaboratively, or *tutor*, by whom she can be aided during her block building activity in the play situation.

The mother then says, "just look. not like that." She takes block K1 and sets it on its Y side on the table and adds "but like that. it is from the top." by rubbing her right index finger on the Y side of the block, then asks “okay?” <03-06> (see Fig.4.24).

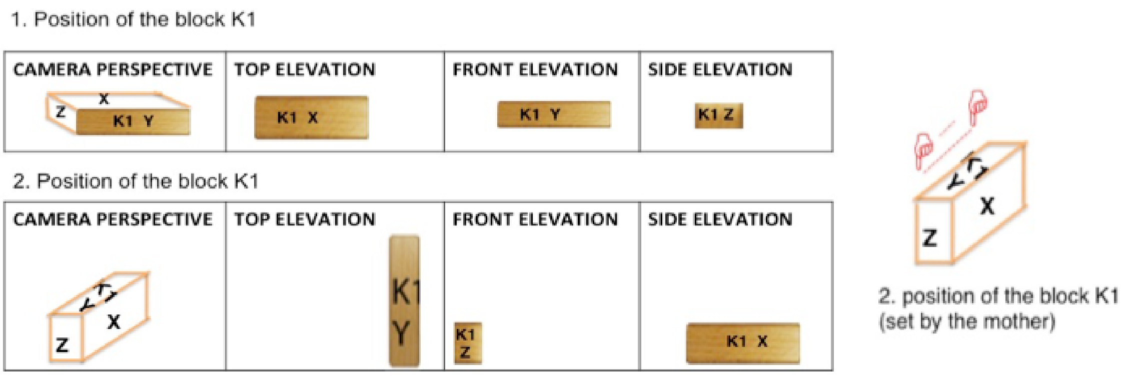


Fig. 4.24. The positions of the first block (K1), which is set by mother and Ayse

Through her first phrase “just look.” <03-04> the mother might mean that Ayse should look at the figure on the chosen card carefully. Most probably she tries to call Ayse’s attention to the position of the block. From a supportive perspective, her reaction refers a kind of supportive activity, namely *prompt after error*, in that she tries to prompt Ayse after she set the block wrongly – to her way of thinking. Moreover, she renders a possible performance for Ayse to think about, how the corpus should be built. From a participatory point of view she might try to fascinate Ayse and enable her an expanded leeway of participation so that she can examine and perform building the corpus correctly. Thereon she adds “not like that.” <03-04> by bringing the block to another position (see Fig. 4.24.) and says “but like that.” <05>. Considering both her utterances “just look. not like that” <03-04>, the reaction of Ayse’s mother can be interpreted as trying to change the subject. Maybe she does not wish to build the corpus together and thus she tries passively to refuse Ayse’s request and to shift the subject to the position of the block. Thereby she might simultaneously try to control the game and to obstruct Ayse’s endeavour of building a corpus together in her turn.

Her utterance “not like that. but like that.” <03-05> by bringing the block to another position can be interpreted as follows:

By saying “not like that.”<03> the mother seems to disapprove of Ayse’s proposal. Whereas she might disaffirm Ayse’s assertion that they “always make alternately brick” <01-02>, she might also disapprove of Ayse’s action in setting the block incorrectly. Most probably she just gives a negative response to Ayse's action that she shouldn't set the block "like that". Turning the block to another position might be an effort to show the right position of the block (see Fig. 4.24). From a supportive perspective, the mother seems to realize two different support activities consecutively: first *disaffirmation* and then *modelling.* Her first reaction implies the supportive activity of *disaffirmation*, which is a type of correction that indicates a definitive negative response, "not". First she expresses her disagreement and then by changing the position of the block and saying "but like that." she seems to *model* – to her way of thinking – the right position of the block which should be set "like that". She seems to realize some *scaffolding* functions too: *marking critical features* and *demonstration.* By means of *marking critical features* she is providing information about Ayse's act, that she shouldn't set the block "like that". Thereby the mother appears to use *maternal planning* and *demonstration* as to how the block actually should be set. In the sense of *demonstration*, she might try to indicate a position that Ayse will be able to imitate in a more appropriate form. On the other hand, her reaction gives the impression of somehow *framing* the situation. The mother seems to take Ayse's interpretation as a reference that she remarks on that the corpus should be built horizontally. Utterance of the mother signalizes that she reacts *disputationally* that she expresses her disagreement and individual decision-making and assertion about the set block. Thus, from a participatory point of view, the mother has the appearance of preventing Ayse from turning up and expressing her own idea. In this respect it seems that the leeway of participation for Ayse is restricted. Furthermore, the mother gives the impression of ruling over the play situation and striving for Ayse to build a corpus with reference to her mother.

The mother adds "it is from the top." by rubbing her right index finger on the Y side of the block <05-06> (see Fig. 4.24). Thereby her reaction reinforces the idea that she tries to show the correct – according to her interpretation – position of the block. In this sense she seems to maintain framing the situation. The mother's statement can be interpreted two different ways:

- One possibility is that she just means that the Y side of the block should be seen from the top instead of the X side, because just before <01-02> Ayse had set the block in such a way that the X side can be seen from the top instead of the Y side.

- Another possibility is that the mother might pay attention to the spatial visualization of the figure on the chosen card. Maybe she tries to emphasize to Ayse that she should build a corpus with the view of top elevation of the figure on the chosen card (see Fig. 4.25.). Namely the elevation of the corpus could be her focus point and thus she might try to let Ayse build the corpus "exactly" how it is seen on the card. For identification in the definition of the blocks, each block is tagged with a letter (see Fig. 4.25). Considering the figure (Fig. 4.25), the set block K1 could represent one of the blocks on the card: a or c or f or g (see Fig.4.25). Thereby the statement "it is from the top." could mean that each set block should be seen from the top as she sees the blocks in the figure on the card. More specifically she might mean that the top elevation of the figure should be taken as reference. Each block should be brought to such a position that their top elevation together should be identical to the front elevation of the figure on the card.

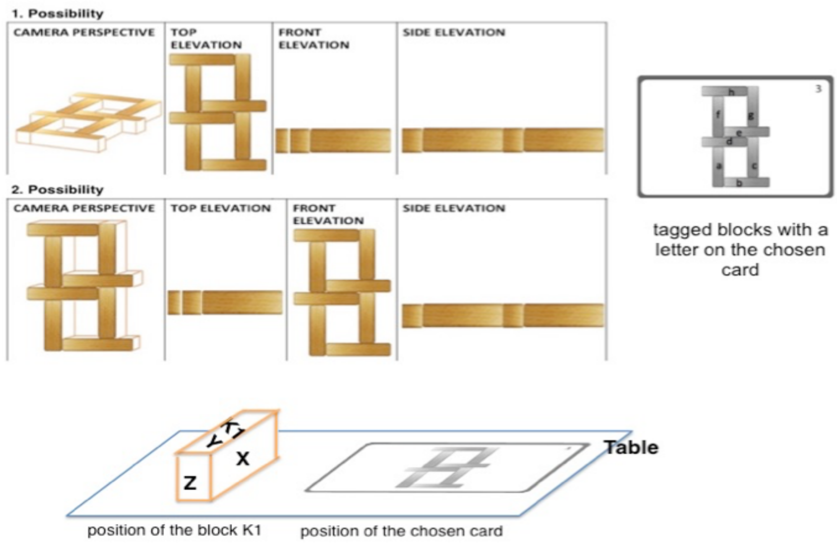


Fig. 4.25. The chosen card with tagged blocks and how it is situated on the table in front of Ayse and her mother.

From a supportive perspective, to say “it is from the top” can be seen as a supportive activity, namely *instruction*, by which the mother suggests a specific strategy for how Ayse should situate the block or blocks in the corpus. From a developmental perspective, she might try to let Ayse understand clearly what she should do in a spatial sense and which elevation she should take as a reference. Thereby she also signalizes thematizing one of the spatial functions – namely *kinaesthetic imagery* – in that she makes up-down/left-right discrimination with respect to the location of their bodies (see section 2.1.). Moreover she seems to render a scaffolding process for Ayse by pointing out the relevant characteristics of the block on the card. In this sense she seems to realize one of the scaffolding functions, namely *marking critical features*. On the other hand, her reaction reinforces the idea that she somehow maintains *framing* the situation.

Thereupon the mother asks Ayse for approval <06>. By posing such a question, she might try to get feedback from Ayse, whether she gets her point and thus she might try to offer Ayse an expanded leeway, in which she can take an opportunity to interrogate her idea and come up with a new idea as an *author* after musing on her *outcomes.* Moreover, she might try to *scaffold* Ayse's skills in such a way that it is possible for Ayse to internalize the spatial knowledge of her mother and convert it into a tool for *conscious control* (see 2.2.2.3.). From a supportive perspective, the mother might try to *engage* Ayse in the play situation by posing such a question to let her feel free to express her own ideas and to become deeply involved in the play situation. Thus the mother's reaction can be seen as one type of supportive action, namely *motivation*.

Taking all the mother's utterances <03-06> into account, she gives the impression of coming up with a geometrical argument that emphasizes the view of the blocks or corpus from the top elevation. In this sense she seems somehow to *frame* the situation, that the corpus should be built horizontally on the table. From a developmental perspective, her reaction might activate both Ayse's geometrical skills that she can use her mother's geometrical approach and produce a 3D corpus with different features.

Ayse utters "hııhıı"<07>, which is a typical Turkish expression of approval. Her reaction can be interpreted as either agreeing with her mother or understanding what the mother points out. By approving *wordlessly* Ayse might not exactly be sure whether her mother approaches the building action comprehensibly. Maybe therefore she is uttering "hııhıı" rather than saying "yes" or "okay". So indeed, considering the position of the block and the figure on the card, it cannot be exactly diagnosed whether the mother set the block right or wrong, because of the ongoing building process. Moreover Ayse's reaction seems to indicate that she instruments her mother's tutoring, asserts a counter idea and turns up by expressing her own idea. In this sense she seems like an *author* and might regard her mother as a *tutor*.

Thereon the mother says, "from the side- the side can be seen" by rubbing her right index finger on the surface of block K1 <08-10>. She might try to keep her argument in line <05> that the Y side of the block "should be seen from the top". Thereby she is once again thematizing the *kinaesthetic imagery* of the figure on the card in that she makes up-down/left-right discrimination with respect to the location of their bodies. Maybe therefore she seems to mean that the other sides can be seen only from the side elevation so that the Y side of the block can be seen only "from the top" elevation <05>. Thereby she might also mean that this surface of the block can be seen more clearly from the top elevation rather than the other elevation types (side elevation and front elevation). From a developmental perspective the mother seems to go on geometrical *framing* how the perspective should be taken to build a corpus to match the figure on the card. Her reaction might be a sign of situating the figure on the playing card in the built corpus, which would rely on developed spatial competences. From a supportive perspective, her declaration indicates a supportive activity, namely *instruction*, by which she suggests a specific strategy for how Ayse should take the perspective and situate the block or blocks in the corpus. Moreover, she seems to render a scaffolding process for Ayse by pointing out the relevant characteristics of the figure on the card. She gives the impression of drawing Ayse's attention to features of the perspective view of the figure which are important or relevant for its completion. In this sense she is performing one of the scaffolding functions, namely *marking critical features*.

Ayse tells her mother “now it is you.” and then puts her right hand next to the set block K1 and adds “you must here.”, while holding her right hand next to K1 and her left hand a bit further and parallel to K1 <11-14> (see Fig. 4.26).

Ayse’s first reaction reinforces the second interpretation in line <01> that she wants to build the corpus together with her mother by taking turns to set blocks one after the other. Maybe therefore she states, “now it is you.” to emphasize that “now it is her (mother’s) move” and the mother has to set a block to build a corpus. From a participatory point of view, Ayse seems to take the role of *author* by coming up with a new idea, whereas her mother discusses the 2D features of the figure on the card. Moreover Ayse ascribes the role of “collaborative game partner” to her mother, with whom she can build the corpus collaboratively.

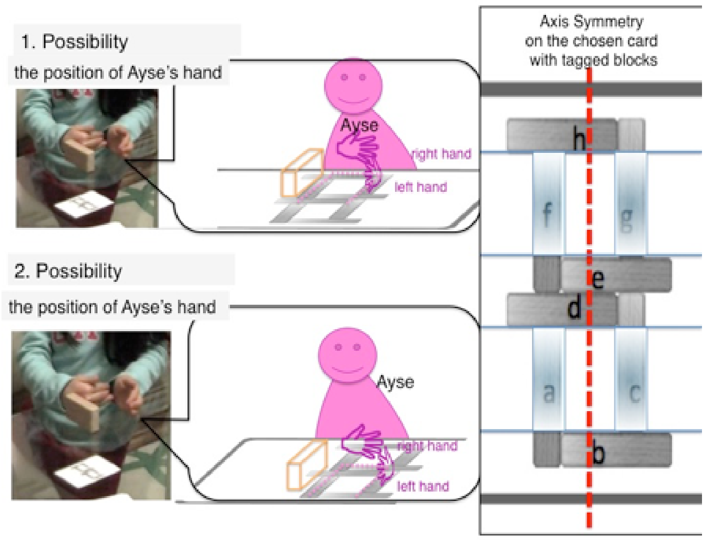


Fig. 4.26. Two possibilities of Ayse’s hand position in the interpretation

Thereon Ayse adds “you must here.” <12>. While she is moving her right hand next to block K1 and her left hand across from it (see Fig. 4.26). Most probably she is using her hand to represent the next blocks in the corpus. In this sense her action can be interpreted in at least the following two different ways (see Fig. 4.26):

- Considering the position of her hands, her right hand might identify block b, her left hand block a and block K1 might identify block c.
- Considering the identification of her hands in another way, her right hand might identify either block e or d, her left hand block f and block K1 might identify block g.

From a developmental perspective, it seems – no matter which interpretation is taken – that Ayse can recognize and apply slides, flips and turns and thereby she could manifest recognition of the symmetry of blocks, either a and c or f and g. Namely Ayse seems to be able to apply transformation and use symmetry to build a corpus identical with the figure on the card. Furthermore, she is making right–left discriminations with respect to the location of her body, so that her left and right hands seem to be used vicariously. She seems to be able to build the structure from the pictured model in her mind and express it with her hand movements. All these particulars signalize Ayse's sufficient spatial abilities. From a participatory point of view, Ayse seems to take the role of *spokesman*, while the mother becomes the *sponsor*. She takes the geometrical *framing* of the mother and expresses the position of the next blocks with her hands pursuant to her argument. She both approaches the block building activity from the spatial perspective and thematizes geometrical *framing* like her mother. Moreover, she engages in the negotiation process critically but constructively and collectively. She offers her mother a hypothesis, which is made publicly accountable, and signalizes her effort to reach agreement with her mother. She seems to negotiate with her mother in an *exploratory* way so that the mother can turn up by setting the blocks as Ayse demonstrated. A working consensus arises between Ayse and her mother.

Meanwhile the mother says "yes. Do we want to do so that we alternate?" <15>. The first utterance of the mother seems an *affirmation*. Maybe she affirms Ayse either to emphasize that she gets her mother's point and has the right point of view, or to confirm that Ayse's hypothesis is right, and she (the mother) should set the next block as Ayse demonstrated. Thereon she poses a question whether they want to alternate in the play situation. Her questioning brings to mind Ayse's phrasing in lines <01-02> where she said "then we always make alternately brick". Most probably the mother tries either to understand clearly or to be sure whether Ayse wants to build the corpus together with her. Therefore, she might pose such a question to make it clear whether she should do the next move or not. The mother's reaction refutes the interpretation <03-04> that she tries to change the subject and does not prefer to build the corpus together. Moreover, by posing such a question she is ascribing the role of "collaborative game partner" to her daughter with whom she can build the corpus collaboratively.

Thereon Ayse says "yes." <16>. Maybe by giving a positive response Ayse might try to emphasize that she wants to build the corpus together by taking turns one after the other. Another possibility is that she might approve her (mother's) move through which she has to set one block to build a corpus. From a participatory point of view, her reaction signalizes that she ascribes the role of "collaborative game partner" to her mother with whom she can build the corpus collaboratively.

The mother puts the second block (K2) on its Y side with the right angle next to the first block (K1) and says "so..and now we have this block." By showing block a on the card <17-19>. Then she adds "and this one." By showing block b on the card <19> and goes on "now you make this one." By showing block c on the card <20> (see Fig. 4.27).

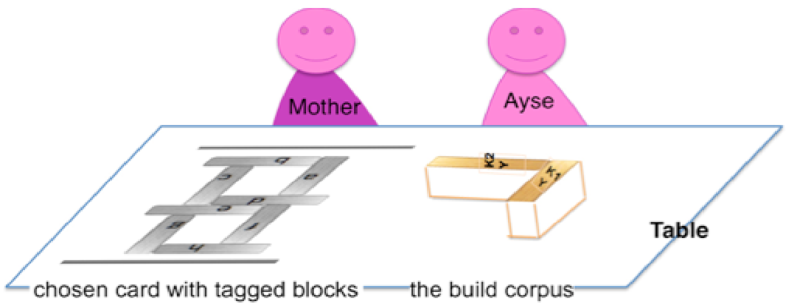


Fig. 4.27. Identification of the first two blocks K1 and K2

The mother puts the second block (K2) on its Y side with the right angle next to the first block (K1), says "so..and now we have this block." by showing block a on the card, then adds "and this one." by showing block b on the card and goes on “now you make this one.” by showing block c on the card <17-20>. Most probably she seeks to identify each set block with one of the blocks on the chosen card. Through her utterances she might also try to emphasize which block to identify with which block on the card. By saying “so..and now we have this block.” <18>, “and this one.” <19> and “now you make this one.” <20>, she seems to explain the previous, the present and the coming moves needed to build a corpus identical to the figure on the card. Considering the reaction of Ayse <11-14>, the mother might see or recognize that the first set block (K1) can be identified with different blocks on the chosen card and maybe therefore she just tries to pinpoint each block on the chosen card, which represents the set block in the building corpus. She lets Ayse know exactly that the first set block (K1) represents block a on the chosen card, whereas the second one (K2) represents block b. Thereby she determines that they have set the first block as a substitute for block a – ”now we have this block.” – and the second block as a substitute for block b – “and this one.”. Thereon she says to Ayse “now you make this one.” by showing her block c on the card. Most probably she means that Ayse should set a block in the building corpus, which represents block c on the chosen card. Thereby she gives the impression of giving Ayse a clue for the next step, which block should be set in the corpus. Moreover, she appears to specify the next move of Ayse, that the set block should represent block c. The mother seems maintain her geometrical *framing* that the (coming) next block should be set horizontally either as block b or c in the figure on the chosen card. From a participatory point of view, she seems to obstruct Ayse from expressing and performing her own idea. In this respect it seems that the leeway of participation for Ayse is somewhat restricted. From a supportive perspective, all explanations of the mother can be deemed a supportive activity, namely *instruction*, through which the mother suggests the use of a specific strategy. She instructs Ayse about the set blocks and the next block. Through her mother’s *framing* Ayse can organize her further building actions pursuant to their *collective argumentation*.

The mother seems to realize a support activity, namely *modelling.* By bringing the second block into position she *models* the way of building the corpus, in which position the coming blocks should be set. The mother seems to realize some *scaffolding* functions too, namely *demonstration* and *marking critical features.* She appears to *demonstrate* how the block actually should be set and how the blocks in the building corpus should be identified with the blocks on the chosen card. In the sense of *demonstration*, she might try to provide a position such that Ayse will then *imitate* it back in a more appropriate form. By means of *marking critical features* she is providing information about Ayse’s previous act, her own act and Ayse’s coming act that the set blocks should be represented with which blocks on the chosen card. From a participatory perspective, the mother seems to be an *expert*, while she is reserving the role of *novice* for Ayse. Moreover, she offers her hypotheses to Ayse about the blocks in the building corpus which represent the blocks on the chosen card.

The reaction of Ayse’s mother confirms the idea in lines <01-02,11-14,15,16> that Ayse wants to build the corpus alternately and the mother gives the impression of acquiescing in her request. In this sense a *working consensus* between Ayse and her mother seems to emerge about building the corpus collectively. Maybe therefore the mother seems to sum up the position of the blocks and emphasizes the next coming block, which should be set by Ayse. Both settle on building the corpus together by taking turns one after the other and attempting to accomplish Ayse’s p-turn by building a corpus identical to the figure on the chosen card. From a participatory point of view, they both appear to be “collaborative game partners” in that they build the figure on the card collaboratively.

Considering lines <1-20>, the negotiation of meaning between Ayse and her mother is a *collective argumentation process* in that they engage collaboratively and communicatively in the horizontal block building activity. They are observing, following decisions, and adjusting themselves to accomplish a corpus which is identical to the figure on the chosen card.

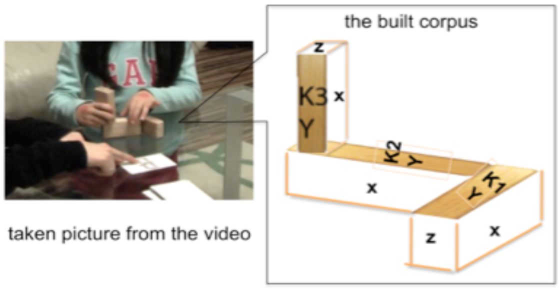


Fig. 4.28. Screen capture from the video

Thereon Ayse takes the third block (K3), puts it directly in the upright position on K2 <21-22> (see Fig. 4.28). From a developmental perspective, Ayse's reaction shows a problem in *perspective taking*.

Her action can be interpreted in three ways:

- As in the beginning of the building action <01>, in the current p-turn of Ayse, she seems intent on setting the block vertically in the corpus, whereas the mother seems to *frame* the situation and to build a horizontal corpus on the table. In this sense Ayse's reaction appears to be in conflict with her previous reaction in line <12> where she transformed a 2D object in her mind and represented it in 3D space. Whereas she seems to be able to build the structure from the pictured model in her mind and express it with her hand movements, through her current action she gives the impression of not being able to perform and build a structure from pictured model.

- Maybe Ayse cannot take in the different dimensions of the blocks. Therefore she might not to be able to rotate the third block in her mind and she thus sets it perpendicularly. Because of either the first or second possibility Ayse might still focus on building the corpus vertically and thus she could try to rotate it perpendicularly.

- Maybe she cannot instrument her mother's idea, to build the corpus horizontally on the table, and therefore she might involuntarily set the block in a vertical position in the building corpus.

- From a participatory point of view, Ayse acts like an *author* as she expresses and performs her own original idea. From a developmental perspective, Ayse's action signalizes a problem at replicating the top elevation of the figure on the card and performing it with 3D objects. Moreover, she might ambiguously interpret her mother's framing in a different way.

Thereon the mother says "no! You should first think about it!"<23>. It seems that the mother does not give Ayse any opportunity to continue her building activity and put forward her further ideas. From a supportive perspective, the mother's reaction implies *disaffirmation*, which is a type of correction that indicates a definitive negative response "no". By saying no the mother might try to give a definitive response to Ayse's action that she cannot set the block in a different way. Her reaction can be interpreted in three different ways: she says no because

- Ayse sets the block congruently with the figure but not in the same place as the figure on the chosen card.

- Ayse sets the block incongruently with the figure but in the same place as the figure on the chosen card.

- Ayse sets the block incongruently with the figure and not in the same place as the figure on the chosen card. If so, the mother does not express any reason for her "no" and she seems obviously to deprive Ayse of becoming informed about the kind of mistake that the mother assumes or she did. In this respect it seems that the leeway of participation for Ayse is somewhat restricted. From a participatory point of view, she deprives Ayse of turning up and expressing her own idea.

Moreover the mother seems to express her disagreements and individual decision-making and adds that Ayse should first think about it. She seems to negotiate with Ayse disputationally and to call Ayse's attention either to the position of the block or to the blocks in the figure on the chosen card. She seems to induce Ayse to think about setting the block either in an identical place or in a congruent way. From a supportive perspective her reaction refers a kind of supportive activity, namely *prompt after error*, in that she tries to prompt Ayse after she set the block in a different way. Moreover she renders a possible performance for Ayse to rethink the position of the block and to open a discussion about the difference between the figure on the chosen card and the built corpus. In this regard she might try to fascinate Ayse and enable her an expanded leeway so that she can examine and perform the rightness of the position of the set block.

By showing the blocks on the card the mother adds “just look, how the block stands that the side can be seen.” <23-25>. She comes up with geometrical *framing* again, as in the previous lines. She seems to reference the side elevation of the figure, whereas her previous arguments she was taking the top elevation of the figure as a reference. Taking account the position of the block K3, it can be said that its Y side can be seen from the front elevation, its Z side can be seen from the top elevation and its X side can be seen from the side elevation (see Fig. 4.29).

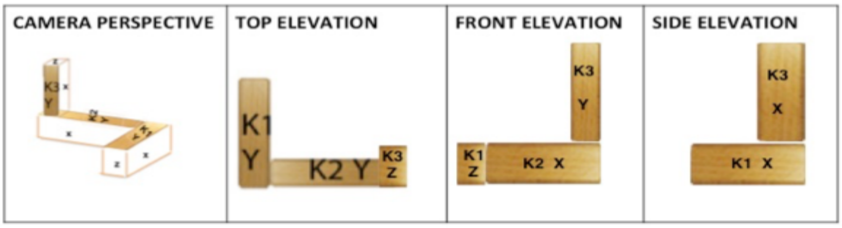


Fig.4.29. The set blocks on the table up to now and their top, front and side elevations from Ayse’s position

In this regard the mother’s reaction can be interpreted as follows: Ayse should look at the position of the block (K3) because

- K3 should be set in such way that its Y side could be seen from the top elevation.
- K3 should be set in such way that its Y side could be seen only from the side elevation.
- K3 should be set in such way that its other sides could be seen only from the side elevation.

- K3 should be set in such way that its other sides could be seen only from the front elevation.
- Current position of the block K3 is such that from the top elevation another side can be seen but not the Y side.
- Current position of the block K3 is such that from the side elevation another side can be seen but not the Y side.
- Current position of the block K3 is in a way that from the front elevation the Y side can be seen but not the other sides.
- In the first two blocks (K1 and K2) – by ruling out the back side (one of the Z side) of K2 – every side except the *bottom* (one of the Y side) can be seen, but at the position of the third block different sides or rather every side except the *back* (one of the Z side) can be seen (see Fig. 4.29).

From the developmental perspective the mother gives the impression of giving a clue by thematizing again the *kinaesthetic imagery* of the figure on the card. She seems to make up-down/left-right discrimination with respect to the location of their bodies. Moreover by emphasizing the visual features of the figure on the card she seems to thematize *perspective taking* of the figure on the card too. From a supportive perspective, her declaration indicates a supportive activity, namely *instruction*, by which she suggests any specific strategy, how Ayse should take the perspective and let stand the block or blocks in the corpus. Moreover she signalizes providing *elaborations* and points out how the block stands. Therefore her utterances are “instructive and conceptually rich” in terms of scaffolding. Her reaction seems to be a prediction for Ayse that she can use for correction to achieve the right built corpus. Thereby the mother gives the impression of fulfilling one of the scaffolding functions, namely *marking critical features*. From a participatory point of view, she gives the impression of offering Ayse such a participation profile that she can muse on her further steps in the building activity, through which she can participate in an expanded leeway, and perform her ideas by taking the role of *author*.

Taking into account the mother's reactions together <23-25>: by disapproving of Ayse's action <23>, calling her attention and giving a clue <23-25>, the mother appears to provide Ayse with different leeways of participation through her one reaction. From a participatory point of view, she gives the impression of acting collaboratively.

Ayse puts K3 in the upright position next to K2 <26>. Taking account the new position of block K3, it can be said that its Y side can still be seen from the front elevation, its Z side can be seen from the top elevation and its X side can be seen from the side elevation (see Fig. 4.30.). Her reaction can be interpreted in four different ways:

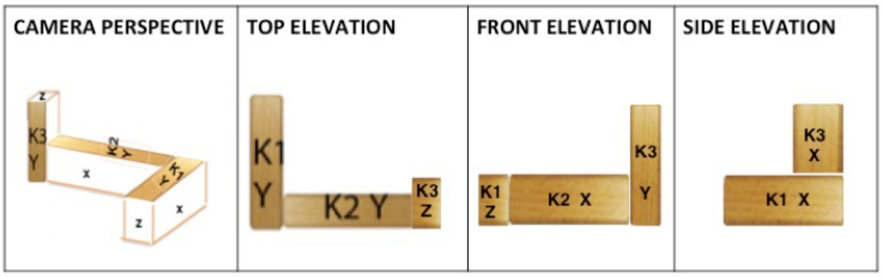


Fig. 4.30. The changed position of the third block and the top, front and side elevations of the blocks from Ayse's position

- Maybe Ayse cannot take the perspective, which her mother just mentioned. The utterance of her mother, "just look, how the block stands that the side can be seen." <23-25>, was not comprehensible enough for her in order to transform a 2D object in her mind and represent it in 3D space through taking the right perspective.
- Maybe she is unable to rotate the third block in her mind and thus sets it again perpendicularly just next to the second block K2.
- As in lines <21-22>, Ayse might still consider on building the corpus vertically and thus seems to set it perpendicularly again.
- Maybe she cannot get her mother's point of view to build the corpus horizontally on the table. Her mother says, "just look, how the block stands that the side can be seen."<23-25>. Because of her comment that the side

can be seen, Ayse might involuntarily set the block to the “side of K2” in a vertical position in the building corpus, so that the side can be seen (see Fig. 4.30.).

From a developmental perspective, Ayse’s reaction reinforces the idea in lines <21-22> that she has a problem in geometrical *perspective taking*. From a participatory point of view, Ayse acts like an *author* [in] that she expresses and performs her own original idea.

The mother says “lay down.”<27>. Thereby she puts in issue the block which Ayse puts on the upright position next to K2 just before <26>. Most probably she means that Ayse should lay the third block (K3) down. By emphasizing the building interpretation in order to construct the right – according to her – corpus, she appears to insist on her geometrical *framing* that the corpus should be built horizontally on the table. Referring to external regulation theories (see 2.2.2.2.3.) her reaction appears *executive* and *directive* in that she seems to provide Ayse with little *autonomy and hardly any responsibility for the learning process*. The mother is adjusting her verbal and nonverbal interventions because of Ayse’s *framing.* Whereas up to now she has acted as a collaborative game partner of Ayse, now she reacts as an *executive* in that she notifies her interpretations more *directively* than before. Regarding family system theories (see 2.2.2.2.2.) and the first observation phase (see section 4.1.1.) it is not surprising that she uses *minimal-effort strategies* and more directivity than the father. The mother’s reaction has the look of *taking on more responsibility for the social exchange*, depending on Ayse’s capacity and performance. In this sense, from the developmental point of view, the mother seems to provide feedback of an *instructional nature, in which interactive content arises richly in terms of positive affect and verbal stimulation during the negotiation process with her daughter.* Within the scope of both approaches the reaction of Ayse’s mother can be interpreted in a way that she might try to *direct* Ayse in terms of scaffolding. Maybe thus she uses an *unmoderated imperative that calls for a verbal response, which is categorized as a direct command*. Regarding family system theories (see 2.2.2.2.2.), she appears to give instructional feedback as a didactic exchange in which she stimulates Ayse verbally that she should lay the block down. Thereby she might also try to *scaffold* Ayse’s skills in such way as to enhance Ayse's to spatial knowledge. She seems to emphasize the next building action as a goal to keep Ayse partly *in the field* by means of *direction maintenance*.

From a supportive perspective, her reaction seems a type of supportive action, namely *provide a solution*. She seems to *provide a solution* for how Ayse actually should set the third block (K3). Thus from a participatory point of view she gives the impression of depriving Ayse of turning up and discussing her own idea further. In this respect it seems that the leeway of participation for Ayse is somehow restricted. Moreover the mother seems to *disagree* with Ayse’s action, to *assert* a *counter* idea and to direct Ayse to lay the block down. Her reaction signalizes that she negotiates with her daughter *disputationally* in the building process <23-27>. From the participatory point of view, the mother puts herself in the role of an *expert,* while assigning the role of *novice* to her daughter.

Thereupon Ayse sets the third block (K3) horizontally on its Y side next to K2 <28> (see Fig. 4.31). By laying the third block down she seems to adapt her mother’s geometrical *framing* to her own building framing, which becomes *standardized* on mother’s interpretation. Thus she is maintaining the geometrical *argument* of her mother and performing her mother’s idea that she should lay the third block down. Thus she gives the impression of imitating and conforming with her mother’s idea. From the participatory point of viewn she appears to take the role of *relayer* while she ascribing the role of *deviser* to her mother. Furthermore she seems to accept her mother’*s tutoring* or *external regulation* by virtue of the fact that she does not go into any further discussion and does exactly what her mother said, in which it might also be seen that she does not reject her mother’s argument.

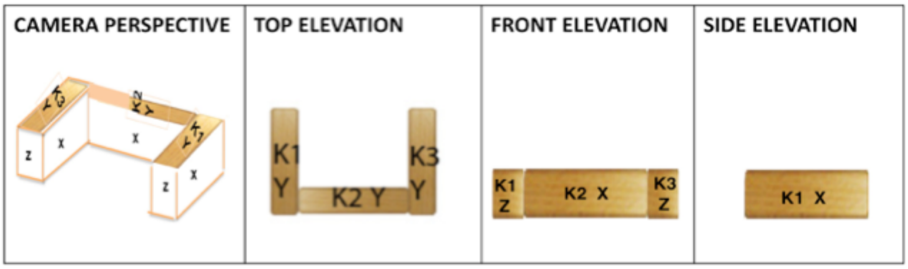


Fig.4.31. The second changed position of the third block and the top, front and side elevations of the blocks from Ayse's position.

It is not obvious from Ayse's action whether she has set the block rightly or wrongly. From a developmental perspective, the possibilities of Ayse's further actions to achieve a corpus identical to the figure on the card can be interpreted in two different ways (see Fig. 4.32):

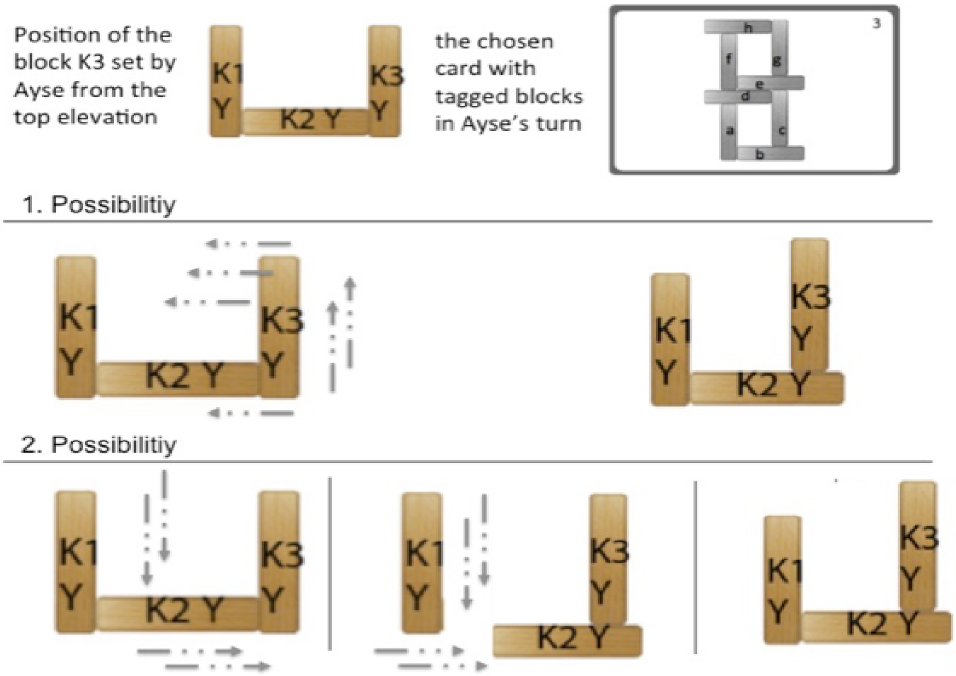


Fig.4.32. Two further possible positions of the third block (K3) from the top elevation.

- She can push the block K3 a bit to the left and set it perpendicularly on K2.
- She can push the block K2 a bit down and set it under K3. Then she can push the block K2 a bit down and set it perpendicularly next to K2.

- With reference to these two possibilities it remains ambiguous whether Ayse sets the block (K3) right or wrong. As things stand, the negotiation process between Ayse and her mother formalize Ayse's further steps and maintain her mother's *geometrical framing*.

The mother says "exactly. and then? I put this here." by showing block b on the card, adds "now you make this block" by showing block c on the card and then looks at Ayse <29-31>. With the word "exactly" the mother might try to approve Ayse's action in such way that she does it right by laying the third block down. Taking into account the new position of block K3, it can be said that its Y side can be seen from the top elevation, its Z side can be seen from the front elevation and its X side can be seen from the side elevation. Thus the new position of the third block and its elevations from different perspectives seem to match up with the mother's previous hints <23-25, 08-10, 05-06>. She might approve Ayse's action, meaning that she set the block "exactly" right in accordance with her *framings*. From a supportive perspective, her reaction can be seen as a type of supportive activity, namely *affirmation*, in that she demonstrates her agreement to Ayse's building activity. Thereon she poses a question, "and then?". By posing such a question she might try to enquire about Ayse's further steps: "and" what you should do "then"?, or

- the next coming block: "and then" which block should be set?, or
- the coming turn: "and then" is it now my turn?, or
- their further steps: "and then" what should we do?, or "and then" which block should we set?

Taking into account the negotiation process up to now (Ayse: "then we always make alternately brick" <01-02> Mother: "yes. do we want to do so that we alternate?" <15> Ayse: "yes." <16>), the reaction of the mother reinforces the idea that they play *collaborative* and build the corpus together. From a participatory point of view, she ascribes the role of *collaborative game partner* to her daughter with whom she builds the corpus together.

Showing block b on the card, she says “I put this here.”<29-30>. Most probably she tries to remark that block K2, which she set, matches block b on the chosen card. Thereon she adds “now you make this block” by showing block c on the card <30-31>. Similarly, she might try to state that block K3, which Ayse just set, should match block c on the chosen card. From a supportive perspective, showing blocks and saying, “I put this here. now you make this block” can be seen as a supportive activity, namely *instruction*, by which she suggests a specific strategy, which block she set and which block Ayse has just situated. She might try to be clear to let Ayse understand what they have done in the spatial sense, which blocks they have set and which elevation they took as a reference. Thereby she thematizes different spatial functions about the representation of 2D objects in 3D space. Her reaction gives the impression of providing *elaborations* that she utters *instructively* and *conceptually rich* in terms of *scaffolding*. From a developmental perspective, her reaction has the look of *predicting positive cognitive outcomes* in Ayse in that she *prompts* Ayse to think about setting the block in an identical place with the congruent elevations. Furthermore she seems to render a scaffolding process for Ayse by pointing out the relevant characteristics of the block on the card. In this sense she seems to realize one of the scaffolding functions, namely *marking critical features*. From a participatory point of view, she appears to be acting *collaboratively*.

Thereupon she looks at Ayse <31> to get some reaction or feedback from her. Ayse looks at her mother and says, “I have already.” <32>. She might mean that she has already

- set block K3 in the building corpus, or
- “made this block”, bearing in mind her mother’s utterance “now you make this block” <30-31>, or
- understood what her mother means, or what they have done, or which elevation they took as a reference,

- known what they have done, or which blocks they have set, or which elevation they took as a reference.

Through her reaction Ayse gives the impression of emphasizing that she picked up the argument of her mother and takes it as a reference. From a developmental perspective, it appears she has already performed her mother's point by laying down the third block (K3). However, she gives the impression of acting as a *picture maker* (see Table 2.4) in that she can use multiple spatial relations by showing flexibility in integrating parts of the structure and thus produce corners in composing of blocks.

Ayse and her mother seem to negotiate *in an exploratory way* in that they engage critically but constructively with each other's ideas, offering justifications and alternative hypotheses <26-32>. Furthermore, their knowledge is made publicly accountable and their reasoning is *more visible in the utterances, and progress results from the eventual agreements reache*d.

The mother says "no. just look, where your block is." by showing block b on the chosen card and adds "yours begins here." by showing the edge of block b on the chosen card <33-34> (see Fig. 4.33.). By giving a definitive negative response she might try to emphasize that Ayse did not set the third block exactly right. From a supportive perspective, her reaction indicates the supportive activity of *disaffirmation*, which is a type of correction that indicates a definitive negative response "no" <33>.

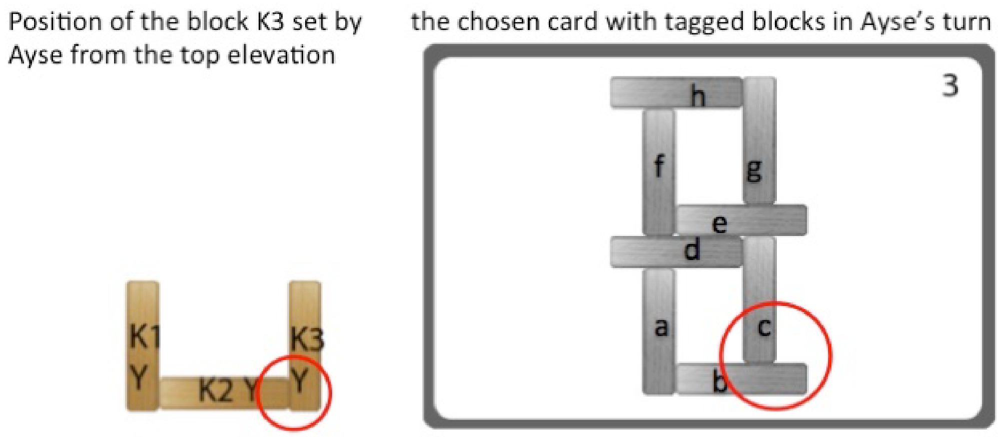


Fig.4.33. The chosen card and the position of the third block.

Thereon she says “just look, where your block is.” <33> by showing block b and adds “yours begins here.” by showing the edge of block b on the chosen card <34>. Most probably she tries to emphasize where the third block (K3) is situated and how actually it has to be set (see Fig. 4.33.). Through her first sentence, “just look, where your block is.” by showing block b, the mother might mean that Ayse should look at the location of the set block in the building corpus. Most probably she tries to call Ayse’s attention to the difference between the corpus and the figure on the card. From a supportive perspective her reaction refers to a kind of supportive activity, namely *prompt after error*, that she tries to *prompt* Ayse after she set the block nonidentically to the figure on the card. Moreover she renders a possible performance for Ayse to rethink about her action and to open a discussion about the difference between the figure on the chosen card and the built corpus.

From a participatory point of view, she might try to fascinate Ayse and enable her an expanded leeway so that she can examine and perform how a block should be set in a right way in order to build a right corpus. Through her second sentence, “yours begins here.” by showing the edge of block b on the chosen card <34>, Ayse’s mother might try to give a clue about the wrongness – according to her interpretation – of the built corpus. Thus, it seems she makes the building action easier for Ayse, by showing the exact difference between the location of block K3 and block c. She seems to take block b on the card as a reference and to check the location of the third block (K3), whether it is set horizontally on the second block (K2) or next to it. From a supportive perspective, her second sentence indicates a supportive activity, namely *instruction*. Thereby she suggests to Ayse a specific strategy in order to help Ayse to set the third block right. Thereby she seems to come up with the geometrical approach. Bearing in mind the possibilities in line <28> that Ayse can go on setting the third block (K3) in her own way, through her second sentence, the mother seems not to give Ayse any further opportunity to think about her further steps. From the developmental perspective, the mother might perceive Ayse as a *picture maker* (see Table 2.4) in that Ayse can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus errs at adding pieces.

From a participatory point of view, the mother deprives Ayse of turning up and expressing her own idea. Thus, it seems that the leeway of participation for Ayse is rather restricted. In summary of the supportive perspective, it seems that Ayse's mother uses three different support activities together in one reaction through her utterances: *disaffirmation, prompt after error* and *instruction*. In terms of *scaffolding,* she might try to help Ayse enact a sequence of actions, in which she acts as a *tutor.* Referring to *scaffolding* functions, the mother provides *elaborations* that she utters *instructively* and that are *conceptually rich*. Her feedback seems to be regulated to a level that Ayse can use it for *correction* to achieve the right built corpus. From a developmental perspective, her reaction shows she is *predicting positive cognitive outcomes* in Ayse. By pointing out the relevant characteristics of the position of the blocks in the corpus and the figure on the card, she seems to carry out one of the *scaffolding* functions, namely *marking critical features*, and to ascribe the role of *tutee* to Ayse. Moreover, from a participatory point of view, she renders Ayse different leeways of participation through the sum of her utterances.

Thereupon Ayse puts K3 on its Y side horizontally on to K2 <35>.

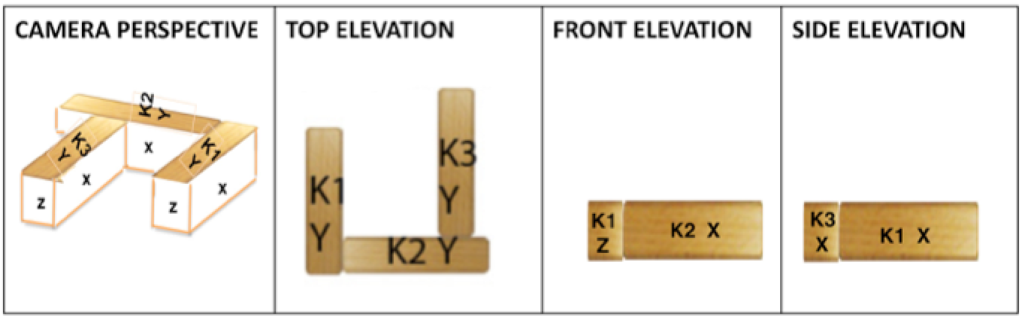


Fig. 4.34. The third changed position of the third block and the top, front and side elevations of the blocks from Ayse's position.

From a developmental perspective, Ayse might apparently take into consideration the *framing* of her mother that the corpus should be built horizontally. In this regard she seems to try setting the block as stated by her mother. Comparing the corpus with the new position of the third block and the figure on the card, the building corpus bears a resemblance to the basic part of the figure on the chosen card. Moreover she gives the impression of becoming a *shape composer* in that she can produce *arches, enclosures, corners, and crosses systematically* (see Table 2.4.).

From a participatory point of view, Ayse seems to take the role of *relayer*, while she ascribing to her mother the role of *deviser.* She takes the idea of placing the building corpus horizontally and sets the third block on to the second block, which her mother states from the beginning of Ayse's p-turn. This strengthens previous interpretations as she gives the impression of adopting her mother's geometrical *framing* to the building activity and performing it in exactly the same way as her mother mentioned.

The mother still shows the edge of block b on the card, says "exactly O.K.! yes. right." and then moves K3 a bit towards to K1 <36-38>. With the words "exactly, O.K., yes, right" the mother gives the impression of approving and accepting Ayse's action of laying the third block K3 on to block K2. Through her utterances she seems to express her agreement. She appears to state that Ayse does "exactly" what she means. From a supportive perspective her reaction can be seen as one of the supportive activities, *affirmation*, as she demonstrates her agreement to Ayse's building activity. Thereon she moves the block K3 a bit towards the block K1. Her reaction gives the impression that she brings the third block a bit closer to the block K1. From a supportive perspective her reaction indicates a supportive activity, namely *providing a solution*. Probably Ayse's action somehow indicates an *error* and thus the mother seems to try producing the right position of the third block by acting but without saying anything about her *error.* Thereby it seems that she creates a conflict between her reactions: She seems to approve Ayse but then somehow adjusts the last position of the third block (K3) to bring it to the correct position. In this sense, between Ayse and her mother there seems to emerge a substantially *disputional* negotiation process <1-34>. In the emerging negotiation process the mother mostly gives the impression of expressing her disagreements, individual decision-makings and short counter-assertions. Bearing in mind the idea of family systems theories, that children become attached to their mothers through consistently *sensitive care* and the mother-child relationship aims to *calm and comfort rather than arouse* (see section 2.2.2.), Ayse's mother gives the impression of making tolerable demands of her daughter and *relying on low-level strateg*y in her geometrical *framing*. Maybe she focuses on having and meeting Ayse's *emotional needs* and therefore she does not criticize further but adjusts the set position of the third block (K3) by herself rather than saying that Ayse should change the position of the set block a bit towards the block K1.

In summary of the supportive perspective, it seems that Ayse's mother uses two different support activities together in one reaction through her utterances and action. Thus, her action can be deemed as a supportive action, which is conjunction of two supportive activities: *affirmation* and *providing a solution*. Maybe through her reaction the mother gives the impression of providing Ayse with somehow *comforting tutoring,* in which a sense of *psychological well-being* of the child as well as a sense of *scaffolding* are aimed for and restored. From a participatory point of view, she gives the impression of acting as a *comforting tutor* and in some measure as a *nurturer* (see 2.2.2.)*,* who provides *emotional support, creates safety*, is available to others, and can be a *mediator*, as well as a *tutor*, who is an *expert* and provides scaffolding to the child.

Considering lines <21-38>, the negotiation process between Ayse and her mother is *collaborative, collective* and *communicative*. This process seems to include deliberate attempts and incidental comments of the mother as she instructs and assigns setting the block to Ayse, while Ayse appears to be observing, following and performing the instructions and directions of her mother. This interpersonal process signalizes a collective participation in which they manage their roles.

Ayse next says something, which is not understandable.

The mother looks at the figure on the card. "<u>Let me look</u>.", she says to Ayse in Turkish and then continues in German "yes right." <40>. She takes the fourth block (K4) and adds "so now I put one- now I make. now I put this block, O.K.?"

while showing block d on the card <41-42>. First the mother utters in Turkish, which could mean a kind of attempt to see the figure on the chosen card. Most probably she tries to look at the figure on the card in order to examine the built corpus so far. Then she goes on talking in German, “yes right.”. Through her reaction she seems to approve

- the idea that the corpus is built till now “right”, or
- Ayse’s action again as in line <36> that she set the third block “right”, or
- their collective building process as they build the corpus in the “right” way together.

From a supportive perspective her reaction can be seen as a type of supportive activity, namely *affirmation*, as she demonstrates her agreement.

Thereon she adds “so now I put one- now I make. now I put this block, O.K.?” by showing block d on the card <41-42>. Most probably she tries to assert block K4, which she will set in the corpus and identify with block d in the figure on the chosen card. From a supportive perspective, showing block d to say “so now I put one- now I make. now I put this block” can be seen as a supportive activity, namely *instruction*, by which she suggests a specific strategy, which block will be set in the corpus during the building action. Her reaction reinforces the idea at line <34> that Ayse’s mother assumes Ayse is a *picture maker* (see Table 2.4) as Ayse can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus errs at adding pieces. From the beginning of the play situation, the mother and daughter speak in German and it may be from her remarks that the mother returns to German after reacting briefly in Turkish. In terms of scaffolding, her reaction gives the impression of providing *elaborations* which she utters *instructively* and are *conceptually rich,* on what she will do and which block she will set in the building corpus. From a developmental perspective, her reaction should produce positive cognitive outcomes in Ayse, as in lines <29-31, 33-34> where she induces Ayse to muse on setting the block in an identical place with the congruent elevations. Moreover, she is providing a scaffolding process for Ayse by pointing out the relevant characteristics of the set block and the identical block on the card. In this sense she gives the impression of performing one of the scaffolding functions, namely *marking critical features*. From the participatory point of view, she seems to act collaboratively but as an expert about the spatial problem, in the course of reserving the role of *tutee* for Ayse.

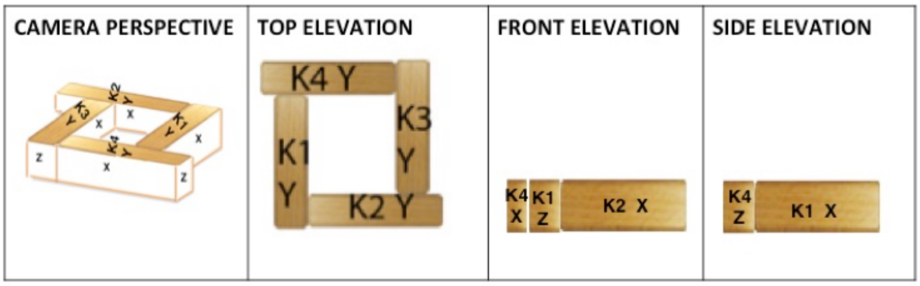


Fig. 4.35. The set fourth block with first three blocks and their top, front and side elevations from Ayse's position.

Thereon she asks Ayse for approval. By posing such a question, she might try to get feedback from Ayse, whether she understands what she means or will do. She seems to offer Ayse an expanded leeway, in which she can take an opportunity for interrogating her idea and coming up with a new idea as an *author* after musing on her outcomes. Moreover, she might try to *scaffold* Ayse's skills in such a way that is possible for Ayse to perform her spatial knowledge. From a supportive perspective, the mother might try to engage Ayse in the play situation by posing such question to let her feel free to express her own ideas and to become deeply involved in the play situation. Thus, her reaction can be seen as one type of supportive action, namely *motivation*.

As Ayse is leaning over the table, she says "hıııııııım", which is a typical Turkish expression for thinking about something <43>. In this sense her reaction can be interpreted as a *consideration* process in that her mother's remarks might make her think either about the coming action or the reaction of her mother. Maybe therefore she leans over the table to get more involved in this *consideration* process or to understand clearly what her mother shows and means.

The mother says “but you have to move away from there.” while she is putting K4 on its Y side on K1 intersecting with K3, and adds in Turkish “then the camera can not detect us.” <44-46>. Taking into consideration the recording position (see Fig. 4.36.), the mother’s reaction can be interpreted as meaning that she tries to make sure they and their building action can be seen in the video-recording.

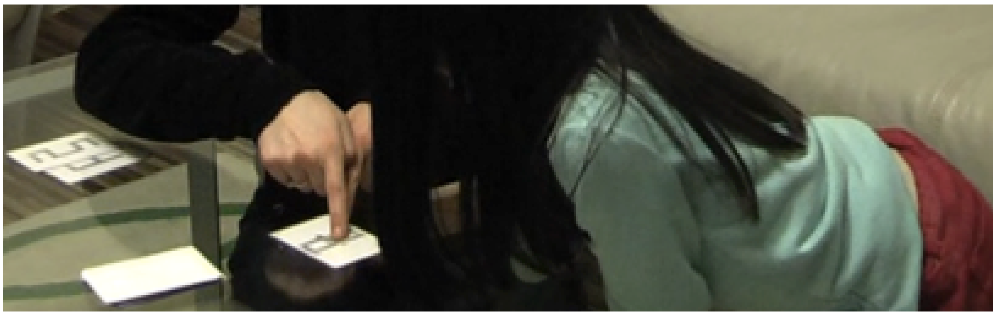


Fig.4.36. The recording position of Ayse and her mother

Another possibility is that she might aim to *calm* and *comfort* Ayse and maybe thus she tries implicitly to predict the consequence of any anticipated action by Ayse, regarding family system theories, whereas she explicity states a reason that the camera can not detect them. The reaction of the mother can be seen as a type of scaffolding function, namely *direction maintenance*, in that she might try to provide comfort for Ayse’s consideration process. Maybe she emphasizes the recording position of the building action to keep Ayse *in the field*, as she accentuates features of the building activity. From a participatory point of view, the mother seems to take the role of *tutor*, whereas she ascribes the role of *tutee* to Ayse.

Ayse utters “ehem” <47> and leans back. Most probably her reaction means she doesn’t know how she should react against her mother’s command. Furthermore, the mother’s tutoring seems to work on Ayse, in that she leans back.

Thereupon the mother says “so O.K.? did you understand it? so now it is your turn which one would you like to put? this one or this one?” by showing blocks e and f on the card <48-50> (see Fig. 4.37.).

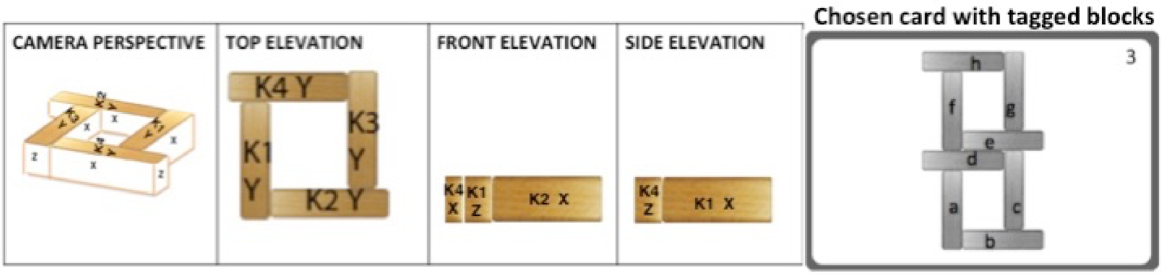


Fig. 4.37. The current built corpus and the chosen card with tagged blocks

The mother seems to ask Ayse for approval: “so O.K.? did you understand it?” <48>. Thereby she might try to check Ayse’s comprehension or to get any feedback from Ayse whether she understands what her mother means or does. By asking Ayse for approval, the mother signalizes that she is cooperative and strives to reach an agreement with Ayse. Furthermore, by posing the question “did you understand it?”, she might make herself sure whether Ayse understood

- that she should be careful about her position in order to let the camera detect them (see lines <44-46>).
- which blocks constitute the current built corpus (see lines <41-42, 33-34, 11-14, 16).
- that she should set the blocks proper to their position in the figure on the card (see lines <33-34>).
- that the blocks should be laid down during the block building activity (see line <27>).

Thereon she says “so now it is your turn which one would you like to put? this one or this one?”<49-50>. In lines <01-02,11-14,15,16> Ayse and her mother negotiated about to building the corpus alternately and they seemed to settle it. Therefore, by saying “so now it’s your turn which one would you like to put?”, most probably she means that Ayse should set one block in the corpus in current time in order to build up the figure on the chosen card. Thereby the reaction of Ayse's mother obviously confirms the idea in the previous lines that they build the corpus alternately together. Moreover she seems to pose a further question which block Ayse wants to set in the corpus. By showing blocks e and f on the chosen card to say "this one or this one?" she signalizes that Ayse should choose one of them to set it in the corpus (see Fig. 4.37.).

- From a participatory point of view, one possibility is that her mother offers Ayse an expanded leeway, in which she can take an opportunity for interrogating her idea and coming up with a new idea as an *author* after musing on her mother's *outcomes*. Moreover she might try to *scaffold* Ayse's skills in such a way that it is possible for Ayse to perform her spatial knowledge. She might try to ensure that the block building activity of Ayse is *directed towards achieving particular outcomes that contribute to completion* of the building corpus in terms of *scaffolding*. By means of *direction maintenance* she seems to emphasize the next building action as a goal to keep Ayse partly *in the field* and to deploy her enthusiasm and sympathy to keep her motivated. From a supportive perspective, the mother might try to *engage* Ayse in the play situation by posing such question to let her feel free to express her own ideas and to become deeply involved in the play situation. Thus her reaction can be seen as one type of supportive action, namely *motivation*.

- Another possibility is that she restricts the leeway of Ayse by asking for a choice whether she wants to set block e or f. So indeed they build the corpus horizontally on the table, so it is unlikely to fall, and hence Ayse can set one of the remaining four blocks (e,f, g,h). In this sense her selection field is more than two blocks (blocks e,f) (see Fig. 4.37.). She might restrict the leeway of participation in that she reduces Ayse's options from four to two. In terms of *scaffolding* she might try to simplify the building activity by reducing the number of constituent acts required to reach the right built corpus. Maybe the mother tries to let Ayse *perfect the component sub-routines* that she can manage in the block building activity, in terms of scaffolding. Thus she seems to realize one of the scaffolding functions, namely *reduction in degrees of freedom.*

In both ways, from a participatory point of view, they look like *collaborative* game partners in that they build the figure on the card collaboratively.

Ayse shows block f and says “these-” <51>. By uttering “these-” while showing block f, she might emphasize that she wants to set a block in the building corpus, which identifies with block f in the figure on the chosen card. Thereby she gives the impression of making her choice and replying to her mother’s question in lines <49-50>. In this regard she seems to specify which block she wants to put in the corpus. Considering her mother’s question, “this one or this one?” in line <50>, Ayse gives the impression of taking the role of *spokesman*, in that she takes the choice offer of her mother, and says “these” which she answers with a different word, whereas she seems to ascribe the role of *sponsor* to her mother. From a developmental perspective, her reaction might show that her mother’s *tutoring* works on Ayse. She gives the impression of understanding and performing the point of view of her mother. She seems to make her choice, which her mother offered.

The mother shows block f in the figure on the card with her right index finger and says “these.” <52>. Her reaction seems to be a repetition in order to clarify whether Ayse wants to set the block, which is identified with f on the chosen card. She takes Ayse’s idea and points out her choice with the same word. From a participatory point of view, she seems to take the role of *relayer,* while Ayse becomes a *deviser.* From a supportive perspective, her reaction might be interpreted as a supportive activity, *re-representation*, in which she remarks on Ayse’s choice with the same word.

In lines <40-51> Ayse and her mother seem to negotiate *in an exploratory way* in that they seem to strive to discuss the alternatives, offering and reasoning hypotheses and striving to reach agreement with each other. They seem to *collaborate* and understand each other’s points of view.

Thereupon Ayse puts the fifth block (K5) at right angles to the block K4 <53> (see Fig. 4.38.). Thereby she seems to perform her own choice. From a participatory point of view, she gives the impression of keeping her role of *author* as she makes her own choice and performs it. From a developmental perspective, her reaction signalizes that she has sufficient spatial abilities at composing a corpus in 3D space and decomposing a figure in 2D space. In this sense, moreover, her reaction reinforces the idea that they negotiate using *exploratory* talk (Fernández et al., 2001, p.42) by collaborating and seeming to understand each other's point of view.

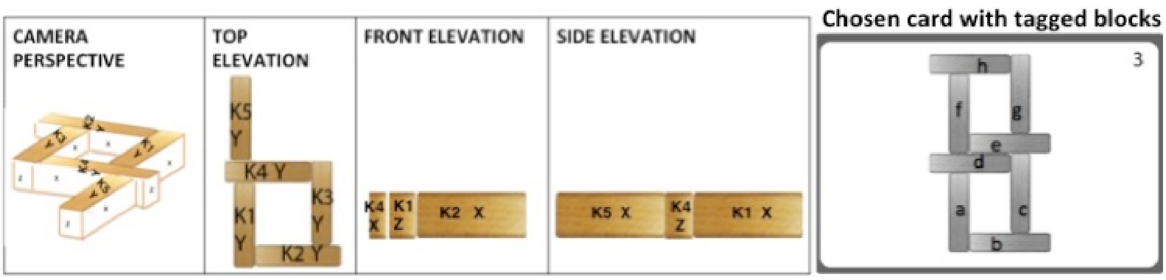


Fig.4.38. The fifth set block in the built corpus and the chosen card with tagged blocks.

The mother says "try to put it exactly at the same straight like these." by pointing at the column of block a and f on the card with her right index finger <54-56>, while Ayse says "noo these." by pointing at block e on the card with her right index finger <57-58> (see Fig.4.39.).

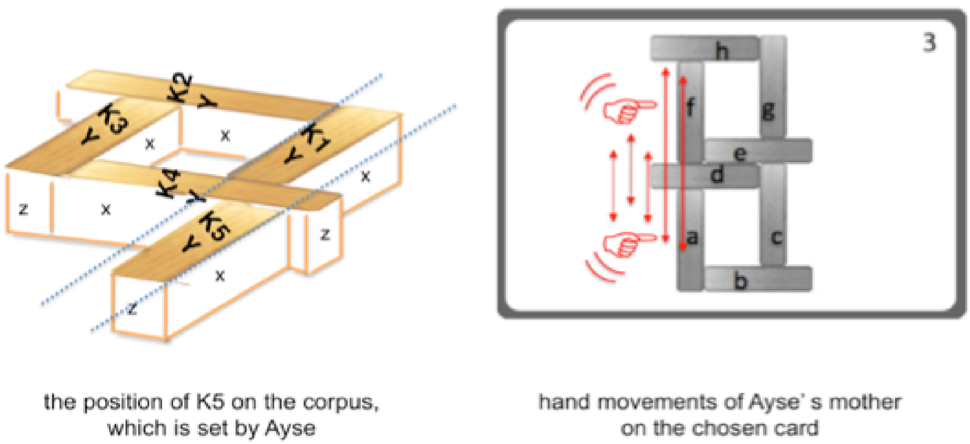


Fig.4.39. How Ayse set the block K5 and how the mother shows the straightness of blocks at the column of block a and f on the card

Pointing at the column of block a and f on the card with her right index finger and saying, “try to put it exactly at the same straight like these.” <54-56>, the mother might mean to let Ayse build a corpus, which should be completely identical to the figure on the card, and to direct Ayse in accordance with the rules. Block (K5) is set in the same way but not in the same direction as K1 (see Fig. 4.39.). Thus the mother seems to lay stress on the direction of two blocks (a and f) in the figure on the chosen card.

Her reaction reinforces the idea at lines <34, 41-42> that she assumes Ayse as a picture maker (see Table 2.4) as Ayse can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus errs at adding pieces. Maybe therefore her mother tries to give such *instruction* in order to make her realize their building actions.

Most probably she means that Ayse should align block K5 with block K1 as seen in the figure on the chosen card. Her reaction shows that the external regulation she *uses phrase in the imperative mood and regulates* Ayse’s building action. Maybe therefore she uses the verb “try” to let Ayse set the fifth block (K5) exactly aligned with the first block (K1). Furthermore, in terms of family system theories, the mother gives the impression of using *directivity* in order to let Ayse build a corpus identical to the figure on the chosen card. In terms of *scaffolding* there are two different possibilities that the mother seems to realize:

- She might try to simplify the building action, through which the set block can be in a same straight with the first block as seen on the card. Thereby her reaction signalizes that she reduces the number of constituent actions required to reach the right built corpus. thus she seems to realize one of the scaffolding functions, namely *reduction in degrees of freedom*. Her reaction can be assumed as a supportive activity, namely *instruction*, through which the mother suggests the use of any specific strategy to build an identical corpus with the figure on the card. Thereby Ayse can organize her further building actions pursuant to the clue or information given by her mother. In that regard she seems to offer Ayse a participation profile such that she can muse on her further steps in the building activity and perform her ideas by taking the role of *author*. The interaction process renders an expanded leeway of participation to Ayse.

- In terms of scaffolding her mother might try to *keep* Ayse *in the field* so that she focuses on setting the blocks completely identical to the blocks in the figure on the chosen card. In a similar manner she might also try to keep Ayse *in pursuit of a particular objective* so that she can *directly* maintain the building activity. Thus she seems to realize one of the scaffolding functions, namely *direction maintenance.* Thereby Ayse can set further blocks aligned with each other in similar situations. The reaction of the mother seems to be a supportive activity, namely *providing a solution*, as she declares the right position of the fifth block in order to let Ayse build a corpus identical to the figure on the chosen card. She indicates the right position of the fifth block (K5), how it should be set exactly. In that regard the interaction process renders a restricted leeway of participation to Ayse. Ayse can perform her mother's ideas by taking the role of *relayer* as she sets blocks in reference to the *directions* of her mother. The mother gives the impression of offering Ayse a restricted participation profile.

In regard to both possibilities, the mother seems to ascribe the role of *novice* and *tutee* to Ayse, whereas she takes the role of tutor, who is an *expert* in introducing spatial tools into the building activity. By using the word "these." <56>, the mother is taking the role of *ghostee* as she uses the same formulation as Ayse, but her finger movements signalize another idea. Whereas Ayse points out one block, the mother seems to emphasize the column of blocks f and a. Thereby she is ascribing the role of *ghoster* to Ayse (see section 2.2.1.).

Concurrently Ayse says says "noo these." by pointing at the block e on the card with her right index finger <57-58>. Her reaction can be interpreted in two different ways.

- By pointing at block e on the card with her right index finger to say "noo these." she might mean that:

- Ayse brings a counter-argument to her mother's that she should try to put K5 aligned with block e on the chosen card.

- Block K5, which is set by Ayse, matches perfectly block e in the figure on the chosen card.

- Ayse changed her mind and wants to set a block which matches block e in the figure on the chosen card. So indeed, at lines <49-50> the mother asked Ayse "so now it's your turn which one would you like to put? this one or this one?" by showing blocks e and f on the chosen card. Ayse might prefer block e instead of block f. Thereby her reaction reinforces the idea in lines <49-50> that Ayse should choose one of them to set it in the corpus. However the current reaction of Ayse seems to diminish the interpretation in lines <51, 53> that she prefers to set a block identified with block f in the figure on the chosen card. From a participatory point of view, Ayse might try to indicate indirectly to her mother that she does not accept the restriction of her leeway and by changing her mind she might try to expand it. By using again the word "these" and showing block e, she gives the impression of taking the role of *ghostee*, while ascribing the role of *ghoster* to her mother, from the participation point of view. She takes her mothers idea into consideration (to set block e or f) and points out her choice with the same word "these" but pointing out one block as in lines <48-58>.

Thereupon the mother says "noo. now you have these." and shows block f on the card while saying, "now you make-" <59-60>. It seems that she does not give Ayse any opportunity to muse on her own choice or argument. From a supportive perspective her reaction indicates in a supportive activity, *disaffirmation*, which is a type of correction that indicates a definitive negative response "no". By reacting with the word "no" she might try to give a definitive negative response that:

- Ayse cannot set a block identified with block e in the figure on the chosen card.

- The block set (K5) by Ayse did not identify with the block e in the figure on the chosen card.

- Ayse cannot change her mind and should keep the position of the block (K5) stable as she set it just before (see <53>).

“now you have these. now you make-” says the mother while showing block f at <59-60>. Most probably she tries to clarify with which block on the chosen card the set block by Ayse is to be identified. The utterances “now you have these.” and “now you make-” look like a repetition of the same issue, which block Ayse should set or which one she would prefer to set (see lines <49-50>). From the supportive perspective, her reaction appears to be one of the supportive activities, *re-representation*. She seems to re-represent the issue in a way that appears more familiar to Ayse. Moreover through her utterances while showing block f in the figure on the chosen card, the mother might try to draw attention to

- a block which Ayse has already set in the corpus that is identified with block f in the figure on the chosen card.

- a block, which is tagged as block f in the figure on the chosen card. Thereby she might mean that Ayse should attend to the block f instead of e. She is simplifying the building action for Ayse. Thereby she appears to let Ayse handle the block building activity more simply. Thus she seems to realize one of the scaffolding functions, namely *reduction in degrees of freedom*.

Regarding Ayse’s utterance in line <57-58>, the mother seems to use the same word and idea of Ayse. She is ascribing the role of *deviser* to Ayse, whereas she seems to take the role of *relayer*. She points out a block by using the word “these”. Therewith the mother might try to draw attention to

- the fifth block (K5) and block f in the figure on the card. By saying “now you have these.” she might put emphasis on the fifth block (K5) that Ayse set and by saying “now you make-” she might emphasize that the fifth block matches block f in the figure on the chosen card.

- blocks, which Ayse just set and which Ayse now will set. By saying “now you have these.” while showing block f, the mother might address the block, which Ayse just set (K5) identifies to the block f, and by saying “now you make-” she might address the next block which Ayse will set. She might *set a specific goal* for Ayse that “now” she should place K5 in a way that matches block f in the figure on the card. Thus she seems to realize one of the scaffolding functions, namely *direction maintenance*.

Taking into account the part between lines <38 and 58>, it seems that Ayse and her mother negotiate collaboratively and communicatively. They seem to structure the play situation by managing each other’s roles and the coming steps in the block building activity. Furthermore they are observing, following decisions, and adjusting themselves to accomplish a corpus which is identical to the figure on the chosen card. Furthermore they are performing the mother’s geometrical *framing.*

Ayse says “no!” <61> at the same time her mother speaks in Turkish: “Allright. Then do it.” <62>. Ayse seems to disaffirm her mother. She might refuse

- to set another block instead of block f.
- to set a block which matches block f.
- to perform the *specific goal*, which her mother meant.

Thereon her mother gives the impression of either approving Ayse’s reaction or reaching agreement with her daughter, and then talks on “Then do it.”. She seems to let Ayse do whatever she wants or set which block she prefers. Her reaction reinforces the idea that she reaches an agreement with her daughter. Moreover she does not enter into any further discussion about the block setting or building activity. From a participatory point of view, she is offering Ayse an expanded leeway, in which she can take an opportunity for interrogating her own idea and coming up with a new idea as an *author* after musing on fulfilling her mother’s geometrical framings. Considering lines <54-61>, the mother and daughter seem to negotiate *disputationally* as they express their disagreements and individual decision-makings, assertions and counter-assertations.

Ayse puts K5 on its Y side horizontally overlapping with K4 (see Fig. 4.40.)

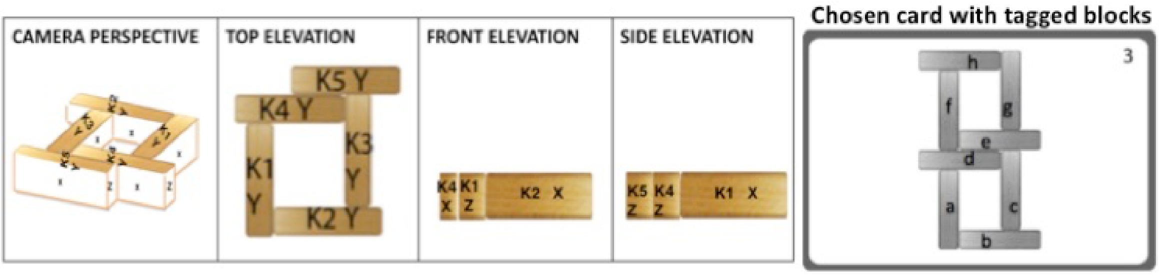


Fig. 4.40. Changing position of the fifth block in the built corpus

Her reaction signalizes that she obviously changes the position of fifth block (K5) and sets it in a position, which matches the position of block e in the figure on the chosen card. Her reaction reinforces interpretations in lines <57-58,61> that she changed her mind and prefers to set a block, which is identified with block e instead of f. So indeed, by showing blocks e and f on the chosen card and asking Ayse "so now it's your turn which one would you like to put? this one or this one?" at lines <49-50>, the mother might *prompt* the idea that Ayse can set the fifth block in a position which matches the position of either block f or block e in the figure on the chosen card. By this means Ayse can make a choice to set a block in a position which she prefers. From a participatory point of view Ayse's reaction strengthens the idea in line <61> that she does not accept the restriction of her leeway and she might try to expand it by changing her mind. By setting the fifth block (K5) in another position she seems to perform one option that her mother offered. The leeway of participation left to Ayse does not appear to be exactly expanded, rather it still looks to be somehow restricted. From a developmental perspective, her reaction shows she has quite sufficient spatial abilities in that she can *relate*, *represent* and *identify* 3D shapes. She seems to *produce arches with vertical and horizontal interior spaces and enclosure corners in vertical and horizontal axes* (see section 2.1.). In this regard Ayse seems to get her mother's point of view and perform it. She seems to *compose* and *decompose building blocks using multiple spatial relations to produce composite shapes.* Therefore, she gives the impression of taking the role as *shape composer* from the developmental point of view (see Table 2.4). From a supportive perspective, her reaction gives the impression that her mother's *re-representation* and *instructions* work on Ayse, evoking her spatial skills so that she can manage to build a corpus horizontally.

Ayse and her mother continue the block building activity and likewise engage in it collaboratively and communicatively. The mother sets the sixth block (K6) horizontally in the building corpus, whereas Ayse sets the remaining two blocks (K7 and K8) like her mother, which might be seen as a confirmation of her mother's *geometrical framing, and* maintaining *collective argumentation* process <64-90>. Ayse rounds the corpus out. All these block building activities lead us to the conclusion that there emerges a *working consensus* in the negotiation process between mother and daughter <1-90> as they set all the blocks horizontally in the corpus (see 2.2.1) and achieve a corpus identical to the figure on the chosen card (see Fig. 4.41). Thereupon Ayse and her mother declare that the corpus is done <91-92>.

Hereby Ayse's p-turn ends, which they accomplish having built an identical corpus to the figure on the chosen card.

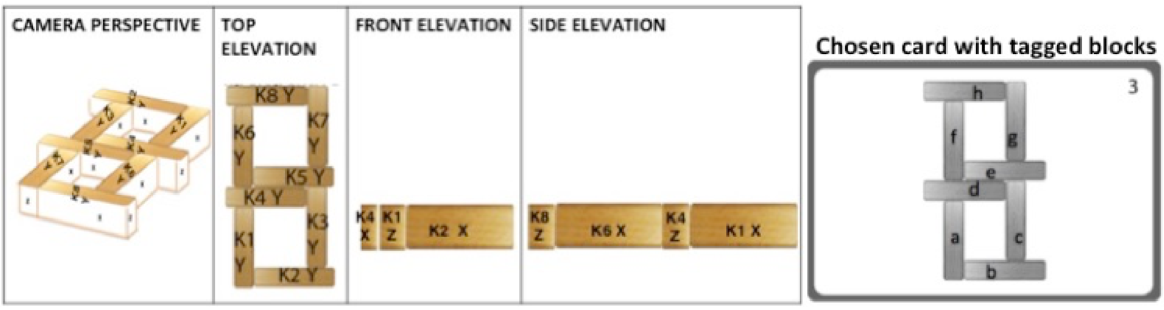


Fig. 4.41. The chosen card and the built corpus from top, front, side elevations

Thereafter the mother chooses a new card from the deck (see Fig. 4.42.) and poses a question what she has to do in her p-turn <100-101>. Ayse's p-turn obviously ends and her mother's p-turn begins.

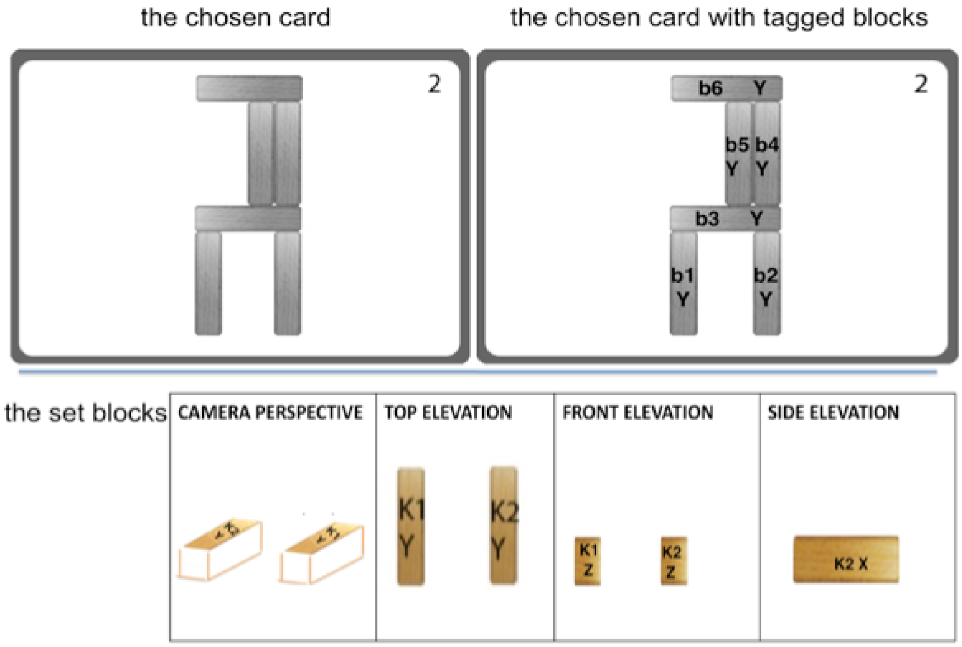


Fig.4.42. The chosen card in the mother's turn and the position of the first two blocks, which are set by Ayse

Ayse asks her mother "should I help you?", then talks on "please it works easily." and takes two blocks from the box, sets them first vertically on their Z sides but then lays them on their Y sides on the table <102-105> (see Fig. 4.43.). Her reaction gives the impression of understanding her mother's point of view in the previous turn and paying attention to setting the blocks horizontally on the table.

From a participatory point of view, Ayse turns up by relaying her mother's idea and setting two blocks in the same way as they did in the previous p-turn. In this sense she gives the impression of taking the role of *relayer* while ascribing the role of *deviser* to her mother. It seems that Ayse and her mother accomplish an *exploratory* negotiation process in that Ayse is collaborating with her mother and they understand each other's points of view.

From here on the chosen part of the transcribed scene will be taken up in detail:

The mother remarks, "you can put in the upright position." <106>. Most probably she means that Ayse can set two blocks in the vertical position instead of horizontal. By using the modal verb "can" the mother obviously gives the impression of offering Ayse an idea to build the corpus vertically instead of horizontally. Considering Ayse's p-turn just before <1-64>, it seems surprising that she proposes building a vertical corpus on the table. Thus the mother's approaches are a new geometrical *framing, which denote building a corpus only vertically* instead of *horizontally.* Her reaction appears to be in contradiction to her previous *framing* in Ayse's p-turn ("it is from the top." <05-06>, "from the side-the side can be seen."<08-10>, "just look, how the block stands that the side can be seen."<23-25>, "lay down" <27>) but by emphasizing another building alternative she seems to build up her spatial skills in mental rotation by shifting to build vertical and horizontal spaces in practice. From a supportive perspective, her reaction can be seen as a supportive activity, namely *instruction*, by which she suggests *a specific strategy*, how Ayse can situate blocks to build an identical corpus to the figure on the chosen card. From a developmental perspective, she might float this idea in order to let Ayse perform building corpuses in two different ways and explore geometrical connections in 3D space. From the developmental perspective, her reaction can be interpreted by means of family system theories (see 2.2.2.) in that the mother provides opportunities for independence of her daughter while still *monitoring* her actions. Thereby in this p-turn she seems to offer Ayse the alternatives that she can build either a vertical or a horizontal corpus, whereas in the previous p-turn she used *directivity* and *minimal-effort strategy* so that Ayse should lay the blocks down (see <27>). By means of scaffolding she seems to emphasize another building possibility as a goal to *keep* Ayse partly *in the field*, which seems to be *direction maintenance*. She might try to ensure that Ayse's block building activity is *directed towards* achieving particular outcomes that *contribute to completion* of the building corpus. Thereby the mother might try to maintain the block building activity of Ayse, in which she can strive to achieve a goal.

From the participatory point of view, she seems to provide Ayse an expanded leeway and *increasing autonomy*, while she is giving different choices to her daughter in the course of a collective argumentation process. The mother gives the impression of putting herself in the role of an *expert*, while she is reserving the role *novice* for her daughter.

Thereon Ayse states, “no then falls down” <107>. Through her reaction, she might mean that she cannot set blocks in the upright position (see <106>), when she does, then they will fall down. Most probably Ayse refers to the static balance of either the building corpus or two set blocks. Considering Ayse’s first observation phase and the previous p-turns of the chosen play situation, it seems surprising that Ayse comes up with such an idea not to build a corpus vertically on the table due to the risk of collapse of the built corpus. Taking into consideration Ayse’s p-turn just before <1-64>,

- she might still take notice of building a corpus horizontally on the table, or
- she has just experienced that building a corpus horizontally is sturdier than building it vertically. Therefore, she might come up with a contra-assesment in order to avoid the hazard of the building corpus falling being aware of the instability of a corpus, which is built vertically on the table.

Considering the previous p-turn, her reaction signalizes that her mother’s tutoring and the *scaffolding* process worked on Ayse, in which they engage collaboratively and communicatively. Moreover, Ayse’s reaction reinforces the idea in the previous p-turn that she has observed and met with decisions. Namely negotiation of taken-as-shared meanings and her mother’s usage of scaffolding functions in the previous p-turn (“it is from the top.” <05-06>, “from the side- the side can be seen.”<08-10>, “just look, how the block stands that the side can be seen.”<23-25>, “lay down” <27>), seem to evoke Ayse’s spatial abilities by letting her perform to build a horizontal corpus. Thereby it seems that Ayse becomes able to represent 3D transformations, regulate their relations and link them with each other (see section 2.1.), which evokes the idea that Ayse is a *shape composer of units of units* from the developmental point of view. Ayse’s performance in both turns indicates that she can conflate and perform spatial issues in a short amount of time. Thus, from a developmental perspective, Ayse shows sufficient spatial abilities by decomposing and composing a spatial field (see Table 2.4). From a participatory point of view, she gives the impression of acting as an *author* by arguing the hazard of the built corpus falling.

Thereupon the mother says “no we put it up” and puts both blocks up so that Ayse can see their Y sides in the upright position in front of her <108-110>. Most probably she *expands* Ayse’s *framing* <107> about building a corpus from horizontal to vertical. By saying “doesn’t fall down” she seems to strengthen her objection “we put it up” while putting two blocks vertically on the table. Regarding her remark “you can set in the upright position.” at line <106>, the word “up” might be short for the word “upright”. Moreover her utterance might mean that they will set either two set blocks or the building corpus in the upright position on the table. Thereby she gives the impression of coming up with the geometrical argument that they should build the corpus vertically on the table. From a developmental perspective her reaction might activate both Ayse’s geometrical skills in that she can use her mother’s geometrical *framing* and produce a 3D corpus with different features. From a supportive perspective she seems to realize two different support activities consecutively: first *disaffirmation* and then *modelling.* Her first reaction indicates a supportive activity, *disaffirmation*, which is a type of correction that indicates a definitive negative response “no”. First she expresses her disagreement and then by changing the position of the blocks and saying “we put it up” she seems to *model* – to her way of thinking – the right position of the blocks that they should be set in the “up”right position. She seems to realize some *scaffolding* functions too: *marking critical features* and *demonstration.* By means of *marking critical features* she provides information about Ayse’s *framing* that either the corpus “doesn’t fall down” or two set blocks “don’t fall down”. Then she appears to use demonstration to show how the blocks actually should be set. By using *directivity* and adjusting a verbal intervention she appears to be depriving Ayse of turning up and expressing her own idea. In this respect it seems that the leeway of participation for Ayse is somehow restricted. Furthermore, the mother gives the impression of ruling over the play situation and striving to let Ayse build a corpus in reference to her mother.

Ayse states “but it falls down.” <111> and then puts a new block, K3, horizontally on K1 and K2 on its Y side <111-112>. Through her utterance <110> Ayse seems to insist in her *argument* at line <107> that the corpus will fall down. Most probably she means that the corpus will fall down if they set the blocks vertically and build a vertical corpus on the table. Moreover, her reaction reinforces the idea in line <107> that Ayse might pay attention to the static balance of either the building corpus or two set blocks. Thereupon she sets the third block horizontally on to two set blocks. She appears not to maintain her argument and not to object to her mother’s argumentation by setting the next block on the blocks. From a developmental perspective her reaction strengthens the idea in line <107> that she has sufficent spatial abilities that she can conflate and perform spatial relations in a short amount of time. She puts the third block in an identical place to the figure on the chosen card (see Fig. 4.43.), while she is still arguing about setting blocks horizontally and building a horizontal corpus on the table. From a developmental perspective, Ayse seems able to *decompose and compose both vertical and horizontal spatial fields* by virtue of a collective argumentation process with her mother in the previous turn. Moreover, she looks to be able to regulate spatial relations at the same time in different ways (see section 2.1.). From a participatory point of view, she gives the impression of acting as an *author* by arguing the falling hazard of the building corpus while she is performing the building action, which is actually based contrary to her argument.

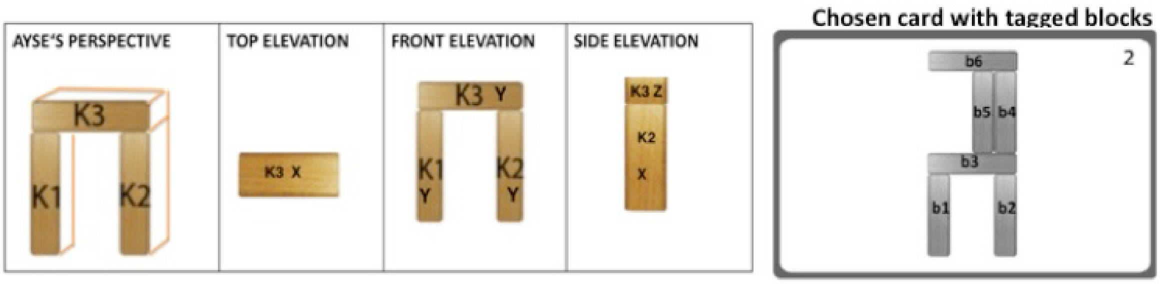


Fig. 4.43. The set blocks so far and the chosen card

Considering lines <106-112>, Ayse and her mother seem to lead a *disputation* negotiation process in which they *express their disagreements and individual decision-makings, assertions and counter-assertations* in the block building activity.

Thereafter Ayse goes on with building action and completes the remaining corpus, while her mother only supplies blocks from the box to Ayse <113-123>. Thereby Ayse builds a vertical corpus on the table, which might be seen as a fulfilling of her mother's *framing* again. In this sense a working consensus seems to arise between Ayse and her mother about building the corpus vertically (see Fig.4.44.). Furthermore, she turns up by relaying her mother's idea and setting two blocks in a same way as they did in the previous p-turn. From a participatory point of view she gives the impression of taking the role of *relayer* while ascribing the role of *deviser* to her mother. It seems that the negotiation process between Ayse and her mother is an *exploratory* negotiation process in that Ayse is collaborating with her mother and they understand each other's points of view.

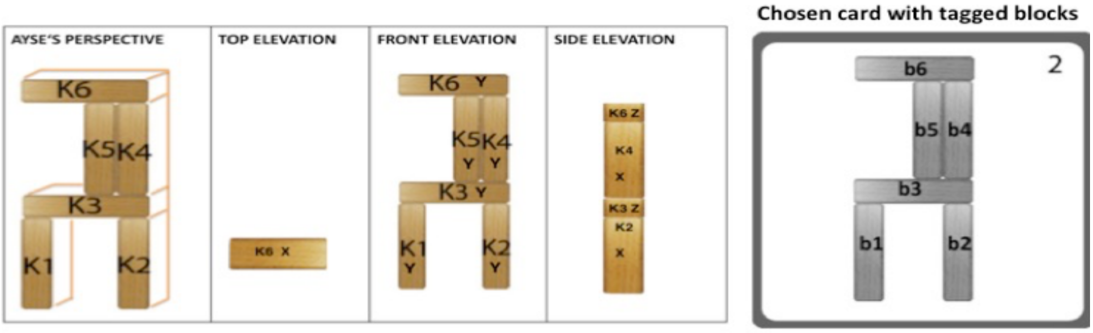


Fig. 4.44. The chosen card and the built corpus from frontal elevation

Ultimately Ayse achieves a vertical corpus identical to the figure on the chosen card, although it is her mother's p-turn actually she should build the corpus according to play rules of "Building 02" (see Fig. 4.44.). Here a *collective argumentation* process leads Ayse and her mother to build corpuses identical with the figures on the chosen cards.

Summing up from perspective of the interactional niche in the second observation period

In the chosen sequence the mother is the official game partner of Ayse and her competitor in the play situation. But situationally she and Ayse decide to build the corpuses together and they carry out a collective building process in both p-turns. Thereby they realize a *collective argumentation* process in which the mother uses some *scaffolding* functions. In the negotiation process between Ayse and her mother, the approach of her mother's geometrical framings enables Ayse to realize building different corpuses by reference to the vertical and horizontal planes. The mother adopts some scaffolding functions in the negotiation process with Ayse, while she is *arguing* about the way of building corpuses with Ayse *collectively*. Whereas in Ayse's p-turn they mostly negotiate *in an exploratory way* by struggling for alternatives, reasoning a hypothesis and reaching an agreement with each other, in the mother's turn they negotiate *disputationally* that they output their disagreements and individual decision-makings, assertions and counter-assertations in the block building activity. In both turns, exploratory and disputional negotiation processes enable Ayse to accomplish different participation profiles, while she is taking all four roles of *production design* in the course of these talks. Therefore, Ayse's participation profile instantly changes.

In both turns, through usage of scaffolding functions, a working census emerges as they perform the mother's geometrical framing which makes Ayse an *externally regulated participant*. Considering the whole interaction process in the chosen play situation, the mother gets a grip on the play situation in that she provides *instructively and conceptually rich elaborations*, while she *argues* with Ayse about building the corpus *collectively*. In this manner the mother seems to facilitate the development of metacognitive skills in Ayse by *using direct elicitations*. Therefore, she seems to take the role of *tutor* and at the same time she acts as *external regulator,* whereas she ascribes the role of *tutee* to Ayse. Concordantly, the *verbal and nonverbal stimulations* and *direct commands* of the mother enable content-related exchanges so that Ayse explores and performs whole spatial consequences in the block building activity, which leads Ayse to the conclusion that a vertical corpus can fall down, but a horizontal corpus cannot. Moreover, through a collective argumentation process, she *performs* building a vertical corpus, which leads Ayse to another conclusion – that the vertical corpus does not always have to fall down. For both matters Ayse turns up, completes building horizontal and vertical corpuses and succeeds in both her own p-turn and the p-turn of her mother.

Table 4.3. The roles taken in the chosen scene of the second observation

	Ayse's turn & Mother's turn
Ayse	Tutee as an external regulated participant
Mother	Tutor as an expert

In the regard to the theoretical point, in this play situation an emergence of "learning-as-participation" methaphor can be seen as Ayse experience all the building actions *by participating actively* and *fully* in the interaction process (see 2.2.2.2.2.).

Within this context there emerges a developmental niche for Ayse.

According to the whole analysis the three components of an interactional developmental niche in Ayse's familial context can be structured as follows:

<u>Component "content":</u>

Allocation x Content: In the chosen scene Ayse and her mother are confronted with a spatial play situation. The chosen play is structured in the mathematical domain of geometry and is based upon the game "Make 'n' Break" (Lawson & Lawson, 2008). The aim of the game is to rebuild the 2D representations from the cards as 3D figures properly with wooden blocks, which are unform in size and weight.

Situation x Content: The chosen play situation enables Ayse and her mother to negotiate interactively about the different ways of the building corpuses. A dyadic interaction process between Ayse and her mother emerges as they thematize intensively different geometrical features of block building activity. The mother's *geometrical framings* enable Ayse to be exposed to experience of building corpuses vertically and horizontally. In this sense, during the block building activity, Ayse and her mother occasionally negotiate in *exploratory* and *disputational ways*, which proceed in the interaction process to a collective argumentation process and lead to the working consensus between Ayse and her mother in the course of both p-turns.

Contribution x Content: Ayse builds a corpus identical to the figure on the card. Through external regulation in both turns, which is served by the mother, Ayse explores and examines directly the stability of the building corpuses. Moreover in the course of building activity she finds that the horizontally built corpus is sturdier and has less risk of falling than the vertical one. By letting her build a horizontal corpus, the mother externally regulates Ayse's spatial abilities as she represents 3D transformations, regulates their relations, links them with each other and comes to conclusion in a short amount of time. Thus Ayse performs spatial relations between 2D and 3D objects and relates some parts with the whole that she can correspondingly change the position of horizontal set blocks and set them vertically in the building corpus. In this regard her *metacognitive awareness is regulated externally* by her mother in that she can compose and decompose 3D structures in horizontal and vertical planes and build them from pictured models.

<u>Component "cooperation":</u>

Allocation x Cooperation: In the play situation Ayse and her mother are game partners. Ayse's mother is the adult person and obviously directs the play situation. Moreover she is Ayse's *official* conversation partner, who allocates the right to take the next p-turn.

Situation x Cooperation: In this dyadic interaction process Ayse and her mother are collaborative game partners. They perform block building activities *collaboratively*. In Ayse's p-turn they discuss building alternatives, reasoning a hypothesis and reaching an agreement with each other which they mostly negotiate *in an exploratory way*, whereas in the mother's turn they negotiate *disputationally* as they express their disagreements and individual decision-makings, assertions and counter-assertations in the block building activity. In her p-turn Ayse takes the idea of her mother and expresses it with her hands as she undertakes the role of *spokesman.* In further building steps the mother takes responsibility and has the original ideas, which are performed by Ayse. In this sense Ayse mostly takes the role of *relayer* in the chosen sequence. Therefore the negotiation processes in the chosen scene generates for Ayse restricted leeways of participation, while she takes only the roles of *author, spokesman, relayer* (see section 2.2.1.) in the course of disputational and exploratory negotiation processes. Her mother comes up with the geometrical *framing* by pointing out the position of the blocks and how they should be set. The mother's *geometrical framings* lead to the emergence of some scaffolding functions, which enable Ayse to be exposed to the experience of building corpuses vertically and horizontally. In this sense the mother takes the role of tutor as an *expert*, while ascribing the roles of *novice* and *tutee* to Ayse.

Contribution x Cooperation: Ayse *collaborates* with her mother in the course of whole block building activities in the play situation. In both turns Ayse apparently cares for her mother's *elaborative descriptions, demonstrations, verbal stimulations and instructions* that *geometrical framings* of her mother enable a learning situation for Ayse. The mother uses instant *directivity*, through which she somehow *externally regulates* Ayse's geometrical arguments so that she experience building corpuses horizontally and vertically. Through *external regulation* from her mother, Ayse actively explores, experiences and performs spatial features in the building activity, which enable her to take different participation profiles that change instantly. Besides this, her mother as an external agent regulates Ayse's actions and further building steps in the block building process. By accepting the geometrical framings of her mother and performing both vertical and horizontal building activities through her mother's *elaborations* and *directives*, Ayse adopts an externally regulated participation profile in the whole interaction process. In this respect Ayse takes the role of *externally regulated* participant, while ascribing the role of *external regulator* to her mother. Thus the interaction process enables Ayse to explore and perform vertical and horizontal ways of building in 3D space.

<u>Component "Pedagogy and Education":</u>

Allocation x Pedagogy and Education: Block building provides a view of children's initial abilities to compose 3D objects. In the chosen game, four goals are pursued: spatial structuring, operating shapes and figures, static balance between blocks, and identifying the faces of 3D shapes with 2D shapes. These competencies reflect an initial development of thinking at the level of relating parts and wholes. The chosen scene refers to exploration and examination of spatial structuring, visualizing, relations and kinaesthetic imagery.

Situation x Pedagogy and Education: In the chosen play situation the mother takes the role of *tutor* as an *expert*, whereas she ascribes the role of *tutee* as a *novice* to Ayse. She strikes a balance between playing with Ayse and at the same time *collaborating* with her. She responds consistently to ongoing building activities and expands on the block building activities in the vertical and horizontal planes. In the negotiation process, she uses *verbal stimulations,* and *elaborative, instructive* and *conceptually rich directive* and *direct commands* that can regulate Ayse's building activities externally. Regarding the six scaffolding functions, the mother exposes Ayse to three of them, namely demonstration, marking critical features and direction maintenance, whereas she does not draw on the other scaffolding functions of "recruitment, frustration control, reduction in degree of freedom" (see Wood et al.,1976):

- **Demonstration:** The mother models the *idealized* position of the blocks to requirements of the building activity so that *they may be imitated by* Ayse in the course of further block building. By changing the position of the blocks she uses *maternal planning* and models the way of building the corpus showing how the coming blocks should be set <03-06,17-20,108-110>. She performs a new *idealized subject* in order to let Ayse *imitate* it back in a more appropriate form.

- **Marking critical features:** The mother obviously emphasizes the geometrical features and different aspects of the building activity that are important or relevant for its completion. The mother accentuates vertical and horizontal and interprets features and aspects of the building activity (see lines <03-06, 08-10,17-20, 23-25, 29-31, 33-34, 41-42, 108-110>) in terms of assisting and monitoring.

- **Direction maintenance:** The mother ensures that in the block building activity Ayse is directed towards achieving particular outcomes that contribute to completion of building vertical and horizontal corpuses. She tries to keep Ayse *in pursuit of a particular objective* so that she can *directly* maintain the building activity and become involved only in the current turn in the play situation. Moreover through her utterances she emphasizes the next building action as a goal to keep Ayse motivated *in the field* (see lines <27, 44-46, 49-50, 54-56,106>).

In this sense the mother fulfils three scaffolding functions. Moreover she and Ayse engage collaboratively and communicatively in the block building activity and are observing, following decisions, and adjusting themselves to accomplish both turns. Regarding family system theories, the roles in the mother-child interaction are altered slightly in that the mother *takes on more responsibility for the social exchange*, and depending on Ayse's *capacity* or *developmental level* the mother *provides feedback of an instructional nature* during the negotiation process with her daughter. Moreover she applies the principle of *contingent shifting*, and *emotional* and *motivational* regulation, while her *directivity* is not consistent with the *minimal-sufficiency* principle of family system theory. In terms of *contingent shifting*, she calibrates the building activity and varies the quality and amount of assistance according to the changing level of Ayse's independent functioning. In terms of emotional and motivational *regulation* the mother perceives and responds in a highly auxiliary manner to Ayse's motivational and emotional signs. By means of the *minimal-sufficiency* principle, she collaborates with Ayse while she uses direct cueing in order to let Ayse perform different geometrical features. The mother uses more directivity and minimal-effort strategies in the negotiation process as she regulates Ayse's building activities *externally*. Thereby the directiveness of the mother's utterances initiates positive cognitive developmental outcomes in Ayse.

Contribution x Pedagogy and Education: In both turns Ayse has learning opportunities for building the right corpus in horizontal and vertical planes. Through the external regulation of her mother in the negotiation process, Ayse participates in the play situation actively so that she can learn to build the blocks vertically and horizontally. In this sense, a collective argumentation process with her mother enables Ayse to reconstruct *geometrical meanings,* that building corpuses can be accomplished not only vertically but also horizontally. The mother's direction gives Ayse less opportunity to build corpuses freely in different ways as Ayse might wish, but through the mother's usage of three scaffolding functions (demonstration, marking critical features, and direction maintenance) she directs and maintains elaborations whereby Ayse's development is facilitated and this feeds into her constructivist and cognitive activities (Bibok et al., 2009). Ayse becomes able to see, interrogate, realize, examine and perform the block building activities during the interaction processes in both p-turns. The *external regulation* of the mother is conducive to "positive cognitive outcomes" in Ayse (Bibok et al., 2009, p.21). On a metacognitive level (Bruner 1996), by providing explicit directions on how to build the corpus, the mother emphasizes crucial actions, guides at key points and indicates alternatives as she leads Ayse to "internalisation of schemes, concepts and reasoning that are the subject of intra-psychic regulations" (Nader-Grosbois et al., 2008; Deci et al., 1996; Vermut, 1998; Boekaerts, 1999). Whereas Ayse "uses unsystematic trial and error at simple addition of pieces" in the beginning of her p-turn (in first p-turn), she becomes able to "produce corners and arches" with vertical and horizontal "internal spaces" at the end of her p-turn (Clements & Sarama, 2015). Thereon in her mother's p-turn (in second p-turn), Ayse argues about building corpuses either vertically or horizontally, which can enhance Ayse's spatial skills like spatial relation, visualization and kinaesthetic imagery (see section 2.1.1.).

Moreover through reaching, grasping, balancing, stacking and moving blocks she gets an opportunity to learn hand–eye coordination and the sense of balance and symmetry. The negotiation process with her mother thus inherently enables Ayse's temporal and representational cognitive development (Bibok et al., 2009).

Interactional niche in the development of Ayse's geometrical thinking and learning occurs very intensively. Due to these three components, the interactional developmental niche in the Kil familial context is structured as follows (Table 4.4):

Table 4.4. NMT-Family Kil in the game "Building 02"

NMT-Family Kil **Building 02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.	Playing with mother	Development of spatial skills and transformational abilities in spatial thinking and learning
aspect of situation	Negotiation between mother and Ayse both exploratory and disputational Working consensus Geometrical framings	Restricted leeways of participation for Ayse Tutor-Tutee	Three scaffolding functions by mother Enabling to perform different spatial features
aspect of Contribution	Exploring stability of the building corpus Representing 3D transformations Regulating and linking spatial relations in a short amount of time	External regulated participant as novice The mother: Expert as external regulator	Building vertically and horizontally identical corpuses to the figures on the chosen cards Learning spatial features

Functioning of MLSS and reflection on NMT_1 and NMT_2 for Ayse

In the first observation period a sequence of a play situation of Ayse and her father is chosen and observed, and in the second observation period a sequence with Ayse and her mother. In both, dyadic interaction processes and intensive negotiation processes arise between Ayse and her parents about building the wooden-block corpuses.

In both play situations Ayse participates actively and takes the role of tutee, whereas the adult person acts as tutor. Moreover, Ayse fulfils different leeways of participation and takes all four profiles of *production design* in the course of her verbal and nonverbal exchanges with an adult person (see section 2.2.1.).

Regarding folk pschology and folk theory, one should also consider the fact that her parents' educational level might have a large influence on the negotiation of taken-as-shared meanings. They have formal education at a high level and the father is engineer and the mother is an experimental chemist in a laboratory. In this sense they are both exposed to the use of different mathematical domains in course of their daily working life. As Bruner (1996) indicated, people are sensitive to and eager to adopt the folkways they see around them, so that Ayse's parents can use highly geometrical features during block play with Ayse. Maybe this is the reson they get to grips with the play situations and adopt scaffolding functions in the negotiation process with Ayse. Their *geometrical framings* enable Ayse to experience different geometrical features. Whereas in the play situation with her father Ayse explores how to build vertical corpuses identical to figures on the chosen cards without needing any long or short blocks, in the play situation with her mother she explores how to build robust corpuses vertically and horizontally to match the figures on the cards. In the course of their interaction processes with Ayse, her parents perform a range of supportive activities: *prompt, prompt after error, affirmation, disaffirmation, provide solution, motivation, conclusion, instruction, modelling, re-representation* (see section 2.2.3.). Regarding family system theories, the mother provides Ayse with *emotional and motivational regulation*, whereas the father *motivates* Ayse in an emotional way and *concludes* the play situation in a positive way (see sections 2.2.2. and 2.2.3. ; see also Acar Bayraktar, 2014a). By her father's emotional regulation, Ayse seems to be encouraged to *adjust the material and local conditions of her learning environment by mobilizing her social environment through communicative solicitations.* She becomes self-regulated through her *mobilized* environment, whereas she is regulated externally in both emotional and motivational ways in the course of playing with her mother. Maybe the combination of supportive activities and all the scaffolding functions used by her father leads Ayse to adjust the conditions of the learning situation. In the game with her mother, regarding family system theories, the mother is seen to take more responsibility and regulate both emotional and motivational fields for Ayse while she is realizing only three scaffolding functions. In this sense the characteristics of the adult-child interactions in the Kil family differ a somewhat. Whereas in the father-Ayse dyad of the first observation phase the support system is mainly generated in the vicinity of self-regulation, in the second observation phase the mother-Ayse dyad the MLSS is characterized by Ayse's external regulation in the course of *collaborative argumentation* with her mother. In this sense both the mother and the father seem to have direct influences on Ayse's *geometrical development*.

In this manner, a MLSS is constituted in the mathematical domain of geometry mainly by Ayse's parents as they render mindfulness of spatial features directly for Ayse. Thus, MLSS is accomplished interactively so that the interactional niche in the development of Ayse's geometrical learning emerges in the long run too.
Regarding the functioning of the MLSS, the following questions arise and should be answered in detail:

- Which kinds of *format* provide a learning situation for Ayse in this familial system? and
- How do these *formats* provide a learning situation in the first and second observation phases?

In accordance with the family system theory, in the Kil family two adaptive and complementary systems can be seen. Through the whole of the first play situation, the father seems to lead Ayse to explore the block building activity "beyond the proximity of the caregiver", whereas the mother drives "proximity to purpose of protection and comfort" for Ayse in that she mostly uses elicitations and directives during the block building activity (Tamis-LeMonda, 2004). The comparison of mother-child and father-child interactions in the Kil family suggests that the father plays a particularly important role in the development of Ayse's "openness to the world" by encourging her to build the figure on the chosen card without needing any short or long block (Paquette, 2004). The father latently encourages Ayse to take risks, while at the same time enabling her to learn to be braver in unfamiliar situations, as well as to stand up for herself. The "activation function" is evoked (Tamis-LeMonda, 2004) by her father as Ayse involves herself in novel experiences in the block building activity, through which an exploratory negotiation process emerges.

With regard to family system theories, the father and mother adopt a multitude of roles within the system of the Kil family. Whereas the father seems to be a moral guide and an activator, the mother looks like an expert, a companion, and a care provider. The father typically engages in didactic behaviours that support Ayse's cognitive development. He realizes all six scaffoling functions as he maintains both verbal and nonverbal behaviours which offer Ayse choices (questions, suggestions or comments) directly relevant to block building activities (Bibok et al., 2009). Thereby he calibrates the building activity, gives indirect cues and varies the quality and amount of assistance according to the changing level of Ayse's independent functioning, while at the same time minimizing his assistance and regulating Ayse's emotional and motivational behaviours. By realizing all six scaffolding functions he activates Ayse's cognitive behaviours and seems to be an activator, while – as a moral guide – encouraging Ayse to build the corpus.

The mother offers some scaffolding functions too but not all of the types. She realizes only three functions, which together generate an externally regulated process for Ayse. As a companion, she perceives and responds to Ayse in a highly auxiliary manner, while as an expert she maximizes her assistance and directs Ayse's performance instead of letting her be "pulled" (Salonen et al., 2007, p.79). As regards family system theory, as a care provider Ayse's mother takes on more responsibility for exchanges in the negotiation process. She seems to aim at calming and comforting Ayse based on the mother-child attachment relationship (see section 2.2.2.). Depending on Ayse's reactions, the mother uses verbal stimulation, direct commands and feedback of an instructional nature, while as a game partner she collaborates with Ayse. In contrast, as a playmate the father activates Ayse's competitiveness. He treats Ayse as his game partner, encourages his daughter just to build the corpus and both of them accomplish their p-turns separately. In the second play situation, in contrast, Ayse and her mother build corpuses together as collaborative game partners, no matter whose p-turn it is. The mother realizes a soft and comforting atmosphere for Ayse, while she fulfils disputation and exploratory negotiation processes in the play situation, whereas the father strives for a balance between competition and cooperation in the play situation with Ayse.

Ayse undergoes both interdependence and independence during play with her parents. While she is playing with her mother, she realizes an interpersonal process, as she collaborates strongly with her mother in the course of participation in shared endeavours, but not independent processes. As an externally regulated participant she is regulated through her mother's directions during the negotiation of taken-as-shared meanings. While she is playing with her father, in contrast, she realizes both independence and interdependence. She experiences independence as a self-regulated participant whe adapts the local conditions of her learning environment and mobilizes her social environment through communicative solicitations, and at same time interdependence, as she has to obey play rules and build corpuses in regular p-turns during the play situation.

With respect to the functioning of the MLSSs of both observation phases, the overall interactional niche in the development of Ayse's geometrical learning (NMT-Family Kil) can be demonstrated in the following way (see Fig. 4.45.):

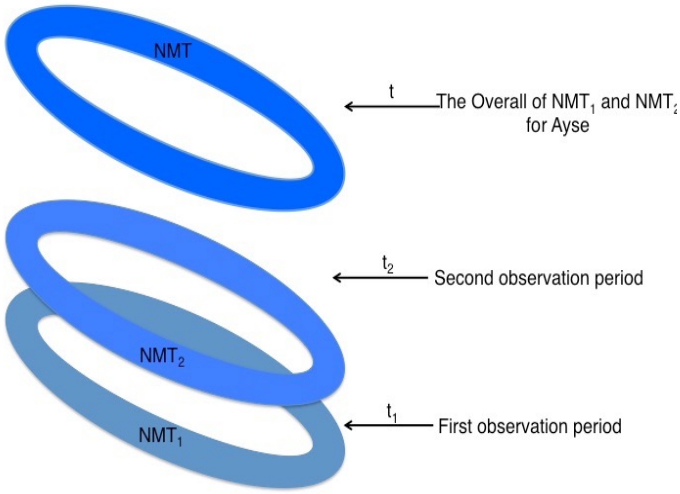


Fig. 4.45. NMT_1 and NMT_2 for Ayse ("NMT-Family Kil")

This diagram shows the relationship between NMT and the time axis, which provides evidence for the Ayse's further development. In the both obversation period Ayse experiences different geometrical features intensively. Therefore the interactional niche in the development of geometrical learning of Ayse in the both observation periods are labelled in blue. In the first observation period she learns block building in the vertical spaces, whereas in the second one learns block building in both vertical and horizontal spaces. Therefore blue colour of NMT1 differs from NTM2. The overall of NTMs of Ayse in the diagram is labelled in shiny blue in order to emphasize the intensity of the interactional niche in the development of geometrical and spatial thinking of Ayse.

To sum up, the directiveness and collaboration of Ayse's mother offer Ayse external regulation, whereas the encouragement and scaffolding of Ayse's father render her environmental self-regulation. Thereby her parents offer formats in which Ayse can explore and perform different spatial features through different regulation styles, while getting emotional reassurance from her parents. This MLSS leads to Ayse having a notably high level of development of spatial awareness.

Bearing in mind that all the cultural, lingual, social and emotional factors are embedded (see sections 2.2. and 1.4.1.), the detailed overall NTM can be presented as follows.

The overall NMT ($NMT_{1+2} = NMT_1 + NMT_2$) for Ayse

Component "content $_{1+2}$":

Allocation x Content: Both games "Building 01" and "Building 02" are allocatively located in the mathematical domain of geometry and based upon performing spatial skills. After choosing one card from the deck, each player should build a corpus related to the figure on the card. Thereby play situations facilitate each player to perform their spatial skills.

Situation x Content: In both chosen play situations the negotiation process between Ayse and the adult person (mother or father) thematizes mainly the spatial features of the built corpuses and in both a working consensus emerges between Ayse and her parent.

Contribution x Content: In both play situations, Ayse contributes actively to the negotiation processes and experiences different spatial features of the building activity. Thereby she explores and examines directly the feasibility, stability and *gestalt* of the building corpus in vertical and horizontal planes. Moreover she becomes able to interrogate the imperfection of the building corpus and try to fix it. She experiences and explores different building varieties with the same length blocks and the static balance to build a robust corpus. In this regard she is engaged in learning opportunities about spatial perspectives.

Component "cooperation$_{1+2}$":

Allocation x Cooperation: In both chosen scenes a child-adult dyad is actualized in that Ayse's "conversation partners" are her parents by means of *recipient* design (see section 2.2.1). In the first observation period Ayse plays only with her father, in the second her mother becomes her game partner.

Situation x Cooperation: In the dyadic interaction processes of both observation periods, each adult person enables Ayse to perform different regulation types by means of *participation* profiles (see section 2.2.1). In the first observation period, Ayse has an expanded leeway of participation. By dint of her father's scaffolding, she can undertake almost all *participation* profiles as *author, ghostee, spokesman, sponsor, relayer, deviser* (see 2.2.1.). In the second observation period, she has a more restricted leeway of participation. By dint of her mother's external regulation she takes on the roles only of *author, spokesman, relayer* (see 2.2.1.). In this sense different leeways of participation are generated. Furthermore by dint of the scaffolding functions used by both parents in the respective observation periods, the adult person takes the role of *tutor*, while Ayse undertakes the role of *tutee.*

Contribution x Cooperation: In both play situations the negotiation processes and building activities between Ayse and the adult person create learning situations for Ayse about spatial perspective. During the interaction processes she has opportunities to perform each block building activity. She participates actively in each play situation so that she frequently becomes engaged in ongoing building activities. In the first observation period she adapts the local conditions of her learning environment and mobilizes her social environment through such communicative solicitations. In the second observation period, in contrast, she performs all the directives given her mother so that she is regulated from an external source. Thereby she acts as an environmentally self-regulated participant in the first observation period and as an externally regulated participant in the second. Therefore different participation profiles are realized in the first and second observation periods. In both Ayse takes the role of tutee, while she ascribes the role of *activator* for her father and *expert* to her mother.

Component "Pedagogy and Education$_{1+2}$":

Allocation x Pedagogy and Education: In both play situations the following goals are pursued: spatial structuring, identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures, using kinaesthetic imagery and spatial visualization, realizing spatial operations. At the age of five, a child can *compose shapes with anticipation, understanding what 3D shape will be produced with a composition of two or more other (simple, familiar) 3D shapes*, whereas from the age of six a child can *make complex structures units by units* (see section 2.1.). Moreover, from the age of five, a child can produce arches with vertical and horizontal spaces. The US National Research Council reports that five-year-old children can understand and replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National Research Council, 2009, p.191). Block building activities enable children to practise their spatial skills. Moreover there can emerge for children an interactional niche for the development of their spatial thinking.

Situation x Pedagogy and Education: In both play situations Ayse directly interacts with the play materials. Through different scaffolding processes initiated by her father and mother, there emerge disputational and exploratory negotiation forms, in which they construct geometrically different argumentations about the block building activity. Whereas the mother enables Ayse to deal with building blocks in vertical and horizontal planes, the father enables Ayse to build corpuses with the blocks of the same length in the horizontal plane. Thereby they shed light mainly on spatial features of block building activity and enable Ayse to perform them. Moreover, by enabling her success in exercising her spatial abilities Ayse's parents initiate positive cognitive developmental outcomes for her.

Contribution x Pedagogy and Education: In both cases it seems that Ayse has sufficiently developed spatial abilities so that she can "see", "interrogate", "realize", "examine" and "perform" the block building activities during the interaction processes. Thus, Ayse vigorously and frequently performs building activities in which she appears as a relatively competent and increasingly autonomous participant. One can assume that this was possible for her because she was simultaneously engaged in a successful learning process about aspects of spatial geometry. During the negotiations of meanings in both cases she gets different learning opportunities from the block building activity. At a metacognitive level, the different regulation types provided to Ayse, enable her to develop her spatial abilities. Whereas at the beginning of both sequences Ayse can use multiple spatial relations to produce arches and corners unsystematically, but makes errors in the addition of pieces as a *picture maker*, due to the regulation types of her parents she becomes able to *compose structures from pictured* *models units by units and* produce arches and corners with vertical and horizontal spaces systematically (Table 2.4). In this sense different regulation types which lead her to different spatial learning outcomes, while she is getting emotional reassurance by her parents.

With respect to the abovementioned three components and their aspects the NMT table for Ayse can be structured in the following way (Table 4.5):

Table 4.5 The overall NMT1-Family and NMT2-Family of Ayse

NMT-Family Kil **Building 01,02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.	Playing with parents	Development of spatial skills and transformational abilities in spatial thinking and learning
aspect of situation	Exploratory and disputational negotiation process *Working consensus* *geometrical framings*	Different leeways of participation: Expanded and restricted Tutor-tutee	Different scaffolding processes by mother & father Enabling to perform different spatial features Enabling the success of spatial abilities Positive cognitive developmental outcomes in Ayse emotional reassurance by her parents.
aspect of contribution	Exploring stability & feasibility of the building corpuses without needing any long or short blocks Representing 3D-transformations in vertical and horizontal planes Regulating and linking spatial relations in a short amount of time	Different types of regulations Activator: environmentall y self-regulated Expert: external regulated	Building vertically and horizontally identical corpuses to the figures on the chosen cards without needing any short or long block Learning different spatial features

In summary, the overview of these three components of NMT-Family Kil leads us to the conclusion that Ayse is involved in the interactive accomplishment of NMT that obviously offers her successful support in the development of the geometrical learning. The situational aspect and the aspect of contribution in the NMT-Kil coalesce dynamically in that mathematical learning support system comes into being as a format in the grey labelled area in the table 4.5. The part of “Contribution x Pedagogy and Education” seems compatible with the mathematical learning situation, in which Ayse benefits from the learning opportunities and explores more than offered. Through her mother’s directiveness and guidance and her father’s encourgement and activation, spatial learning opportunities are provided, of which Ayse avails herself by taking on different regulation profiles. A MLSS is constituted so that Ayse can actively explore and learn different spatial features.

4.2. The Ak Family

The Ak family are GermanTurks who live in a major German city. Aleyna is our focus child. She is a single child and can speak German and rudimentary Turkish. Her parents grew up in Germany and can speak German and rudimentary Turkish. They speak with their daughter Aleyna mostly in German. The mother has 12 years of formal education. At both observation periods she did not go out to work and was the “major caregiver” (Barnard & Solchany, 2002) of Aleyna in the family. The father has 15 years of formal education in Germany. He chose not to declare his current employment. Aleyna’s parents did not want to give any further information about their careers nor about their private life, including their care-giving system in the family, for erStMaL-FaSt.

In the first observation phase Aleyna is aged four years and seven months. Her father could not attend that meeting, so Aleyna performs all the play situations with her mother (cf. Acar, 2011a; Acar Bayraktar; 2012b, in press-b; Acar Bayraktar & Krummheuer, 2011). In the game “Building 01”, Aleyna and her mother play four rounds in total. They do not play turn-by-turn but rather they build all the figures from the chosen cards together. In that respect Aleyna and her mother fulfil a collective working process in the block building action. However, Aleyna has a restricted leeway because of the lack of geometrical coordination between mother and daughter. Her mother turns up with *arithmetical framings* and encourages Aleyna to perform them, which leads Aleyna to take them as a reference. In this sense the *emotional regulation* of Aleyna's mother often comes to the fore, so that her mother lets Aleyna perform block building activities by means of her (mother's) *framings*. Maybe this is the reason they mostly build corpuses that are not identical with the figures on the chosen cards during the whole of the playing activities (cf. Acar 2011, Acar Bayraktar 2012b, in press-b; Acar Bayraktar & Krummheuer 2011, 2014). *Emotional regulation* from the mother also consists of many positive feedbacks given to Aleyna in the building activities. Thus, the negotiation process between Aleyna and her mother emerges "cumulatively" so that they engage in the negotiation process positively but uncritically on what Aleyna either says or does (Fernández et al., 2001, p.42). In that respect Aleyna stands at the *centre* of her mother's interest. Therefore, Aleyna seems to be a "central participant" during the whole interaction process in the game (Lave & Wenger, 1991, p.36). She is kept at the centre of her mother's concern with respect to each individual's "place" in it and their negotiation process is characterized by the mother's repetitions and confirmations (ibid.). Regarding the standard developmental phases of geometrical and educational issues (see sections 2.1. and 2.2.) it seems that mother has insufficient spatial-geometrical abilities for negotiating with Aleyna about geometrical *meanings.* She gives Aleyna imperfectly geometrical experiences in the building activities during the play situations (cf. Acar 2011, Acar Bayraktar 2012b, in press-b; Acar Bayraktar & Krummheuer 2011, 2014). Precisely Acar Bayraktar argues that Aleyna's mother cannot be assumed to be an adult person, who has sufficient spatial skills to be able to aid her daughter's spatial and geometrical development during the block building activities (2012b, p.45). Moreover, Acar Bayraktar and Krummheuer assert that supportive effects on the development of spatial skills in the interaction process between mother and daughter are so limited, that the MLSS in this familial context leads to an undesirable development at Aleyna's spatial skills (2011, 2014). Mostly Aleyna's mother overemphasizes block counting activities during the negotiation process about building corpuses so that the interactional niche in the development of Aleyna's arithmetical learning emerges perfectly (cf. Acar 2011, Acar Bayraktar 2012b,in press-b; Acar Bayraktar & Krummheuer 2011, 2014). Through negotiations between mother and daughter a collective problem-solving process is elaborated, in which arithmetical activities come to the fore. In that respect in the play situation MLSS refers to only the mathematical domain "numbers and operations" instead – as allocatively intended – of "spatial thinking" (Acar Bayraktar & Krummheuer, 2011, 2014). In the sense of the functioning of the MLSS, the mother supplies only counting activities for Aleyna so that an NMT (Interactional Niche in the development of Mathematical Learning) emerges only in the arithmetical sense. In Acar Bayraktar and Krummheuer (2014, p.6; 2011, p.168), the interactional niche for Aleyna is reflected in only two aspects but the deeper analysis here led us to the conclusion that Aleyna contributes actively to the negotiation processes with her mother and thus she characterizes a different aspect, namely contribution, with respect to three components of the interactional niche.

Referring to the works of Acar Bayraktar and Krummheuer, Aleyna's NMT table is structured as follows (Table 4.6) with all their reflections (2014, p.6; 2011, p.168):

Table 4.6 NMT of Aleyna in the game "Building 01" (Acar Bayraktar & Krummheuer 2014, p.6; & 2011, p.168)

NMT-Family Ak **Building 01**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Mathematical domain: "Geometry and spatial thinking", using spatial skills in the building activity	Playing with mother	Theory of the development of spatial skills and spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.

aspect of situation	Negotiation with mother about the built corpus and the amount of blocks, Cumulative negotiation process between Aleyna and her mother	Restricted leeway of participation	Focus on block counting activities by mother Collective block building process in arithmetical activities, Infinitesimal spatial skills
aspect of Contribution	Particularly experiencing arithmetical features, imperfectly geometrical experiences	Central participant	Having strong arithmetical skills. Learning opportunities only in arithmetic

Bearing this NMT table in mind, I now present another scene of Aleyna from the second observation phase. In the second observation phase the meeting takes place in the playroom at the Goethe University and Aleyna attends the meeting with both her parents. She plays one game with her father, then with her mother. Afterwards they play one game together as a child-mother-father tetrad, which facilitates the emergence of *polyadic interaction* process.

In the second observation phase Aleyna is aged six years and one month. For the coming analysis, a scene from the game “Building 02” is chosen, which is materialized in the second observation phase. Aleyna’s game partner is only her father and her mother accompanies them behind the cameras by watching and making interpretations during their negotiations (see Fig. 4.46.). Therefore, the mother is spectator, who can see the chosen cards, building corpuses and whole building activities of Aleyna and her father in the course of the play situation. In this sense the mother seems to participate as an “over-hearer” in their play situation in that she does not take part in it directly but can be tolerated by Aleyna and her father (Krummheuer & Brandt, 2001).

In total, Ayse and her father play five rounds by turns in the following recording position (Fig. 4.46).

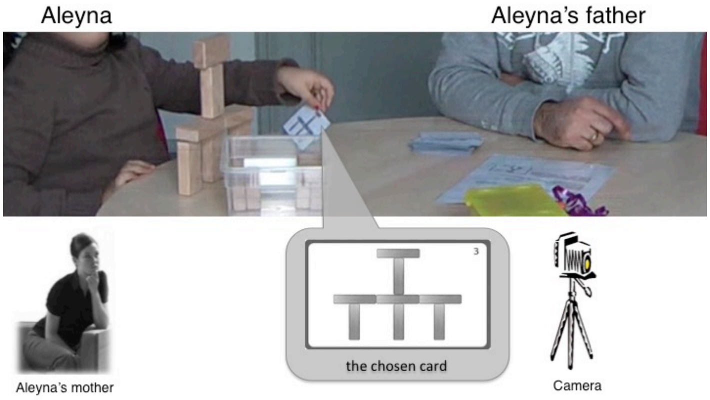
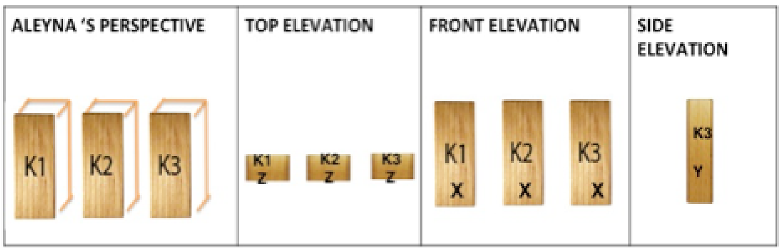
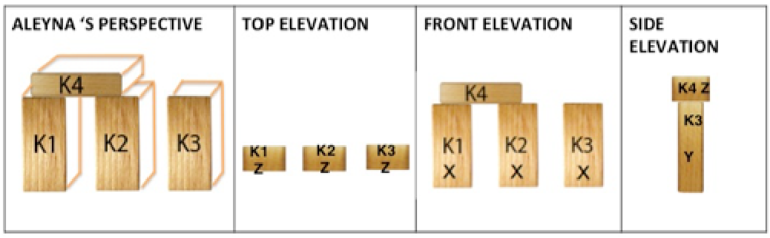
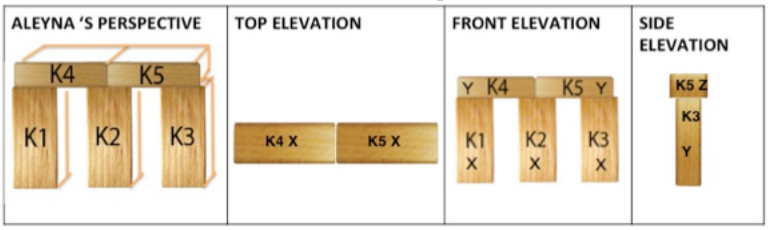
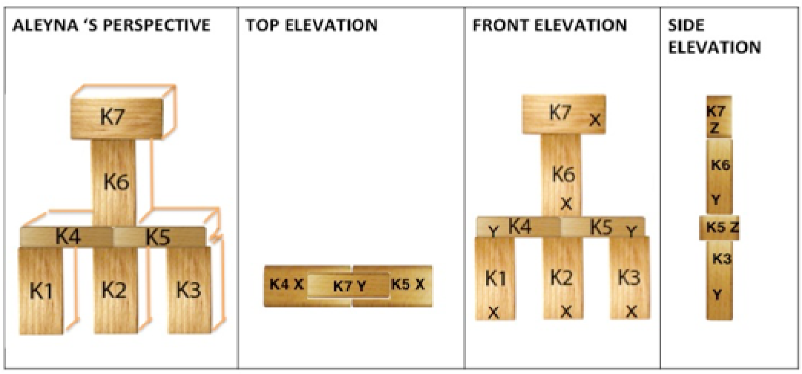


Fig. 4.46. Recording position of the game "Building 02"

The chosen and transcribed scene begins with Aleyna and is the first round. She chooses a card, which is shown in Fig. 4.46. She starts to build the figure up and sets the blocks in the upright position on the table, while her father reads the instruction manual of the game. During the interaction process they use German language and occasionally Turkish language by switching (see 1.4.1.1.). In the chosen transcript German speech is written in normal font and Turkish speech is underlined.

Transcript

1			Aleyna	*takes a block, K1, from the pile with her right hand and*
2				*puts it vertically on its Z side then takes another block,*
3				*K2, from the pile, puts it vertically on its Z side*
4				*next to K1 in a parallel direction. She takes another block,*
5				*K3, puts it vertically on its Z side and next to K2*
6				*in parallel* 
7				Should I broad? *(sic) looks at her father*
8			Father	*barely visible reaction, raises his eyebrows, turns his head*
9				*to the right and goes on looking at the game instructions*
10			Aleyna	*pushes the box a bit farther, takes a block from the box*

11				sets K4 horizontally on K1 K2 on their Z sides like that? 
12		#	Mother	yes! but straight, right?
13		#	Father	but.. *goes on looking at the game instructions*
14			Aleyna	looks at her father, K4 falls down
15			Father	but.. *nods* O.K. build. build it again.
16	01: 22		Aleyna	*pushes two blocks, K1 and K2, together; looks at her mother*
17				should I together?*separates two blocks from each other again*
18				*then puts K4 horizontally on K1 and K2 on its X side again.*
19			Mother	slow and concentrated..exactly. and bring it a bit closer.
20			Aleyna	*takes another block from the box*
21			Mother	yes. exactly.
22			Aleyna	*puts K5 on its Y side horizontally bonded to K4 on K2 and K3* 
23				Puh! *wipes her forehead with the back of her right hand,*
24	01. 48			*takes K6, sets it in upright position on K4 and K5 on Z side,*
25				*takes K7 sets it horizontally centred on K6 on Y side* 

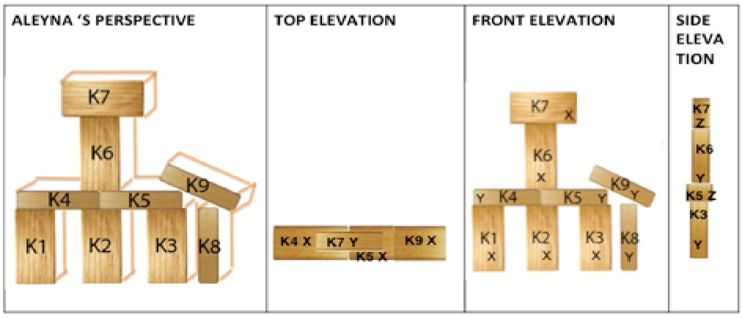
26				*looks at camera then looks at the card*
27			Father	is it correct? No.
28			Mother	just look at it accurately, Aleyna. there are two blocks, on
29				it- or? there comes one more block up on it.
30			Father	O.K. be quiet. don't interfere. *looks at mother*
31			Aleyna	*grimaces* noooooooo! it is correct.
32			Mother	just look
33			Father	*shows with his right index finger on the card* just look
34		#		there are three parts. one two three.
35		#	Mother	it is not true like that. honey?
36		<	Aleyna	I've- *opens her mouth,looks grimly,handles K8 with righthand*
37		<	Father	yes. you lose. *takes the card away*
38		>	Aleyna	*takes the card furiously with her left hand from her father*
39		>	Father	now it is daddy's turn-
40			Aleyna	nooo! *sets K8 on the Z Side near K3*
41			Father	but it can't be played like that
42			Aleyna	*puts K9 diagonally on K8 and K5* 
43			Father	no not like that.. not like that *smiles*
44			Aleyna	*lays K9 under* ıııııhhhhhh!
45		#	Father	O.K. now it is daddy's turn. *removes K6 and K7 and puts them*
46				*into the box*
47	02. 25	#	Mother	never mind next round.

Fig. 4.47. Transcription of first round.

Interaction analysis

The chosen and transcribed scene begins with Aleyna's building action. She takes a block, K1, from the pile with her right hand and puts it vertically on its Z side then takes another block, K2, from the pile, puts it vertically on its Z side next to K1 in a parallel direction. Then she takes another block, K3, and puts it vertically on its Z side and next to K2 in parallel. In this sense, Aleyna successively takes three blocks from the box and places each one vertically on its Z side parallel with each other <1-6> (see Fig. 4.48.).

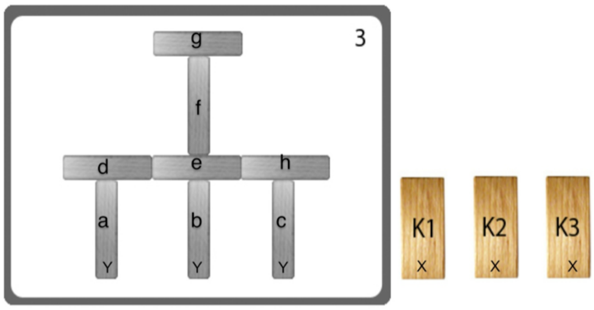


Fig. 4.48 The chosen card and the front elevation of basic part of corpus

Aleyna situates basic three blocks widely (with the X side) instead of narrowly (with the Y side) (see Fig. 4.48 and Fig.4.49) with respect to the front elevation of the built corpus. Therefore it seems that the sides of the blocks on the built corpus (K1, K2, K3) are not identical with the sides of the blocks on the card (a,b,c) (see Fig. 4.48) and thereby the faces of blocks on the built corpus seems not to be congruent with the faces of blocks on the figure.

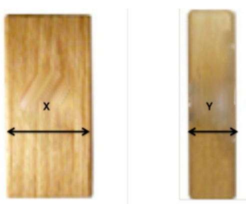


Fig. 4.49. The wide side (X) and narrow side (Y) of building blocks from the front elevation

From a participatory point of view, Aleyna seems to take the role of *author*. By setting the first three blocks parallel to each other on the table, she gives the impression of coming up with a totally new idea. From a developmental perspective, her reaction signalizes that she is a *piece assembler* by building vertical components within a building through limited range (see Table 2.4). In this sense she gives the impression of *not being able to distinguish* the *congruence by comparing all attributes and all spatial relationships* (see section 2.1.) so that she could *realize* or *see* the difference between wide and narrow side of blocks in the figure on the chosen card.

She asks, “Should I broad?” and looks at her father <7>. Her question might be a sign of trying to get feedback from her father, if she is building the corpus right. The question itself, “should I broad?”, can be interpreted as follows:

- “Broad” can refer to the distance between each block. The question “should I broad?” can be interpreted as asking if she has to make the distance a bit greater between the three blocks.

- “Broad” could mean the distance between Aleyna and the corpus. The question “should I broad?” can interpreted as asking whether she has to build the figure a bit further away from her.

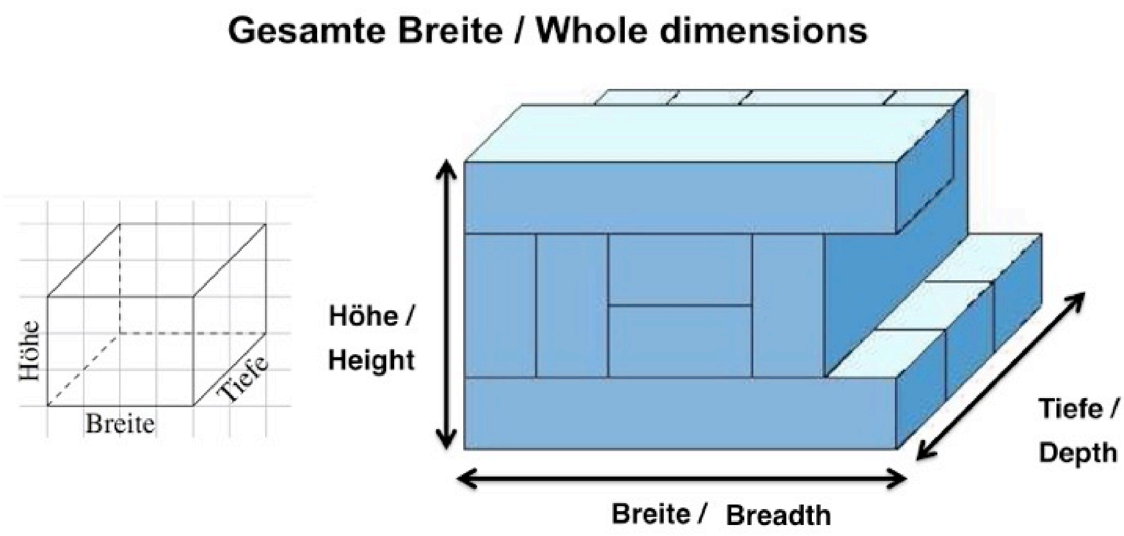


Fig. 4.50 The meaning of “*gesamte Breite*”

- The wording used by Aleyna has another meaning in German. She says originally, “Soll ich breit auf-?”. In this sense the word “breit” has two meanings in the German language:

- Occasionally this word can be used to express the whole dimensions of a body of structure. As “gesamte Breite” it refers to the three dimensions in German: “Höhe-Tiefe-Breite” (height-depth-breadth) (see Fig. 4.50). With the question “soll ich breit auf-?” Aleyna might try to ask either if she should think about the dimensions of the corpus or if she is using the correct faces of the blocks.
- As seen in Fig. 4.50 the translation of the word “breit” into English can also be “breadt”, which is used also adjectivally as a dimension. Considering this translation, her interrogative sentence can be interpreted as a question, “should I breadt?”. Taking into an account that each block has three dimensions, it may be understood that she might ask, how she should set each block by using the wide (X) side or the narrow (Y) side (see Fig. 4.50.).

By posing this type of request, Aleyna shows *environmental self-regulation* in that she tries to adapt either the *material* or *local conditions* in the play situation or to initiate referential joint attention in the play situation with her partner, i.e. the father (see 2.2.2.3.). Moreover, asking a question might signalize that what she tries to do is to get some feedback from her father. From a participatory point of view, Aleyna seems to takes the role of *author* (see 2.2.2.2.) by posing this question, whereas she predicates the father’s role as *tutor* in the play situation in that she seems to strive to discuss the alternatives and reach an agreement with her father.

Aleyna’s father raises his eyebrows and then turns his head to the right and goes on looking at the game instructions <8-9>. His reaction can denote that

- he is interested in looking at or reading the instruction manual, thus he does not really observe what Aleyna builds, or
- he does not have enough spatial competence to give any other feedback to his daughter. Therefore, he might dally with the instruction manual.

It seems that the father reacts directly to Aleyna's question but does not provide the required response to it. He seems to leave Aleyna's leeway open so that she can think, explore and examine her building activities by herself. From a participatory perspective, he might take the role of *tutor,* while ascribing the role of *tutee* to Aleyna. On the other hand, by not providing a response to Aleyna's question he might reject the *tutoring* role which is ascribed to him by Aleyna.

Aleyna pushes the box a bit further, takes a block from the box and sets it, K4, horizontally on K1 and K2 on the Y side <10-11> (see Fig. 4.51). In comparison with Aleyna's first building activity, here it seems that she is now able to represent block sides at the detailed level. From a developmental perspective Aleyna seems to act as a *picture maker* (see Table 2.4) in that she can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus somehow errs in adding pieces. She should *have the ability to balance blocks intuitively* by her age (see Table 2.4) and thus she might have pay attention to the static balance of the corpus in lines <1-6>. Maybe therefore she prefers to set the three basic blocks of the corpus a bit wider apart, to mitigate the hazard of the blocks falling. Moreover, she sets the block vertically on on its Y side, which might stand sturdy on the Z sides of K1 and K2. She can keep the static balance of the corpus by placing block K4 to the *off centre* of K1 and K2 in a sturdy way (see Fig. 4.51).

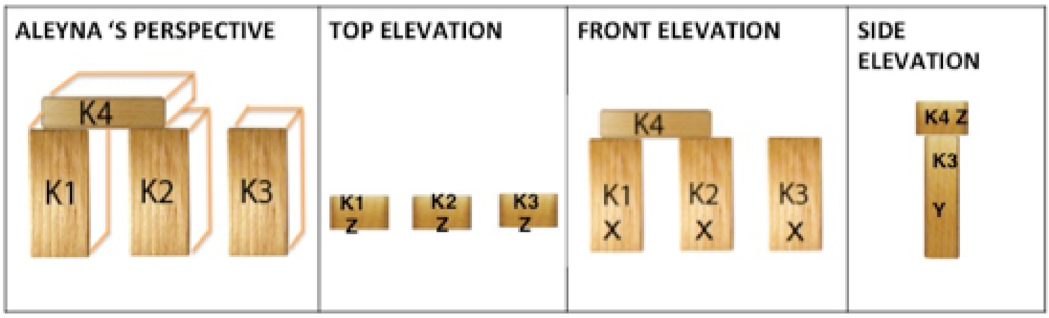


Fig. 4.51. The chosen card and the front elevation of basic part of corpus

In this respect Aleyna's action might refute the idea at lines <1-6> that she is not able to distinguish the *congruence by comparing all attributes and all spatial relationships* (see Table 2.4) so that she could *realize* or *see* the difference between the wide and narrow sides of the blocks in the figure on the chosen card. From a participatory point of view, it seems that Aleyna keeps her role of *author* in that she goes on setting blocks and constructing her own corpus. Maybe her father's reaction of not putting his ideas into words might work on Aleyna so that she can go on building a robust corpus in her own way.

Then she asks "like that?" <11>. By posing this question, Aleyna reinforces the interpretation at line <7> that she might be trying to get some feedback from her father and tags her father as a *tutor*. Thereby she attempts again to gain feedback from her father. While she goes on building, she might await somecommentary from her father, which she did not receive just before. Her question can be also a kind of repetition of her question in <7>, whether she should set the wide (X) or narrow (Y) side of the block or if she is setting the block with the correct face to the corpus.

Instantly, Aleyna's mother says "yes! but straight, right?" <12>. She says originally "ja! aber gerade, gell?". This expression has two meanings in German:

- The translation of the word "*gerade*" into English can be "straight". Considering this translation, her question can be interpreted as "yes! but straight, right?". By saying "yes" Aleyna's mother might demonstrate her agreement to Aleyna's action and then by uttering "but straight, right?" might give a clue that she should actually set the blocks straight. Thus the mother comes up with the geometrical argument that emphasizes the straightness of the set blocks. In this sense her reaction may be interpreted in a way that she *frames* the situation as a geometric one.
- The translation of the word "*gerade*" into English can also be "just". From this point of view the mother's reaction can be translated as follows: "yes just go on, right?". Considering this translation, it might be interpreted that Aleyna's mother might mention that Aleyna should go on building the corpus in the right way as she has done before.

With regard to the game rules, Aleyna's only game partner is her father and her mother can only accompany them from behind the cameras. Nevertheless, the mother seems to deny being a *spectator* in the interaction routines and appears to be trying to take on the role of tutor instead of the father. Moreover she seems to render a scaffolding process for Aleyna by pointing out the relevant characteristics of the set blocks in the building corpus. In this sense she seems to realize one of the scaffolding functions, namely *marking critical features*. From a supportive perspective her reaction can be seen as the supportive activity of *affirmation* (see section 2.2.3.) in that she demonstrates her agreement to Aleyna's building activities and gives the feedback anyhow, which Aleyna has been trying to get from her father for a little while (see lines <7,11>). Moreover, she might try to fascinate Aleyna and enable her an expanded leeway so that she can think over building the corpus right. From the participatory point of view, the mother gives the impression of being a *tutor,* reserving the role of *tutee* for Aleyna.

At the same time Aleyna's father reacts too. He says "but.." Unfortunately, he does not complete the sentence but goes on looking at the game instructions <13>. The word "but" might signalize his disagreement with either

- the mother's affirmation, or
- his inconvenience because of Aleyna's action, or
- the role stealing of Aleyna's mother.

By going on looking at the game instructions, his action reinforces the interpretation in lines <8-9> that he is currently interested only in the instruction manual. Regarding family systems theory (see section 2.2.2.), it does not seem to be surprising that we identify breakdowns between Aleyna and her father during the negotiation process in the play situation. Keeping in mind the father's reaction in both lines <13> and <8-9> and family system theories, his reaction appears quite different from the reaction of Aleyna's mother.

Aleyna looks at her father, as K4 falls down <14>. Her reaction might strengthen the interpretation that she tries to get some feedback from her father. Furthermore she might try to get the attention of her father by letting block K4 fall down. By letting the block fall, which she had placed to the off centre of K1 and

K2 in a sturdy way at lines <10-11>, she might try to caution her father, that he should deal with his daughter instead of looking at the instruction manual. In terms of *environmental self-regulation*, her reaction reinforces the idea in line <7> that she adapts local conditions in the play situation and *mobilizes her social environment through communicative solicitations.* In this sense her utterance might show that she asks her father for help or some feedback by posing a question or letting a block fall down. From a developmental perspective, her action signalizes at lines <10-11> that she can examine and perform the activity of setting blocks so that the structure is either robust, or insecure, or fallen. From a participatory point of view, she seems to ascribe the role of *tutor* to her father. By using *referential nonverbal acts*, Aleyna might try to negotiate *with her father about the block building activity,* through which they can engage in the play situation constructively and cooperatively.

The father nods and continues “O.K. build. build it again.”<15>. Probably he passes over to the falling of K4 and offers Aleyna to fix the demolished corpus. Another possibility is that he shuts his eyes to the fallen block and encourages Aleyna up to build the corpus from the beginning. In both ways it seems that he gives Aleyna an opportunity to build the corpus again, in order either to examine how she builds the corpus or to let Aleyna perform her own corpus by being held under the auspices of the father. Additionally the falling of K4 might give him a reason or occasion to say that she may build the corpus again. In this manner he might get the chance again to observe how she builds the corpus. Another probability is that the built corpus might be obviously *wrong* and the father might try to give Aleyna an opportunity to build the figure again by uttering somehow as a kind of warning. From a participatory point of view, it seems that the father tries to get a grip on the play situation and at the same time to take the role of tutor, while ascribing the role of *tutee* for Aleyna. Moreover, through his reaction he provides Aleyna an expanded leeway whereby she can explore, perform and examine further building possibilities in detail. From a supportive perspective, the father’s reaction could be an *instruction*, by which he suggests the use of some specific strategy. In this regard he seems to suggest Aleyna should either fix the demolished corpus or rebuild the corpus from the beginning, which looks like a supportive activity. In terms of *scaffolding,* he might try to put forward a *recruitment* by saying “O.K. build. build it again.”. Namely he gives the impression of enlisting the adherence work in the current play and let Aleyna undertake to build the corpus correctly. Maybe he tries to *ensure that she does not give up*, in terms of the scaffolding. In that respect he gives the impression of negotiating with Aleyna *in an exploratory way* by sharing relevant information (“the card looks like that”) with her.

Aleyna pushes two blocks (K1 and K2) together, looks at her mother and asks, “should I together?”<16-17>. It seems that Aleyna intends to rebuild corpus from the beginning by pushing two blocks as though changing the places of the two blocks. By posing a question to her mother Aleyna might try to get some clue from her. In this respect it seems that either she gives up seeking feedback from her father or seeks to be tutored by her mother instead as in line <12>. Maybe therefore she denies her father’s role as *tutor* and tries to reserve it for her mother. By posing such a question, her reaction shows *environmental self-regulation* in that she tries either to adapt *material* or *local conditions* in the play situation or initiate referential joint attention in the play situation with the partner, i.e., the father (see section 2.2.3.).

Then she moves two blocks a bit further from each other and puts K4 horizontally on K1 and K2 on its X side again <17-18>. She seems to struggle to fix the building corpus and setting the fallen block again. However, she sets the fallen block back in its previous place, which reinforces the idea that she pays attention to the static balance of the corpus in lines <1-6, 10-11>. From a developmental perspective, she acts as a *picture maker* (see Table 2.4) in that she can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus somehow errs at adding pieces. Aleyna should *have the ability to balance blocks intuitively* by her age (see Table 2.4). From a participatory point of view, Aleyna seems to take the role of *relayer* as she sets block K4 as she did in lines <10-11>.

Her mother makes a reply: “slow and concentrated..exactly. <u>and bring it a bit closer.”</u> <19>. The mother might be trying either to give a clue or to motivate Aleyna by saying “slow”, “concentrated” and “exactly”. The first two utterances, “slow” and “concentrated”, together might refer to *emotional regulation* for Aleyna so that she is able to build the figure *right* by being slow and concentrated. With the word “exactly” the mother might also try to *approve* and *motivate* Aleyna more than the father. From a supportive perspective, she uses two different supportive activities together; while she is motivating Aleyna *emotionally*, she approves her at the same time. Thereby her reaction can be interpreted as two types of supportive activities together: *motivation* and *affirmation* (see section 2.2.3). Bearing in mind family system theory, it is accepted that children become attached to their mothers through consistently *sensitive care* and the mother-child relationship aims to *calm and comfort rather than arouse* (see section 2.2.2.). In this regard Aleyna’s mother gives the impression of making tolerable demands on her daughter and *relying on low-level strateg*y in her geometrical *framing*. Maybe she focuses on meeting Aleyna’s *emotional needs* and therefore she does not criticize her further but tries to let Aleyna adjust the set position of the fourth block (K4) by herself rather than saying that Aleyna should change the position of the block. Considering the first observation phase as well, the reaction of Aleyna’s mother does not seem surprising, as she tries to give Aleyna positive feedback and thereby to *regulate* her emotionally again. Moreover Aleyna’s effort to adapt *local conditions* and to initiate referential joint attention in the play situation, seems to work on her mother in that she tries to *regulate* Aleyna’s emotional need (see section 2.2.3.). In this regard her reaction confirms the idea that Aleyna *mobilizes her social environment through communicative solicitations.*

Straight afterwards, she says to Aleyna in Turkish that she should bring “it a bit closer” <19>, which could mean a kind of direction to let her build the corpus correctly. Most probably she gives Aleyna a clue to as where or how she should set the block/s. The pronoun “it” might refer to an object which should be moved from its current place and brought “a bit closer” to another object. In that sense the mother might accept one stable object and one dynamic one. In this respect her speech can be interpreted as either that Aleyna should bring

- the block, either K1 or K2 or K3, a bit closer to another block/s, either K1 or K2 or K3, or
- the part of corpus (K1, K2 and K4 together) a bit closer to block K3, or
- block K3 a bit closer to the part of corpus (K1, K2 and K4 together)
- the corpus a bit closer to her.

From a supportive perspective to say "bring it a bit closer" can be seen as a supportive activity, namely *instruction*, by which she suggests some specific strategy for how Aleyna should move and situate the block or blocks in the corpus. From a developmental perspective she might try to evoke Aleyna's spatial skills and thus to encourage Aleyna to build the corpus right by saying what she should do in the spatial sense. Maybe therefore Aleyna's mother utters in Turkish language, which contingently works on Aleyna. Maybe she tries to emphasize the right way of setting the block/blocks to keep Ayse *in the field*, whereas she accentuates features of the fixing activity. Her reaction can be seen as a type of scaffolding function, namely *direction maintenance*, in that she might try to keep Aleyna in pursuit of a particular objective during block building. From a supportive perspective, her action is *providing a solution*. Probably Aleyna's action somehow indicates an *error* and thus the mother seems to try to produce the right position of the block or blocks without giving any further detail about her *error.* Taking the mother's utterances all together into account, she is calling it to Aleyna's attention, in order to aid her to avoid another collapse of the corpus. Maybe through her reaction, the mother gives the impression of somehow providing Aleyna with *comforting tutoring,* in which a sense of *psychological well-being* of the child as well as a sense of *scaffolding* are intended and restored. From a participatory point of view, she gives the impression of acting as a *comforting tutor* and in some measure as a *nurturer* (see section 2.2.2.) who provides *emotional support, creates safety*, is available to others, and can be a *mediator*, as well as a *tutor*, who is an *expert* and provides scaffolding to the child. In this regard her reaction confirms the idea that Aleyna can *mobilize her social environment through communicative solicitations.*

Aleyna takes another block from the box <20>. It seems that she attempts to go on building the corpus, by taking the next block. Moreover, her reaction indicates that her mother's *nurturing or comforting tutoring* works on Aleyna so that she can go on with the building action. Her reaction also signalizes that by her mother's *direction maintenance in line<19>* Aleyna is kept in pursuit of a particular objective during block building so that she goes on with the building action.

The mother reacts "yes. exactly."<21>. She thus gives the impression of approving Aleyna's action. From a supportive perspective her reaction can be seen as a type of supportive activity, namely *affirmation*, in that she demonstrates her agreement with Aleyna's building action. In terms of scaffolding, she might also try to keep Aleyna *in the field* so that she should continue building the corpus. In this manner she might try to keep Aleyna *in pursuit of a particular objective* so that she can proceed *directly* with the building activity. Therefore, the mother employs a *scaffolding* function, namely *direction maintenance.* From a participatory point of view, she gives the impression of acting as a *comforting tutor*, and in some measure as a *nurturer* (see sections 2.2.2. and 2.2.3.)*,* who provides *emotional support* and *creates safety.* Furthermore, she is taking the role of *tutor* instead of the father, as in lines <12,19>.

Regarding the previous lines up to now, the negotiation process between Aleyna and her parents seems to be generated cumulatively, characterized by repetitions, confirmations and elaborations. They fulfil the negotiation process positively but uncritically on what the others do. Moreover a working consensus seems to emerge about the mother's *tutoring* in the play situation, although she is not the "official" game partner of Aleyna.

Aleyna puts K5 horizontally bonded to K4 on K2 and K3 on its Y side <22>. Then she voices "Puh!" and wipes her forehead with the back of her right hand <23>. Aleyna's reaction can be interpreted as an achievement of a tough task. The *tough task* can be understood as Aleyna either building the corpus or keeping the static balance of blocks with each other. She sets the block, K5, in a similar way as she set K4, so that she performs static balance of blocks in the corpus. From a developmental perspective, Aleyna can act as a *picture maker* (see Table 2.4) in that she can use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus somehow errs at adding pieces. Moreover she reinforces the idea in the previous lines that she pays attention to the static balance of the corpus. From a participatory point of view, Aleyna keeps her role of *author* by setting K5 in the same way as she did in the previous lines. Her reaction reinforces the idea that her mother's *tutoring* or *nurturing* works on Aleyna in that she keeps on with her building activities.

Thereupon Aleyna takes K6, sets it in an upright position on K4 and K5 on its Z side, and then takes K7, and sets it horizontally centred on K6 on Y side <24-25> (see Fig. 4.2.5.).

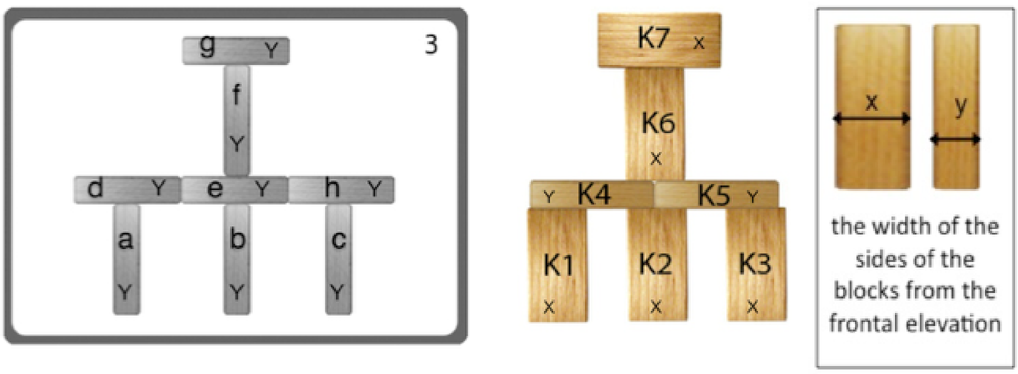


Fig. 4.52. The chosen card, the front elevation of the built corpus from Aleyna's perspective, and wide /narrow sides of blocks

The faces of the blocks in the corpus do not match the faces of the blocks on the figure depicted on the card (Fig. 4.52). While only two blocks (K4, K5) (cf. d, e, h) are set in the right horizontal way, the other blocks (K1, K2, K3, K6, K7) (cf. a, b, c, f, g) are set in a different way (see Fig. 4.52). Although the blocks on the card are seen only with the narrow sides (Y), Aleyna's corpus is built differently with the blocks, of which wide sides (X) and narrow sides (Y) are seen in the corpus. In the middle of Aleyna's corpus (see Fig. 4.52) there are only two blocks instead of three (cf. K4, K5, d, e, h). Thus, the built corpus is not identical with the figure.

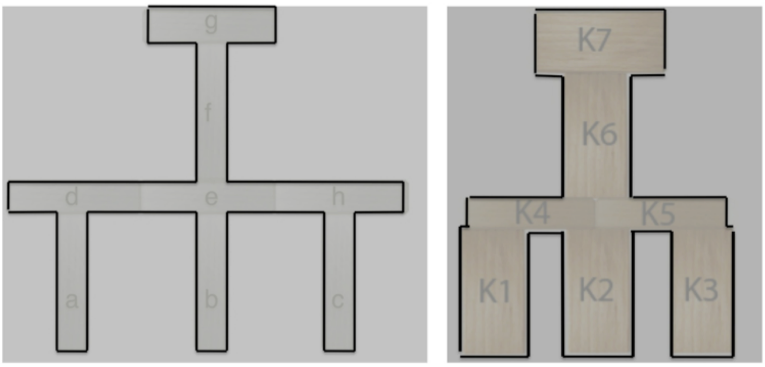


Fig. 4.53. *Gestalt* of the chosen card and the corpus built by Aleyna

From a developmental perspective, Aleyna's building action reinforces the idea that she is a *picture maker* (see Table 2.4). She seems to be able to match shapes using *gestalt configuration* but unable to *compose and decompose complex towers* in great detail (see section 2.1.). In this regard the difference between the chosen card and the built corpus can be interpreted as follows:

- Aleyna might pay attention only to the static balance between the blocks. Instead of situating each block on one block, she might prefer to situate K4 on two blocks (K1 and K2), and K5 on two blocks (K2 and K3) (see Fig. 4.53), on which both blocks can be in static balance and might not be at risk of falling. By using the wide sides of the blocks (K1, K2, K3) she makes sure that K4 and K5 take up more space on them and assures the stability of a robust corpus. Similarly Aleyna sets K6 by using its wide side, which also takes up more space on K4 and K5. Following that she places K7 again in such a way that it can take more space on K6, which might assure Aleyna about its stability. In this regard she might adapt her idea to the *gestalt* of the corpus and pay attention only to the *gestalt* of the figure on the chosen card. Maybe therefore she composes and decomposes the figure in her mind according to the static balances of the blocks and thus the *gestalt* of the corpus roughly resembles the figure and the corpus is robust enough (see Fig. 4.53.).

- Focusing on the stability of the built corpus, Aleyna might err by comparing faces of blocks so that she might not able to match narrow and wide shapes. Another possibility is that she might overlook the narrow and wide sides of the blocks. In this respect Aleyna's action reinforces the idea at lines <1-6> that she is not able to distinguish the *congruence by comparing all attributes and spatial relationships* (see section 2.1.) so that she cannot *realize* or *see* the difference between wide and narrow sides of blocks in the figure on the chosen card.

In sum it seems that the built corpus is not identical to but roughly resembles the 3D figure that is represented as a 2D projection on the card.

Aleyna looks at the camera, then at the chosen card <26>. Her reaction might show that she has finished building the corpus and thus might give an eye to the chosen card one last time to examine the resemblance between the built corpus and the figure on the card.

Her father asks Aleyna whether she has built the corpus correctly <27>. By posing this question he might try to *prompt* Aleyna to work independently and to be able to justify herself on her own. From a supportive perspective his reaction refers a kind of supportive activity, namely *prompt after error*, in that he might try to prompt Aleyna after her *error* about setting blocks with different faces. He seems to render a possible performance for Aleyna to rethink the position of the blocks and to open a discussion about the difference between the figure on the chosen card and the built corpus. In this regard he might try to interest Aleyna and offer her an expanded leeway so that she can examine the built corpus and correct it. In terms of scaffolding, he might try to keep Aleyna *in the field* so that she will focus on the built corpus again. In this manner he might also try to keep Ayse *in pursuit of a particular objective* so that she can *directly* maintain the building activity. Through his reaction, the father employs a *scaffolding* function, namely *direction maintenance.* He seems to take the role of *tutor*, while ascribing the role of *tutee* to Ayse.

Straight afterwards, the father answers his own question by saying “no” <27>. He gives the impression of not giving Aleyna any opportunity to answer the question or think about an answer. From a supportive perspective, his reaction indicates a supportive activity, namely *disaffirmation*, which is a type of correction that indicates a definitive negative response: “no”. By answering his own question he might try to give a definitive response that the built corpus is wrong. Furthermore, he does not express any other reason why the corpus is wrong. It seems that he obviously deprives Aleyna of becoming informed about “the right corpus” or the kind of mistake she made. Thus, the father seems to express his disagreement and individual decision-making and thus seems to negotiate with Aleyna *disputationally*. In terms of scaffolding, he declares obviously the wrongness of built corpus, which is important and relevant for its completion. In this regard he seems to realize a kind of scaffolding function, namely *marking critical features.* From a participatory point of view, he deprives Aleyna of turning up and expressing her own idea. In this respect it seems that the leeway of participation for Aleyna is restricted. Further, from a participatory point of view, he seems to keep on taking the role of *tutor*, while ascribing the role of *tutee* to Aleyna. Moreover, the mother’s emotional regulation and cumulative tutoring to Aleyna might evoke the father’s tutoring functions to Aleyna in that he comes up with it, although he didn’t really engage in the play situation up till now <27>. For that matter, by posing a question and by answering his own question, he might try latently to highlight the point that he is the only game partner of Aleyna and not her mother.

Following this, Aleyna’s mother says: “just look at it accurately, Aleyna. there are two blocks, on it- or? There comes one more block up on it.” <28-29>. Through her first sentence “just look at it accurately, Aleyna.” the mother might mean that Aleyna should look at the built corpus carefully. Most probably she tries to call Aleyna’s attention to the difference between the corpus and the figure on the card. By saying “just look at it accurately, Aleyna” <28> she seems to *motivate*

Aleyna in a positive way by encouraging her to examine the built corpus. From a supportive perspective her reaction can be seen as a supportive activity, namely *motivation*. From another point of view her reaction might refer another kind of supportive activity, namely *prompt after error*, in that she tries to prompt Aleyna after she has built a corpus not identical to the figure.

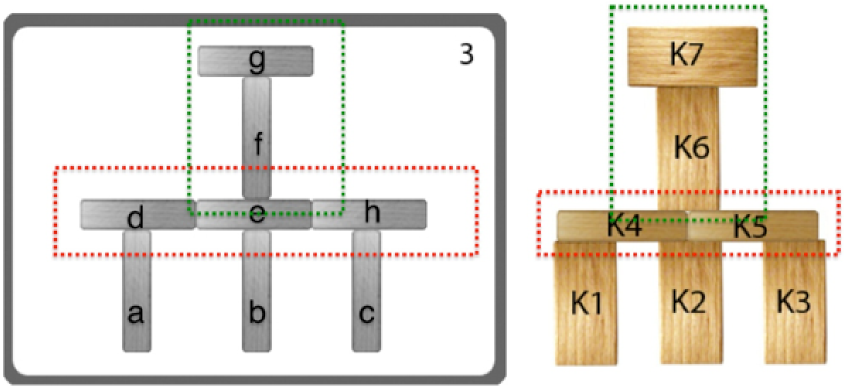


Fig. 4.54. The chosen card and the front elevation of the built corpus from Aleyna's perspective

By saying "there are two blocks, on it- or?" Aleyna's mother might try to give a clue about the wrongness of the built corpus. She marks how many blocks actually should be put either in the middle part of the built corpus (outlined with red) or in the top of the built corpus (outlined with green) (see Fig. 4.54.). From a developmental perspective, Aleyna's mother comes up with *arithmetical framing* (see section 2.2.) about the amount of blocks in a part of the corpus. Thereby she *frames* the situation as a numeric one. Thereupon she comes up with further numerical argument and keeps the arithmetical framing by saying that, "There comes one more block up on it." <29>. In this sense she might try to lay stress on the amount of blocks missing from the built corpus. However, she does not give a definitive target of the location, upon where one more block should be set. Most probably she talks about the middle part of the built corpus, in which there are only two blocks instead of three (see Fig. 4.54.).

The mother renders a possible performance for Aleyna to rethink her corpus and to open a discussion about the difference between the figure on the chosen card and the built corpus. From a participatory point of view she might try to fascinate Aleyna and enable her an expanded leeway so that she can examine and explore the way of building a right corpus. By *marking critical features* she accentuates numerical features of the built corpuses. Through her utterances, she seems to shift the geometrical issue of the play situation to the arithmetical one. Instead of arguing about the geometrical features of the built corpus, she suggests rather a numeric analytical approach by pointing out the amount of the horizontal blocks. The argument is made up of only arithmetical approaches, which touches on *folk psychology* and *folk pedagogy* (see section 2.2.). Bearing in mind Aleyna's first observation phase, it does not seem surprising that the mother comes up with a numerical approach instead of a spatial one. Most probably she tries to overemphasize block counting activities again. In the game "Building01" arithmetical activities came to the fore in the collective block building process of Aleyna and her mother, which touched on the mathematical domain of *numbers and operations* (see section 2.1.) and led the MLSS (see section 2.3.) to the same domain. In that respect the mother might try to *fulfil* her argument in a similar way. From a supportive perspective her reaction indicates a supportive activity, namely *instruction.* Thereby she might suggest to Aleyna some specific strategy, which refers to a numerical aspect and consists of arithmetical *framing.* In this sense the mother's assessment seems to implicate relatively weak spatial arguments out of which Aleyna's spatial competences might evolve. From a participatory point of view, the mother seems to take the role of tutor, whereas she ascribes the role of *tutee* to Ayse (as in lines <12,19,21>).

Straight afterwards, the father says in Turkish "<u>O.K. be quiet. don't interfere.</u>" while looking at the mother <30>. His reaction can be interpreted as a punctuation or warning for the mother to be quiet and not to interfere in Aleyna and her father's play situation. In terms of scaffolding, he might try to *control* any *frustration* from Aleyna and try to ensure that she does not give up. Namely he might predict the consequence of Aleyna's anticipated action as that she might give up, and thus he tries to use a direct elicitation that the mother should be quiet in order to let Aleyna to think about the built corpus. By his relatively harsh reaction, he also might indicate that he disagrees with his wife's *numerical framings*. He might mean that she should be quiet and not interfere in their play through her numerical framings. Furthermore, he might lay stress on the mother's taking the role of *tutor.* Maybe he tries to show his role in the current play situation more obviously as the official game partner of Aleyna. Regarding family systems theory (see sections 1.5. and 2.2.2.), his reaction might refer to keeping his *privacy and family system constant* and thus he speaks to his wife in Turkish, instead of in German. He might also try to keep for himself the mother's role, as the one who can provide emotional regulation to Aleyna. Thereby he might also try to let the mother take back her *reserved* role as *spectator*. Up till now the father seems a rule-bound person in the game, in that he is interested only in the instruction manual (see lines <13, 8-9>), and prefers to adhere to the game rules. In this play situation his game partner is only Aleyna and according to the rules of the father-child dyad instead of parents-child triad, he should perform this play situation. He might try to keep the mother away from their game and to reserve the role of spectator for her by means of adhering to the rules of the game.

Aleyna grimaces and says "noooooooo! it is correct." <31>. Unfortunately, it is not clear enough whether she reacts against either her mother's commentary or her father's. But as a consequence, she does not accept the criticisms and she says that the built corpus is correct.

Shortly after this, the mother says to Aleyna: "just look" <32>. The mother thus calls Aleyna's attention to the built corpus. She might wish to emphasize that Aleyna should look at the built corpus carefully again. Her reaction brings to mind the interpretation at line <28> that she might try to point out to Aleyna the difference between the figure on the card and the corpus. From a supportive perspective her reaction is *prompt after error*, which is a type of supportive activity. It seems that without mentioning any specific strategy the mother *prompts* Aleyna to look at the corpus and to think about it again. Thus, she might also try to show the father that she denies his warning or endeavour to render her passive. From a participatory point of view, it seems that she still tries to take the role of tutor, while she is ascribing the role of *tutee* to Ayse.

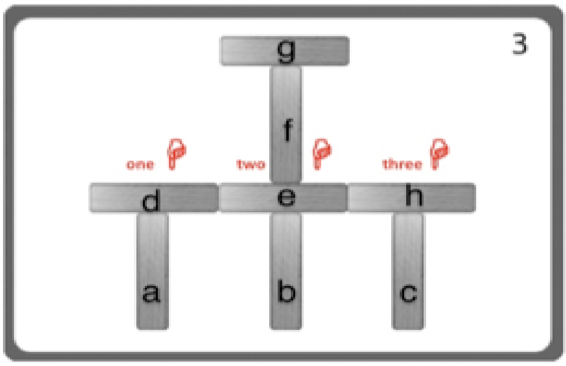


Fig.4.55. The father shows with his right index finger the blocks on the card by counting

Afterwards the father shows with his right index finger the blocks on the card while saying, “just look there are three parts. one two three.” <33-34> (see Fig.4.55.). While he was speaking, the mother says to Aleyna at the same time: “it is not true like that. honey?” <35>.

Most likely the father takes on the *framing* of his wife and tries to call Aleyna’s attention in a similar way as the mother. He counts the blocks d,e,h (see Fig.4.55.) by pointing with his index finger. Thereby he seems influenced by his wife as he tries to show Aleyna the different parts of the figure in comparison to the built corpus. His reaction might be interpreted as an attempt to show and to make it clear how many blocks have to stand in the middle of the corpus (see Fig.4.55.). Hence, he might begin with similar speech to the mother by counting and pointing out the amount of the blocks in order to go into particulars of the built corpus. Regarding family system theory, fathers tend to make more requests for information (e.g. reading the instructional manual in lines <8-9> and <13>), to give more exact and elaborative descriptions of the cards, and to use a greater proportion of verbalizations describing form, shape and direction relations than mothers (e.g. questioning and answering his own question in line <27>) in the course of the negotiation process. In this regard the father gives the impression of offering further details by one-to-one counting of blocks on the card, whereas the mother only gives broad information about the amount and position of the blocks (cf. lines <19, 28-29,32>. Regarding both aspects, Aleyna’s father seems to be elaborating the description and re-representing the problem to emphasize the missing block in the built corpus. From a developmental perspective, the father gives the impression of coming up with the numerical argument and *reframing* the argument by elaborating and stating more precisely the numerical feature of the built corpus. In this regard his reaction takes the mother's *framing* as a reference and suggests again an analytical approach by counting the horizontal blocks as his wife did in lines <28-29>. Thus, he re*frames* the situation through numeric analytical approach rather than spatial approach in the same way the mother did <28-29>. The argumentation process about the built corpus seems to be made up of only arithmetical and analytical approaches, which touches on folk psychology and folk pedagogy. From a participatory point of view, the father gives the impression of taking the role *spokesman*, while he ascribes the role of *sponsor* to his wife (see section 2.2.1.). From a supportive perspective, he seems to take the numerical *framing* of the mother and re-represent it. In this regard his reaction might refer to a supportive action, namely *re-representation.* In terms of scaffolding his reaction can be seen as a prediction for Aleyna that she can use it for correction to achieve the right built corpus. In this sense he is providing *elaborations* and pointing out the amount of the standing blocks in instructive and conceptually rich utterances. In this sense he employs one of the scaffolding functions, namely *marking critical features*. He remarks on the critical feature of the built corpus. From a participatory point of view, he gives the impression of offering Aleyna such a participation profile that she can muse on the built corpus, through which she can participate in an expanded leeway, and perform her ideas by taking the role of *author*.

Afterwards Aleyna says: "I've-" and opens her mouth <36>. But she doesn't complete the sentence. She looks grimly at her father and handles a new block, K8, with her right hand <36>. It seems that she picks up on the argument of her parents and maybe thus she doesn't complete her sentence. Regarding family system theory, it does not come as surprise that communicative breakdowns arise in the negotiation process. Another possibility is that she is denying her parents' argument about the corpus being not identical with the figure on the chosen card. She appears to grasp her parents' point of view and seem to fulfil a *disputational* negotiation process with her father in which they express their disagreement and individual decision-making and assertions about the set block.

At the same time, the father says to her instantly: “yes. you lose.” and takes the card away <37>. Obviously, the father interrupts the ongoing negotiating process and concludes that her p-turn is over. Another possibility might be that the father tries again (see line <30>) to play strictly by the rules and the run of p-turns. Regarding family system theory, the father might try to complete the game in the shortest amount of time and to keep the p-turns of the play situation as short as possible. He might have not wish to waste time with keeping on negotiating about the built corpus and thus might try to push Aleyna eventually to finish her turn. So indeed, he does not enter into any further discussion about the built corpus. Maybe for the father, the game is more important than Aleyna’s mathematics learning. From a supportive perspective, he realizes a support activity, namely *conclusion.* Through his utterance, “yes. you lose.”, and by taking the chosen card away, the father appears to *conclude* Aleyna’s turn and sum up that she has lost her turn. His reaction might show that he assumes that Aleyna’s turn is over. In this regard the negotiation process between Aleyna and her father appears to be generated *disputationally* in that they express their disagreement and individual decision-making and assertion. From the participatory point of view, he gives the impression of being a *tutor,* while reserving the role of *tutee* for Aleyna. By taking her card away, he gives the impression of depriving Aleyna of turning up and expressing her own idea. In this respect it seems that the leeway of participation for Aleyna is restricted. On the other hand, he might be trying to motivate Aleyna to feel an ambition to win the game. In terms of family system theory, the father’s reaction can be viewed as fostering Aleyna to encourage risk taking, to stand up for her beliefs and to display courage in the face of unfamiliar events. In this sense the father's reaction may refer to the effort to keep Aleyna motivated and *in the field*, in terms of scaffolding. Thus, he might try to *control* Aleyna’s *frustration*. From the participatory point of view, he takes the role of *tutor*, while ascribing the role of *tutee* to Aleyna. By reason of his directness, the father’s tutoring seems to be served a bit sharply. In this sense his reaction might restrict Aleyna's leeway of participation. From another point of view, his reaction can be seen as a type of emotional regulation so that she can turn up by persisting in opposition of her father's idea and try to expand her leeway by fixing the error of the built corpus. In this respect he might latently allow Aleyna to expand her leeway and turn up.

Aleyna takes the card furiously with her left hand from her father <38>, while he is saying "now it is daddy's turn-"<39>. By taking the card from his hand, Aleyna might be trying to show or emphasize that she does not accept his assessment and restriction. From a participatory point of view, she shows signs of denying her father's reaction <37>. At the same time, her father says that it is his turn. His reaction reinforces the idea (line<37>) that he *concludes* Aleyna's p-turn, sums up that Aleyna lost her p-turn and emphasizes that his p-turn now begins. From a supportive perspective he seems to realize a support activity, namely *conclusion.* From a participatory point of view, he tries to maintain his *tutoring* in order latently to allow her to open her own leeway and to turn up persistently. Thus, he might rather gently oblige Aleyna to rethink the built corpus and might enable an expanded leeway of participation for her.

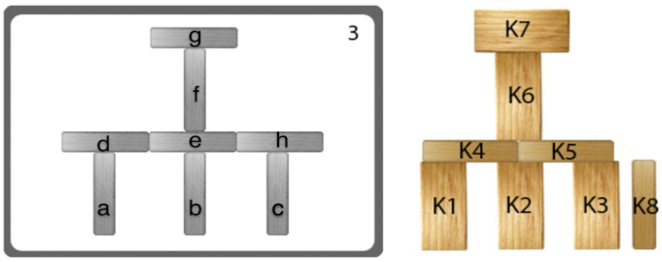


Fig.4.56. Aleyna re-builds the corpus (the front elevation from Aleyna's perspective)

Aleyna says "nooo!" and sets K8 on the Z side near K3 <40> (see Fig.4.56). Through her reaction Aleyna might be trying to go on showing her father that she denies his assessment and restriction. In this regard she seems not to accept her lose and she tries to win the game. Her ambition seems to be evoked by her father's *tutoring* in line <37>. Through her father's restriction of her own leeway, Aleyna might persist in opposition of her father's assertion that her p-turn is over. Maybe therefore she continues the building action and tries to show that she has not lost the turn. In this regard her reaction reinforces the idea in line <36> that she picks out her parents' arithmetical *framings* and she thus handles a new block to fix her own *fault*, that one block is missing in the middle part of the built corpus. In this context her mother's *framing* and her father's *reframing* seem to work on Aleyna, which causes her to muse on the rightness of the built corpus. She gives the impression of realizing an *environmental self-regulation* process, in which she *mobilizes her social environment* through *communicative solicitations* e.g. asking for help or a clue, searching for any approval or disapproval for her reactions or initiating referential joint attention on the block building process with her parents (see section 2.2.). She appears able to recognize her *fault* and to try to fix it by setting a new block, K8, next to K3 (see Fig.4.56). By setting another block in the corpus, Aleyna comes up with geometrical framing, so that she does not come up with any *arithmetical argument* but rather a *geometrical* one.

Regarding geometrical developmental theories, Aleyna acts as a *picture maker* as she cannot represent blocks at the detailed level of shapes but rather can produce structures unsystematically with an error (see Table 2.4).

However, unlike the chosen card, Aleyna put one block (K8) next to K3 in the rebuilt corpus, which actually does not exist in the original figure on the card (see Fig.4.57.). Although there have to be three blocks at the base of the figure on the card, which is marked with the yellow line (a, b, c) (see Fig.4.57.), there are four blocks (K1, K2, K3, K8) in the rebuilt corpus.

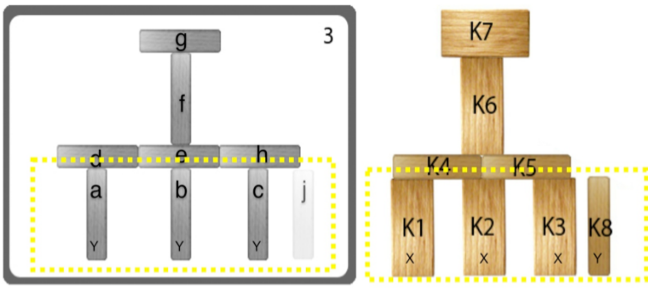


Fig.4.57. The comparison between the chosen card and the first step of the rebuilding (the front elevation from Aleyna's perspective)

The faces of the blocks in the built corpus mostly do not match the faces of the blocks on the card. Interestingly Aleyna sets this new block, K8, in such a way that the front elevation of the faces of the block is identical with the other basic blocks in the figure on the card (a,b,c), although it seems redundant (see block j) in the rebuilt corpus (see Fig. 4.57). From a participatory point of view, Aleyna acts like an *author* in that she expresses and performs her own original idea, by setting the fourth block at the base of the built corpus. Moreover her reaction reinforces the idea that she negotiates with her parents *disputationally* about the built corpus. In this sense she seems to express her disagreement and individual decision-making assertion about the set block.

The father says “but it can’t be played like that” <41>. Probably he means that she is not obeying the rules and is not allowed to play like that. Aleyna’s reaction shows that she puts in a claim for rebuilding the corpus, maybe therefore the father tries to emphasize that she oversteps the rules. In this regard his expression fortifies the preceding interpretation in line <37> that the father attaches great importance to the rules of the game and the run of p-turns. Moreover his reaction reinforces the idea that they negotiate disputationally about the built corpus so that they express their disagreement and individual decision-making. From a supportive perspective, his reaction can be seen as a supportive activity, namely *instruction*, by which he suggests a specific strategy in which way Aleyna should not proceed with her block building action. From a participatory point of view, his reaction brings to the mind the preceding interpretations <37,39> that he might try either to deprive Aleyna of turning up and expressing her own idea or to motivate her in such a way that she might feel an ambition to win the game. From this point of view, he gives the impression of being a *tutor,* while reserving the role of *tutee* for Aleyna. Moreover the father’s reaction may reflect his effort to keep Aleyna motivated and *in the field*, in the terms of scaffolding. Thus he might try to *control* Aleyna’s *frustration* so that through his emotional regulation Aleyna can turn up with persisting in opposition to her father’s idea and might try to expand her leeway by fixing the wrongly built coprus. In this respect he might latently let Aleyna expand her leeway and turn up.

Aleyna goes on to rebuild the corpus and puts K9 diagonally on K8 and K5 <42> (see Fig.4.58.). Her reaction might show that she does not care about either the rules or her father's objection. As in line <40>, she might try to express latently to her father that she denies his assessment and restriction. In this sense her reaction shows she does not accept her loss and is still trying to win her p-turn. Through the father's reaction, her ambition might be evoked and thus she might persist in carrying on building until she builds the corpus right. In this regard *the frustration control* of her father works on Aleyna in terms of scaffolding. Moreover she seems to *stand up for her own belief* and *to display courage* to deny her father's *assessment*, in terms of family system theory. Most probably she is deeply involved in the correction of her initial construction and thus she does not accept her loss and wants to accomplish her p-turn. Her parents' *scaffolding* seems to work on Aleyna in that she comes to realize the fault of the built corpus. Moreover, Aleyna's effort to fix the fault reinforces the idea in line <40> that she comes up with geometrical framing. From a participatory point of view, she is taking the *numerical framings* of her parents as a reference and trying to make the built corpus accord with these *framings* by fixing the third block up in the middle part of the corpus. In this regard she seems to take the role of *ghostee,* whereas she ascribes the role of *ghoster* to her parents. Moreover, her reaction reinforces the idea that she adapts *local conditions* of her learning environment and mobilizes her social environment through such communicative solicitations. Through her increasing autonomy, Aleyna performs her parents' regulation, and she gets learning opportunities for building the right corpus.

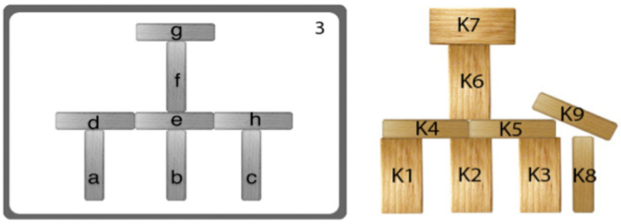


Fig.4.58. The chosen card with the named blocks and the front elevation of rebuilt corpus from Aleyna's perspective

Comparing the rebuilt corpus with the figure on the card (see Fig.4.58.), the rebuilt corpus bears slight resemblance to the figure on the chosen card: In the middle part of the figure on the chosen card there should be three horizontal blocks (d, e, h), which are marked with the red line (see Fig.4.58.), and similarly in the middle part of the rebuilt corpus there are three blocks (K4, K5, K9) too. Thereby it seems that Aleyna has placed the right amount of blocks in the middle part of the corpus. Furthermore, this signalizes that she adapts the *framing* of her mother and the *reframing* of her father. Maybe therefore she appears to be coming up with the numeric analytical approach, in that she *frames* the situation not only geometrically but also arithmetically like her parents. The rebuilt corpus bear partial resemblance to the figure on the chosen card. The middle part of the built corpus consists of three blocks (K4, K5, K9), which makes it resemble the figure on the chosen card. This context clearly shows that the arithmetical *framings* of her parents work on Aleyna so that the amount of the blocks in the middle part of the built corpus (K4,K5,K9) is identical to the number in the middle part of the figure on the card (d,e,h) (see Fig.4.59.).

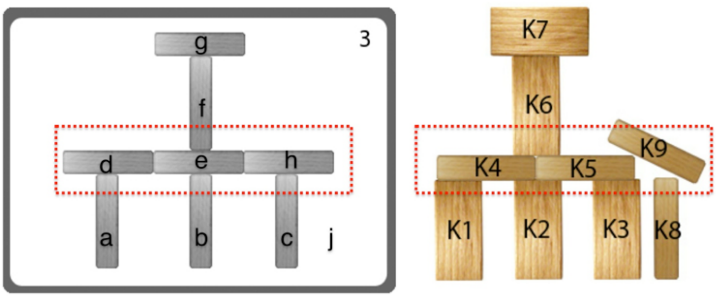


Fig.4.59. The second comparison between the chosen card and the front elevation of rebuilt corpus from Aleyna's perspective

Nevertheless, the third block (K9) is set up in a different way. To be specific, while block h stands horizontally on the basic part of the figure, block K9 stands slanted in the corpus, whereas all three blocks in the middle part of the built corpus should be set straight. Alayna's reaction reinforces the idea in lines <24-25>, that she might pay attention only to the static balance between the blocks. Most probably she tries to keep all the blocks in static balance and assure the stability of a robust corpus. In this regard she might try to keep block K9 steady and to situate it off centre by setting it obliquely on K5 and K8 in order to avoid the risk of it falling. Thus, the action of setting K9 on a slant evokes the idea at lines <10-11>, <17-18>, <22>, <24-25> that Aleyna pays attention to the static balance of the block. Moreover, grounding the fourth block (K8) under K9, gives the impression she is able to *recognize simple 2D shapes* and *compose* them in 3D spaces, in terms of developmental perspective (see Table 2.4.). In this regard Aleyna can add a new block to the built corpus and so rebuild the corpus. Moreover, her reaction signalizes that Aleyna has quite strong spatial abilities, through which she tries somehow to overcome her error.

Straight after this, her father reacts: "no not like that.. not like that" and then he smiles <43>. Through his utterances he might mean that

- Aleyna has not set up blocks K8 and K9 in the right way, or
- Rebuilding activity is not permitted in the game, thus she is not allowed to do something "like that", or
- Aleyna has rebuilt the corpus "not" in a similar way "like" it seems on the card.

From a supportive perspective, the father gives Aleyna a type of negative response, which includes explicit *no* and *not*, thus using a supportive activity, namely *disaffirmation.* Thereby he expresses his disagreement and individual decision-making which strengths the idea (see lines <27, 37, 39, 41>) that he negotiates with Aleyna *disputationally*. Moreover, he does not go any further details about the built corpus and thus he seems to try to deprive Aleyna of arguing about the rebuilt corpus. In terms of scaffolding his reaction can be seen as a remark for Aleyna that she cannot fix the fault of the corpus in this way and achieve a correctly built corpus. In this sense he is providing *elaborations* and pointing out one feature of the building action. He gives the impression of fulfilling one of the scaffolding functions, namely *marking critical features*. He remarks on one of the critical features of the built corpus that its wrongness cannot be fixed like that. So, he is *restricting* the *leeway* of Aleyna's *participation*, while he is taking over the role of tutor and ascribing the role of *tutee* to Aleyna. Moreover, his reaction reinforces the idea that Aleyna's enterprise can be regulated through her environment. By adapting the *local conditions* of her learning environment and mobilizing her social environment through such communicative solicitations, Aleyna provides for herself a learning situation in which her father informs her about *critical features* of the built corpus.

Aleyna takes block K9 back and says "ıııııhhhhhh!" <44>. This utterance is a typical type of negation in Turkish. By reacting like that, Aleyna might be insisting on either the rightness of the built corpus, or the success of her turn. Moreover, her reaction reinforces the idea that she is still striving to adapt *local conditions* to her learning environment and mobilizing her social environment through such communicative solicitations. Through her increasing autonomy, Aleyna performs her parents' regulation, and she gets learning opportunities for building the right corpus. In this sense she might try to deny her father's *disaffirmation*s.

Thereon her father says "O.K. now it is daddy's turn." by removing K6 and K7, while the mother says "Never mind- next round." <45-47>. The father repeats his *disaffirmation* in line <39> and indicates that it is time for his turn, while the mother is trying to regulate either Aleyna or the father emotionally. From a supportive perspective, the father's reaction seems to be a type of supportive activity, namely *conclusion,* whereby he concludes Aleyna's turn and takes his turn, while the mother employs *motivation*, in that Aleyna should not mind about her loss and she can win in the next round. Regarding family system theory, the father's reaction reinforces the idea in lines <39,43> that he is pushing the rules to let Aleyna's turn end, whereas the mother's reaction strengths the idea (see <12,19,21,28-29>) that she focuses on meeting Aleyna's emotional needs and aims to comfort her. From a participatory point of view, the father is taking the role of *tutor*, and ascribing the role of *tutee* to Aleyna, while the mother seems to take the role of *nurturer*. However, Aleyna's leeway of participation seems to be restricted so that she is deprived of turning up, discussing the built corpus further and completing her p-turn successfully. Moreover, a working interim between her parents and Aleyna about the wrongness of the built corpus and the expiration of Aleyna's turn emerges unambiguously.

Summing up from the perspective of the interactional niche in the second observation period

In the chosen sequence the father is the official game partner of Aleyna and her mother is a spectator, from an allocative scope. Situationally, the mother is embroiled with the negotiation process between father and daughter in the course of the game, although she is not an official game partner. In the chosen sequence Aleyna adapts *material* and *local conditions* in the play situation and initiates referential joint attention in the play situation with her game partner father. By posing questions, searching for approval or disapproval of her reactions she adjusts to her environment for a learning situation. Before the father, her mother comes forth, ensures Aleyna an *emotional regulation* and *tutors* Aleyna in a *cumulative* way at the beginning of the chosen sequence. Thereon her father gets a grip on the play situation, *tutors* Aleyna in a *disputational* way whereby he pushes the rules of the game. Her parents' arithmetical *framings, the* emotional regulation of her mother and the disputational tutoring of her father enable Aleyna to turn up and try to fix her fault. The negotiation processes with her father and mother render Aleyna to have occasionally different leeways of participation. Whereas Aleyna's mother mostly takes the role of *nurturer* in the play situation, her father acts as a *tutor* through his directness. The parental relationship and their family system cause Aleyna to have imperfect geometrical and mostly arithmetical experiences with the building activities during the play situation. Aleyna's parents overemphasize block counting activities during the negotiation process of building corpuses so that interactional niche in the development of arithmetical learning of Aleyna emerges perfectly.

Table 4.7. The roles taken in both turns in the chosen scene of the second observation

	Aleyna's turn
Aleyna	Environmental self-regulatory participant and tutee
Father	Tutor
Mother	Spectator and Nurturer

According to the whole analysis, the three components of an interactional developmental niche in Aleyna's familial context can be structured as follows:

<u>Component "content":</u>

Allocation x Content: In the chosen scene Aleyna and her parents are confronted with a mathematical and spatial play situation. The chosen game is structured in the mathematical domain of geometry and is based upon the game "Make 'n' Break" (Lawson & Lawson, 2008). The aim of play is to rebuild 2D representations of the cards as 3D figures properly with provided wooden blocks, which are of the same size and weight.

Situation x Content: The chosen play situation enables Aleyna and her parents to negotiate interactively about building the corpus correctly with the right amount of blocks. A polyadic interaction process between Aleyna and her parents emerges as her parents thematize intensively arithmetical features of block building activity. Whereas Aleyna comes up with *geometrical framings.* her parents' *arithmetical framings* enable to fix the fault of the built corpus. In this sense during the block building activity, Aleyna and her parents occasionally negotiate *cumulatively* and *disputationally*, which leads by the interaction process to a working interim between Aleyna and her parents about the wrongness of the built corpus and Aleyna's loss of her p-turn.

Contribution x Content: Aleyna experiences building a robust corpus and getting partly similar *gestalt* with the figure on the card. She performs the spatial relations between 2D and 3D objects and relates some parts with the whole. She sets the blocks in such a way that they can take up more space on each other and thus have more stability. Mostly she uses the different faces of the blocks and seems to pay attention to the static balance of the blocks in the building corpus. She builds a robust corpus, which is not completely identical with the figure on the chosen card. Through her geometrical and her parents' arithmetical framings she tries to fix the fault in the built corpus.

<u>Component "cooperation":</u>

Allocation x Cooperation: The chosen game is structured in the mathematical domain of geometry and enables each family member to set out her own spatial and furthermore mathematical skills. Thereby each family member enables our focus child to have some learning opportunities in different mathematical domains. In the chosen play situation, Aleyna and her father are supposed to be game partners and the mother is supposed to be a spectator.

Situation x Cooperation: In this polyadic interaction process Aleyna's father is her game partner, whereas the mother acts rather as a *nurturer* who participates directly in the negotiation process between Aleyna and her father. Thereby both father and mother take the role of tutor*,* while they are ascribing the role of *tutee* to Aleyna. In the interaction process different tutoring types of parents emerge which enables Aleyna occasionally different leeways of participation. Through her mother's *comforting tutoring,* the mother prevents emotional regulation so that Aleyna undertakes the role of *author*, whereas her leeway of participation becomes restricted by virtue of her father's *tutoring*. In this regard her mother seems to focus on meeting Aleyna's emotional needs. Maybe thus she does not criticize her daughter and the negotiation process with her daughter proceeds in a *cumulative* way. In this sense she takes the role of *nurturer,* who provides emotional support. Through her father's *tutoring*, Aleyna is encouraged in risk taking, standing up for her beliefs and displaying courage in the face of unfamiliar events. In the course of the disputational and cumulative negotiation process between Aleyna and her parents, Aleyna is exposed to exploring the fault of a built corpus. In terms of folk psychology and folk pedagogy, Aleyna's parents focus on arithmetical and numerical subjects in block play. In this sense they come up with *numerical framings,* which prevents Aleyna to taking the role of *ghostee*, while she ascribes the role of *ghoster* to her parents. Thereby Aleyna brings her geometrical and her parent's arithmetical framings together as she goes to the effort of fixing the third block up in the middle part of the corpus. In this regard her parents take the role of *tutors*, while they ascribe the role of *tutee* to Aleyna.

Contribution x Cooperation: Aleyna negotiates with her parents in the course of the whole block building activities. She builds a robust corpus, which is not identical with the figure on the chosen card. Apparently, she cares about her father's and mother's tutoring which constitutes *environmental self-regulation* for Aleyna. Thereby she becomes able to mobilize her social environment through communicative solicitations. She asks for help or a clue, searches for any approval or disapproval for her reactions, and initiates referential joint attention on the block building activity occasionally with her father and mother. In this regard she ascribes the role of tutor to her parents latently, while she procures herself self-regulatory participation in the whole interaction process so that she performs her own geometrical ideas by means her parent's framings. Through their framings Aleyna actively explores, experiences and performs arithmetical features of the building corpus and becomes able to explore the fault of the built corpus and fix it.

<u>**Component "Pedagogy and Education":**</u>

Allocation x Pedagogy and Education: Block building provides a view of children's initial abilities to compose 3D objects. In the chosen game, four goals are pursued: spatial structuring, operating shapes and figures, static balance between blocks, and identifying the faces of 3D shapes with 2D shapes. The US National Research Council reports that five-year-old children can understand and replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National

Research Council, 2009, p.191). The chosen scene refers to exploration and examination of spatial structuring, visualizing and kinaesthetic imagery.

Situation x Pedagogy and Education: In the chosen play situation the father and the mother strike a balance between *tutoring* and *nurturing*. While the father is playing with Aleyna, her mother tries to meet her emotional needs. The father does not respond contingently to Aleyna's ongoing building activity but the mother takes this role by virtue of tutoring Aleyna in a cumulative way. In that regard father and mother simultaneously and occasionally realize some of the scaffolding functions through which both verbal and nonverbal behaviours are maintained.

- **Recruitment:** The father involves Aleyna's interest in the play situation and adherence to the requirements of the play situation. By saying, "OK.build.build it again" <15> he weans Aleyna away from abandonment of her turn and ensures that she does not give up building the corpus.

- **Marking critical features:** At the beginning of the sequence, the mother comes up with quasi geometrical framings by saying "but straight, right?"<12> and <u>"bring it a bit closer"</u> <19>. Thereon **t**he mother and father obviously emphasize arithmetical features of the building activity that are important or relevant for its completion. They accentuate only the amount of the blocks in the building activity and by announcing the loss of her p-turn and declaring the wrongness of the built corpus her parents provide information about her p-turn (see lines <28-29,33-34,41,43, 45-46>).

- **Frustration control:** The mother manages and regulates Aleyna's emotional reactions to difficulties in the building activity in order to maintain her commitment to finishing and achieving her p-turn. In this sense she meets Aleyna's emotional needs and provides emotional regulation. In terms of family system theory, the mother aims for Aleyna's comfort, whereas her father encourages her to *stand up for her own belief and to display courage to deny father's assessment* (see lines <19,21,32,35,3745-47>).

- **Direction maintenance:** Her parents ensure that in the block building activity Aleyna is directed towards achieving particular outcomes that contribute to completion of building the right corpus. In this sense they try to keep Aleyna *in pursuit of a particular objective* so that she can *directly* maintain the building activity and become involved only in the current turn in the play situation (see lines <21,27>.

In this sense Aleyna's parents fulfil four scaffolding functions. Moreover, they prompt her by instructing her about the amount of blocks in the middle part of the built corpus and try to call her attention to the numerical perspective, which touches on folk psychology and folk pedagogy. Whereas the mother is the first participant by *framing* the situation, the father comes later, *framing* the situation by re-representing the mother's numerical framing. In this sense he re*frames* the situation through the numeric analytical approach rather than spatial approach in a same way as the mother did. The father gives the impression of taking the role of *spokesman*, while he ascribes the role of *sponsor* to his wife (see section 2.2.1.). The argumentation process about the built corpus seems to be made up of only arithmetical and analytical approaches, which leads Aleyna to think about the missing block in the middle of the corpus. Neither her mother nor her father exactly aids Aleyna to explore how the corpus actually should be built. In terms of folk psychology and folk pedagogy (see section 2.2.), Aleyna's parents pay attention to the arithmetical and numerical subjects of block building activity. Taking folk theories into consideration, one can speak about the common education system globally, whereby each individual, whether of high or low educational level, is exposed to learning "numbers and operations" in mathematics classrooms. Consequently, every individual in the community can or should know the numbers and be able to do basic addition and subtraction at least. In this sense, Aleyna's parents prefer to approach her in terms of analytical perspective and *frame* the situation in a numerical way. Moreover, they reiterate the numeric analytical skills and intensify the one-to-one counting. The negotiation process between the mother and Aleyna emerges in a cumulative way, which is generated positively but uncritically by repetitions, confirmations and elaborations. The negotiation process between the father and Aleyna arises in a disputational way, which is characterized by disagreements and individualized decision-making, short assertions and counter-assertions of Aleyna and her mother. The negotiation process between the parents and their daughter thus corresponds to the transfer of responsibility for managing the block building activity and self-regulated learning for Aleyna.

Contribution x Pedagogy and Education: In the chosen play situation Aleyna seems to accept her parents' different styles of *tutoring,* while she takes the role of *tutee*. She adapts *material* and *local conditions* in the play situation and initiates referential joint attention in the play situation with her parents. Moreover, she mobilizes her social environment through communicative solicitations. By posing questions, and seeking approval or disapproval for her actions, she adjusts her environment to a learning situation. Regarding geometrical developmental theories, Aleyna acts as *piece assembler* and *picture maker*, as she can build vertical components within a building though with a limited range, use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus somehow errs at adding pieces. Nevertheless, she can attend to the static balance of the corpus and thence come up with geometrical arguments. However, she is not able to distinguish the congruence by comparing all attributes and all spatial relationships so that she *realize* or *see* the difference between the wide and narrow sides of blocks in the figure on the chosen card. This point eludes the parents too. They intensify only one-to-one counting and summing up the amount of blocks by means of the mathematical domain of numbers and operations. Nevertheless, Aleyna experiences and explores different building varieties and the static balance to build the robust corpus. The arithmetical framings of her parents let Aleyna think about the fault of the built corpus which enables her in a very weak sense to see, interrogate, realize, examine and 'perform' the block building activities during the interaction processes. In this regard the arithmetical framings of her parents and the geometrical framings of Aleyna enable her to perform her own idea and to get learning opportunities in conjunction with the numeric analytical *framings* of her parents. Through her increasing autonomy for building the right corpus, slightly geometrical and highly arithmetical development are facilitated in terms of NMT.

With reference to these three components, the interactional developmental niche in the Ak family context is structured as follows:

Table 4.8. NMT-Family Ak in the game "Building 02"

NMT-Family Ak **Building 02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes	Playing with father, Mother as a spectator	Theory of the framing on the development of spatial visualization and spatial senses
aspect of situation	Negotiation between father, mother and Aleyna about the built corpus Cumulative and disputational negotiation process; Working interim	Different leeways of participation offered by Aleyna's parents Aleyna: tutee Father: tutor Mother: comforting tutor as a nurturer	Some scaffolding functions by parents Enabling to perform numerical features Folk Psychology and Folk Pedagogy Numerical-Arithmetical Framings of Parents
aspect of Contribution	Exploring static balance to build the robust corpus Examining the resembling *gestalt* of the figure and the corpus through framing of parents	Environmental self regulatory	Getting learning opportunities highly in arithmetic slightly in geometry Struggling for her rights Displaying her courage Examining the diagonal static balance

Functioning of MLSS and reflection on NMT_1 and NMT_2 for Aleyna:

In the first observation period Aleyna plays only with her mother, collaboratively, and dyadic interaction processes emerge, whereas in the second observation she plays with her father while her mother, as spectator, offers nurturing, so polyadic interaction processes emerge. In both play situations Aleyna engages in the negotiation process actively and takes the role of tutee, whereas the adult person acts as tutor. Moreover Aleyna fulfils different leeways of participation and takes some profiles of *production design* in the course of her verbal and nonverbal outputs with an adult person (see 2.2.2.). Her parents get to grips with the play situations and adopt some scaffolding functions in the negotiation process with Aleyna. Their *arithmetical framings* enable Aleyna to experience different *arithmetical* features in the block building situation, although allocatively intended negotiations about a spatial interpretation of the play situation, however, do not come to the fore. In the course of interaction processes with Aleyna, her parents perform similar supportive activities, although they approach the negotiation process in different ways. Whereas Aleyna and her mother perform the negotiation process *cumulatively*, with Aleyna and her father generate a *disputational* negotiation process. The mother negotiates with Aleyna positively but uncritically. Their negotiation process is characterized by *repetitions, confirmations and elaborations*. Moreover, the orientation of cumulative negotiations is towards *solidarity and participants achieve agreement without critiques or reasons being given.* In this sense Aleyna takes the role as central participant in that she is the centre of the interest of her mother during the whole interaction process in the first observation period. Thereby the mother *collaborates* with Aleyna while *nurturing* her during the whole play situation in the first observation, whereas she meets the *emotional needs* of Aleyna while pointing out amount of blocks needed in the middle part of the corpus in the second observation. The negotiation process between Aleyna and her father is characterized by *disagreements and individualized decision-making, and short assertions and counter-assertions.* The orientation of disputational negotiation is *more individualized and competitive.* Each participant aims to win and thus there are no attempts to construct joint understanding or to reason together.

Interestingly the father and mother both use similar supportive activities (*prompt, prompt after error, affirmation, disaffirmation, motivation, instruction, modelling, conclusion, provide solution*, and *re-representation)*, although they realize different types of negotiations of taken-as-shared meanings (see section 2.2.1.). By the mother's emotional regulation in both observation phases, Aleyna seems to be encouraged to *adjust the material and local conditions of her learning environment by mobilizing her social environment through communicative solicitations.* So she becomes self-regulated through her *mobilized* environment. Interestingly, her parents' realization of some scaffolding functions seems to lead Aleyna to adjust conditions for a geometrical learning situation so that she can explore how to build a robust corpus through arithmetical framings by her parents. Moreover, they enable Aleyna to reiterate the numeric analytical skills and intensify one-to-one counting and summing up situations.

Whereas in the first observation phase the support system is mainly generated in the vicinity of central participation of Aleyna, in the second observation phase the parents-Aleyna triad characterizes the MLSS through environmental self-regulation by Aleyna. In this sense both the mother and the father seem to have direct influences on Aleyna's arithmetical *development*, while having indirect influences on her *geometrical development.* In this manner, the MLSS is mainly constituted in the mathematical domain "numbers and operations" by Aleyna's parents, and they render directly the mindfulness of numerical features for Aleyna. Thus, MLSS is accomplished interactively so that the interactional niche in the development of *arithmetical* learning of Aleyna emerges in the long run, while her geometrical learning emerges in a very limited fashion.

Regarding the functioning of the MLSS, the following questions arise and will be answered in detail:

- Which kinds of *format* provide a learning situation for Aleyna in this familial system? and,

- How do these *formats* provide a learning situation in the first and second observation phases?

In accordance with family system theory, in the Ak family two adaptive and complementary systems can be seen: Through the whole play situation, the father seems to lead Aleyna to explore arithmetical features in the block building activity as "beyond the proximity of the caregiver", whereas the mother drives "proximity to the purpose of protection and comfort" for Aleyna in that she tries to regulate her emotionally and arithmetically during the block building activity (Tamis-LeMonda, 2004, p.220). Moreover her father seems to foster Aleyna to *encourage risk taking*, to *stand up for her beliefs* and to *display courage in the face of unfamiliar events*, in comparison with the *comforting tutoring,* through which the mother provides *emotional support, creates safety* and *meets emotional needs in balanced manner.* In this regard Aleyna's parents seem to interact with and care for her in distinctly different but "*complementary"* ways (Bornstein & Sawyer 2008, p.386). In accordance with family system theory, the relationship of Aleyna's parents seems to affect Aleyna's arithmetical development directly in the course of both play situations. The different communication styles of her mother and father with Aleyna enable her to engage in block building activities also to regulate her environment for her own geometrical development.

Bearing in mind that all these cultural, lingual, social, and emotional factors are embedded, different styles of parental *tutoring* constitute a format that provides for Aleyna a learning situation from an arithmetical and somewhat spatial perspective. Within this context the MLSS leads notably to Aleyna's high arithmetical development, but it does little for her spatial development.

With respect to the functioning of the MLSS in both observation phases, the overall Interactional Niche for Aleyna (NMT-Family Ak) can be presented in the following way (see Fig.4.60.):

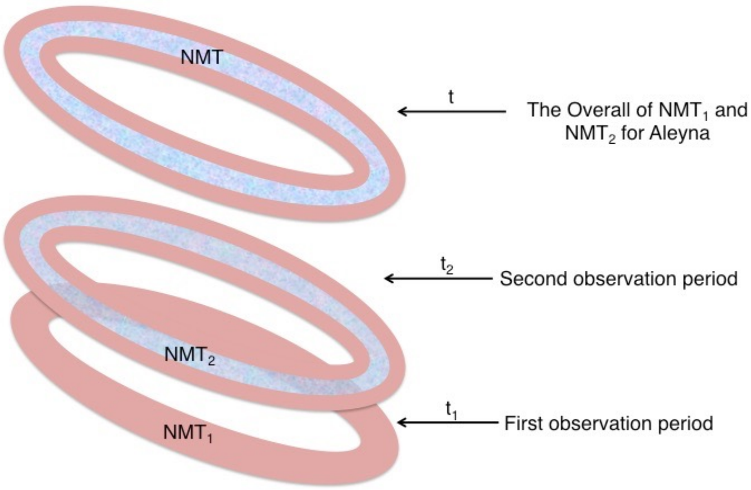


Fig.4.60. NMT_1 and NMT_2 for Aleyna ("NMT-Family Ak")

This diagram shows the relationship between NMT and the time axis, which provides evidence for the Aleyna's further development. In the first obversation period Aleyna experiences numeric analytical activities perfectly and geometrical activities imperfectly. In the second observation period *arithmetical* learning emerges in nonlimited fashion, whereas geometrical learning for Aleyna emerges in a very limited fashion. Thus, the interactional niche in the development of *arithmetical* learning of Aleyna is labelled in pink, while the interactional niche in the development of geometrical learning of Aleyna is labelled in light blue. In this regard the overall of NTMs of Aleyna in the diagram is labelled in the colours, which refer to the combination of colours of both NTMs.

The overall NMT (NMT_{1+2} = NMT_1 + NMT_2) for Aleyna

__Component "content $_{1+2}$":__

Allocation x Content: Both games, "Building 01 & 02", are allocatively located in the mathematical domain of geometry and based upon performing spatial skills. After choosing one card from the deck, each player should build a corpus related to the figure on the card. Thereby play situations facilitate each player to perform their spatial skills.

Situation x Content: Between Aleyna and the adult person (mother, father) mainly numeric analytical features of the built corpuses are thematized. Mother and father proceed negotiation process in virtue of their numerical *framings* as reiterating Aleyna's numeric analytical skills and intensifying one-to-one counting.

Contribution x Content: Both in the selected scene of "Building 01" and in her own turn in "Building 02" Aleyna contributes actively to the negotiation processes. In any event she experiences numeric analytical features of the built corpuses. Thereby she can also examine the amount of blocks in the built corpuses. Moreover, she becomes able to interrogate the imperfection of the built corpuses. She experiences and explores different types of building and the static balance to build the robust corpus. In this regard she gets learning opportunities from numerical and somewhat spatial perspectives through the tutoring of either her mother or father or both.

Component "cooperation$_{1+2}$":

Allocation x Cooperation: In both chosen scenes child-adult interaction is actualized. While in the first scene Aleyna plays only with her mother, in the second observation period her father becomes her game partner while the mother attends as a spectator. In each play situation Aleyna game partners are respectively her mother, father and parents as a triad.

Situation x Cooperation: In the dyadic and polyadic interaction processes of both observation periods, each adult person reserves the role of *tutee* for Aleyna and that of tutor for themselves. Through their different kinds of tutoring Aleyna participates in both play situations in more or less restricted ways in that she can only as an *author* and/or *ghostee* in terms of production design (see 2.2.1.). The numerical framings of her parents in both observation phases lead Aleyna to participate in a rather restricted way as she cannot experience the geometrical features of block building activities intensively.

Contribution x Cooperation: Negotiation processes and building activities between Aleyna and each adult person in both play situations offer Aleyna learning situations from a numeric analytical perspective. During the interaction processes she gets opportunities to perform each building activity. Whereas she stands at the *centre* of her mother's interest in the first observation period, she adjusts the material and local conditions of her learning environment by mobilizing her social environment through communicative solicitations.

Component "Pedagogy and Education$_{1+2}$":

Allocation x Pedagogy and Education: In both play situations the following goals are pursued: spatial structuring, identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures, using kinaesthetic imagery and spatial visualization, and realizing spatial operations. The US National Research Council reports that five-year-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National Research Council 2009, p.191). Block building activities enable children to practice their spatial skills so an interactional niche in the spatial thinking of children can emerge.

Situation x Pedagogy and Education: In both play situations Aleyna directly acts with the play materials, whereas the adult person indirectly uses playing cards and blocks as negotiation elements. Through different scaffolding processes initiated by her father and mother, there emerge disputational and cumulative negotiation forms, in which they construct highly arithmetical and rarely geometrical argumentations about the block building activity. In terms of folk psychology and folk pedagogy, Aleyna's parents pay attention to the arithmetical and numerical subjects of block building situations. In this sense they shed light mainly on the numerical features of building corpuses and assist Aleyna to perform them. By enabling the success of her exercise of numerical abilities, Aleyna's parents initiate positive cognitive outcomes her arithmetical development.

Contribution x Pedagogy and Education: In both cases it seems that Aleyna has strong spatial abilities in that she can see, interrogate, realize, examine and perform the block building activities during the interaction processes. Aleyna vigorously and frequently performs building activities in which she appears as an increasingly autonomous participant. During the negotiations of taken-as-shared meanings in both cases she gets learning opportunities for how many blocks should be used to build the right corpus. One can assume that this was possible for her because she was simultaneously engaged in a learning process about aspects of numbers of operations. During the negotiations of meanings in both cases she gets learning opportunities about numerical issues, while she contributes to both play situations in different ways. In the first observation phase Aleyna is placed at the centre of her mother's interest and thus gets learning opportunities only in mathematical domain numbers and operations instead of geometry. In the second observation phase Aleyna adjusts the *material* and *local conditions* in the play situation and mobilizes her social environment through communicative solicitations. Thereby she gets good learning opportunities in the mathematical domain of numbers and operations but less so in geometry. She brings the arithmetical framings of her parents into the block building activity so that she becomes able to interrogate, realize, represent and perform the spatial features of the built corpus to a limited extent. In this regard by providing different regulation types to Aleyna, her parents enable her to develop her numeric abilities highly and her spatial abilities slightly at a metacognitional level. Regarding geometrical developmental theories, Aleyna is a *piece assembler* as she is not able to *realize* or *see* the difference between the wide and narrow sides of blocks in the figure on the chosen card. In continuing the play situation in the second observation period, Aleyna attends to the static balance of the corpus which makes her come up with geometrical arguments. In this sense Aleyna begins to act as a *picture maker*, as she can build vertical components within a building though with a limited range, use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus somehow errs at adding pieces. Due to the regulation types of her parents, she becomes able, weakly, to compose structures from pictured models and to produce arches and corners with vertical and horizontal spaces systematically (Tables 2.3 and 2.4). In this sense different regulation types lead her to different numerical learning outcomes, while she is slightly adjusting local conditions to learn geometric features.

With respect to the above mentioned and reflected three components and their aspects, the NMT table for Aleyna can be structured in the following way (Table 4.9):

Table 4.9 The overall NMT1-Family and NMT2-Family of Aleyna

NMT-Family Ak **Building 01,02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Mathematical domain: “Geometry and spatial thinking”, using spatial skills at the building activity	Playing with parent	Theory of framing on the development of spatial visualization and spatial senses
aspect of situation	Negotiation between father, mother and Aleyna about the built corpus cumulatively and disputationally Working interim	Different leeway of participation but both restricted Parents: Tutor Aleyna: Tutee	Framing of parents enables Aleyna to examine numeric analytical and spatial features of the built corpuses Folk Psychology and Folk Pedagogy
aspect of contribution	Exploring static balance to build the robust corpus Exploring and examining different building varieties Explicitly experiencing numeric analytical features	Different leeway of participation: Central and environmental self-regulation	Getting learning opportunities notably high in arithmetic, low in geometry

In sum, the overview of these three components of NMT-Family Ak leads us to the conclusion that Aleyna is involved in the interactive accomplishment of NMT that obviously offers her successful support in the development of numerical learning but very weak support in the development of geometrical learning. The aspects of situation and contribution in the NMT coalesce dynamically so that a MLSS comes into being as a format in the grey labelled area in the table 4.9. The part of “Contribution x Pedagogy and Education” seems compatible with the mathematical learning situation, in which Aleyna benefits from the learning opportunities and explores more than is offered. Through negotiating with her mother cumulatively and with father disputationally, Aleyna has learning opportunities. A MLSS is constituted so that Aleyna can actively explore and learn spatial features to a limited extent but numerical features intensively.

4.3. The Gül Family

The Gül family are German Turks who live in a major German city. Berk is our main focus child and can speak German and rudimentary Turkish. He has an elder brother, Can, who is 13 years old and attends secondary school in Germany (*Hauptschule*). He can speak both German and Turkish fluently. The mother grew up in Turkey and moved to Germany after her marriage. She can speak Turkish and rudimentary German. She completed 12 years of primary and secondary school in the Turkish school system. Currently she works in a post office. The father grew up in Germany and can speak German and rudimentary Turkish. He had 15 years of formal education in Germany and attended a *Realschule* (a type of secondary junior high school for ages from 10 to 16) in Germany. Normally in German students complete Realschule after 10 years of formal education (primary plus secondary), which means Berk’s father failed and was obliged to repeat five years during his schooling. Currently he works as a bus driver. The father’s mother has a close relationship with Gül family and cares for Berk and his elder brother when they come home from school. Thus she can be named as a third person in the Gül Family, who is really close to the children. Berk’s grandmother can speak Turkish, rudimentary German and a little English, and has a low level of education, having attended school for five years in Turkey.

In the first observation phase Berk is aged four years and six months. The meeting with Berk’s family takes places in the kindergarten, which Berk attends daily. Berk's elder brother and his mother attend the meeting and in total they play three different games as triads, in which *polyadic interaction* emerges. In the game "Building 01", they play 10 rounds by turns in total and the negotiation process proceeds mostly in Turkish. Most probably, because of the mother's rudimentary knowledge of German they negotiate in Turkish in order to let the mother understand each utterance. The turn of mother in the first round was observed and analysed by Acar Bayraktar and Krummheuer (2011). Eventually the negotiation process between Berk, his mother and brother emerges as "exploratory" as they engage in the negotiation process critically but constructively (Fernández et al., 2001, pp.42-43). They offer justifications and alternative hypotheses, while they are overcoming challenges. They perform collective argumentation in that they offer hypotheses, which can be made publicly accountable, and try to reach an agreement with each other. The peer related co-construction process between mother and elder brother constitutes a type of supportive action namely *modelling*, which enables Berk to see and explore the different manners of the building corpus in 3D space. Thereby their reasonings become more visible in the negotiation of taken-as-shared meanings and progress results from the eventual agreements reached. Berk's mother and elder brother realize one of the scaffolding functions, namely *demonstration*, which assists Berk to examine different types of building and experience spatial features in an implicit and a particular way. By witnessing all the situated activities of his elder brother and mother, Berk is exposed to learning identical corpuses with the figures on the chosen cards. In this regard the exploratory negotiation process that emerges between elder brother and mother makes it possible for Berk to join in this process smoothly. Thus, they ascribe the role of "legitimate peripheral participant" to Berk, whereas Berk participates as an "over-hearer" and "observer" (Acar Bayraktar & Krummheuer, 2011, p.165) in the negotiation process between his mother and elder brother.

Regarding all these facts, the mother and the elder brother realize a MLSS for Berk and a NMT for Berk emerges (Acar Bayraktar & Krummheuer, 2011, p.167). Thus, the NMT table can be shown in Table 4.10 (2011, p.168):

Table 4.10 NMT of Berk in the game "Building 01" (Acar Bayraktar & Krummheuer, 2014)

NMT-Family Gül **Building 01**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Mathematical domain: "Geometry and spatial thinking", using spatial skills at the building activity	Playing with elder brother and mother	Theory of the development of spatial skills and spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.
aspect of situation	Exploratory negotiation process between brother, mother and Berk About the built corpus *Collective argumentation*	Legitimate peripheral participation	*Peer related co-construction and a scaffolding process by the function demonstration* Enabling to perform different spatial features
aspect of contribution	Exploring and examining different building varieties. Implicitly and particularly experiencing spatial features	Over-hearer Observer	Witnessing of all the situated activities of brother and mother Learning spatial features

Bearing in mind this NMT table (Acar Bayraktar & Krummheuer, 2011), now I present another scene of Berk from the second observation phase. In the second observation phase the meeting takes place at Berk's house. Berk's parents, his elder brother and his grandmother – called "granny" – attend the meeting. Berk and his elder brother play the game once with his mother, once with his father and once with his granny, hence in all cases as triads. Similarly, they play one game as the two parents and the two brothers, without granny, as a tetrad, which also facilitates the emergence of a polyadic interaction process.

The game 'Building 02' from the second observation period

For the coming analysis, a scene from the game "Building 02" is chosen, which is materialized in the second observation phase when Berk is six years of age. In this game, Berk's game partners are his elder brother and their grandmother ("granny"). In total, they play 10 rounds by turns. The transcribed scene is from the second round, which begins with Berk and ends with granny's turn. For the transcription a part of Berk's turn and a part of granny's turn are chosen, which are named the first part and the second part.

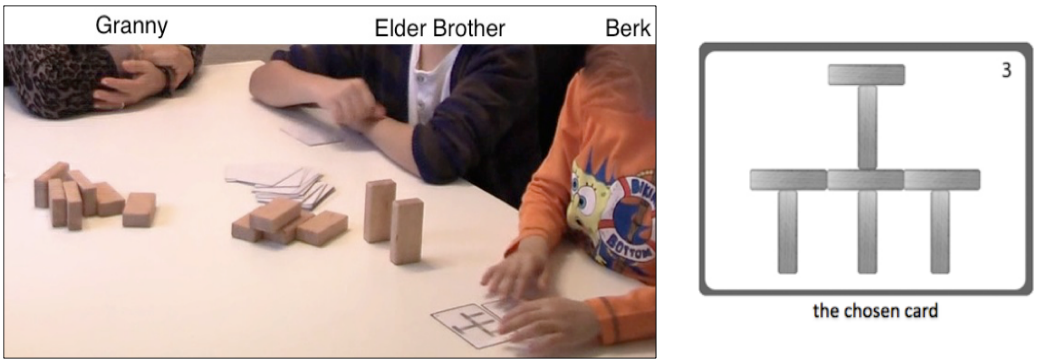
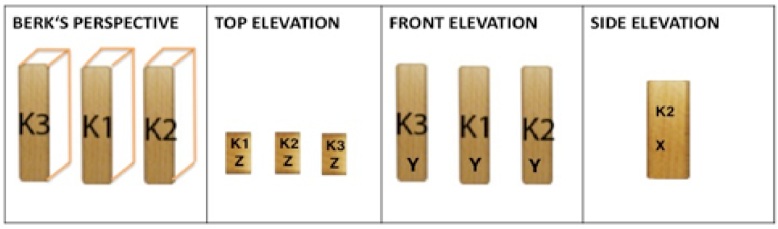
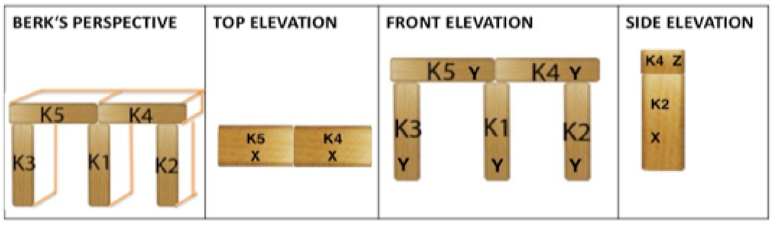
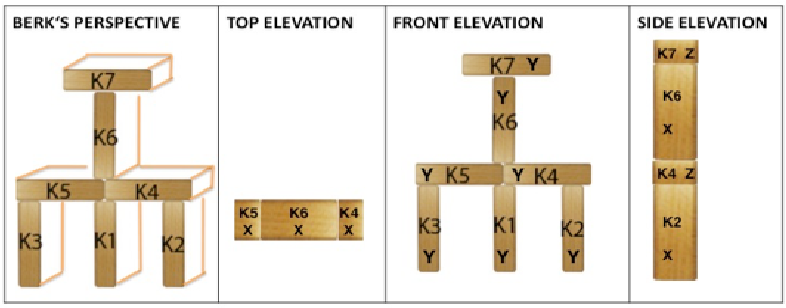


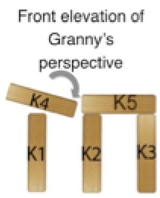
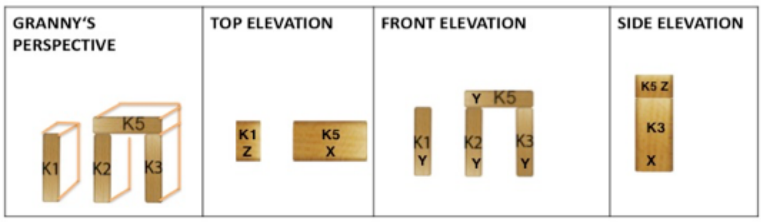
Fig. 4.61. Recording position of the game "Building 02"

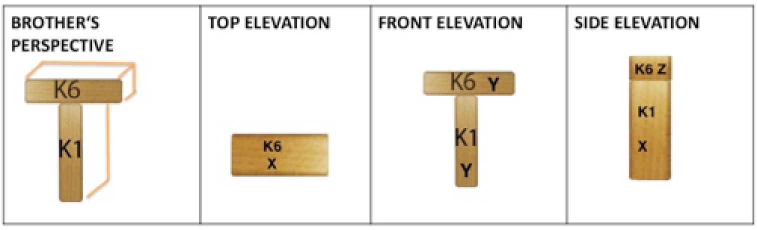
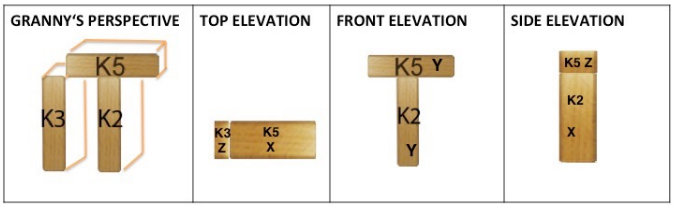
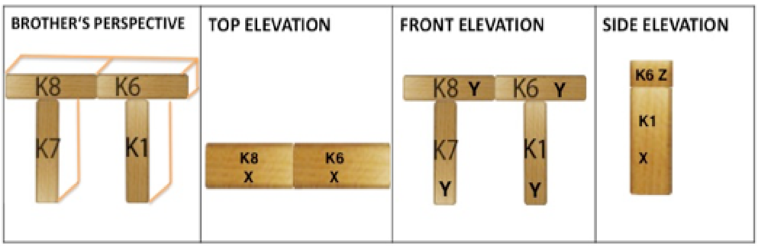
In the first part Berk and in the second part granny choose the same card, shown above (see. Fig.4.61.). After Berk's turn he puts the chosen card back in the deck, then his brother's turn comes. After his turn, granny's turn comes. She picks up the card, which lies on the top of the deck, which was just put back by Berk two turns before. At the beginning of the chosen scene, Berk shows the card to his game partners and then starts to build the figure up immediately. Up to the chosen part, he has puts one block vertically after the other block on their Z side next to each other in a parallel direction. He builds the corpus in the upright position on the table. During the interaction process they use German language and occasionally Turkish language by switching (see 1.4.1.1.). In the transcript German speech is written in normal font and Turkish is underlined. Sometimes, the granny speaks in English, which is underlined twice.

Transcript

1	01:58		Berk	*moves the card about 10 cm to the left, checks*
2				*the chosen card, takes another block, K3, from the pile*

3				*and puts it vertically on its Z side next to K1 in a* 
4				*parallel direction. Holds another block, K4, from the pile*
5				*of blocks and puts it horizontally on K1 and K2 on its*
6				*Y side. takes another block,K5, from the pile of blocks*
7				*with his right hand and puts K5 horizontally bonded to K4* 
8				*on K1 and K3 on its Y side*
9			Granny	yes!
10			Brother	*smiles*
11			Berk	*takes another block, K6, from the pile of blocks with*
12				*his right hand and centres it in upright position on K4*
13				*and K5 on its Z side, then takes another block K7, from*
14				*the pile of blocks with his right hand and sets it*
15				*horizontally centred on K6 on Y side.* 
16			Brother	wrong. *looks and smiles at granny*
17			Granny	is it wrong? *looks and smiles at Berk's brother*
18			Brother	*nods and looks at Berk, laughing*
19			Granny	you did it wrong. *laughing, looks at Berk*

20			Brother	wrong. *laughing, looks at Berk*
21			Berk	*holds the chosen card and looks at it* why?
22			Brother	*wipes the surface of blocks K4, K5 from the corpus* this is
23	02:25			in the middle and there are three of them. *takes card away*
500	After this explanation the p-turn of Berk's brother comes. He builds the corpus identical with the figure on the chosen card. Then comes granny's turn as the last player in the second round. She chooses the same card from the pile of cards as Berk's. She starts to build the figure. While she is setting block K5, block K4 falls down from the corpus and Berk's brother laughs. She takes block K4 in her right hand and asks if the corpus is wrong, which she built up till that time:  			
501			Granny	wrong 
502			Brother	yes.
503			Granny	*holds K4, moves K1 a bit left, looks at the chosen card*
504				why here? *shows on the card*
505			Brother	*shows on the card* there are three pieces just look
506			Granny	Here. There are three pieces.
507			Brother	*shows on the card* no. upon it. in the middle there are
508				three pieces.
509			Granny	*holds K5 with her right hand above K2 and K3, holds K4*
510				*with her left hand above K1* I see. *moves K2 a bit left*
511				*with the tip of her right index finger. Swings K5 above*
512		<		*K2 and K3 around, while she goes on holding K4 above K1*
513				*and K2. Takes another block, K6, with the tip of her index*
514				*finger from the pile of blocks and holds it centred*

515				above K2. how three pieces?
516		<	Brother	*holds his head down laughing* you put it in the middle
517				holds K2
518			Granny	*puts K6 centred on K2 while holding K4 and K5 still. K6*
519				*falls down. directly puts K5 centred on K2.*
520			Brother	like that. *takes K1 in front of him and puts K6*
521		>		*horizontally centred on K1* 
522		>	Granny	*moves K3 closer to K2* 
523	3:30		Brother	*takes K7 from the pile of blocks and puts it next to K1*
524				*in a parallel direction.*
525				*takes K8 and sets it horizontally on K7* 
526			Granny	*sets K4 at the right side of K7 and puts K5 horizontally*

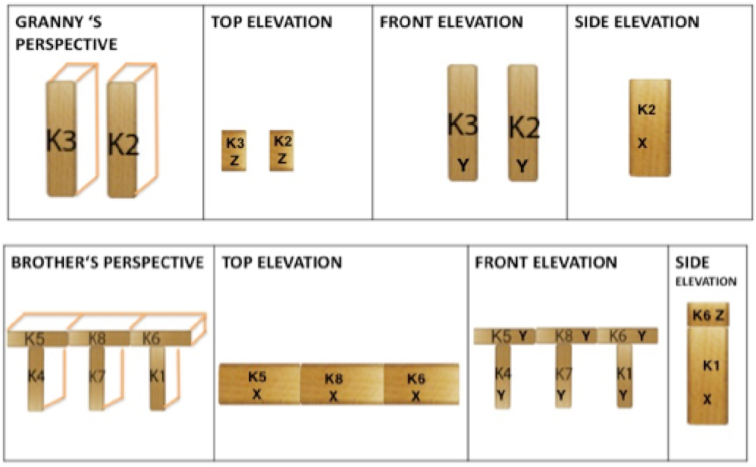
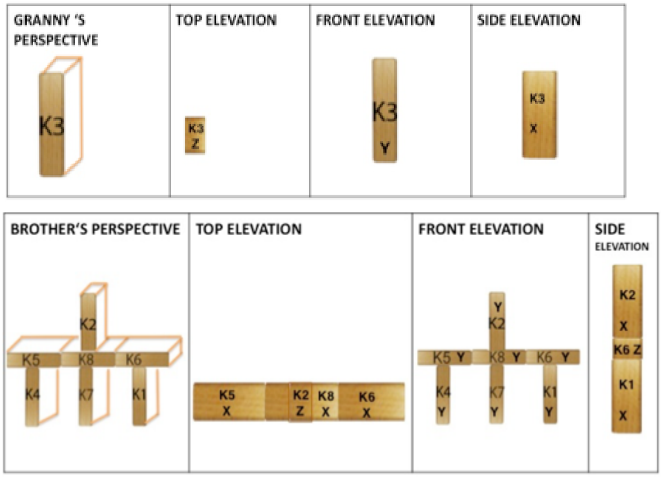
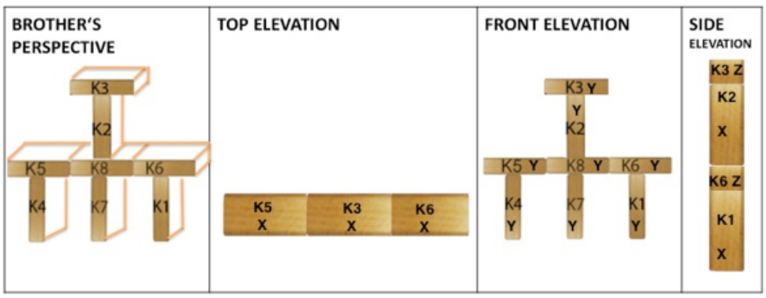
527				*centred on K4* one more 
528				isn't it?
529			Brother	*nods* too far..yes.
530		<	Granny	yes *takes K2 and puts it vertically centred on K8* 
531		<	Berk	granny says yes
532			Granny	*laughs, takes K3 and puts it horizontally on K2* 
533				Okay?
534	03:52		Brother	*nods*

Fig. 4.62. Transcription of second round.

Interaction analysis

Without any interruption Berk begins to build the corpus vertically on the table. He puts the third block (K3) vertically next to and parallel to the first two blocks <01-04>. Then he puts two blocks (K4, K5) bonded to each other horizontally on these three blocks (K1, K2, K3) <04-08>. From a participatory point of view, Berk looks like an *author* in that he expresses his own idea about the built corpus. Berk's leeway seems quite open as he participates actively in the situation.

His granny says "yes!" <09>. Most probably she approves what Berk builds or how he acts. Maybe she tries to highlight her grandson's achievement and thus expresses her approval in English. From a supportive perspective, she seems to demonstrate her agreement by approving either his action or the built corpus. In this sense her reaction seems a type of supportive action, namely *affirmation*. However the corpus built so far by Berk does not seem completely identical with the figure on the card. In this regard what granny approves does not seem clear enough. Maybe she really finds the built corpus to be right, or she pretends to assume that the built corpus is right.

The elder brother smiles <10>. His reaction might mean that either granny's English approval or her taking the role of *tutor* or the corpus built by Berk please him.

Berk goes on building the corpus. He centres a block (K6) on the conjunction of both blocks K4 and K5, vertically in the same direction as K1<11-13>. Then he sets another block (K7) up on K6 horizontally in a parallel direction to K4 and K5 <13-15> (see Fig. 4.63.).

The built corpus bears some resemblance to the figure on the chosen card. Regarding the front elevation of the built corpus, the faces of all the set blocks (Y sides) seem identical to the faces of the blocks (Y sides) in the chosen card.

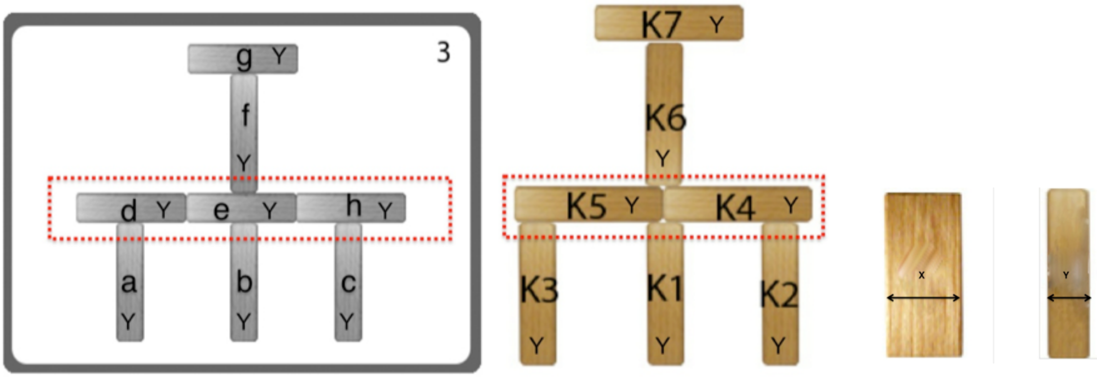


Fig. 4.63. The chosen card and the built corpus with the named blocks and sides from the front elevation and the wide side (X) and narrow side (Y) of building block

However, the amount of blocks and the appearance of the built corpus is not identical. More precisely in the middle part of the figure on the chosen card is a longer horizontal part (cf. d, e, h), which is marked with the red line (see Fig. 4.63.) and consists of three blocks with two juts on the both right and left sides (half parts of d and h) of the figure on the chosen card. Apparently Berk built this part of the corpus a bit differently. In this sense the block K4 might refer to either block e or h in the chosen card, and K5 might refer to either block d or e. Regarding these two features, the middle part of the built corpus consists of only two blocks (K4, K5), which means lacks the juts on the left and right sides.

In this regard the difference between the chosen card and the built corpus can be interpreted as follows (Fig. 4.64):

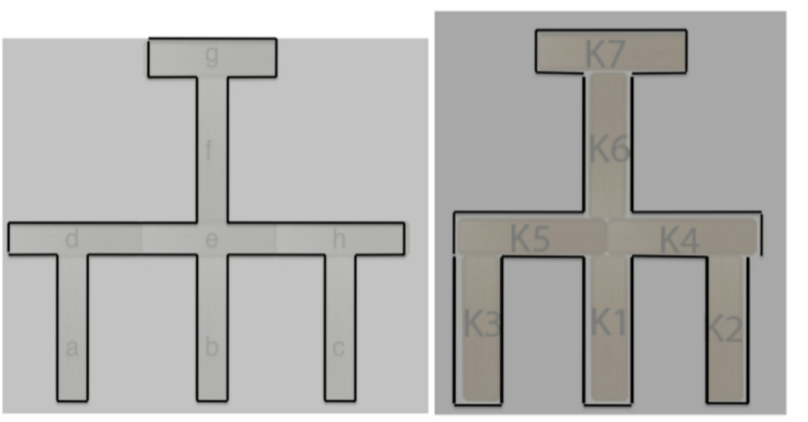


Fig. 4.64. *Gestalt* of the chosen card and the built corpus

- Berk might pay attention only the static balance between the blocks. Instead of situating each block on one block, he might prefer to situate K4 on two blocks (K1 and K2), and K5 on two blocks (K1 and K3) (see Fig. 4.64.), on which both blocks can be in static balance and are not at risk of falling. In this regard he might adapt his idea to the *gestalt* of the corpus and pay attention only to the *gestalt* of the figure on the chosen card. Maybe therefore he composes and decomposes the figure in his mind according the static balances of the blocks and thus the *gestalt* of the corpus resembles the figure and the corpus can be robust (see Fig. 4.64.).

From a developmental perspective, Berk is a *shape composer* (see Table 2.4). He seems to be able to match shapes using *gestalt configuration* and to *compose towers* but not in great detail (see section 2.1.). In this sense he seems to be able to use multiple spatial relations extending in multiple directions and use systematic trial but is not able to add block pieces unit by unit in great detail. However, he should *have the ability to balance blocks intuitively* by his age (see Table 2.4) and thus it might be that he pays attention to the static balance of the corpus. From a participatory point of view, Berk takes the role of *author*, who comes up with a totally new idea by building a corpus resembling the figure on the card.

The elder brother smiles and tells granny in Turkish that Berk built the corpus wrong <16>. Appropriate to his age, the elder brother seems able to compose and decompose complex shapes units of units. He can distinguish the difference between the built corpus and the figure on the chosen card. From a supportive perspective, his reaction indicates the supportive activity of *disaffirmation*, which is a type of correction and a definitive negative response indicating an incorrect reaction. From a participatory point of view, the elder brother takes the role of *tutor,* which is not usual according to socio-constructivist approaches (see section 2.2.). Nevertheless, regarding family system theory, it is also not surprising that an elder brother takes the responsibility while he is playing with his younger brother (see section 2.2.).

Thereupon granny smiles at him and poses a question in Turkish whether the built corpus is wrong <17>. Posing the question in Turkish might be due to the granny's weakness in German. In this sense she gives the impression of trying to perform the negotiation process with her grandchildren in Turkish, although they can also speak fluent German. From a participatory point of view she seems to take the role of *tutee*, while she is ascribing the role of tutor to Berk's brother. Considering family system theory, granny might try to pretend to act as a *tutee* too, because she is one of the *primary caregivers* of her grandchildren, the creator of *parenting patterns* in their nuclear family and the *sufficient* person, in terms of pedagogical approaches (see sections 2.2.2 and 2.2.3.). In this regard she gives the impression of reserving the role of tutor especially for the elder brother, so that he can be answerable to his younger brother as an *adviser* (see section 2.2.2.). Moreover she might also try to enable the elder brother to take more responsibility and to show concern for his younger brother. One possibility is that she does not really apprehend the wrongness of the way of building or that she has not enough spatial skills to grasp Berk's block building activity. Regarding family system theory, the granny has a *low-level education* and can be responder less than the elder brother (see sections 1.4.1 and 2.2.2.).

Berk's brother replies to granny by nodding <18>. He appears to agree with granny that the built corpus is wrong and to show his agreement by nodding. Moreover, he gives the impression of continuing his previous argument in line <16> that Berk built the corpus wrong. From a supportive perspective his reaction can be seen as a type of supportive activity, namely *affirmation*, as he demonstrates his agreement with granny.

Thereafter granny declares that Berk built the corpus wrong <19>, while she and his brother go on to laugh at Berk <15-19>. Granny gives the impression of declaring that the built corpus is wrong. In this sense she seems to take the idea of her elder grandson in line <16>, that Berk built the corpus wrongly, and to express it in a same way as he did. From a participatory point of view, she is taking over the role of *relayer,* while ascribing the role of *deviser* to her elder grandson. Moreover, she is perceiving Berk as a *tutee*. From a supportive perspective her reaction is a supportive activity, namely *disaffirmation*, in that she indicates the incorrectness of the built corpus.

The elder brother repeats granny's declaration while laughing <20>. His reaction seems a confirmation of granny's declaration. He keeps his idea constant and in participatory terms keeps his role as *deviser* by repeating his claim in line <16>. In this respect he keeps the role, which granny seems to assign him, as *being answerable to his younger brother* as an *adviser*. From a supportive perspective he still indicates the wrongness of the built corpus, which refers to a type of support, namely *disaffirmation.* During this *disaffirmation* process the laughter of the elder brother and granny can be interpreted in different ways: They might laugh at Berk to

- make him think that he has built the corpus wrong, or
- show that he lost his p-turn, or
- distract his attention away from the building activity in order for him not to realize the wrongness of the built corpus, or
- hide their own stress over whether he can build the corpus right.

The negotiation process between granny, the elder brother and Berk seems to be fulfilled *disputationally* in that they express their disagreements and individual decision-making assertions about the built corpus.

Berk holds the chosen card, looks at it and asks "why?" <21>. Holding the chosen card and looking at it might be a check in order to see what the difference between the figure on the chosen card and the built corpus is. Maybe Berk tries to find out his mistake by looking at the card more closely. Thereupon he asks the reason for the declaration of his granny and elder brother or why the built corpus is wrong. Thus, his reaction might mean that either he asks them indirectly for help to build the corpus correctly or tries to get a clue for how he can build the corpus right. Furthermore, his movements signalize that he starts to think about the built corpus and tries to check what has gone wrong with the card or corpus. In this context the reactions of granny and elder brother seem to work on Berk, causing him to muse on the wrongness of the built corpus. By posing such question, he shows signs *environmental self-regulation* in that he tries either to adapt the *material* or *local conditions* in the play situation or to initiate referential joint attention in the play situation with these partners (see section 2.2.3.). Berk gives the impression of *mobilizing* his *social environment through communicative solicitations* by posing such question and seeking an answer to his question. From a participatory point of view, he seems to take the role of *tutee*, while ascribing the role of *tutor* to either his elder brother or granny. Moreover, checking the card and posing a question after his brother's declaration show him accepting his elder brother as an *adviser.*

The elder brother gives Berk feedback immediately and rubs the surface of blocks K4, K5, K6 of the corpus. Then he says, "this is in the middle and there are three of them"< 22-23>. His answer can be interpreted in different ways:

- "This is in the middle": "In the middle" could be either the facet of blocks d, e, h in the figure or K4 and K5 the facet of blocks in the corpus, which are shown with red lines Fig. 4.63. In all probability, he compares block e or block h in the figure with block K4 in the corpus. When the figure and the corpus are compared, it can be seen that one block is missing in the built corpus. In this regard the utterance "This is in the middle" can be interpreted as follows:

I. "This block (K4) has to be normally between the blocks d and h, so that K4 could be in the middle of corpus". If one more block were not missing in the corpus, then block K4 would have been in the middle of the corpus. Thus, block e represents block K4 in the corpus (see Fig. 4.64.). The missing block in the corpus is identified with block h in the figure.

II. "This block (e) is in the middle of the corpus. Here it has to be one more block in the middle." If this block were not missing in the corpus, then between blocks K4 and K5 would have been one more block in the middle of the corpus. Thus, block e represents the missing block, while K5 represents block d and K4 represents block h in the corpus (see Fig. 4.64.). The missing block in the corpus is identified with block e in the figure.

- "There are three of them" can be interpreted as follows:

I. “There are three blocks (d, e, h) in the middle of the figure (see Fig. 4.64.). He means there also have to be three from each block in the corpus.”

II. “The block (K4) has to be in the middle of the corpus thus in total there have to be three blocks. But here there are two. Actually there have to be three blocks.”

III. “There have to be three blocks and K4 has to be in the middle, but Berk used only two blocks.”

IV. “The block (e) is in the middle of the figure. On the figure there are three blocks in the place (which has a red line in Fig. 4.63. But there are two in the corpus. Actually there have to be three blocks.” (see Fig. 4.64.)

- “This is in the middle and there are three of them” can be interpreted as follows: Block K6 stands in the middle of the corpus and there have to be two more blocks the same as K6, then there will be three all together.

Most likely the elder brother tries to show and tell Berk what is wrong with the built corpus. In this sense he gives the impression of fulfilling one of the scaffolding functions, namely *marking critical features*. He remarks on the critical feature of the built corpus, how many blocks should stand in the corpus and how many are in the middle part of the figure on the card. From a participatory point of view, he gives the impression of taking the role of *tutor*, while ascribing the role of *tutee* to his younger brother. Thereby he also acts as an *author*, in that he comes up with an original new idea.

Through his utterances, the brother seems to begin with the geometrical issue of the play situation and then shift to the arithmetical one. By pointing out the middle part of the built corpus he takes the geometrical approach. Then he emphasizes the amount of the blocks in this part. In this regard he suggests a numeric analytical approach by arguing the amount of the horizontal blocks. His argument is made up of both geometrical and arithmetical approaches, which relate to *folk psychology* and *folk pedagogy* (see section 2.2.). Moreover, his reaction gives the impression he has sufficient knowledge of geometry and numbers. From a supportive perspective his reaction indicates a supportive activity, namely *instruction.* Thereby he seems to suggest to his brother a specific strategy, which refers to both geometrical and numerical aspects and consists of both geometrical and arithmetical *framings.* However, in the instruction he gives, the elder brother seems not to take note of Berk's comprehension.

Thereafter the elder brother takes the chosen card away. Obviously, the elder brother interrupts the ongoing negotiating process and might conclude that Berk's p-turn is over. He might have enough of wasting time with keeping on negotiating about the built corpus and thus might try to push his younger brother to finish his turn. So indeed, the elder brother does not go into any further discussion about the built corpus. From a supportive perspective he seems to realize a support activity, namely *conclusion.* By taking the chosen card away, the brother seems to *conclude* Berk's turn and sum up that he has lost his p-turn. Therefore, his reaction can be seen as a *conclusion.* In this regard the negotiation process between elder brother and Berk appears to be generated *disputationally* in that they express their disagreement and individual decision-making assertions. From the participatory point of view, he gives the impression of being a *tutor,* while reserving the role of *tutee* for Berk. By taking the card away, he gives the impression of depriving Berk of turning up and expressing his own idea. In this respect Berk's leeway of participation seems to be restricted so that he is deprived of turning up, discussing the built corpus further and achieving his p-turn. A *working interim* between granny, elder brother and Berk about the wrongness of the built corpus and the expiration of his turn emerges unambiguously.

As the last player in the second round, the granny picks the top card from the pile. It is the same card that was chosen two turns before in the same round by Berk <500>. She starts to build the corpus. While she is setting block K5, block K4 falls down from the corpus and Berk's elder brother laughs <500>. Granny takes the block K4 in her right hand and asks if the corpus is wrong <501>, which she has built up to that time.

Granny seems to build the corpus in the same way as Berk did (see <7>). Thus, she gives the impression of imitating Berk's building process so she does not perform any original idea. From the participatory point of view, she seems to take the role of *relayer*, while ascribing the role of *deviser* to Berk.

- One possibility is that she does not get the point made by the elder brother in Berk's turn or she does not have sufficient spatial skills to achieve the block building activity. Regarding family system theory, granny has a *low education level* and typically would be less responsive than the elder brother (see sections 1.4.1 and 2.2.2.). Maybe she thus poses a question in order to understand what she is doing wrong in the building action. In this sense she seems to act as an *environmental self-regulator.* She is adapting local conditions in the play situation and trying to mobilize her elder grandson through communicative solicitation by posing such a question and seeking approval of her action.
- Another possibility is that she tries to create a learning situation for Berk. Considering family system theory (see section 2.2.2.), granny is one of the *primary caregivers* of her grandchildren, the creator of *parenting pattern* in their nuclear family and the *sufficient* person in terms of pedagogical approaches. In this regard she might try to pretend to be a *tutee* and create a learning situation for Berk, in which the elder brother might give more detailed information about the right way of building an identical corpus to the figure on the card. In this sense her reaction indicates a method with a particular aim, namely a *tactic*, in order to create a learning situation for Berk. From a participatory point of view, granny takes the role of *tutee*, and ascribes the role of *tutor* to the elder brother, while Berk also seems to take the role of *tutee*.

Brother says "yes." <502>. He gives the impression of approving granny's question. From a supportive perspective his reaction can be seen as a type of supportive activity, namely *affirmation*, in that he demonstrates his agreement with the idea that the corpus is wrong. From a participatory point of view the elder brother seems to keep his role as *tutor*, while ascribing the role of *tutee* to both his granny and younger brother. He seems to act like an *author* as he expresses and performs his own original idea.

Granny moves K1 a bit to the left, while she is still holding K4 <503>. Then she looks at the chosen card and asks “why here?” by showing the figure on the card <504>. One possibility is that she does not get the point of the elder brother’s view and thus asks a question. In this sense her reaction shows *environmental self-regulation* in that she tries to mobilize her elder grandson through communicative solicitation by posing such a question and searching for an approval of her action. Another possibility is that she tries to create a learning situation for Berk, as in line <501> where the elder brother gives more detailed information about the right way of building a corpus identical to the figure on the card. From a participatory point of view, she tries to reserve the role of *tutor* to the elder brother, while ascribing the role of *tutee* to Berk and herself. Moreover, she acts like an *author* when she expresses and performs herown original idea by moving block K1.

The elder brother responds in Turkish, “there are three pieces just look” by showing the figure on the card <505>. His reaction reinforces the observation in line <16> that granny cannot speak fluent German and thus they perform the negotiation process in Turkish. His argument strengthens the idea in lines <22-23> that there has to be three of each block in the corpus. The word “pieces” might refer to the “block” in the figure on the card. His response can be interpreted in two different ways:

- The elder brother might give the clue that granny should look at three blocks in the figure, which are outlined in red on the card (see Fig. 4.65). In this part of the corpus, as built by granny (Fig. 4.65), there are only two blocks instead of three. Thus, he might mean that she should look carefully at the figure on the card.
- The elder brother might point out three blocks of the figure, which are outlined in yellow on the card (see Fig. 4.65). Maybe he gives a response to her

question <504>, there are three blocks both in the base and also in the middle of the corpus (see Fig. 4.65). Therefore, she should look at the card carefully.

The elder brother gives the impression of fulfilling one of the scaffolding functions, namely *marking critical features*. He remarks on the critical feature of the built corpus, how many blocks exist either in the base or in the middle of the figure on the card. By *marking critical features* he also accentuates the numerical features of the built corpus. Through his utterances, she seems to shift the geometrical issue of the play situation to the arithmetical one.

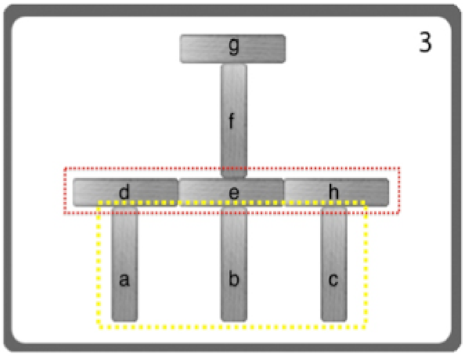


Fig. 4.65. Alternative sets of three blocks which elder brother might indicate

Instead of arguing about the geometrical features about the built corpus, he suggests rather a numeric analytical approach by arguing the amount of the blocks. The argument is made up of only arithmetical approaches, which relate to *folk psychology* and *folk pedagogy* (see section 2.2.). From a supportive perspective, his reaction indicates a supportive activity, namely *instruction.* Thereby he seems to suggest to his granny a specific strategy, which refers to a numerical aspect and consists of arithmetical *framing.* From a participatory point of view, he gives the impression of taking the role of *tutor*, while ascribing the role of *tutee* to granny. Regarding lines <22-23>, he seems to keep his role as *author*, in that he comes up with an original new idea. Furthermore, he appears to perform a *disputational* negotiation process with granny in which he expresses his disagreement and individual decision-making assertion about the set blocks.

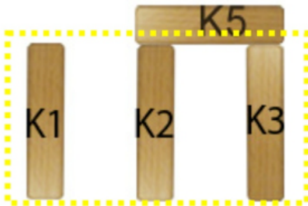


Fig. 4.66. The built corpus and the base of the corpus outlined in yellow (from front elevation of granny's perspective)

Granny reacts immediately and says in Turkish "here. There are three pieces." <506>. Her reaction in Turkish reinforces the observation (<1-17>) that she prefers to conduct her conversation with her elder grandson in Turkish due to her rudimentary German. Considering the corpus built up to now, granny seems to emphasize three blocks in the base of this corpus, which are outlined in yellow in Fig. 4.66. In this regard she gives the impression of highlighting that there is already three blocks in the built corpus.

- One possibility is that she might *really* comprehend what the elder brother mentioned "with three blocks" and means that she has already set three blocks. As already mentioned in the previous lines <500-501>, she might not have sufficient spatial skills to proceed with the block building activity. Regarding family system theory, the granny has a *low education level* and could be responder less than the elder brother for Berk (see sections 1.4.1 and 2.2.2.2.). In this sense her reaction indicates *environmental self-regulation* in that she tries to mobilize her elder grandson through communicative solicitation by posing questions and seeking his approval of her action. In this regard, from a participatory point of view she seems to take the role of *tutee*, while ascribing the role of *tutor* to Berk's brother.
- Considering family system theory (see 2.2.2.), another possibility is that she might *pretend* not to comprehend what the elder brother mentioned, as a *tactic*, and latently she might try to ask what he means with three blocks. In this sense she gives the impression of reserving the role of *tutor* especially for the elder brother, so that he can be responder for his younger brother as

an *adviser* (see 2.2.2.) and let him to explain in detail what was wrong with Berk's corpus. From this point of view, she might also try to enable the elder brother to take more responsibility and show concern for his younger brother, while trying to create a learning situation for Berk. In this regard, from a participatory point of view she seems to take the role of *tutee*, while ascribing the role of *tutor* to Berk's brother.

In any event, granny's reaction signalizes that she tries to understand what he means by the three blocks at line <505>, as he already mentioned a similar thing in Berk's turn too (see lines <22-23>). From a participatory point of view, the granny seems to take the role of *relayer,* while she ascribes the role of *deviser* to her elder grandson (see 2.2.2.2.). She takes the arithmetical idea of the elder brother and points out the relevant characteristics of the built corpus with the same words as the brother did. Moreover she employs a scaffolding function, namely *marking critical features*, in that she points out the relevant characteristics of the built corpus and the amount of blocks. By *marking critical features* she accentuates the numerical features of the built corpus. Through her utterances, she seems to take the arithmetical approach by arguing the amount of the blocks, which was raised by the elder brother just before <505>. The argument is made up exclusively of arithmetical approaches, which relates to *folk psychology* and *folk pedagogy* (see section2.2.). From a supportive perspective, her reaction might be interpreted as a supportive activity, namely *re-representation,* in which she represents once again the elder brother's idea and utterances.

The elder brother says, in Turkish, "no. upon it. in the middle there are three pieces." <507-508> by showing the figure on the chosen card. His reaction reinforces the idea that granny just meant three blocks in the base of the corpus she built and the elder brother gets her point of view and comes up with a more precise argument. In this regard he progresses from his *numerical framing* by instrumenting a geometrical approach. By pointing out the "middle" part of the built corpus he comes up with the geometrical approach. In this regard he gives the impression of suggesting that the three blocks, which he mentioned in Berk's turn <22-23>, are placed in the middle of the corpus on the card. In this regard he seems to repeat his argument in lines <22-23>. Moreover, considering granny's reaction just before <506>, his argument emphasizes that there are three blocks not only in the base of the corpus but also in the middle of it. His argument is made up of both geometrical and arithmetical approaches. Furthermore, from a supportive perspective, his reaction indicates a supportive activity, namely *re-representation.* Thereby he seems to re-represent his argument in a more detailed way, which consists of both geometrical and numerical *framings.* From a participatory point of view, he takes the role of *tutor*, while ascribing the role of *tutee* to granny.

Furthermore, the reaction of the elder brother gives the impression of that granny's *tactic* (see line <503>) works on him in that he begins to explain what he means in more detail, which can give rise to a learning situation for Berk. Besides this, they (granny and elder brother) seem to perform the negotiation process in an *exploratory* way (see 2.2.3.2.).

Granny takes and holds K5 with her right-hand above K2 and K3. At the same time, with her left hand she holds K4 above K1 <509-510>. Then she says in German, "I see". Most probably she tries to emphasize that she understood what elder brother meant just before <510>. By switching language, she might try to underline that she can also speak a bit of German and is able to give a type of reaction in German at least. She moves K2 a bit to the left with the tip of her right index finger, then moves K5 above K2 and K3 around, while she goes on holding K4 above K1 and K2 <510-513>. She keeps on building the corpus and takes another block, K6, from the pile of blocks and holds it centred above K2 <513-515>. Suddenly she poses a question to the elder brother in Turkish: "how three pieces?" <515>. Her question seems to refer to a *numerical framing* in terms of folk pedagogy and folk psychology. By posing the question she might try to find out what the elder brother exactly means by saying "three pieces" (see <507-508>) and "there are three of them" (see <22-23>). Regarding this, her reaction suggests two interpretations:

- she does not get the elder brother's point and thus asks a question in order to understand what the brother exactly means with "three pieces". Maybe she did not get the elder brother's point in Berk's turn, or she does not sufficient spatial skills to proceed with the block building activity. Regarding family system theory, the granny has a *low education level* and she may not be able to manage block building activities (see sections 1.4.1, 2.1.4. and 2.2.2.). In this sense she seems to act as an *environmental self-regulator.* She is adapting local conditions in the play situation and trying to mobilize her elder grandson through communicative solicitation by posing a question and seeking approval of her action.

- she pretends not to get the of elder brother's point and asks a question, in answer to which the elder brother gives more detailed information about his argument. In this sense her reaction seems that she goes on with her *tactic*, in order to create a learning situation for Berk. Regarding scaffolding theories, she might try to keep the elder brother in the field in order for him to draw Berk up towards a higher level of understanding.

From a participatory point of view, granny is taking the role of *tutee*, and ascribing the roles of *tutor* to the elder brother and *tutee* to Berk.

Elder brother holds his head down, laughing, and directs her by saying, in Turkish, "you put it in the middle" and takes K2 in his hand <516-517>. To hold his head down and to laugh might refer to his astonishment at granny's lack of judgement, regarding the standard socio-constructivist approaches (see section 2.2.). The elder brother might have such a *standard idea* that granny "has to be" an "expert" and she "has to achieve" "everything" better than "children" (see section 2.2.). Therefore, the elder brother might be astonished or confused at how *an adult person* cannot understand – although they negotiate in her mother language instead of in German – what he means or what is wrong with the built corpus. Besides, his statement "you put it in the middle" might be a definite explanation of what granny did wrong in the built corpus. Maybe he tries to use clear portraiture for granny and thus conducts the negotiation process in Turkish as he did in <16-17> and <505-508>. Moreover, he comes up with the geometrical argument by emphasizing "you put it in the middle".

Granny puts K6 centred on K2 while still holding K4 and K5. K6 falls down. Thereafter she puts K5 directly centred on K2 <518-519>. Correspondingly elder brother says “like that” and takes K1 and puts K6 horizontally centred on K1 in front of him <520-521> (see Fig.4.67.)

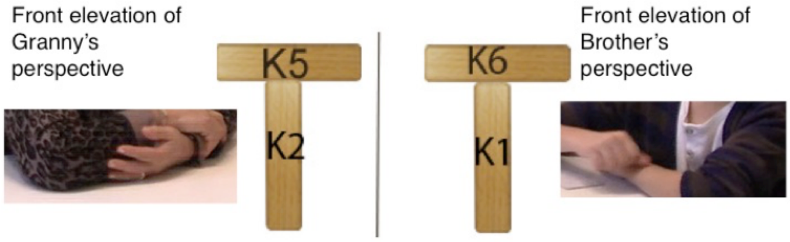


Fig.4.67. Built Corpuses by granny and the elder son from the frontal elevation

Probably he shows granny the way of building the corpus right. Maybe he tries to inform granny by acting instead of telling i.e. using any language. In this regard he seems to *model* the right positions of the blocks which should be set “like that”.

Hereby granny and elder brother together seem to realize a *scaffolding* function, namely *demonstration.* In the sense of demonstration, they might try to provide each other and Berk with a position in which they become able to ‘imitate’ it back in a more appropriate form. From a supportive perspective the reaction of granny and the elder brother is a supportive activity, namely *modelling,* in that the elder brother and granny model an action for each other, and Berk can observe and imitate. The blocks set by granny and the elder brother are shown in the following (see Fig. 4.67.). Moreover, they come up with geometrical *framing* and expresses the right position of the blocks by setting the real blocks pursuant to the argument of the elder brother. They approach the block building activity from a geometrical perspective. Moreover, they engage in the negotiation process critically but constructively and collectively. They offer a hypothesis, which is made publicly accountable, and signalize striving to reach agreement with each other. Granny and the elder brother seem to negotiate with each other *in an exploratory way* that their ongoing action which can aid Berk to explore and examine how to situate each block correctly. From a participatory point of view, they ascribe the role of *collaborative game partner* to each other as they build the right corpus collaboratively, while ascribing the roles of *legitimate peripheral participant* to Berk. Moreover, granny is imitating the building action of the elder brother in that she doesn't perform any original idea. From the participatory point of view, she seems to take the role of *relayer*, while ascribing the role of *deviser* to the elder brother.

Granny moves K3 closer to K2 <522> (see Fig.4.68.). Regarding the previous interpretations at lines <500-501,503, 506, 509-515>, her reaction reinforces two ideas, that

- she might *really* try to build the figure as her elder grandson showed. In this regard her reaction seems like *environmental self-regulation* in that *s*he adapts local conditions in the play situation and tries to mobilize her elder grandson through communicative solicitation.
- she might *pretend* to try to build the figure as her elder grandson showed. In this sense she might be seen as carrying on her tactic of creating a learning situation for Berk. Regarding scaffolding theory, she might try to keep the elder brother in the field in order to let him draw Berk up towards a higher level of understanding by carrying on the *modelling* process.

From a participatory point of view, granny takes the role of *tutee*, while ascribing the role of *tutor* to the elder brother and that of *tutee* to Berk.

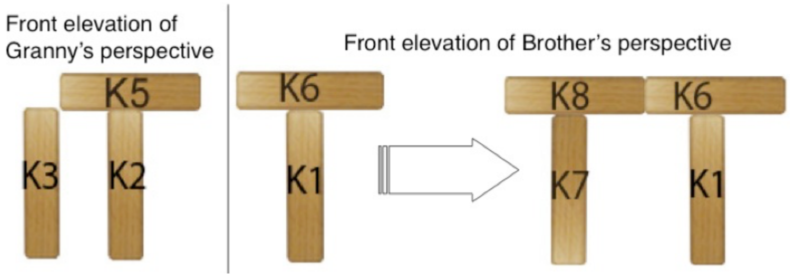


Fig. 4.68. Built corpus by granny, while the elder brother goes on building a corpus in front of him.

The elder brother takes K7 from the pile of blocks, puts it next to K1, in parallel, and then takes K8, and sets it horizontally centred on K7 <523-525> (see Fig.4.68.). His reaction gives the impression of being an ongoing action of setting blocks in the right way in order to reach a corpus identical to the figure on the chosen card. He employs a support activity, namely *modelling.* By going on setting further blocks he *models* the way of the building the right corpus. Most probably, the elder brother tries to show directly to granny, and concomitantly to Berk, the right way of building the corpus. In terms of scaffolding, the elder brother *demonstrates* how blocks should be set actually. By this *demonstration*, he might try to provide such a position for granny that she can get his point of view and *imitate* it back in a more appropriate form. He seems to construct his argument from the geometrical perspective; he is trying to perform his argument in 3D space. He *frames* the situation according to the mathematical domain of geometry. Thereby the elder brother seems to be able to represent 3D transformations, regulate their relations and link them with each other (see Table 2.4). He is an *units of units shape composer* from the developmental point of view (see Table 2.4). His performance in both turns leads us to the idea that he can master, conflate and perform spatial issues in a short amount of time. Thus, from a developmental perspective, he has sufficient spatial abilities for decomposing and composing the spatial field and regulating spatial relations (see Table 2.4). From a participatory point of view, he gives the impression of acting as an *author* by starting the modeling process. Moreover, he gives the impression of taking over the role of *tutor*, while he is ascribing the role of *tutee* to the granny and his younger brother. He seems latently to prove that he can be answerable to his younger brother as an *adviser*. From a developmental perspective, the ongoing action of the elder brother enables Berk to explore and steadily to examine the correct situation of each block.

Thereon granny sets K4 at the right side of K7, puts K5 horizontally centred on K4 and then asks the elder brother “one more isn’t it?” <526-527>. Her reaction gives the impression of maintaining the building action of the elder brother. By setting two more blocks next to the built corpus, she shows she has got the point of her elder grandson and carries on the building action in order to achieve the correctly built corpus. In this sense granny gives the impression of going on to imitate the building action of the elder brother in that she doesn’t perform any original idea. From the participatory point of view she seems to take the role of *relayer*, while ascribing the role of *deviser* to the elder brother.

From a supportive perspective, her reaction looks like a supportive activity, namely *modelling*, in that she models the building action, how each block should be situated, to build the right corpus. Thereby she sustains the ongoing action of the elder brother at lines <523-524>. By posing the question “one more isn’t it?”, she might try to ask whether one more structure should come next in the built corpus in order to achieve having three blocks in the middle of built corpus regarding the previous arguments of her elder grandson (see lines <22-23,505,507-508>). Her reaction recalls previous interpretations <500-501,503, 506, 509-515,522> that either she can really, or pretend to, get some clue from the elder brother, whether she maintains the right building activity. From a participatory point of view, she seems like either an *environmental self-regulator,* who is adapting local conditions in the play situation by posing questions and seeking approval, or a *tutee*, who is less expert and needs some help (see 2.2.2.2.). Furthermore, she might try to complete her p-turn successfully and thus to build the figure correctly regarding the geometrical framings of her elder grandson. In this sense her reaction can be perceived as a confirmation of the *geometrical framing of the elder brother.* By this means, granny seems to carry on one of the scaffolding functions, namely *demonstration,* so that she gives the impression of showing how and where blocks should be set. She might try to provide such a position for Berk and the elder brother so that they can *imitate* it back in a more appropriate form.

Regarding lines <523-528>, granny and the elder brother seem to perform an *exploratory* negotiation process in that they both engage critically but constructively with each other’s ideas and offer justifications cooperatively and collectively (see section 2.2.1.). Granny gives the impression of being dedicated, with her elder grandson, to the common pursuit of the correct corpus and thus they both perform a *collective argumentation process* in that they engage collaboratively and communicatively. Moreover, they seem to ascribe the role of *legitimate peripheral participant* to Berk since he can witness all their building activities. Moreover, in the negotiation process, Berk is not directly addressed or considered in the same manner to take part in the conversation between the elder brother and granny, but he is tolerated by them. In this sense they seem to create a learning situation for Berk, while they are ascribing to him the role of *legitimate peripheral participant* in the interaction process, whereas Berk reserves the role of *over-hearer* and *observer* for himself by observing and witnessing their negotiation process.

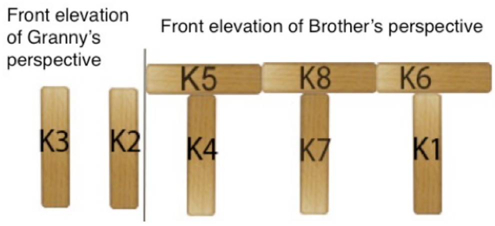


Fig.4.69. The corpus built by granny and elder brother

The elder brother nods and says, in Turkish, "too far..yes." <529>. Most probably he tries to *tutor* granny as to how she should set the blocks. By saying "too far" he might mean that she sets blocks K4 and K5 too far either from each other or from the built corpus. From a supportive perspective, his reaction seems a conjunction of two supportive activities: *instruction* and *affirmation.* First, he is *instructing*, that either the set blocks or the corpus are too far from other blocks. Then he seems to *affirm* granny as if she has set the two blocks right. From a participatory point of view, the elder brother seems to take the role of *tutor*, while ascribing the role of *tutee* to granny and Berk.

Granny says "yes", in English, and then takes K2 and puts it vertically centred on K8 <530> (see Figure 4.3.9.). Switching language from Turkish to English seems an emotional expression for her. Maybe granny estimates her achievement according to the elder brother's *affirmative* answers and thus she congratulates her perseverance by saying yes in English. Moreover, her reaction brings to mind that she used the same utterance in line <9>. Maybe she has the same argument as in line <9>, that she approves her success as she did in Berk's turn. From a supportive perspective, her reaction is a supportive activity, namely *affirmation*, in that she *affirms* herself that she goes on setting the blocks in the corpus right.

At the same time Berk remarks in Turkish that his granny says “yes” in English <531>. Granny laughs, takes K3 and puts it horizontally on K2 <532-533>. Then she asks elder brother if it is okay <532-533> (see Fig. 4.70.).

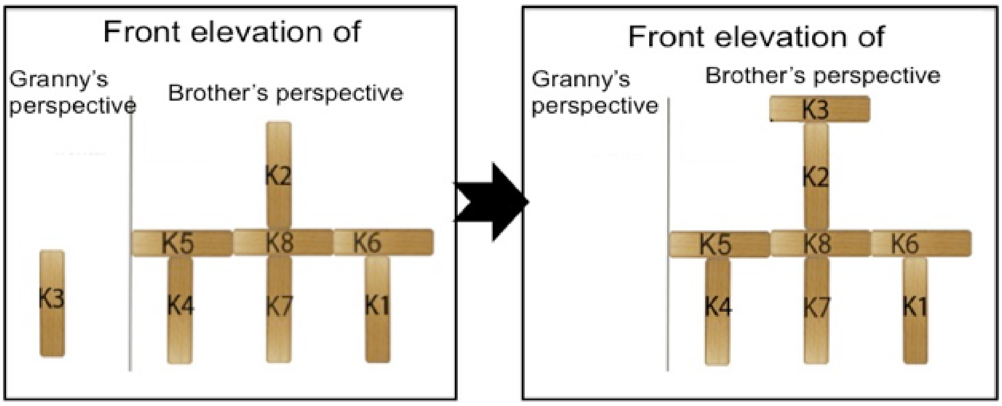


Fig. 4.70. The final building actions by granny

Maybe Berk is surprised at his granny’s reaction and thus makes fun of her by remarking on what she has done. Thereon granny laughs. Her reaction reinforces the idea that Berk makes fun of her and they are both having fun about it.

At the same time granny continues her building action, while she is asking whether the built corpus is okay <530-533>. Her reaction brings to mind previous interpretations <500-501,503, 506, 509-515,522, 526-527> that either she can really, or she can pretend to get a clue from the elder brother, whether she maintains the building activity right and the built corpus is okay. From a participatory point of view, she seems like either an *environmental self-regulator,* who is adapting local conditions in the play situation by posing questions and asking for approval, or a *tutee*, who is who is less expert and needs some help (see 2.2.2.). Furthermore, she might try to complete her p-turn successfully and needs confirmation from the elder grandson. By posing a question she may try to go on providing the position as in the previous lines (eg. <501,504,506,515,527>), in order to let the elder brother give more precise information about the building activity for his younger brother as an *adviser*. Granny seems to carry on one of the scaffolding functions, namely *demonstration,* so that she seems to go on showing how and where blocks should be set. She seems to go on providing a learning opportunity for Berk so that he can explore the right way of building a corpus identical to the figure on the card. From a participatory point of view, granny takes the role of *tutee*, while ascribing the roles of *tutor* to the elder brother and *tutee* to Berk.

Thereafter the elder brother nods <534>. His reaction shows approval that granny set the last two blocks in the right way and built a corpus identical to the figure on the card. From a supportive perspective, is a supportive activity, namely *affirmation*. He gives the impression of affirming either the last building action of granny or granny's achievement of the correctly built corpus. Moreover, the *collective argumentation process* with her elder grandson seems to lead granny to build a corpus identical with the figure on the chosen card. The negotiation process between granny and the elder brother seems to lead them to the working consensus that they built the corpus right. From a participatory point of view, the elder brother is taking the role of *tutor*, while ascribing the role of *tutee* to his granny and Berk.

Ultimately granny's p-turn ends.

Summing up from perspective of the interactional niche in the second observation period

In the chosen sequence, first Berk's p-turn and then granny's turn are observed, in which a polyadic interaction process emerges. From an allocative scope the elder brother and granny are official game partners of Berk and competitors in the play situation. Situationally Berk's turn differs from granny's turn, although the same playing card is chosen. In Berk's turn, he builds a corpus that resembles but is not identical to the figure on the card. In this sense he takes the role of *author* by performing his original idea but not *full participant* because of building a corpus that resembles but is not identical to the figure on the card (see section 2.2.). In his p-turn, the negotiation process between elder brother, granny and Berk is generated *disputationally* that granny and the elder brother express their disagreement and individual decision-making, assertions and somehow it restricts Berk's leeway of participation at the end of his p-turn. A *working interim* emerges between granny, elder brother and Berk unambiguously that the corpus built by Berk is wrong. Thereby Berk is deprived of turning up, discussing the built corpus further and achieving his p-turn, while he seems to accept the tutoring of his elder brother.

	Berk	**Elder brother**	**Granny**
Berk's Turn	author tutee	full participant tutor	relayer tutee
Granny's Turn	co-hearer, observer, tutee legitimate peripheral participant,	full participant tutor	relayer tutee

Table 4.11 The roles taken in both turns in the chosen scene

In granny's turn, granny and the elder brother build the corpus together. The negotiation process between granny and the elder brother seems to be accomplished in an *exploratory* way in that they are collaborating, reaching an agreement with each other and understanding each other's points of view. Moreover, they negotiate critically but collectively and constructively. The negotiation process between granny and the elder brother seems to lead them to the working consensus that they built the corpus correctly. They get a grip on the play situation and, in a relatively resolute manner, they realize a *collective argumentation* process and latently adapt some scaffolding functions into the play situation for Berk. They use both *numerical* and *geometrical framings* which let Berk be exposed to experiencing the way of building the corpus correctly, which he could not achieve in his turn. In this sense, an *exploratory* negotiation process between granny and the elder brother enables Berk to accomplish different participation profiles. While family members reserve for Berk the role of *legitimate peripheral participant*, he takes the roles of *over-hearer* and *observer*. Moreover, in both turns the elder brother takes the role of *tutor* and *full participant,* whereas he seems to reserve the role of *tutee* for Berk and granny. It is unclear whether granny – as an adult person – *pretends to need* or *really needs* tutoring from her elder grandson.

One possibility, considering family system theory (see 2.2.2.), is that granny uses a *tactic* as a creator of the *parenting pattern* in the nuclear family and the *sufficient* person in child-rearing. Thereby she can try to give the elder brother such a leeway that he can act as a *socializing agent, a social teacher* and a *model for social behaviour* for his younger brother. He can be answerable to his younger brother as an *adviser.* In this regard she can enable the elder brother to take more responsibility and show more concern for his younger brother, while she is creating a learning situation for Berk.

Another possibility is that she does not really get the wrongness of the way of building, or she has not enough spatial skills to grasp Berk's block building activity. Regarding family system theory, granny has a *loweducation level,* she may be less responser than the elder brother (see 1.4.1 and 2.2.2.) and thus she lets the elder brother *tutor* both Berk and herself.

In any event she takes the role of *relayer* in that she imitates the idea of Berk and the elder brother. This could be either as a *tactic* or *real action* of granny in that she reserves the role of tutor for her elder grandson, while ascribing the role of *tutee* to Berk. Moreover, Berk accepts the tutoring of his elder brother too. Through both turns Berk can experience making an error, then experience the reason for his error and investigate ways of fixing it, so that the interactional niche in the development of Berk's geometrical and arithmetical learning emerges perfectly.

Due to these notations the three components of an interactional developmental niche in the familial context can be structured as in follow:

<u>Component "content":</u>

Allocation x Content: In the chosen scene Berk and his family members are confronted with a mathematical play situation which offers opportunities to negotiating interactively for family members. The chosen play is structured in the mathematical domain of geometry and based upon the game "Make 'n' Break" (Lawson & Lawson, 2008). The aim of play is to rebuild 2D representations of the cards as 3D figures accurately with provided wooden blocks, which are of uniform size and weight.

Situation x Content: In the chosen play situation different types of negotiation processes emerge between family members. Whereas in Berk's turn they negotiate *disputationally* in that granny and the elder brother express their disagreement, decision-making and assertions through *numerical* framings, in granny's turn they generate an *exploratory* negotiation process through their *geometrical* and *numerical* framings. Moreover, in granny's turn a collective argumentation process is realized in that granny and the elder brother negotiate critically but reach an agreement collectively and constructively. Thereby in Berk's turn a *working interim* emerges unambiguously that he built the corpus wrong and was then deprived of turning up and discussing the built corpus, while with granny a working consensus arises about the way of building the right corpus.

Contribution x Content: In Berk's turn, he realizes the spatial relations between 2D and 3D objects so that he can relate some parts with the whole. He matches shapes using *gestalt configuration, composes corpuses* and uses multiple spatial relations extending in multiple directions through a systematic trial. He experiences building a robust corpus and getting similar *gestalt* with the figure on the card. In granny's turn, Berk witnesses all the situated activities of the elder brother and granny; he experiences and explores the static balance of the corpuses and the way of composing and decomposing of the corpuses units of units.

<u>Component "cooperation":</u>

Allocation x Cooperation: The chosen scene consists of two turns of one round, in which a polyadic interaction process emerges. The game partners of Berk are his elder brother and granny.

Situation x Cooperation: In this polyadic interaction process, Berk's brother and granny are collaborative game partners. In Berk's turn, Berk acts as an *author* so that he builds a corpus through his original ideas but his leeway is somehow restricted by granny and the elder brother so that he cannot discuss the built corpus later on. In granny's turn, granny and the elder brother reserve the role of *legitimate peripheral participant,* for whom there is an opening a way of gaining access to sources for understanding through growing involvement. Moreover, he is exposed to their *numerical* and *geometrical framings* and building activities, which can be a learning situation for him.

Regarding family system and scaffolding theories, it is obscure whether granny – as an adult person – pretends to need or really needs the tutoring of her elder grandson. When she does really need tutoring, however, she is as environmental self-regulator that she adjusts material means and local conditions of her learning environment by mobilizing her social environment through communicative solicitations e.g. asking for a clue from the elder brother, seeking approval or disapproval for her reactions, or initiating referential joint attention on the task with the elder brother. Maybe she does not have sufficient spatial skills to grasp Berk's block building activity because of her *low education level* and thus is less responsive than the elder brother (see section 1.4.1 and 2.2.2.2.)

When granny pretends to need tutoring from her elder grandson, then she acts as an *expert*, who is a wise adult, and lets the elder brother to be answerable to his younger brother as an *adviser*. Thereby she can try to let the elder brother such a leeway that he can act as a *socializing agent, a social teacher* and a *model for social behaviour* for his younger brother. In this regard she can enable the elder brother to take more responsibility and show concern for his younger brother, while she is creating a learning situation for Berk.

In any event the elder brother ascribes the role of *tutee* to Berk and granny, whereas he takes the role of *tutor*; in both turns Berk seems to accept the tutoring of his elder brother. Moreover, in boths turn the elder brother is a *full participant,* who has the mastery of geometrical and numeric arithmetical knowledge and skills required in the play situation.

In both turns granny imitates the elder brother, so that she neither takes responsibility nor have originality in her utterances and actions. Thus, granny takes the role of *relayer*.

Contribution x Cooperation: In his own turn, Berk acts as an *author* in that he builds a corpus through his original ideas and comes up with a totally new idea by building a corpus resembling the figure on the card.

In granny's turn, Berk takes the roles of *over-hearer* and *observer* so that he witnesses the collective argumentation process. By observing or rather witnessing of all the situated activities of his elder brother and granny Berk becomes able to examine his corpus with the last built corpus, so he can pick up some ideas that change his interpretation of the problem situation and to realize the mistake that he made. Moreover, the *numerical* and *geometrical framings* of granny and the elder brother let Berk be exposed to experiencing the way of building a corpus correctly, which he could not achieve in his turn. Considering the dynamics of the interactional turn-taking process, the commitment of a recipient to listen, observe, witness and pursue the ongoing actions implies a certain degree of attentiveness, which can be seen as a positive condition of initiating a learning process. Thereby the elder brother gives somehow more detailed information about the building activity in the collective argumentation process with granny, which leads Berk to organize information in his mind by relating concepts together and to achieve situated learning.

Component "Pedagogy and Education":

Allocation x Pedagogy and Education: Block building provides a view of children's initial abilities to compose 3D objects. In the chosen game, four goals are pursued: spatial structuring, operating shapes and figures, static balance between blocks, and identifying the faces of 3D shapes with 2D shapes. The US National Research Council reports that five-year-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National Research Council, 2009, p.191). The chosen scene refers to exploring and examining spatial structuring, visualizing and kinaesthetic imagery.

Situation x Pedagogy and Education:

In the chosen play situation, the elder brother takes the role of *tutor*, whereas he ascribes the role of *tutee* to Berk and granny. More specifically, regarding folk psychology and folk pedagogy, the elder brother seems the only person who is answerable to his younger brother. The elder brother is of secondary school age. He is the only person in the family who at that time attends formal education in a secondary school and he is actively exposed to geometrical and numeric analytical features in the mathematics classroom. As Bruner (1996) indicated, he is sensitive to and eager to adopt the folkways he sees around him. The elder brother seems to be answerable to his younger brother and he realizes a teaching situation for his younger brother in the same manner that he (the elder one) experiences it in the school. Regarding folk psychology and folk pedagogy, the granny can also have ingrained cultural beliefs about the mind that the elder brother os the responsible one for the youngest child in the family and should take on this responsibility. Moreover, considering folk theories, one can speak about the common education system in the world, that everyone, whether of high or low educational level, is exposed to learning “numbers and operations” at a basic level. In this regard, granny most probably does know the numbers and can do addition and subtraction, at least, so that she can come up with the numeric analytical approach during the block play situation so that she can accompany her elder grandson's *numerical framings,* whereas she does not or cannot really come up with any geometrical argument in the negotiation of taken-as-shared meanings. Considering these ideas, two possibilities come forth: One possibility is that granny does not really get the wrongness of the way of building or she has not enough spatial skills to grasp Berk’s block building activity. Regarding family systems because of granny’s *low education level* and folk theories she may be less responsive than the elder brother (see 1.4.1 and 2.2.2.2.). Another possibility is an effort to let the elder brother be answerable, responsible, a *socializing agent, a social teacher* and a *model for social behaviour* for his younger brother as an *adviser*, in terms of family system theory. In both ways her reaction and the reaction of the elder brother lead the interaction process to the occurrence of some scaffolding function.

Regarding the six scaffolding functions, the elder brother and granny expose Berk to two, namely *demonstration* and *marking critical features*, whereas they do not draw on the other scaffolding functions, namely, “recruitment, frustration control, direction maintenance, reduction in degree of freedom” (see Wood et al., 1976):

- **Marking critical features:** The elder brother obviously emphasizes *numeric analytical* and *geometrical* features of the built corpuses and comes up with different aspects of the building activity that are important or relevant for its completion. Thereby granny and the elder brother *frame* the play situation with *numerical* and *geometrical arguments,* while they negotiate about building activities in both Berk’s and granny’s turns. Specifically, the elder brother accentuates the amount of the blocks in the middle of the built corpus and interprets features and aspects of the building activity (see lines <22-23,505-517,529>). In this sense they prompt Berk by instructing about the amount of blocks in the middle part of the built corpus and try to call his attention to the numerical perspective, which touches on folk psychology and folk pedagogy.

- **Demonstration:** The elder brother and granny model the *idealized* position of the blocks to requirements of the building activity, which can be *imitated* in the course of further block building process. By changing the position of the blocks they use *maternal planning* and model the correct way of building the corpus in terms of which way the coming blocks should be set <520-528,530-533>. Thus, they perform a new *idealized subject* in granny’s turn in order to let Berk *grasp* the right way of building the corpus in a more appropriate form.

Berk’s turn differs from granny’s turn, although the same playing card is chosen. In Berk’s turn, the negotiation process between the elder brother, granny and Berk is generated *disputationally* in that granny and the elder brother express their disagreement and individual decision-making assertions. In granny’s turn, the *exploratory* negotiation process between granny and the elder brother seems to be accomplished in that they collaborate, reach an agreement with each other, understand each other's points of view and build the corpus together. In this sense they negotiate critically but collectively and constructively, so that they realize a collective argumentation process, while they are *modelling* block building activity. In the negotiation process, the elder brother and granny use *verbal stimulations* and *direct but not elaborative commands*. Moreover, in both turns, granny and the elder brother realize different supportive activities such as *prompt, prompt after error, affirmation, disaffirmation, instruction, modelling, conclusion* and *re-representation.* Thereby they create for Berk a learning situation in which he can witness and observe this negotiation process and explore the right way of building the corpus.

Contribution x Pedagogy and Education:

In Berk's turn, he realizes the spatial relations between 2D and 3D objects, relates some parts with the whole and experiences building a robust corpus and getting similar *gestalt* with the figure on the card. Thus, Berk gives the impression of acting as a *shape composer.* Thereby he seems able to match shapes using *gestalt configuration, use multiple spatial relations, use systematic trial* and *compose corpuses* but not unit by unit in great detail. In the chosen play, spatial structuring, identifying the faces of 3D shapes to 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures, are pursued. Although Berk does not directly operate with shapes and figures physically in granny's turn, he is steadily informed by his elder brother and granny at exploring and examining the process of situating each block. Thereby he can structure shapes and figures in his mind as an observer who can participate with keen attention to ongoing events and provide a learning process for himself. In this respect Berk gets learning opportunities: how the position of the blocks can be represented and how spatial relations and kinaesthetic imagery can be configured. Through the supportive activity of *modelling*, granny and the elder brother enable Berk clearly to observe and thus to explore the setting each block right and balancing them intuitively. By observing or rather witnessing of all these situated activities of the elder brother and granny, Berk becomes able to examine his corpus in comparison with the corpus they have just built and to realize the difference between them. Two scaffolding functions realized by granny and the elder brother enable Berk to observe, grasp, realize and explore the way of building the right corpus, which he could not build in his p-turn. Furthermore, he gets a chance to compare his corpus and the right corpus in his mind and to find out his mistake. These opportunities enable Berk to appropriate for himself a learning situation. Namely he witnesses all the activities of elder brother and granny, which enable him a *situated learning* (see section 2.2.1.).

The interactional niche in the development of Berk's geometrical thinking and learning occurs latently. Due to these three components, the interactional developmental niche in the Gül family context is structured as follows (Table 4.12):

Table 4.12. NMT of Berk in the game "Building 02"

NMT-Family Gül **Building 02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Geometry, spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.	Playing with elder brother and granny	Theory of the development of spatial skills and spatial structuring: identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly and indirectly operating shapes and figures
aspect of situation	Disputational and explatory negotiation process between Berk, elder brother and granny Working interim and working consensus Geometrical and numerical *Framings*	Different leeways of participation: restricted leeway for Berk Legitimate peripheral participant tutor-tutee	Two scaffolding functions by granny and the elder brother Directly acting with the play materials Enabling to examine his corpus with the last built corpus and to realize the mistake

aspect of Contribution	Exploring static balance to build the robust corpus; Examining the resembling gestalt of the figure and the corpus Overhearing all the situated activities of elder brother and granny.	author in his own turn Over-Hearer and observer in granny's turn	Witnessing of all the situated activities of elder brother and granny. Situated Learning Learning opportunities about the way of building right and robust corpus

Functioning of MLSS and overview of NMT_1 and NMT_2 for Berk:

In the first observation period a sequence of a play situation of Berk, his elder brother and mother was chosen and observed. In the second observation period a game with Berk, his elder brother and grandmother is examined. In both observation periods, polyadic interaction processes emerge and intensive negotiation processes arise between the adult person and elder brother about building activities and built corpuses.

In both play situations Berk participates actively and the role of *tutee* is ascribed to him by adult person and his elder brother, whereas the elder brother acts as a *tutor*. Regarding family system theory, the elder brother *elicits many more explanations from his younger sibling than from the adults* and seem to *enjoy a privileged teaching status* as tutor (see sections 2.2.2. and 2.2.3.). Furthermore, Berk accepts the tutoring of his older sibling, who is answerable and responsible to his younger brother. In this regard, both adult persons offer an open leeway for the elder brother so that he can be somehow answerable to his younger as an *adviser* and also act as a *socializing agent, a social teacher* and a *model for social behaviour* for his younger brother, while creating a learning situation for Berk. Thereby both adult persons take either the role of *tutee* or *expert* in both observation phases.

In both play situations, the adult person and the elder brother get a grip on the play situations and adopt some scaffolding functions in the negotiation process. Their *numeric analytical* and *geometrical framings* enable Berk to experience different geometrical features. In the play situation with his mother and elder brother, Berk observes and explores different building alternatives of each figure on the chosen card, whereas in the play situation with granny and elder brother Berk observes and explores the way of building corpuses identical to the figures on the chosen cards. In the course of the interaction process, the adult person and elder brother perform similar supportive activities. They use *prompt, prompt after error, affirmation, disaffirmation, instruction, modelling, conclusion, providing solution, motivation, and re-representation* (see section 2.2.3.) during negotiation of taken-as-shared meanings.

In this sense the characteristics of the adult-child-sibling interaction in the Gül family do not differ from each other. In both situations the combination of supportive activities and some scaffolding functions used by family members leads Berk to different learning situations, so that he participates in both play situations as *observer and over-hearer,* while taking the role of *legitimate peripheral participant.* In terms of situated learning, Berk can observe and witness all their block building activities during negotiations of taken-as-shared meanings. Berk fulfils leeways of participation in the same way in the course of verbal and nonverbal outputs of the adult person-elder sibling dyad. This dyad enhances Berk's own geometrical development by observing and grasping all situated activities that take place. Through observing and witnessing the adult person-elder sibling dyad Berk experiences a learning situation. In this sense the adult person-elder sibling-Berk triad generates a MLSS for Berk by the occurrence of *geometrical* and *numeric analytic framings.*

Regarding the functioning of the MLSS, the following questions arise and their answers are given in detail:

- Which kinds of *format* provide a learning situation for Berk in this familial system? and,
- How do these *formats* provide a learning situation in the first and second observation phases?

In accordance with family system theory, two adaptive and complementary systems can be seen in the Gül family.

Regarding family system theory and family members' regulation (see 2.2.3.), in the interaction process between the adult person and the elder brother emerges a mutual adaptation, in which they enable Berk to experience a learning situation in a *complementary* way*.* By observing and overhearing, Berk witnesses all these situated activities and becomes involved in this interaction process. In terms of family system theory, the elder brother engages in teaching roles in the family and functions as a *tutor* or an *adviser* to his younger brother during social interactions in the family system (see 2.2.2.2.1.). He is *responsible* for his younger brother in the family, while Berk is more likely to be a *follower*, an *observer* and a *learner* (see 2.2.2.2.1.). The elder brother has a crucial role in the negotiation of taken-as-shared meanings by initiating collective argumentation processes with an adult person. Specifically, his individually uttered or enacted arguments are reciprocated by an adult person. Their arguments and framings bring on the dynamic of the interaction process, in which Berk and the mentioned dyad involve and engage conjointly. In this sense the elder brother-adult dyad serves as a bridge between learning action and Berk, which influences *Berk to organize information in the mind by relating concepts together*. Thereby this occurrence provides Berk such a leeway, that he can have *some periods of solitude* to combine all arguments and framings *from his own head with the ideas that his partners have in theirs*. Moreover it lets Berk cultivate and furnish *all ideas with those models and techniques for how to operate on his own*. He becomes involved in the situation by accepting and walking on this *bridge towards the learning action.* In this sense such bridge between learning action and Berk, which is served by the elder brother-adult dyad can be defined as a format, which is "standardized, initially microcosmic interaction pattern between the elder brother-adult dyad and Berk that contains demarcated roles that eventually become reversible"(Krummheuer & Schütte, in press).

With respect to the functioning of the MLSS in both observation phases, the overall Interactional Niche for Berk (NMT-Family Gül) can be presented in the following way (Fig. 4.71):

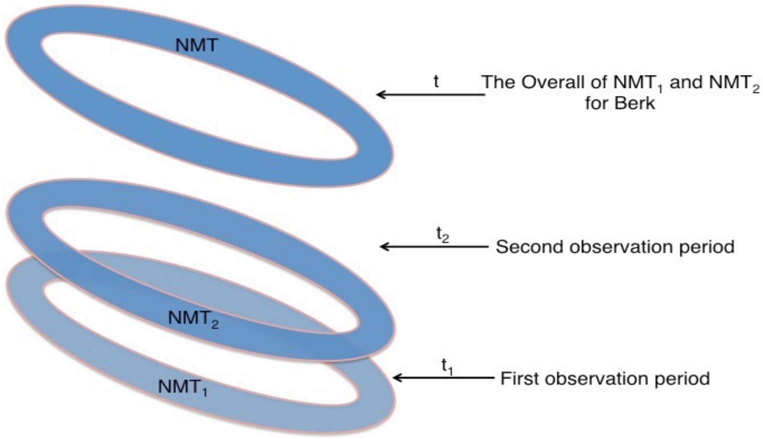


Fig. 4.71. NMT ($NMT_1 + NMT_2$)for Berk (“NMT-Family Gül”)

This diagram shows the relationship between NMT and the time axis, which provides evidence for the Berk’s further development. In the first obversation period Berk experiences geometrical activities less intensive then in the second obversation period. Therefore, the interactional niche in the development of geometrical learning of Berk in the first observation period is labelled in light blue, whereas the interactional niche in the development of geometrical learning of Berk in the second observation period is labelled in dark blue. In both periods Berk also experiences numerical features of block building activities in a limited fashion, which is labelled in pink in the diagram (see Fig.4.71.). In this regard the overall of NTMs of Berk in the diagram is labelled in the colour, which is combination of colours of both NTMs.

Bearing in mind that all cultural, lingual, social and emotional factors are embedded (see sections 2.2. and 1.4.1.), similar regulation by family members the legitimate peripheral participation of Berk constitute a format which provides Berk with a learning situation from a spatial and numeric analytical perspective. Within this context the MLSS is notably high in the spatial and low in the numeric analytical development of Berk. In this regard the detailed overall NTM can be furnished as follows:

The reflection of NMT ($NMT_{1+2} = NMT_1 + NMT_2$) for Berk

<u>Component "content$_{1+2}$":</u>

Allocation x Content: Both games, "Building 01 & 02", are structured in the mathematical domain of geometry and based upon performing spatial skills. After choosing one card from the deck, each player should build a corpus related to the figure on the card. Thereby play situations facilitate each player to perform their spatial skills.

Situation x Content: Between Berk, his elder sibling and an adult person (mother, grandmother), mainly geometrical and fewer numeric analytical features of building action and built corpuses are thematized. The elder brother, mother and grandmother generalize the negotiation process by their geometrical and numerical *framings as reiterating Berk's geometrical and numeric analytical skills.*

Contribution x Content: Both in the scene from "Building 01" and in his own turn in "Building 02", Berk contributes to the negotiation processes. In any event he is exposed to the experience of geometrical and numeric analytical features of the building actions and built corpuses. Thereby he can explore different building alternatives of the figures on the chosen cards, static balances and *gestalts* to build robust corpuses and ways of building corpuses correctly. Moreover he becomes able to distinguish some imperfections in the built corpuses during the process of negotiation between family members. Thereby he gets learning opportunities from geometrical and numeric analytical perspectives through the tutoring of his elder brother and the negotiation of taken-as-shared meanings between family members.

<u>Component "cooperation$_{1+2}$":</u>

Allocation x Cooperation: In both chosen scenes, adult-sibling-child interaction is actualized. While in the first scene Berk plays with his elder brother and mother, in the second observation period he plays with his elder brother and grandmother. In each play situation Berk's game partners are respectively dyads of his mother-brother and his grandmother-brother. In both chosen scenes the triad setting “child-sibling-adult (a family member)” is actualized.

Situation x Cooperation: In both observation phases, polyadic interaction processes emerge between Berk, his brother and the adult person. In both cases, family members shape Berk’s options for participation in such a way that they reserve the role of legitimate peripheral participant for Berk. The triad ambience enable Berk to observe, to see, to overhear and to explore different spatial features so that he can concern himself with the learning offerings, which are provided by elder brother, mother and granny. In both observation phases the elder brother takes the role of tutor and the adult person acts as a tutee. In terms of production design (see section 2.2.1.), in Berk’s own turns family members restrict his leeway so that he can act only as an *author and ghostee.*

Contribution x Cooperation: Negotiation processes and building activities between elder brother and each adult person in both play situations offer Berk different learning situations that he can arrogate to himself each building activity of the family members and negotiations of taken-as-shared meanings. Berk participates in the play situations as a recipient of a motional, verbal and nonverbal actions so that he can pick up some ideas that change his interpretation of the problem situation. In the first observation phase, the negotiation process of elder brother and mother enables Berk to *receive* all spatial features of their building activities. Considering the dynamics of the interactional turn-taking process, the commitment of a recipient to listen, observe and witness and pursue the ongoing actions implies a certain degree of attentiveness, which can be seen as a positive condition of initiating a learning process. Thus, they shape Berk’s options for participation in such a way that he seems only a recipient of their activities. Similarly in the granny’s turn in the second observation phase, such a negotiation process occurs between elder brother and granny that Berk seems again as a recipient of their activities. Thus Berk observes, sees, overhears and thus can explore different spatial features so that he can concern himself with the with the learning offerings, which are provided by elder brother, mother and granny.

Component "Pedagogy and Education$_{1+2}$":

Allocation x Pedagogy and Education: In both play situations the following goals are pursued: spatial structuring, identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures, using kinaesthetic imagery and spatial visualization, and realizing spatial operations. The US National Research Council has reported that five-year-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the relating parts and wholes level (National Research Council 2009, p.191). Therefore block building activities enable children to practise their spatial skills. So for children an interactional niche in the spatial thinking of children can emerge.

Situation x Pedagogy and Education: In both play situations all family members interact with the play materials. Through similar scaffolding processes initiated by dyads of elder brother-mother and elder brother-grandmother, disputational and exploratory negotiation forms emerge, in which highly geometrical and some numeric arithmetical argumentations about the block building activity are constructed. In terms of folk psychology and folk pedagogy, family members shed light mainly on the geometrical and numerical features of the building activities. During collective argumentation processes between the family members, the same supportive activities are realized, which lead Berk to learn geometrical and numerical notations. In addition to this, elder brother-child-adult triads accomplish interaction processes by constructing and co-constructing taken-as-shared meanings. Berk accepts his elder brother's tutoring, through which the mastery of required geometrical and numeric arithmetical knowledges and skills are revealed in both play situations. Moreover each adult person seems to act as a *tutee*, while ascribing the role of *tutee* to Berk as well.

Contribution x Pedagogy and Education: In both cases Berk witnesses all the situated activities of elder brother-adult person dyads. Thus Berk witnesses matching and balancing shapes, using *gestalt configuration* and *multiple spatial relations,* and *composing corpuses* unit by unit in great detail. Moreover, he overhears all the arguments of the elder brother-adult person dyads, so that he is exposed to geometrical and numeric analytical framings. In both observation phases Berk gives the impression of acting as a *shape composer.* In the course of both play situations he can structure shapes and figures in his mind and participate with keen attention to ongoing events of elder brother-adult person dyads. Furthermore, he can realize the spatial relations between 2D and 3D objects in great detail, while relating some parts with the whole.

With respect of the above mentioned and reflected three components and their aspects, the NMT table for Berk can be structured in the following way (Table 4.13):

Table 4.13 The overall of NMT_1 and NMT_2 of Berk

NMT-Family Gül **Building 01,02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation	Mathematical domain: “Geometry and spatial thinking”, using spatial skills at the building activity	Playing with elder brother, adult person	Theory of development of spatial skills and spatial structuring: identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly and indirectly operating shapes and figures
aspect of situation	Disputational and explatory negotiation process between Berk, elder brother, adult person about the built corpuses Working interim and working consensus	Legitimate peripheral participation	Modelling, by which adult person and elder brother directly acts with the play materials. Enabling to realize the difference between right- and wrong-built corpuses Situated Learning for Berk

	Geometrical and Numerical Framings		
aspect of contribution	Exploring static balance to build the robust corpus; Exploring and examining different building varieties, experiencing spatial features	Listener / Over-Hearer and Observer	Learning opportunities, how the position of the blocks can be represented and how spatial relations and kinaesthetic imagery can be configured. *Witnessing of* all the situated activities of brother and adult person.

In sum, the overview of these three components of NMT-Family Gül leads us to the conclusion that Berk is involved in the interactive accomplishment of NMT that obviously offers him successful support in the development of geometrical learning. The situational aspect and the aspect of contribution in the NMT-Berk coalesce dynamically so that a MLSS comes into being as a format in the grey labelled area of Table 4.13. The part of “Contribution x Pedagogy and Education” seems compatible with the mathematical learning situation in which Berk benefits from the learning offerings, which are provided by elder brother, mother and granny. Through disputational and exploratory negotiation processes with family members, learning opportunities are provided for Berk. A MLSS is constituted such that Berk can explore and learn spatial features intensively and also some numerical features.

4.4. Comparison

Regarding the above analyses, the results of reconstructions of all the chosen sequences of the three families are compared in this section. For this, the concept of interactional niche is used for an elaborative and well-structured comparison model, which specifies the three components of NMT: content, cooperation, and pedagogy/education.

Component “content”:

Allocation x Content: Both games, “Building 01 & 02”. are allocatively located in the mathematical domain of geometry and based upon performing spatial skills. After choosing one card from the deck, each player should build a corpus related to the figure on the card. Thereby play situations facilitate each player to perform their spatial skills.

Situation x Content: In the case of Aleyna playing with her mother, a *working interim* emerges that all the corpuses are built correctly, whereas when she plays with her father a working interim emerges that she built the corpuses incorrectly. Moreover, the negotiation process between Aleyna and her mother occurs in a *cumulative* way as they build and do everything right. In the case of Ayse, in both observation phases, in contrast to the play situations of Aleyna, a *working consensus* emerges in which all players reach an agreement whether the player built the corpus correctly. In the case of Berk, similar to Aleyna, *working interims* emerge in his own p-turns, that he built the corpuses incorrectly, but he could not get any detailed further information as to why he built it that way or how he could fix it. In this sense *disputational* negotiation processes emerge in Aleyna’s own p-turns while playing “Building 02” with her father and in Berk’s own p-turns in both “Building 01” and “Building 02”. Whereas in other family members’ p-turns Berk is exposed to an *exploratory* negotiation process, Aleyna does not have this opportunity. Like Berk, Ayse performs an *exploratory* negotiation process with her father so that she can explore what was the fault in the built corpuses, or or how she can fix it, whereas she uses a *disputational* negotiation process with her mother. Ayse’s parents come up with *geometrical framings,* whereas Aleyna’s come up with *numeric analytical framings.* In both of her play situations, *working interims* emerge between Aleyna and her parents, whereas there is a *working consensus* between Ayse and her parents. In both play situations, Berk’s brother and adult persons (granny and mother) come up with highly geometric but rarely numeric analytical *framings*. Moreover, *working interims* emerge between them in all Berk’s p-turns, whereas in their own turns *working consensuses* emerge between them in both observation phases.

Contribution x Content: In both games, spatial structuring, identifying the faces of 3D shapes to 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures are pursued. In both observation phases, Berk, Ayse and Aleyna perform the spatial relations between 2D and 3D objects and relate some parts with the whole. They can all see, interrogate, realize, examine and perform the block building activities during the interaction processes in different intenseness. Whereas Ayse is always able to building corpuses identical to the figures on the chosen cards, Berk and Aleyna are not. Moreover, they all experience and explore different varieties of building, static balances and *gestalts* of the blocks to build robust corpuses. Berk matches shapes using *gestalt configuration, composes corpuses* and uses multiple spatial relations extending in multiple directions through a systematic trial, whereas Aleyna builds vertical components within a building though with limited range, uses multiple spatial relations extending in multiple directions but does not use systematic trial and thus somehow errs at adding pieces. Ayse *composes structures from pictured models and* produces arches and corners with vertical and horizontal spaces.

<u>Component "cooperation":</u>

Allocation x Cooperation: Both games, "Building 01" and "Building 02", are designed for five rounds of play in total. Each player should play by turns in each round. In this regard each player is a competitor and official game partner of the others. What the experimental design of erStMaL-FaSt project required is that Aleyna and Ayse, as only children, should perform one play situation with their fathers, one with their mothers and one with both parents, whereas Berk with his brother should perform one play situation with their father, one with their mother, one with both parents and one with their grandmother (see section 1.4.3.).

Situation x Cooperation: In both play situations of the Gül family polyadic interaction processes emerge, whereas in Kil family dyadic interaction processes emerge. In the Ak family dyadic interaction process emerges in the first observe phase, whereas in the second phase a polyadic one emerges, which was not anticipated. Aleyna and her father choose the game "Building02" to play as a pair. In this regard Aleyna's game partner is only her father and her mother accompanies them behind the cameras by watching and making interpretations during their negotiations. Therefore the mother is a spectator, who can see the chosen cards, building corpuses and whole building activities of Aleyna and her father in the course of the play situation. Moreover, the Ak family accomplish four rounds in the game "Building 01" and five rounds in "Building 02" by p-turns, i.e., alternately, as anticipated in the settings of the mathematical play situations "Building 01" and "Building 02". Similarly, Ayse and her father proceed five by p-turns, i.e., alternately in the game "Building 01", whereas in the game "Building 02" Ayse and her mother realize a collective argumentation process in that they build each corpus together without abiding by the game rules. The Gül family accomplishes more than five rounds by p-turns, i.e., alternately as anticipated in the settings of "Building 01" and "Building 02". In playing "Building 01" Berk's official game partners are his mother and brother, whereas his grandmother and brother are his official game partners in "Building 02". In both the Kil and Ak families adult persons act as *tutor*, while ascribing the role of *tutee* to the child (Aleyna and Ayse). In contrast, in the Gül family, Berk's brother takes the role of *tutor*, while ascribing the role of *tutee* to his younger brother and the adult person (mother and grandmother), which is unusual in a socio-contructivist approach. More precisely Ayse's father takes the role of *activator*, whereas Aleyna's mother acts as a *nurturer* and Berk's elder brother as an *adviser* in the course of play situations. In both play situations, her parents provide Aleyna a restricted leeway so that she can only be *author* or *ghostee* in terms of production design (see 2.2.1.). In Berk's own turns, family members restrict his leeway too, so that he can only be an *author* or *ghostee* in terms of production design (see 2.2.1.), while they ascribe the role of *legitimate peripheral participant* to Berk in their own turns. In contrast to Berk and Aleyna, Ayse has an expanded leeway in that she can discuss the built corpuses with her parents. But more precisely, she has more restricted leeway while she is playing with her mother in that the only *participation* profiles she has are: *author, spokesman,* or *relayer*. While she is playing with her father, she can undertake almost all the *participation* profiles: *author, ghostee, spokesman, sponsor, relayer, deviser* (see section 2.2.1.).

Contribution x Cooperation: In both play situations Berk takes the role of *author* in his own p-turns, in that he performs his own idea, whereas in others' p-turns he is an *over-hearer* and *observer* who receives and witnesses all the spatial features of building activities of family members. Ayse playing "Building 01" and Aleyna playing "Building 02" are environmental self regulators. They both adjust the material and local conditions of their learning environments by mobilizing their social environment through communicative solicitations. In playing "Building 01" Aleyna takes the role of *central participant*, as she places herself at the centre of her mother's interest, while in playing "Building 02" Ayse is regulated by her mother externally. In this sense Ayse takes the role of *externally regulated participant* so that she acts in a way that her mother directs.

<u>Component "Pedagogy and Education":</u>

Allocation x Pedagogy and Education: Spatial structuring, identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly or indirectly operating shapes and figures, using kinaesthetic imagery and spatial visualization, realizing spatial operations are the goals of in both block plays. After the age of three, children's developmental level can be characterized as *picture maker*, meaning that they can at least use multiple spatial relations extending in multiple directions but cannot use systematic trial and thus they may err at adding pieces (see Table 2.4.). Children of about the ages of four and five, as *3D shape composer*, can compose shapes with anticipation, produce arches, enclosures, corners systematically. Children aged six and over, as *3D shape composer-units of units*, can build adult-like structures with blocks and make complex towers or other structures, which involves multiple levels with ceilings and other substructures. (see Table 2.4). Furthermore, over the age of six, children can produce arches with vertical and horizontal spaces. The US National Research Council has reported that five-year-old children can understand and replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the level of relating parts and wholes (National Research Council, 2009, p.191). Therefore, block building activities enable children to practise their spatial skills. Block play contributes to children gaining physical, social, emotional and cognitive growth (see section 2.1.).

Situation x Pedagogy and Education: In the play situations of all the families some scaffolding functions are realized. In the Kil family, the father offers all the scaffolding functions (recruitment, frustration control, reduction in degree of freedom, marking critical features, direction maintenance, demonstration) whereas Ayse's mother realizes just three (demonstration, marking critical features, direction maintenance). In "Building 02" Aleyna's parents realize four scaffolding functions together (marking critical features, direction maintenance, recruitment, frustration control), whereas in play situations with Berk his family members realize only two of them (demonstration and marking critical features). In all families the usage of the scaffolding function "marking critical features" is the most common. Regarding folk psychology and folk pedagogy, it does not seem surprising that the family members come up with arguments in areas in which they are competent and accentuate certain features of the block building activity that are relevant to them. In this regard such *markings* provide information about the issues, how they interpret the current issue, and which results they yield. More precisely, Ayse's mother very often uses directivity and minimal-effort strategies in the negotiation process with Ayse and thus she regulates Ayse's building activities *externally,* whereas Aleyna's mother negotiates with Aleyna cumulatively so that they generate the negotiation process positively but uncritically by repetitions and confirmations from the mother. In this sense Aleyna's mother meets her emotional needs and she is exposed to her mother's *nurturing* and *numeric analytical framings* in both play situations. Like Aleyna's mother, her father comes up with the numeric analytical framings in terms of folk psychology and pedagogy. Moreover, he seems to try regulate Aleyna emotionally but not in the same way as mother. Rather he seems to regulate Aleyna in such a way that she can turn up with persisting in opposition to her father’s idea and might try to expand her leeway by fixing the faults of the built corpus. Here the negotiation process between Aleyna and her father is disputational and is characterized by disagreements, short assertions and counter-assertions of her father, whereas Ayse’s father responds in an auxiliary manner to her motivational and emotional needs in terms of emotional and motivational regulation. In this regard Aleyna's parents strike a balance between *tutoring* and *nurturing* in the game “Building 02” in that they realize four scaffolding functions together, whereas Ayse’s father and mother realize scaffolding functions separately in different observations. In terms of folk psychology and folk pedagogy, Berk’s family members shed light mainly on the geometrical and numerical features of the block building activities, whereas Aleyna’s family refer only to numerical features and Ayse’s family only to geometrical ones. Thereby Ayse is exposed to building vertical and horizontal corpuses without needing any long or short blocks, while Aleyna only experiences counting and calculating the amount of the blocks either in the figure on the card or in the built corpuses, although her built corpuses are wrong. Similar to Aleyna, Berk is also exposed to amount of the blocks in the building activities, which characterizes numerical features of block building activities. What differs from Aleyna is that he can explore geometrical features of block building activities in all p-turns as well. Both Aleyna and Berk experience building blocks only vertically, whereas Ayse experiences building blocks both vertically and horizontally while she is playing with her mother. Moreover, Ayse performs a collective argumentation process with her mother in that they build the figures on the cards collectively and cooperatively. Like Ayse, Berk’s family members perform a collective argumentation process as well but without Berk’s active participation. This means Berk takes the role of legitimate peripheral participant instead of active participant. In this regard family members enable *situated learning* for Berk in that he can witness all the building activities of the adult person and his elder brother. Thereby the elder brother proceeds with modelling process with the adult person, while marking critical features of the building activities. In this regard the elder bother negotiates mainly about the geometrical issues with the adult person, while they are using some *numeric analytical framings* in terms of folk psychology and folk pedagogy. All the family members of Ayse, Aleyna and Berk enable these children to gain learning opportunities about building robust and correct corpuses, exploring static balance of the blocks and different spatial features during both play situations. Moreover, they use similar supportive activities in similar ways, namely *prompt, prompt after error, affirmation, disaffirmation, providing solution, motivation, conclusion, modelling, instruction* and *re-representation,* although they all realize different types of negotiations processes between each other.

Contribution x Pedagogy and Education: In both play situations Berk and Aleyna become able to interrogate the imperfection of the building corpuses, whereas Ayse already does. Whereas Aleyna experiences highly numeric analytical features of built corpuses and rarely geometrical ones, Berk experiences highly geometrical features of built corpuses and rarely numeric analytical ones. Differently from Berk and Aleyna, Ayse experiences geometrical features of building activities very intensively. Aleyna's parents enable her to develop her numeric abilities strongly and her spatial abilities weakly. While at the beginning of the observations Aleyna acts as a *piece assembler* in that she is not able to *realize* or *see* the difference between the wide and narrow sides of blocks in the figure on the chosen card, she becomes aware of the static balance of the corpus which makes Aleyna come up with geometrical arguments as a *picture maker.* So Aleyna becomes able to build vertical components within a building though with limited range, and use multiple spatial relations extending in multiple directions, but she cannot use systematic trial and thus may err at adding pieces. Ayse’s parents enable her to develop her spatial abilities very intensively. Whereas at the beginning of both sequences Ayse acts as a *picture maker* in that she can use multiple spatial relations to produce arches and corners unsystematically but makes errors in addition of pieces, through the regulation types of her parents she acts as a *shape composer-units of units* so that she becomes able to *compose structures from pictured models unit by unit* and produce arches and corners with vertical and horizontal spaces systematically.

Differently from Aleyna and Ayse, Berk witnesses matching and balancing shapes, using *gestalt configuration* and *multiple spatial relations,* and *composing corpuses* unit by unit in great detail. In both observation phases Berk acts as a *shape composer*, in that he can match shapes using *gestalt configuration, use multiple spatial relations, use systematic trial* and *compose corpuses* but not unit by unit in great detail. Compared with Aleyna and Ayse, Berk does not directly operate with shapes and figures physically in others' p-turns, but he can be steadily informed about the block building activities by overhearing and observing his elder brother and an adult person negotiating taken-as-shared meanings. By participating with keen attention to the ongoing events of his elder brother and the adult person, Berk provides a learning process for himself whereby he can learn to *compose structures from pictured models unit by unit* and produce arches and corners systematically. Thus Berk, Aleyna and Ayse vigorously and frequently experience building activities through which they appear as increasingly autonomous participants. Moreover, they can investigate and predict the results of combining, subdividing, and changing shapes; explore properties of geometric shapes, transformations and their relationships to the concepts of area; apply the concepts of symmetry, similarity and congruence; and describe geometry in nature and real-world applications by using models and manipulations. In addition, playing with family members at block building enables Ayse, Aleyna and Berk to practise their spatial skills, and thus to gain more or less physical, social, emotional and cognitive growth (see section 2.1.). Thereby in any event they get different learning opportunities either about mathematics or social life or both.

In this regard to all three children, there emerge interactional niches in the development of mathematical thinking of different intensity.

Regarding these insights the NMT tables of Berk, Aleyna and Ayse can be assembled as follows:

Table 4.13 The overall NMTs of Family Kil, Gül and Ak

NMT-Families **Building 01,02**	**component:** **content**	**component:** **cooperation**	**component:** **pedagogy and education**
aspect of allocation **The Kil Family** **The Ak Family** **The Gül Family**	Geometry, Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3D shapes with 2D shapes.	Playing with family members	Theory of the development of spatial skills and spatial structuring: identifying the faces of 3D shapes with 2D shapes, relating parts and wholes, replicating the perspectives of different viewers, directly and indirectly operating shapes and figures
aspect of situation **The Kil Family**	Exploratory and disputational negotiation process *Working consensus* *Geometrical framings*	different leeways of participation: Expanded and restricted Parents: Tutor Ayse: Tutee	Different scaffolding processes by mother & father Enabling to perform different spatial features Enabling the success of spatial abilities Positive cognitive developmental outcomes in Ayse emotional reassurance by her parents.
aspect of situation **The Ak Family**	Negotiation between father, mother and Aleyna about the built corpus *cumulatively and disputationally* *Working interim*	Different leeway of participation but both restricted Parents: Tutor Aleyna: Tutee	Framing of parents provide Aleyna to examine numeric analytical and spatial features of the built corpuses Folk Psychology and Folk Pedagogy
aspect of situation **The Gül Family**	Disputational and explatory negotiation process between Berk, elder brother, adult person about the built corpuses Working interim and working consensus Geometrical and Numerical Framings	Legitimate peripheral participation Brother: Tutor Adult person: Tutee Berk:Tutee	Modelling, by which adult person and elder brother directly acts with the play materials. Enabling to realize the difference between right- and wrong-built corpuses. Situated Learning for Berk

aspect of Contribution **Ayse** **The Kil Family**	Exploring stability & feasibility of the building corpuses without needing any long or short blocks Representing 3D transformations in vertical and horizontal planes Regulating and linking spatial relations in a short amount of time	Different types of regulations Environmental self-regulated External regulated	Building vertically and horizontally identical corpuses to the figures on the chosen cards without needing any short or long block Learning different spatial features
aspect of **contribution** **Aleyna** **The Ak Family**	Exploring static balance to build the robust corpus, Exploring and examining different building varieties, Explicitly experiencing numeric analytical features	Different leeway of participation: Central and Environmental self-regulated	Getting learning opportunities notably high in arithmetic, infinitesimally in geometry
aspect of Contribution **Berk** **The Gül Family**	Exploring static balance to build the robust corpus, Exploring and examining different building varieties, experiencing spatial features	Listener / Over-Hearer and Observer	Learning opportunities, how the position of the blocks can be represented and how spatial relations and kinaesthetic imagery can be configured. *Witnessing of* all the situated activities of brother and adult person.

In the functioning of the MLSS, different kind of *formats* (see section 2.3.) provide learning situations for Berk, Aleyna and Ayse.

In the Kil family, Ayse's parents and Ayse herself enable her to experience and explore different kinds of geometrical features to a large degree. Her parents have a high level of formal education: her father is an engineer and her mother is an experimental chemist in a laboratory. In this sense they are both accustomed to using different mathematical domains in the course of their everyday work. Hence, as Bruner (1996) indicated, they are sensitive to and eager to adopt the folkways they see around them, so that they can use highly geometrical features during block play with Ayse. Thereby her mother's *directiveness*, *collective argumentation* process and Ayse's *external regulation,* and through her father's *emotional and motivational regulations*, his usage of *all six function*s of *scaffolding* and Ayse's *environmental self-regulation* lead Ayse to learning situations in which she can learn geometrical features intensively. While receiving different *reassurances* from her parents, Ayse explores and performs different spatial features. More precisely, while Ayse is playing with her parents, either she or her parent adopts a pattern of interaction by making appropriate changes in the definition of the situation to a commonly shared interpretation. Ayse uses this patterned process of negotiation as her MLSS, and the changes in her definition of the situation are an expression of her cognitive achievement of adaptation so that she can act with increasing autonomy in the evolving format in the situation. Thereby she is learning mathematics.

In the Ak family, Aleyna's parents and Aleyna herself enable her to experience and explore both strongly numerical and weakly geometrical features. Aleyna's father and mother act in a complementary way that meets her emotional needs in a balanced manner, while they are exposing Aleyna to numeric analytical framings. By coming up with numerical framings, her parents emphasize numbers and use counting as an integral part of their interaction with the child (Pound, 2006, p.51; 2008; Clements & Sarama, 2014; Casey et al., 2008; see also Blevins-Knabe, 2008; Devlin, 2010; Acar, 2011a,b; Tiedemann, 2012; Schuler, 2013; Newcombe, 2010, 2013). Aleyna's parents have a high level of education and can master "numbers and operations" and "geometry" at a basic level at least (Clements & Sarama, 2014; Sperry Smith, 2012; Pound, 2006, 2008). With regard to folk psychology and folk pedagogy (Bruner, 1994), Aleyna's parents might regard mathematics as composed only of numbers and operations and thus might overlook the geometrical features in the negotiation of taken-as-shared meanings. Aleyna learns strongly numerical and weakly geometrical issues. She can adapt herself to such a pattern of interaction, by making appropriate changes in her definition of the situation to a commonly shared interpretation. Thereby she uses the patterned process of negotiation as her

MLSS, and the changes in her definition of the situation are an expression of her cognitive achievement of adaptation. In the situation, she can act with increasing autonomy in the evolving format, i.e., the child is learning mathematics.

In the Gül family, Berk's brother, an adult person and Berk himself enable him to experience and explore both strongly geometrical and weakly numerical features. Differently from the Kil and Ak families, the elder brother takes the tutoring role instead of an adult person. Conversely, the adult person acts as a tutee as if they accompany Berk, while he takes the roles of listener, over-hearer and observer. Not only Berk but also an adult person can learn something in the play situation during negotiating of taken-as-shared meanings. Berk's brother is at the age of formal, secondary education and he is actively exposed to geometrical and numeric analytical features during mathematics lessons. With reference to folk psychology and folk theory (Bruner, 1996), the elder brother seems sensitive to and eager to adopt the folkways that he sees in the school classroom, while granny is adopting the folkways by virtue of her cultural psychology (Bruner, 1996). Thereby the elder brother becomes the only person answerable to his younger brother. Thereby this triad serves as a format for Berk so that he can learn mathematics in a situated way. More precisely Berk gets the chance to adapt himself to such a pattern of interaction by making appropriate changes in his definition of the situation to a commonly shared interpretation. Then he can use this patterned process of negotiation as his MLSS, and the changes in his definition of the situation are an expression of his cognitive achievement of adaptation. He is a participant in this mathematical discourse, which is tantamount to learning to think in a mathematical way, and thus Berk learns mathematics.

Regarding all these facts, the situational aspect and the aspect of contribution in the NMT-Family Kil, Ak and Gül coalesce dynamically and the MLSS comes into being as a format in the grey labelled area. The part of "Contribution x Pedagogy and Education" seems compatible with the mathematical learning situation in which Ayse, Aleyna and Berk benefit from the learning opportunities and explores more mathematical features than offered ones. Different emergent interaction processes lead Berk, Aleyna and Ayse to different occurrences of learning (see Figure 4.72.). In any event children somehow ensure learning situations for themselves.

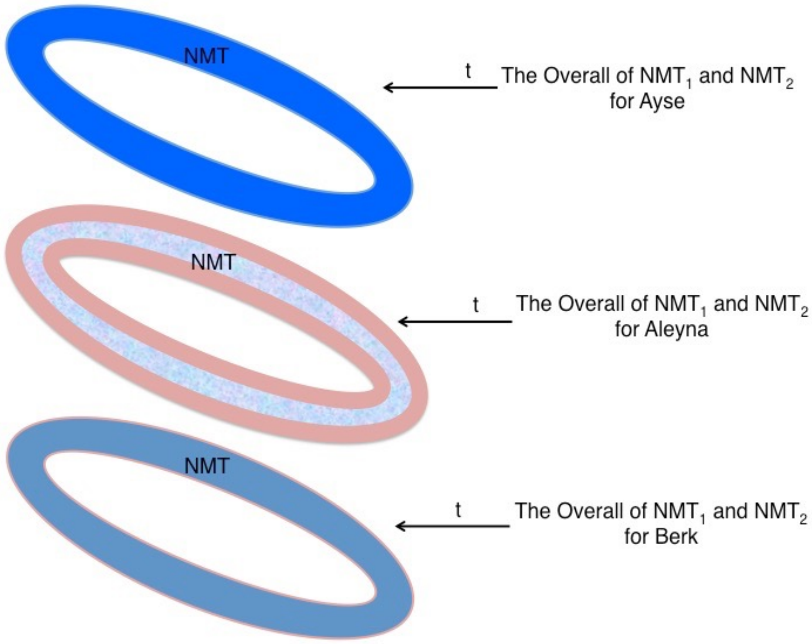


Fig. 4.72. NMTs of the Kil, Gül and Ak Family

4.5. Summary

In the chosen sequences different types of MLSS are observed. They all emerged particularly as a combination of aspects of situation and contribution of the children. Whereas Ayse, Berk, and Aleyna contribute and participate in the play situations in various ways, they receive different learning offerings from family members. Ayse contributes to the play situation as environmentally self-regulated and externally regulated while playing with her parents. Aleyna contributes to the play situation as a central participant and is environmentally self-regulated while playing with her parents. Berk contributes to the play situation as over-hearer and observer while he is playing with his brother and family members (granny/mother) and they are ascribing the role of *legitimate peripheral participant* to him. All three children are exposed to family members' regulation, while their interaction processes are bound up with their family system dynamics, folk psychologies and folk pedagogies of family members, and their competences. In this sense the MLSSs of the three children occur in the interface between the aspects of situation and contribution. Namely, in each family, the individuals adapt themselves to patterns of interaction, by making appropriate changes in their definition of the situation to a commonly shared interpretation. Thereon they use this patterned process of negotiation as their MLSS. Changes in their definitions of the situation become an expression of their cognitive achievement of adaptation. In any event all the play situations enable Ayse, Aleyna and Berk to learn mathematics and realize physical, social, emotional and cognitive growth.

5. RESULT, DISCUSSION AND CONCLUSION

5.1. Result and Discussion

This study is principally based on “interaction theory on mathematics teaching and learning” (Krummheuer, 1992, 1995, 1997a, 1999b, 2000a,b,c, 2002, 2007a,b, 2011a,b,c,d, 2012, 2013a,b, 2014, 2015; Brandt 2002, 2004; Cobb & Bauersfeld, 1995; Jungwirth & Krummheuer, 2008; Krummheuer & Brandt, 2001; Krummheuer & Fetzer, 2005; Krummheuer & Naujok, 1999; Voigt, 1995). As a concept of the study, interactional niche in the development of mathematical learning in the familial context is determined. In this regard, mathematics learning is perceived as a social and active process in that children interact with family members and actively construct meanings, while they participate in increasingly substantial ways in the re-enactment of established mathematical play situations. Therefore I regard the mathematics learning of a child as a dual process, in which the child’s cognitive construction of knowledge and his/her increasingly autonomous participation are accomplished during block play. The empirical findings in my study strengthen these theoretical approaches and lead me to several conclusions.

Theoretically, scaffolding is defined as “adult controlling of elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence” (Wood et al., 1976, p.90). Wood and colleagues (1976) define scaffolding as a process that may be subdivided into six scaffolding functions. Most researchers perceive and measure scaffolding by the accomplishment of these six functions (Anghileri, 2006; Bibok et al., 2009; Bliss at al., 1996; Fernández et al., 2001; Hammond & Müller, 2012; Nader-Grosbois et al., 2008; Renninger & Granott, 2005; Salonen et al., 2007). The empirical findings in my study indicate that not all scaffolding functions have to be fulfilled to offer the child a learning situation. Regarding the interaction processes between Ayse and her mother, Aleyna and her parents, and Berk, his brother and the adult person, it seems that usage of two or three scaffolding functions by game partners can expand the interaction process and can shed light on negotiation of taken-as-shared meanings. Through mutual adaptation, game partners interpret the ongoing play situation according to sensible and tenable occurrences for them and develop their content-related understandings, in terms of folk psychology and folk pedagogy (Bruner, 1996). As in the findings of Bruner (1996), in my study also the three focus children show a striking interest in the activities of their family members, while each of the family members reflects a variety of assumptions about children. In this sense, my study reveals that all family members offer different learning opportunities not only to the child but also to each other with respect to their folk pedagogies, while they are participating in the joint creation of interactions.

According to Bruner (1996), “empty vessels can be filled with knowledge that only adults can provide” (pp.47-49), however, this idea differs from the idea in this study. From the standard social constructivist perspective, the adult person has a standard role model for the child as a mentor, who is an experienced and trusted adviser (cf. Brandt 2004, 2006, 2013, 2014; Bruner, 1978, 1983, 1985a, 1986, 1990, 1996, 2002; Cobb, 2000a, b, c; Cobb & Bauersfeld, 1995; Cobb et al., 1993, 2000; Ernest, 1998, 1990, 1991, 1998, 2006, 2010; Palincsar,1998; Rogoff,1981,1990, 1991, 1995, 1998, 2002; Rogoff et al., 2007, 2011; Rogoff & Lave, 1984; Rogoff & Wertsch, 1984; Sfard, 2001, 2008; Tiedemann, 2010a, b, 2012a, b, 2013). However, in my study, not only for the child but also for other family members, whether adult or a child, a learning situation can occur and this situation can be created by a child as an expert. In this regard, as well as an adult person, a child can be also a mentor and an experienced and trusted adviser, as revealed in the Gül family. Moreover, my analysis of the Ak family divulges the possibility that the adult person might not have enough competence and experience in geometry and thus cannot be a trusted adviser about the geometrical features of block play at all. Thereby I propose these two findings in contrast to the standard and usual socio-constructivist approaches that an adult person can be also a novice instead of the child in geometrical activities.

Keeping this idea in mind, my study also reveals that individual experiences and behaviours in everyday life rub off on the negotiation process between individuals. The level of education and occupations of family members and the dynamics of family systems take place in the course of interaction and constitute such interaction patterns, which underscore the developmental importance of coordination, dynamic match, i.e., reciprocity, mutuality and synchrony of family members' and children's behaviours. Maybe therefore not all scaffolding functions have to be fulfilled in order to achieve a learning situation. The factors of the roles taken can change dynamically and mutually so that individuals can facilitate different types of learning and the way of negotiating can take place in different characters.

Additionally, my study shows that the interdependence of the roles and functions of all family members affect the interaction process between the child and family members, as referred to in family system theory (Abramovitch et al., 2014; Azmitia & Hesser, 1993; Barnard & Solchany, 2002; Bornstein & Sawyer, 2008; Bornstein, 1989, 2002a,b,c,d,e; Brahier, 2009; Collins et al., 2002; Coll & Pachter, 2002; Cox & Paley, 2003; Crawford, 2012; Daniels et al., 1985; Dench & Ogg, 2002; Dombeck & Wells-Moran, 2006; Doron, 2009; Goodfellow, 2010; Goodfellow & Laverty, 2003; Herzog, 1998; Howe, Brody & Recchia, 2006; Laakso, 1995; Lamb, 1981; McGillicuddy-DeLisi, 1988; Morgaine, 2001; Mullis & Mullis, 1986; Paquette,1994, 2004; Parke, 2002; Pepler et al., 1981; Perner, Ruffman & Leekam 1994; Tamis-LeMonda, 2004; Salonen et al., 2007; Silverstein & Ruiz, 2006; Smith, 2005; Tomasello et al., 1990; Vallacher & Nowak, 1994). So indeed, while a mother aims to provide calm and emotional comfort for her child, a father might aim to arouse childrens' activation, as in the cases discussed above. Here the way of putting such relations forward, can differ from each other in each family, although the main idea is kept constant that mothers are emotional comfort providers, while fathers foster their children's openness to the world. One mother can only take her child and her emotional needs to the centre of her interest instead of playing (cf. Ak family in section 4.2.), whereas another mother offers her child a learning situation in geometry while meeting with the emotional needs of the child at the same time (cf. Kil family in section 4.1.). Similar to this, one father can encourage his child to succeed in dealing with difficulties and to learn geometrical features (cf. Kil family in 4.1.), whereas another father might play with his child as a competitor and let his child to experience the need to stand up for her own beliefs and face unfamiliar occurrences and her own mistakes, while the child is justifying herself and taking risks in new sets of circumstances (cf. Ak family in section 4.2.). In connection with this, both siblings and grandparents might take on such responsibilities and act in similar way to parents. Thereby grandparents can be contributors to the childcare system, and older siblings might can take such roles as experts, while the younger ones can be novices in any block play situations (cf. Gül family in section 4.2.). Referencing my analysis in this study (cf. Acar, 2011a,b; Acar Bayraktar, 2012a,b, 2014a,b,c,d, in press-a,b; Acar Bayraktar & Krummheuer, 2011, 2014) one can argue that each family member both jointly and separately produce different playing atmospheres, while they are playing as pairs or as triads or as a whole family together. In this sense each interaction process with different family members can differ from each other, but all can work complementarily, variously and positively in order to offer a child a learning situation. Regarding this, in addtion to works of Tiedemann (2013; 2012a,b; 2010), my study reveals that MLSSs are not only created by child-mother dyads and not only relate to a *situational* context but also can be created by a child with other family members relating to *situated* and *situational* contexts.

Furthermore, in my study all children participate in the familial play situations variously, whereas family members offer them either restricted or expanded leeways of participations during block play situation. This issue leads me to crucial points in my study:

- The first one is the importance of the "participation metaphor" of Sfard (1998, 2001, 2005, 2015). Sfard (1998) indicates that "participation gives prominence to the aspect of mutuality characteristic of the part-whole relation and makes salient the dialectic nature of the learning interaction" (Sfard, 1998, p.6). So indeed, in my study too, it is obvious to see that the whole and the parts affect and inform each other, while family members are playing together. Thereby my study strengthens the idea of Sfard (1998, 2001, 2005,

2015) that learning is becoming a participant in any certain activity. Ayse, Aleyna and Berk take part in both block play situations and ensure different characters of learning situations with the accompaniment of other family members. They seem to accept and engage in the learning offerings provided by participating and becoming one of the greater part of interaction processes in the familial contexts. In this sense the first and crucial step is “to participate” in the mathematical practices with family members, and this is one of the contributions of this study.

- The second one is that my study reveals that children contribute to play situations in the same or similar ways, but different participation profiles of production and recipient designs occur during interaction processes. In my opinion this means that each child can participate in mathematical discourse in a similar way but each child does not get the same mathematical offerings or experience the same mathematical features. The child or the family members can adjust the material means and local conditions of the child’s learning environment by mobilizing his or her social environment through communicative solicitations. From my point of view, this can be seen as family members’ regulation or the child’s self-regulation but, in any event, family members negotiate between themselves in terms of their family subsystems, their folk pedagogy and folk psychology. This mutual dynamic leads them to shape such play situations in a different way. In this sense, in my view, having the same regulation types or participation profiles does not lead children to the same learning situation. Instead, family dynamics, folk pedagogy of family members, their education level and relationship with each other, and the child’s contribution are intertwined with accomplishing a learning situation for the child.

Besides, from my point of view, such learning situations can be accomplished no matter which language is used by the families. As my empirical findings indicate, the negotiation process does not have to be realized in speech, but rather can be realized by verbal, nonverbal and also motional ways. In contrast to Krummheuer and Brandt (2001), the constitution of interaction process among child and family members does not require acoustical appearance as a “sounding box” or as syntactical construction with certain words and expressions. But rather, from my point of view, the thematic/semantic contribution to the negotiation of taken-as-shared meanings can be in a different way, namely “modelling”, in that one can express oneself by modelling a behaviour or acting in a different way without any sound. Thereby I can talk about “the use of nonverbal act”, which can be perceived as the kinesics construction and the appearance of expressions instead of any syntactical constructions. My empirical findings reinforce this idea that one should debate the characters of the production profiles of the individuals not only in language and syntax levels but also in motional level as body language, in contrast to work of Krummheuer and Brandt (2001).

From this point of view I should emphasize that the negotiation process cannot be realized only by “talking” but also through different types of syntactic and kinesics manifestations. Emotional and motivational reassurances can occur in a motional way, such as pinching a cheek or slapping the other’s head or patting each other on the back etc. In my view, family members can be exposed to any supportive activity by others who can either try to motivate or conclude the situation in order to output their interpretations during the interaction processes. In this regard I think that motional outputs represent not only a modelling action but might also represent either motivating or concluding actions. Furthermore, both types of outputs of syntactic and kinesic manifestations can be revealed in a complementary way. For example, by saying to somebody “yes very good.” and then pinching his cheek might refer to the re-representation of the syntactical construction with a kinesical construction. In this way, from my point of view, an individual can express and reinforce his interpretation with the help of any other expression styles with which he re-represents his reaction in order to aid others to understand his intentions. In this sense, I carry this idea into the process of a block play and consider the work of Bjorklund et al. (2004). Whereas in their work Bjorklund and colleagues code parental behaviours and categorize them into six groups – “prompt, prompt after error, affirmation, disaffirmation, provide answer and instruction, modelling, re-representation” (Bjorklund, Hubert & Reubens, 2004, p.351) – I would rather call them supportive activities of family members and adapt them to the block play situation by adding three more activities namely *motivation*, *conclusion* and *re-representation.* Family members realize ten types of supportive activities during interaction process in block play (see Table 5.1.):

Table 5.1. Generalized supportive activities of family members in block play

Supportive Activities	**Description**
Prompt	Suggestion by the family member as to the generation of an answer without mention of use of any specific strategy. (e.g. How should you put the block?)
Prompt after error	Child makes error in calculation and family member prompts for a rebuilding. (e.g. Are you sure?, just look accurately at it!, be careful!, ok do it again!)
Affirmation	The family member demonstrates agreement with child's answer or response to a math problem. (e.g. That's right!, yes it has to be stand like that!, exactly!)
Disaffirma-tion	This is a type of correction, a definitive negative response indicating an incorrect response; disaffirmation could be an explicit "no". (e.g No, that is not right!; The corpus is wrong!; You built it wrong!; No, not like that!)
Provide so-lution	The family member provides the child with the correct solution. The family member spontaneously produces the answer.
Motivation	The family member motivates the child in a positive or negative way. It can be in different ways: by consoling, angering, encouraging, criticizing, insulting etc. (e.g.: verbal: You are doing perfect!; In the next turn you will get better!; We have five rounds! motional: pinching other's cheek, slapping the other's head, patting someone on the back)
Conclusion	The family member concludes the situation or gives commentary about the current situation. (e.g. Now it is daddy's turn!; You lose!, You get 3 points!; It can't be played like that!)
Modelling	The demonstration of a strategy independent of instruction, that is, the family member models a behaviour for the child to observe and imitate (e.g. family members build the right or similar corpus in front of the player)
Instruction	The family member suggests the use of a specific strategy (e.g. Maybe you should put this block in between, or? ; When you take one block back, do you have the right corpus?; There are two blocks, on it- or?; There comes one more block up on it.)

Re-repre-sentation	When the family member re-represents the problem in a way that is more familiar to the child. This can be verbal+verbal or verbal+motional or motional+motional (e.g. holding up blocks to represent to addend: one two three!)

In contrast to the idea of Bjorklund and colleagues (2004) that parents are "sensitive to the cognitive and social demands" of their children in different contexts and "engage their children more in the math context than in the game context and vary their behaviour accordingly" (p.355), my analysis leads me to the idea that not only parents but also other family members vary their supportive activities accordingly in block play situations and are sensitive to the cognitive and social demands of children in the familial contexts. In my point of view these supportive activities would be helpful to define and examine dynamic interaction process between family members and focus child in detail.

In my analysis I also see that the usage of all these supportive activities is common, although interaction processes can be characterized in different ways. This fact leads me to two different points:

- One is that these supportive activities can occur in the same way, although the observation groups have different characteristics (for example, in one family it is parent-child dyad, in another parent-sibling-child triad). Here the idea of Fernández and colleagues (2001) about "asymmetrical and symmetrical" interactions comes to mind:

  > Where, as an "asymmetrical" interaction, a teacher might explicitly plan how to show children an idealized version of a problem to help them understand it, in symmetrical talk the idealized version often emerges in an unplanned way through attempts by children to share understandings and to explain solutions as they work together. (Fernández et al., 2001, p.53).

  In this regard one can say that my analyses and observation groups do not exactly fit with this idea. But rather my observations lead me to the idea that an "asymmetrical" interaction can emerge in an unplanned way through attempts by children and family members to share understandings and to explain solutions while they are playing together. In this sense on one point I agree with Fernández and colleagues (2001) that "without needing any conscious intention" the family members can scaffold the development of each other's understanding by acting as a tutor (Fernández et al., 2001, p.53).

- The second point is that the usage of such supportive activities does not lead the interaction processes to a common style of negotiating. In this sense the work of Fernández and colleagues helps me to categorize negotiating styles into three groups: *disputational, cumulative,* and *exploratory* (Fernández et al., 2001, p.53). As before mentioned, I regard the negotiation process as not realization of "talking" but the realization of syntactic and kinesic manifestations. In this regard the recognition of different negotiation types helps me to determine the interaction processes more clearly. With reference to the work of Fernández et al. (2001) these styles can be characterized as follows:

  a) **Negotiating taken-as-shared meanings disputationally:** This kind of negotiation consists of disagreements, individualized decision-making, counter-assertions through verbal, nonverbal and motional outputs. In this type of negotiating each individual seems to perceive others as competitive or incompetent.

  b) **Negotiating taken-as-shared meanings cumulatively:** This kind of negotiation consists of agreements, positive and uncritical assertions through verbal, nonverbal and motional outputs such as repetitions, confirmations and elaborations. In this type of negotiating each individual seems to achieve agreement without any critiques.

  c) **Negotiating taken-as-shared meanings in an exploratory way:** This kind of negotiation consists of alternative hypotheses, offered justifications, rationalized practices and common pursuit of the best solutions. Its orientation is being constructive and cooperative so that all relevant features about the thematized issue can be shared. In this type of negotiating individuals seem to achieve agreement through contribution by verbal, nonverbal and motional outputs.

Regarding these three negotiating styles, I think that the nature of family relationships determines the flux of the interaction, while the negotiation processes differs from each other and can be divided into subgroups in an interaction process. On other words, family patterns can characterize and bring about all the different negotiating styles in one play situation. In this sense, from my point of view, the negotiation process do not have to be determined by one kind of negotiating style. For example, the mother negotiates with her child cumulatively, while the father negotiates with his child disputationally during interaction processes in a same play situation (see the Ak family in section 4.2.). Another example might be the play situation in the Gül family, where family members negotiate with the child disputationally, whereas they are negotiating with each other in an exploratory way in the same play situation (see 4.3.). In this regard, my analyses reveal that the negotiation process does not have to be accomplished with one type of negotiation style but rather can be accomplished in a complementary way, while different negotiating styles emerge during family interaction. Furthermore, Fernández and colleagues (2001) indicate that in exploratory talk children support each other and so they travel further in an intellectual sense (p.53). In contrast to this idea, my analyses in the current study reveal that the negotiation of taken-as-shared meanings can be achieved not only in an exploratory but also in a complementary way with each other, which enables all family members to travel further in an intellectual sense. From my point of view, different framings can be combined through such negotiation processes so that the child can learn something from the block play situation.

In addition to two points above mentioned, my findings are that the collective argumentation process does not have to be explorative. In addition to the idea of Krummheuer, that the "collective argumentation" process is explorative and rhetorical, which refers to the "isolated metacommunicative activity that emerges [in] everyday classroom activities and follows an ordinary action when the validity of the claimed argumentation is doubted or challenged" (Krummheuer,1995, p.232; see also 1992, 1997; Brandt 2002, 2004; Brandt & Krummheuer 2001; Cobb & Bauersfeld, 1995), my analysis of the Kil family (see section 4.1.1.) reveals that through negotiating disputationally, the individuals can also accomplish a collective argumentation process. In this regard, from my point of view, individuals can cooperate, trying to adjust their intentions and interpretations, verbally and nonverbally, and can present the rationalities while they are negotiating disputationally.

Keeping all results above mentioned in mind, my study also reveals that bilingualism and biculturalism do not have any remarkable effect on the learning of the child in a block play situation in the familial context. In contrast to the idea of Bornstein and Bohr (2011) that culture and language influence a wide array of family functions including roles, decision-making patterns, and cognitions and practices related to child-rearing and child development, my analysis leads me to the idea that Turkish ethnicity, being German-Turks, speaking both or more languages do not put forward any remarkable influence on family functions. Occasionally family members can switch either their languages or cultural attitudes and so they can take advantage of being either bilingual or bicultural, which enable individuals to be able to accomplish the negotiation process in a complementary way. From my point of view, in any event, family members negotiate between themselves in terms of their family subsystems, their folk pedagogy and folk psychology and this leads them to the achievement of mutual dynamics in the family. In this regard, in contrast to the idea of Suárez-Orozco and Suárez-Orozco (2001) and the idea of Civil and colleagues (2005) that there is a gap between immigrant parents and their children in their cultural and lingual worlds, my analysis leads me to the idea that lingual and cultural factors do not seem to demolish neither the negotiation process nor the flux of interaction between family members. There are mutual adaptations not only between parents and children but also between other family members and children instead of having gap between themselves in the interaction processes. In this sense interaction processes can be characterized in different ways but learning situations are accomplished no matter which language and cultural attitudes are used by the families. Therefore, either bilingualism or biculturalism do not have any high remarkable effect on the emergence of interactional niche in the development of geometrical and spatial thinking in the German Turk familial context and the functioning of a MLSS.

Regarding all the above-mentioned points, another finding that I want to emphasize is the importance of block play. Although block play refers to geometrical play, blocks as play materials enable children to experience different mathematical domains. My analyses in the current study reveal that children can experience some numeric analytical features while they are exploring geometrical features. In this regard blocks as play materials enable them to exercise the same features in numbers and operations, while focusing on a building action by constructing shapes, forms or any world that they can imagine. This fact confirms the idea of Clements and colleagues about mathematical domains (Clements, Sarama & DiBiase, 2004) that numbers are used to quantify properties of geometrical objects, while geometric objects provide models for number and operations. So indeed, in Gül and Ak families, children are exposed to the use of numerical and geometrical features at the same time by virtue of the geometrical and numerical framings of family members.

Furthermore, my analyses strongly support the idea of Bullock (1992) that block play contributes to children's physical, social, emotional and cognitive growth. So indeed, in all three families, in block play situations children can experience taking turns, sharing and respecting the rights of others, and learn to cooperate while exploring, matching and classifying the sizes, shapes, distances and proportions of block structures. Furthermore such block play situations enable all three children (Aleyna, Ayse and Berk) to learn patience, increase independence and to experience a sense of accomplishment. In this sense my observations steer me towards the same opinion as Bullock (1992) that blocks provide many opportunities for children to develop in a variety of ways, and also with Sperry Smith (2012) that block building is a valuable activity for children to express themselves and the world they live in, while they are building many wildly imaginative structures.

Consequently, all above mentioned facts lead me to the result that even within a constant mathematical domain, the emerging NMT-Family can be structured in a different way through alternations of the social settings and might require different kinds of adaptations. Therefore, each NMT-Family differs from each other, while the nature of different family relationships determines the emergence style of NMT-Family. In this regard, from my point of view, the NMT-Family can not be standardized or determined by certain kind of NMT style. This result strengthens the idea of Bruner that "there is not one kind of learning and any learner has a host of learning strategies at [his/her] command" (1985a, p.8). Furthermore, MLSS varies depending on the determination of family systems and different kind of regulations (self-regulation and family members' regulation) in the flux of the interaction as well. Each family member adapts themselves to such a pattern of interaction, by making appropriate changes in their definition of the situation to a commonly shared interpretation. Then they use this patterned process of negotiation as their MLSS, and the changes in their definitions of the situation are an expression of their cognitive achievement of adaptation. In this sense the MLSSs of the three children occur in the interface between the aspects of situation and contribution in the NMT table.

5.2. Conclusion and Future Directions

In conclusion, mathematical play situations conducted in the familial context seem to be a possible contribution to the child's mathematical development. While children experience different learning opportunities during block play with family members, they are exposed to learning about giving, receiving, sharing and expressing their ideas and feelings. Play is a social act for the child and it gives children an opportunity to think, to talk, to learn and perhaps, as Bruner said (1983), to be themselves. In this way, they develop in mathematical and in cognitive and social-emotional competences as well. Their participation, negotiation and interaction processes can emerge differently, but such play situations enable the children to gain different learning opportunities. There occur

MLSSs and interactional niche in the development of spatial thinking for Ayse, Aleyna and Berk respectively.

Most children grow up in a closely linked network of family members, where early learning occurs within play activities. With the participation of each family member, block play can be productive and fruitful for the child. Regardless of whether the family member has adequate knowledge about geometrical activities, the interaction process can lead the child to learn something. Family members can impart knowledge to each other and provide new interpretations, which strongly and constructively support the child's development in block play. In this way, a block play situation with family members facilitates the child's exploration and way of using his or her mind (Bruner, 1983) through linking his or her own ideas with others. Different family members are likely to provide many learning opportunities about mathematical ideas. Thus, the more children are exposed to block play with family members, the better they can develop in mathematics before entering primary school and can reach relatively high levels of achievement and learning.

In this regard the effect of block play with family members can be traced further, when the children enter primary school and secondary school as well. Thereby the further effects of block play can be observed in the long run.

As another future direction of my study, the answers to the question: "How do bilingualism and biculturalism affect the learning of the child in a block play situation?" can be searched deeply. To answer both questions, in future research, German families, Turkish families without any immigration background and German families with immigration backgrounds in Turkey should be observed and analysed in great detail. Thereby one can provide the required responses to the following two questions:

- How can the influences of being German, Turk and German-Turk on the geometrical and spatial development of children be shaped?

- Which learning opportunities does familial environment give German, Turk and German-Turk children to experience geometry and spatial abilities during block play?

In this regard the effect of ethnic background and mono- and bi-lingualism in both Turkish and German communities can be discussed in detail. Moreover, these factors can be reflected on the school education of German, Turk and German-Turk children. It is important to answer the questions on how parents and teachers can support the mathematical development of German Turk children and how parents and practitioners can lighten the load for these children, offering them a more secure and consistent mathematical experience at home and in school.

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