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Statistical coalescence model analysis of J/ψ production in Pb + Pb collisions at 158 A GeV

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Abstract

Production of J/ψ mesons in heavy ion collisions is considered within the statistical coalescence model. The model is in agreement with the experimental data of the NA50 Collaboration for Pb + Pb collisions at 158 A GeV in a wide centrality range, including the so-called "anomalous" suppression domain. The model description of the J/ψ data requires, however, strong enhancement of the open charm production in central Pb + Pb collisions. This model prediction may be checked in the future SPS runs.

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Production of charmonium states J/ψ and ψ' in nucleus–nucleus collisions has been studied at CERN SPS over the previous 15 years by the NA38 and NA50 Collaborations. This experimental program was mainly motivated by the suggestion [1] to use the J/ψ as a probe of the state of matter created at the early stage of the collision. The original picture [1] (see also [2] for a modern review) assumes that charmonia are created exclusively at the initial stage of the reaction in primary nucleon–nucleon collisions. During the subsequent evolution of the system, the number of hidden charm mesons is reduced because of: (a) absorption of pre-resonance charmonium states by nuclear nucleons (normal nuclear suppression), (b) interactions of charmonia with secondary hadrons (comovers), (c) dissociation of $c\bar{c}$ bound states in deconfined medium (anomalous suppression). It was found [3] that J/ψ suppression with respect to Drell–Yan muon pairs measured in proton–nucleus and nucleus–nucleus collisions with light projectiles can be explained by the so-called "*normal*" (due to sweeping nucleons) nuclear suppression alone. In contrast, the NA50 experiment with a heavy projectile and target (Pb + Pb) revealed essentially stronger J/ψ suppression for central collisions [4–7]. This *anomalous* J/ψ suppression was attributed to formation of quark–gluon plasma (QGP) [7], but a comover scenario cannot be excluded [8].

A completely different picture of charmonium production was developed recently within several model approaches [9–14]. In contrast to the standard approach, hidden charm mesons are supposed to be cre-

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ated at the hadronization stage of the reaction due to coalescence of c and $\bar{c}$ quarks created earlier. In this case the J/ψ yield is not restricted from above by the normal nuclear suppression curve. Therefore, neither anomalous suppression nor enhancement are excluded.

In the present Letter we consider the statistical coalescence model (SCM) [10,11] of charmonium production. We assume that c and $\bar{c}$ are created at the initial stage of the reaction in primary hard parton collisions. We neglect creation of $c\bar{c}$ pairs after the hard initial stage as well as their possible annihilation. Then, the number of charmed quark–antiquark pairs remain approximately unchanged during subsequent stages. They are distributed over final hadron states at the hadronization stage in accord with laws of statistical mechanics. The SCM provides an excellent quantitative description of the NA50 data on centrality dependence of J/ψ production in Pb + Pb collisions at SPS, provided that the number of nucleon participants is not too small ($N_p \gtrsim 100$). The peripheral collision data can be explained qualitatively.

If creation of heavy quarks is indeed a hard process only, the average number $\langle c\bar{c}\rangle_{AB(b)}$ of produced $c\bar{c}$ pairs must be proportional to the number of primary nucleon–nucleon collisions. Then the centrality dependence of $\langle c\bar{c}\rangle_{AB(b)}$ can be calculated in Glauber's approach:

$$\langle c\bar{c}\rangle_{AB(b)} = \sigma_{c\bar{c}}^{NN} T_{AB}(b). \tag{1}$$

Here b is the impact parameter, $T_{AB}(b)$ is the nuclear overlap function (see appendix) and $\sigma_{c\bar{c}}^{NN}$ is the $c\bar{c}$ production cross section for nucleon–nucleon collisions. As discussed in Ref. [15], deconfined medium can substantially modify charm production in hard collisions at SPS. Therefore, $\sigma_{c\bar{c}}^{NN}$ in $A + B$ collisions can be different from the corresponding cross section measured in a nucleon–nucleon collision experiment. The present analysis considers $\sigma_{c\bar{c}}^{NN}$ as a free parameter. Its value is fixed by fitting the NA50 data.

Event-by-event fluctuations of the number of $c\bar{c}$ pairs follow the binomial distribution, which can be safely approximated by the Poisson distribution because the probability to produce a $c\bar{c}$ pair in a nucleon–nucleon collision is small:

$$P_k(b) = \exp\left[-\langle c\bar{c}\rangle_{AB(b)}\right] \frac{[\langle c\bar{c}\rangle_{AB(b)}]^k}{k!}, \tag{2}$$

where $P_k(b)$ is the probability to produce k $c\bar{c}$ pairs in an $A + B$ collision at impact parameter b. Assuming exact $c\bar{c}$-number conservation during the evolution of the system, the SCM result for the average number of produced J/ψ per $A + B$ collision is given by [11]

$$\langle J/\psi\rangle_{AB(b)} = \langle c\bar{c}\rangle_{AB(b)}\left[1 + \langle c\bar{c}\rangle_{AB(b)}\right] \frac{N_{J/\psi}^{\mathrm{tot}}}{(N_O/2)^2} + o\left[\frac{N_{J/\psi}^{\mathrm{tot}}}{(N_O/2)^2}\right]. \tag{3}$$

Here

$$N_O = \sum_{j=D,\bar{D},D^*,\bar{D}^*,\ldots} N_j \tag{4}$$

is the total open charm *thermal* multiplicity. The sum runs over all known (anti)charmed particle species [16]. The total J/ψ multiplicity includes the contribution of excited charmonium states decaying into J/ψ:

$$N_{J/\psi}^{\mathrm{tot}} = \sum_{j=J/\psi,\chi_1,\chi_2,\psi'} R(j) N_j. \tag{5}$$

Here $R(j)$ is the decay branching ratio of the charmonium j into J/ψ: $R(J/\psi) \equiv 1$, $R(\chi_1) \approx 0.27$, $R(\chi_2) \approx 0.14$ and $R(\psi') \approx 0.54$. The multiplicities N_j are found in the grand canonical ensemble formulation of the equilibrium hadron gas model:

$$N_j = V n_j(T, \mu_B) = \frac{d_j V e^{\mu_j/T}}{2\pi^2} T m_j^2 K_2\left(\frac{m_j}{T}\right). \tag{6}$$

Here V and T are the volume[1] and temperature of the HG system, m_j and d_j denote, respectively, the masses and degeneracy factors of particles. K_2 is the modified Bessel function. The chemical potential μ_j of the particle species j in Eq. (6) is defined as

$$\mu_j = b_j \mu_B + s_j \mu_S + c_j \mu_C. \tag{7}$$

Here b_j,s_j and c_j represent the baryon number, strangeness and charm of the particle j, respectively. The baryonic chemical potential μ_B regulates the baryonic density. The strange μ_S and charm μ_C chemical potentials are found by requiring zero value for the total strangeness and charm in the system. In

[1] We use ideal HG formulas and neglect excluded volume corrections.

our consideration we neglect small effects of a non-zero electrical chemical potential.

We assume that the chemical freeze-out occurs close to (or even coincide with) the hadronization (where charmonia supposedly formed). Therefore for the thermodynamic parameters T and μ_B we use the chemical freeze-out values found [17] by fitting the HG model to the hadron yield data in Pb + Pb collisions at SPS:

$$T = 168\ \text{MeV}, \qquad \mu_B = 266\ \text{MeV}. \tag{8}$$

Uncertainties in the freeze-out parameters exist due to time evolution of the system through the phase transition [18] and because of possible change of effective hadron masses in hot and dense hadron medium [19]. To check robustness of the predictions, an independent parameter set [20] is also used. It has been obtained by assuming strangeness and antistrangeness suppression by factor γ_s:

$$T = 158\ \text{MeV}, \qquad \mu_B = 238\ \text{MeV},$$
$$\gamma_s = 0.79. \tag{9}$$

The system is assumed to freeze-out chemically at some common volume. This is fixed by the condition of baryon number conservation:

$$N_p(b) = V n_B(T,\mu_B) = V \sum_{j=N,\bar{N},\Delta,\bar{\Delta},\ldots} b_j n_j(T,\mu_B). \tag{10}$$

Here $N_p(b)$ is the number of participating nucleons at impact parameter b, $n_B(T,\mu_B)$ is the net baryon density at the chemical freeze-out. The sum in Eq. (10) runs over all (anti)baryon species. The linear relation (10) between N_p and V as well as constant values of T and μ_B parameters are assumed for all collisions with different values of impact parameter b.

Eq. (3) gives the *total* number of produced J/ψ's. They decay into $\mu^+\mu^-$ with the probability $B_{\mu\mu}^{J/\psi} = (5.88 \pm 0.10)\%$ [16]. Only the fraction η of $\mu^+\mu^-$ pairs that satisfies the kinematical conditions

$$0 < y < 1, \tag{11}$$

$$-1/2 < \cos\theta < 1/2 \tag{12}$$

can be registered by the NA50 spectrometer. Here y stands for the rapidity of a $\mu^+\mu^-$ pair in the center-of-mass frame of colliding nuclei. θ is the polar angle of the muon momentum in the rest frame of the pair. An estimate of η is impossible without detailed information about the hydrodynamic expansion of the system and the conditions at the thermal freeze-out. We shall therefore treat η as one more free parameter.

In the NA50 experiment the Drell–Yan muon pair multiplicity (either measured or calculated from the minimum bias data) is used as a reference for the J/ψ suppression pattern. Similarly to $c\bar{c}$ pairs, the number Drell–Yan pairs is proportional to the number of primary nucleon–nucleon collisions:

$$\langle DY' \rangle_{AB(b)} = \sigma_{DY'}^{NN} T_{AB}(b), \tag{13}$$

where $\sigma_{DY'}^{NN}$ is the nucleon–nucleon production cross section of $\mu^+\mu^-$ Drell–Yan pairs. The prime means that the pairs should satisfy the kinematical conditions of the NA50 spectrometer (11) and (12). As the Drell–Yan cross section is isospin dependent, an average value is used:

$$\sigma_{DY'}^{NN} = \frac{\sigma_{DY'}^{AB}}{AB}. \tag{14}$$

For the case of Pb + Pb collisions, $A = B = 208$ and $\sigma_{DY'}^{\text{PbPb}} = 1.49 \pm 0.13$ μb [5].

Hence, the quantity to be studied is the ratio

$$R(b) = \frac{\eta B_{\mu\mu}^{J/\psi} \langle J/\psi \rangle_{AB(b)}}{\langle DY' \rangle_{AB(b)}} = \eta B_{\mu\mu}^{J/\psi} \frac{\sigma_{c\bar{c}}^{NN}}{\sigma_{DY'}^{NN}} \left(1 + \sigma_{c\bar{c}}^{NN} T_{AB}(b)\right) \frac{N_{J/\psi}^{\text{tot}}}{(N_O/2)^2}. \tag{15}$$

It is convenient to rewrite the last expression in a simpler form

$$R(b) = C \frac{1 + \sigma_{c\bar{c}}^{NN} T_{AB}(b)}{N_p(b)} \tag{16}$$

and treat C and $\sigma_{c\bar{c}}^{NN}$ as free parameters. In this form our fitting procedure does not depend on chemical freeze-out conditions. The new free parameter C is connected to η by the expression

$$C = \eta B_{\mu\mu}^{J/\psi} \frac{\sigma_{c\bar{c}}^{NN}}{\sigma_{DY'}^{NN}} \frac{n_{J/\psi}^{\text{tot}}(T,\mu_B) n_B(T,\mu_B)}{(n_O(T,\mu_B)/2)^2}. \tag{17}$$

Here we have introduced the total open (anti)charm density: $n_O = N_O/V$ and total J/ψ "density": $n_{J/\psi}^{\text{tot}} = N_{J/\psi}^{\text{tot}}/V$. The relation between C and η does depend on freeze-out conditions, but our calculations with the parameter sets (8) and (9) have shown that this dependence is not essential.

In the NA50 experiment, the neutral transverse energy of produced particles E_T was used to measure centrality of the collisions. This variable, however, provides a reliable measure of the centrality only if it does not exceed a certain maximum value: $E_T \lesssim 100$ GeV (see also Refs. [21,22]). To show this we have calculated the dependence of the average number of participants on the transverse energy $\overline{N_p}(E_T)$.

The conditional probability to measure some value of E_T at fixed impact parameter b is given by a Gaussian distribution:

$$P(E_T|b) = \frac{1}{\sqrt{2\pi q^2 a N_p(b)}} \times \exp\left(-\frac{[E_T - qN_p(b)]^2}{2q^2 a N_p(b)}\right). \tag{18}$$

Analyzing the experimental situation, we are interested in a quite opposite question: how events with fixed E_T are distributed with respect to the centrality. The answer is

$$P(b|E_T) = \frac{b P(E_T|b) P_{\text{int}}(b)}{\int_0^{+\infty} db\, b P(E_T|b) P_{\text{int}}(b)}, \tag{19}$$

where $P_{\text{int}}(b)$ stands for the probability (see appendix) that two nuclei at fixed impact parameter b interact (at least one pair of nucleons collides). The average number of participating nucleons at fixed E_T is then given by the expression:

$$\overline{N_p}(E_T) = \int_0^{+\infty} db\, N_p(b) P(b|E_T) = \frac{\int_0^{+\infty} db\, b N_p(b) P(E_T|b) P_{\text{int}}(b)}{\int_0^{+\infty} db\, b P(E_T|b) P_{\text{int}}(b)}. \tag{20}$$

The parameter values $q = 0.274$ GeV and $a = 1.27$ [23] are fixed from the minimum bias transverse energy distribution.

The result is shown in Fig. 2. As is seen, the transverse energy is simply related to the number of participants $E_T = q\overline{N_p}$ in the domain $E_T \lesssim 100$ GeV. Outside of this domain $\overline{N_p}$ does not change essentially as E_T grows. Therefore the data at $E_T > 100$ GeV do not represent centrality dependence of the J/ψ suppression pattern but rather its dependence on fluctuations of the stopping energy at *fixed* number of participants. In principle, influence of such fluctuations on J/ψ multiplicity can be studied in the framework of our model, but information concerning the corresponding fluctuations of the chemical freeze-out parameters T and μ_B would be needed. Experimental data that would allow to extract this information (hadron yields at extremely large transverse energy) are not available at present. Therefore, we restrict our analysis to centrality dependence of J/ψ production and do not use the data corresponding to large transverse energies $E_T > 100$ GeV.

On the other hand, the SCM is not expected to describe small systems. This can be seen from ψ' data [10]. In the framework of SCM the multiplicity of ψ' is given by the formula (3) with the replacement $N^{\text{tot}}_{J/\psi} \to N_{\psi'}$. Therefore, the ψ' to J/ψ ratio as a function of centrality should be constant and equal to its thermal equilibrium value. The experimental data [24] (see also a compilation in Ref. [10]) are consistent with this picture only at rather large ($N_p \gtrsim 100$) numbers of participants [25].

Hence, the applicability domain of the model is limited to

$$27 < E_T < 100 \text{ GeV}. \tag{21}$$

Note that the most precise and abundant NA50 data (see Fig. 1) correspond to this kinematical region.

At $E_T \lesssim 100$ GeV the formula (16) and the equation

$$E_T = qN_p(b) \tag{22}$$

give a parametric dependence of the ratio R on the transverse energy. This dependence for the parameter set

$$C = (2.59 \pm 0.25) \times 10^3, \quad \sigma^{NN}_{c\bar{c}} = (34 \pm 10)\ \mu\text{b} \tag{23}$$

is plotted in Fig. 1. The free parameters were fixed by fitting three sets of NA50 data [6,7] within the applicability domain (21) of the model by the least square method. The model demonstrates excellent agreement with the fitted data ($\chi^2/\text{dof} = 1.2$).

Extrapolation of the fit to peripheral collisions reveals sharp increase of the ratio (15) with decreasing

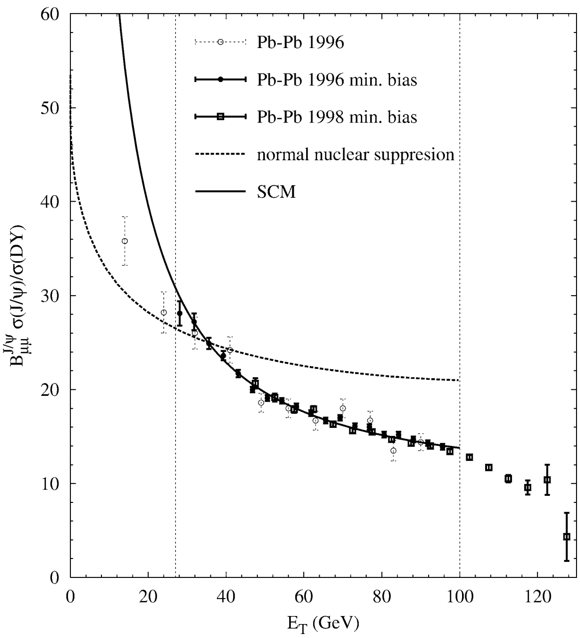

Fig. 1. The dependence of J/ψ over Drell–Yan multiplicity ratio on the transverse energy. The normal nuclear suppression curve is obtained at $\sigma_{\text{abs}} = 6.4$ mb, where σ_{abs} is the absorption cross section of preresonant charmonia by nuclear nucleons. Two vertical lines show the applicability domain of the model under consideration, see the text for details.

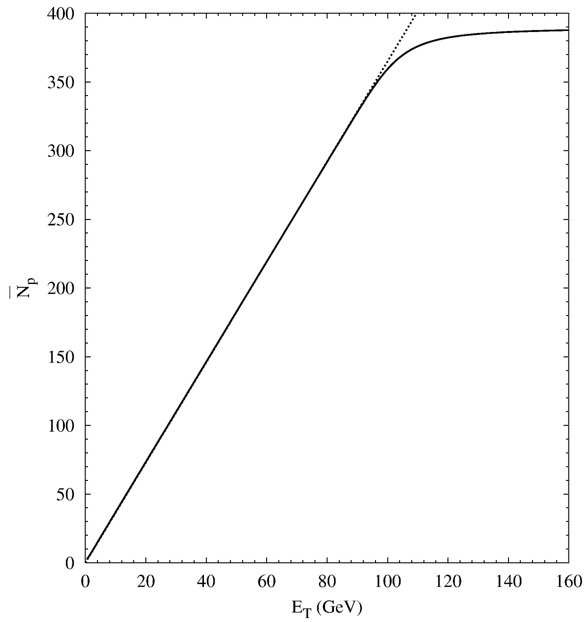

Fig. 2. The dependence of the average number of nucleon participants on the transverse energy. The dotted straight line corresponds to Eq. (22).

N_p. Such behavior in the SCM can be understood as the following. The smaller is the volume of the system the larger is the probability that c and $\bar{c}$ meet each other at hadronization stage and form a hidden charm meson. As is seen from Fig. 1, this is not supported by the data: the SCM curve lies above the experimental points in the low E_T region. On the other hand, the normal nuclear suppression model also fails to explain the leftmost point from the 1996 standard analysis set and two leftmost points from the 1996 minimum bias set. Those theoretical calculations underestimate the experimental values. It is natural to assume that an intermediate situation takes place.[2] Some fraction of peripheral Pb + Pb collisions result in formation of deconfined medium. In these collisions charmonia are formed at the hadronization stage, and their multiplicities are given by SCM. The rest collisions (we shall call them 'normal collisions') do not lead to color deconfinement, therefore charmonia are formed exclusively at the initial stage and then suffer normal nuclear suppression. The experiment measures the average value, which lies between the two curves.

The fraction of 'normal' events decreases with growing centrality. Their influence on J/ψ production becomes negligible at $N_p \gtrsim 100$. To check this we repeated the above fitting procedure using only the experimental data corresponding to $N_p > 200$. The quality of the fit is only slightly better: $\chi^2/\text{dof} = 1.1$, the parameter values $C = (2.73 \pm 0.40) \times 10^3$ and $\sigma_{c\bar{c}}^{NN} = (31 \pm 12)$ µb are consistent with the analysis of the full data set (23).

Our picture is also supported by ψ' data. The normal nuclear suppression influence nascent charmonia before the formation of meson states. Therefore its effect on ψ' is the same as on J/ψ. The multiplicity ratio of ψ' to J/ψ in 'normal' nuclear–nuclear collisions should be the same as in nucleon–nucleon collisions and should not depend on the centrality. In the

[2] Similar combination of standard and SCM production mechanisms has been considered in Ref. [26] for central Pb + Pb collisions. It was not, however, checked whether this approach is able to describe the centrality dependence of the J/ψ suppression pattern in (semi)central collision region.

framework of SCM, the ψ' to J/ψ ratio, as was explained above, should be equal to its thermal equilibrium value, which is a few times smaller than the corresponding value for 'normal' collisions. As the fraction of 'normal' events decreases, the measured ratio should decrease and then become constant and equal to its thermal value. The experimental data [24] indeed demonstrate such behavior [10,25].

The present analysis predicts strong enhancement of the total number of charm. From a pQCD fit of available data on charm production in $p+N$ and $p+A$ collisions, one could expect $\sigma_{c\bar{c}}^{NN} \approx 5.5$ µb at $\sqrt{s} = 17.3$ GeV. Our result (23) is larger by a factor of 4.5–8.0, which is around the upper bound of the charm enhancement estimated in Ref. [15].

Formation of deconfinement medium can change not only the total number of charmonia and open charm particles but also their rapidity distributions. For direct charmonium production in hard parton collisions, dimuon pairs satisfying the kinematical conditions (11) and (12) account for a fraction of about $\eta_{\text{hard}} \approx 0.24$ in the total number of pairs originating from J/ψ decays. (The value was found using Schuler's parameterization [27].) Our result (23) corresponds to $\eta \approx 0.14$, which is by a factor of about 0.6 smaller. This difference can be attributed to broadening of the J/ψ rapidity distribution. It is natural to expect similar modification of the open charm rapidity distribution. Because of this modification the open charm enhancement within a limited rapidity window can, in general, differ from the one for the total phase space. Assuming that the broadening for the open charm is approximately the same as that for J/ψ, one obtains open charm enhancement by a factor of about 2.5–4.5 within the rapidity window (11), which is consistent with the indirect experimental result [28].[3]

In conclusion, we have shown that the NA50 data on centrality dependence of the J/ψ and ψ' production in Pb + Pb collisions [6,7,24] are consistent with the following scenario.

The deconfined medium, which is formed in a Pb + Pb collision, prevents formation of charmonia at the initial stage of the reaction. Instead, hidden charm mesons are created at the hadronization stage due to coalescence of created earlier c and $\bar{c}$ quarks. Within this scenario, the color deconfinement does not necessary lead to suppression of J/ψ. Both suppression and enhancement are possible [30]. If the number of nucleon participants is not too small ($N_p \gtrsim 100$), the number of produced J/ψ is smaller than in the case of normal nuclear suppression, therefore *anomalous suppression* is observed. As color deconfinement is present in most collision events for $N_p \gtrsim 100$, our model reveals excellent agreement with the experimental data in this centrality domain. The statistical coalescence model does not describe the NA50 data for the peripheral Pb + Pb collisions. It seems that the fraction of events producing the deconfinement medium is not dominating there and most of peripheral collisions follow the normal nuclear suppression scenario. Still, the presence a fraction of abnormal events could reveal itself in the deviation of the J/ψ data up from the normal nuclear suppression curve.

Our model analysis predicts rather strong enhancement of the open charm. This effect can also be related to the color deconfinement [15]. The enhancement within the rapidity window $0 < y < 1$ is consistent with the indirect NA50 data [28]. A direct measurement of the open charm would be very important for checking the above scenario.

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[3] Our previous study [11] was based on the J/ψ multiplicity data [29], which were extracted from the NA50 data [6,7] assuming narrow rapidity distribution of J/ψ. In this case, the charm enhancement in the total phase space is by a factor of about 2–3.5 and does not differ from the enhancement in the limited rapidity domain, but the number of $c\bar{c}$ pairs should grow faster than the number of nucleon–nucleon collisions. Either data on rapidity distribution of J/ψ or precise data on centrality dependence of the open charm multiplicity would help us to decide, which of these two versions is preferable.

Appendix A. Nuclear geometry

The spherically symmetrical distribution of nucleons in the Pb-208 nucleus can be parameterized by a two-parameter Fermi function [31] (this parameterization is also known as the Woods–Saxon distribution):

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r-c}{a}\right)\right]^{-1} \tag{A.1}$$

with $c \approx 6.624$ fm, $a \approx 0.549$ fm and ρ_0 is fixed by the normalization condition:

$$4\pi \int_0^\infty dr\, r^2 \rho(r) = 1. \tag{A.2}$$

The nuclear thickness distribution $T_A(b)$ is given by the formula

$$T_A(b) = \int_{-\infty}^{\infty} dz\, \rho\left(\sqrt{b^2+z^2}\right), \tag{A.3}$$

and the nuclear overlap function is defined as

$$T_{AB}(b) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, T_A\left(\sqrt{x^2+y^2}\right) \times T_B\left(\sqrt{x^2+(y-b)^2}\right). \tag{A.4}$$

From Eq. (A.2), one can deduce that the above functions satisfy the following normalization conditions:

$$2\pi \int_0^\infty db\, b\, T_A(b) = 1,$$
$$2\pi \int_0^\infty db\, b T_{AB}(b) = 1. \tag{A.5}$$

In Glauber's approach the average number of participants ('wounded nucleons') in $A+B$ collisions at impact parameter b is given by [32]

$$\widetilde{N}_p(b) = A \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy\, T_A\left(\sqrt{x^2+y^2}\right) \times \left\{1 - \left[1 - \sigma_{NN}^{\text{inel}} T_B\left(\sqrt{x^2+(y-b)^2}\right)\right]^B\right\} + B \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy\, T_B\left(\sqrt{x^2+(y-b)^2}\right) \times \left\{1 - \left[1 - \sigma_{NN}^{\text{inel}} T_A\left(\sqrt{x^2+y^2}\right)\right]^A\right\}. \tag{A.6}$$

Here $\sigma_{NN}^{\text{inel}}$ is the nucleon–nucleon total inelastic cross section.

At large impact parameter, the nuclei may do not interact at all. Therefore $\widetilde{N}_p(b) \to 0$ at $b \to \infty$. If one interested in the average number of participants, provided that an interaction between two nuclei has taken place, the relevant quantity is

$$N_p(b) = \widetilde{N}_p(b)/P_{\text{int}}(b), \tag{A.7}$$

where

$$P_{\text{int}}(b) = 1 - \left[1 - \sigma_{\text{inel}}^{NN} T_{AB}(b)\right]^{AB} \tag{A.8}$$

is the probability for nuclei A and B to interact at impact parameter b. Although $N_p(b)$ differ from $\widetilde{N}_p(b)$ at large b: $\widetilde{N}_p(b) \to 2$ at $b \to \infty$, they are almost identical for more central collisions.

The average number of nucleon–nucleon collisions can be calculated from

$$\widetilde{N}_{\text{coll}}(b) = AB\sigma_{\text{inel}}^{NN} T_{AB}(b). \tag{A.9}$$

Provided that an interaction between two nuclei has taken place, the above formula should be modified as

$$N_{\text{coll}}(b) = \widetilde{N}_{\text{coll}}(b)/P_{\text{int}}(b). \tag{A.10}$$

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