Critical exponents of the statistical multifragmentation model

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Abstract

For statistical multifragmentation model the critical indices α', β, γ', δ are calculated as functions of the Fisher parameter τ. It is found that these indices have different values than in Fisher's droplet model. Some peculiarities of the scaling relations are discussed. The basic model predicts for the index τ a narrow range of values, $1.799 < \tau < 1.846$, which is consistent with two experiments on nuclear multifragmentation.

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1. Introduction

The statistical multifragmentation model (SMM) [1,2] rather successfully explains the multiple production of intermediate mass fragments [3,4]. On the other hand, it is also an example of a statistical model exhibiting a phase transition (PT) of the liquid–gas type [4,5]. Therefore, the SMM was studied extensively in order to clarify not only the relationship between multifragmentation and the nuclear liquid–gas PT, but also to elucidate the connection of the SMM to both Fisher's droplet model (FDM) [6] and the lattice gas model [7,8]. For the last purpose a simplified version of the SMM was suggested [9,10].

In recent works [11,12] an exact analytical solution of a simplified version of the SMM was found in the thermodynamic limit. This model is mathematically very similar to the original FDM. However, it differs from the FDM in two aspects: (i) the nonzero volume of the nucleons is taken into account, and (ii) the free surface energy is non-negative. These two main differences generate distinct results concerning the properties of the critical point: the SMM predicts the existence of a critical point for the parameter $\tau \leqslant 1$ and a tricritical point for $1 < \tau \leqslant 2$ (see below), whereas for the FDM this parameter is assumed to be larger, $\tau > 2$. Such a difference is not surprising since the FDM considers the droplets to have zero volume, i.e., its validity is restricted to low particle densities. Consequently, the FDM conclusions regarding the critical behavior must be considered as mainly suggestive in relation to the behavior of real fluid systems or of more realistic models. Therefore, it is necessary to study how the critical indices of the SMM relate to those of the FDM. It will be shown that the critical indices in the SMM and the FDM have different values, i.e., these models belong to different universality classes.

Another important issue is the comparison of the critical indices of the SMM with the corresponding values found in numerical studies [13] and in exper-

iments. As we shall show the experimental data for the index τ are consistent with our predictions for the SMM.

The Letter is organized as follows. In Section 2 a brief description of the simplified version of the SMM is given. In Section 3 some useful formulae are derived and the critical indices of this model are found. The scaling laws and the relations between the critical indices of the SMM are discussed in Section 4. The conclusions are given in Section 5.

2. The statistical multifragmentation model

In Refs. [11,12] a simplified version of the SMM is solved analytically with the method suggested in [14]. The interaction of the fragments is taken into account by replacing the total volume V by the free volume $V_f \equiv V - bA$, where A is the number of nucleons in the system and b is the eigenvolume of a nucleon. In the grand canonical ensemble the pressure of the system is found for the liquid and gaseous phase

$$p_l(T, \mu) = \frac{\mu + W(T)}{b},$$
$$p_g(T, \mu) = T\mathcal{F}(p_g, T, \mu), \tag{1}$$

respectively, where

$$\mathcal{F}(p_g, T, \mu) \equiv g(T)\Big[z_1 e^{\frac{\nu - W(T)}{T}} + \Sigma_{\mathbf{0}}(\varepsilon, \nu)\Big] \tag{2}$$

and

$$\Sigma_{\mathbf{q}}(\varepsilon, \nu) \equiv \sum_{k=2}^{\infty} k^{q-\tau} e^{\frac{\nu}{T}k - \frac{a(\varepsilon)}{T}k^{\sigma}}, \tag{3}$$

with $g(T) \equiv [mT/(2\pi)]^{3/2}$ and non-negative integer q. The mass of a nucleon is denoted as m, $z_1 = 4$ is the number of spin-isospin states of a nucleon, $\nu \equiv \mu + W(T) - bp_g$ is the shifted chemical potential of the gaseous phase and $W(T)$ is the free energy per particle inside of a fragment. The critical temperature of the system is denoted as T_c and $\varepsilon \equiv (T_c - T)/T_c$ defines the reduced temperature. The actual parameterization of $W(T)$ and the free surface tension $a(\varepsilon)$ can be found in Ref. [11,12]. In this Letter it is assumed that $W(T)$ and all its derivatives are regular functions of temperature and that the free surface tension obeys $a(\varepsilon) = a_0\varepsilon^{\zeta}$ for small positive values of ε and vanishes for $\varepsilon \leqslant 0$. The surface area of a fragment of k nucleons is taken in the general form k^{σ}. In the SMM

one usually chooses $\zeta = 5/4$ and $\sigma = 2/3$, but in the following treatment $\zeta \geqslant 1$ and $0 < \sigma < 1$ will be regarded as free parameters.

The particle densities in the liquid and in the gaseous phase are $\rho_l \equiv (\partial p_l/\partial\mu)_T = 1/b$ and $\rho_g \equiv (\partial p_g/\partial\mu)_T = \rho_{id}/(1 + b\rho_{id})$, respectively. The density

$$\rho_{id}(T, \mu) \equiv g(T)\Big[z_1 e^{\frac{\nu - W(T)}{T}} + \Sigma_{\mathbf{1}}(\varepsilon, \nu)\Big] \tag{4}$$

has the meaning of the ideal gas density in the limit $b \to 0$.

When both pressures coincide, $p_l = p_g$, $\nu = 0$, the liquid–gas PT occurs. This equality defines the PT equilibrium curve $\mu^*(T)$ in the (T, μ)-plane. For $T < T_c$ the surface tension $a(\varepsilon)$ is nonzero and the PT is of 1st order, since $\rho_{id}(T, \mu) < \infty$ and consequently $\rho_g < \rho_l$ on the curve $\mu^*(T)$. The critical point at $T = T_c$ corresponds to the critical density $\rho_c = \rho_l = 1/b$. Since at the critical point $\nu = 0$ and the surface tension vanishes, it follows from Eqs. (1) and (2) that for $\tau \leqslant 1$ the function $\mathcal{F}$ is divergent and, therefore, p_c is infinite. For $1 < \tau \leqslant 2$ one finds similarly that $p_c < \infty$, $\rho_c = 1/b$. In the present parameterization of the surface tension the SMM exhibits a 2nd order PT for $T > T_c$ (see Ref. [11,12] for more details). The general dependence of the phase structure of the SMM for $T > T_c$ on the parameterization of the surface tension and additional refinements of the model are the theme of a separate work. In this Letter the analysis of the critical exponents of the SMM is restricted to $T \leqslant T_c$. For $\tau \geqslant 2$ on the phase equilibrium line both $\mathcal{F}(p^*(T), T, \mu^*(T))$ and $\rho_{id}(T, \mu^*(T))$ are finite for any finite temperature, i.e., the mixed phase region extends formally to infinite temperatures. Therefore, this case exhibits no critical point at all in contrast to the FDM and is not considered in the following.

3. The critical indices of the SMM

The critical exponents α', β and γ' describe the temperature dependence of the system near the critical point on the coexistence curve $\nu = 0$

$$c_\rho \sim \begin{cases} |\varepsilon|^{-\alpha}, & \text{for } \varepsilon < 0, \\ \varepsilon^{-\alpha'}, & \text{for } \varepsilon \geqslant 0, \end{cases} \tag{5}$$

$$\Delta\rho \sim \varepsilon^{\beta}, \quad \text{for } \varepsilon \geqslant 0, \tag{6}$$

$$\kappa_T \sim \varepsilon^{-\gamma'}, \quad \text{for } \varepsilon \geqslant 0, \tag{7}$$

where $\Delta\rho \equiv \rho_l - \rho_g$ defines the order parameter, $c_\rho \equiv \frac{T}{\rho}\left(\frac{\partial s}{\partial T}\right)_\rho$ denotes the specific heat at fixed particle density and $\kappa_T \equiv \frac{1}{\rho}\left(\frac{\partial\rho}{\partial p}\right)_T$ is the isothermal compressibility. The shape of the critical isotherm for $\rho \leqslant \rho_c$ is given by the critical index δ

$$p_c - \tilde{p} \sim (\rho_c - \tilde{\rho})^\delta \quad \text{for } \varepsilon = 0. \tag{8}$$

The tilde indicates $\varepsilon = 0$ hereafter.

The calculation of α and α' requires the specific heat c_ρ. With the formula [15]

$$\frac{c_\rho(T,\mu)}{T} = \frac{1}{\rho}\left(\frac{\partial^2 p}{\partial T^2}\right)_\rho - \left(\frac{\partial^2\mu}{\partial T^2}\right)_\rho \tag{9}$$

one obtains the specific heat on the PT curve by replacing the partial derivatives by the total ones [16]. The latter can be done for every state inside or on the boundary of the mixed phase region. For the chemical potential $\mu^*(T) = bp^*(T) - W(T)$ one gets

$$\frac{c_\rho^*(T)}{T} = \left(\frac{1}{\rho} - b\right)\frac{d^2 p^*(T)}{dT^2} + \frac{d^2 W(T)}{dT^2}, \tag{10}$$

where the asterisk indicates the condensation line ($\nu = 0$) hereafter. Fixing $\rho = \rho_c = \rho_l = 1/b$ one finds $c_{\rho l}^*(T) = T\frac{d^2 W(T)}{dT^2}$ and hence obtains similarly for both α and α'

$$\alpha = \alpha' = 0. \tag{11}$$

To calculate β and γ' the behavior of the series

$$\Sigma_{\mathbf{q}}(\varepsilon,0) = \sum_{k=2}^{\infty} k^{q-\tau} e^{-A\varepsilon^\zeta k^\sigma} \tag{12}$$

for small positive values of ε should be analyzed ($A = a_0/T_c$). In the limit $\varepsilon \to 0$ the function $\Sigma_{\mathbf{q}}(\varepsilon,0)$ remains finite, if $\tau > q + 1$, and diverges otherwise. For $\tau = q + 1$ this divergence is logarithmic. The case $\tau < q + 1$ is analyzed in the following.

With the substitution $z_k \equiv k(A\varepsilon^\zeta)^{1/\sigma}$ it follows

$$\Sigma_{\mathbf{q}}(\varepsilon,0) = \left(A\varepsilon^\zeta\right)^{\frac{\tau-q}{\sigma}} \sum_{k=2}^{\infty} z_k^{q-\tau} e^{-z_k^\sigma}. \tag{13}$$

Since $\Delta z_k = (A\varepsilon^\zeta)^{1/\sigma} \equiv \Delta z$, the series on the r.h.s. of (13) converges to an integral for $\varepsilon \to 0$

$$\Sigma_{\mathbf{q}}(\varepsilon,0) = \left(A\varepsilon^\zeta\right)^{\frac{\tau-q-1}{\sigma}} \int\limits_{2(A\varepsilon^\zeta)^{1/\sigma}}^{\infty} dz\, z^{q-\tau} e^{-z^\sigma}. \tag{14}$$

The assumption $q - \tau > -1$ is sufficient to guarantee the convergence of the integral at its lower limit. Because of the finite value of the above integral one obtains the following result for $\varepsilon \to 0$

$$\Sigma_{\mathbf{q}}(\varepsilon,0) \sim \begin{cases} \varepsilon^{\frac{\zeta}{\sigma}(\tau-q-1)}, & \text{if } \tau < q+1, \\ \ln|\varepsilon|, & \text{if } \tau = q+1, \\ \varepsilon^0, & \text{if } \tau > q+1. \end{cases} \tag{15}$$

For the calculation of δ one obtains similarly

$$\Sigma_{\mathbf{q}}(0,\tilde{\nu}) \sim \begin{cases} \tilde{\nu}^{\tau-q-1}, & \text{if } \tau < q+1, \\ \ln|\tilde{\nu}|, & \text{if } \tau = q+1, \\ \tilde{\nu}^0, & \text{if } \tau > q+1. \end{cases} \tag{16}$$

From $\Delta\rho = \frac{1}{b}\frac{1}{1+b\rho_{id}}$ it follows $\Delta\rho \sim \rho_{id}^{*-1}$ for $\varepsilon \to 0$. Therefore, inserting $q = 1$ into Eq. (15) immediately gives β

$$\beta = \frac{\zeta}{\sigma}(2-\tau). \tag{17}$$

From $\rho_g = \rho_{id}/(1+b\rho_{id})$ and Eq. (4) it follows that

$$\kappa_T = \frac{g(T)}{T}\frac{\rho_g}{\rho_{id}^3}\left[z_1 e^{\frac{\nu-W(T)}{T}} + \Sigma_{\mathbf{2}}(\varepsilon,\nu)\right]. \tag{18}$$

Using Eq. (15) one easily finds γ' from Eq. (18)

$$\gamma' = \frac{2\zeta}{\sigma}\left(\tau - \frac{3}{2}\right). \tag{19}$$

Near the critical point it follows $\rho_c - \tilde{\rho} \sim 1/\tilde{\rho}_{id}$. With (4) one finds using Eq. (16) $\rho_c - \tilde{\rho} \sim [\Sigma_{\mathbf{1}}(0,\tilde{\nu})]^{-1} \sim \tilde{\nu}^{2-\tau}$. Substituting $1 = e^{\frac{-\tilde{\nu}}{2T_c}k} e^{\frac{\tilde{\nu}}{2T_c}k}$ yields for small values of $\tilde{\nu}$

$$p_c - \tilde{p} \cong g(T_c)T_c\left[-\frac{z_1}{T_c}\tilde{\nu} + \sum_{k=2}^{\infty} k^{-\tau}\left(1 - e^{\frac{\tilde{\nu}}{T_c}k}\right)\right]$$

$$\cong g(T_c)\left[-z_1\tilde{\nu} - \tilde{\nu}\sum_{k=2}^{\infty} k^{1-\tau} e^{\frac{\tilde{\nu}}{2T_c}k}\right]$$

$$\sim \tilde{\nu}\Sigma_{\mathbf{1}}\left(0,\frac{\tilde{\nu}}{2}\right) \sim \tilde{\nu}^{\tau-1}, \quad \text{for } \tilde{\nu} < 0. \tag{20}$$

Combining the result above with the expression found for $\rho_c - \tilde{\rho}$ one obtains the critical index δ

$$\delta = \frac{\tau-1}{2-\tau}, \tag{21}$$

which is independent on the choice of ζ and σ.

4. Scaling relations of the SMM

In the special case $\zeta = 2\sigma$ the well-known exponent inequalities proven for real gases by

Fisher [17]: $\qquad \alpha' + 2\beta + \gamma' \geqslant 2,$ (22)

Griffiths [18]: $\quad \alpha' + \beta(1+\delta) \geqslant 2,$ (23)

Liberman [19]: $\quad \gamma' + \beta(1-\delta) \geqslant 0,$ (24)

are fulfilled exactly for any τ. (The corresponding exponent inequalities for magnetic systems are often called Rushbrooke's, Griffiths' and Widom's inequalities, respectively.) For $\zeta > 2\sigma$, Fisher's and Griffiths' exponent inequalities are fulfilled as inequalities and for $\zeta < 2\sigma$ they are not fulfilled. The contradiction to Fisher's and Griffiths' exponent inequalities in this last case is not surprising. This is due to the fact that in the present version of the SMM the critical isochore $\rho = \rho_c = \rho_l$ lies on the boundary of the mixed phase to the liquid. Therefore, in expression (2.13) in Ref. [17] for the specific heat only the liquid phase contributes and the proof of Ref. [17] following (2.13) cannot be applied for the SMM. Thus, the exponent inequalities (22) and (23) have to be modified for the SMM. Using Eqs. (11), (17), (19) and (21) one finds the following scaling relations

$$\alpha' + 2\beta + \gamma' = \frac{\zeta}{\sigma} \quad \text{and} \quad \alpha' + \beta(1+\delta) = \frac{\zeta}{\sigma}. \quad (25)$$

Liberman's exponent inequality (24) is fulfilled exactly for any choice of ζ and σ.

Since the coexistence curve of the SMM is not symmetric with respect to $\rho = \rho_c$, it is interesting with regard to the specific heat to consider the difference $\Delta c_\rho(T) \equiv c_{\rho_g}^*(T) - c_{\rho_l}^*(T)$, following the suggestion of Ref. [16]. With Eq. (10) and noting that $1/\rho_g^* - b = 1/\rho_{id}^*$ it follows

$$\Delta c_\rho(T) = \frac{T}{\rho_{id}^*(T)} \frac{d^2 p^*(T)}{dT^2}. \quad (26)$$

The most divergent term in Eq. (26) yields for $\zeta > 1$

$$\alpha_s' = \begin{cases} 2 - \frac{\zeta}{\sigma}, & \text{if } \tau < \sigma + 1, \\ 2 - \frac{\zeta}{\sigma}(\sigma + 2 - \tau), & \text{if } \tau \geqslant \sigma + 1. \end{cases} \quad (27)$$

Then it is $\alpha_s' > 0$ for $\zeta/\sigma < 2$. Thus, approaching the critical point along any isochore within the mixed phase region except for $\rho = \rho_c = 1/b$ the specific

heat diverges for $\zeta/\sigma < 2$ as defined by α_s' and remains finite for the isochore $\rho = \rho_c = 1/b$. This demonstrates the exceptional character of the critical isochore in this model.

In the special case that $\zeta = 1$ one finds $\alpha_s' = 2 - 1/\sigma$ for $\tau \leqslant 1 + 2\sigma$ and $\alpha_s' = -\beta$ for $\tau > 1 + 2\sigma$. Therefore, using α_s' instead of α', the exponent inequalities (22) and (23) are fulfilled exactly if $\zeta > 1$ and $\tau \leqslant \sigma + 1$ or if $\zeta = 1$ and $\tau \leqslant 2\sigma + 1$. In all other cases (22) and (23) are fulfilled as inequalities. Moreover, it can be shown that the SMM belongs to the universality class of real gases for $\zeta > 1$ and $\tau \geqslant \sigma + 1$.

The comparison of the above derived formulae for the critical exponents of the SMM for $\zeta = 1$ with those obtained within the FDM (Eqs. 51–56 in [6]) shows that these models belong to different universality classes (except for the singular case $\tau = 2$).

Furthermore, one has to note that for $\zeta = 1$, $\sigma \leqslant 1/2$ and $1 + \sigma < \tau \leqslant 1 + 2\sigma$ the critical exponents of the SMM coincide with those of the exactly solved one-dimensional FDM with non-zero droplet-volumes [16].

For the usual parameterization of the SMM [1] one obtains with $\zeta = 5/4$ and $\sigma = 2/3$ the exponents

$$\alpha_s' = \begin{cases} \dfrac{1}{8}, & \text{if } \tau < \dfrac{5}{3} \\ \dfrac{15}{8}\tau - 3, & \text{if } \tau \geqslant \dfrac{5}{3} \end{cases}, \qquad \beta = \frac{15}{8}(2-\tau),$$

$$\gamma' = \frac{15}{4}\left(\tau - \frac{3}{2}\right), \qquad \delta = \frac{\tau - 1}{2 - \tau}. \quad (28)$$

The critical indices of the nuclear liquid–gas PT were determined from the multifragmentation of gold nuclei [20] and found to be close to those ones of real gases. The method used to extract the critical exponents β and γ' in Ref. [20] was, however, found to have large uncertainties of about 25 per cents [21]. Nevertheless, those results allow us to estimate the value of τ from the experimental values of the critical exponents of real gases taken with large error bars. Using Eqs. (17), (19) and (21) one can generalize the exponent relations of Ref. [16]

$$\tau = 2 - \frac{\beta}{\gamma' + 2\beta} \quad \text{and} \quad \tau = 2 - \frac{1}{1 + \delta} \quad (29)$$

for arbitrary σ and ζ. Then, one obtains with [22] $\beta = 0.32 - 0.39$, $\gamma' = 1.3 - 1.4$ and $\delta = 4 - 5$ the

estimate $\tau = 1.799 - 1.846$. This demonstrates also that the value of τ is rather insensitive to the special choice of β, γ' and δ. Theoretical values for β, γ' and δ for Ising-like systems within the renormalized ϕ^4 theory [23] lead to the narrow range $\tau = 1.828 \pm 0.001$.

The critical indices of the SMM were studied numerically in Ref. [13]. The version V2 of Ref. [13] corresponds precisely to our model with $\tau = 0$, $\zeta = 5/4$ and $\sigma = 2/3$, but their results contradict to our analysis. Their results for version V3 of Ref. [13] are in contradiction with our proof presented in Ref. [11]. There it was shown that for non-vanishing surface energy (as in version V3) the critical point does not exist at all. The latter was found in [13] for the finite system and the critical indices were analyzed. Such a strange result is, probably, due to finite volume effects, although some doubts about the validity of the methods used to extract the critical indices (especially, τ) remain.

It is widely believed that the effective value of τ defined as $\tau_{\text{eff}} \equiv -\partial \ln n_k(\varepsilon)/\partial \ln k$ attains its minimum at the critical point (see references in [24]). This has been shown for the version of the FDM with the constraint of sufficiently small surface tension $a \cong 0$ for $T \geqslant T_c$ [25] and also can be seen easily for the SMM. Taking the SMM fragment distribution $n_k(\varepsilon) = g(T)k^{-\tau} \exp[\frac{\nu}{T}k - \frac{a(\varepsilon)}{T}k^\sigma] \sim k^{-\tau_{\text{eff}}}$ one finds

$$\tau_{\text{eff}} = \tau - \frac{\nu}{T}k + \frac{\sigma a(\varepsilon)}{T}k^\sigma \implies \tau = \min(\tau_{\text{eff}}),$$

(30)

where the last step follows from the fact that the inequalities $a(\varepsilon) \geqslant 0$, $\nu \leqslant 0$ become equalities at the critical point $\nu = a(0) = 0$. Therefore, the SMM predicts that the minimal value of τ_{eff} corresponds to the critical point.

In the E900 $\pi^- +$ Au multifragmentation experiment [26] the ISiS Collaboration measured the dependence of τ_{eff} upon the excitation energy and found the minimum value $\min(\tau_{\text{eff}}) \cong 1.9$ (Fig. 5 of Ref. [26]). Also the EOS Collaboration [24] performed an analysis of the minimum of τ_{eff} on Au + C multifragmentation data. The fitted τ_{eff}, plotted in Fig. 11.b of Ref. [24] versus the fragment multiplicity, exhibits a minimum in the range $\min(\tau_{\text{eff}}) \cong 1.8$–1.9. Both results contradict the original FDM [6], but agree with the above estimate of τ for real gases and for Ising-like systems in general.

5. Conclusions

The critical indices of an exactly soluble version of the SMM are derived. The inclusion of excluded volume effects generates a principal difference between the SMM and the FDM — these models belong to different universality classes. It is found that for the critical exponent α' obtained traditionally on the critical isochore Fisher's and Griffiths' exponent inequalities are broken. The modification of the definition of α' in the spirit of Ref. [16] recovers both scaling relations. The scaling laws (29) connecting τ with β, γ' and δ are generalized for arbitrary σ and ζ. The ranges $\tau = 1.799$–1.846 for real gases and $\tau = 1.828 \pm 0.001$ for Ising-like systems are estimated. It is shown that experimental data on nuclear multifragmentation agree well with these ranges. Therefore, it is possible that the values for the exponent $\tau < 2$ seen in the data evidence that the critical point of the nuclear liquid–gas phase transition has been reached experimentally.

Acknowledgements

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