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Interacting hadron resonance gas meets lattice QCD

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ABSTRACT

We present, in the framework of the interacting hadron resonance gas, an evaluation of thermodynamical quantities. The interaction is modelled via a correction for the finite size of the hadrons. We investigate the sensitivity of the model calculations on the radius of the hadrons, which is a parameter of the model. Our calculations for thermodynamical quantities as energy and entropy densities and pressure are confronted with predictions using the lattice Quantum Chromodynamics (QCD) formalism.

1. Introduction

One of the major goals of ultrarelativistic nuclear collision studies is to obtain information on the QCD phase diagram [1]. A promising approach is the investigation of hadron production. Hadron yields measured in central heavy ion collisions from AGS up to RHIC energies can be described very well [2–9] within a hadro-chemical equilibrium (also called hadron resonance gas or statistical) model. The main result of these investigations is that the extracted temperature values rise rather sharply from low energies on towards a center-of-mass energy per colliding nucleon pair $\sqrt{s_{NN}} \simeq 10$ GeV and reach, for higher collision energies, constant values near $T = 160$–165 MeV, while the baryochemical potential μ_b decreases monotonically as a function of energy. The Hagedorn limiting temperature [10] behavior suggests a connection to the phase boundary between the hadronic world and the deconfined phase. It was, indeed, argued [11] that the quark–hadron deconfinement phase transition drives the equilibration dynamically, at least for SPS energies and above. For lower energies, the quarkyonic state of matter [12] could complement this picture. The conjecture of the triple point [13] between hadronic, deconfined and quarkyonic matter was put forward in this context. In a recent study [14] it is argued, however, that the chemical frezee-out region at large μ_b is not close to the phase boundary.

Theoretical investigations of the QCD phase diagram are an important priority for ongoing research. While effective field theories need to be employed to model QCD in the strongly interacting regime for finite μ_b [15–17], QCD calculations on the lattice are an increasingly reliable approach for $\mu_b \simeq 0$. Employing calculations of QCD on lattice all groups predict indeed a steep increase of thermodynamical quantities near a critical temperature for deconfinement, T_c. Until recently, values for T_c between 151 MeV [18] and 192 MeV [19] were obtained. New results on larger lattices and with quark masses approaching the physical values [20,21] lead to better agreement on the T_c value in the range 155–160 MeV, while important details of lattice QCD calculations continue to be addressed. In this context, the hadron resonance gas (HRG) is used by lattice QCD groups as reference for their calculations in the hadronic sector [22,23]. This follows earlier ideas of [24] and involves modelling of quark mass dependence in the hadron resonance gas model to account for the finite lattice spacing [22–24]. Conversely, lattice data are used to constrain effective models based on hadronic resonances [25] to describe hadronic matter near T_c.

The details of the hadron resonance gas (or statistical) model are important too, as we have recently shown in [9], where we demonstrated that the completeness of the hadron spectrum involved in calculations is important for a precise description of data in nucleus–nucleus collisions. The aim of this Letter is to confront our HRG model calculations with lattice QCD predictions for thermodynamical observables in the hadronic sector. In particular, we investigate the effect of the excluded volume correction employed in the model to approximate a short-range repulsive hadron–hadron interaction. Such excluded volume corrections were first introduced in [26], albeit not yet in a thermodynamically consistent way. A thermodynamically consistent approach was first developed in [27] and will be the basis for our investigations.

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2. Model description

We restrict ourselves here to the basic features and essential results of the statistical model approach. A complete survey of the assumptions and results, as well as of the relevant references, is available in Ref. [28].

The basic quantity required to compute the thermal composition of hadron yields and the thermodynamical quantities is the partition function $Z(T,V)$. In the grand canonical (GC) ensemble, the partition function for a particle species i in the limit of large volume takes the following form ($k=\hbar=c=1$):

$$\ln Z_i^{id.gas} = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2\,\mathrm{d}p \ln\bigl[1\pm\exp\bigl(-(E_i-\mu_i)/T\bigr)\bigr], \tag{1}$$

from which the particle density n_i, the partial pressure P_i, the energy density ε_i and the entropy density s_i are then calculated according to:

$$n_i^{id.gas}(T,\mu_i) = N_i/V = \frac{T}{V}\left(\frac{\partial \ln Z_i^{id.gas}}{\partial\mu}\right)_{V,T} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}, \tag{2}$$

$$P_i^{id.gas}(T,\mu_i) = \frac{T}{V}\ln Z_i^{id.gas} = \pm\frac{g_i T}{2\pi^2}\int_0^\infty p^2\,\mathrm{d}p \ln\bigl(1\pm\exp\bigl[-(E_i-\mu_i)/T\bigr]\bigr), \tag{3}$$

$$\varepsilon_i^{id.gas}(T,\mu_i) = E_i/V = -\frac{1}{V}\left(\frac{\partial \ln Z_i^{id.gas}}{\partial(1/T)}\right)_{\mu/T} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}E_i, \tag{4}$$

$$s_i^{id.gas}(T,\mu_i) = S_i/V = \frac{1}{V}\left(\frac{\partial(T\ln Z_i^{id.gas})}{\partial T}\right)_{V,\mu} = \pm\frac{g_i}{2\pi^2}\int_0^\infty p^2\mathrm{d}p\left(\ln\bigl(1\pm\exp\bigl[-(E_i-\mu_i)/T\bigr]\bigr) \pm \frac{E_i-\mu_i}{T(\exp[(E_i-\mu_i)/T]\pm 1)}\right), \tag{5}$$

where $g_i=(2J_i+1)$ denotes the spin degeneracy factor, T is the temperature and $E_i=\sqrt{p^2+m_i^2}$ is the total energy. The $(+)$ sign corresponds to fermions and $(-)$ corresponds to bosons. For the hadron species i of baryon number B_i, third component of the isospin I_{3i}, strangeness S_i, and charm C_i, the chemical potential is $\mu_i=\mu_b B_i+\mu_{I_3}I_{3i}+\mu_S S_i+\mu_C C_i$. The chemical potentials related to baryon number (μ_b), isospin (μ_{I_3}), strangeness (μ_C) and charm (μ_C) ensure the (on average) conservation, in the collision, of the respective quantum numbers: i) isospin: $V_{cons}\sum_i n_i I_{3i}=I_3^{tot}$, with $V_{cons}=N_B/\sum_i n_i B_i$; ii) strangeness: $\sum_i n_i S_i=0$; iii) charm: $\sum_i n_i C_i=0$. The (net) baryon number N_B and the total isospin I_3^{tot} of the system are input values which need to be specified according to the colliding nuclei and rapidity interval studied. Taking into account the conservation laws i)–iii), the freeze-out temperature T, the baryochemical potential μ_b and the fireball volume at chemical freeze-out V are the parameters of the model, which are obtained from fits to experimentally measured hadron yields.

The following hadrons (number of species, not counting g_i) are included in the calculations: i) mesons: non-strange (123), strange (32), charm (40), bottom (28); ii) baryons: non-strange (48), strange (48), charm (32), bottom (14). The corresponding anti-particles are of course also included. Their characteristics, including a rather complete set of decay channels (all strong and electromagnetic decays), are implemented according to the 2008 PDG compilation[1] [29], with hadron masses reaching 3 GeV. We use vacuum masses for all hadrons.

Usually, whenever thermal fits are performed, the finite widths of resonances are taken into account in the density calculation by an additional integration, over the particle mass, with a Breit–Wigner distribution as a weight [8]. For the range of temperatures investigated in this work the effect of the finite resonance widths is small and to save computing time we have not employed the additional integration in the present calculations except where stated otherwise.

3. Interactions in the hadron gas model

When comparing thermodynamical quantities computed within the framework of the hadron resonance gas model with results obtained using lattice QCD methods one has to decide how to incorporate interactions among the hadrons. One approach is to use results obtained by the authors of [30–32] where two-body collisions are taken into account through scattering phase shifts. Here the interaction measure (the 2nd virial coefficient) is related to the derivative of the phase shifts with respect to energy. To compute the thermodynamics of the interacting hadron resonance gas in this way one would need knowledge of the energy dependence of all phase shifts. At first glance this seems quite impractical. An interesting result was, in this context, obtained in [33]. These authors show by explicit construction that, for simple systems such as gases of pions, pions and nucleons, and pions, kaons, and nucleons, the equation of state of the interacting system is obtained by adding the relevant resonances, the ρ and $f^0(980)$ mesons, the Δ baryon, the K^* meson, to the list of particles and by computing the partition function of the enlarged gas assuming no interactions.

This interesting result has led some authors [22,23] to argue that the thermodynamics of the interacting hadron resonance gas is well approximated, via the Dashen, Ma and Bernstein theorem [31,32], by that of the non-interacting case, provided that *all* states (resonances) are included in the partition function. It is one of the goals of this Letter to address the accuracy of this approximation within the framework of our interacting HRG model. Even at the formal level, there are points to be considered. First, the η, ω, η', ϕ and a_0 mesons cannot be treated like this [33]. Second, the baryon–baryon interaction is largely repulsive, with no known resonance structure,[2] see, e.g. Fig. 8 of [33]. More importantly, the approach of [31,32] is, as also discussed there, a low density approach, relevant for dilute systems. At temperatures near T_c, the temperature of the phase boundary between hadron gas and quark–gluon plasma, the hadron resonance gas is not dilute anymore. As will be shown below, overall densities exceed 0.5 fm^{-3} and total baryon densities are close to normal nuclear matter densities of 0.15 fm^{-3}. This implies that for the whole range of baryon chemical potentials considered here the baryon densities near T_c

[1] The 2010 PDG compilation contains updates in the hadron mass spectrum, but these are expected to have a minor influence on our results.

[2] We neglect here the deuteron in the $^3S_1$ state. In the baryon–antibaryon system there is likely no short-range repulsion and this leads to a small correction which is discussed below.

are close to or exceed the critical value worked out in [33] above which the virial expansion breaks down. In this environment also the concept of asymptotic states needed for the S-matrix approach of [30–32] is ill defined.

We therefore explore in the following, in addition to the 'free' hadron resonance gas, also the thermodynamic properties of a hadron resonance gas in which short-distance repulsion is explicitly taken into account using the thermodynamically consistent excluded volume approach developed in [27]. In essence this amounts to a Van der Waals construction. This is implemented according to [27,34] in an iterative procedure for the total pressure as:

$$P(T,\mu_1,\ldots,\mu_m) = P^{id.gas}(T,\hat{\mu}_1,\ldots,\hat{\mu}_m) \tag{6}$$

where $P^{id.gas} = \sum_i P_i^{id.gas}(T,\hat{\mu}_i)$ and for each particle i the chemical potential at a given iteration is recalculated as:

$$\hat{\mu}_i = \mu_i - V_{eigen,i} P(T,\mu_1,\ldots,\mu_m). \tag{7}$$

This approach yields the following formulae for the particle densities n_i, the total energy density ε and the total entropy density s expressed as a function of the respective quantities in the ideal gas, which are given in (2)–(5):

$$n_i = n_i(T,\mu_1,\ldots,\mu_m) = \left(\frac{\partial P}{\partial \mu_i}\right)_T = \frac{n_i^{id.gas}(T,\hat{\mu}_i)}{1+\sum_k V_{eigen,k} n_k^{id.gas}(T,\hat{\mu}_k)}, \tag{8}$$

$$s = s(T,\mu_1,\ldots,\mu_m) = \left(\frac{\partial P}{\partial T}\right)_{\mu_1,\ldots,\mu_m} = \frac{\sum_i s_i^{id.gas}(T,\hat{\mu}_i)}{1+\sum_k V_{eigen,k} n_k^{id.gas}(T,\hat{\mu}_k)}, \tag{9}$$

$$\varepsilon = \varepsilon(T,\mu_1,\ldots,\mu_m) = \frac{\sum_i \varepsilon_i^{id.gas}(T,\hat{\mu}_i)}{1+\sum_k V_{eigen,k} n_k^{id.gas}(T,\hat{\mu}_k)} \tag{10}$$

where $V_{eigen,i} = 4\cdot 4\pi R_i^3/3$ is the eigenvolume of a hadron[3] with radius R_i. We checked numerically that thermodynamical consistency, expressed by $\varepsilon = Ts - P + \sum_i \mu_i n_i$, is well fulfilled by our calculations, explicitly confirming the consistency of the procedure [27] used for the excluded volume correction.

For the radius parameter R_i, governing the excluded volume calculation, we follow the earlier arguments in [4]. There it was argued that, for baryons, the radius is given by the hard-core repulsive interaction as extracted from nucleon–nucleon scattering [35], giving a radius of about 0.3 fm. Values for other baryons should be similar. For mesons, in the absence of detailed information on their interactions at short distance, we assign the same radius value, based on the similarity of the meson charge radii compared to baryons and on the energy dependence of the pion–nucleon phase shifts [36]. For illustration, we have included the case of $R = 0$ for mesons, although we believe that the physical case is for mesons with repulsive core radius R comparable to baryons.

For the baryon–antibaryon system there are likely no short-range repulsive interactions, because of annihilation processes, which are by construction included in the hadron resonance gas

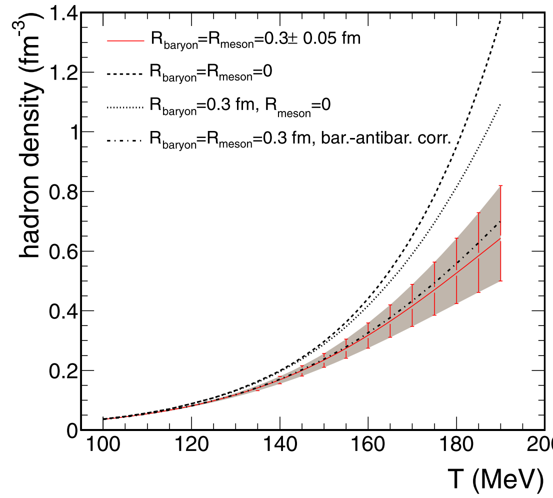

Fig. 1. Total hadron densities as a function of temperature, calculated for the hadron resonance gas model at $\mu_b = 0$, with and without excluded-volume corrections. A radius value of 0.3 fm was employed, with the band spanning ±0.05 fm around this value. The case of $R_{\rm meson}$=0 is shown with the dotted line, while the dot-dashed line denotes the effect of the baryon–antibaryon annihilation correction.

at equilibrium. We have modelled the absence of short-range repulsion in a schematic way, introducing a correction factor k_{anni} for the (anti)baryon eigenvolume based on the expression:

$$k_{anni} = 1 - \frac{2n_{\rm baryons} n_{\rm antibaryons}}{(n_{\rm baryons}+n_{\rm antibaryons})^2}, \tag{11}$$

where $n_{\rm baryons}$ is the density of baryons and $n_{\rm antibaryons}$ the density of antibaryons. As shown by the results presented in the following, the absence of short-range repulsion in the baryon–antibaryon system leads to only a small correction, since mesons dominate at small μ_b and since there are very few antibaryons at large μ_b.

For the rest of the Letter we show results of our calculations for radii in the range of 0.3 ± 0.05 fm, along with the cases discussed above and contrast those with the case of no interaction ($R_i = 0$). The value of 0.3 fm, common for mesons and baryons, was also used whenever we performed thermal fits to hadron abundancies [4,6,8,9]. In the description of hadron yields with the statistical model, the excluded volume correction leads to a larger volume parameter, while the fit temperature and baryochemical potential are unchanged compared to the case of fitting hadron ratios [8], for the case of identical R_i for mesons and baryons. The implication of a pion radius different from all other hadrons for the description of data has been studied by Yen et al. [34].

In Fig. 1 we present the temperature dependence of the hadron densities calculated with our model, with and without interactions modelled via excluded-volume corrections, as well as with the cases of $R_{meson} = 0$ and of absence of short-range interactions for baryon–antibaryon pairs. This illustrates our remarks above, namely that, while at low temperatures (low densities) there is no difference between the case of excluded volume correction and the case of free hadron resonance gas, the difference becomes appreciable for T above 130–140 MeV, which is significantly below the value for the critical (crossover) temperature $T_c \simeq 160$ MeV. Near T_c, the hadron resonance gas becomes manifestly dense, with the mean distance between hadrons getting significantly smaller than twice the hadron radius. All approximations appropriate for the dilute gas, discussed above, break down and the non-interacting hadron gas is not a suitable approach anymore.

We illustrate, for the energy density, ε, in Fig. 2 the sensitivity of the calculations on the radii of the eigenvolume for mesons

[3] Consider a particle with radius R in the hard sphere model. Then no other particle can come closer than a distance $2R$. Per pair the excluded volume is $4\pi(2R)^3/3$, leading to $V_{eigen} = 4\cdot 4\pi R^3/3$ for the particle.

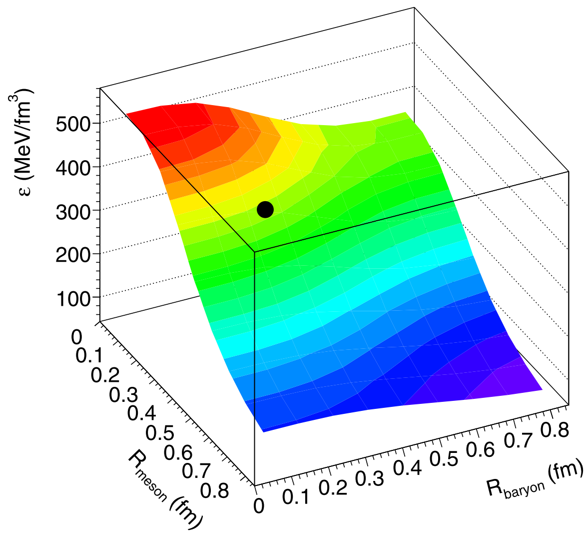

Fig. 2. The energy density of a hadron gas as a function of the radius (for the eigenvolume calculation) for mesons and baryons, at a temperature value of 164 MeV and $\mu_b = 0.8$ MeV. The dot indicates the radius value of 0.3 fm which we employ as default.

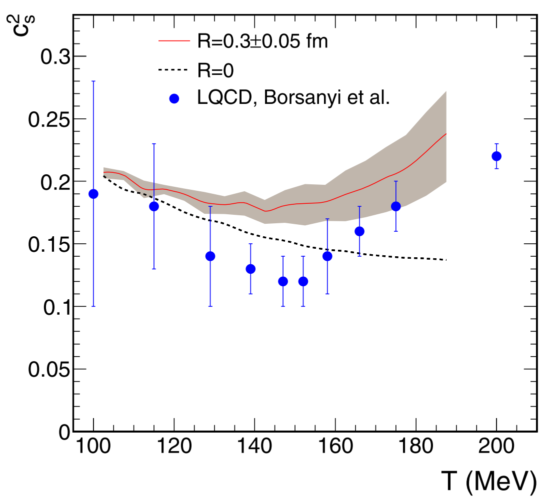

Fig. 3. The adiabatic speed of sound (in units of velocity of light, $c = 1$) squared as a function of temperature, calculated for the hadron gas model, with and without excluded-volume corrections. Our calculations are compared to lattice QCD calculations of Borsányi et al. [37].

and baryons, R_{meson} and R_{baryon}, respectively. The calculations have been performed for $T = 164$ MeV, corresponding to the limiting temperature reached in heavy-ion collisions [9] and for $\mu_b = 0.8$ MeV, the value expected for the LHC energy according to Ref. [9]; calculations for $\mu_b = 0$ lead to identical results. We observe a strong influence of the excluded volume correction on the energy density and this is the case for all other thermodynamical quantities. Due to the larger abundance of mesons (and in particular of pions) in the hadron gas at these values of T and μ_b, the sensitivity on R_{meson} is more pronounced.

As pointed out earlier [27,33], a possible problem of the hadron gas model with excluded volume corrections is acausal behavior (speed of sound larger than velocity of light). As we demonstrate in Fig. 3, our model is not plagued by such a behavior (for the case $\mu_b \simeq 0$ considered here). The adiabatic speed of sound, c_s, is calculated as:

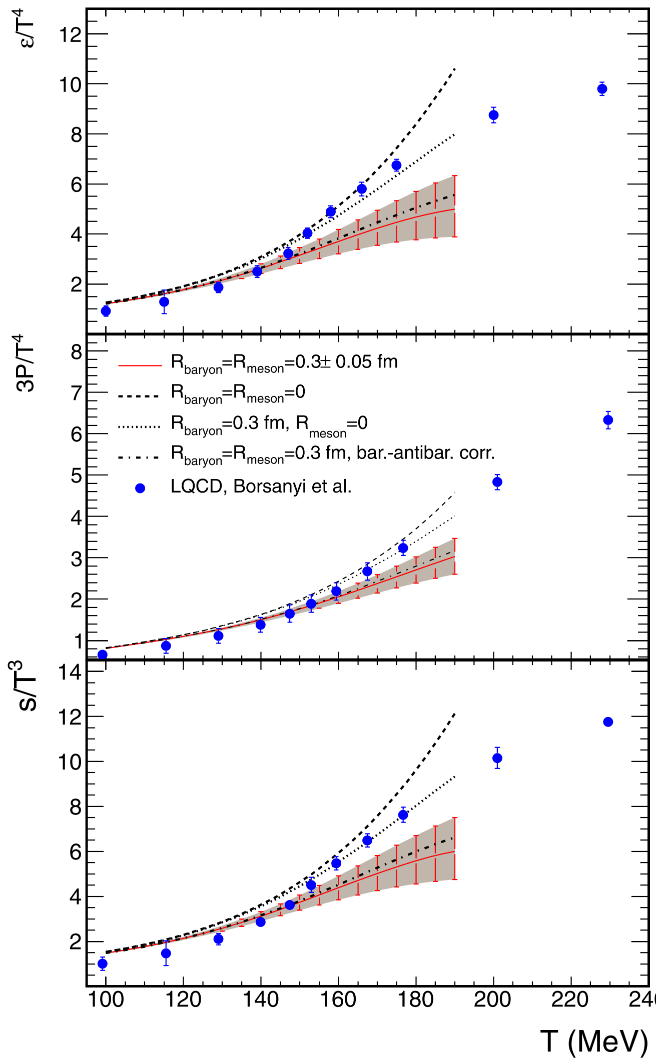

Fig. 4. Temperature dependence of thermodynamical quantities. The calculations with the hadron gas model are shown without (dashed line) and with (band for $R = 0.3 \pm 0.05$ fm) the excluded volume correction. The case of $R_{meson} = 0$ is shown with the dotted line, while the dot-dashed line denotes the effect of the baryon–antibaryon annihilation correction. They are compared to LQCD results of Borsányi et al. [37].

$$c_s^2 = \left(\frac{\mathrm{d}\ln s}{\mathrm{d}\ln T}\right)^{-1}. \tag{12}$$

It exhibits a shallow minimum as a function of T for the case of excluded volume corrections. Our calculations are compared to lattice QCD calculations [37]. In general, a good agreement between our and the lattice result is observed. We note that our calculations predict a shallow minimum for T around 140–150 MeV, while the lattice values indicate a more pronounced dip and exhibit a speed of sound value smaller than the hadron resonance gas with interactions. It would be interesting to see if the corresponding low temperature part of the equation of state (EoS) would lead to changes in hydrodynamic calculations, where generally the EoS shown in Fig. 6 or Ref. [22] is used.

4. Hadron resonance gas and lattice QCD results

In the following we compute, in the HRG model, thermodynamical quantities with and without excluded volume corrections and compare the results to predictions from lattice QCD. In Fig. 4

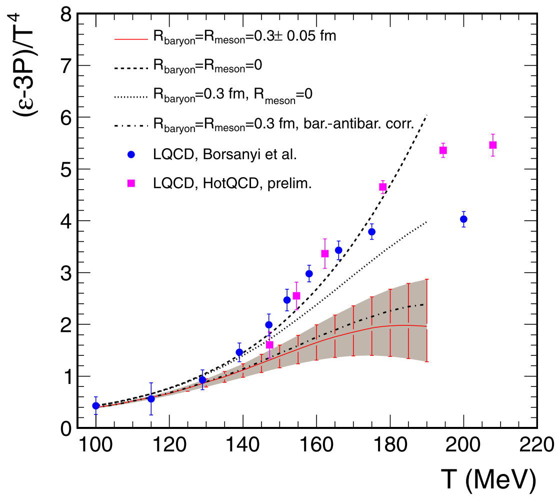

Fig. 5. Temperature dependence of the trace anomaly. The calculations within the hadron gas model (lines, as in Fig. 4) are compared to LQCD calculations of Borsányi et al. [37] and HotQCD Collaboration [38] (preliminary results).

we show the temperature dependence of energy density, pressure, and entropy density, each normalized to appropriate powers of the temperature.

The case without interactions (no excluded volume correction) has the expected strong dependence on temperature. As noted early on by Hagedorn [10], a limiting temperature, also called "Hagedorn temperature", of $T_H \simeq 200$ MeV arises for calculations of thermodynamical quantities within the HRG model if one assumes a hadron mass spectrum which increases exponentially with particle mass. Such exponential behavior is consistent with the present knowledge of hadron resonances [29] up to 2.0–2.5 GeV in mass. At this temperature all thermodynamical quantities for the HRG without excluded volume corrections diverge. We note in passing that all thermal model calculations without excluded volume corrections become meaningless for temperature values close to T_H. In the course of investigations reported in [9] we realized that this implies a practical limitation to temperatures below 175 MeV as all calculations for higher temperatures become very sensitive to details of the mass spectrum for masses larger than 3 GeV.

For the case of calculations employing finite hadron volume corrections the Hagedorn infinities are tamed. This was already noted by Hagedorn [10] who was the first to introduce excluded volume corrections [26]. Our findings substantiate this and imply that the Hagedorn limiting temperature is an artifact of the usage of the free hadron resonance gas description at temperatures where the implicit approximations for dilute systems are manifestly inappropriate.

For temperatures below 120 MeV the HRG model results with and without excluded volume correction almost coincide, see Fig. 4. For larger temperatures, the HRG with interactions yields, in our view, a realistic description of the hadronic phase. Therefore, in the confined regime, lattice QCD calculations of thermodynamical variables should give results in agreement with the interacting HRG. The expectation is that, as soon as effects of deconfinement become important in the lattice QCD results, they should increasingly exceed the HRG values. In Fig. 4 the most recent predictions of lattice QCD are compared to the HRG results. Indeed, below $T = 150$ MeV good agreement between results of lattice QCD [37] and the interacting HRG is found. On the other hand, effects of the onset of deconfinement [20,21] are apparent for T in excess of 150 MeV.

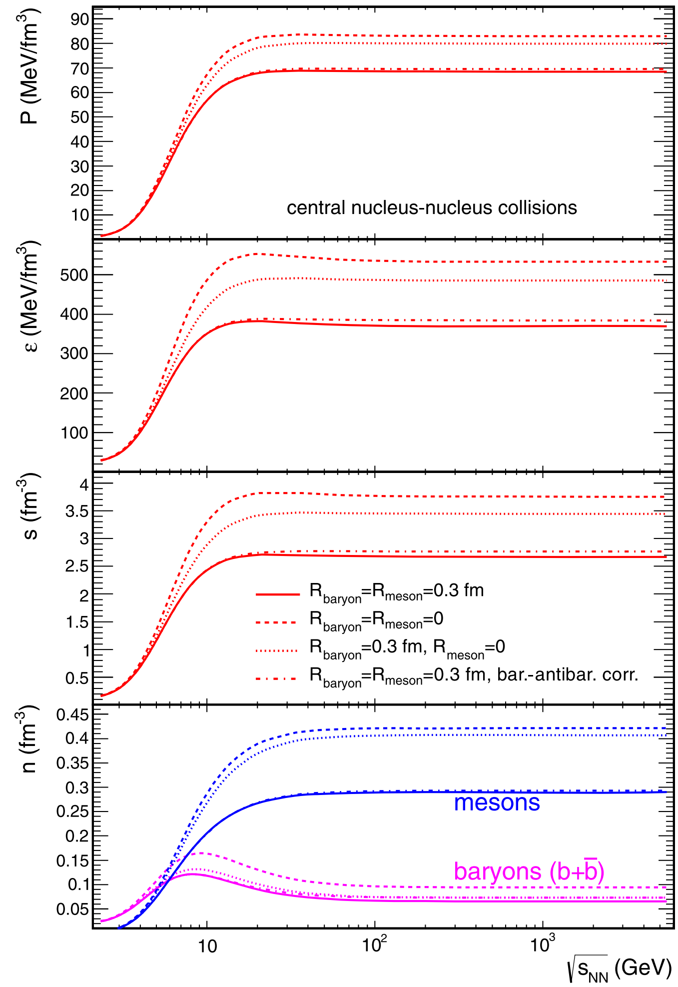

Fig. 6. Energy dependence of energy density, pressure, entropy density and baryon and meson densities at chemical freeze-out in central nucleus–nucleus collisions. The full lines are for the excluded volume corrections, the dashed line without, while the case of $R_{meson} = 0$ is shown with the dotted line and the dot-dashed line denotes the effect of the baryon–antibaryon annihilation correction.

In Fig. 5 we compare our calculations for the trace anomaly $\varepsilon - 3P$ (normalized to T^4) to LQCD results [37,38]. We see an agreement between LQCD data and our calculations for the interacting hadron gas only up to $T = 140$ MeV for the data of Borsányi et al. [37]. For the (preliminary) data of the HotQCD Collaboration [38] we have used for illustration the set for *hisq* action with $N_t = 6$, but we note that other available sets are in agreement with those within the errors [38].

We turn now to the energy dependence of the thermodynamical quantities at chemical freeze-out in central nucleus–nucleus collisions [9]. The degree of stopping of the colliding nuclei, which is energy dependent, brings some uncertainty in the choice of N_B and I_3^{tot}. As we study central collisions of heavy nuclei (Au or Pb) and focus on data at midrapidity, we have chosen $N_B = 400 \cdot \mu_b/938$ MeV and $I_3^{tot} = -40 \cdot \mu_b/938$ MeV, reflecting that μ_b traces stopping of the two colliding nuclei. The sensitivity of the thermodynamical quantities on N_B and I_3^{tot} is, however, small.

In Fig. 6 we show the thermodynamical quantities as a function of collision energy for chemical freeze-out in central nucleus–nucleus collisions. The trends seen in Fig. 6 reflect primarily the sharp increase of the temperature at chemical freeze-out (determined from fits of experimental data up to $\sqrt{s_{NN}} = 200$ GeV [9]) followed by a saturation above $\sqrt{s_{NN}} \simeq 10$ GeV. The characteristic energy dependence of the baryon density, exhibiting a maximum around 8 GeV, is determined by the increase of T combined with the strong decrease of μ_b with energy, as discussed in [9]. The effect of interactions leads to up to 30% reduction of the thermodynamical quantities at chemical freeze-out in nucleus–nucleus collisions.

5. Summary

We have presented an evaluation of thermodynamical quantities in the framework of the interacting hadron gas model, incorporating all known hadrons with masses reaching 3 GeV. A Van der Waals-type interaction is modelled via an excluded volume correction. Thermodynamic consistency is ensured by construction and the model exhibits proper causal behavior.

The resulting values for the thermodynamical quantities increase, already for temperatures of 130–150 MeV, i.e. significantly below T_c, much less steeply than in case of a free hadron resonance gas. Near T_c the free hadron resonance gas calculations already show signs of the Hagedorn divergence. Comparisons of lattice QCD results with the free hadron resonance gas in this temperature regime are therefore in our view problematic. Our results imply the need to consider the hadron resonance gas with interactions, beyond the usual implementations based on the Dashen, Ma and Bernstein theorem.

On the other hand, lattice QCD simulations start to be precise enough to reproduce the complete hadron gas at low temperatures. The apparent rise of the lattice QCD results above the HRG results is a clear indication of the onset of deconfinement not contained in the latter. In our view, the lattice results show genuine quark and gluon degrees of freedom in the vicinity of the (crossover) transition. In this temperature range, the lattice results produce thermodynamical quantities well above our predictions for the interacting hadron resonance gas. Our findings also imply that the Hagedorn limiting temperature is an artifact of the usage of the free hadron resonance gas description at temperatures where the gas becomes manifestly dense.

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References

[1] P. Braun-Munzinger, J. Wambach, Rev. Mod. Phys. 81 (2009) 1031, arXiv:0801.4256.
[2] P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, Phys. Lett. B 344 (1995) 43, arXiv:nucl-th/9410026;
P. Braun-Munzinger, J. Stachel, J.P. Wessels, N. Xu, Phys. Lett. B 365 (1996) 1, arXiv:nucl-th/9508020.
[3] J. Cleymans, D. Elliott, H. Satz, R.L. Thews, Z. Phys. C 74 (1997) 319, arXiv:nucl-th/9603004.
[4] P. Braun-Munzinger, I. Heppe, J. Stachel, Phys. Lett. B 465 (1999) 15, arXiv:nucl-th/9903010.
[5] J. Cleymans, K. Redlich, Phys. Rev. C 60 (1999) 054908, arXiv:nucl-th/9903063.
[6] P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B 518 (2001) 41, arXiv:hep-ph/0105229.
[7] F. Becattini, M. Gaździcki, J. Manninen, Phys. Rev. C 73 (2006) 044905, arXiv:hep-ph/0511092.
[8] A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 772 (2006) 167, arXiv:nucl-th/0511071.
[9] A. Andronic, P. Braun-Munzinger, J. Stachel, Phys. Lett. B 673 (2009) 142, arXiv:0812.1186;
A. Andronic, P. Braun-Munzinger, J. Stachel, Phys. Lett. B 678 (2009) 516 (Erratum).
[10] R. Hagedorn, CERN-TH-4100/85, 1985.
[11] P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B 596 (2004) 61, arXiv:nucl-th/0311005.
[12] L. McLerran, R. Pisarski, Nucl. Phys. A 796 (2007) 83, arXiv:0706.2191.
[13] A. Andronic, et al., Nucl. Phys. A 837 (2010) 65, arXiv:0911.4806.
[14] S. Floerchinger, Ch. Wetterich, Nucl. Phys. A 890–891 (2012) 11, arXiv:1202.1671.
[15] K. Fukushima, T. Hatsuda, Rep. Progr. Phys. 74 (2011) 014001, arXiv:1005.4814.
[16] K. Fukushima, Phys. Lett. B 695 (2011) 387, arXiv:1006.2596.
[17] T.K. Herbst, J.M. Pawlowski, B.-J. Schaefer, Phys. Lett. B 696 (2011) 58, arXiv:1008.0081.
[18] Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B 643 (2006) 46, arXiv:hep-lat/0609068.
[19] M. Cheng, et al., Phys. Rev. D 74 (2006) 054507, arXiv:hep-lat/0608013.
[20] Y. Aoki, et al., JHEP 0906 (2009) 088, arXiv:0903.4155.
[21] A. Bazavov, et al., Phys. Rev. D 85 (2012) 054503, arXiv:1111.1710.
[22] P. Huovinen, P. Petrecky, Nucl. Phys. A 837 (2010) 26, arXiv:0912.2541.
[23] S. Borsányi, et al., JHEP 1009 (2010) 073, arXiv:1005.3508.
[24] F. Karsch, K. Redlich, A. Tawfik, Eur. Phys. J. C 29 (2003) 549, arXiv:hep-ph/0303108;
F. Karsch, K. Redlich, A. Tawfik, Phys. Lett. B 51 (2003) 67, arXiv:hep-ph/0306208.
[25] L. Turko, D. Blaschke, D. Prorok, J. Berdermann, Acta Phys. Pol. B Proc. Suppl. 5 (2012) 485, arXiv:1112.6408.
[26] R. Hagedorn, J. Rafelski, Phys. Lett. B 97 (1980) 136.
[27] D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, Z. Phys. C 51 (1991) 485.
[28] P. Braun-Munzinger, K. Redlich, J. Stachel, invited review in: R.C. Hwa, X.N. Wang (Eds.), Quark Gluon Plasma 3, World Scientific Publishing, 2004, nucl-th/0304013.
[29] C. Amsler, et al., Particle Data Group, Phys. Lett. B 667 (2008) 1.
[30] E. Beth, G.E. Uhlenbeck, Physica 4 (1937) 915.
[31] R. Dashen, S.-K. Ma, H.J. Bernstein, Phys. Rev. 187 (1969) 345.
[32] R. Dashen, S.-K. Ma, Phys. Rev. A 4 (1971) 700.
[33] R. Venugopalan, M. Prakash, Nucl. Phys. A 546 (1992) 718.
[34] G.D. Yen, M.I. Gorenstein, W. Greiner, S.N. Nang, Phys. Rev. C 56 (1997) 2210, nucl-th/9711062.
[35] A. Bohr, B. Mottelson, Nuclear Structure, vol. 1, Benjamin, New York, 1969, p. 251.
[36] L.D. Roper, R.M. Wright, B.T. Feld, Phys. Rev. 138 (1964) 190.
[37] S. Borsányi, et al., JHEP 1011 (2010) 077, arXiv:1007.2580.
[38] A. Bazavov, P. Petreczky, HotQCD Collaboration, PoS Lattice 2010 (2010) 169, arXiv:1012.1257;
W. Soldner, HotQCD Collaboration, PoS Lattice 2010 (2010) 215, arXiv:1012.4484.