International Center for Insurance Regulation
Goethe University Frankfurt

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

ICIR Working Paper Series No. 29/17

Edited by Helmut Gründl and Manfred Wandt

Rising Interest Rates, Lapse Risk, and the Stability of Life Insurers

Elia Berdin, Helmut Gründl, Christian Kubitza*

This version: July 2017

Abstract

This paper investigates the effects of a rise in interest rate and lapse risk of endowment life insurance policies on the liquidity and solvency of life insurers. We model the book and market value balance sheet of an average German life insurer, subject to both GAAP and Solvency II regulation, featuring an existing back book of policies and an existing asset allocation calibrated by historical data. The balance sheet is then projected forward under stochastic financial markets. Lapse rates are modeled stochastically and depend on the granted guaranteed rate of return and prevailing level of interest rates. Our results suggest that in the case of a sharp increase in interest rates, policyholders sharply increase lapses and the solvency position of the insurer deteriorates in the short-run. This result is particularly driven by the interaction between a reduction in the market value of assets, large guarantees for existing policies, and a very slow adjustment of asset returns to interest rates. A sharp or gradual rise in interest rates is associated with substantial and persistent liquidity needs, that are particularly driven by lapse rates.

Keywords: Interest Rate Risk, Lapse Risk, Life Insurance

*All authors are affiliated at the International Center for Insurance Regulation, Goethe-University Frankfurt, Theodor-W.-Adorno Platz 3, D-60629 Frankfurt am Main, Germany. Elia Berdin is also affiliated with Assicurazioni Generali S.p.A. The findings, views and interpretations expressed herein are those of the author and should not be attributed to Assicurazioni Generali S.p.A. Corresponding authors e-mail: Christian Kubitza *kubitza@finance.uni-frankfurt.de*, Elia Berdin *Elia.Berdin@generali.com*.

1 Introduction

Insurers provide essential services to economies by assuming, transferring and diversifying risks. Life insurers in particular promote economic growth by efficiently allocating assets and providing funding to other financial and non-financial companies as well as states. Thereby, the size of life insurers is substantial even when compared to banks.[1] The interconnectedness of (life) insurers has been increasing in the last decades, resulting in (life) insurers being important nodes in the global financial system. For example, Billio et al. (2012) find that insurance companies became more interconnected within the financial system in general. Foley-Fisher et al. (2016) examine the role of securities lending by life insurers for the functioning of securities markets.

Subsequent to the 2007/08 financial crisis, life insurers have been struggling with and adjusting to exceptionally low interest rates for approximately 10 years. Now, they are facing a new challenge: the risk of rising interest rates. At first sight, rising interest rates might stabilize the balance sheet of life insurers by increasing fixed income returns and decreasing the present value of future liabilities. Such a development would benefit in particular life insurers that are exposed to low interest rates due to long term financial guarantees sold in the past, e.g. guaranteed minimum rates of return. This is especially true in many European jurisdictions, e.g. Germany, where products embedding a minimum guaranteed rate of return are particularly popular. Typically, life insurance policies with a minimum guaranteed rate of return are long term contracts, in which the guaranteed rate of return is set at inception and it cannot be changed until maturity. As a result, life insurers selling long term guaranteed business are particularly exposed to low interest rates as the guarantees sold in the past become expensive to get funded (Berdin and Gründl (2015), Berdin et al. (2016)).

Nonetheless, life insurers in such low interest rate environment face an additional risk: life insurance policies issued at current market rates may become less attractive to policyholders as soon as interest rates rise and new savings opportunities yield higher returns. This might result in increased lapse rates (Feodoria and Förstemann (2015)). More formally, one might think of an

[1] In 2015 US life insurers held 6,3 billion US dollar in assets and 5 billion US dollar life insurance and annuity liabilities, while US depository institutions held 14,2 billion US dollar in assets and 8 billion US dollar in savings deposits Board of the Governors of the federal reserve system (2016).

endowment life policy with minimum guaranteed rate of return[2] as a put option which loses value as soon as prevailing market rates are higher than the minimum guaranteed rate of return (Albizzati and Geman (1994)). 90% of all life insurance contracts sold by European life insurers can be lapsed with a penalty lower than 15% of the policy value (see European Systemic Risk Board (2015)). Overall, this may incentivize a large fraction of policyholders to lapse their life insurance contracts in case of a steep increase in interest rates, which in turn may pose a risk on the insurer's liquidity and solvency, and may even endanger financial stability, as argued by the European Systemic Risk Board (2015) and European Central Bank (2017). Both recent reports stress the potential risk of increasing lapse rates due to a rise in interest rates and the resulting consequences for the liquidity and solvency situation of European insurance companies, which also carries over to other jurisdictions. Due to their high interconnectedness and importance as intermediaries, potential liquidity and solvency problems of life insurers may spread out to other financial institutions and, thereby, endanger financial stability.

The contribution of this article is to shed more light on the joint impact and interrelation between interest rates, lapse risk, as well as the liquidity and solvency situation of life insurers. For this purpose, we model a stylized financial market and a life insurance company that sells endowment life insurance policies. This model allows us to include potential portfolio effects between legacy business and newly sold insurance contracts as well as between existing and new asset investments. To assess the solvency situation of this stylized life insurer, we compute risk-based capital requirements for market risk and lapse risk based on a market-consistent valuation of assets and liabilities. Our approach is based on the European solvency regime, Solvency II. The market oriented and principles based view of Solvency II as well as its total balance sheet approach are similar to capital standards of New Zealand as well as Switzerland, as Eling and Holzmüller (2008) point out. For determining capital requirements, the US RBC standard employs a similar static factor model, that, however, is not taking an insurer's total balance sheet into account to the extent as Solvency II. The model is calibrated with German data, since endowment life contracts with annual guaranteed rates have been very popular in Germany and, thus, German life insurers are particularly exposed to the resulting risks.

[2] In this study we focus on the saving phase of such contracts. In this phase, policyholders pay periodic premiums, the insurer invests the premium payments and pays the contract value at maturity. If policyholder lapse before maturity, they receive the current (book value) of the policy.

The impact of rising interest rates on a life insurer's solvency after a period of particularly low interest rates is not immediately clear. In particular, it depends on the interplay of two major effects: a valuation effect and a cash-flow effect. On the one hand, rising interest rates yield smaller market-consistent values for both assets and liabilities of an insurance company. Given the longer duration of a life insurer's liabilities than assets, the valuation effect leads to an increase in the value of equity (i.e. own funds in the terminology of Solvency II). On the other hand, this duration gap does not reflect the evolution of an insurer's cash-flows, which are heavily affected by high lapse rates. When interest rates increase, market consistent values of contracts (i.e. liabilities) might drop below recovery values. In such a situation, lapses decrease an insurer's own funds. Moreover, large lapse rates for contracts with comparably small guarantees increase the average guarantee in-force, thereby increasing capital requirements and reducing profits.

Our study contributes in three ways to understanding the impact of interest rate changes for life insurers. Firstly, we modify the model of a life insurance company as presented in Berdin and Gründl (2015), Berdin (2016) and Berdin et al. (2016) to include lapse risk. In particular, we demonstrate how to perform a balance sheet and market-consistent valuation of life insurance liabilities in the presence of lapse risk. Secondly, we develop a model for policyholders' individual probability to lapse an insurance contract, i.e. the *lapse rate*, based on the contract's guaranteed rate of return and market interest rates as well as contract age. The rationale is similar to Feodoria and Förstemann (2015), who show that it is rational for policyholders to lapse contracts when interest rates exceed a certain threshold. Thirdly, to be able to model different interest rate environments, we extend the Hull-White model for interest rates. In contrast to other common interest rate models (as the CIR (Cox et al. (1985)) or Vasicek (Vasicek (1977)) model), our approach directly specifies the evolution of average interest rates while the model still yields an arbitrage-free yield curve, based on the model developed by Hull and White (1990). We initialize the model with interest rates as of 2015 and calibrate it to yield either a sudden or gradual increase in interest rates over time. The Vasicek model calibrated to low interest rates serves as a benchmark environment.

In general, we find that a life insurer's solvency improves with rising interest rates in the long run. However, in the short run life insurers are particularly vulnerable towards interest rate driven lapse risk. A sudden increase in interest rates, in particular, is related to a sharp drop in solvency ratios, i.e. in the ratio of own funds to the capital requirement, and substantial liquidity needs. Although the insurer's solvency is more stable when interest rates gradually increase, its liquidity situation depletes over time.

Several implications can be drawn from our study. Most importantly, we show that the sensitivity of lapse rates towards market rates is an important driver for the solvency of insurers. Therefore, it is important for life insurers and regulators to support mechanisms that protect the ability of insurers to provide recovery values resulting from increasing lapses. Life insurers can expect free cash flows to become negative for a substantial amount of time, which also impacts the profitability of these companies.

We focus on a life insurance company selling endowment life contracts with an annual guaranteed rate and surplus participation. The rationale of policyholders decreasing their investment in life insurance endowment policies due to positive shocks on interest rates is substantiated by numerous theoretical and empirical studies (Dar and Dodds (1989), Kim (2005), Kuo et al. (2003), Kiesenbauer (2012), Russell et al. (2013), Russo et al. (2017)) and commonly referred to as *interest rate hypothesis*. Barsotti et al. (2016) develop a model that is similar to our model for lapse rates but additionally account for correlation and contagion effects among policyholders. In contrast, we rely on a very basic model in order to focus on effects solely stemming from an increase in interest rates without imposing additional assumptions on policyholder behavior.

Some intuition about the impact of lapse risk on the solvency of life insurers is provided by Le Courtois and Nakagawa (2009) and Buchardt (2014). However, none of the mentioned studies embed a model for lapse risk into a balance sheet model for life insurers that simultaneously models book values and market-consistent values of the asset and liability side and evaluates the resulting solvency situation under risk-based capital requirements from a portfolio perspective. For example Russo et al. (2017) find that the best estimate for liabilities under Solvency II increases when taking interest rate sensitive lapse rates into account. However, in our model this only holds in the first years of the simulation, but due to a large negative free cash flow the aggregate best estimates of liabilities under interest rate sensitive lapse rates decrease below the value with constant lapse rates. We argue that it is important to take such effects into account in order to develop a complete picture about the solvency situation of life insurers.

The article proceeds as follows: Section 2 revisits our model of a life insurer and describes how we simulate, calculate, and calibrate the dynamics of the insurer's balance sheet, financial market, and policyholders' lapse behavior. Section 3 discusses the results and Section 4 concludes.

2 The Model

In this section we briefly describe the model of a stylized life insurer we employ. For further details we refer to Berdin and Gründl (2015) and Berdin (2016).

2.1 Assets

The financial market model consists of a short rate model for interest rates, spreads for different bond categories, and distributional assumptions for stock and real estate returns. The short rate model is given by the Hull-White model (Hull and White, 1990). This model drives the evolution of interest rates. In order to simulate rising interest rates, we model the time-dependent level of mean reversion as an increasing function, which is given by[3]

$$\theta(t) = \gamma + (\beta - \gamma)\left(1 - \frac{1}{1 + e^{-b(t-h)}}\right). \tag{1}$$

Further details on the short-rate model and its calibration can be found in Section 3.1.1 and Appendix B.

As in Berdin (2016) and in Berdin et al. (2016), spreads for sovereign and corporate bonds are modeled by truncated Ornstein-Uhlenbeck processes and calibrated with historical data. Stock and real-estate returns follow Geometric Brownian Motions that are also calibrated by historical data. Finally, all stochastic processes are correlated through a Cholesky decomposition of the diffusion terms.

The insurance company invests into 4 different asset classes: 1) German, French, Dutch, Italian, Spanish sovereign bonds, 2) stocks, 3) real estate, and 4) AAA, AA, A, BBB corporate bonds. The weights for each asset class (as in market values) are calibrated based on the results of the 2014 insurance stress tests by the European Insurance and Occupational Pensions Authority (EIOPA) (2014a) and are reported in Table 1. For sovereign bonds we select the last bonds with 20 YTM

[3] β and γ are the initial and long-term level of mean reversion, respectively, b describes the skewness of mean reversion, and h is a shift parameter.

during the last 20 years to represent the different coupons held in the portfolio. Each coupon has a different remaining time to maturity such that the oldest coupon in the sovereign bond portfolio is due in 1 year and the youngest in 20 years. The weights are chosen in order to represent the modified duration as in European Insurance and Occupational Pensions Authority (EIOPA) (2014a). We follow the same calibration strategy as for sovereign bonds and assume that corporate bonds portfolio are held to maturity and time to maturity at purchase is 10 years. Due to the absence of data, we calibrate real estate and stock weights in order to yield a plausible home bias of 60% for German real estate and stocks and distribute the remaining weights equally.

The reported initial asset allocation yields an asset duration of 8.26 years, which is in line with reports by the German Insurance Association (GDV) and European Insurance and Occupational Pensions Authority (EIOPA) (2014a). During the evolution of the model, we assume that the portfolio weights remain constant relative to the market values of the respective assets. This investment strategy is plausible for insurers to maintain a similar level of investment risk and risk-based capital requirement for assets over time.

2.2 Liabilities

Initially, the insurance company's back book is simulated by accumulating previously closed contracts for the past 30 years. These contracts entail historical values of the guaranteed rate and realized profit participation of endowment life insurance contracts in Germany. In each year, one cohort of contracts matures, while one cohort of new contracts is sold. Thereby, the level of the guaranteed rate of newly sold contracts is based on the evolution of the technical rate which depends on current and past interest rates.

The lifetime of each insurance contract is assumed to equal 30 years at contract inception. However, we assume that each year each policyholder might lapse her life insurance contract with a certain probability λ. The lapse probability λ is assumed to either equal the average lapse rate in 2015, 2.68%, as reported by the German Insurance Association (GDV), or depend on the market's risk free rate as described in Section 2.4.

Suppose that policyholders can lapse a contract directly at the begin of year t and receive the current accumulated funds less a haircut, i.e. they receive ϑV_t with $\vartheta \in (0, 1]$. The lapse rate equals λ. We assume that the insurer sets λ equal to the average lapse rate in the previous year

Asset Portfolio Weights	
Sovereigns w_{sov}	56.7%
Corporate w_{corp}	34.3%
Stocks w_{stocks}	5.6%
Real Estate $w_{\text{real estate}}$	3.4%
Sovereign Portfolio	
German Sovereigns/All Sovereigns w_{DE}	88.18%
French Sovereigns/All Sovereigns w_{FR}	2.95%
Dutch Sovereigns/All Sovereigns w_{NL}	2.95%
Italian Sovereigns/All Sovereigns w_{IT}	2.95%
Spanish Sovereigns/All Sovereigns w_{ES}	2.95%
Corporates Portfolio	
AAA/All Corporates w_{AAA}	23.6%
AA/All Corporates w_{AA}	16.85%
A/All Corporates w_{A}	33.71%
BBB/All Corporates w_{BBB}	25.84%
Stocks and Real Estate Portfolios	
German/Portfolio $w_{\text{s/re DE}}$	60%
French/Portfolio $w_{\text{s/re FR}}$	10%
Dutch/Portfolio $w_{\text{s/re NL}}$	10%
Italian/Portfolio $w_{\text{s/re IT}}$	10%
Spanish/Portfolio $w_{\text{s/re ES}}$	10%

Table 1: Initial asset allocation based on European Insurance and Occupational Pensions Authority (EIOPA) (2014a).

when calculating the market-consistent value of liabilities since it does not know the particular dynamics of the lapse rate. The dynamics of the accumulated funds for a specific contract are given as $V_{t+1} = (1+\tilde{r}_{t+1})V_t$, where $\tilde{r}_{t+1}$ is the stochastic growth in year $t+1$. At contract maturity policyholders receive the accumulated funds V_T if they do not lapse before.

Under German GAAP accounting standards, the book value of liabilities is computed with $\tilde{r}_{t+1} \equiv r_{t+1} \equiv r^G$, where r^G is the rate annually guaranteed to policyholders. Thus contracts are valued as if future profit participation would equal the guaranteed rate. Due to competition among life insurers, we assume that the guarantee of newly sold life insurance contracts equals the current maximum technical interest rate for discounting policy reserve as set by the regulator.[4] As discussed in Eling and Holder (2012) and Berdin and Gründl (2015), r_G^h follows in 0.5% steps 60% of the 10 year moving average of past AAA German sovereign rates (i.e. the reference rate) with maturity 10 years.

[4] Eling and Holder (2012) discuss how technical rates differ in different jurisdictions in Europe and the U.S.

Liabilities are discounted under German GAAP accounting with the maximum technical rate at contract inception if it is smaller than the current reference rate. Then we yield the book value of liabilities as

$$L_t^{BV} = V_t \ \vartheta(1-(1-\lambda)^{T-t}) + (1-\lambda)^{T-t}. \tag{2}$$

However, the insurer must build up an additional reserve if the reference rate for calculating the technical rate falls below the guaranteed return. In this case, liabilities are discounted with the reference rate.[5]

The present value of a contract's future cash flows at time t is given as

$$PV_t = V_t \mathbb{E}_t^{\mathbb{Q}} \left[\sum_{i=1}^{T-t} \lambda(1-\lambda)^{i-1} \frac{\vartheta}{(1+r_{i-1,t})^{i-1}} \prod_{j=1}^{i-1}(1+\tilde{r}_{t+j}) + \frac{(1-\lambda)^{T-t}}{(1+r_{i-1,t})^{i-1}} \prod_{j=1}^{i-1}(1+\tilde{r}_{t+j}) \right]. \tag{3}$$

As required under Solvency II, the market consistent value of liabilities is the sum of the best estimate of future cash flows and a risk margin for non-headgeable risks, i.e. $L_t^{MV} = PV_t(1+RM)$. The discount rate under Solvency II corresponds to the risk-free rate linearly extrapolated to the ultimate forward rate (UFR) of 4.2% as given by EIOPA for maturities of 60 years or longer.[6] It is likely that the UFR will increase following substantial interest rate rises. Therefore, the discount rate for life insurance liabilities in our model, R_L, equals the maximum of extrapolated discount rates with an UFR of 4.2% and the actual risk-free rate.

The annual growth of accumulated funds is given as the maximum of guaranteed annual return and profit participation, i.e. $\tilde{r}_{t+j}^h = \max(r_G^h, r_{S,t+j}^h)$, where r_G^h is the guaranteed annual return for cohort h and $r_{S,t+j}^h$ the profit participation of cohort h in year $t+j$. The profit participation, $r_{S,t-s}^h$, usually originates from the asset return of the insurer in year $t-s$ weighted by the insurer's actuarial reserves (see Berdin and Gründl (2015) for further details).[7]

Hence, to compute the market-consistent value of liabilities, the insurance company needs to estimate the distribution of the future growth of accumulated funds, $\tilde{r}_{t+j}^h$. This estimation is

[5] Details are discussed in Berdin and Gründl (2015).

[6] In April 2017 EIOPA announced a change in the UFR from 4.2% to 4.05% to be applied from 2018 on. However, a downward change of the UFR will not substantially impact our results since we focus an on interest rate rise, that is likely to be accompanied by an upward adjustment of the UFR.

[7] For example, German life insurers are legally obliged to distribute at least 90% of their investment surplus to policyholders. In our model, we stay with this minimum level of profit participation.

complicated by the fact that future levels of profit participation depend on the realization of asset returns as well as the balance sheet and portfolio dynamics of the insurance company. We assume that the insurer does not know the distributional characteristics we impose in our model. As in practice, the insurer needs to rely on its own estimation of the future evolution of its balance sheet. For this purpose, we assume that the insurer extrapolates the past realizations of $r^h_{S,t+s}$ according to the following model:

$$\hat{r}^h_{S,t+s} = \beta_{t,0} + \beta_{t,1} f(s). \tag{4}$$

For estimating and predicting the profit participation, the insurer relies on the past 10 years. By changing the function $f(\cdot)$ the insurer is able to control the degree of conservatism in the prediction. The larger $|f'|$, the more severe are predicted changes in the profit participation. Since it seems unreasonable to predict severe changes for years that are far ahead, we require that $f'(x) \to 0$ for $x \to \infty$ and assume that $f(x) = \log(x)$.

Solvency II requires an additional risk margin for the market consistent value of life insurance liabilities to account for potential costs of running off the insurance business in case of defaults (European Insurance and Occupational Pensions Authority (EIOPA) (2014b)). In the model we assume the risk margin to be a deterministic share of best estimates equal to 1.83%. This value corresponds to the average level of the risk margin in Europe according to the European Insurance and Occupational Pensions Authority (EIOPA) (2011, p. 52).

2.3 Solvency Situation

The stylized insurer's solvency situation is based on the market consistent value of assets and liabilities. This is consistent with the European solvency regime, Solvency II. Own funds are given as the difference between the market consistent value of assets and liabilities.

The insurer's solvency capital requirement (SCR) is calculated by employing the Solvency II standard formula as described by the European Insurance and Occupational Pensions Authority (EIOPA) (2014b).[8] It is calibrated in order to correspond to the capital an insurer would need to hold to limit its default probability over the next year to 0.5%, i.e. its Value-at-Risk at the 99.5%

[8] Eling and Holzmüller (2008) discuss similarities and differences between Solvency II, the U.S. RBC standard as well as insurance regulation in New Zealand and Switzerland.

confidence level. In accordance with the bottom-up approach of Solvency II, the overall capital requirement is calculated by aggregating the capital requirement for individual risks by

$$SCR = \sqrt{\sum_i SCR_i^2 + 2\sum_{i \neq j} \rho_{i,j} SCR_i SCR_j}, \tag{5}$$

where $\rho_{i,j}$ is the correlation between risks i and j. SCR_i corresponds to the solvency capital requirement in module i, that equals the solvency capital requirement of its submodules as aggregated in the same way. In this study we include the market risk module with sub-modules for interest rate, equity, property and spread risk, and the life risk module with the sub-module for lapse risk. Market risk is the largest risk for European life insurers followed by life risks (European Insurance and Occupational Pensions Authority (EIOPA) (2011)) since they are heavily exposed to investment risk. We include the lapse risk submodule in order to assess the effect of changing lapse rates on the composition and size of the overall solvency capital requirement as well as on the insurer's solvency situation.

The solvency capital requirement for lapse risk is determined as the maximum of a capital requirement for a downward lapse rate shock, $Lapse_{down}$, for an upward lapse rate shock, $Lapse_{up}$, and a mass lapse event, $Lapse_{mass}$. A downward lapse rate shock adversely affects an insurer's solvency if market consistent values of liabilities exceed recovery value in case of lapses. In this case, lapses increase the insurer's own funds by exchanging the large market consistent value of liabilities with the small recovery value, and thus are beneficial for its solvency. Therefore, $Lapse_{down}$ is applicable only for such contracts without a positive lapse strain (Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) (2009)).[9] For these contracts, the capital requirement is given by the change in own funds if lapse rates permanently decrease to 50% of the currently assumed lapse rate to calculate the insurance liabilities. Similarly, $Lapse_{up}$ is the change in own funds if lapse rates permanently increase by 50% of the currently assumed lapse rate for contracts with a positive lapse strain. Finally, $Lapse_{mass}$ is the change in own funds if lapse rates permanently increase to 40% for contracts with a positive lapse strain.

In our model we determine the level of initial own funds such that a predetermined initial

[9]The lapse (or surrender) strain is defined as the difference between the recovery value and the current best estimate of liabilities (excluding the risk margin).

solvency ratio is fulfilled. The initial solvency ratio is set to 120%, which roughly corresponds to the median solvency ratio without transitional measures of German life insurers on March 31, 2016 (German Federal Financial Supervisory Authority (BaFin) (2016)).[10]

The insurer pays out dividends only if its free cash flow is positive. If this pre-condition is fulfilled the insurer pays out the minimum of the free cash flow and the maximum possible amount to maintain a solvency ratio of 100%. Dividend policies that depend on the solvency ratio and free cash flow are very common for European insurance companies.[11]

2.4 Lapse Risk

To illustrate the rationale for lapsing an endowment life insurance contract, consider a simple endowment life insurance contract purchased at time $t = 0$ that pays the gross return e^{Tr_G} at time T, where r_G is the annual guaranteed rate. The present value of this contract at time t equals $e^{Tr_G-(T-t)r_f}$, where r_f is the risk free rate. We assume that a policyholder has the opportunity to lapse this contract, receive the recovery value ϑe^{tr_G} at time t, and invest it into a risk free asset. The present value of this investment opportunity is equal to the recovery value ϑe^{tr_G}, where $1 - \vartheta \in (0, 1)$ is a lapse penalty.

In this setting, a risk neutral investor lapses her insurance contract if the present value upon lapsation is larger than the present value of the contract, i.e.

$$\vartheta e^{tr_G} > e^{Tr_G-(T-t)r_f}, \tag{6}$$

which is equivalent to

$$\frac{-\log \vartheta}{T-t} < r_f - r_G. \tag{7}$$

Thus, for a sufficiently large difference between risk free and guaranteed rate (i.e. a small excess guaranteed rate), lapsing the contract is optimal. A smaller contract age, t, increases the minimum

[10]The standard formula allows for several transitional measures in order to ease the transition from the previous regulatory regime, Solvency I, to Solvency II. These measures would blur our results and the immediate risks we identify in this study. Therefore, we do not include them in our model.

[11]For example, Allianz SE pays out 50% of the net income only if it can maintain a solvency ratio above 160% (`https://www.allianz.com/en/investor_relations/share/dividend`).

risk free (maximum guaranteed) rate such that lapsing is optimal. This results from the lapse penalty that reduces the recovery value. The absolute cost of lapsing ($(1-\vartheta)e^{tr_G}$) are larger for older contracts, which decreases the present value of lapsation relative to the value of the contract.

To summarize, we find that policyholders have a larger incentive to lapse an endowment life insurance contract when interest rates increase relative to guaranteed rates or contracts grow older. The results from this simple model are in accordance with various empirical studies (for example see Dar and Dodds (1989), Kiesenbauer (2012), or Eling and Kiesenbauer (2014)).[12]

We embed these stylized facts into a model for the lapse rate, that reflects a policyholder's individual likelihood to lapse her insurance contract. Clearly, there are numerous other factors that influence policyholder lapse behavior.[13] We account for these 1) by modeling the probability of lapsation instead of a binary decision to lapse or not to lapse, and 2) by including a policyholder individual fixed effect c that varies across policyholders. The policyholder fixed effect accounts for different policyholder specific factors that influence lapse risk, as for example her financial or family situation. Due to the absence of data about lapse penalties, we do not include it in our lapse rate model. Instead, we assume a lapse penalty of $1-\vartheta = 0.01$ in our baseline calibration and conduct a sensitivity analysis towards this parameter.[14]

For every cohort of contracts, we assume that lapse rates exponentially depend on the difference between the cohort's guaranteed rate and the current market's risk-free spot rate for the same maturity, given by the excess guaranteed rate $\Delta r_t^h = r_G^h - r_f(t)$ (Dar and Dodds (1989), Eling and Kiesenbauer (2014)).

Note that the decision to lapse is not affected by the expected distribution of the insurer's surpluses to policyholders due to three main reasons: First, due to the long duration of the insurer's asset investments, the insurers return on assets adjusts very slowly to market conditions. Thus, policyholders can profit more from rising interest rates when investing immediately into risk free bonds instead of waiting until the profit participation adjusts. Second, the level of profit participa-

[12] In the example above, contract age is positively related to a lapse decision if interest rates are larger than the guaranteed rate, $r_G < r_f$. However, the mentioned studies consistently find a negative relation between lapse rates and contract age. Possible reasons go beyond a present value perspective and include the counseling and service quality of insurers. A large lapse rate for relatively young contracts might result from policyholders realizing that they actually do not need the contract or they cannot afford premium payments, inappropriate advise of a salesperson, or liquidity needs in relatively young ages of policyholders that usually coincide with young policy ages.

[13] For an overview see Eling and Kochanski (2013).

[14] Note that the lapse penalty is a direct subsidy to the insurer's equity, since we do not include any costs in the model.

tion is generally smaller than market rates when interest rates are rising, since insurers distribute only part of their profit. Finally, due to lacking financial literary and the complexity of insurance products and companies, policyholders might not be able to actually infer the distribution of future surpluses. Predicting future levels of profit participation is particularly complicated as it depends not exclusively on market conditions and investment behavior but also on the evolution of the insurer's full balance sheet and managerial decisions.

In line with intuition developed above, we include the current contract age, ΔT_t^h, as an important factor for the lapse rate. Policyholders' lapse behavior is also affected by age, income, financial literary or financial advice (Kim (2005), Kiesenbauer (2012), Nolte and Schneider (2017)). To also account for these additional factors, we include a policyholder-specific risk factor that is normally distributed across policyholders, $c \sim \mathcal{N}(\mu_c, \sigma_c^2)$.

The model the policyholder-specific lapse rate at the beginning of period t as

$$\lambda_t^h(\Delta r_t^h, \Delta T_t^h) = a + e^{c - e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}. \tag{8}$$

The log-log-form of the lapse rate is also used by Kim (2005). It allows us to compute a closed-form for the distribution of average lapse rates, which simplifies its calibration (see Appendix C and below). Moreover, it yields a natural lower and upper bound for the resulting lapse rate.

Historical lapse rates suggest that lapse rates are strictly larger than zero (Geneva Association (2012)). Hence, we set the minimum lapse rate a to 1%. The detailed calibration of the model is described in Appendix C. In the following we provide a brief overview.

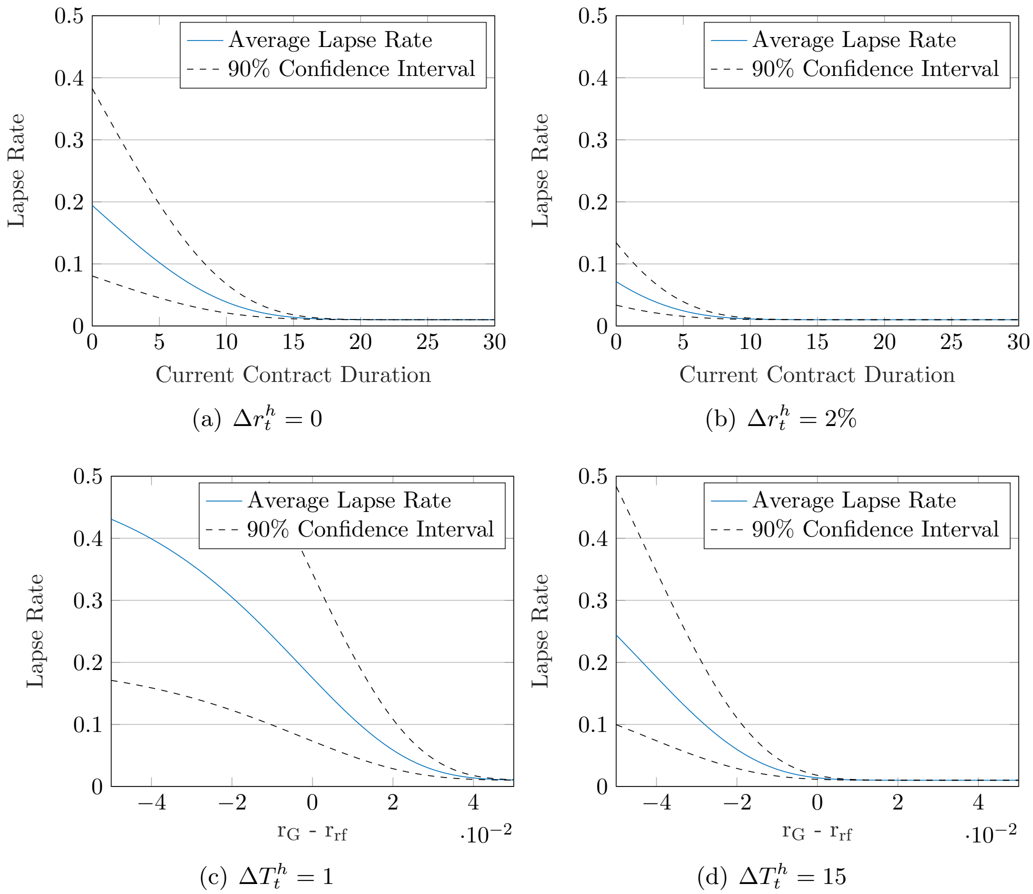
We calibrate lapse rates based on historical average lapse rates and newly sold contracts for endowment life policies in Germany as reported by the German Insurance Association (GDV) (2016). In line with observations on the German market, we assume that the guaranteed rate of each cohort equals the maximum technical rate in Germany, which is publicly observable.[15]

By aggregating the average lapse rate of each cohort, we yield the average lapse rate across cohorts implied by our model. The parameters of our model are calibrated in order to match the average lapse rate across cohorts in our model with the average lapse rate across cohorts reported by the German Insurance Association (GDV). The resulting parameters are $a = 1\%$, $\mu_c = -0.8199$,

[15] The technical rate is determined by the German regulator and called *Höchstrechnungszins*. More details on the technical rate are discussed in Section 2.2.

$\sigma_c = 0.5062$, $d_1 = 0.3707$, and $d_2 = 0.1053$.

Figure 1 depicts the average lapse risk across one cohort for different differences between guaranteed and market rate as well as different current contract ages. In comparison with historically observed lapse rates in Germany as well as other countries, the lapse rates implied by our model seem very reasonable (Tsai et al. (2002), Kuo et al. (2003), Geneva Association (2012), Eling and Kiesenbauer (2014)).



(a) $\Delta r_t^h = 0$ (b) $\Delta r_t^h = 2\%$

(c) $\Delta T_t^h = 1$ (d) $\Delta T_t^h = 15$

Figure 1: Lapse rate calibration: average and 90% confidence intervals for different levels of the difference between guaranteed and market rate and different levels of current contract age.

3 Results

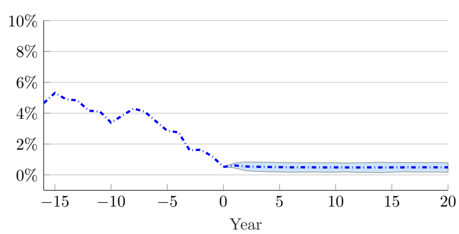
3.1 Interest Rate and Lapse Behavior

3.1.1 Interest Rate Environments

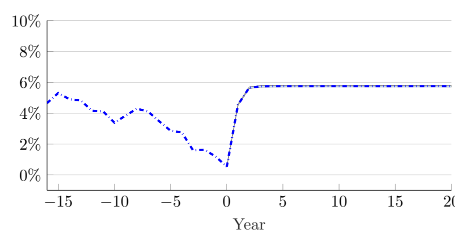
We calibrate three different interest rate environments. The first environment is calibrated to match the yield curve of risk-free interest rates (AAA German sovereign bonds) as per end 2015. Average interest rates fluctuate around this level according to a Vasicek model.

In the second and third interest rate environment interest rates increase. For the second environment we model a sharp increase in interest rates. In the first two years the risk-free rate with a maturity of 10 years (10 year risk-free rate in the following) rises from approximately 1% to 6%. From the following years on average interest rates are constant and volatility is very small. Such a shock is not unrealistic, considering that, for example, the key interest rate in Japan (given by the Bank of Japan) increased from 2.5% to 6% between 1988 and 1990.

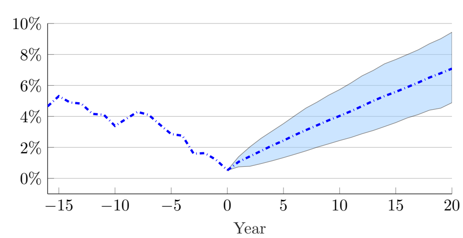
The third interest rate environment displays a gradual increase in interest rates starting with approximately 1% in $t = 0$ and increasing on average by approximately 0.3 percentage points each year. Figure 2 depicts the evolution of the 10 year risk-free rate.



(a) Environment (1)



(b) Environment (2)



(c) Environment (3)

Figure 2: Median risk-free interest rate with 10 years maturity from year 0 on. Subsequent to year 0 the German sovereign bond yield with 10 years maturity from 1999 to 2015 is shown, where the value at time 0 corresponds to the value in 2015.

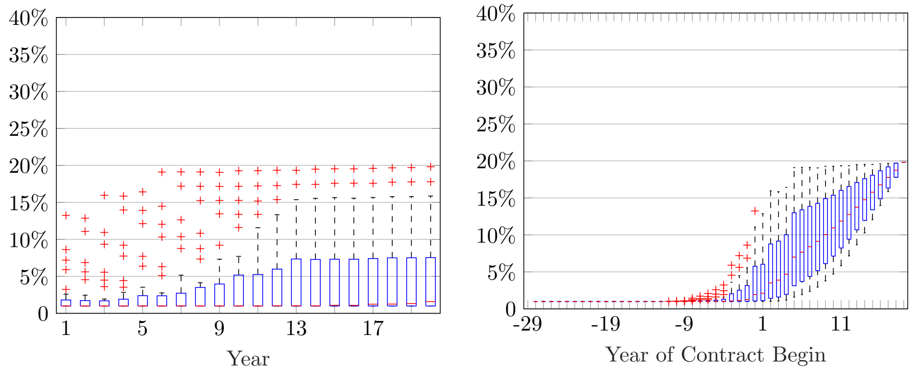
3.1.2 Lapse Behavior

For each interest rate environment we simulate the evolution of the life insurer's balance with two different specifications. In the first specification lapse rates stay constant at the average lapse rate in 2015, which was 2.86%. Thus, each policyholder lapses her policy individually with a probability of 2.86%. In the second specification lapse rates are interest rate sensitive as described in Section 2.4. Thus, it is more likely for each policyholder to lapse her policy if the policy duration is shorter and if the gap between the 10 year risk-free rate and the individual guaranteed rate increases.

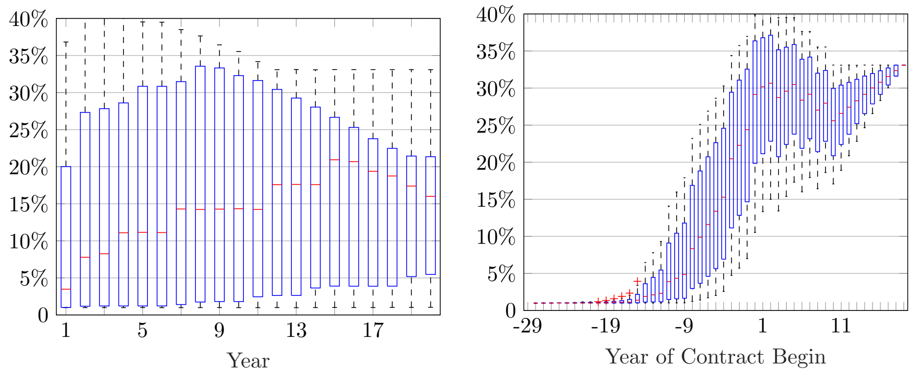
Figure 3 depicts the distribution of lapse rates over time and over cohorts of contracts in the second specification. On the left hand side, each boxplot refers to one year in the evolution of the model and displays the distribution (in particular the lower quartile, median, and upper quartile) of lapse rates during this year. In the first environment the guaranteed rate of different cohorts as well as the risk-free rates stabilize over time and, thus, lapse rates depend on contract age exclusively. The outliers in the boxplot show that cohorts that just purchased a contract lapse this contract with up to 20% in the next few years. In contrast, half of the cohorts (with a longer contract age) lapse with less than 2% probability.

On the right hand side, Figure 3 seems to indicate that there is an upward trend in the lapse rates per cohort. Cohorts that purchased a contract later in time have a larger median probability to lapse during the observed lifetime. However, this effect is mainly driven by the fact that for most cohorts we observe different lifetimes. For example, for a contract purchased in the beginning of $t = 1$ we observe the evolution of lapse rates until a contract age of 20 years at the end of $t = 20$, for a contract purchased in $t = 2$ we observe 19 years, etc. Since policyholders a more likely to lapse in the first years of their lifetime, later cohorts display larger observed levels of lapse rates than earlier cohorts. Thus, the right hand side is less informative for an individual interest rate environment but, instead, highly informative when comparing it between different interest rate environments.

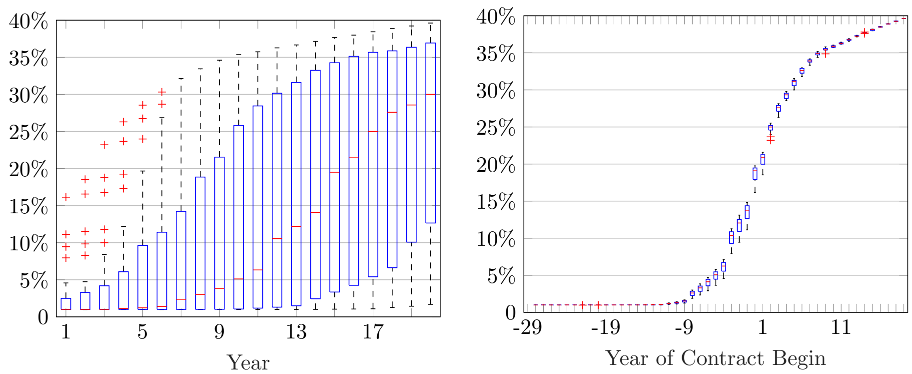
In Environment (2) lapse rates per cohort are sharply increasing. The right hand side of Figure 3 shows that every cohort is more likely to lapse than in the first environment. Lapse rates increase up to contracts purchased in $t = 1$, then slightly decrease and increase again. This behavior is a result from the sudden interest rate shock in 2016. Contracts that are in place in 2016 face a sharp



(a) Environment (1)



(b) Environment (2)



(c) Environment (3)

Figure 3: Distribution of lapse rates across cohorts at each point in time (left side) and across time for each cohort (right side) for interest rate sensitive lapse rates. Each box consists of lower, median, and upper quartile, crossed points are outliers.

increase in lapse rates since market rates suddenly increase far above the guaranteed rates of these contracts. Since it takes time until guaranteed rates of newly sold contracts adjust to market rates, lapse rates stay at similar levels for the following cohorts. The slight upward trend for the last purchased cohorts can be contributed to the effect of observing just the first few years for these contracts, as explained above.

When comparing the left hand side of Figure 3 for Environments (1) and (2), we find a sharp increase in the variation of lapse rates. Contracts with a large contract age are not as sensitive towards the increase in interest rates in 2016 as contracts with a small contract age. Therefore, lapse rates for newly purchased contracts rise up to 40% while lapse rates for very old contracts (more than 50% of the contract portfolio) stay below 5%. In the following years the median lapse rate of the insurer still increases, although interest rates stay constant. This results from two effects: Firstly, guaranteed rates of newly sold contracts adjust very slowly to the new level of interest rates. Thus, lapse rates for newly sold contracts are still very high. Secondly, old contracts with large guaranteed rates mature and are replaced with contracts with smaller guaranteed rate, which are more likely to lapse.

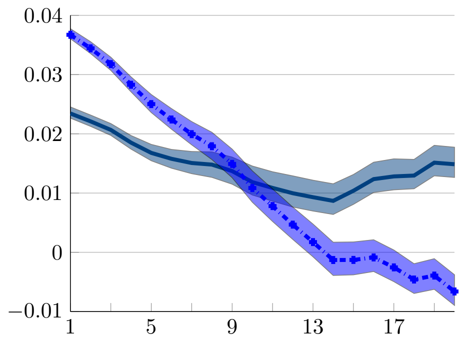
The gradual increase in interest rates in Environment (3) results in a very different evolution of lapse rates. In this environment, the effect of the increase in market rates relative to guaranteed rates offsets the effects of the increase in contract age with respect to lapse rates. Consequently, the lapse rate for each cohort is relatively stable over time. However, there is a large increase in the lapse rate across cohorts: Contracts that were sold later are associated with a larger lapse rate. A part of this result can again be explained by different observed lifetimes, as described above. The other part is explained by a growing difference between the risk-free rate and the guaranteed rate of newly sold products.

3.2 Balance Sheet Variables

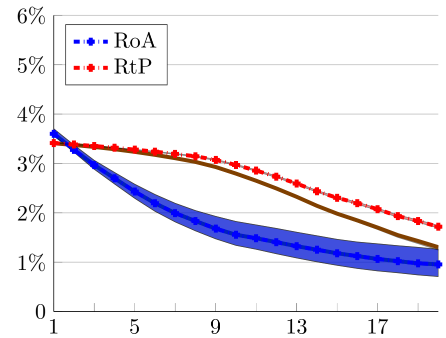
3.2.1 Environment (1)

Figure 4 (a) depicts the development of the free cash flow (FCF) under a constant lapse rate and under stochastic lapses. Under a protracted period of low rates, guarantees gradually converge to market rates and, thus, lapse rates stabilize. However, since old contracts with larger guarantees mature, lapse rates slightly increase and, thus, cash outflows increase under interest rate sensitive lapse rates.

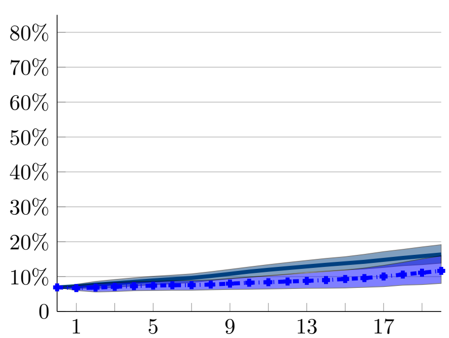
Figure 4 (b) shows the evolution of both the return on assets and the return granted to pol-



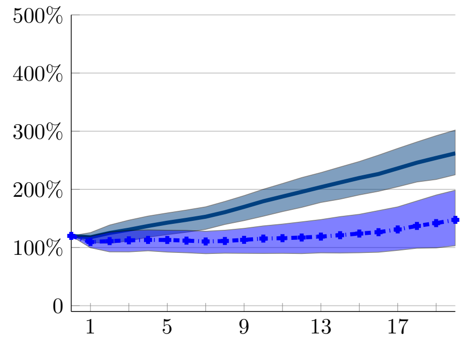
(a) Free Cash Flow relative to the initial Book Value of Assets at $t = 0$



(b) Return on Assets and Return to Policyholders



(c) Own Funds relative to the Market Value of Assets



(d) Solvency Ratio

Figure 4: Environment (1) In the first specification (straight line) lapse rates are constant to 2.86%. The crossed line depicts the second specification with interest rate sensitive lapse rates. The median and 90% confidence interval are reported.

icyholders.[16] Under interest rate-sensitive lapses we can observe that the liability portfolio is substantially more expensive due to the fact that under a protracted period of low rates lapses tend to occur to those contracts which have a relatively low guarantee since older and higher guarantees offer a much better return compared to market rates.[17] In turn, this increases the average guarantee in the liability portfolio across a large number of the simulated paths and it mirrors the much worse FCF dynamics observable in figure (a) under stochastic lapses.

Finally, figure 4 (c) and (d) depict both the evolution of the Own Funds and of the Solvency Ratio. The insurer's solvency ratio is the ratio of the insurer's own funds and solvency capital requirement (SCR). The latter aggregates the capital requirement for lapse risk as well as market risk (including interest rate, equity, property, and spread risk). The Solvency Ratio in particular,

[16]Note that the Return granted to policyholders does not include recovery values.

[17]In other words, this is equivalent to say that the put options that policyholders hold vis-á-vis shareholders are deeply in the money.

clearly reflects the underlying dynamics between cash flows and lapses: in fact, with sensitive lapses we can observe a lower solvency ratio over time as a result of a more expensive liability portfolio, that coupled with low achievable returns on financial markets, substantially erodes the solvency position of the insurer over time.

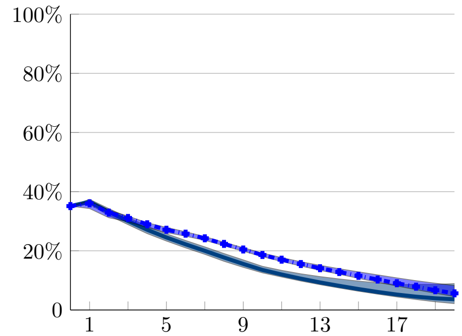


Figure 5: Environment (1) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

This increase in the solvency ratio is mainly driven by a reduction in the required solvency capital requirement for lapse risk over time, as depicted in Figure 5. This decrease is mainly driven by old contracts with large guarantees maturing and, thus, decreasing the exposure of the insurance company to the sudden lapse of these policies.

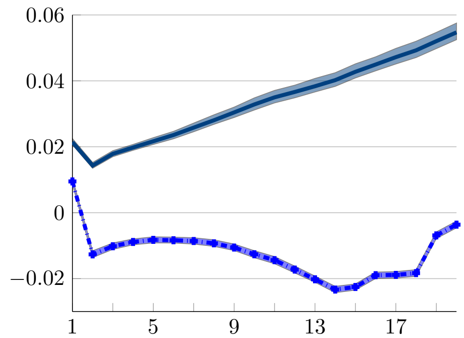
3.2.2 Environment (2)

Figure 6 (a) depicts the evolution of the insurer's free cash flow. Clearly, cash outflows exceed cash inflows substantially if lapses are interest rate sensitive in contrast to a constant lapse rate. The sudden rise in interest rates in this environment increases lapse rates especially for those cohorts of contracts with relatively low guarantees and long remaining durations. These are the contracts that lose most value due to the existence of higher return opportunities in the market. Thus, as lapses suddenly increase, the insurer has to serve extraordinary recovery payments, and cash outflows also increase beyond cash inflows. This jeopardizes the liquidity position of the insurer.

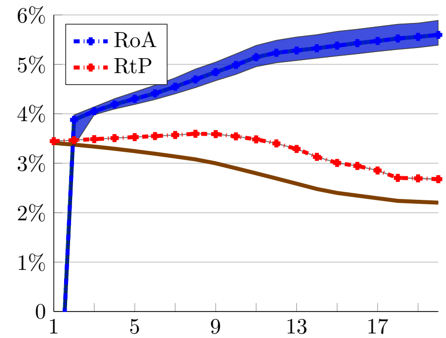
In Figure 6 (b) we observe that a sudden rise in interest rates is accompanied by a substantial drop in the insurer's return on assets[18]. This results from enormous depreciations on bonds in particular, that result from rising interest rates. As we move further in time, the return on assets

[18]The median return on assets is -4.5% in period 1 in both specifications.

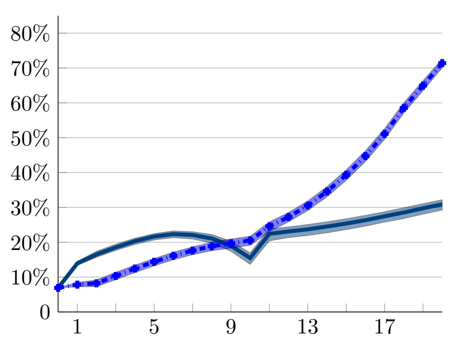
gradually increases, particularly due to increasing coupon payments from newly bought bonds. The sudden rise in interest rates is accompanied by an increase in lapse rates. The increase in interest rate sensitive lapses compared to constant lapses causes the average annual return granted to policyholders to be much higher. This results again from contracts with smaller guarantees being more likely to lapse and contracts with larger guarantees staying in the insurer's contract portfolio. Consequently, this effect increases the cost of financing guarantees for the insurer.



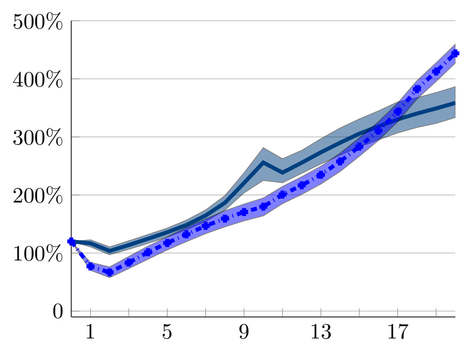
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).



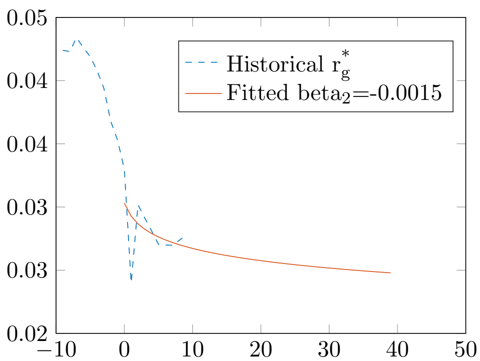
(c) Own Funds relative to the Market Value of Assets



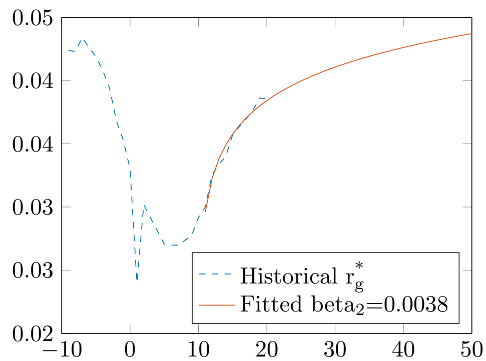
(d) Solvency Ratio

Figure 6: Environment (2) In the first specification (straight line) lapse rates are constant to 2.86%. The crossed line depicts the second specification with interest rate sensitive lapse rates. The median and 90% confidence interval are reported. The median return on assets is -4.5% in period 1 in both specifications.

Finally, Figures 6 (c) and (d) depict the evolution of the insurer's own funds and Solvency Ratio, respectively. As life insurer liabilities usually display a larger duration than its assets, an increase in the risk-free rate is accompanied by valuation benefits for the insurer's own funds, which can be seen in the specification with constant lapse rates. Interestingly, the own funds show a downward peak in year 10 of the simulation. This peak results from a change in the predicted profit participation. As described in Section 2.2, the insurer predicts the future distribution of surplus to policyholders by extrapolating this value based on the last 10 years. In the first years of Environment (2), interest rates experience a large upward shock but stabilize afterwards. The asset return of the insurer only gradually adjusts to the new level of interest rates and, consequently, the level of profit participation adjusts even slower. As a result, predicted levels of profit participation which contribute to the market consistent value of liabilities are still very low for the first years. They gradually adjust upwards until they change in shape from decreasing to increasing in year 10. We show this behavior in Figure 7. Consequently, liabilities increase until $t = 10$. This finding highlights the enormous sensitivity of market-consistent valuation towards an insurer's methodology to compute market-consistent values for the profit participation of life insurance contracts.



(a) In year 9.



(b) In year 20.

Figure 7: Environment (2) Observed and predicted profit participation which contribute to the market-consistent value of life insurance liabilities.

This sensitivity is reduced by large lapse rates of contracts with small guarantees under interest rate sensitive lapses. Such lapse behavior increases the average guaranteed rate in the insurer's contract portfolio and, hence, the gap between average guaranteed rate and surpluses distributed to policyholders. Nonetheless, in the first years following an upward interest rate shock own funds are smaller for interest rate sensitive lapse rates. The intuition is that, with larger interest rates, lapsing contracts display larger guarantees and, thus, book values for these contracts (discounted with the guaranteed rate) are larger than market-consistent values. In our model, recovery values for lapsing contracts equal book values.[19] Therefore, the insurer is paying part of the recovery

[19] This is consistent with current European legislation. In fact, insurers are able to subtract only occurred expenses from book values when paying recovery values. Since we assume zero expenses (and no upfront loading on premiums), our assumptions are in line with current practices.

values by using own funds. However, guarantees adjust upwards until book values are in fact smaller than market values. In this case, the insurer records a surplus in terms of own funds for each lapsing contract. Consequently, own funds are larger in the long run for interest rate sensitive lapses in comparison to those with constant lapse rate.

In contrast to increasing own funds, the Solvency Ratio (SR) is decreasing for the first years following a positive interest rate shock. This is due to an increase in the SCR. As Figure 8 indicates, the SCR mainly increases due to an increase in the capital requirement for lapse risk when lapse rates are constant but not for interest rate sensitive lapse rates. This results again from market-consistent contract values dropping below recovery values (i.e. book values) when lapses are constant. The resulting gap is accounted for by an increasing capital requirement. In contrast, interest rate sensitive lapses increase the average guarantee of contracts and, thus, recovery values are larger and the capital requirement for lapse risk is smaller.

As the market-consistent value of liabilities increases in the first 10 years, the market consistent value of liabilities converges to the recovery value for potential lapses. Therefore, less capital is needed to account for an increase in lapse rates, which is shown in Figure 8. This decreases the solvency capital requirement and increases the solvency ratio. With increasing surpluses recovery values drop far below market consistent life insurance liabilities from year 10 on. This increases the risk of a sudden drop in lapse rates, that would be accompanied by a sudden rise in liabilities. To account for this risk, Solvency II requires additional capital, which is illustrated by the increase in capital requirements for life risks in year 11 in Figure 8.

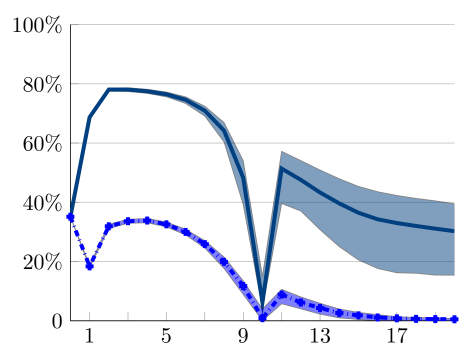


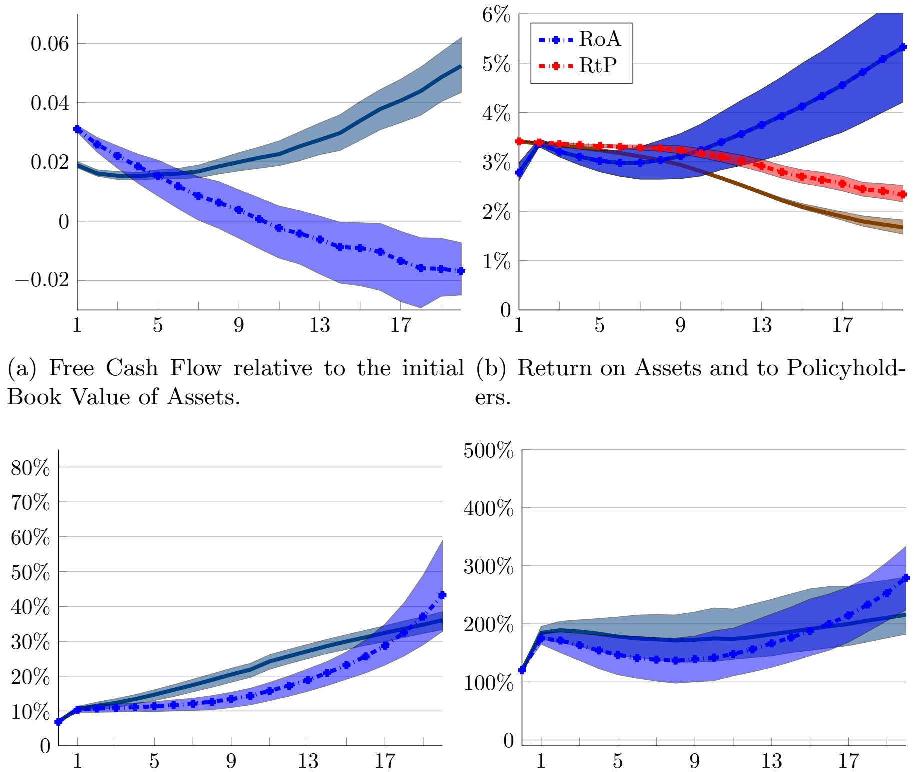
Figure 8: Environment (2) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

3.2.3 Environment (3)

Under Environment (3), the evolution of the free cash flow as depicted in Figure 9 (a) is similar to the evolution of the free cash flow under Environment (2). Not surprisingly, rising interest rates generate higher cash outflows if policyholders react to new market conditions by lapsing those policies with relatively lower guaranteed interest rates. Under Environment (3) we can observe how a slow but steady increase in interest rates drives up the probability of lapsing over time, as depicted in Figure 3. Hence, cash outflows increase slowly over time. It is interesting to observe in Figure 9 (a) that the (median) liquidity position of the life insurer under interest rate sensitive lapses becomes worse than the position of the life insurer facing a constant 2.68% lapse rate when interest rates tend toward 3% in year 5.

Different balance sheet items of the life insurer adjust with different speed to rising interest rates. In Figure 9 (b) we can observe the insurer's return on assets increasing above the average return granted to policyholders beyond the ninth year of the simulation. The increasing trend in interest rates under Environment (3) slowly increases the return of the insurer's reinvested assets and, thereby, pushes up the total return of the asset portfolio. The development of the return granted to policyholders is consistent with the observations in Environment (2): When lapse rates increase with interest rates, we tend to observe higher lapse rates for relatively smaller guarantees, which in turn increases the average in-force guaranteed interest rate compared to the case with constant lapses. Clearly, the absolute size of the liability portfolio changes substantially, with constant lapses implying a liability portfolio much greater in volume. Interestingly, the return for policyholders does neither with constant nor with interest rate lapse rates increase in this environment. In contrast, the rising return on assets is not accompanied by a rising return for policyholders. This illustrates the extraordinary slow speed of guarantees adjusting to changes in interest rates, that cannot be compensated by surpluses distributed to policyholders.

Finally, in Figure 9 (c) and (d) we can observe the evolution of the insurer's own funds and Solvency Ratio. When lapses are interest rate sensitive, the insurer's solvency situation is slightly worse than with constant lapse rates. This again results from smaller own funds, that suffer from large cash outflows, and increasing guarantees in-force.



(a) Free Cash Flow relative to the initial Book Value of Assets.

(b) Return on Assets and to Policyholders.

(c) Own Funds relative to the Market Value of Assets.

(d) Solvency Ratio.

Figure 9: Environment (3). In the first specification (straight line) lapse rates are constant to 2.86%. The crossed line depicts the second specification with interest rate sensitive lapse rates. The median and 90% confidence interval are reported.

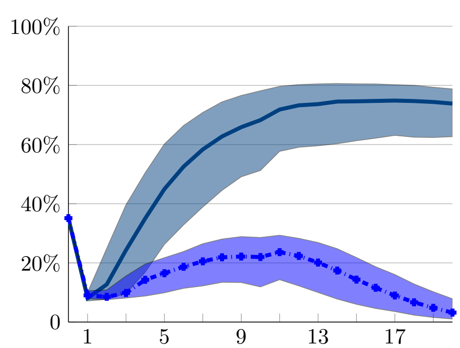


Figure 10: Environment (3) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

4 Sensitivity Analysis

Figures for all sensitivity analyses can be found in Appendix A.

4.1 Demand

In our baseline analysis we assume that each year a constant number of new policyholders n_{new} purchases new insurance contracts. However, similar to lapse rates, demand for endowment life contracts is likely to decrease when risk-free rates increase beyond guaranteed rates. In this case, the liability portfolio of the insurance company changes: Less new contracts enter the portfolio and, thus, the average guarantee is potentially larger. Nevertheless, by allowing for lapsation, our baseline calibration already accounts for potential, although time lagged, demand adjustments.

We examine the impact of a changing demand by employing our empirically calibrated lapse function, as $\lambda_t^h(\Delta r_t^h, 0)$ is the probability that a policyholder lapses her contract immediately upon purchase. As before, we assume that n_{new} potentially new policyholders enter the market. For each potential new policyholders $1, ..., n_{new}$, we compute the individual probability of purchase, which is $1 - \lambda_t^h(\Delta r_t^h, 0)$. Then, the expected demand at time t (for cohort t) equals $n_{new}\left(1 - \mathbb{E}[\lambda_t^t(\Delta r_t^t, 0)]\right)$.

Figure 11 shows the evolution of demand over time. Similar to lapse rates, in Environment (1) median demand slightly declines and then stays at a constant level since average guarantees and average interest rates converge. In contrast, the interest rate shock in Environment (2) substantially reduces demand that does not recover due to the slow adjustment of guarantees to increased interest rates. The gradual increase in interest rates in Environment (3) is accompanied by a gradual decrease in demand since the gap between average guarantee and interest rate increases.

Although demand in this specification is substantially changing over time, Figures 12 to 14 show that a changing demand has a negligible impact on our baseline results. This is mainly due to the implicit demand function as given by lapse rates in the baseline calibration.

4.2 Lapse Penalty

To assess the sensitivity of our results with respect to the lapse penalty, we set the recovery value in case of lapsation to $\vartheta = 85\%$ of the current accumulated funds, which corresponds to a lapse penalty of 15%.[20] In our model, this parameter does not affect lapse rates, since the lapse rate model is empirically based on the actual lapse penalty in the German market, that we are not able to observe.

Penalty costs enter our model via a reduction in recovery values upon lapses. Thus we expect a reduction in the insurer's cash outflows and an increase in the insurer's own funds. Moreover, a lapse penalty also reduces the expected future cash flows to policyholders and, thereby, the book and market value of liabilities. This effect is also likely to decrease the solvency capital requirement. In summary, we expect an improvement in the insurer's liquidity and solvency condition in comparison to our baseline calibration.

Figure 15 depicts central balance sheet variables of the insurer when interest rates stay at low levels in Environment (1). As expected, the insurer's own funds and solvency ratio increase with the lapse penalty. In contrast, we do not identify a noteworthy change in the insurer's liquidity condition. In Environment (1) the insurer's liquidity situation is mainly driven by a deteriorating return on assets and the gap between the return on assets and to policyholders. Thus the main effect of a larger lapse penalty on an insurer's balance sheet is a reduction in the value of liabilities, that is related to a slight increase in the ratio of own funds to total assets and the solvency ratio.

With low interest rates the market-consistent value of the contracts is substantially larger than the accumulated funds. Thus, in Environment (1) the main lapse risk is that less policyholders than expected lapse their insurance contracts. If the lapse penalty increases, the value of liabilities decreases. Hence, an increase in the value of liabilities due to an unexpected decrease in lapse rates is positively related to the lapse penalty. This effect increases the sensitivity of the insurer's solvency situation towards the lapse rate. The relative solvency capital requirement for lapse risk increases accordingly, as Figure 15 shows.

With a sharp rise of interest rates in Environment (2) the liquidity and solvency situation is substantially driven by unexpectedly large recovery values accompanied by a decrease in the market consistent value of liabilities. Therefore, the insurer's liquidity situation improves particularly in the first years following the interest rate shock, as Figure 16 shows. As in Environment (1) the insurer's own funds and value of liabilities decrease with a larger lapse penalty, and thus the solvency ratio

[20] The European Systemic Risk Board (2015) reports that 90% of all European life insurance contracts can be lapsed with a penalty lower than 15%.

substantially improves.

In the first years, market consistent values drop below recovery values and thus an increase in lapse rates is the main driver for lapse-related solvency capital requirements. A larger lapse penalty reduces this risk since it reduces recovery values. Thus, the relative capital requirement for lapse risk reduces with the lapse penalty. In contrast, in the last years guaranteed rates and expected profit participation adjust to the higher level of interest rates, and market consistent values increase above accumulated funds. Similar to Environment (1), a decrease in lapse rates becomes the main lapse risk and thus the lapse penalty increases the relative solvency capital requirement for lapse risk.

Figure 17 illustrates the case of a gradual increase in interest rates in Environment (3). As before, the liquidity and solvency situation improves with the larger lapse penalty. The solvency ratio increases substantially and the likelihood of a critical solvency situation becomes negligible. The improvement in the insurer's liquidity situation is smaller. Since the market consistent value of liabilities only gradually declines over time, the larger penalty increases the relative capital requirement for lapse risk in the first years where a decrease in lapse rates is the main driver for lapse-related capital requirements. With market consistent values dropping below the contract's accumulated funds over time, this effect reverses.

In conclusion, we find that the effect of a large lapse penalty mainly improves the insurer's solvency situation. In contrast, it is not able to prevent the meltdown of the insurer's liquidity. Moreover, when interest rates are small in comparison to the guaranteed return to policyholders, a lapse penalty increases the insurer's sensitivity towards lapse risk since the insurer is adversely affected particularly by a reduction in lapse rates.

4.3 Lapse Risk

4.3.1 Excess Guaranteed Rate

Referring to the lapse rate in Section 2.4, we decrease d_1 in the lapse rate in Equation (8) to assess the impact of policyholders' sensitivity to a difference between guaranteed and market rate. To yield comparable levels for the initial average lapse rates, we recalibrate the remaining parameters by setting $d_1 = 0.15$ and the baseline lapse rate to 2.68% and running our calibration algorithm without adjustments of d_1. The resulting parameters are $a = 0.01\%$, $\mu_c = -1.47$, $\sigma_c = 0.2132$, and $d_2 = 0.0661$. Nonetheless, the initial average lapse rate implied in the model equals 2.23% in contrast to 2.68%.

As Figure 18 shows, lapse rates with a smaller sensitivity towards the difference of guarantee and market rates follow a similar pattern but are substantially less volatile and smaller in Environments 1 and 2 than in our baseline calibration. Intuitively, with a smaller sensitivity d_1 the policyholder reaction to changes in interest rates is less pronounced.

Across all environments we find that the median return to policyholders is smaller. This results from policyholders with small guarantees not lapsing as likely as in the baseline calibration. As Environment (1) does not experience any interest shock, the smaller average guarantee granted to policyholders slightly increases the insurer's solvency ratio in the long run, as depicted in Figure 19, and the reduces the relative solvency capital requirement for lapse risk, as depicted in Figure 20.

Figures 21 and 22 show the evolution of the insurer's key variables in Environments (2) and (3), respectively. As lapse rates are substantially smaller, recovery values are smaller and thus the free cash flow is larger. A negative free cash flow in particular is now very unlikely, which reflects a substantially improved liquidity situation in comparison to the baseline calibration. However, the insurer still suffers a substantial drop in the free cash flow that might result in liquidity problems. The solvency situation slightly changes in the direction of the solvency situation with a constant lapse rate: In the first years own funds and the solvency ratio are slightly larger and in the last years they are smaller than under the baseline calibration. Nonetheless, the median solvency ratio in Environment (2) still drops below the critical threshold of 100% in the second year after an upward interest rate shock.

Figure 23 depicts the relative solvency capital requirement for lapse risk in Environments (2) and (3). In both environments the relative capital requirement is larger with a smaller sensitivity towards the excess guaranteed rate. This results from the small level of lapse rates: On the one hand, a decrease in lapse rates yields a unproportionally larger increase in the value of liabilities (see 2.2). On the other hand, the risk of a mass lapse scenario is larger.

4.3.2 Contract Age

To isolate the impact of contract age on lapse risk, we conduct a sensitivity analysis with $d_2 = 0$ in the lapse rate in Equation (8). To yield comparable levels of the initial average lapse rate, we recalibrate the remaining parameters by setting $d_2 = 0$ and the baseline lapse rate to 2.68% and running our calibration algorithm without adjusting the level of d_2. The resulting parameters are $a = 0.01\%$, $\mu_c = -2.8925$, $\sigma_c = 0.165$, and $d_1 = 0.1958$. Nonetheless, the initial average lapse rate implied in the model is slightly smaller.[21]

Contract age serves as a catalyst of lapse rates for relatively young contracts. Thereby it also amplifies policyholders' sensitivity towards interest rate shocks. As Figure 24 shows, lapse rates with a smaller sensitivity towards contract age follow a similar pattern but are substantially less volatile and smaller in Environments 1 and 2 than in our baseline calibration. Thus, with a smaller sensitivity d_2, the policyholder reaction to changes in interest rates is less pronounced.

Consequently, across all environments we find that the median return to policyholders is smaller. This results from policyholders with small guarantees not lapsing as likely as in the baseline calibration. As Environment (1) does not experience any interest shock, the smaller average guarantee granted to policyholders slightly increases the insurer's solvency ratio in the long run as depicted in Figure 25.

Figures 27 and 28 show that the decrease in the sensitivity towards contract age results in similar effects as a decrease in the sensitivity towards the excess guaranteed rate when interest rates increase in Environments (2) or (3). The liquidity position improves in comparison to the baseline calibration. Particularly during the first years the solvency situation is slightly improved while it is slightly worse during later years in comparison to the baseline calibration. Moreover, the relative level of the solvency capital requirement for lapse risk in Figure 29 is larger than under the baseline calibration due to the small level of interest rates as in Section 4.3.1.

In conclusion, we find that a smaller level of the lapse rate's sensitivity towards contract age or the excess guaranteed rate particularly improves the insurer's liquidity via a reduction in lapse rates. The sensitivity of the lapse rate influences the insurer's solvency to a much smaller extent. This finding indicates that an increase in lapse rates due to rising interest rates affects the

[21]The initial average lapse rate implied in the model equals 2.23% in contrast to 2.68%.

insurer's liquidity in particular while the insurer's solvency situation mainly depends on the interaction between the book and market consistent value of assets and liabilities during interest rate movements.

5 Conclusion

In this article we examine the impact of rising interest rates and accompanying lapse risk on an insurer's liquidity and solvency situation. Thereby we focus on endowment life contracts due to their particular interest rate exposure. To assess the solvency situation we calculate the market consistent value of assets and liabilities and a solvency capital requirement based on the European regulatory framework Solvency II. This balance sheet model allows us to identify the interaction between an insurer's backbook of life insurance contracts, newly sold contracts and the development of the insurer's investment portfolio.

We calibrate our model based on the situation of an average German life insurer in 2015, since the German market for endowment life insurance is particularly large. Lapse rates in our model decrease with the excess guaranteed rate, i.e. the difference between the guaranteed rate and market risk free rate, and with contract age. We calibrate the lapse rate model based on lapse rates observed in Germany during the last decade.

We find that a sudden severe upward shock in interest rates is related to a very detrimental effect on the liquidity position of life insurers. Our results suggest that an average insurance company would need approximately 20 years until cash inflows compensate the enormous cash outflows resulting from paying out recovery values and depreciations related to a sudden increase in risk-free interest rates from 1% to 6% for 10 years time to maturity. The insurer's solvency situation is endangered for the first five years following the interest rate shock as expensive life insurance contracts with large guarantees in-force stay in the insurer's contract portfolio and those with small guarantees lapse.

Gradually increasing interest rates, however, do not endanger an insurer's solvency situation to the same extent, but still have a substantial impact on its liquidity situation. Long-term investments make it difficult for insurers to profit from gradually increasing interest rates to the same extent as increasing lapse rates would require. Therefore, the free cash flow gradually decreases for a considerable amount of time.

A central driver for our results is an increase in lapse rates particularly for contracts with small guaranteed returns. Policyholders with these contracts have a large incentive to lapse as interest rates increase. Thereby, mostly contracts with large guarantees are left in the insurer's contract portfolio. These contracts, however, require a larger capital requirement and are more expensive to fund than contracts with smaller guarantees. Moreover, due to the typically long asset duration the insurer's return on assets adjusts very slowly to new interest rates. Therefore, the free cash flow as well as the insurer's solvency ratio come under pressure.

The sensitivity analysis indicates that a lapse penalty is particularly able to improve an insurer's solvency situation since it reduces the value of liabilities and increases own funds via smaller recovery values. An improvement in the liquidity situation can only be achieved by substantially smaller lapse rates than empirically observable.

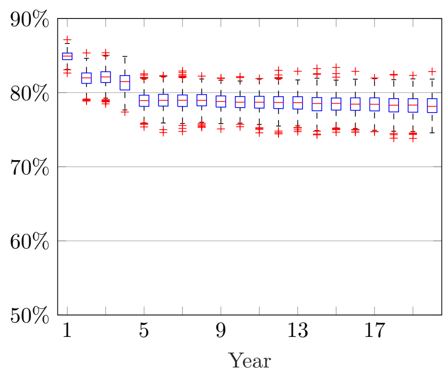
Our findings have important implications for insurance companies, policyholders, and policymakers. They indicate that all stakeholders need to prepare for negative shocks on the solvency situation of life insurers in case of rising interest rates. A gradual rise in interest rates might as well lead to a period of approximately 10 years with decreasing solvency ratios and substantial cash outflows.

As (life) insurance companies constitute a vital part of the global financial system, rising interest rates can have a substantial impact on financial stability. The liquidity problems insurers would face in case of a sudden increase in lapse rates are likely to be related to massive capital outflows that would have to be financed by selling insurers' assets. On top on these sales it is likely that insurers would reduce their activity in new investments. Such developments embed the potential for loss spirals in market (particularly bond) prices and financing problems of infrastructure projects. Similarly, insurers might reduce their financing of banks that heavily rely on debt-financing by insurance companies.

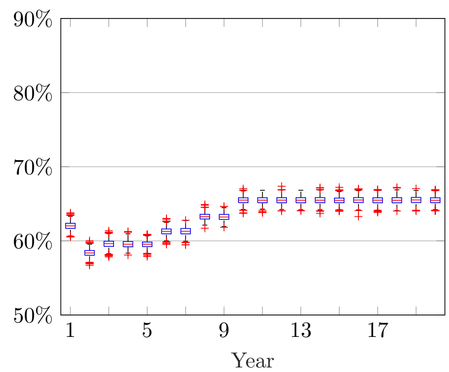
Due to the high interconnected of life insurers in the global financial system, their possible liquidity needs are likely to affect other financial institutions as well. For example, counterparty credit risk of other financial institutions towards insurers might substantially increase. Such an increase in credit risk might be accompanied by demands for additional collateral for security lending, central clearing as well as other financial securities and instruments. Calls for additional collateral would further deteriorate life insurers' liquidity position. Due to the recent low interest rate environment, it is not unlikely that insurers increasingly engaged in such alternative investments to increase investment returns. Thus liquidity needs of life insurers might go along with losses in such investments and thereby endanger financial stability.

Appendix

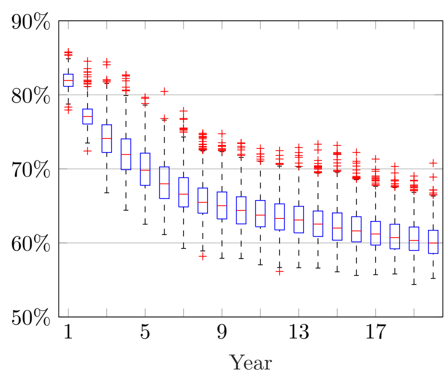
A Figures



(a) Environment (1)

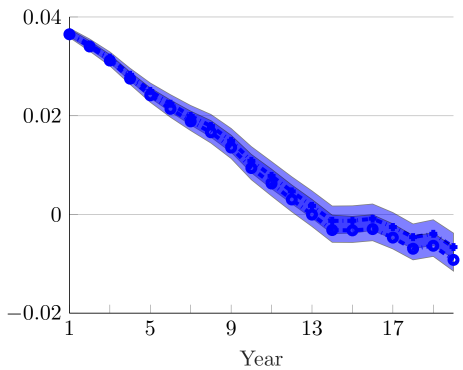


(b) Environment (2)

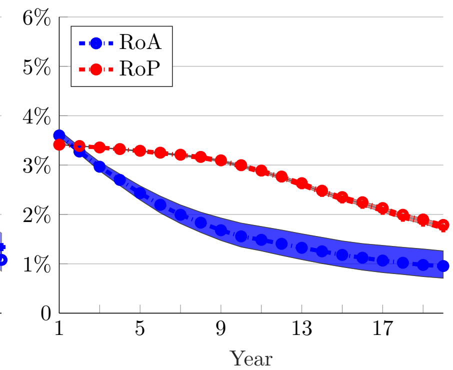


(c) Environment (3)

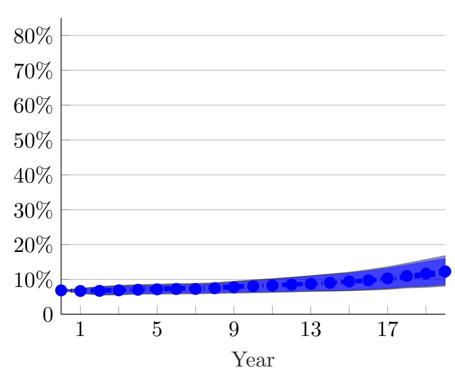
Figure 11: Demand as a fraction of maximum possible demand, where $1 - \lambda(\Delta r_t^h, 0)$ is the likelihood of each consumer to buy an insurance contract. Each boxplot depicts the distribution of new policyholders in a specific year in the model.



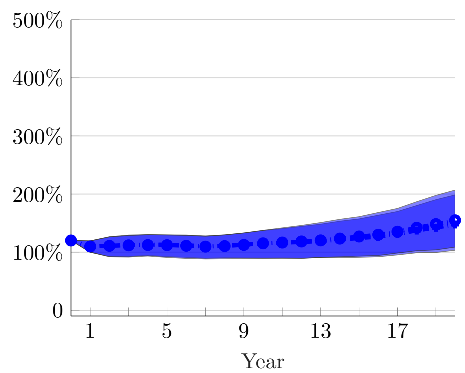
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Return on Assets (blue) and to Policyholders (red).

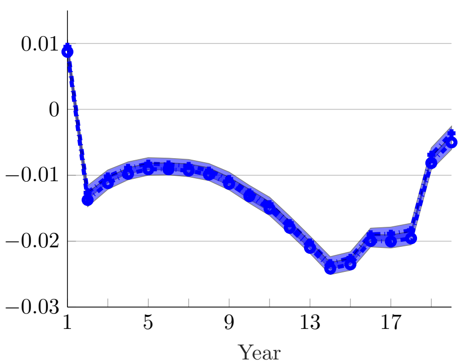


(c) Own Funds relative to the Market Value of Assets.

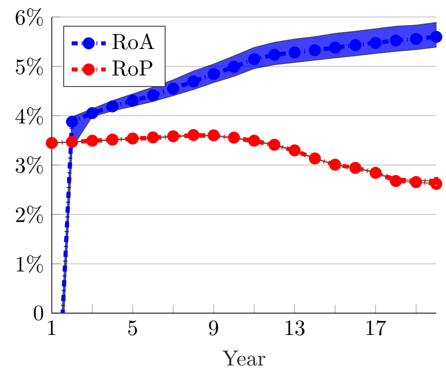


(d) Solvency Ratio.

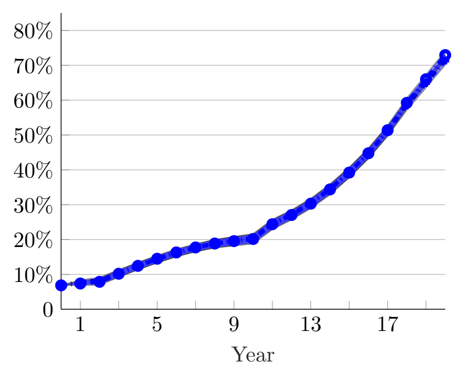
Figure 12: Environment (1) with demand function, where $1 - \lambda(\Delta r_t^h, 0)$ is the likelihood to buy an insurance contract. The crossed line depicts the second specification with interest rate sensitive lapse rates in the baseline model. The dotted line depicts the second specification with interest rate sensitive lapse rates and interest rate sensitive demand. The median and 90% confidence interval are reported.



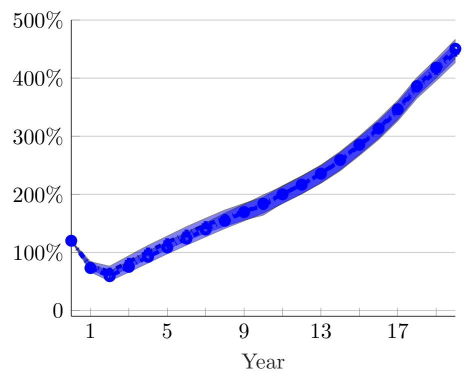
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).

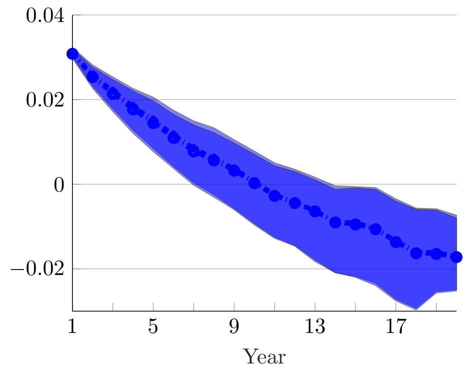


(c) Own Funds relative to the Market Value of Assets

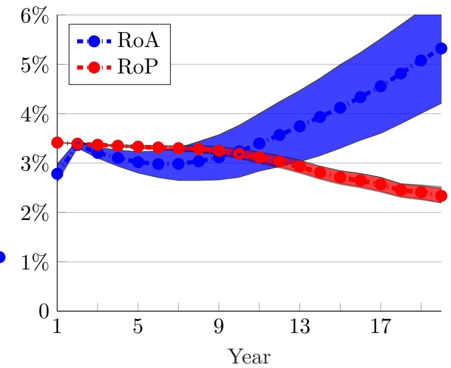


(d) Solvency Ratio

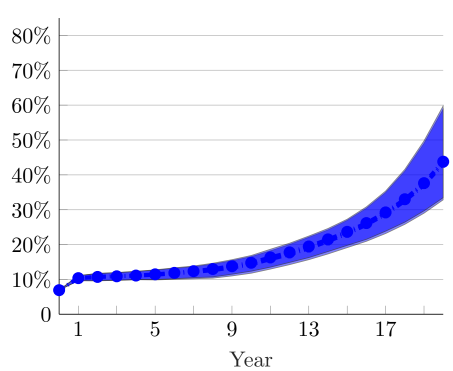
Figure 13: Environment (2) with demand function, where $1 - \lambda(\Delta r_t^h, 0)$ is the likelihood to buy an insurance contract. The crossed line depicts the second specification with interest rate sensitive lapse rates in the baseline model. The dotted line depicts the second specification with interest rate sensitive lapse rates and interest rate sensitive demand. The median and 90% confidence interval are reported.



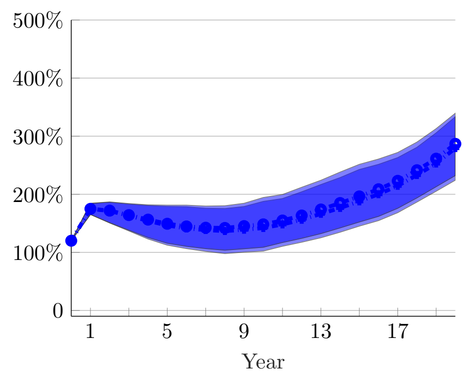
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).

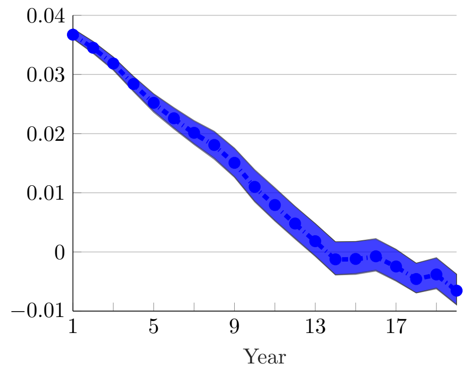


(c) Own Funds relative to the Market Value of Assets

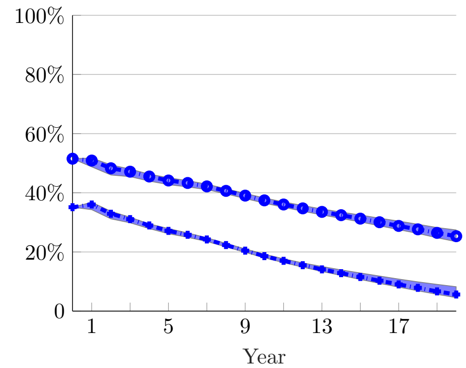


(d) Solvency Ratio

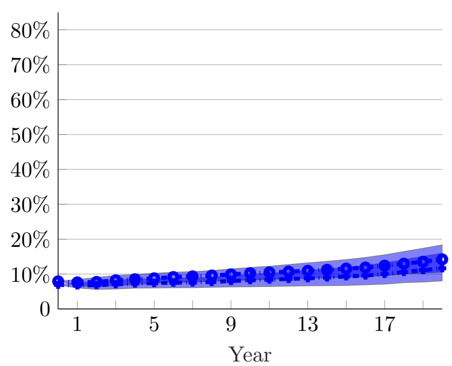
Figure 14: Environment (3) with demand function, where $1 - \lambda(\Delta r_t^h, 0)$ is the likelihood to buy an insurance contract. The crossed line depicts the second specification with interest rate sensitive lapse rates in the baseline model. The dotted line depicts the second specification with interest rate sensitive lapse rates and interest rate sensitive demand. The median and 90% confidence interval are reported. The median return on assets is -4.5% in period 1 in both specifications.



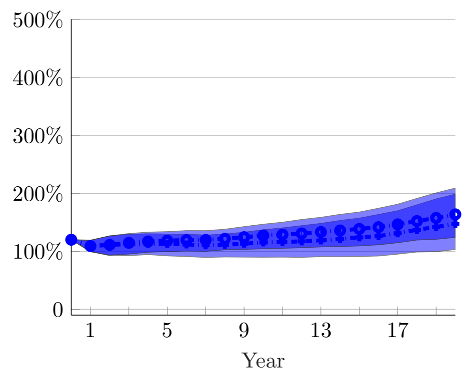
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

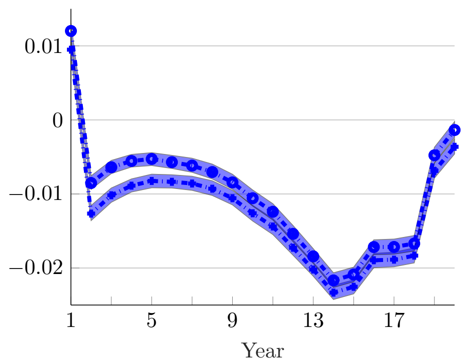


(c) Own Funds relative to the Market Value of Assets

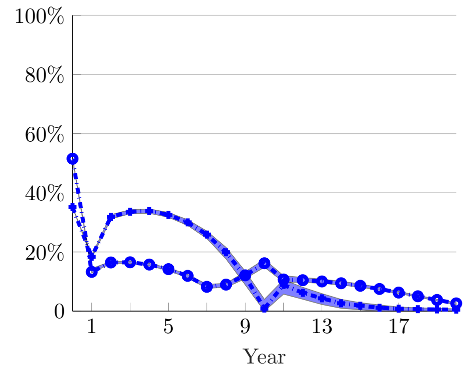


(d) Solvency Ratio

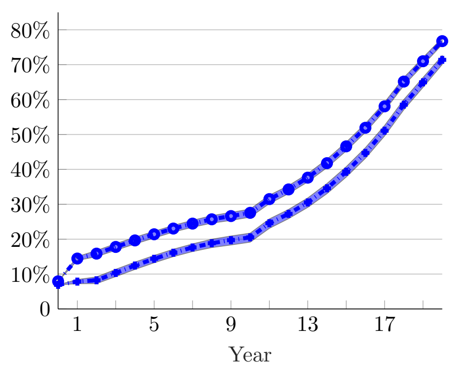
Figure 15: Environment (1) with haircut $1 - \vartheta = 0.15$. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates and haircut $1 - \vartheta = 0$. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and haircut $1 - \vartheta = 0.15$. The median and 90% confidence interval are reported.



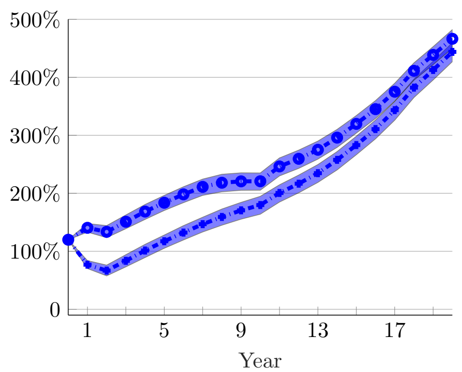
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

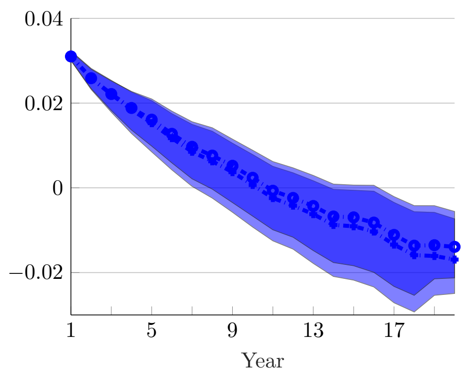


(c) Own Funds relative to the Market Value of Assets.

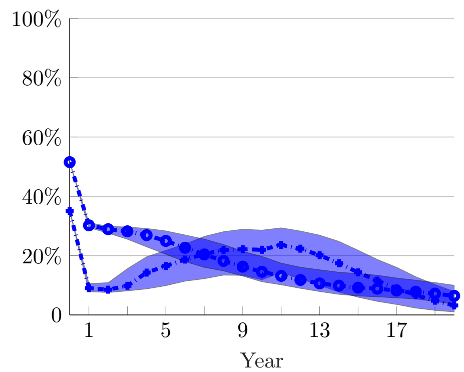


(d) Solvency Ratio.

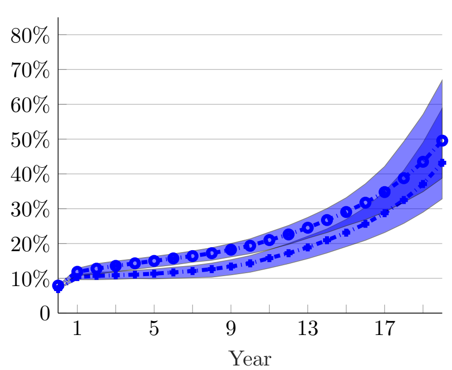
Figure 16: Environment (2) with haircut $1-\vartheta = 0.15$. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates and haircut $1-\vartheta = 0$. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and haircut $1-\vartheta = 0.15$. The median and 90% confidence interval are reported.



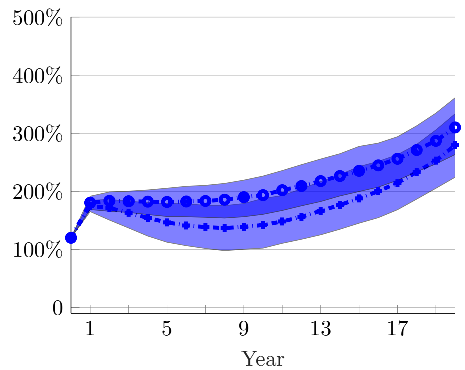
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk).

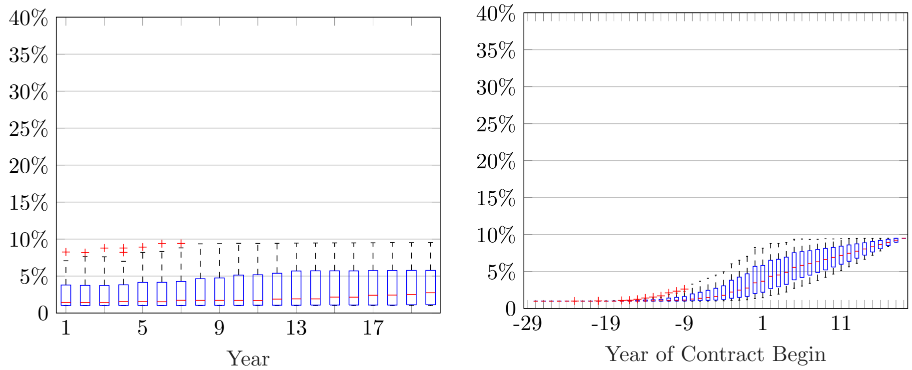


(c) Own Funds relative to the Market Value of Assets.

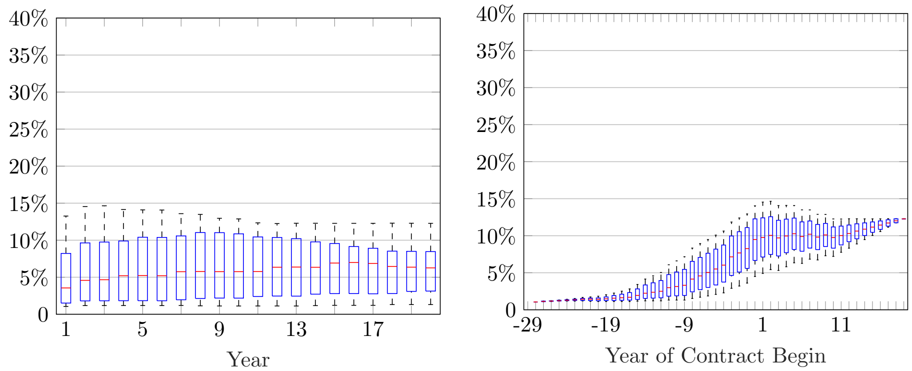


(d) Solvency Ratio.

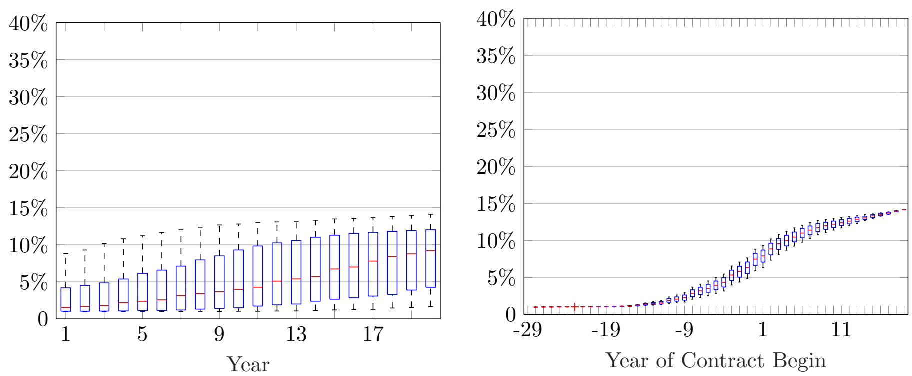
Figure 17: Environment (3) with haircut $1 - \vartheta = 0.15$. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates and haircut $1 - \vartheta = 0$. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and haircut $1 - \vartheta = 0.15$. The median and 90% confidence interval are reported.



(a) Environment (1)

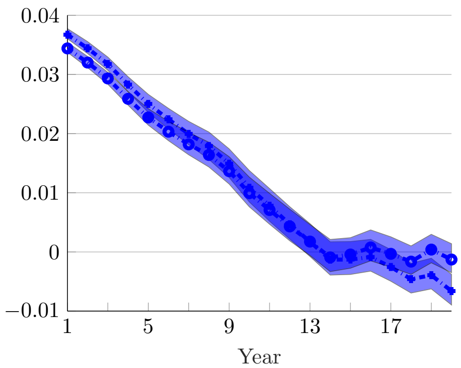


(b) Environment (2)

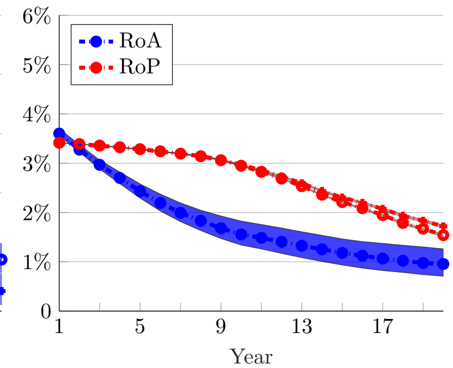


(c) Environment (3)

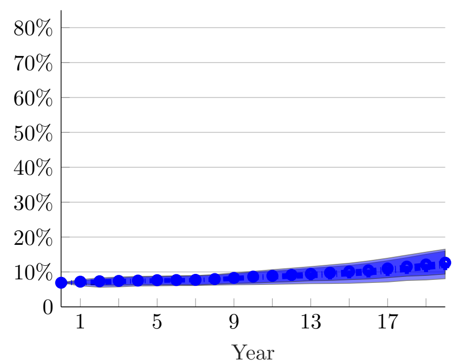
Figure 18: Distribution of lapse rates across cohorts at each point in time (left side) and across time for each cohort (right side) for interest rate sensitive lapse rates with smaller sensitivity towards the excess guaranteed rate. Each box consists of lower, median, and upper quartile, crossed points are outliers.



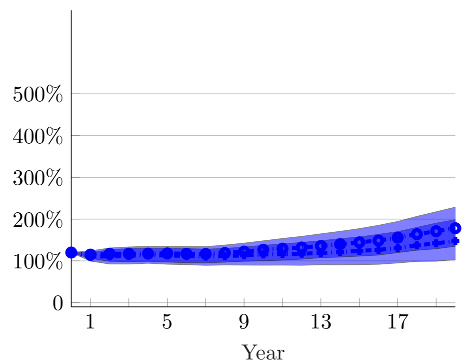
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).



(c) Own Funds relative to the Market Value of Assets



(d) Solvency Ratio

Figure 19: Environment (1) with smaller sensitivity towards the excess guaranteed rate. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and smaller sensitivity towards the excess guaranteed rate.The median and 90% confidence interval are reported.

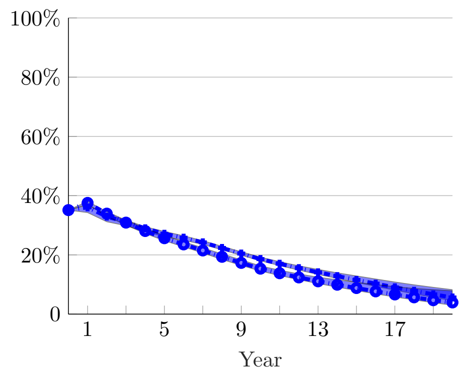
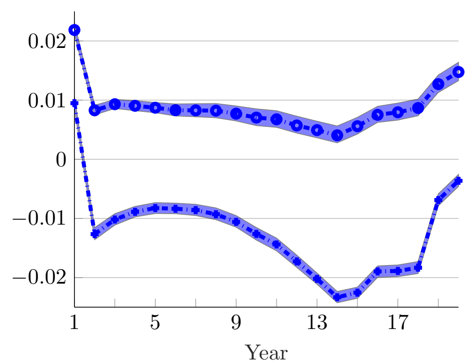
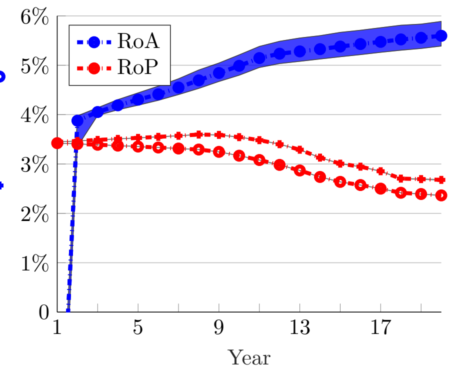


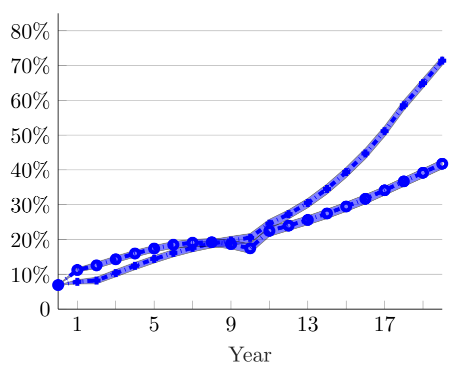
Figure 20: Environment (1) with smaller sensitivity towards the excess guaranteed rate. Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and smaller sensitivity towards the excess guaranteed rate.



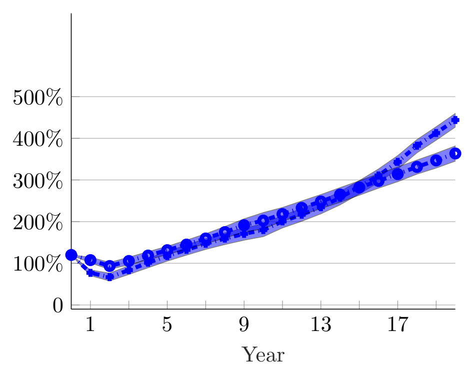
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Return on Assets and to Policyholders.

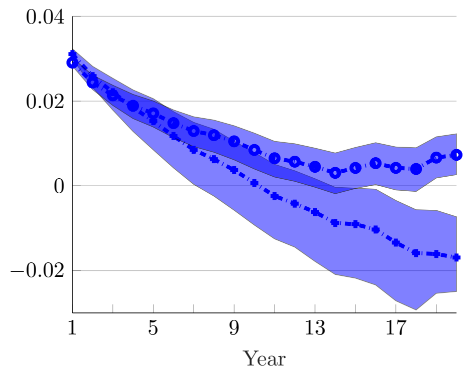


(c) Own Funds relative to the Market Value of Assets.

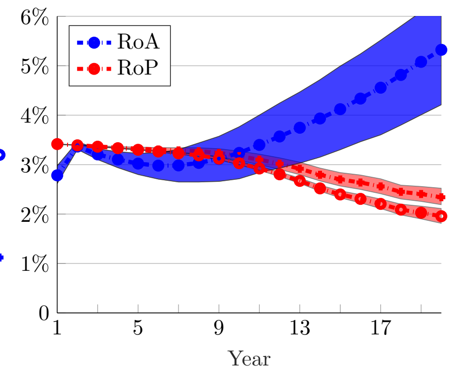


(d) Solvency Ratio

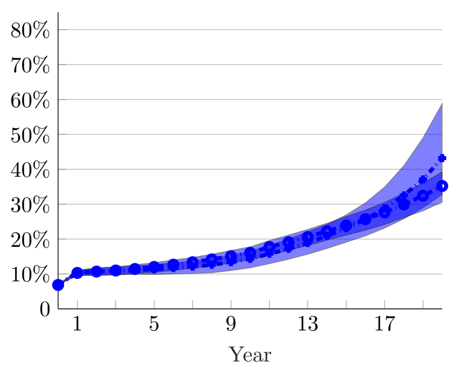
Figure 21: Environment (2) with smaller sensitivity towards the excess guaranteed rate. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and smaller sensitivity towards the excess guaranteed rate. The median and 90% confidence interval are reported. The median return on assets is -4.5% in period 1 in both specifications.



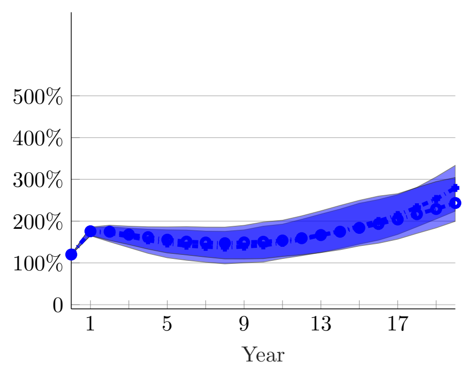
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Return on Assets and to Policyholders.



(c) Own Funds relative to the Market Value of Assets.



(d) Solvency Ratio.

Figure 22: Environment (3) with smaller sensitivity towards the excess guaranteed rate. The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and smaller sensitivity towards the excess guaranteed rate. The median and 90% confidence interval are reported.

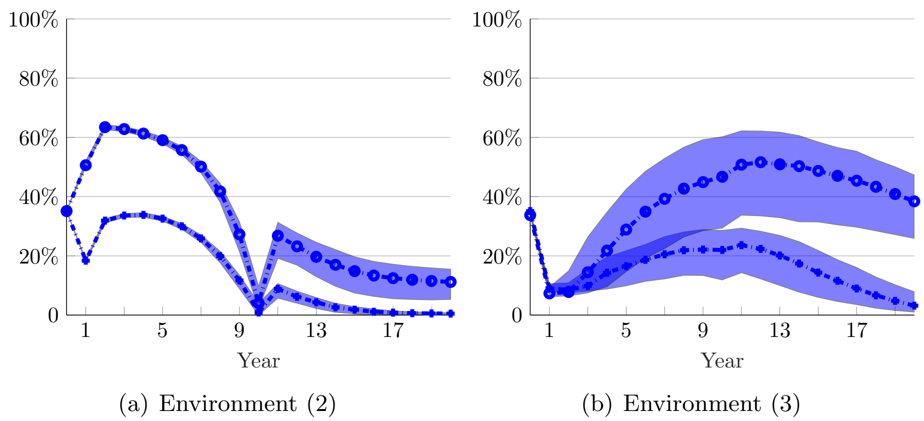
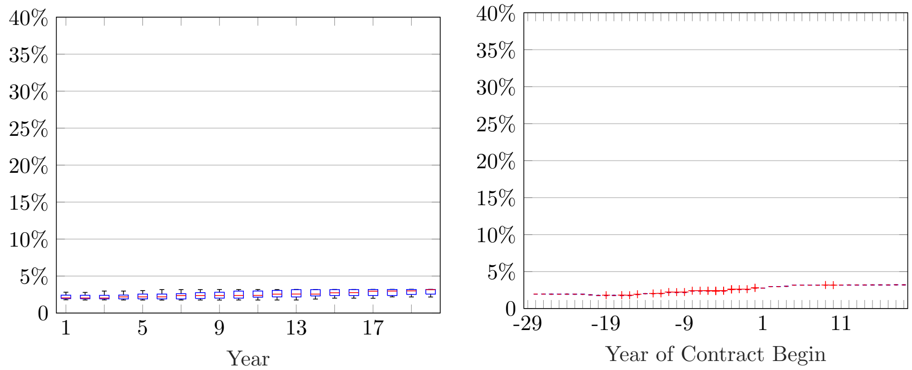
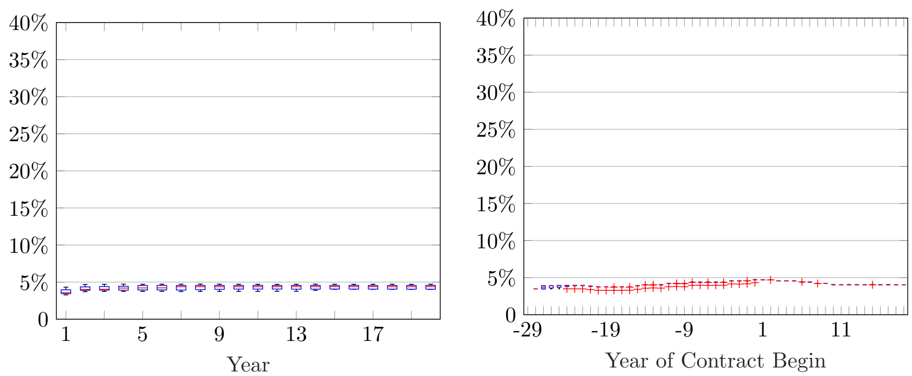


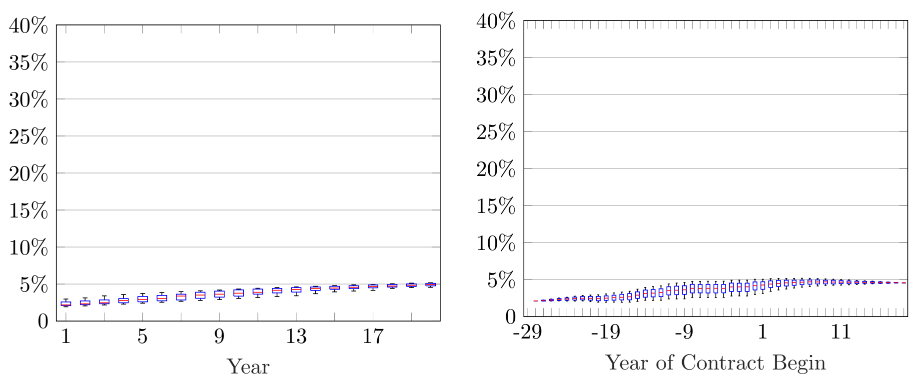
Figure 23: Environment (2) and (3) with smaller sensitivity towards the excess guaranteed rate. Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and smaller sensitivity towards the excess guaranteed rate.



(a) Environment (1)

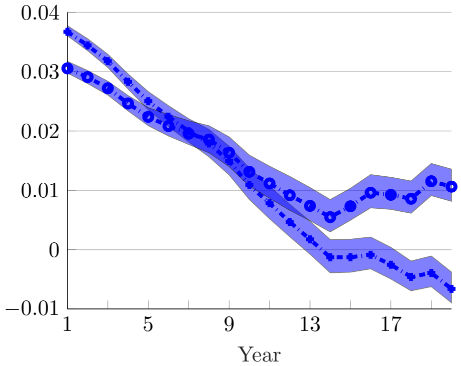


(b) Environment (2)

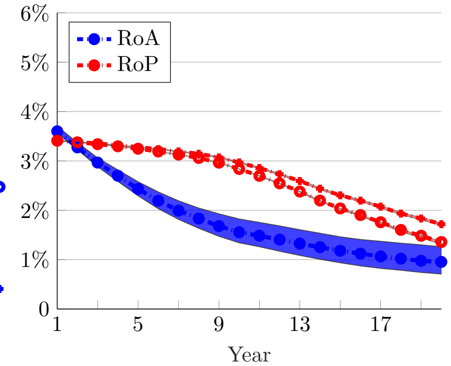


(c) Environment (3)

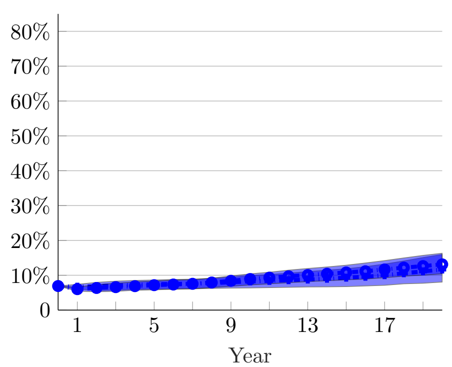
Figure 24: Distribution of lapse rates across cohorts at each point in time (left side) and across time for each cohort (right side) for interest rate sensitive lapse rates with no sensitivity towards contract age ($d2 = 0$). Each box consists of lower, median, and upper quartile, crossed points are outliers.



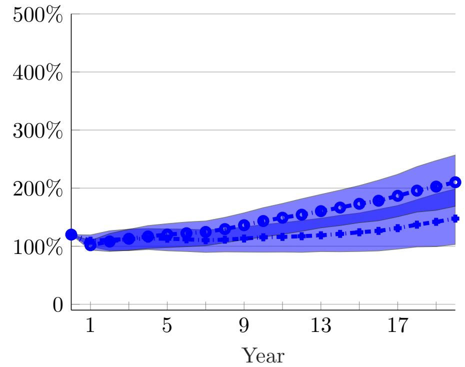
(a) Free Cash Flow relative to the initial Book Value of Assets.



(b) Return on Assets and to Policyholders.



(c) Own Funds relative to the Market Value of Assets.



(d) Solvency Ratio.

Figure 25: Environment (1) with no sensitivity towards contract age ($d2 = 0$). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and no sensitivity towards contract age. The median and 90% confidence interval are reported.

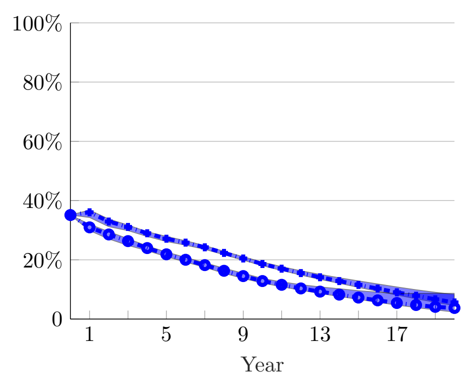
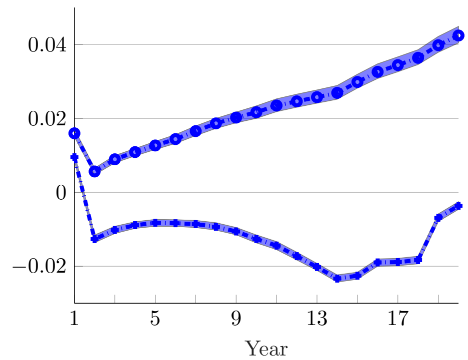
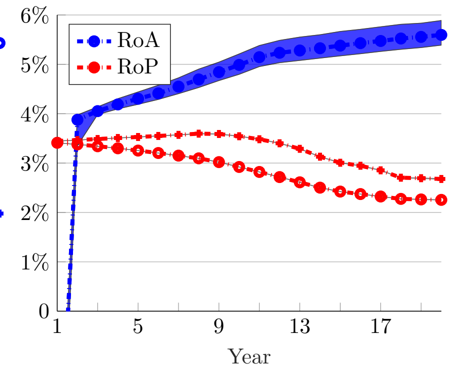


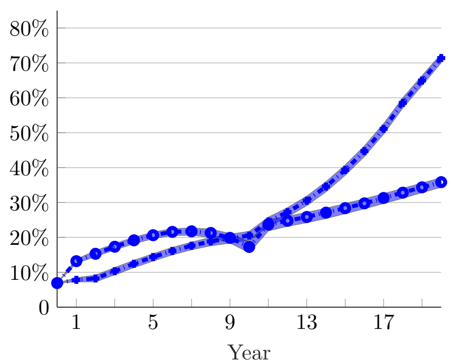
Figure 26: Environment (1) with smaller sensitivity towards the excess guaranteed rate. Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and no sensitivity towards contract age.



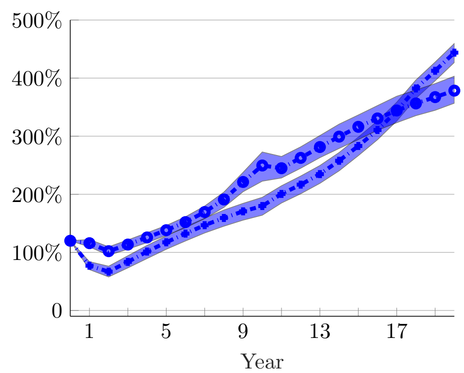
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).

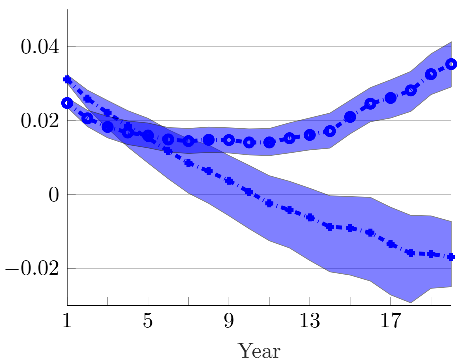


(c) Own Funds relative to the Market Value of Assets

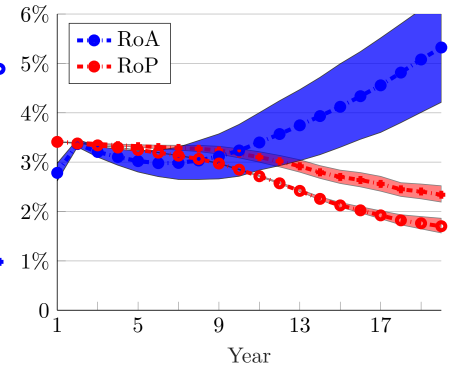


(d) Solvency Ratio

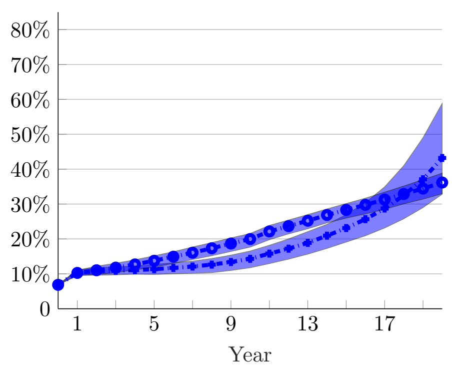
Figure 27: Environment (2) with no sensitivity towards contract age ($d2 = 0$). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and no sensitivity towards contract age.The median and 90% confidence interval are reported. The median return on assets is -4.5% in period 1 in both specifications.



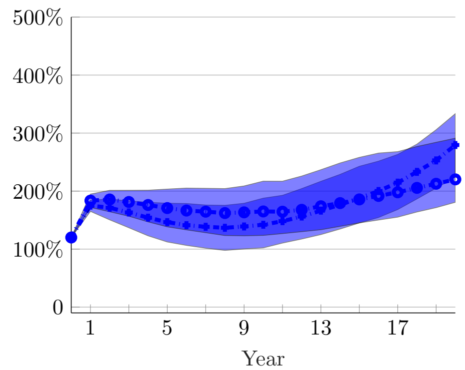
(a) Free Cash Flow relative to the initial Book Value of Assets



(b) Return on Assets (blue) and to Policyholders (red).



(c) Own Funds relative to the Market Value of Assets



(d) Solvency Ratio

Figure 28: Environment (3) with no sensitivity towards contract age ($d2 = 0$). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and no sensitivity towards contract age. The median and 90% confidence interval are reported.

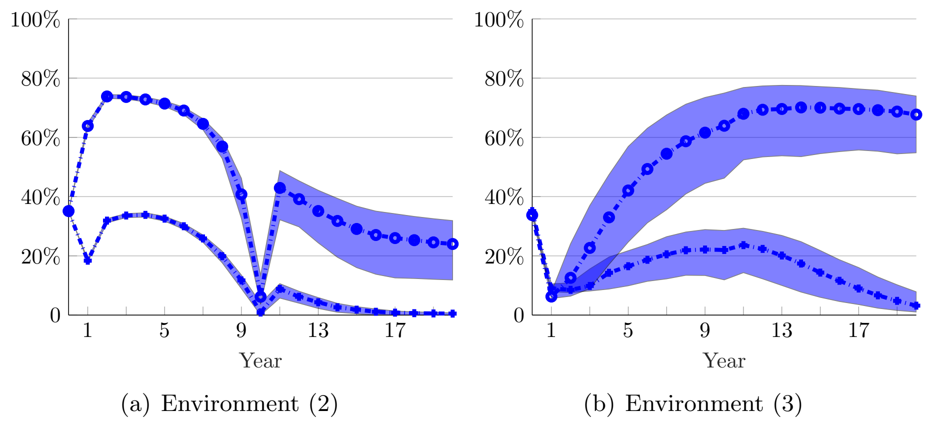


Figure 29: Environment (2) and (3) with smaller sensitivity towards the excess guaranteed rate. Solvency capital requirement for lapse risk relative to the total solvency capital requirement (that comprises market and lapse risk). The crossed line depicts the second baseline specification with interest rate sensitive lapse rates. The circled line depicts the sensitivity analysis with interest rate sensitive lapse rates and no sensitivity towards contract age.

B The Short-Rate Model and its Calibration

In the Hull-White model, the short-rate dynamics are given by

$$dr(t) = \alpha_r(\theta(t) - r(t))dt + \sigma_r dW_r(t) \quad \text{with } r(0) = r_0, \tag{9}$$

where $W_r(t)$ is a standard Brownian motion, $r(t)$ is the instantaneous (short) interest rate at time t, $\alpha_r > 0$ is the speed of reversion, $\sigma_r > 0$ the volatility and $\theta(t)$ the (non-constant) level of mean reversion. The stochastic differential equation (9) can be solved, which yields (cf. Brigo and Mercurio (2006))

$$r(t) = r_0 e^{-\alpha_r t} + \alpha_r \int_0^t e^{-\alpha_r(t-u)}\theta(u)\, du + \sigma_r \int_0^t e^{-\alpha_r(t-u)}\, dW_r(u). \tag{10}$$

Thus, the short-rate is normally distributed, i.e. $r(t) \sim \mathcal{N}(\mu_t, \sigma_t^2)$, with parameters

$$\mu_t = \mathbb{E}[r(t)] = r(0)e^{-\alpha_r t} + \alpha_r \int_0^t \theta(u)e^{-\alpha_r(t-u)}\, du \tag{11}$$

$$\sigma_t^2 = \text{var}(r(t)) = \frac{\sigma_r^2}{2\alpha_r}\left(1 - e^{-2\alpha_r t}\right). \tag{12}$$

In this interest-rate model, the price $P(t,\tau)$ of a zero-coupon bond at time t with time to maturity τ is given by (cf. Hull and White (1990) and Brigo and Mercurio (2006))

$$P(t, t+\tau) = A(t, t+\tau)e^{-r(t)\, B(\tau)}, \tag{13}$$

where

$$B(\tau) = \frac{1 - e^{-\alpha_r \tau}}{\alpha_r}$$

$$\text{and} \quad A(t, t+\tau) = \exp\left(\frac{\sigma_r^2}{2\alpha_r^2}(\tau - B(\tau)) - \frac{\sigma_r^2}{4\alpha_r}B^2(\tau) - \alpha_r \int_t^{t+\tau} \theta(u)B(t+\tau-u)\, du\right).$$

Hence, the continuously compounded spot rate at time t for time to maturity τ is given by

$$\hat{r}_{f,\tau}(t) = -\frac{1}{\tau}\log P(t, t+\tau) = \frac{B(\tau)r(t) - \log A(t, t+\tau)}{\tau} \tag{14}$$

and the equivalent annually-compounded spot rate is given by

$$r_{f,\tau}(t) = e^{\hat{r}_{f,\tau}(t)} - 1 = \left(\frac{e^{B(\tau)\, r(t)}}{A(t, t+\tau)} \right)^{1/\tau} - 1. \tag{15}$$

To yield rising interest rates, we choose the mean reversion level to be

$$\theta(t) = \gamma + (\beta - \gamma) \left(1 - \frac{1}{1 + e^{-b(t-h)}} \right). \tag{16}$$

We calibrate the initial short-rate $r(0)$, speed of mean reversion α_r, volatility σ_r and mean reversion parameters γ, β, b and h by means of weighted least squares in order to match historical short rate volatility, the yield curve implied by German government bonds in 2015 as initial risk-free yield curve, and a target interest rate level. The resulting calibration for Environment (2) and 3 as well as the calibration of the Vasicek model in Environment (1) are reported in Table. The calibration of the Vasicek model is based on the adverse scenario in Berdin et al. (2016) but exhibits a smaller long-term equilibrium and volatility in order to obtain a better to fit to interest rates in the year 2015. Since our model of the insurance firm and the financial market does not differentiate between interest rates under risk-neutral and real world measures and a market price of risk is captured by the mean-reversion function of the Hull-White-Model, we assume a market price of risk $\lambda = 0$ for the Hull-White model.

Environment	(1)	(2)	(3)
θ	-1%		
$r(0)$	-0.08%	-1.3559%	3.8%
α_r	0.5462	2	0.0095
σ_r	1%	0.131%	0.3%
λ	-1	0	0
β		1.4224	-0.5
γ		0.575	0.4167
b		5	10
h		0	0

Table 2: Calibration of the Short-Rate Model in Environments (1), (2), and (3).

C Calibration of Lapse Rate Behavior

Conditional on the policyholder-specific risk factor the average lapse rate across all cohorts at time t is given as

$$\lambda_t = a + e^c \frac{1}{\sum_h n_t^h} \sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}, \tag{17}$$

where n_t^h is the number of contracts of cohort h in place at time t. Therefore, unconditionally the logarithmic excess lapse rate is normally distributed,

$$\tilde{\lambda}_t = \log(\lambda_t - a) = c + \log\left(\frac{1}{\sum_h n_t^h}\right) + \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right) \sim \mathcal{N}\left(\mu_t,\, \sigma_t^2\right) \tag{18}$$

where

$$\mu_t = \mu_c + \log\left(\frac{1}{\sum_h n_t^h}\right) + \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right) \tag{19}$$

$$\text{and} \quad \sigma_t^2 = \sigma_c^2. \tag{20}$$

The results of earlier studies suggest that the relationship between lapse rate and current contract age is negative (Eling and Kiesenbauer (2014)). In other words, policyholders are more likely to lapse their contract shortly after purchase than shortly before maturity. Hence, we expect $d_2 > 0$. Moreover, we expect that $d_1 > 0$ since a larger difference between guaranteed and market rate $\Delta r_t^h = r_G^h - r_{fc}(t)$ is likely to decrease lapse rates since it increases the value of the policy to the policyholder.

For given sensitivities d_1 and d_2, the Maximum-Likelihood estimates for μ_c and σ_c are

The Maximum-Likelihood function for $\tilde{\lambda}_t$ is given by

$$\mathcal{L} = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{1}{2\sigma_c^2}(\tilde{\lambda}_t - \mu_t)^2} \tag{21}$$

$$= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{1}{2\sigma_c^2}\left(\tilde{\lambda}_t - \mu_c - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right)^2} \tag{22}$$

and the log-Likelihood is given as

$$l = \sum_{t=1}^{T} - \log\left(\sqrt{2\pi}\sigma_c\right) - \frac{1}{2\sigma_c^2}\left(\tilde{\lambda}_t - \mu_c - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right)^2. \quad (23)$$

Therefore, the Maximum-Likelihood (ML) estimate for μ_c satisfies

$$\frac{dl}{d\mu_c} = \sum_{t=1}^{T} \frac{2}{2\sigma_c^2}\left(\tilde{\lambda}_t - \mu_c - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right) = 0, \quad (24)$$

or equivalently

$$\mu_c = \frac{1}{T}\sum_{t=1}^{T}\left(\tilde{\lambda}_t - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right). \quad (25)$$

The ML estimate for σ_c is given as

$$\frac{dl}{d\sigma_c} = \sum_{t=1}^{T} -\frac{1}{\sigma_c} + \frac{1}{\sigma_c^3}\left(\tilde{\lambda}_t - \mu_c - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right) = 0, \quad (26)$$

or equivalently

$$\sigma = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\tilde{\lambda}_t - \mu_c - \log\left(\frac{1}{\sum_h n_t^h}\right) - \log\left(\sum_h n_t^h e^{-e^{d_1 \Delta r_t^h + d_2 \Delta T_t^h}}\right)\right)^2}. \quad (27)$$

However, it is not possible to determine closed-form ML estimates for d_1 or d_2. Hence, we rely on a numerical procedures. In particular, d_1 is set to a least-squares estimator to minimize deviations between historically observed and model implied average lapse rates conditionally on μ_c and σ_c. After updating d_1, μ_c and σ_c are updated with the ML estimator. This procedure is repeated until convergence of μ_c and σ_c.

Afterwards, d_2 is increased in small steps if the model implied average lapse rate across all cohorts with the initial values of the insurer and financial market in our model differs from the observed lapse rate in 2015, 2.86%. Upon an increase of d_2 all other parameters are updated as described above.

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