International Center for Insurance Regulation
Goethe University Frankfurt

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

ICIR Working Paper Series No. 20/16

Edited by Helmut Gründl and Manfred Wandt

Systemic Risk: Time-Lags and Persistence*

Christian Kubitza, Helmut Gründl§

This version: May 2016

Abstract

Common systemic risk measures focus on the instantaneous occurrence of triggering and systemic events. However, systemic events may also occur with a time-lag to the triggering event. To study this contagion period and the resulting persistence of institutions' systemic risk we develop and employ the Conditional Shortfall Probability (CoSP), which is the likelihood that a systemic market event occurs with a specific time-lag to the triggering event. Based on CoSP we propose two aggregate systemic risk measures, namely the Aggregate Excess CoSP and the CoSP-weighted time-lag, that reflect the systemic risk aggregated over time and average time-lag of an institution's triggering event, respectively.

Our empirical results show that 15% of the financial companies in our sample are significantly systemically important with respect to the financial sector, while 27% of the financial companies are significantly systemically important with respect to the American non-financial sector. Still, the aggregate systemic risk of systemically important institutions is larger with respect to the financial market than with respect to non-financial markets. Moreover, the aggregate systemic risk of insurance companies is similar to the systemic risk of banks, while insurers are also exposed to the largest aggregate systemic risk among the financial sector.

Keywords: Contagion Period, Spillover Effects, Systemic Risk, Financial Crisis, Financial Markets

JEL Classification: G01, G14, G15, G20

* We are grateful for helpful comments and suggestions by Elia Berdin, Mario Brandtner, Wolfgang Kürsten and Christian Pigorsch. Any errors are our own.

§ Both authors are affiliated with the International Center for Insurance Regulation, Goethe-University Frankfurt, Theodor-W.-Adorno Platz 3, D-60629 Frankfurt am Main, Germany. E-mail: *kubitza@finance.uni-frankfurt.de*.

1 Introduction

According to the Committee on Capital Markets Regulation (2009, p. ES-3) systemic risk is "the risk of collapse of an entire system or entire market, exacerbated by links and interdependencies, where the failure of a single entity or cluster of entities can cause a cascading failure". This definition was also adopted by the Financial Stability Board (2011, p. 5) in the sense, that an institution or market is considered as being systemic if its "failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion". Thus, systemic risk is usually connoted with the risk of contagion and spillover effects.[1] These effects may result from direct linkages between firms, i.e. counterparty contagion. Additionally, contagion may evolve from indirect linkages due to the exposure to common risk factors (see Chan-Lau et al. (2009)), for example asset prices, or (over-)reactions of market participants like fire sales of securities, bank runs, or insurance runs, in particular through mass surrenders in life insurance.[2]

As Harrington (2009) points out, contagious reactions typically evolve over time and, thus, responses to shocks may be delayed: Since systemic distress is a result of triggering events that cause (the lack of) cash-flows - or the information about (the lack of) cash-flows -, systemic risk migrates from institution to institution through their interconnectedness.[3] Still, the efficient markets hypothesis seems to provide a reason against the occurrence of delayed responses to shocks: In (semi-)strongly efficient markets, all (publicly available) information are reflected in current prices (see Fama (1969)). Thus, markets would react immediately to the information about shock events. However, not all information about the (own) exposure to shock events may be available immediately, such that a proper market reaction may be delayed. This idea is similarly to the concept of

[1] In this article we solely focus on contagion and spillover effects of adverse effects, which is in contrast to other articles incorporating both positive and negative spillovers (for example see International Monetary Fund (2016) and references therein).

[2] Harrington (2009) finds that the economics literature mainly distinguishes between four sources of systemic risk: asset price contagion, counterparty contagion, contagion due to uncertainty and opacity of information, and irrational contagion. Another classification is given by Arregui et al. (2013), who differentiate between contagion caused by direct bilateral exposure across institutions and contagion caused by indirect exposure to common risk factors. Vital features of systemic crises are also described by Marshall (1998).

[3] This effect is often referred to as domino or cascade effect (for example see Committee on Capital Markets Regulation (2009) or Smaga (2014)).

information contagion studied by Acharya and Yorulmazer (2002), and is also partly described by Harrington (2009) as *contagion due to uncertainty and opacity of information*, where the information about shocks creates uncertainty about the effects on counterparties. Similarly, there may be uncertainty about the effects on the own institution: The high complexity and opaqueness of interconnected markets and institutions, particularly in the financial services sector (see Adrian et al. (2014), Arora et al. (2009) or Moghadam and Viñals (2010)), but also complex products like hybrid debt (e.g. CoCo-Bonds), reinsurance, specific forms of parent-subsidiary relationships, captives and other forms of capital transfer mechanisms make it difficult to fully assess the impact of external events immediately. Thus, also wake-up call effects, i.e. contagion caused by the reassessment of assets in response to trouble in one country or sector (see Ahnert and Bertsch (2015)), may not happen simultaneously.

Moreover, in inefficient markets not all information about shocks are reflected immediately, which provides another reason for the occurrence of time-lags between triggering and systemic events. As Fama (1969) already points out, markets are clearly not efficient in the strong form, i.e. not all private information is immediately reflected in market prices. There exist several findings challenging the efficient market hypothesis also in its weak and semi-strong form. For example, Dong et al. (2013) find persistent price patterns for several global markets that violate the weak-form efficient market hypothesis. Similarly, Billio et al. (2010) find that stock returns exhibit large autocorrelations with a lag of one month particularly during the 2007-2009 crisis. Non-zero autocorrelations indicate that market frictions prevent arbitrage opportunities from being completely exploited (see for example Farmer and Lo (1999), Grossmann and Stiglitz (1980) or Lo (2004) and references therein). These frictions may be caused by regulation, transaction costs or borrowing constraints, but also by the high complexity and opaqueness of interconnected markets and institutions, as outlined above.

We conclude that systemic contagion effects have an inherent time dimension. They start with one or multiple triggering events and result in the infection of capital markets and even the whole economy.[4] For this reason, one might expect that systemic risk measures incorporate the timing

[4]Such contagion cascades with time-lags are also recognized by Pericoli and Sbracia (2003) and are similar to

of triggering and subsequent systemic events, i.e. the buildup component of systemic risk. Interestingly, all recently proposed cross-sectional risk measures that play an important role in the discussion about the measurement of systemic risk do not take such time-lags into consideration (for an overview see Bisias et al. (2015)).[5] The most prominent method that, in contrast, considers a time-lag between return series is the Granger causality test (see Granger (1969)). This test is for example employed by Chen et al. (2013) and Billio et al. (2010), who both find that the influence of banks on insurers is more persistent than vice versa. Nonetheless, the Granger causality test does not assess the magnitude of spillover effects between triggering and systemic events. In contrast, it focuses on a model for the whole time series distribution of market returns.

Hence, the major part of the literature focuses on simultaneous movements of returns, i.e. correlation. This observation motivates our article, in which we develop measures for systemic risk that explicitly take the time-lag between triggering and systemic events into account. Hereby, we aim to describe the timing dimension of systemic risk. In our approach we build on recent literature aiming to measure systemic risk by using stock returns (see, e.g. Acharya et al. (2012), Adrian and Brunnermeier (2014) or Black et al. (2016)), which reflect market participants' expectations on the future firm policy and policy-decisions (like, e.g., bailouts), but also, e.g., herding behavior.

To conclude, a thorough analysis of the contagion period between triggering and systemic events is still missing. We address this issue by proposing and employing the Conditional Shortfall Probability (CoSP), which is the likelihood of a systemic event occurring with a specific time-lag to a triggering event. The underlying rationale of CoSP is very similar to ΔCoVaR of Adrian and Brunnermeier (2014). However, CoSP has several advantages over ΔCoVaR, in particular a substantially smaller estimation error and, thus, a larger reliability in the sense of Danielsson et al. (2015). Moreover, it is independent from market volatility. Motivated by its properties, we use CoSP to define measures of the systemic risk aggregated over time and the systemic time-lag,

the phenomenon of *mutual excitation* in financial modeling, for example see Aït-Sahalia et al. (2015) and references therein.

[5] Our understanding of a time-lag between trigger and systemic events is different from forward-measures as e.g. the Forward-ΔCoVaR: Forward-measures focus on a forecast of future values of a systemic risk measure (in the case of Forward-ΔCoVaR the future value of ΔCoVaR is forecasted, see Adrian and Brunnermeier (2014)), whereas the risk measure itself still focuses on simultaneous movements of market's and institution's returns.

namely the Aggregate Excess CoSP and the CoSP-weighted time-lag.

In addition to identifying systemically relevant institutions, the need for considering the informativeness of systemic risk measures (e.g. with confidence bounds) is stressed by many authors and regulatory authorities (for example see Danielsson et al. (2016)). However, only few articles deal with the estimation error of employed systemic risk measures.[6] A reason might be given by the additional computational effort for estimating confidence intervals. In contrast, we present a closed-form for the lower bound of significance for the CoSP, which is valid under rather weak assumptions. Thus, by applying CoSP one is able to distinguish between institutions with significant and non-significant systemic importance.

We conduct an empirical analysis of the proposed risk measures that focuses on the structural differences between the systemic risk of banks, brokers, insurance and non-financial companies. The systemic relevance of (life) insurance institutions, in particular, has recently caused massive disputes. Due to the long-term investments on the one hand, and the insurers' core business activities being fundamentally different from the banking business on the other hand, many authors argue that (life) insurers are less systemically relevant than banks (for example see Haefeli and Liedtke (2012), Harrington (2009), or Thimann (2014)). This view is also supported by the econometric analysis of Acharya et al. (2010), but it is not generally agreed on. In particular, Billio et al. (2010) and Adrian and Brunnermeier (2014) find that the systemic risk in the insurance sector is generally not smaller than in the banking sector, whereas Weiß and Mühlnickel (2014) conclude that the systemic risk triggered by insurance institutions is mainly driven by the insurer's size. Clearly, the specific systemic role of insurance institutions is still not clear and is causing controversial discussions between academics, regulators and insurance institutions.[7]

By considering the timing dimension of systemic risk, our article provides a broader perspective on the systemic riskiness of financial and non-financial companies. We find that systemic shocks

[6] Exceptions include Benoit et al. (2013), Danielsson et al. (2015), Danielsson et al. (2016), Danielsson and Zhou (2015), Guntay and Kupiec (2014), and Löffler and Raupach (2013).

[7] For example, MetLife filed a lawsuit against its systemic-risk label, which MetLife also won in 2016, see Harris and Chiglinsky (2016).

are often spread among institutions and markets with a time-lag significantly different from zero. Approximately 15% of the financial institutions in our sample are significantly systemically important with a non-zero time-lag for the financial sector and 27% of the financial institutions are significantly systemically important with a non-zero time-lag for the American non-financial sector. In particular, the systemic risk of insurance institutions is similar to banks, whereas their exposure to systemic risk is larger than the exposure of banks and brokers.[8]

The remainder of the paper is organized as follows. Section 2 gives an overview of existing systemic risk measures. In Section 3 we introduce new systemic risk measures that incorporate time-lags. Section 4 describes the data sample which we use for our empirical analysis in Section 5. The final Section 6 concludes and gives an outlook to future research directions.

2 Traditional Measures for Systemic Risk

Bisias et al. (2015) report four main cross-sectional systemic risk measures: distress insurance premium (see Huang et al. (2009)), marginal expected shortfall (see Acharya et al. (2010)), co-risk (see Chan-Lau et al. (2009)) and ΔCoVaR (see Adrian and Brunnermeier (2014)).[9] The distress insurance premium (DIP) is a hypothetical insurance premium against catastrophic losses in the financial/banking system and is estimated by using CDS spreads (see Huang et al. (2009)). As Black et al. (2016) show in an empirical analysis, the DIP is mainly driven by risk-neutral probabilities of default, thus, mainly reflects the firm-specific risk. The Marginal Expected Shortfall (MES) quantifies the immediate exposure of a financial institution to the market's risk. Co-Risk is very similar to CoVaR, but examines CDS spreads instead of returns.

In the following, we will review ΔCoVaR in more detail to motivate our own approach. ΔCoVaR measures the change in the market's risk conditional on a financial institution being in distress. Hereby, the market's risk conditional on a specific event E, $\mathrm{CoVaR}_E(q)$, is defined as the Value-at-

[8] Throughout the article we refer to depository institutions as *banks* and security and commodity brokers as *brokers*.

[9] Danielsson et al. (2016) give a general classification of these measures.

Risk (VaR) of the conditional distribution of the market return r^M, i.e.

$$\mathbb{P}\left(r^M \leq \text{CoVaR}_E(q) \mid E\right) = q. \tag{1}$$

Then, ΔCoVaR is the difference between the market's CoVaR conditional on a triggering event TE^I and a benchmark event BM^I, i.e.

$$\Delta\text{CoVaR} = \text{CoVaR}_{TE^I}(q) - \text{CoVaR}_{BM^I}(q). \tag{2}$$

Adrian and Brunnermeier (2014) define the triggering event as the institution's return being at the $VaR(q)$, i.e. $TE^I = \left\{r^I = VaR^I(q)\right\}$, and the benchmark event as the institution's return being at the median state, i.e. $BM^I = \left\{r^I = VaR^I(0.5)\right\}$, which yields

$$\Delta\text{CoVaR}^{=}(q) = \text{CoVaR}_{r^I = VaR^I(q)}(q) - \text{CoVaR}_{r^I = VaR^I(0.5)}(q). \tag{3}$$

However, with this definition more severe losses than $VaR^I(q)$ are not considered as triggering event. Hence, it may be appropriate to study a more general version of the triggering event. For this purpose, Ergün and Girardi (2013) propose

$$\Delta\text{CoVaR}^{\leq}(q) = \text{CoVaR}_{r^I \leq VaR^I(q)}(q) - \text{CoVaR}_{r^I \in [\mu^I \pm \sigma^I]}(q), \tag{4}$$

where μ^I and σ^I are the mean and standard deviation for the return of the institution, respectively. The change in the triggering event definition from being exactly at the VaR to being at or below the VaR also effects the consistency of CoVaR: Mainik and Schaanning (2014) show that $\text{CoVaR}_{r^I \leq VaR^I(q)}(q)$ is a continuous and increasing function of the dependence parameter between r^I and r^M, while $\text{CoVaR}_{r^I = VaR^I(q)}(q)$ is not.

By conditioning on the triggering event, on first sight it seems that ΔCoVaR is based on a causal relationship between institution and market. However, a series of theoretical results indicate that ΔCoVaR is the result of the co-movement of (tail-)returns: For example, this can directly be verified for $\Delta\text{CoVaR}^{=}$ in case of bivariate normally distributed returns, for which Adrian and

Brunnermeier (2014) show that

$$\Delta\mathrm{CoVaR}^{=} = \sigma^{M}(-\Phi^{-1}(q))\rho^{I,M}, \tag{5}$$

where σ^M is the standard deviation of market returns, Φ^{-1} is the inverse of the cumulative density function of the standard normal distribution, and $\rho^{I,M}$ is the correlation coefficient between market and institution returns.

More generally, Benoit et al. (2013) find that $\Delta\mathrm{CoVaR}^{=}$ is proportional to the institution's firm-specific risk if the dependence between financial asset returns is linear. In this case, the proportionality coefficient depends on the market's volatility and the correlation between market's and institution's returns. In other words, $\Delta\mathrm{CoVaR}^{=}$ is not able to identify a causal relationship between systemic and triggering event since it solely focuses on simultaneous events with exceptionally high losses of both the market and institution. Any contagion effects in the sense that a high loss of an institution can trigger losses of a market with a time-lag are, therefore, not captured. Thereupon, in Section 3 we develop measures of systemic contagion periods by including time-lags into systemic risk measures.

Moreover, several studies indicate that the estimation error of $\Delta\mathrm{CoVaR}^{=}$ makes it a rather unreliable systemic risk measure (for example see Castro and Ferrari (2012), Danielsson et al. (2015) or Guntay and Kupiec (2014)). As a consequence, we develop a risk measure that exhibits a smaller estimation error and that is, thus, more reliable.

3 Measuring Lagged Systemic Risk

3.1 The Conditional Shortfall Probability

The Conditional Shortfall Probability (CoSP) is a systemic risk measure that explicitly accounts for time-lags between triggering and systemic events. Consistent with $\Delta\mathrm{CoVaR}^{\leq}$, we interpret the occurrence of one of the $q^I \cdot 100\%$ smallest institution returns as a proxy for a triggering event and one of the $q^M \cdot 100\%$ smallest returns as a proxy for a systemic market event.[10] To assess the time-lagged influence of an institution's triggering event on the market, we define the Conditional Shortfall Probability (CoSP) by

$$\psi_\tau(q^M, q^I) = \mathbb{P}\left(SE^M_\tau \mid TE^I\right) = \mathbb{P}\left(r^M_\tau \leq VaR^M(q^M) \mid r^I \leq VaR^I(q^I)\right). \tag{6}$$

In this definition r^M_τ is the market return τ days after the institution's return r^I, while SE^M_τ is the systemic market event τ days after the triggering event TE^I. Thus, ψ_τ is the likelihood of an exceptionally small market return occurring τ days after an exceptionally small return of an institution.

The identification of the VaR-levels q^M and q^I is both necessary and challenging: For one exemplary market the 1% smallest market returns may relate to a systemic event, e.g. *market failure*. In contrast, all 5% smallest market returns may be systemic for a different market, for example due to a larger market capitalization. For other markets that exhibit no systemic risk q^M would equal zero. Thus, q^M reflects a market's systemic risk probability. Analogously, q^I reflects the institution's distress probability.

Eventually, the choice of q^M and q^I depends on the respective definition of systemic risk and triggering events, as well as the respective market's and institution's properties. However, presently there is no common agreement on the definition and level of market-specific systemic risk and institution-specific distress probabilities. For this reason, we set $q^M = q^I = q$ in line with $\Delta\text{CoVaR}^{\leq}$ and denote $\psi_\tau(q) = \psi_\tau(q, q)$ in the empirical analysis.

The idea behind CoSP is very similar to the idea of the distress spillover measure by Chan-Lau et al. (2012). However, Chan-Lau et al. (2012) base their measure on Merton's model for defaults and, thus, by their definition stress events only depend on balance sheet data and the model's assumptions about the evolution of assets and liabilities but not on market behavior. Moreover, the authors do not consider different time-lags between triggering and systemic events. The idea

[10] Note that the definition of CoSP also allows for different definitions of triggering and systemic events.

behind CoSP is also similar to the idea of $\Delta\text{CoVaR}^{\leq}$. In fact, one might also define a time-lagged CoVaR^{τ}_{E} by

$$\mathbb{P}\left(r^{M}_{\tau} \leq \text{CoVaR}^{\tau}_{E}(q) \mid E\right) = q. \tag{7}$$

Then, $\text{CoVaR}^{\tau}_{r^I \leq VaR^I(q)}$ and ψ_τ are properties of the same conditional distribution, namely the q-quantile and tail probability. When considering the same market, CoSP and $\Delta\text{CoVaR}^{\leq}$ also generate the same order of institutions according to systemic risk if the conditional market returns stochastically dominate each other.[11] However, the interpretation for the two measures is different: CoSP captures the likelihood of a predefined stress event (i.e. the size of the conditional distribution's tail), while ΔCoVaR reflects the additional tail risk of a triggering event (i.e. the difference between quantiles).

Clearly, ΔCoVaR depends on market volatility: If market returns are more volatile, the change in tail risk, as measured by ΔCoVaR, is larger.[12] Consequently, the systemic risk implied by ΔCoVaR is larger on more volatile markets, ceteris paribus. Therefore, the understanding of systemic risk by ΔCoVaR does not only involve the occurrence of systemic events, as e.g. market failure. It goes one step further and also quantifies the results of these events in terms of the change in tail returns. Hence, an institution may impose a very large threat towards one specific market in terms of spillover effects, while the resulting systemic risk implied by ΔCoVaR may be very small due to a small overall market volatility.

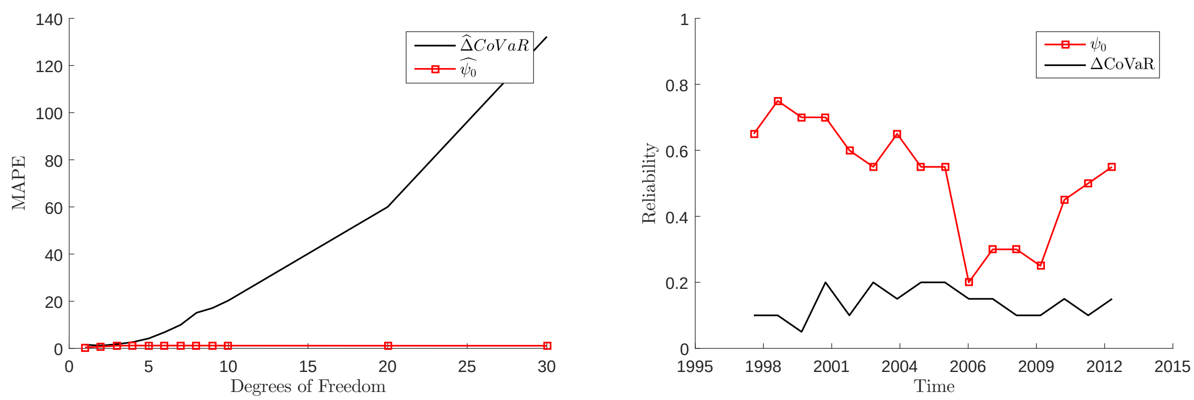
Still, systemic events like market distress or failure have important implications, not only in terms of lost return. For example, a small return loss on a largely capitalized market may be more adverse than a large return loss on a less capitalized market. In addition, implications of a market's distress may involve the interconnectedness with other markets and industries, but also affect political and socioeconomic dimensions. For example, Chan-Lau (2010) suggests to regulate too-connected-to-fail institutions based on societal losses. Consequently, it may not be sufficient to

[11] We discuss this property in Appendix A.3.

[12] For the Gaussian case the results of spillover effects measured by $\Delta\text{CoVaR}^{=}$ are solely driven by market volatility, as Equation (5) shows.

assess the implications of market distress solely by changes in tail returns, as ΔCoVaR does. Alternatively, we suggest to disentangle the threat of a systemic market event, i.e. market distress, from its implications. This idea is reflected by CoSP, which is independent from market volatility since systemic events are assumed to depend on the market's return quantile. More specifically, CoSP quantifies the risk of a systemic market event without already incorporating the impact of such an event. Thus, with CoSP it is possible to compare different markets in terms of their fragility towards spillover effects. As a prerequisite for financial regulation, one may then assess the impact of systemic events in terms of the change in tail risk, but also other socioeconomic and political factors.

CoSP has several advantages from a statistical point of view: Firstly, the standard error of CoSP is smaller.[13] Thus, less data is needed to estimate CoSP. Consequently, CoSP exhibits a larger reliability than $\Delta\text{CoVaR}^{\leq}$ in the sense of Danielsson et al. (2015) as we show in Appendix C.2 and in Figure 1. Secondly, asymptotic confidence bounds for CoSP are available in closed form, which permits to assess the statistical significance of CoSP in a straightforward manner.



(a) Mean Absolute Percentage Error (MAPE) of $\hat{\psi}_0$ and $\widehat{\Delta\text{CoVaR}}^{\leq}$ for student-distributed returns.

(b) Reliability of CoSP and $\Delta\text{CoVaR}^{\leq}$.

Figure 1: Estimation Error and Reliability of CoSP and $\Delta\text{CoVaR}^{\leq}$ for a sample size of 2500 (a description of the error and reliability measure can be found in Appendix C.2).

It is worthwhile to take a second look at the interpretation of CoSP, since ψ_τ, at first sight, seems to be the probability that a triggering event takes exactly τ days to affect the market. However, one specific triggering event may contribute to several lags.

[13]In Section C.1 we compare the standard error and reliability of CoSP and ΔCoVaR.

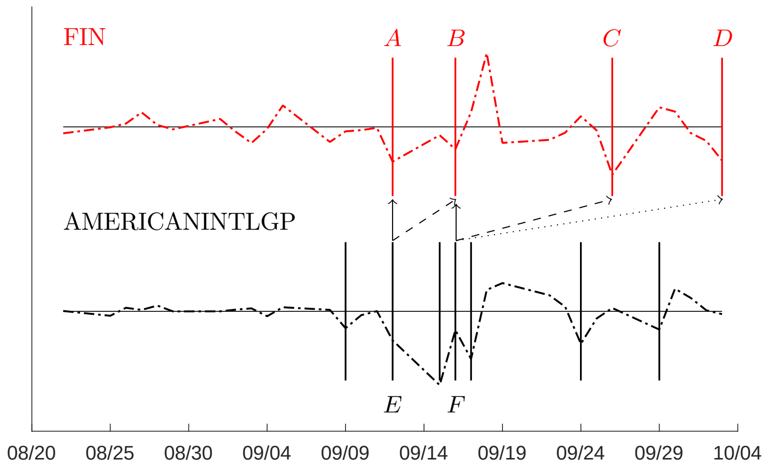


Figure 2: Returns of the FIN index and AIG during the Financial Crisis from August 20, 2008 to October 10, 2008. Vertical lines indicate that the respective return is below the VaR(0.01), i.e. triggering and systemic events.

Figure 2 depicts an exemplary sequence of events: E and A as well as F and B occur simultaneously, thus, with lag $\tau = 0$ (straight arrows). These are the only two events the ΔCoVaR or ψ_0 would capture. But there seems to be no compelling reason, why E and F should not also contribute to subsequently occurring systemic events B, C and D. We capture these effects by ψ_τ. Should a triggering event TE^I, instead, contribute utmost to one special lag $\tilde{\tau}$? To answer this question, consider the definition of systemic risk by the Financial Stability Board (2011, p. 5), who argue that an institution is also considered as systemically important if it triggers broader contagion. Clearly, *broader contagion* may result in several systemic market events caused by one (or more) triggering events. Hence, triggering events may generally contribute to the buildup and broader contagion of systemic risk, thereby affecting all subsequent systemic events. Moreover, different triggering and systemic events might also be caused by a common factor, even if there was no causal relationship between the events. This phenomenon is described by Adrian and Brunnermeier (2014) as being *systemic as part of a herd* and, also, captured by CoSP.

Still, observations for different time-lags may also occur randomly and without any structural relationship or common risk factor of institution and the market. The reference level of CoSP allows to differentiate between these randomly occurring events and unusually often occurring time-lags.

This reference level is given by the VaR-level q^M, since TE^I and SE^M_τ are independent if, and only if,

$$\psi_\tau(q^M, q^I) = q^M. \tag{8}$$

In Appendix A.2 we derive this property and give an example with independent student-distributed returns. Also, in Appendix A we discuss several other properties of CoSP.

The reference level q^M allows to identify time-lags that occur unusually often, i.e. observations that are not caused by independent events. Therefore, we identify an institution as systemically important at lag τ, if $\psi_\tau(q^M, q^I) > q^M$. Furthermore, for very large lags one would intuitively presume that the influence of the institution's return r^I on the market return r^M_τ diminishes, i.e. that TE^I and SE^M_τ become independent.[14] Hence, we conjecture (and find this confirmed in the empirical analysis) that

$$\lim_{\tau\to\infty} \psi_\tau(q^M, q^I) = q^M. \tag{9}$$

In conclusion, we interpret CoSP as a measure for the systemic influence an exceptionally small return of a institution has on the market: If $\psi_\tau(q^M, q^I)$ declines slowly, more systemic events occur with larger lags and, thus, the influence of the institution's triggering events lasts longer. However, not only the speed of decline but also the size of $\psi_\tau(q^M, q^I)$ is important to consider, since larger values for $\psi_\tau(q^M, q^I)$ indicate a larger risk.

3.2 Aggregate Systemic Risk and the CoSP-weighted Time-Lag

Since both the speed of decline and amplitude of CoSP reflect the persistence of an institution's triggering event, we suggest to compute the size of the area between the reference level q^M and CoSP $\psi_\tau(q^M, q^I)$ as a measure for the aggregate systemic risk, i.e.

$$\bar{\psi} = \int_0^\infty \psi_\tau(q^M, q^I) - q^M \, d\tau. \tag{10}$$

[14] This behavior may for example result from arbitrage opportunities being exploited in the long-run.

We call $\bar{\psi}$ the **Aggregate Excess CoSP** and employ this systemic risk measure two assess the total systemic impact of triggering events. In Section B.3 we propose an estimation procedure for $\bar{\psi}$. A second measure is the **CoSP-weighted time-lag**, which is given as

$$\bar{\tau} = \frac{1}{\bar{\psi}} \int_0^\infty \tau \, \left(\psi_\tau(q^M, q^I) - q^M \right) \, d\tau. \tag{11}$$

This measure is an average of all time-lags, which are weighted with their contribution to the Aggregate Excess CoSP.[15] Hence, it is essentially a measure for the (weighted) time-lag between triggering and systemic event. A major advantage of $\bar{\tau}$ is that it is measured in time units (e.g. days). In Section B.4 we propose an estimation procedure for $\bar{\tau}$.

3.3 The Contagion Period

Another quantity of interest is the contagion period τ^* that lies between a triggering event and the first systemic market reaction. We study a discrete approximation of the probability distribution of the contagion period τ^*, which is[16]

$$F_{\tau^*}(x) = \mathbb{P}(\tau^* = x) = \mathbb{P}\left(r_x^M \le VaR^M(q^M) \,\text{and}\, r_0^M, ..., r_{x-1}^M > VaR^M(q^M) \mid r^I \le VaR^I(q^I) \right). \tag{13}$$

To examine the properties of F_{τ^*} one can apply the whole toolbox of statistics. For example, one may study the mean, standard deviation, quantiles, etc. However, the estimation of F_{τ^*} may lead to substantial outliers and, thus, we do not study the mean contagion period. Instead, we focus on the median value $\tau^*_{0.5}$, i.e. the median contagion period between triggering and first systemic event.

[15] Note that $\bar{\tau}$ exhibits analogies to the duration concept of Macaulay (1938).

[16] Note that F_{τ^*} is indeed a probability distribution if almost surely at some point in time a systemic market event occurs, which seems reasonable. In this case,

$$1 = \mathbb{P}\left(\bigcup_{x \ge 0} \left\{ r_x^M \le VaR^M(q^M) \,\text{and}\, r_0^M, ..., r_{x-1}^M > VaR^M(q^M) \mid r^I \le VaR^I(q^I) \right\} \right) = \sum_x F_{\tau^*}(x). \tag{12}$$

3.4 Estimation of CoSP

To estimate $\psi_\tau(q^M, q^I)$ we employ historical simulation (HS).[17] The use of this simplified approach is particularly motivated by the fact that, as to our knowledge, this is the first study about the interdependence of lagged tail-returns. Thus, it seems unreasonable to impose distributional or modeling assumptions.[18] Additionally, it seems intuitive that systemic market events mostly occur in times with large volatility. In other words, the maximum return level that corresponds to systemic market distress, $VaR^M(q^M)$, should not depend on the current volatility level but on the (time-)unconditional volatility. Therefore, we employ the (time-)unconditional Value-at-Risk to estimate CoSP.

The estimation procedure is similar to Acharya et al. (2010) and described in Section B.1 in more detail. The smaller the VaR-levels q^M and q^I are, the less observations there are and, thus, the smaller is the estimation precision. To isolate significantly large values from noise, we compute a lower bound of significance for $\hat{\psi}_\tau(q^M, q^I)$ in Appendix B.2, which is given by

$$k^*_\tau(q^M, q^I) = \frac{1}{n_\tau q^I} \left(F^{-1}_{Bin(n_\tau, q^M q^I)}(1-\alpha) + 1 \right), \tag{14}$$

for a significance level $\alpha \in (0, 1)$. Due to the estimation error of $\hat{\psi}_\tau(q)$, we suggest to fit $\hat{\psi}_\tau(q)$ for $\tau = 1, 2, ...$ to the following function:

$$H(\tau) = q^M + e^{-a\tau^2 + b\tau + c}, \quad \text{with } a > 0,\ b, c \in \mathbb{R}. \tag{15}$$

In other words, we assume that ψ_τ declines exponentially and find this confirmed in the empirical analysis. Note that we do not include co-movements at the time-lag $\tau = 0$ in the fitting procedure, since these do not reflect persistence and, particularly, do not steadily continue ψ_τ, as the results in Section 5 suggest. The choice of H has several advantages: Since H is always larger than q^M but converges to q^M, we only capture systemically important lags, i.e. fit the part of $\hat{\psi}_\tau(q^M, q^I)$

[17]There exist several studies discussing and improving the statistical properties of HS and other estimation approaches for time series of returns, for example Danielsson and Zhou (2015), Hendricks (1996), Hull and White (1998), Kuester et al. (2006) or Pritsker (2001). However, these focus on quantile or moment estimation and do not consider a time-lag and, thus, cannot be applied for the estimation of CoSP.

[18]Note, that by employing HS we do not need to assume that the full bivariate return distribution is stationary. In contrast, it is sufficient to assume that solely the dependence between lagged tail returns is stationary over time.

that lies above q^M. However, if $\hat{\psi}_\tau(q^M, q^I) < q^M$ for many lags τ, we have $H \approx q^M$, indicating that the systemic risk is zero.

We employ the fitted CoSP, $H(\tau)$, instead of $\hat{\psi}_\tau$ to assess the significance of systemic importance. This is particularly motivated by the reason that the significance bound k^*_τ is only an asymptotic bound (see Appendix B.2). Therefore, an institution is classified as significantly systemically important if $H(\tau) \geq k^*_\tau$ for at least one time-lag $\tau > 0$. As before, we do not include co-movements at $\tau = 0$, since these do not necessarily reflect a causal relationship between triggering and systemic events. In contrast, for non-zero time-lags triggering events cannot be caused by systemic events.

4 Data and Descriptive Statistics

Our data sample includes all historical daily returns of 917 publicly traded financial institutions that are classified as banks (i.e. depository institutions), brokers (i.e. security and commodity brokers), or insurers in Datastream.[19] Moreover, we examine the returns of 36 non-financial companies that are selected according to market capitalization.[20] All returns are daily for the period from November 21, 1995 to November 20, 2015. To avoid endogeneity, we compute our own market indices that exclude the currently considered institution. The computational procedure is described in Appendix D.1. In Appendix D.2 in Figure 12 (a) we show the resulting indices for banks (BAN), brokers (BRO), insurers (INS) and the whole financial market (FIN) if no institution is excluded from the index. Furthermore, we consider three continent-specific indices for non-financial companies, namely indices for the Americas (AMC), Asia (ASIA), and Europe (EU), which are shown in Appendix D.2 in Figure 12 (b).

For the firm-specific analysis we exclude institutions with less than 1750 observations. Then, 725 institutions remain in the sample.[21] In Appendix D.2 we report the descriptive statistics for the returns of all institutions included in the data sample. To this end, we estimate the mean and standard deviation of returns for all single institutions and report the distribution of these estimates

[19] The names of the 10 largest institutions in each subsector included in the sample are reported in Table 3.
[20] The non-financial companies' names are reported in Table 2.
[21] After excluding institutions with too few observations 442 banks, 106 broker, 141 insurer and 36 non-financial companies remain in the sample. Still, we compute the sector-specific indices with all available institutions.

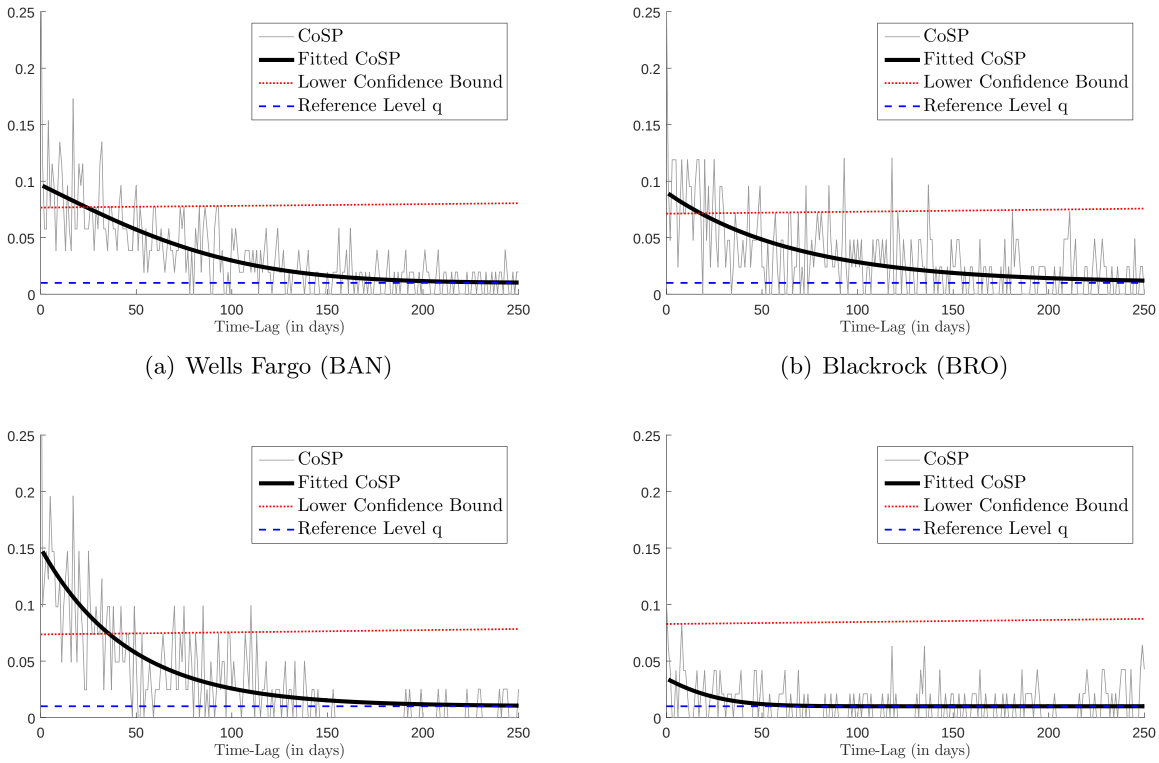
among the subsectors banks (BAN), brokers (BRO), insurers (INS), and non-financial companies (NoFIN) in Figure 13. Moreover, Table 4 shows the mean, standard deviation and quantiles for different indices. Hereby, all financial indices are computed as described in Appendix D.1 without excluding any institution.

The mean institution returns are approximately zero for all institutions, whereas 95% of the institutions exhibit a standard deviation of returns between 1% and 4%. Finally, in Figure 13(c) we show the distribution of empirical correlation coefficients between single institutions and the financial market index. Interestingly, the correlation between the returns of non-financial companies and the financial market is substantially larger than the correlation between the returns of banks and the financial market.

5 Empirical Findings

We apply the methodology as described in Section 3 and examine the aggregated systemic risk and time-lag of systemic risk for the data sample described in Section 4. As described in Section 3.1 for the systemic risk and institution distress probability we set $q^M = q^I = q$ and use $q = 1\%$ as reference level, which seems reasonable to capture exceptionally small returns. To compute the lower bound of significance we use the significance level $\alpha = 1\%$.

In Figure 3 we show the CoSP with respect to the financial index for several institutions that exhibit a typical pattern. More examples are shown in Appendix E and can be provided by the authors on request. As described in Section 3.1, the fitted CoSP is declining and converges to the reference level q for $\tau \to \infty$. Furthermore, for Wells Fargo, Blackrock, and Metlife the CoSP is significantly larger than the reference level for several lags $\tau > 0$. Thus, there is statistically significant influence between triggering events and systemic events with a time-lag. However, there are substantial differences between the institutions: For the exemplary non-financials the CoSP declines relatively fast and is rather small, whereas the CoSP for financial institutions seems to be larger and of slowly declining shape.



(a) Wells Fargo (BAN)

(b) Blackrock (BRO)

(c) Metlife (INS)

(d) Amazon.com (NoFIN)

Figure 3: CoSP triggered by exemplary institutions w.r.t. the FIN index.

We begin our analysis by examining the dependence between the measures of co-movement, in particular the CoSP without time-lag, i.e. ψ_0, MES, and $\Delta\text{CoVaR}^{\leq}$.[22] Across all observations with respect to the financial index the correlation between ψ_0 and MES is -81.92%, and between ψ_0 and $\Delta\text{CoVaR}^{\leq}$ it is -79.17%.[23] The levels of correlation are very similar with respect to other indices. Thus, all three measures ψ_0, MES and $\Delta\text{CoVaR}^{\leq}$ result in very similar levels of systemic risk.

However, the correlation is substantially smaller if considering the Aggregated Excess CoSP $\bar{\psi}$ instead of ψ_0. For the aggregate systemic risk with respect to the financial index the correlation with MES is -48.92% and the correlation with $\Delta\text{CoVaR}^{\leq}$ is -57.21%. For other indices the values are similar. We conclude that the Aggregate Excess CoSP is able to capture a dimension of systemic risk that traditional systemic risk measures of co-movement like MES or $\Delta\text{CoVaR}^{\leq}$ are

[22] For all measures we set the VaR-levels equal to 1%. The results can be found in Table 5 in Appendix E.

[23] Note that, by definition, the larger MES and $\Delta\text{CoVaR}^{\leq}$ are the smaller is the anticipated systemic risk, whereas for ψ_0 a large value indicates a large systemic risk. Therefore, a negative correlation of the measures' values indicates a positive relationship with respect to the level of systemic risk.

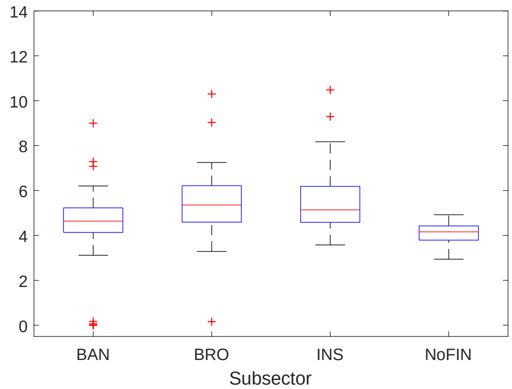
not able to fully reflect.

Secondly, we exclude the number of institutions that do not exhibit significant lagged systemic risk. As described in Section 3.4, an institution is classified as significantly systemically important (s.s.i.) if $H(\tau) \geq k_\tau^*$ for at least one lag $\tau \in \{1, 2, 3, ...\}$. According to this criterion, roughly 15% of the financial institutions in the sample are significantly systemically important for the financial sector. Interestingly, more financial institutions are s.s.i. for the American non-financial sector (27%) and the European non-financial sector (18%), while less of the financial institutions are s.s.i. for the Asian non-financial sector (13%). Moreover, a substantially larger fraction of insurance institutions is classified as significantly systemically important for the financial sector (34%) than of brokers (21%) and banks (15%). The fraction of significantly systemically important institutions for each (sub-)sector with respect to the different indices is shown in Appendix E in Figure 17.

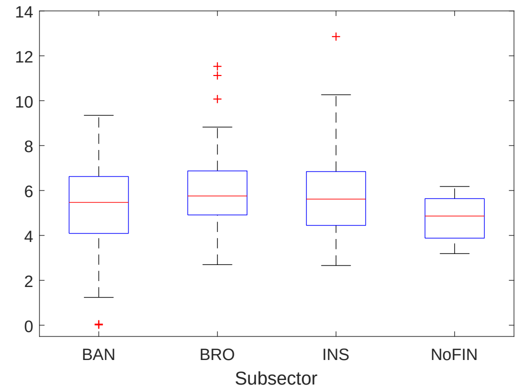
Thirdly, we focus on the measures for the aggregate systemic risk, the contagion period, and systemic time-lags. In Figure 4 we show the distribution of the Aggregate Excess CoSP for all significantly systemically important institutions with respect to the indices for banks (BAN), brokers (BRO), insurers (INS), and the American non-financial index. No subsector exhibits an Aggregate Excess CoSP that is significantly different from the others. However, the Aggregate Excess CoSP triggered by non-financial companies is slightly smaller with respect to all indices. Moreover, the Aggregate Excess CoSP triggered by systemically important brokers is slightly larger, while the Aggregate Excess CoSP triggered by insurance institutions is similar to that of banks.

This finding corresponds to the results of Billio et al. (2010) and Adrian and Brunnermeier (2014). However, it is in contrast with the results of Cummins and Weiss (2014), who argue that systemic risk within the insurance industry is considerably smaller than between insurance and banking industry. Still, similar to Chen et al. (2013) we find that the Aggregate Excess CoSP triggered by systemically important banks with respect to the insurance market is slightly larger than vice versa.[24] Interestingly, the ranking of subsectors is substantially different if studying the

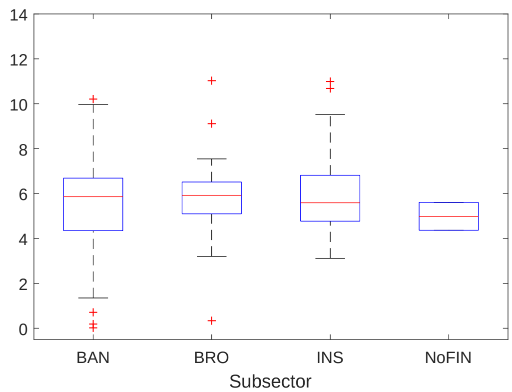
[24]The median Aggregate Excess CoSP triggered by systemically important banks w.r.t. the insurance market is 5.86, whereas the median Aggregate Excess CoSP triggered by systemically important insurers w.r.t. the banking market is 5.14.



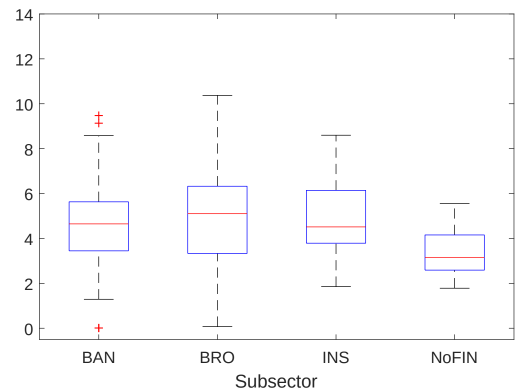
(a) Aggregate Excess CoSP w.r.t. the BAN index.



(b) Aggregate Excess CoSP w.r.t. the BRO index.



(c) Aggregate Excess CoSP w.r.t. the INS index.

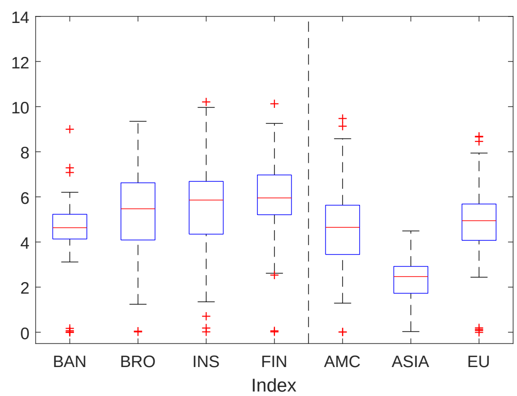


(d) Aggregate Excess CoSP w.r.t. the American NoFIN index.

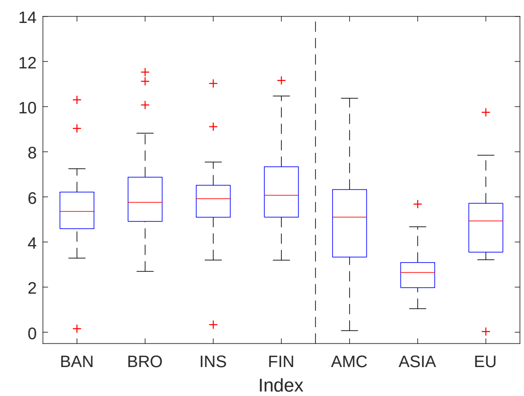
Figure 4: Aggregate Excess CoSP w.r.t. the BAN, BRO, INS and American NoFIN indices triggered by significantly systemically important institutions of the subsectors BAN, BRO, INS and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, q_1 and q_3, and the maximum whiskers' length is $1.5(q_3 - q_1)$.

$\Delta\text{CoVaR}^{\leq}$ instead of the Aggregate Excess CoSP, as shown in Figure 18 in Appendix E. In this case, non-financial companies are found to trigger the largest systemic risk, particularly on the financial market. In contrast, banks are found to trigger the smallest risk w.r.t. the American non-financial market, while insurer trigger the smallest risk w.r.t. the financial market.

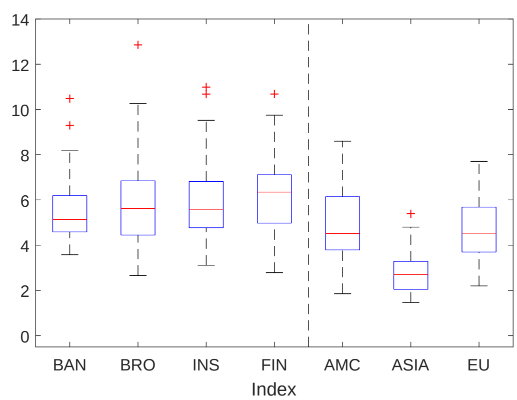
To study a market's exposure towards systemic risk (i.e. its susceptibility), Figure 5 depicts the distribution of the Aggregate Excess CoSP among systemically important institutions by comparing the systemic risk triggered by one subsector with respect to different indices. In general, we find that the systemic risk with respect to non-financial markets is smaller than for the financial sector. Clearly, the Aggregate Excess CoSP with respect to the Asian non-financial market is the smallest for all subsectors, whereas the Aggregate Excess CoSP with respect to the American and European non-financial market is approximately equal if triggered by financial institutions. However, the Aggregate Excess CoSP triggered by non-financials is slightly larger with respect to the European non-financial market than with respect to the American non-financial market. Regarding the financial sector, the Aggregate Excess CoSP with respect to the banking sector is the smallest among the financial subsectors while it is particularly large with respect to the insurance sector.



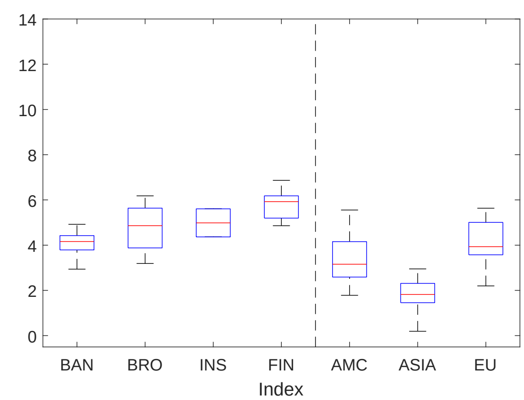
(a) Aggregate Excess CoSP triggered by banks.



(b) Aggregate Excess CoSP triggered by brokers.



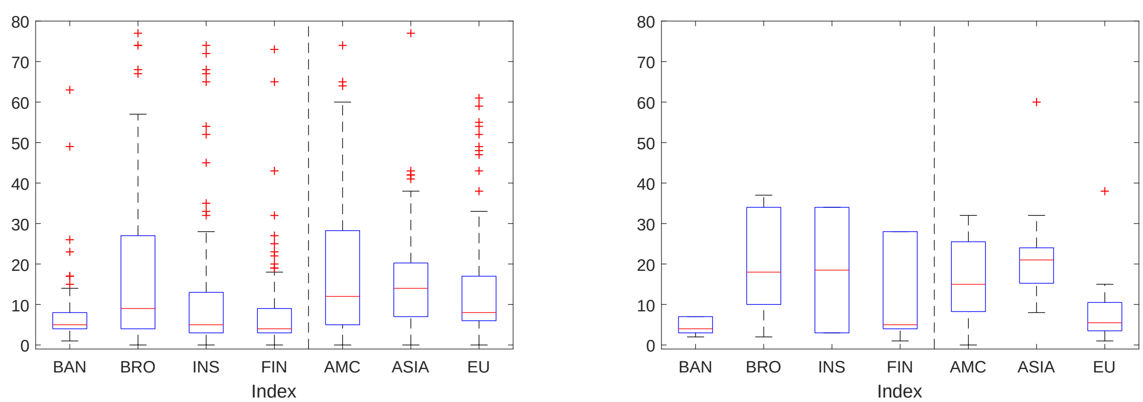
(c) Aggregate Excess CoSP triggered by insurance institutions.



(d) Aggregate Excess CoSP triggered by non-financial companies.

Figure 5: Aggregate Excess CoSP w.r.t. different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by significantly systemically important banks, brokers, insurers, and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, q_1 and q_3, and the maximum whiskers' length is $1.5(q_3 - q_1)$.

Next, we examine the contagion period and CoSP-weighted time-lag. To this end, we start with studying the median contagion period between all stress (triggering) events of systemically important institutions and the first (subsequently or simultaneously) occurring systemic market event. Figure 6 clearly shows that the median contagion period is different from zero for more than 75% of all systemically important institutions with respect to all indices. In general, the banking, insurance, and overall financial markets show systemic distress after a median time of approximately 5 days, subsequently to a triggering event of systemically important financial institutions. In contrast, the median contagion period for these institutions is approximately 10 days for systemic events on the brokerage, American, or European non-financial market, and 15 days for the Asian non-financial market. For triggering events of systemically important non-financial companies the median contagion period is larger, particularly for systemic events on the brokerage, insurance, American, or Asian non-financial market.

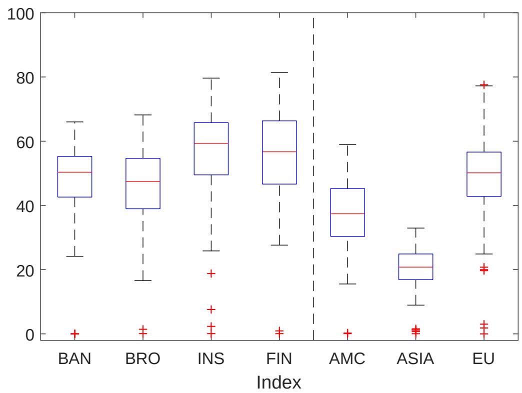


(a) Median contagion period triggered by financials. (b) Median contagion period triggered by non-financials.

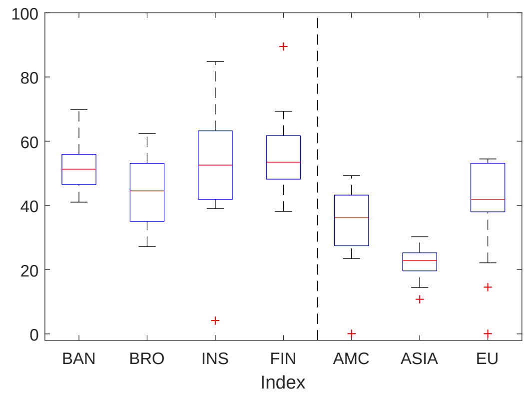
Figure 6: Median contagion period w.r.t. different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU triggered by significantly systemically important financials and non-financials. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, q_1 and q_3, and the maximum whiskers' length is $1.5(q_3 - q_1)$.

The median contagion period measures the time-lag between stress events of an institution and a market, but the respective stress events might also occur randomly. Therefore, we also study the CoSP-weighted time-lag, i.e. the weighted average time-lag between triggering and systemic events, whereby the weighting factors are the contribution of the single time-lags to the Aggregated Excess CoSP. In Figure 7 we show the distributions of the CoSP-weighted time-lag for systemically important institutions. In contrast to the median contagion period, the CoSP-weighted time-lag is smaller with respect to non-financial markets than for financial markets. Thus, it takes longer to observe a systemic event on non-financial markets after a triggering event, whereas the systemic risk associated with these large time-lags is rather small. Regardless of the triggering institution, the CoSP-weighted time-lag is particularly large for systemic risk with respect to the insurance market.

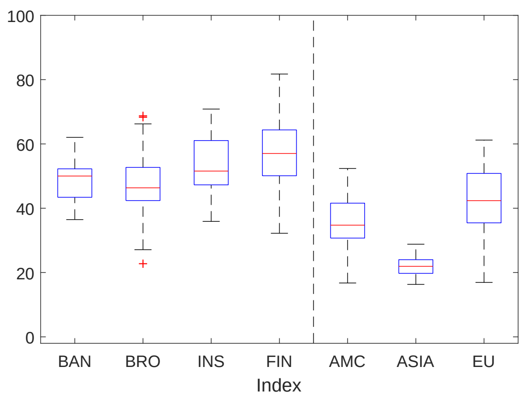
Most interestingly, the CoSP-weighted time-lag indicates a systemic risk cascade: If systemic risk is triggered by financial institutions, firstly brokers are systemically affected, then banks, and, lastly, insurance companies. In contrast, if systemic risk is triggered by non-financial companies, firstly banks are systemically affected, then brokers, and, lastly, insurance companies. A rationale for this cascade may be the investment behavior of the three types of financial institutions: Since brokers tend to invest on a shorter time horizon than insurance companies, the distress of other institutions might also affected brokers earlier than insurers. Banks are particularly exposed to the distress of non-financial companies due to loans to non-financial companies, which might explain why the CoSP-weighted time-lag is the smallest for banks if triggered by non-financial companies.



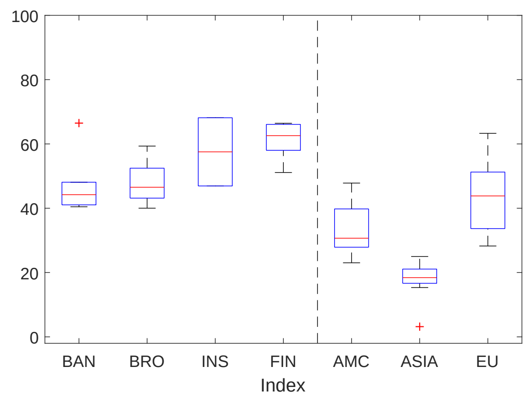
(a) CoSP-weighted time-lag triggered by banks.



(b) CoSP-weighted time-lag triggered by brokers.



(c) CoSP-weighted time-lag triggered by insurers.



(d) CoSP-weighted time-lag triggered by non-financials.

Figure 7: CoSP-weighted time-lag w.r.t. different financial indices BAN, BRO, INS, FIN and non-financial indices AMC, ASIA, EU, triggered by significantly systemically important banks, brokers, insurers, and non-financial companies. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, q_1 and q_3, and the maximum whiskers' length is $1.5(q_3 - q_1)$.

6 Conclusion

Since cash-flows and information may take time to spread within and across (financial) markets, systemic risk is not only the risk of simultaneously occurring extreme events. It is also the risk of systemic market (participants') reactions that occur with a time-lag to the triggering event. To our knowledge, this is the first article that explicitly studies the properties of this systemic contagion period, i.e. the timing dimension of systemic risk. We review the most common systemic risk measure, ΔCoVaR, and propose a new systemic risk measure that exhibits a smaller estimation error and a larger reliability than ΔCoVaR. This new systemic risk measure is the Conditional Shortfall Probability (CoSP), $\psi_\tau(q^M, q^I)$, which is the likelihood that a systemic market event occurs τ days after a triggering event of the institution. If CoSP equals its reference level q^M, triggering and systemic events are independent. In typical cases, $\psi_\tau(q^M, q^I)$ is exponentially declining and converges to the reference level. The significance bound of CoSP allows to identify institutions that are significantly systemically important.

Motivated by the properties of CoSP, we define two aggregate risk measures: The Aggregate Excess CoSP, $\bar{\psi}$, and the CoSP-weighted time-lag, $\bar{\tau}$. Aggregate Excess CoSP is the area between $\psi_\tau(q^M, q^I)$ and the reference level q^M. Thus, it reflects the overall systemic influence of an institution on the market, i.e. the aggregate systemic risk triggered by this institution. The CoSP-weighted time-lag is the weighted average of all time-lags. The weighting factors are the relative contributions of the time-lags to the Aggregate Excess CoSP. Both measures are constructed from CoSP but represent different dimensions of systemic risk. Together, the measures capture the overall persistence of systemic risk triggered by an institution.

In the empirical analysis we study all significantly systemically important institutions in the global subsectors banks, brokers, insurance, and non-financial companies. Hence, we are among the first to differentiate between spillover effects between and within the financial sector and the real industry. In particular, we find that 27% of the financial institutions in our sample are significantly systemically important for the American non-financial sector, whereas 15% of the financial institutions in our sample are significantly systemically important for the (global) financial sector.

However, the results show that the Aggregate Excess CoSP triggered by systemically important institutions is larger for financial markets than for non-financial markets. In particular, the Aggregate Excess CoSP is very small with respect to the Asian non-financial market and particularly large with respect to the insurance market. Brokers tend to trigger a slightly larger Aggregate Excess CoSP than banks or insurers. Interestingly, the Aggregate Excess CoSP of insurance institutions is comparable to that of banks.[25] Still, we find that banks trigger a larger systemic risk on the insurance market than vice versa.

By studying the systemic contagion period we find that approximately 10 days after a stress event of systemically important financial institutions we can observe a systemic event on the brokerage or non-financial markets. This time-lag is only approximately 5 days for the banking, insurance, and overall financial market. In contrast, the systemic risk associated with large time-lags with respect to the non-financial market is rather small, as the CoSP-weighted time-lag shows. Moreover, the CoSP-weighted time-lag reveals a systemic cascade effect for the financial sectors: Systemic risk triggered by financial institutions tends to influence brokers firstly, secondly banks, and, thirdly, insurance institutions. The different investment behavior of the different types of institutions may serve as a reason for this observation.

In conclusion, we find significant evidence that triggering events systemically affect the market with different time-lags. Further research on this topic can take various forms. For example, a more detailed analysis of the relationship between measures for lagged systemic risk and measures for instantaneous systemic risk (e.g. ψ_0, MES or ΔCoVaR) may shed light on the relationship between systemic contagion and (the tail of) co-movements of returns. Also, further differentiating between the types of institutions and geographic differences may make differences in their systemic importance more clear. Moreover, the drivers for systemic risk and contagion periods may be fundamentals like leverage ratios, asset duration, firm size, or financial interconnectedness structures that need to be identified in order to understand the underlying rationale of the timing dimension of systemic risk.

[25] This result is similar to Adrian and Brunnermeier (2014) and Billio et al. (2010), but is contrary to Berdin and Sottocornola (2015) and Cummins and Weiss (2014).

Appendix

A Properties of CoSP

The conditional shortfall probability (CoSP) is given as

$$\psi_\tau(q^M, q^I) = \mathbb{P}\left(r_\tau^M \leq VaR^M(q^M) \mid r^I \leq VaR^I(q^I)\right). \tag{16}$$

Thus, $\psi_\tau(q) = \psi_\tau(q, q)$ is very similar to the coefficient of lower tail dependence. In particular, the latter is the limit of $\psi_\tau(q)$ as q approaches 0, i.e.

$$\lambda_\tau = \lim_{q \to 0^+} \psi_\tau(q), \tag{17}$$

where λ_τ is the coefficient of lower tail dependence between r^I and r_τ^M (see McNeil et al. (2015, p.247)).

A.1 Symmetry

In general, we have

$$\psi_\tau(q^M, q^I) = \mathbb{P}\left(SE_\tau^M \mid TE^I\right) = \mathbb{P}\left(r_\tau^M \leq VaR^M(q^M) \mid r^I \leq VaR^I(q^I)\right) \tag{18}$$

$$= \frac{\mathbb{P}\left(SE_\tau^M,\, TE^I\right)}{q^I} = \frac{q^M}{q^I}\mathbb{P}\left(TE^I \mid SE_\tau^M\right). \tag{19}$$

Thus, $\psi_\tau(q^M, q^I)$ and $\mathbb{P}\left(TE^I \mid SE_\tau^M\right)$ are proportional, whereas $\psi_\tau(q, q)$ is equal to $\mathbb{P}\left(TE^I \mid SE_\tau^M\right)$. If $\tau > 0$, the latter probability, $\mathbb{P}\left(TE^I \mid SE_\tau^M\right)$, cannot be interpreted in a causal sense, i.e. SE_τ^M can not have caused TE^I since it happened later in time. Still, $\mathbb{P}\left(TE^I \mid SE_\tau^M\right)$ is the likelihood that the institution exhibits an extraordinarily small return τ days before a systemic market event SE_τ^M. From this perspective, $\psi_\tau(q)$ may also be interpreted as the likelihood of a triggering event of a specific institution given a systemic market event.

In contrast, the symmetry of $\psi_0(q)$ is very reasonable, since it is the result of co-movements between r^M and r^I. In other words, one can in general not identify a causal relationship between the events. This co-movement is also reflected in other systemic risk measures like MES or ΔCoVaR$^=$ in the sense that these are proportional to the institution's firm-specific risk if the dependence between financial asset returns is linear.[26]

A.2 Independence

By definition, the triggering event $TE^I = \left\{r^I \leq VaR^I(q^I)\right\}$ and systemic event $SE_\tau^M = \left\{r_\tau^M \leq VaR^M(q^M)\right\}$ are stochastically independent if, and only if,

$$\mathbb{P}\left(SE_\tau^M,\, TE^I\right) = \mathbb{P}\left(SE_\tau^M\right)\, \mathbb{P}\left(TE^I\right), \tag{20}$$

which is equivalent to

$$\psi_\tau(q^M, q^I) = \frac{\mathbb{P}\left(SE_\tau^M,\, TE^I\right)}{\mathbb{P}\left(TE^I\right)} = \mathbb{P}\left(SE_\tau^M\right) = q^M. \tag{21}$$

Hence, we have $\psi_\tau(q^M, q^I) = q^M$ if, and only if, triggering event TE^I and systemic event SE_τ^M are stochastically independent.

In Figure 8 we show the estimate for CoSP with $q^M = q^I = q = 1\%$ for 5219 independent observations for r^M and r^I that were drawn from a student-t(5) distribution.[27] Clearly, there exist numerous observations of events SE_τ^M and TE^I for all lags τ. However, $\hat{\psi}_\tau$ fluctuates around the reference level q and is almost always below the confidence bound. In this case one would reason that r^I is not systemically important for r^M for any lag. This conclusion is also supported by the fitted CoSP, which stays constant at the reference level $q = 1\%$.

[26] This is a main finding of Benoit et al. (2013).

[27] 5219 is the maximum number of available observations in our data sample.

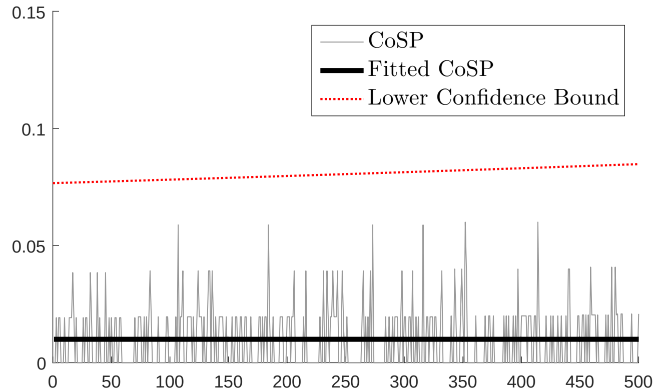


Figure 8: Estimated CoSP, $\hat{\psi}_\tau(0.01)$, for independent student-t(5) distributed returns.

A.3 Systemic Risk Rankings of CoSP and ΔCoVaR

In the following we examine the relationship between two different returns r^{I_1} and r^{I_2} and a market return r^M. For simplicity, we focus on the case with $q^M = q^I = q$. Under the assumption that $r_\tau^M \mid r^{I_1} \le VaR^{I_1}(q)$ first-order stochastically dominates $r_\tau^M \mid r^{I_2} \le VaR^{I_2}(q)$, i.e. for all $x \in \mathbb{R}$

$$\mathbb{P}\left(r_\tau^M \le x \mid r^{I_1} \le VaR^{I_1}(q)\right) \;\le\; \mathbb{P}\left(r_\tau^M \le x \mid r^{I_2} \le VaR^{I_2}(q)\right), \tag{22}$$

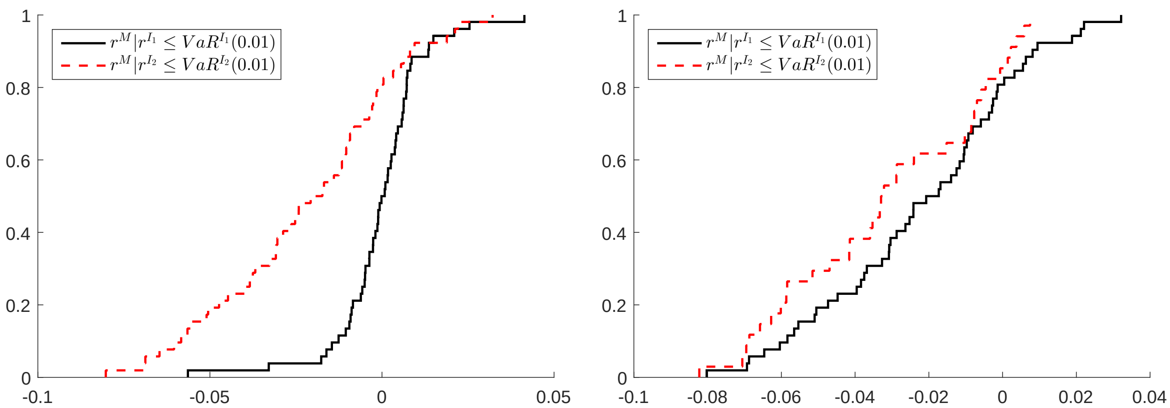
we have $\psi_\tau^{I_1}(q) \le \psi_\tau^{I_2}(q)$. Moreover, for CoVaR^τ we have

$$\mathrm{CoVaR}^\tau_{r^{I_1} \le VaR^{I_1}(q)}(q) \;\ge\; \mathrm{CoVaR}^\tau_{r^{I_2} \le VaR^{I_2}(q)}(q). \tag{23}$$

Hence, with respect to both risk measures I_2 is more systemically important than I_1. Also, if the market risk conditional on the benchmark events is approximately equal, i.e. $\mathrm{CoVaR}^\tau_{BM^{I_1}}(q) \approx \mathrm{CoVaR}^\tau_{BM^{I_2}}(q)$, for $\Delta\mathrm{CoVaR}^{\le}_\tau$ we have

$$\Delta\mathrm{CoVaR}^{\le, I_1}_\tau(q) \;\ge\; \Delta\mathrm{CoVaR}^{\le, I_2}_\tau(q). \tag{24}$$

The condition that $r_\tau^M \mid r^{I_1} \le VaR^{I_1}(q)$ first-order stochastically dominates $r_\tau^M \mid r^{I_2} \le VaR^{I_2}(q)$ can often be observed for financial return series. In Figure 9 we show two exemplary empirical cumulative density functions (ecdf) for lag $\tau = 0$ for the unconditional and conditional returns of the financial index. Particularly in the lower tail $r^M | r^{I_1} \le VaR^{I_1}(0.01)$ stochastically dominates $r^M | r^{I_2} \leq VaR^{I_2}(0.01)$.[28]



(a) I_1 is Bank of Greece and I_2 is AIG

(b) I_1 is AIG and I_2 is CME Group

Figure 9: Empirical CDF of returns from the financial index conditional on institutions' financial distress.

B Estimation Procedure

B.1 Estimation of CoSP

The Maximum-Likelihood estimator for ψ_τ is given as

$$\hat{\psi}_\tau^{(ml)} = \frac{1}{\sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{r_t^I \leq \widehat{VaR}^I(q^I)\right\}}} \sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{r_{t+\tau}^M \leq \widehat{VaR}^M(q^M),\, r_t^I \leq \widehat{VaR}^I(q^I)\right\}}, \tag{25}$$

where the Value-at-Risk estimate is the $[nq^x]$-th smallest observation for return r^x, $\widehat{VaR}^x(q^x) = r^x_{([q^x n])}$. If the triggering events $\{r_t^I \leq \widehat{VaR}^I(q^I)\}$ are independently distributed with the same probability of occurrence, it follows from the strong law of large numbers that almost surely

$$\sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{r_t^I \leq \widehat{VaR}^I(q^I)\right\}} \underset{n\to\infty}{\to} q^I(n-\tau). \tag{26}$$

However, the actual number of observed triggering events for $t \in \{1, n-\tau\}$ may be very small or large in comparison to $q^I(n-\tau)$, which may lead to large estimation errors. Therefore, we smooth

[28]Note that stochastic dominance in the lower tail is sufficient to obtain the same order of the institutions.

the estimator for CoSP with the asymptotic number of observed triggering events, i.e.

$$\hat{\psi}_\tau = \frac{1}{\frac{1}{2}\left(\sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{r_t^I \leq \widehat{VaR}^I(q^I)\right\}} + q^I(n-\tau)\right)} \sum_{t=1}^{n-\tau} \mathbb{1}_{\left\{r_{t+\tau}^M \leq \widehat{VaR}^M(q^M),\, r_t^I \leq \widehat{VaR}^I(q^I)\right\}}. \tag{27}$$

Similar to $\hat{\psi}^{(ml)}$, this estimator is also asymptotically unbiased.

B.2 Lower Bound of Significance for $\hat{\psi}_\tau$

Denote by $n_\tau = n - \tau$ the number of available observations for lag τ. Since almost surely we have

$$\sum_{t=1}^{n_\tau} \mathbb{1}_{\left\{r_t^I \leq \widehat{VaR}^I(q^I)\right\}} \underset{n\to\infty}{\to} q^I n_\tau, \tag{28}$$

we use the following approximation for $\hat{\psi}_\tau$ in this section:

$$\hat{\psi}_\tau \approx \frac{\sum_{t=1}^{n_\tau} \mathbb{1}_{\left\{r_{t+\tau}^M \leq \widehat{VaR}^M(q^M),\, r_t^I \leq \widehat{VaR}^I(q^I)\right\}}}{n_\tau q^I}. \tag{29}$$

Thus, the lower bound of significance will be an asymptotic bound. Under the assumption that $\mathbb{1}_{\left\{r_{t+\tau}^M \leq VaR^M(q^M),\, r_t^I \leq VaR^I(q^I)\right\}}$ are iid for $t = 1, .., n_\tau$, it follows

$$\sum_{t=1}^{n_\tau} \mathbb{1}_{\left\{r_{t+\tau}^M \leq \widehat{VaR}^M(q^M),\, r_t^I \leq \widehat{VaR}^I(q^I)\right\}} \sim Bin(n_\tau, \psi_\tau q^I), \tag{30}$$

where $Bin(n,p)$ is the Binomial distribution. Hence, under the null hypothesis $H_0 : \psi_\tau = q^M$, i.e. that systemic event SE_τ^M and triggering event TE^I are independent, we have

$$n_\tau q^I \hat{\psi}_\tau \sim Bin(n_\tau, q^M q^I). \tag{31}$$

The null hypothesis if $\hat{\psi}_\tau \geq k_\tau^*$ is rejected with a significance level of $\alpha \in (0,1)$. Thus, an asymptotic lower bound for the rejection area, k_τ^*, can be computed as follows:

$$\alpha = \mathbb{P}_{H_0}\left(\hat{\psi}_\tau \geq k_\tau^*\right) = \mathbb{P}_{H_0}\left(\sum_{t=1}^{n_\tau} \mathbb{1}_{\left\{r_{t+\tau}^M \leq \widehat{VaR}^M(q^M),\, r_t^I \leq \widehat{VaR}^I(q^I)\right\}} \geq n_\tau q^I k_\tau^*\right) \tag{32}$$

$$= 1 - F_{Bin(n_\tau, q^M q^I)}(n_\tau q^I k_\tau^* - 1) \tag{33}$$

$$\Leftrightarrow \quad 1 - \alpha = F_{Bin(n_\tau, q^M q^I)}(n_\tau q^I k_\tau^* - 1) \tag{34}$$

$$\Leftrightarrow \quad n_\tau q^I k_\tau^* - 1 = F^{-1}_{Bin(n_\tau, q^M q^I)}(1-\alpha) \tag{35}$$

$$\Leftrightarrow \quad k_\tau^* = \frac{1}{n_\tau q^I}\left(F^{-1}_{Bin(n_\tau, q^M q^I)}(1-\alpha) + 1\right), \tag{36}$$

where $F^{-1}_{Bin(n_\tau, q^M q^I)}$ is the (lower) inverse cumulative distribution of the Binomial distribution.

B.3 Estimation of the Aggregate Excess CoSP

To account for estimation errors, we employ the fitted CoSP H (as described in Section 3.4) for lags $\tau \geq 1$ to estimate the Aggregate Excess CoSP. For the CoSP at lag $\tau = 0$ we include $\hat{\psi}_0$. Then, the estimator for the Aggregate Excess CoSP is given as

$$\bar{\psi} = \hat{\psi}_0(q^M, q^I) - q^M + \int_1^\infty H(\tau) - q^M \, d\tau. \tag{37}$$

Firstly, note that

$$\int H(\tau) - q^M \, d\tau = \int e^{-a\tau^2 + b\tau + c} \, d\tau = e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \operatorname{erf}\left(\frac{2a\tau - b}{2\sqrt{a}}\right). \tag{38}$$

Therefore, if $a > 0$,

$$\int_1^T e^{-a\tau^2 + b\tau + c} \, d\tau = e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \left(\operatorname{erf}\left(\frac{2aT - b}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{2a - b}{2\sqrt{a}}\right)\right) \tag{39}$$

and

$$\bar{\psi} = \left(\hat{\psi}_0(q^M, q^I) - q^M\right) + e^{c + \frac{b^2}{4a}} \sqrt{\frac{\pi}{4a}} \left(1 - \operatorname{erf}\left(\frac{2a - b}{2\sqrt{a}}\right)\right), \tag{40}$$

since $\lim_{x\to\infty} erf(x) = 1$. However, if $a = 0$, we have

$$\int H(\tau) - q^M \, d\tau = \int e^{b\tau+c} \, d\tau = \frac{1}{b} e^{b\tau+c}, \tag{41}$$

thus, if $b < 0$,

$$\int_1^T e^{b\tau+c} \, d\tau = \frac{1}{b} \left(e^{bT+c} - e^{b+c} \right) \tag{42}$$

and

$$\overline{\psi} = \left(\hat{\psi}_0(q^M, q^I) - q^M \right) - \frac{e^{b+c}}{b}. \tag{43}$$

B.4 Estimation of the CoSP-weighted time-lag

To account for estimation errors, we employ the fitted CoSP H (as described in Section 3.4) for lags $\tau \geq 1$ to estimate the CoSP-weighted time-lag. Then, the estimator for the CoSP-weighted time-lag is given as

$$\bar{\tau} = \frac{1}{\bar{\psi}} \int_1^\infty \tau \left(H(\tau) - q^M \right) d\tau. \tag{44}$$

Firstly, note that

$$\int \tau (H(\tau) - q^M) \, d\tau = \int \tau e^{-a\tau^2 + b\tau + c} \, d\tau = \frac{1}{4a^{3/2}} \left(\sqrt{\pi} b e^{\frac{b^2}{4a}+c} \operatorname{erf}\left(\frac{2a\tau - b}{2\sqrt{a}} \right) - 2\sqrt{a} e^{\tau(b-a\tau)+c} \right). \tag{45}$$

Therefore,

$$\overline{\tau} = \frac{1}{\bar{\psi}} \lim_{L\to\infty} \frac{1}{4a^{3/2}} \left(\sqrt{\pi} b e^{\frac{b^2}{4a}+c} \operatorname{erf}\left(\frac{2aL - b}{2\sqrt{a}} \right) - 2\sqrt{a} e^{L(b-aL)+c} \right) \tag{46}$$

$$- \frac{1}{\bar{\psi} 4a^{3/2}} \left(\sqrt{\pi} b e^{\frac{b^2}{4a}+c} \operatorname{erf}\left(\frac{2a - b}{2\sqrt{a}} \right) - 2\sqrt{a} e^{b-a+c} \right) \tag{47}$$

$$= \frac{1}{4a^{3/2}\bar{\psi}} \left(\sqrt{\pi} b e^{\frac{b^2}{4a}+c} \left(1 - \operatorname{erf}\left(\frac{2a - b}{2\sqrt{a}} \right) \right) + 2\sqrt{a} e^{b-a+c} \right). \tag{48}$$

However, if $a = 0$, we have

$$\int \tau(H(\tau) - q^M)\, d\tau = \int \tau e^{b\tau + c}\, d\tau = \left(\frac{\tau}{b} - \frac{1}{b^2}\right) e^{b\tau+c}, \tag{49}$$

thus, if $b < 0$,

$$\overline{\tau} = \frac{1}{\bar{\psi}} \int_1^\infty \tau e^{b\tau+c} d\tau \tag{50}$$

$$= \frac{1}{\bar{\psi}} \left[\lim_{L\to\infty} \left(\frac{L}{b} - \frac{1}{b^2}\right) e^{bL+c} - \left(\frac{1}{b} - \frac{1}{b^2}\right) e^{b+c} \right] = \frac{1-b}{\bar{\psi} b^2} e^{b+c}. \tag{51}$$

C Standard Errors and Reliability of ΔCoVaR and CoSP

In this section we examine the standard errors and reliability of HS estimators for $\Delta\text{CoVaR}^{\leq}$ and ψ_τ. For simplicity, we focus on lag $\tau = 0$ (i.e. co-movements), since the computation and results for all other lags are equivalent. As in the previous sections, the VaR-level is set to 1% for both measures.

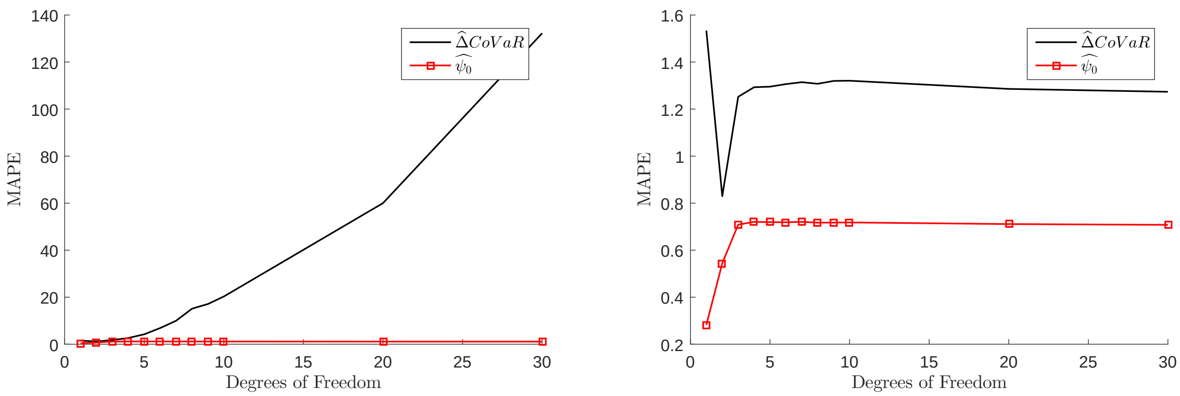
C.1 Standard Errors

Firstly, we perform a Monte-Carlo analysis in two steps: In the first step, we study the mean absolute percentage errors (MAPE) of the risk measures for returns that are student t-distributed. To this end, we estimate the covariance matrix of the firm's and financial index' returns from our data sample by means of the method of moments (see Section D.2).[29] Then, we draw samples from the student-distribution by employing the Cholesky composition of the resulting covariance matrix. The number of samples per iteration of the Monte-Carlo algorithm is set to the maximum number of observations in our data set, which is $n = 5219$, but we also study the implications of a smaller sample size of $n = 2500$. For N realizations (Monte-Carlo iterations) for the estimator ϑ the mean absolute percentage error (MAPE) of the estimator is given as (for example see Tsay (2010, p.217))

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\hat{\vartheta}_i^{(n)} - \bar{\vartheta}^{(n)}}{\bar{\vartheta}^{(n)}} \right|, \tag{52}$$

[29] In line with Section D.2 we set $\sqrt{\text{var}(r^I)} = 0.0236$, $\sqrt{\text{var}(r^M)} = 0.013$ and the correlation to 0.25.

where $\hat{\vartheta}_i^{(n)}$ is the i-th realization of the estimator (either $\widehat{\Delta\text{CoVaR}}^{\leq}$ or $\hat{\psi}_0$) and $\bar{\vartheta}^{(n)}$ the average realized value of the estimator. The MAPE can be interpreted as the average absolute deviation relative to the true value of ϑ. Since the latter is not known, we approximate this true value by $\bar{\vartheta}^{(n)}$.



(a) Sample size: 2500

(b) Sample size: 5219

Figure 10: MAPE of $\widehat{\Delta\text{CoVaR}}^{\leq}$ and $\hat{\psi}_0$ for student-distributed returns.

We show the resulting MAPE for different degrees of freedom (which correspond to the tail-size of the distribution) in Figure 10. Clearly, the MAPE of $\hat{\psi}$ is substantially smaller than the MAPE of $\widehat{\Delta\text{CoVaR}}^{\leq}$. Interestingly, for very small degrees of freedom (i.e. a very heavy tail) the estimation error for $\widehat{\Delta\text{CoVaR}}^{\leq}$ is particularly large and decreases with increasing degrees of freedom, while the estimation error for $\hat{\psi}_0$ is particularly small for small degrees of freedom.

As a second step, we apply a nonparametric bootstrap algorithm to draw samples from the historical returns of exemplary institutions and the financial index. As before, the sample size in each bootstrap step is set to $n = 5219$ and we take $N = 100000$ bootstrap samples. In Table 1 we show the resulting MAPE for $\widehat{\Delta\text{CoVaR}}^{\leq}$, $\widehat{\text{CoVaR}}_{r^I \leq VaR(0.01)}$, $\widehat{\text{CoVaR}}_{r^I \in [\mu^I \pm \sigma^I]}$ and $\hat{\psi}_0$. Clearly, the estimation error of $\hat{\psi}_0$ is substantially smaller for all considered institutions. Moreover, $\widehat{\text{CoVaR}}_{r^I \leq VaR(0.01)}$ has an enormously large estimation error: For some institutions the mean absolute error is 100 times as large as the mean value of the systemic risk measure. This result highly questions the use of $\widehat{\text{CoVaR}}_{r^I \leq VaR(0.01)}$ and, thus, is in line with the findings of Castro and Ferrari (2012), Danielsson et al. (2015) and Guntay and Kupiec (2014).

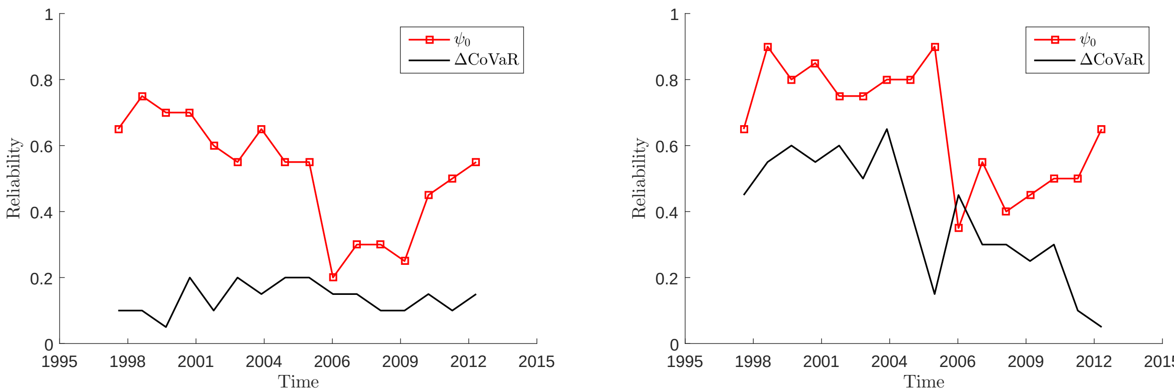
	$\Delta\widehat{\text{CoVaR}}^{\leq}$	$\widehat{\text{CoVaR}}_{r^I \leq VaR(0.01)}$	$\widehat{\text{CoVaR}}_{r^I \in [\mu^I \pm \sigma^I]}$	$\hat{\psi}_0$
WELLSFARGOCO	1.227	41.073	1.963	1.002
JPMORGANCHASECO	1.214	47.543	2.022	1.003
BANKOFAMERICA	1.235	96.071	1.942	0.996
CITIGROUP	1.236	182.269	1.818	0.998
COMMERZBANK	1.208	71.541	1.607	0.995
BANKOFGREECE	1.261	23.876	1.297	0.990
AMERICANINTLGP	1.261	95.391	1.725	0.994
METLIFE	1.222	164.295	1.919	1.001
AXA	1.194	54.827	1.850	1.000
ALLIANZ	1.204	126.014	1.876	0.997
ZURICHINSURANCEGROUP	1.217	110.828	1.832	0.996
GOLDMANSACHSGP	1.206	116.844	1.977	0.999
MORGANSTANLEY	1.214	74.781	1.854	1.000
BLACKROCK	1.207	31.515	1.745	0.999
CHARLESSCHWAB	1.205	59.548	1.668	1.002
CMEGROUP	1.209	29.375	1.718	1.005

Table 1: MAPE of $\Delta\widehat{\text{CoVaR}}^{\leq}$, $\widehat{\text{CoVaR}}_{r^I \leq VaR(0.01)}$, $\widehat{\text{CoVaR}}_{r^I \in [\mu^I \pm \sigma^I]}$ and CoSP for bootstrap samples of size $n = 5219$.

C.2 Reliability

In this section we compare the reliability of $\Delta\text{CoVaR}^{\leq}$ and ψ_0. For this purpose we employ the framework proposed by Danielsson et al. (2015). In this framework an institution is classified as causing systemic risk if its probability to be among the most risky institutions is larger than 90% according to a given systemic risk measure. Then, the reliability of this risk measure is given as the fraction of institutions identified as *guilty* among all most risky institutions. More specifically, in the baseline calibration 10% of the institutions are assumed to be *most risky*. Thus, the reliability is given as the fraction of institutions identified to be among the most 10% risky institutions with a probability larger than 90%. For a motivation for this framework and details regarding the computational implementation we refer to Danielsson et al. (2015).

To compare the reliability of $\Delta\text{CoVaR}^{\leq}$ and CoSP we mainly follow the calibration of Danielsson et al. (2015): The reliability is computed for returns in rolling windows of 5 years, for each window we only consider the 200 firms with the highest market capitalization at the end of the window, and 10% of the institutions are assumed to be *guilty*. However, for our bootstrap algorithm we use a simple non-parametric bootstrap with 100000 iterations and blocks with sample sizes of 2500 or 5219 observations. The resulting reliability of the risk measures is shown in Figure 11. Clearly, CoSP is substantially more reliable than $\Delta\text{CoVaR}^{\leq}$, particularly in the case of less data.



(a) Sample size: 2500

(b) Sample size: 5219

Figure 11: Reliability of CoSP and $\Delta\text{CoVaR}^{\leq}$.

D Data and Methodology

D.1 Market Indices

To account for endogeneity of publicly available market indices, i.e. the issue that institutions may already be incorporated in the index, we compute own market indices similarly to Chan-Lau (2010). To this end, we denote by $MC_t^{(i)}$ the market capitalization of institution i at time t, i.e. $MC_t^{(i)} = P_t^{(i)} \cdot Shares_t^{(i)}$, where $P_t^{(i)}$ is the stock price and $Shares_t^{(i)}$ the number of shares at time t. Moreover, by $TR_t^{(i)}$ we denote the total (dividend-adjusted) return index of institution i.[30] A market is denoted by a subset $\mathbb{S} \subseteq \{1, ..., M\}$, i.e. the institutions that are included in the market. Then, the index for market $\mathbb{S}$ excluding institution j is given as the weighted average of the total

[30]The total return index reflects the evolution of the stock price assuming that dividends are re-invested to purchase additional units of equity.

return indices:

$$INDEX_t^{\mathbb{S}|j} = INDEX_{t-1}^{\mathbb{S}|j} \sum_{s \in \mathbb{S} \setminus \{j\}} \frac{MC_{t-1}^{(s)}}{\sum_{s \in \mathbb{S} \setminus \{j\}} MC_{t-1}^{(s)}} \frac{TR_t^{(s)}}{TR_{t-1}^{(s)}}. \tag{53}$$

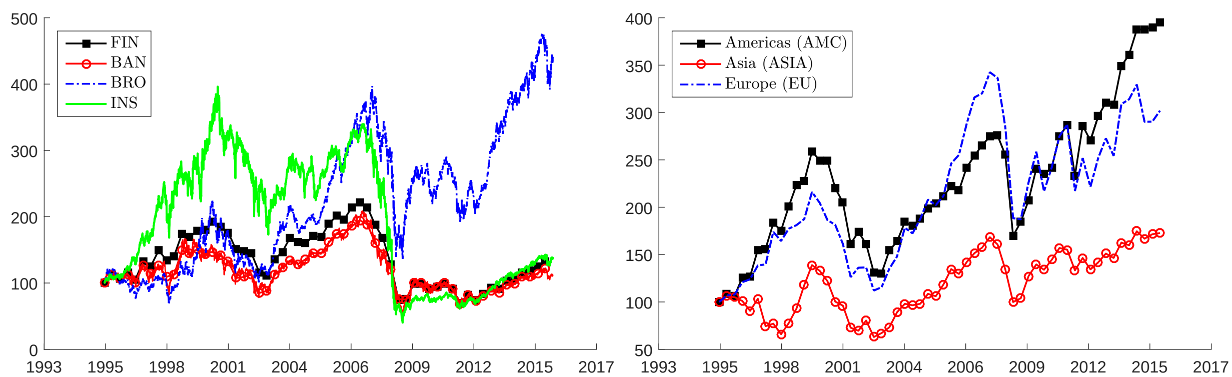
To adjust for different currencies, we calculate the market capitalization in US dollar. Therefore, the time t price of institution s is given by

$$P_t^{(s)} = \tilde{P}_t^{(s)} / ER_t^{(s)}, \tag{54}$$

where $\tilde{P}_t^{(s)}$ is the time t price in currency $\tilde{C}$ and $ER_t^{(s)}$ is the exchange rate from currency $\tilde{C}$ to US Dollar at time t. Finally, the market return is computed as

$$r_t^M = r_t^{\mathbb{S}|j} = \log \left(\frac{INDEX_t^{\mathbb{S}|j}}{INDEX_{t-1}^{\mathbb{S}|j}} \right). \tag{55}$$

D.2 Data and Descriptive Statistics



(a) Financial indices. (b) Non-financial indices.

Figure 12: Financial and non-financial indices.

Company name	**Industry**	**Country**	SRR^{BAN}	SRR^{BRO}	SRR^{INS}	SRR^{AMC}	SRR^{ASIA}	SRR^{EU}
ALPHABET 'A'	Consumer Electronics	USA		2				
AMAZON.COM	E-Commerce, Consumer Electronics	USA					13	
APPLE	Consumer Electronics	USA						
ASTRAZENECA	Health, Pharmaceuticals	UK						
AT&T	Telecommunications	USA						
BP	Oil & Gas	UK				16	10	
BRITISH AMERICAN TOBACCO	Consumer Goods	UK				12	5	
COCA COLA	Consumer Goods	USA				11		
DAIMLER	Automotive	Germany	4	8		8	4	7
DIAGEO	Consumer Goods	UK						
E ON (XET)	Energy	Germany				9	7	5
ENI	Oil & Gas	Italy		7		7		4
EXXON MOBIL	Oil & Gas	USA		3		2		3
GAZPROM	Oil & Gas	Russia		5	2	6	2	
GENERAL ELECTRIC	Energy, Oil & Gas	USA	3	4	1	3		2
GLAXOSMITHKLINE	Health, Pharmaceuticals	UK						
INTEL	Consumer Electronics	USA						
L'OREAL	Consumer Goods	France						
LVMH	Consumer Goods	France				17		

Company name	**Industry**	**Country**	SRR^{BAN}	SRR^{BRO}	SRR^{INS}	SRR^{AMC}	SRR^{ASIA}	SRR^{EU}
MICROSOFT	Consumer Electronics	USA				4	9	
NESTLE 'R'	Consumer Goods	USA						
NOKIA	Telecommunications	Finland						
NOVARTIS 'R'	Pharmaceuticals	Switzerland				18		
PROCTER & GAMBLE	Consumer Goods	USA						
RIO TINTO	Metals, Mining	UK	1	1		1	1	1
ROCHE HOLDING	Pharmaceuticals	Switzerland				15	11	8
ROYAL DUTCH SHELL A	Oil & Gas	UK	2	6		5	3	6
SAMSUNG ELECTRONICS	Consumer Electronics	South Korea						
SANOFI	Pharmaceuticals	France						
SIEMENS (XET)	Energy, Infrastructure	Germany	5	9		10	6	
TELEFONICA	Telecommunications	Spain					12	
TOTAL	Oil & Gas	France				14	8	
TOYOTA MOTOR	Automotive	Japan				19		
VODAFONE GROUP	Telecommunications	UK						
VOLKSWAGEN	Automotive	Germany	6	10		13		
WAL MART STORES	Retail	USA						

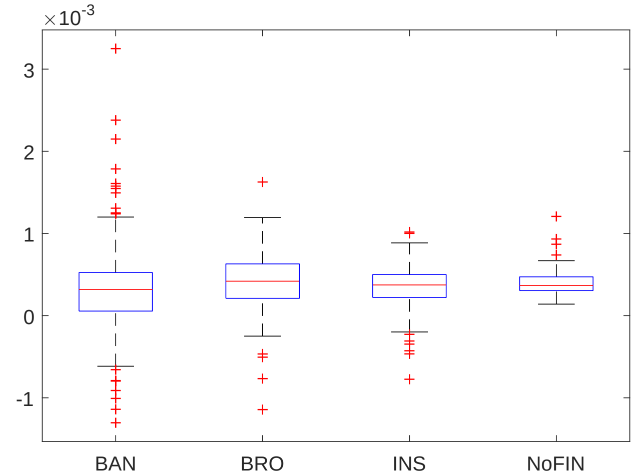
Table 2: Non-financial companies included in the data sample sorted according to market capitalization. SRR^M corresponds to the rank of a company among all non-financial companies according to the Aggregate Excess CoSP w.r.t. the market M. If the respective company is not significantly systemically important, the corresponding cell is left empty.

Banks	**Insurance Companies**	**Brokers**
WELLSFARGOCO	BERKSHIREHATHAWAYB	GOLDMANSACHSGP
JPMORGANCHASECO	BERKSHIREHATHAWAYA	MORGANSTANLEY
BANKOFAMERICA	ALLIANZ	BLACKROCK
CHINACONBANKH	AMERICANINTLGP	CHARLESSCHWAB
CITIGROUP	AXA	CMEGROUP
HSBCHOLDINGS	METLIFE	HONGKONGEXSCLEAR
COMMONWEALTHBKOFAUS	ZURICHINSURANCEGROUP	INTERCONTINENTALEX
MITSUBISHIUFJFINLGP	PRUDENTIALFINL	FRANKLINRESOURCES
ROYALBANKOFCANADA	ACE	NOMURAHDG
BANCOSANTANDER	SWISSRE	MACQUARIEGROUP

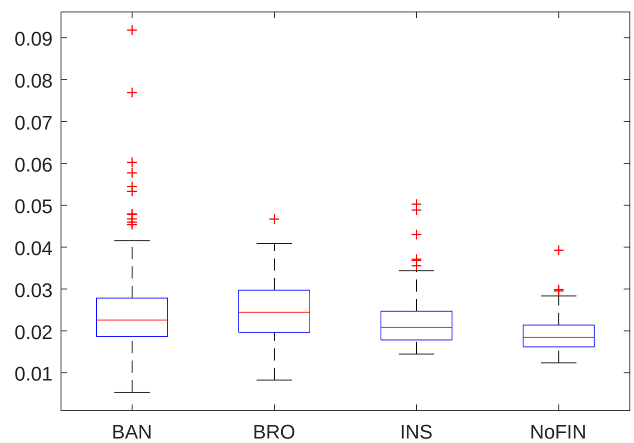
Table 3: Names of the ten largest institutions (by market capitalization in November 2015) in each subsector.

Index	**No. of institutions**	$\overline{r}$	$\sqrt{\mathrm{var}(r)}$	$r_{0.1}$	$r_{0.5}$	$r_{0.9}$
BAN	567	2.29e-05	0.013	-0.015	0.000	0.014
BRO	151	2.85e-04	0.014	-0.015	0.001	0.015
INS	199	6.36e-05	0.015	-0.015	0.000	0.015
FIN	917	4.17e-05	0.013	-0.014	0.001	0.014
AMC NoFIN	1265	2.56e-04	0.012	-0.013	0.001	0.012
ASIA NoFIN	1514	9.42e-05	0.011	-0.013	0.000	0.013
EU NoFIN	1902	1.99e-04	0.012	-0.013	0.001	0.013

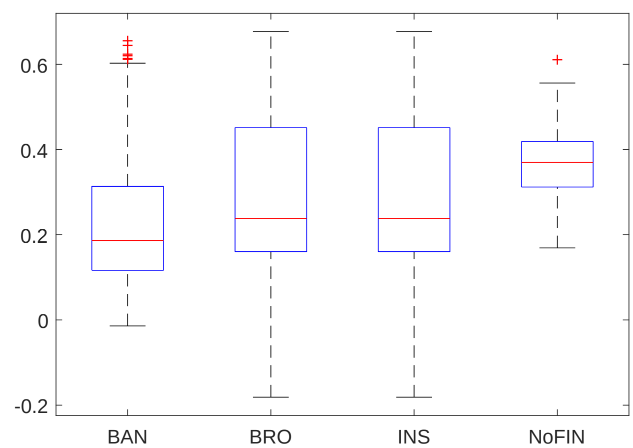
Table 4: Mean, standard deviation and quantiles of different index returns.



(a) Empirical mean return for different subsectors



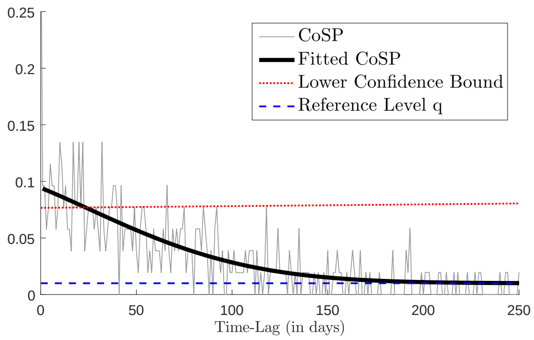
(b) Empirical standard deviation for different subsectors



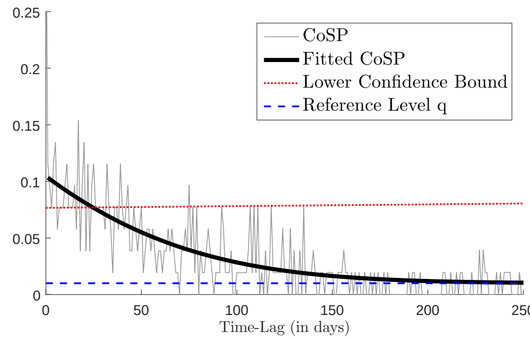
(c) Empirical correlation between r^I and r^{FIN}

Figure 13: Distribution of mean and standard deviation of returns, and correlation between institution returns and returns of the financial index.

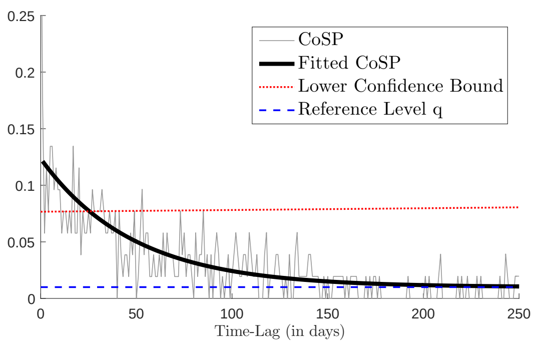
E Additional Figures



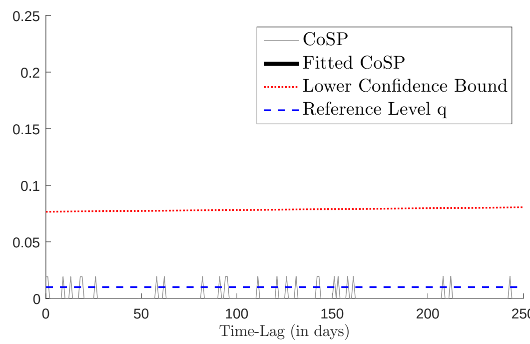
(a) Bank of America (BAN)



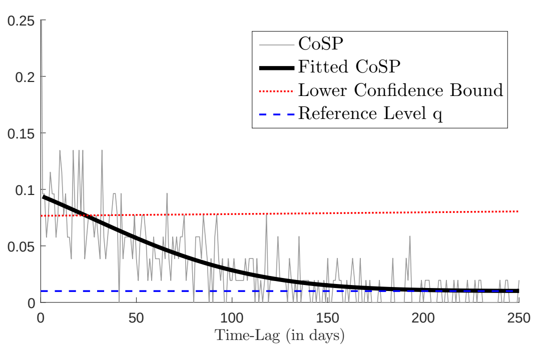
(b) JP Morgan Chase (BAN)



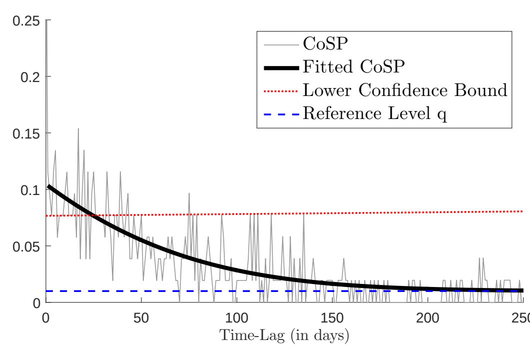
(c) Citigroup (BAN)



(d) Bank of Greece (BAN)

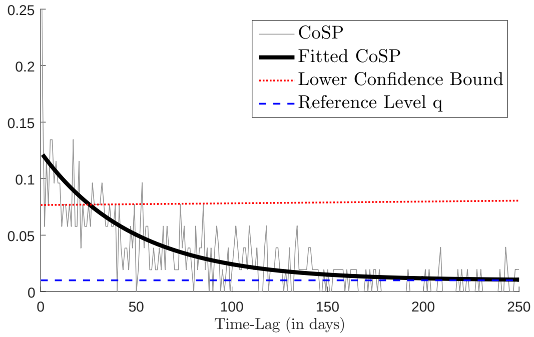


(e) Goldman Sachs (BRO)

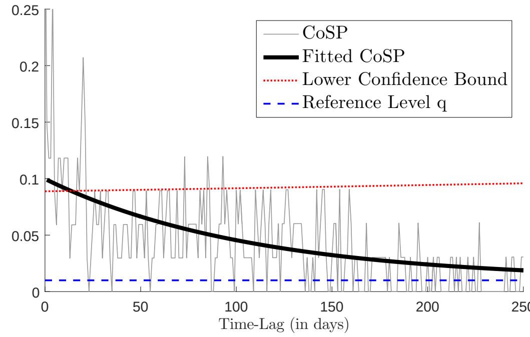


(f) Morgan Stanley (BRO)

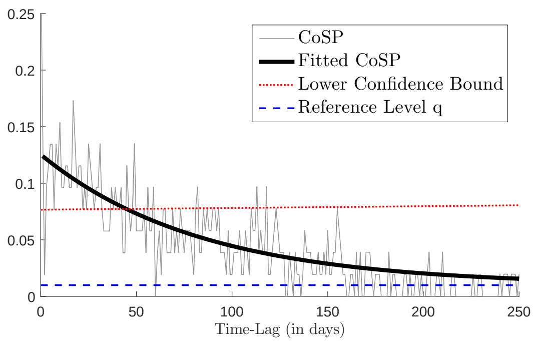
Figure 14: CoSP triggered by exemplary banks and brokers w.r.t. the FIN index.



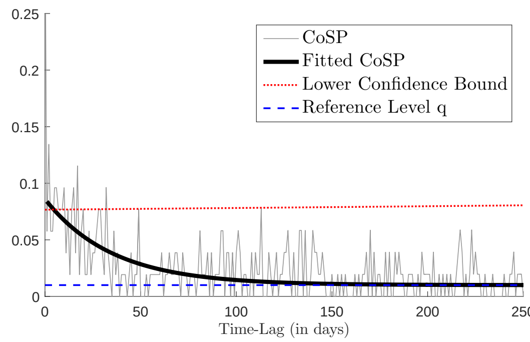
(a) Charles Schwab (BRO)



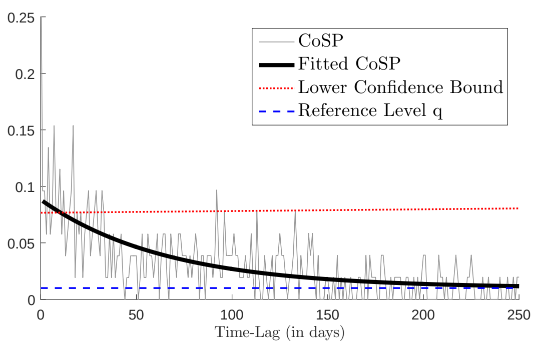
(b) CME Group (BRO)



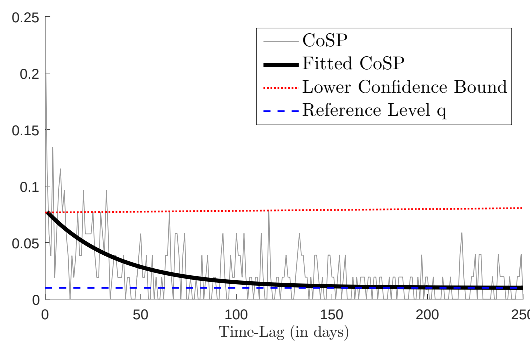
(c) AIG (INS)



(d) Allianz (INS)

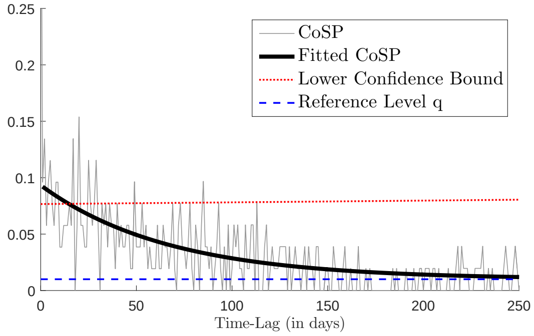


(e) AXA (INS)

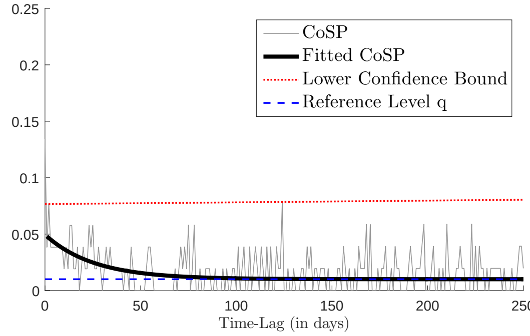


(f) Zurich Group (INS)

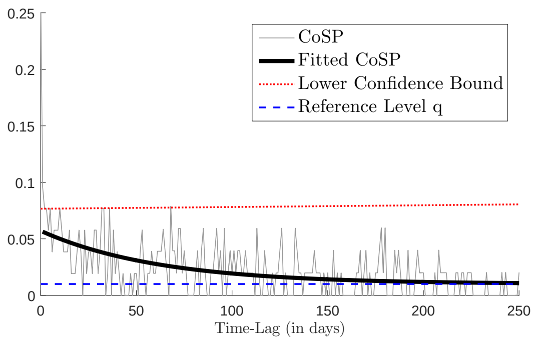
Figure 15: CoSP triggered by exemplary brokers and insurance institutions w.r.t. the FIN index.



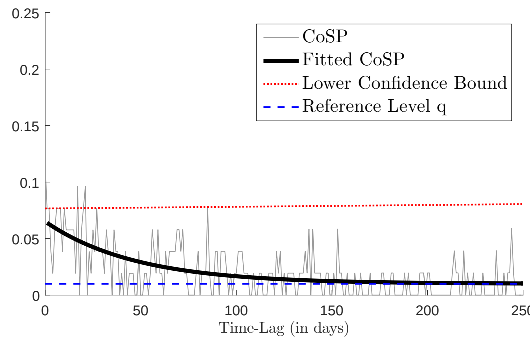
(a) General Electric (NoFIN)



(b) Apple (NoFIN)



(c) BP (NoFIN)



(d) Coca Cola (NoFIN)

Figure 16: CoSP triggered by exemplary non-financial companies w.r.t. the FIN index.

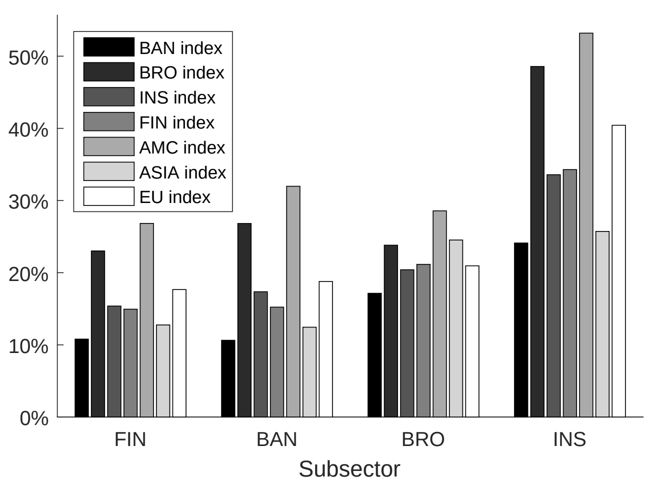
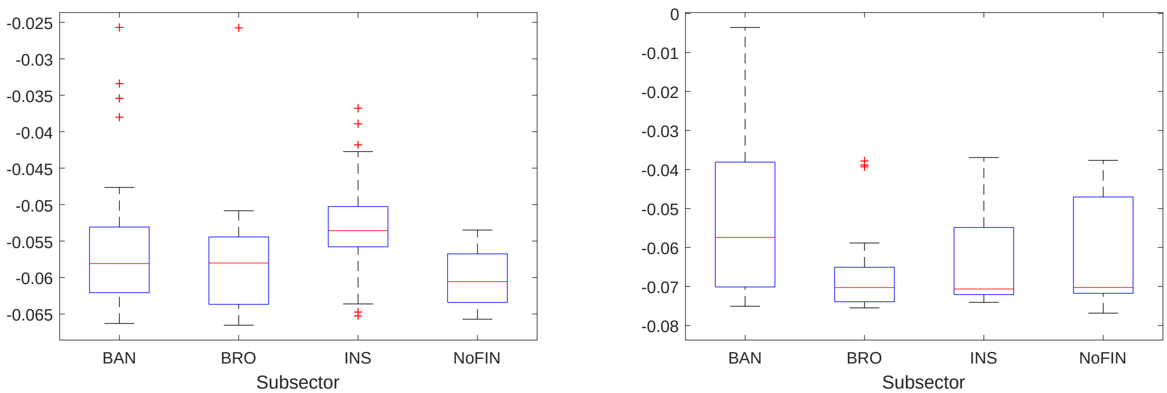


Figure 17: Fraction of significantly systemically important institutions among the subsectors FIN, BAN, BRO, INS w.r.t. the BAN, BRO, INS, FIN, American, Asian and European non-financial index.

	-MES	$-\Delta$CoVaR	AggExcCoSP $\bar{\psi}$
ψ_0	81.92%	79.17%	60.79%
-MES		67.81%	48.92%
-ΔCoVaR			57.21%

Table 5: Empirical correlation between different systemic risk measures with respect to the FIN index across the full sample.



(a) $\Delta\text{CoVaR}^{\leq}$ w.r.t. the FIN index.

(b) $\Delta\text{CoVaR}^{\leq}$ w.r.t. the American NoFIN index.

Figure 18: $\Delta\text{CoVaR}^{\leq}$ w.r.t. the FIN and American NoFIN indices triggered by significantly systemically important institutions of the subsectors BAN, BRO, INS and NoFIN. For each box, the central mark is the median, the edges are the 25th and 75th percentiles, q_1 and q_3, and the maximum whiskers' length is $1.5(q_3 - q_1)$.

References

Acharya, V., Engle, R., and Richardson, M. (2012). Capital shortfall: A new approach to ranking and regulation systemic risk. *American Economic Review: Papers & Proceedings*, 102(3):59–64.

Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. (2010). Measuring systemic risk. *Working Paper*.

Acharya, V. and Yorulmazer, T. (2002). Information contagion and inter-bank correlation in a theory of systemic risk. *London Business School Working Paper*.

Adrian, T. and Brunnermeier, M. (2014). Covar. *Federal Reserve Bank of New York Staff Reports*.

Adrian, T., Covitz, D., and Liang, N. (2014). Financial stability monitoring. *Federal Reserve Bank of New York Staff Reports*.

Ahnert, T. and Bertsch, C. (2015). A wake-up call theory of financial contagion. available at `http://www.christophbertsch.com/data/wake-up_call.pdf` (last checked: April 29, 2016).

Aït-Sahalia, Y., Cacho-Diaz, J., and Laeven, R. J. (2015). Modeling financial contagion using mutually exciting jump processes. *Journal of Financial Economics*, 117:585–606.

Arora, S., Barak, B., Brunnermeier, M., and Ge, R. (2009). Computational complexity and information asymmetry in financial products. available at `http://www.princeton.edu/~markus/research/papers/complexity_asymmetric_information.pdf` (last checked: February 27, 2016).

Arregui, N., Norat, M., Pancorbi, A., and Scarlata, J. (2013). Addressing interconnectedness: Concepts and prudential tools. *IMF Working Paper*, (13/199).

Benoit, S., Colletaz, G., Hurlin, C., and Perignon, C. (2013). A theoretical and empirical comparison of systemic risk measures. *HEC Paris Research Paper*, (FIN-2014-1030).

Berdin, E. and Sottocornola, M. (2015). Insurance activities and systemic risk. *ICIR Working Paper Series*, (19).

Billio, M., Lo, A. W., Getmansky, M., and Pelizzon, L. (2010). Econometric measures of connectedness and systemic risk in the finance and insurance sectors. *MIT Sloan Research Paper*, (4774-10).

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2015). A survey of systemic risk analytics. *Office of Financial Research, Working Paper*.

Black, L., Correa, R., Huang, X., and Zhou, H. (2016). The systemic risk of european banks during the financial and sovereign debt crises. *Journal of Banking & Finance*, 63:107–125.

Castro, C. and Ferrari, S. (2012). Measuring and testing for the systemically important financial institutions. *Working Paper of the National Bank of Belgium*, (228).

Chan-Lau, J. A. (2010). Regulatory capital charges for too-connected-to-fail institutions: A practical proposal. *IMF Working Paper*, (98).

Chan-Lau, J. A., Espinosa, M., Giesecke, K., and Solé, J. A. (2009). Assessing the systemic implications of financial linkages. *IMF Global Financial Stability Review*, 2.

Chan-Lau, J. A., Mitra, S., and Ong, L. L. (2012). Identifying contagion risk in the international banbank system: An extreme value theory approach. *International Journal of Finance and Economics*, 17:390–406.

Chen, H., Cummins, J. D., Viswanathan, K. S., and Weiss, M. A. (2013). Systemic risk and the interconnectedness between banks and insurers: An econometric analysis. *The Journal of Risk and Insurance*, 81(3):623–652.

Committee on Capital Markets Regulation (2009). The global financial crisis - a plan for regulatory reform. available at `http://capmktsreg.org/reports/the-global-financial-crisis-a-plan-for-regulatory-reform/` (last checked: April 21, 2016).

Cummins, J. D. and Weiss, M. A. (2014). Systemic risk and the us insurance sector. *Journal of Risk and Insurance*, 81(3):489–582.

Danielsson, J., James, K. R., Valenzuela, M., and Zer, I. (2015). Can we prove a bank guilty of creating systemic risk? a minority report. *Journal of Money, Credit, and Banking*, (Forthcoming).

Danielsson, J., James, K. R., Valenzuela, M., and Zer, I. (2016). Model risk of risk models. *Journal of Financial Stability*, Forthcoming.

Danielsson, J. and Zhou, C. (2015). Why risk is so hard to measure. *SRC Discussion Paper*, (36).

Dong, H., Bowers, H. M., and Latham, W. R. (2013). Evidence on the efficient market hypothesis from 44 global financial market indexes. available at `http://lerner.udel.edu/sites/default/files/ECON/PDFs/RePEc/dlw/WorkingPapers/2013/UDWP13-07.pdf` (last checked: April 20, 2016). *University of Delaware, Department of Economics Working Paper*, (13-07).

Ergün, A. T. and Girardi, G. (2013). Systemic risk measmeasure: Multivariate garch estimation of covar. *Journal of Banking & Finance*, 37:3169–3180.

Fama, E. (1969). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance*, 25(2):383–417.

Farmer, D. and Lo, A. W. (1999). Frontiers of finance: Evolution and efficient markets. *Proceedings of the National Academy of Sciences*, 96:9991–9992.

Financial Stability Board (2011). Policy measures to address policy measures to address systemically important financial institutions. available at `http://www.fsb.org/wp-content/uploads/r_111104bb.pdf?page_moved=1` (last checked: February 3, 2016).

Granger, C. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica*, 37(3):424–438.

Grossmann, S. J. and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. *The American Economic Review*, 70(3):393–408.

Guntay, L. and Kupiec, P. (2014). Taking the risk out of systemic risk measurement. available at `http://ssrn.com/abstract=2375236` (last checked: April 20, 2016).

Haefeli, D. and Liedtke, P., editors (2012). *Insurance and Resoultion in Light of the Systemic Risk Debate*. The Geneva Association.

Harrington, S. E. (2009). The financial crisis, systemic risk, and the future of insurance regulation. *Journal of Risk and Insurance*, 76(4):785–819.

Harris, A. and Chiglinsky, K. (2016). Metlife defeats u.s. government's too-big-to-fail labeling. available at `http://www.bloomberg.com/news/articles/2016-03-30/metlife-wins-court-ruling-removing-fsoc-s-too-big-to-fail-tag-imeyio6z` (last checked: April 12, 2016).

Hendricks, D. (1996). Evaluation of value-at-risk models using historical data. *Economic Policy Review*, 2(1).

Huang, X., Zhou, H., and Zhu, H. (2009). A framework for assessing the systemic risk of major financial institutions. *Journal of Banking & Finance*, 33:2036–2049.

Hull, J. and White, A. (1998). Incorporating volatility updating into the historical simulation method for value at risk. *Journal of Risk*, 1:5–19.

International Monetary Fund (2016). Global financial stability report - potent policies for a successful normalization. *World Economic and Financial Surveys*.

Kuester, K., Mittnik, S., and Paolella, M. (2006). Value-at-risk prediction: A comparison of alternative strategies. *Journal of Financial Econometrics*, 4(1):53–89.

Lo, A. W. (2004). The adaptive markets hypothesis: Market efficiency from an evoevolution perspective. *Journal of Portfolio Management*, 30:15–29.

Löffler, G. and Raupach, P. (2013). Robustness and informativeness of systemic risk measures. *Bundesbank Discussion Paper*, (4).

Macaulay, F. R. (1938). *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States Since 1856*. NBER Books.

Mainik, G. and Schaanning, E. (2014). On dependence consistency of covar and some other systemic risk measures. *Statistics & Risk Modeling*, 31(1):49–77.

Marshall, D. (1998). Understanding the asian crisis: Systemic risk as coordination failure. *Economic Perspectives. Federal Reserve Bank of Chicago*, 22(3):13–28.

McNeil, A. J., Frey, R., and Embrechts, P. (2015). *Quantitative Risk Management - Concepts, Techniques and Tools.* Princeton University Press.

Moghadam, R. and Viñals, J. (2010). Understanding financial interconnectedness. available at `http://www.imf.org/external/np/pp/eng/2010/100410.pdf` (last checked: February 10, 2016).

Pericoli, M. and Sbracia, M. (2003). A primer on financial contagion. *Journal of Economic Surveys*, 17(4):571–608.

Pritsker, M. (2001). The hidden dangers of historical simulation. *Finance and Economics Discussion Series, Federal Reserve Board*, (27).

Smaga, P. (2014). The concept of systemic risk. *SRC Special Paper*, (5).

Thimann, C. (2014). How insurers differ from banks: Implications for systemic regulation. *SRC Special Paper*, (3).

Tsay, R. S. (2010). *Analysis of Financial Time Series.* John Wiley & Sons, Inc., third edition.

Weiß G. and Mühlnickel, J. (2014). Why do some insurers become systemically relevant? *Journal of Financial Stability*, 13:95–117.