ICIR Working Paper Series No. 32/17

Edited by Helmut Gründl and Manfred Wandt

Do consumers benefit from more informed firms?*

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August 20, 2017

Abstract

We study the impact of estimation errors of firms on social welfare. For this purpose, we present a model of the insurance market in which insurers face parameter uncertainty about expected loss sizes. As consumers react to under- and overestimation by increasing and decreasing demand, respectively, insurers require a safety loading for parameter uncertainty. If the safety loading is too small, less risk averse consumers benefit from less informed insurers by speculating on them underestimating expected losses. Otherwise, social welfare increases with insurers' information. We empirically estimate safety loadings in the US property and casualty insurance market, and show that these are likely to be sufficiently large for consumers to benefit from more informed insurers.

Keywords: Price Uncertainty, Parameter Uncertainty, Economics of Information

JEL Classification: D11, D83, G14, G20, L11

* I am grateful for helpful comments and suggestions by Helmut Gründl, Annette Hofmann, Michael Hoy, and Petra Steinorth.

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1 Introduction

With globalization and digitalization rocketing in the 1990s and the new millennium, a fast increasing amount of information spreads more easily within and across economies. These information include empirical observations, expert opinion, experimental evidence, survey results and numerous other types of public and privative information. It cannot only be used to improve decision-making or risk management, but also for pricing goods. Financial products in particular (e.g. swaps, loans, options, or insurance products) are often characterized by uncertain future costs (i.e. cash expenses). Since firms do usually not know the exact distribution of costs, they must estimate it. For example, insurance companies need to estimate the frequency and severity of claims, brokers the distribution of cash flows of financial products, and banks the likelihood of credit defaults. The resulting uncertainty about the distributional parameters is commonly called *parameter uncertainty* (or *estimation risk*).

Although the amount of available information has been increasing particularly in the recent decades, information is not unconstrained. For example, the cyber insurance market still misses broad supply as information about cyber losses is hard to gather and the risk's characteristics are frequently changing.[1] Due to the very large resulting parameter uncertainty, most insurance companies refrain from offering cyber insurance (Lloyd's and Cyence (2017)). This example shows that the amount of information available to firms can play a pivotal role in financial markets.

Starting with Blackwell (1953) and Stigler (1961), the economic impact of market participants' information has been studied in various settings. In this article, we extend previous literature by exclusively focus on the amount of information available to firms to estimate expected costs. We study the impact of a firm's level of parameter uncertainty on social utilitarian welfare exemplary for the case of insurance markets. As insurance companies naturally need to estimate expected losses of policyholders in order to determine the actuarially fair insurance premium, parameter uncertainty is a central element of insurance markets. We give some intuition for this rationale by studying parameter uncertainty in the cyber and motor insurance market in Section 2. Although both insurance lines exhibit very different characteristics in terms of market coverage and availability of information, we show that parameter uncertainty is important not only for emerging risks as cyber losses but also for very common insurance products as motor insurance. Nonetheless, our results also apply to various other markets for financial products that share similar characteristics.

[1] Lloyd's and Cyence (2017) estimate that only 7% in case of a mass vulnerability due to an error or weakness in a software code are covered by insurance as of 2017.

The core of this article consists of two parts: In Section 3 we present a theoretical model for the insurance market that incorporates an insurer's parameter uncertainty. As an insurer's information is constrained, prices are random *ex ante* (i.e. before information is known) as they depend on the realization of information and the resulting estimate for expected losses. The more information are available for the firm to estimate expected losses, the smaller is the level of parameter uncertainty (i.e. the *estimation error*), and therefore price volatility.

We identify two main effects of an insurer's parameter uncertainty on social welfare: On the one hand, if insurers are more informed, the level of parameter uncertainty is smaller and ex ante prices are less volatile. On the other hand, a larger level of parameter uncertainty increases the insurer's estimation error and, thus, it is likely that it over- and underestimates expected costs to a larger degree. Consumers might ex ante speculate on the insurer underestimating expected costs. Then, they would profit from a small price in case of underestimation and react to overestimation by reducing demand.

Hence, demand is smaller in case of overestimation than in case of underestimation. We show that insurers need to require a safety loading on prices in order to yield nonnegative ex ante expected profits. If the safety loading is large enough, the second effect of parameter uncertainty on social welfare disappears and parameter uncertainty exclusively impacts the volatility of consumers' wealth. As these are risk averse, social welfare is strictly decreasing with a firm's level of parameter uncertainty and, therefore, increasing with its amount of information.

If, however, insurers require a smaller (or no) safety loading, there is a positive ex ante expected transfer of wealth from insurers to consumers. We show that, the smaller the safety loading and the risk aversion of consumers, the more likely it is that the second effect of parameter uncertainty dominates the first. In this case, social welfare is increasing with an insurer's level of parameter uncertainty as consumers speculate on the insurer underestimating expected costs. We illustrate this finding by examining a safety loading implied by cost of capital for a solvency capital requirement that takes parameter uncertainty into account. The capital requirement is calibrated in line with the European solvency regime Solvency II, although this does currently not account for parameter uncertainty. We find that the resulting safety loading is unlikely to be large enough to prevent consumers from speculating on an insurer underestimating expected losses for reasonable levels of risk aversion.

In response to these theoretical results, in Section 4 we empirically examine prices for property and casualty insurance in the U.S. insurance market. We identify a relative safety loading of up to 100 units of loss volatility for insurance lines with a large parameter uncertainty that results from a small amount of available observations (as earthquake insurance) or from large uncertainty due to the involvement of human behavior (as fidelity or surety insurance). Moreover, insurance lines with larger aggregate premium payments are likely to exhibit a small safety loading (and vice versa), since there are typically more information about losses for common insurance products (as private passenger auto physical damage insurance).

Finally, we compare the empirically estimated safety loadings with the theoretical safety loading implied by our model. Our findings suggest that empirical safety loadings are sufficiently large to account for changing consumer demand in response to over- and underestimation in most insurance lines. Thus, consumers do likely not benefit from parameter uncertainty and, therefore, more information to firms increases social welfare.

Our model confirms the intuition, that firms compete over information as they face an inherent incentive to gather information to compensate for parameter uncertainty. Nonetheless, consumers might not necessarily benefit from an increase in a firm's amount of information. In contrast, if firms do not demand a sufficiently large safety loading for parameter uncertainty, less risk averse consumers prefer firms to obtain less information about the products they sell in order to speculate on the firm underestimating expected costs. Therefore, consumers that are less risk averse are more hesitant to share information with firms than more risk averse consumers.

To achieve large safety loadings, policymakers could impose solvency requirements that penalize parameter uncertainty, as considered by Venezian (1983) and Fröhlich and Weng (2015) but currently not implemented by any solvency regime for financial institutions. The resulting capital requirement would prevent a firm's default in case of underestimation up to a specific confidence level. However, we find that the price increase resulting from cost of capital is likely to be too small to prevent consumers from benefiting from a firm's parameter uncertainty. Thus, classical measures of insurance regulation are not able to incentivize less risk averse in sharing information with firms if firms do not require a sufficiently large safety loading on their own.

Our model extends previous literature on the economics of information, starting with Blackwell (1953), who shows that more precise public information increases welfare - the *Blackwell effect*. The intuition is that information about supply and demand of other market participants decreases search costs, and thereby increases social welfare. If, in contrast, participants are perfectly informed about supply and demand but uninformed about their own endowment and investments, Hirshleifer (1971) shows that more public information may limit trade opportunities for risk sharing and, thus, be associated with a negative value - the *Hirshleifer effect*. Eckwert and Zilcha (2003) show in a general equilibrium model that the Blackwell effect dominates the Hirshleifer effect, i.e. welfare increases with information, if consumers are sufficiently risk-averse. Our model differs from the previous ones by not considering a change in public information but assuming that consumers are perfectly informed while firms are uncertain about future costs. This is similar to the setting studied by Riley (1979). Nonetheless, our result for small safety loadings is similar to the finding of Eckwert and Zilcha (2003) that information increases welfare for sufficiently risk averse consumers, although Eckwert and Zilcha (2003) do not consider parameter uncertainty about expected costs and its impact on prices but uncertainty about an aggregate risk.

An alternative way to describe our setting is to call firms *ambiguous* with respect to costs and consumers *ambiguous* with respect to prices. In this case, ambiguity refers to uncertainty about the distributional parameters of costs and prices, respectively. In contrast to the large strain of literature about decisions under ambiguity, most prominently influenced by the model Klibanoff et al. (2005), in our model ambiguity for consumers is resolved by observing prices before they determine optimal demand.[2] In contrast, the evaluation of ex ante welfare is affected by ambiguity, as prices are random ex ante. Ambiguity for firms is resolved exclusively in the case of perfect information. Otherwise, they remain ambiguous about costs, while we vary the degree of ambiguity (i.e. parameter uncertainty). As we study ex ante expected profits, we assume that firms evaluate prospects consistent with Bayesian expectations. This corresponds to ambiguity neutrality in the sense of Klibanoff et al. (2005). If we assumed firms to be ambiguity averse in the sense of Klibanoff et al. (2005) instead, this would create an additional penalty for uncertainty in ex ante expected profits, and, hence, increase the optimal amount of a firm's information and safety loadings.[3]

For the sake of simplicity, we assume that all insurers in our model obtain the same estimate for expected losses and due to competition offer the same price. Intuitively, the positive impact of parameter uncertainty on welfare would be larger, if insurers yielded less dependent estimates and consumers would be able to search for the lowest price. The resulting price dispersion might be persistent due to heterogeneity of products, conditions and participants that change over time, as well as agents that face costly search for information about prices. Diamond (1971) shows that price dispersion can in particular result from search restrictions of consumers. Stigler (1961), Stigler (1962), McCall (1965), Telser (1973), and Salop and Stiglitz (1982) consider the search for prices in a market that displays price dispersion for similar goods. They find that price dispersion can be highly persistent, particularly when supply and demand conditions or the mix of participants vary over time. The fundamental reason is that consumers will engage in price search only as long as marginal gains from this search are larger than marginal costs.As price dispersion and price search together increase price elasticity of demand for products from a particular firm, we expect necessary safety loadings to increase with price dispersion.

We extend the previous literature by directly modeling a firm's uncertainty about expected costs,

[2]For an overview of models for decision making under ambiguity see Etner et al. (2012).
[3]This rationale is in line with the results of Snow (2010), who shows that ambiguity aversion increases the value of information for decision makers.

which yields ex ante random prices. This contrasts the aforementioned studies, that start with a given distribution of prices. Sandmo (1971), Polborn (1998), and Wambach (1999) consider uncertain costs for risk averse firms. Their main result is that prices under cost uncertainty are larger than the competitive price without cost uncertainty, and optimal output is smaller. Our model similarly implies that prices are larger if firms face parameter uncertainty about costs. However, this result comes without assuming risk averse firms and, thus, is more general. Moreover, in contrast to the aforementioned studies we focus on the welfare implications of a change in price dispersion caused by a change in parameter uncertainty.

Our model of an insurance market is based on the models of Doherty and Schlesinger (1995), who study the implications of stochastic loss size for insurance demand, and that of Doherty and Schlesinger (1983), who study insurance demand if consumers face an uninsurable background risk. In line with their studies, we make two assumptions: 1) The occurrence as well as loss size is random and 2) consumers face a hidden background risk that is unobservable by insurers. While these assumptions do not alter our general results, they serve the purpose to prevent the insurer from adapting prices to consumer behavior. For example, if loss sizes were deterministic and the insurer knew the loss probability, it could infer the expected loss by observing one individual loss. Moreover, if consumers would not face an additional, hidden background risk, the insurer would be able to infer if and by how much it over- or underestimated the expected loss and adjust prices accordingly. In contrast, in our model insurers can solely employ information about the losses itself (e.g. historical observations) but not consumer behavior to learn about the loss distribution. This second assumption remarkably simplifies our model as we do not have to incorporate the mechanism how insurers learn about the loss distribution by observing consumer behavior. However, it does not impact our results as such mechanism would only alter but not remove parameter uncertainty.

Central in our model is the asymmetry between the information of firms and consumers. As we focus on changes in the amount of an insurer's information about losses, for simplicity we assume that consumers are certain about the distribution of losses.[4] However, in some markets, information does usually change for a large amount of (or even all) market participants and not just a subset, for example on stock exchanges. Barry (1978) studies parameter uncertainty of investors in such markets and concludes that parameter uncertainty does not affect predictions about market equilibria based on the Capital Asset Pricing Model (CAPM). This, however, changes when estimation risk differs across securities (Klein and Bawa (1977), Barry and Brown (1985)) or across agents (Coles et al. (1995)). Boyle and Ananthanarayanan (1977) show that parameter uncertainty about the variance in option valuation produces biased option values. These results for security markets are in line with our finding that parameter uncertainty has an important

[4]We do not expect our results to change if consumers face parameter uncertainty as well, since the general mechanism in our model - that consumers change demand upon prices and thus might speculate on underestimation - remains the same.

impact on market equilibria.

We proceed as follows. Section 2 gives a definition of parameter uncertainty and intuition about its relevance. Section 3 introduces the theoretical model and presents our main results. In Section 4 we empirically derive safety loadings for parameter uncertainty in the US property and casualty insurance market and compare them with our model. Section 5 contains concluding remarks.

2 Parameter Uncertainty

2.1 Definition of Parameter Uncertainty

Suppose that a firm faces (future) costs that are distributed according to $F_{\tilde{C}}$, $\tilde{C} \sim F_{\tilde{C}}$. If costs are not deterministic, the firm is uncertain about the realization of costs. For example, consider an insurance contract for a loss $\tilde{C} \sim F_{\tilde{C}}$. Then, at the time of contract purchase the actual indemnity payment $\tilde{C}$ is uncertain.

Parameter uncertainty is a second layer of uncertainty that arises, if $F_{\tilde{C}}$ is unknown. In the example, the insurer might be uncertain about the parameter values of $F_{\tilde{C}}$. For example, the insurer might need to rely on an estimate for expected losses, $\tilde{\vartheta}$, instead of knowing the actual expected loss, $\mu = \mathbb{E}[\tilde{C}]$. The resulting parameter uncertainty, or *estimation risk*, is in a Maximum-Likelihood context often measured by the standard error of the estimator $\tilde{\vartheta}$, $se(\tilde{\vartheta})$. In a Bayesian context, parameter uncertainty yields a probability distribution of possible values for μ. In special cases (e.g. a Gaussian framework with non-informative prior distribution), the standard deviation of μ in the Bayesian case is equivalent to the standard error of the Maximum-Likelihood estimator $\tilde{\vartheta}$.

As the distribution of costs can usually not be determined with certainty, essentially all financial products are subject to parameter uncertainty - although to a different degree. Typically, for products with a large parameter uncertainty there is not a large amount of information, experience, or historical observations available, or they are subject to frequent changes in their characteristics. An example from the insurance market is cyber risk. With companies becoming more and more depending on computational advancements, they also face a larger risk of losses due to attacks on their cyber infrastructure. In a recent policy report, Lloyd's and Cyence (2017) estimate that large cyber attacks can have a similar impact as hurricanes in terms of economic losses. Thus, the potential market for cyber insurance is very large.

However, the cyber insurance market is still largely untapped, as Lloyd's and Cyence (2017) estimate that only 17% of economic losses in case of a hack that takes down a cloud service provider and 7% in case of a mass vulnerability due to an error or weakness in a software code are covered by insurance as of 2017.[5]

[5] See also PwC (2015) and TaylorWessing (2015) for a discussion about the size and potential of the cyber insurance

A decisive reason for the lacking supply of cyber insurance is the large uncertainty regarding its impact. For example, Lloyd's and Cyence (2017) estimate a 95% confidence interval for the industry-wide loss in the scenario of a cloud provider hack as between $ 15.6bn and $ 121.4bn. This high level of uncertainty results in particular from the lack of data and data sources about cyber risk and its dependence on various risk factors. The same applies to operational and reputational risks as well as rare disasters as hurricanes and earthquakes.

Additionally, the characteristics of cyber risk are frequently changing over time. Rapid changes in technology as well as immense changes in the usage of technology substantially reduce the number of historically observed cyber losses that can be used to predict future losses. Moreover, the impact of human behavior on cyber attacks further increases uncertainty, as it is usually harder to predict and subject to potentially very rapid changes over time, as well. Another profound example for a change in risk characteristics is the risk of terror attacks. As Cummins and Lewis (2003) point out, the 2001 terrorist attacks on the World Trade Center sparked parameter uncertainty of insurers about terrorism losses. Until 2001, terrorism losses did mostly not occur on US soil, and deviations from this observation were viewed as aberrations by US insurers. Thus, insurance companies essentially neglected a potential exposure to terrorist attacks in the United States. However, the 2001 terrorist attacks seemed to represent a change in the distribution of terrorism losses in the United States, while this incident was the only observation for such a loss.

Consequently, with the 2001 terrorist attack insurers suddenly faced tremendous parameter uncertainty about the new distributional parameters for the risk of terrorism losses on US soil. In response, reinsurers and insurers excluded or significantly restricted terrorism coverage from most policies (Cummins and Lewis (2003)). This result is in line with the intuition from Froot et al. (1993) and Froot and O'Connell (2008) that the price of insurance is increasing with the volatility of the predictive loss distribution due to costs of external funding. If volatility is too large, supply essentially breaks down. It seems important to note that this effect is not necessarily related to a change in the volatility of losses itself. In contrast, a larger parameter uncertainty is sufficient, as it increases the volatility of the insurer's estimate for future losses.

2.2 Parameter Uncertainty and Statistical Inference

Assume that an firm offers a product that generates costs $\tilde{C}$ for the firm at time $t = 1$. In a risk-neutral world the present value at time $t = 0$ of costs is then given as $\mu = \mathbb{E}[\tilde{C}]$ (without loss of generality we ignore discounting). If the expected value of costs, $\mu = \mathbb{E}[\tilde{C}]$, is unknown to the firm, the firm might

market.

estimate expected costs based on observations $c_1, ..., c_n$ by employing the sample mean,

$$\vartheta_n = \frac{1}{n}\sum_{j=1}^{n} c_j. \tag{1}$$

A common measure for parameter uncertainty is the standard error of the sample mean, $\varepsilon = \sqrt{var(\vartheta_n)/n}$, as it reflects the dispersion of the estimator if observations are identically and independently distributed. If ε is small, there is less parameter uncertainty about the true parameter value and, thus, it is unlikely that $(\vartheta_n - \mu)^2$ is large.

In a Gaussian setting, ε can also be interpreted as the standard deviation of a subjective distribution about the true parameter value. The fiducial argument from Fisher (1930) provides an intuitive justification[6]: If $c_1, ..., c_n \overset{iid}{\sim} \mathcal{N}(\mu, \sigma^2)$, then $\vartheta_n - \mu \sim \mathcal{N}(0, \sigma^2/n)$. Therefore, conditional on the estimate ϑ_n it is $\mu \sim \mathcal{N}(\vartheta_n, \sigma^2/n)$. The last step involves a change in perspective - from the estimator to the true parameter being unknown. In this setting, $\varepsilon^2 = \sigma^2/n$ is the variance of a *posterior* (i.e. conditional on observed data) belief about the true expected costs, μ, conditional on the estimate ϑ_n. Conditional on the estimate, the *posterior predictive* distribution of costs, $\tilde{C}$, is given by $\mu + \sigma Z \sim \mathcal{N}(\vartheta_n, \sigma^2 + \sigma^2/n)$, where $Z \sim \mathcal{N}(0,1)$. Thus, parameter uncertainty increases uncertainty about future costs, and thereby cost volatility.

The same distributions arise in a Bayesian framework with non-informative prior $\pi(\mu) \propto 1$: Assume that the firm has a prior belief about expected costs, $\mu \sim p(\mu)$, before having seen information. By acquiring information, the firm forms a new belief, the *posterior belief*, which is based on the prior belief and new information, and is computed according to Bayes' theorem:

$$p(\mu \mid c_1, ..., c_n) = \frac{p(c_1, ..., c_n \mid \mu)p(\mu)}{p(c_1, ..., c_n)}. \tag{2}$$

For normally distributed data, the posterior mean is a weighted average of the prior mean and the sample mean of the data, ϑ_n. If one assumes the non-informative prior $p(\mu) \propto 1$,[7] the posterior mean is distributed as

$$\mu \mid c_1, ..., c_n \sim \mathcal{N}\left(\vartheta_n, \sigma^2/n\right) \tag{3}$$

[6]A discussion on the fiducial argument can also be found in Zabell (1992).

[7]This prior is to reflect that the firm does not have any prior information and weighs each possible value of expected costs equally. Although $(\mu) \propto 1$ is not a proper probability distribution, it yields a proper posterior probability distribution.

and the posterior predictive distribution is

$$\tilde{C} \mid c_1, ..., c_n \sim \mathcal{N}\left(\vartheta_n, \sigma^2 + \sigma^2/n\right). \tag{4}$$

As before, we find the standard error, $\varepsilon = \sqrt{\sigma^2/n}$, in the variance of the posterior distribution.

2.3 Parameter Uncertainty in Motor Insurance

The level of parameter uncertainty is inherently linked to the amount of information about a particular risk. From this point of view, losses in motor insurance should relate to a small level of parameter uncertainty compared to other insurance lines, as it is one of the most common insurance products purchased. According to the German Insurance Association (GDV) (2016), 112 million motor insurance contracts where in force in 2015 in Germany. This corresponds to roughly one third of all property and casualty contracts in Germany.[8] From this perspective, we would expect the parameter uncertainty in motor insurance to be rather small.

However, as motor insurance losses are also associated with a large number of risk characteristics, an observation gives information only for risks that share the same characteristics, i.e. that are in the same risk pool. In the following we show that pooling of motor insurance risks increases parameter uncertainty for this insurance line to a potentially very large extent. Since the risk factors employed by insurers to pool insurance contracts in practice are not public, we study the characteristics that are required by Germany's largest[9] motor insurance company, HUK Coburg, to determine a contract's insurance premium as of the end of 2016.[10] HUK Coburg requires at least 15 categorical (e.g. car type or car usage) and 5 ordered (continuous or discrete) risk factors (e.g. policyholder age or car age) for the calculation of premiums. Solely accounting for the 15 categorical factors and their levels yields approximately 3.6×10^{10} different contract pools.[11] Insurance companies also differentiate premiums with respect to approximately 415 geographical regions in Germany.[12] When also accounting for these geographic factors we yield roughly 1.5×10^{13} different contract pools.

In the following we estimate the number of observed losses in each of these pools per year. According to the German Insurance Association (GDV) (2016), the average number of motor insurance claims

[8]In Table 8 in Appendix D we provide the average aggregate annual premiums in different property and casualty insurance lines in the US. Motor insurance is among the lines with the largest premium income as well.

[9]according to the number of insurance contracts

[10]The insurance premium for motor insurance from HUK Coburg can be calculated on its website at `https://www.huk.de/tarifrechner.jsp`.

[11]We make very conservative assumptions about the actually employed number of levels per risk factor. The different risk factors together with our underlying assumptions can be found in Appendix A in Table 4.

[12]The geographical regions are reported by the German Insurance Association (GDV), see `http://www.gdv.de/2016/08/regionalklassen-in-der-kfz-versicherung-2017/`.

(averaged across the years 1995, 2000, 2005, 2010, and 2015) is roughly 9.16 million per year. The actual number of traffic accidents is substantially smaller and according to the German Federal Statistical Office (2016) approximately equals 2.6 million in 2016. However, one accident might trigger several claims for different persons and vehicles involved. Therefore, we will focus on the number of actual claims.

As a starting point, we assume that the historical claims are uniformly distributed across the different risk categories. We have at best 9.2×10^6 historical claims per year in 3.6×10^{10} different pools. If claims were uniformly distributed across pools, insurance companies observed on average 2.5×10^{-4} claims per pool per year. This finding would imply that an insurer would have to wait on average for 4 thousand years to observe one claim per pool. Clearly, the parameter uncertainty would be immense.

There are three critical simplifications underlying this calculation: Firstly, historical claims are unlikely to be uniformly distributed across contract pools. In contrast, it is likely that certain pools (for example for very common car types) exhibit substantially more claims than other pools (for example vintage vehicles and luxury cars). However, this imbalance decreases parameter uncertainty exclusively for the pools with more observations. It also increases parameter uncertainty in pools with scarce observations. Since the standard error is proportional to $1/\sqrt{n}$, and thus convex and decreasing in the amount of information, n, we might actually expect the increase in parameter uncertainty for pools with scarce observations to be much larger than the reduction in parameter uncertainty for pools with more observations in comparison to the case where observations are uniformly distributed across pools.

Secondly, insurance companies might deem consumers in different pools to be alike in their loss distributions and combine them in a joint pool. This would result in a larger pool with one premium but potentially more heterogeneous consumers. As the previous analysis shows, such joint pools would need to be very large in order to contain parameter uncertainty. The more pools are combined, the more heterogeneous are consumers in the joint pool. Thus, the larger are associated costs of adverse selection (Rothschild and Stiglitz (1976)) and the smaller are diversification benefits. Therefore, it is unlikely that insurers construct pools that are large enough to effectively diminish parameter uncertainty.

Thirdly, numerous risk factors are missing in our baseline estimation that yields 3.6×10^{10} insurance pools, for example geographic factors, policyholder and car age, mileage, or yearly driving performance. Although these factors are not necessarily categorical but continuous, including them only increases parameter uncertainty. For example, when including geographic discrimination, we yield on average 6×10^{-7} observations for each pool in each year. In this case, an insurer would on average have to wait for 1.7 million years to observe one loss per pool if observations were uniformly distributed across pools.

3 A Model for the Insurance Market

3.1 Setup

We assume risk neutral insurers and a continuum of consumers with mass 1 that maximize expected utility for a twice continuously differentiable and strictly concave von Neumann-Morgenstern utility function $u(w)$ with $u' > 0$, $u'' < 0$, and $u''' \geq 0$.[13] The timeline is as follows: First, insurers estimate the expected loss, second they offer insurance at unit price P, third consumers decide about optimal insurance coverage, finally losses occur. We evaluate social welfare at the beginning of the model, i.e. before estimation takes place.

Each consumer faces the risk of a random loss. Our model for losses is based on the framework of Doherty and Schlesinger (1995), who study the introduction of severity risk in models for an insurance market. In state 1 the loss with random size $\tilde{L} = L + \kappa$ occurs, where $L > 0$ and $\kappa \sim F_\kappa$ with $\mathbb{E}[\kappa] = 0$.[14] In state 2 no loss occurs. State 1 occurs with probability p, state 2 with probability $1 - p$. Hence, we distinguish between the frequency (i.e. loss probability) and severity (i.e. loss size) of losses. Then, the uncertain loss (i.e. 'costs') is $\tilde{C} = \mathbb{1}_{\{\text{state 1}\}}\tilde{L}$ and the expected loss is equal to $\mu = p\mathbb{E}[\tilde{L}] = pL$.

Insurers sell protection against the loss $\tilde{C}$. We assume that they know the loss probability, p, but not the expected loss size, L.[15] Based on the insurer's information about the loss size it estimates the expected loss size. We assume that each amount of information, n, relates to a distinct level of parameter uncertainty $\varepsilon = \varepsilon(n)$, such that the insurers' estimate for the expected loss size equals either $\vartheta = L + \varepsilon$ or $\vartheta = L - \varepsilon$ with probability 1/2. Therefore, ϑ is an unbiased estimator for the loss size with standard error ε. The estimator for the expected loss is $\tilde{\vartheta} = p\vartheta$ with standard error $p\varepsilon$. We assume that more information, n, relates to a smaller standard error, $\frac{d\varepsilon}{dn} < 0$.

In our model the variance of losses conditional on the expected loss size, L, equals

$$\mathbb{E}\left[(\tilde{C} - \mathbb{E}[\tilde{C} \mid L])^2 \mid L\right] = p(1-p)L^2 + p\sigma_L^2, \tag{5}$$

where $\sigma_L^2 = var(\tilde{L})$ is the variance of the loss size if a loss occurs. Ex ante, i.e. under parameter

[13] The assumption that $u''' \geq 0$ ensures that expected marginal wealth is not decreasing with the price of insurance. The necessity arises as we will assume a stochastic loss size. If $u''' > 0$, consumers are prudent, i.e. an increase in mean-preserving risk raises expected marginal utility (Eeckhoudt and Gollier (2005)).

[14] Alternatively, we might directly specify the loss size distribution $\tilde{L} \sim F_{\tilde{L}}$ with $\mathbb{E}[\tilde{L}] = L$.

[15] Note that mean-preserving uncertainty about p would not change the distribution of losses in the 2-state example: If $p \sim F_p$, the loss distribution is determined by $\mathbb{P}(\tilde{C} \leq x) = \mathbb{P}(\text{state 1})\mathbb{P}(\tilde{L} \leq x) + (1 - \mathbb{P}(\text{state 1}))\mathbb{1}_{0 \leq x} = \int p \int^x dF_{\tilde{L}} + (1 - p)\mathbb{1}_{0 \leq x} dF_p = \mathbb{E}[p]\mathbb{P}(\tilde{L} \leq x) + (1 - \mathbb{E}[p])\mathbb{1}_{0 \leq x}$. Thus, mean-preserving uncertainty about p does not impact the loss distribution.

uncertainty, it is straightforward to show that the variance of losses is equal to

$$\mathbb{E}\left[(\tilde{C} - \mathbb{E}[\tilde{C} \mid \vartheta])^2\right] = p(1-p)(L^2 + \varepsilon^2) + p\sigma_L^2. \tag{6}$$

Therefore, the variance under parameter uncertainty is equal to the variance without parameter uncertainty plus an additional term that accounts for parameter uncertainty, $p(1-p)\varepsilon^2$. This is analogous to the posterior predictive distribution for the Bayesian Normal model.

For the sake of simplicity, we assume that all insurers offer insurance to the same price. Hence, they all either under- or overestimate expected costs. In practice different insurance companies might yield different estimates due to different information. However, if risks are publicly observable (as catastrophes) or different insurers gain information from the sources (as from central data providers), it is likely that their estimates for expected losses are highly correlated.[16]

Conditional on its set of information, the insurer sets the price equal to its estimate for expected losses plus an additional safety loading, $P = p\vartheta + s(\varepsilon)$ (without loss of generality, we assume a zero risk-free rate). Upon observing prices, consumers decide about purchasing the product. If consumers purchase $q \geq 0$ units of the insurance contract, it pays $q\tilde{L}$ in state 1 and zero otherwise. For consumers, the expected loss size, L, is immediately known at the beginning of the period.[17] They believe that underestimating prices does not affect product quality, and, hence, always prefer a smaller price.[18] Ex ante, i.e. before firms place their offers, consumers are uncertain about the firms' information set and thus prices are random. However, observing prices does not change the consumers' belief about L, as they are certain about the distribution of $\tilde{C}$.[19]

Consumers also face a hidden background risk, η, i.e. an uninsurable risk, that might or might not be correlated with losses $\tilde{C}$. As Doherty and Schlesinger (1983) show, in the presence of background risk full insurance coverage is not necessarily optimal in case of a fair premium. In contrast, if the background risk is positively (negatively) correlated with losses, more (less) insurance can be optimal. We assume that the insurer does not know the characteristics of this background risk. This assumption

[16]Nevertheless, in Appendix C we conduct a sensitivity analysis towards the assumption of perfectly correlated prices by studying a model with two insurers that yield different estimates and consumers that face search costs. The sensitivity analysis shows that the less correlated prices are the more can consumers benefit from parameter uncertainty.

[17]The model also allows for the consumer's belief about L to be biased. For our results to hold it is only necessary that consumers, or at least the financial planner that maximizes social welfare, are certain about L ex ante.

[18]This assumption is reasonable for goods that are produced before they are sold (e.g. automobiles). For financial products (e.g. insurance or future contracts), however, underestimating costs (future cash flows in this context) impairs the financial institution's ability to serve its future obligations arising from the financial product it sold. In this article, we assume that consumers do not account for this case. This simplification might in practice result from consumers trusting insurance supervision and guarantee mechanisms (e.g. governmental guarantee schemes or central clearing) that secure future payments, as well as financial literacy.

[19]Extensions of this model might involve an update of the consumers' belief based on prices, for example in the sense of Spence (1974).

seems reasonable as, although insurers in practice frequently acquire information about risk factors that are positively correlated with losses, they rarely are aware of a consumer's other risks to wealth. Due to the assumption of such a background risk, insurers cannot infer from observed demand whether they under- or overestimated expected losses.[20]

After purchasing q units of the product, consumers' (uncertain) wealth at the end of the period is given as

$$w = w_0 - qP - (1-q)\tilde{C} - \eta, \tag{7}$$

where $w_0 \geq 0$ is an initial wealth endowment, q is (relative) insurance coverage, $\tilde{C}$ is the random insurable loss, P is the unit price for insurance, and η is the uninsurable background risk. Note that our model can easily be applied to other financial products. For example, in the case of a loan, the (negative) price, P, would reflect the loan, and the (negative) cash flow, $\tilde{C}$, would reflect the (uncertain) repayment and interest. In state 2 the borrower would default and, hence, not repay the loan.

Our objective is to study the marginal ex ante expected utility for an increase in parameter uncertainty, $\frac{dEU}{d\varepsilon}$, where $EU = \mathbb{E}[\mathbb{E}[w \mid P]]$ is the ex ante expected utility before insurers place their offer. From this ex ante perspective, the price either equals $P_- = p(L-\varepsilon) + s(\varepsilon)$ or $P_+ = p(L+\varepsilon) + s(\varepsilon)$ in case of under- or overestimation, respectively. Conditional on the realization of the price, consumers maximize expected utility by purchasing either q_- or q_+ quantities of the product. If $\frac{dEU}{d\varepsilon} > 0$, social welfare is ex ante increasing with the level of parameter uncertainty, i.e. consumers benefit from the firm being less informed, and vice versa.

For simplicity, the described model only involves one time period. It can easily be embedded in a multi-period model, where firms update their beliefs at the beginning of each period. Parameter uncertainty would still not vanish in a multi-period model, since firms are unlikely to employ all historical information to estimate costs if the distribution of costs varies over time. For example, Barry (1978) shows that the posterior mean of a Normal distribution is given as a geometrically weighted average over past observations when the distribution's mean value is subject to independent shocks. In this case, the most recent observations have substantially more weight than older observations, and the level of parameter uncertainty converges to a positive constant larger than zero.

[20] In the case without background risk, insurers could observe that they overestimated expected costs if consumers demanded less than full insurance coverage in the presence of no safety loading, and that they underestimated expected costs if consumers demanded (more than) full coverage. Insurers might use these information to decrease parameter uncertainty. Nevertheless, prices would not convergence as long as the distributional parameters of losses would change over time (Barry (1978)).

3.2 Expected Profits

Assume that $\mu = pL = \mathbb{E}[\tilde{C}]$ are the true expected costs. Then, if prices, P, were equal to estimated expected costs, $P = \tilde{\vartheta}$, *ex ante* expected profit would be

$$\mathbb{E}[\Gamma] = \mathbb{E}\left[q\left(\tilde{\vartheta} - \tilde{C}\right)\right] = -p\varepsilon\frac{q_- - q_+}{2} < 0. \tag{8}$$

Therefore, since demand is larger in case of underestimation than overestimation, ex ante expected profits are negative and decreasing with parameter uncertainty.

Hence, it is reasonable to assume that firms compensate negative ex ante expected profits with a safety loading on expected costs, such that prices are equal to $P = \tilde{\vartheta} + s$. Then, ex ante expected profits are given as

$$\mathbb{E}[\Gamma] = \mathbb{E}\left[q\left(\tilde{\vartheta} + s - \tilde{C}\right)\right] = s\frac{q_- + q_+}{2} - p\varepsilon\frac{q_- - q_+}{2}. \tag{9}$$

In a competitive market with zero ex ante expected profits, the safety loading compensates for parameter uncertainty such that

$$s = p\varepsilon\frac{q_- - q_+}{q_- + q_+}. \tag{10}$$

In other words, s ensures that the net present value of purchasing insurance is zero, ex ante. The resulting prices are $P_- = p(L - \varepsilon) + s$ and $P_+ = p(L + \varepsilon) + s$ in case of under- and overestimation, respectively, and, thus, the safety loading changes the average price but not the volatility of prices. It thereby prevents a transfer in expected wealth from firms to consumers, as $\mathbb{E}[q(P - \tilde{C})] < 0$ for $\mathbb{E}[P] < \mu + s$. Since s compensates negative ex ante expected profits that increase with ε, it is increasing with ε as well.

Since parameter uncertainty increases prices via the safety loading, competition incentivizes firms to acquire information in order to reduce parameter uncertainty, and thereby the average price. Nevertheless, insurers will only purchase an additional amount of information if marginal cost of information does not exceed the reduction in the safety loading. Since 1) information is likely to be restricted and, thus, 2) marginal cost of information is likely to increase for large amounts of information, firms are likely to retain a positive level of parameter uncertainty.

Remark 3.1 (Quantity restrictions.). *Instead of a safety loading on prices, insurers might as well restrict demand in case of underestimation such that $q_- \leq q_+$. Then, demand is strictly equal to q_+ in any case and ex ante expected profits are $\mathbb{E}[q(P - \tilde{C})] = 0$. However, since safety loadings are more easily observable* *in empirical insurance prices than restrictions in demand, in this study we focus on a safety loading on prices in order to enable the empirical analysis in Section 4.*

3.3 Social Welfare

If insurers correctly anticipate that they might over- or underestimate expected losses, they account for the resulting changes in consumer demand by increasing prices. In the previous section we have shown that the resulting safety loading on prices is such that consumers are not able to extract positive expected gains from speculating on the insurer underestimating expected losses. Therefore, the insurer's parameter uncertainty exclusively increases the risk of consumer's wealth. Thereby, it reduces social welfare, as the following lemma shows.

Lemma 1. *Assume that insurance prices are such that $P = p\vartheta + s$, where $s = p\varepsilon\frac{q_- - q_+}{q_- + q_+}$. Then, ex ante expected utility is decreasing with the level of parameter uncertainty, ε.*

Proof. Ex ante expected utility is given as

$$EU = \mathbb{E}\left[u\left(w_0 - q(P - \tilde{C}) - \tilde{C} - \eta\right)\right]. \tag{11}$$

As we have seen in Section 3.2, the safety loading ensures that $\mathbb{E}[q(P-\tilde{C})] = 0$. Consider an increase in the level of parameter uncertainty, ε. As $\mathbb{E}[q(P-\tilde{C})] = 0$ while prices as well as demand gain in volatility, the increase in parameter uncertainty is exclusively an increase in risk in the sense of Rothschild and Stiglitz (1970). As Rothschild and Stiglitz (1970) show, this decreases welfare for risk averse individuals. □

In other words, social welfare is increasing with the amount of information available to firms. The intuition behind this result is that expected consumer wealth does not change while it gains in risk. As consumers are risk averse, less information to firms reduces social welfare.

3.4 Small Safety Loadings

In practice, insurers need to estimate the demand function of consumers in order to properly set the safety loading for parameter uncertainty. Thus, there is an estimation error to the safety loading itself. While overestimating the safety loading does not change the main result (i.e. consumers still benefit from more informed firms), underestimating it enables a transfer of ex ante expected wealth from firms to consumers. Following this intuition, in this section we assess the impact of different levels of the safety loading for social welfare.

The following lemma is instrumental for our analysis and establishes a baseline condition for a firm's parameter uncertainty to increase welfare.

Lemma 2. *Assume that the unit price equals P_- and P_+ in case of under- and overestimation, respectively. Then, ex ante expected utility is increasing with parameter uncertainty if and only if*

$$-q_- \frac{dP_-}{d\varepsilon} \mathbb{E}\left[u'(w_-)\right] > q_+ \frac{dP_+}{d\varepsilon} \mathbb{E}\left[u'(w_+)\right]. \tag{12}$$

Proof: See Proof 1 in Appendix B.

Typically, we expect that consumers benefit from parameter uncertainty in case of underestimation, while utility is smaller in case of overestimation. As Lemma 2 shows, for parameter uncertainty to be beneficial for consumers, the marginal increase in expected utility in case a firm underestimates expected costs ($P = P_-$) needs to be larger than the marginal decrease in expected utility in case of overestimation ($P = P_+$).[21]

Example 1 (Risk neutral consumers)**.** *An extreme case are risk-neutral consumers. For simplicity, we assume that quantities are restricted to $0 \leq q \leq 1$. Then, for a given price P, risk neutral consumers maximize $EU = w_0 + q(\mu - P) - \mathbb{E}[\tilde{\eta}]$. Assume that $\mu < P_+$ for any $\varepsilon > 0$. Then, $q_+ = 0$ and ex ante expected wealth equals*

$$\mathbb{E}[w] = w_0 + \frac{1}{2} q\left(\mu - P_-\right) - \mathbb{E}[\eta]. \tag{13}$$

If insurers anticipate the missing demand in case of overestimation, the safety loading is properly set such that ex ante expected profits are zero and equals $s = p\varepsilon$ (see Section 3.2). Then, the price in case of underestimation equals $P_- = \mu$. In this case, risk neutral consumers are indifferent between different levels of the firm's parameter uncertainty.

In contrast, if the marginal safety loading is smaller, such that $\frac{dP_-}{d\varepsilon} < 0$, it is $q_- = 1$ and $\mathbb{E}[w]$ is increasing with ε. In this case, risk-neutral consumers benefit from an increase in parameter uncertainty.

If parameter uncertainty is sufficiently large, overestimated prices exceed the consumers' reservation price. In this case, welfare increases with parameter uncertainty if underestimated prices decline with parameter uncertainty. The next Corollary 1 extends this rationale from Example 1 for risk averse consumers.

[21] Note that this result comes without specifying the particular estimation procedure or how prices (and a potential safety loading) react to parameter uncertainty.

Corollary 1. *Assume that prices are given by $P = p\vartheta + s(\varepsilon)$. Define $\underline{\varepsilon} > 0$ and $\bar{\varepsilon} > 0$ as the smallest level of parameter uncertainty such that optimal demand is zero in case of overestimation and underestimation, respectively, and $\varepsilon^* = s'^{-1}(p)$. Then, ex ante expected utility is increasing for $\varepsilon \in [\underline{\varepsilon}, \min\{\bar{\varepsilon}, \varepsilon^*\})$ if the safety loading is convex, and increasing for $\varepsilon \in [\max\{\underline{\varepsilon}, \varepsilon^*\}, \bar{\varepsilon})$ if the safety loading is concave.*

Proof: See Proof 2 in Appendix B.

If the marginal safety loading is sufficiently small and parameter uncertainty is sufficiently large such that demand is zero in case of overestimation, social welfare is increasing with a firm's level of parameter uncertainty. Analogously to Example 1, under these assumptions parameter uncertainty decreases prices in case of underestimation, which then is the only situation in that consumers buy insurance.[22]

Example 1 shows that risk neutral consumers can benefit from an insurer's parameter uncertainty only if the marginal safety loading is small enough such that prices in case of underestimation decline with parameter uncertainty. If instead, the marginal safety loading is large enough, ex ante expected utility is decreasing with an insurer's parameter uncertainty.

Corollary 2. *Assume that the marginal safety loading is larger than p, $s'(\varepsilon) > p$. Then, ex ante expected utility is decreasing with parameter uncertainty.*

Proof. This follows from Lemma 2, as $-q_- \frac{dP_-}{d\varepsilon} < 0 < q_+ \frac{dP_+}{d\varepsilon}$. □

If insurers do indeed know the true level of expected costs, there is no parameter uncertainty ($\varepsilon = 0$). This case corresponds to a perfect information setting. The next corollary shows that in a situation with perfect information, welfare decreases with parameter uncertainty if insurers demand a positive safety loading for parameter uncertainty.[23] In other words, with a positive marginal safety loading, perfect information locally maximizes social welfare.

Corollary 3. *Assume that prices are given as $P = p\vartheta + s(\varepsilon)$ and that the marginal safety loading is positive in case of perfect information, $s'(0) > 0$. Then, $EU'(0) < 0$ and there exists $\hat{\varepsilon} > 0$ such that ex ante expected utility satisfies $EU(\varepsilon) < EU(0)$ for all $\varepsilon \in (0, \hat{\varepsilon})$.*

Proof: See Proof 3 in Appendix B.

The intuition of Corollary 3 is the following: An increase in parameter uncertainty is related to an increase in the average price, as given by $\mathbb{E}[P] = \mu + s(\varepsilon)$, since $s' > 0$. In case of perfect information, the increasing average price is not compensated by a sufficiently large increase in utility in case of underestimation, as marginal utility does not depend on information in case of perfect information.

[22] As expected, the safety loading that is required to yield nonnegative ex ante profits of the insurer (see Section 3.2) does not fulfill the requirements of Corollary 1 as $q_+ = 0$ for $\varepsilon > \underline{\varepsilon}$ and, hence, $s = p\varepsilon \frac{q_-}{q_-} = p\varepsilon$ and $s' = p$.

[23] Note that this result does not hold in Example 1 as demand is not continuously decreasing with price for risk neutral consumers and thus expected wealth is not differentiable at $\varepsilon = 0$.

The following corollary extends the conditions for social welfare to increase with parameter uncertainty. Most importantly, we show that consumers benefit from parameter uncertainty as long as the marginal and absolute level of the safety loading as well as risk aversion is sufficiently small.

Corollary 4. *Define by* $\underline{\varepsilon} > 0$ *and* $\bar{\varepsilon} > 0$ *the smallest level of parameter uncertainty such that consumers do not purchase any insurance coverage in case of overestimation and underestimation, respectively, and* $\varepsilon^* = s'^{-1}(p)$*. Let* $s'(0) > 0$ *and define*

$$\hat{\varepsilon} = \min\left\{\varepsilon > 0 : -q_- \frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\right] = q_+ \frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\right] > 0\right\} \tag{14}$$

and $\min(\emptyset) = \infty$*.*

a) Either $\hat{\varepsilon} < \underline{\varepsilon}$ *or* $\hat{\varepsilon} = \infty$*.*

b) If risk aversion is sufficiently small and the safety loading is convex, ex ante expected utility is increasing for $\varepsilon \in (\hat{\varepsilon}, \min\{\bar{\varepsilon}, \varepsilon^*\})$*. If the safety loading is concave, ex ante expected utility is increasing for* $\varepsilon \in (\max\{\varepsilon^*, \hat{\varepsilon}\}, \bar{\varepsilon})$*.*

c) If $s(0) = 0$ *and* $s'(\varepsilon) \leq \gamma < \infty$ *for all* ε*, then* $\lim_{\gamma\to 0}\hat{\varepsilon} \to 0$*.*

d) If risk aversion is sufficiently small, ex ante expected utility is increasing in ε *for* $\varepsilon \in \{\varepsilon > \hat{\varepsilon} : s'(\varepsilon) < p$ *and* $s(\varepsilon) \leq p\varepsilon\}$*.*

Proof: See Proof 5 in Appendix B.

As already proven in Corollary 3, under perfect information, $\varepsilon = 0$, welfare is decreasing with parameter uncertainty if there is a positive marginal safety loading. The reason is, that for small ε the increase in the average price via the safety loading is not sufficiently compensated by an increase in expected utility in case of underestimation. In Corollary 4, $\hat{\varepsilon}$ is the minimum level of parameter uncertainty such that expected utility indeed compensates for the increasing average price. Intuitively, the critical level, $\hat{\varepsilon}$, is decreasing with the marginal safety loading, as Corollary 4 c) shows. For a large level of parameter uncertainty, $\varepsilon > \hat{\varepsilon}$, welfare is increasing with ε under the conditions that the marginal and absolute level of the safety loading as well as risk aversion are sufficiently small. In this case, the increase in welfare due to a larger expected utility in case of underestimation exceeds the reduction in welfare due to a larger average price.

Example 2. *We illustrate the result from Corollary 4 with a numerical example. For this purpose, we study a representative CRRA-maximizing consumer with a coefficient of relative risk aversion equal to*

$\gamma = 0.8$. *This level of risk aversion is consistent with the results from Harrison and Rutström (2008), who estimate risk aversion based on experimental evidence. The consumer's endowment wealth is equal to* $w_0 = 100$, *and losses occur with probability* $p = 0.2$ *and exhibit a normally distributed loss size with expected value* $L = 50$ *and standard deviation* $\sigma_L = 10$.[24] *For simplicity, we assume the absence of any background risk.*

We assume that it is not possible for consumers to purchase excess insurance, i.e. we assume that indemnity payments are bounded from above by occurred losses, $q \leq 1$, *as it can also be observed in practice. The reason is that with excess insurance, consumers benefit from losses, which increases the incentive for consumers to increase the loss probability or size. This situation is commonly referred to as moral hazard (Shavell (1979)).*

In Section 3.2 a safety loading for parameter uncertainty directly originates from uncertain demand that is anticipated ex ante by the insurer. In this example, we assume that the insurer does not anticipate the uncertainty in demand. Instead, we assume that a safety loading results from costs of a capital for a solvency capital requirement that accounts for parameter uncertainty. More specifically, we assume that the insurer is subject to a solvency constraint that requires a maximum probability of default of $\psi \times 100\%$,

$$P\left(q\tilde{C} \geq qp\vartheta + E\right) \leq \psi, \tag{15}$$

where $E \geq 0$ *is the insurer's endowment with external capital (i.e. equity). Such a constraint would, e.g., result from a risk-based regulatory capital requirement that takes parameter uncertainty into account.*[25] *The constraint might as well serve as an internal risk management measure. As the provision of the required external capital is costly, insurance prices increase.*

We assume that the insurer sells a sufficiently large number of independent insurance contracts such that the de Moivre-Laplace theorem applies. In line with the fiducial argument as well as the Bayesian posterior predictive distribution described in Sections 2.2 and 3.1, total losses are conditional on the insurer's estimate, ϑ, *approximately normally distributed,*

$$q\tilde{C} \sim \mathcal{N}(qp\vartheta,\, q^2(p(1-p)(\vartheta^2 + \varepsilon^2) + p\sigma_L^2)). \tag{16}$$

[24] This level of standard deviation and expected loss size is in line with typical loss distributions. For example, a popular dataset covering 1,500 US indemnity losses that is e.g. analyzed in Frees and Valdez (1998) displays an average loss of 41.21 and standard deviation 10.27 US dollar. Thus, the ratio of standard deviation to expected loss size is roughly equal to 0.25, as in our example.

[25] For example, the European regulatory regime Solvency II requires insurers to remain solvent with at least 99.5% probability, i.e. $\psi = 0.5\%$. However, currently it does not take parameter uncertainty into account. Hoy (1988) derives insurance prices when insurers face a solvency constraint that is similar to ours.

This distribution results from the insurer's subjective distribution of losses after observing information that lead to the estimate ϑ. The volatility of the predictive distribution takes parameter uncertainty into account, as measured by ε. The capital requirement to satisfy the solvency constraint is then given by[26]

$$E = q\sqrt{p(1-p)(\vartheta^2+\varepsilon^2)+p\sigma_L^2}\,\Phi^{-1}(1-\psi). \tag{17}$$

It is straightforward to verify that $\frac{dE}{d\varepsilon} > 0$ and $\frac{d^2E}{d\varepsilon^2} < 0$. Hence, E is convex and monotonically increasing in ε.

To simply the calculation and illustration, we refrain from a stochastic safety loading as implied by its dependence on the estimate ϑ, and assume that $\vartheta = L$ for the purpose of calculating the safety loading.[27] *We assume that the insurer conservatively holds enough equity, E, to satisfy maximal demand, $q = 1$. The cost of capital rate is given by $\chi \in (0,1)$. Analogously to Hoy (1988), the resulting safety loading on insurance prices is*

$$s(\varepsilon) = \chi\sqrt{p(1-p)(L^2+\varepsilon^2)+p\sigma_L^2}\,\Phi^{-1}(1-\psi). \tag{18}$$

We assume that the cost of capital rate is equal to $\chi = 0.06$, and that the insurer's probability of default is constrained by $\psi = 0.005$. These values correspond to the calibration of the European regulatory framework Solvency II as specified by the European Insurance and Occupational Pensions Authority (EIOPA) (2014).

We study different relative levels of parameter uncertainty, $\tilde{\varepsilon} = \varepsilon/L$. The relative level of parameter uncertainty such that the marginal safety loading equals the marginal decrease in underestimated prices, $s'(\varepsilon) = p$, is given as

$$\tilde{\varepsilon}^* = \frac{1}{L}\sqrt{\frac{p(1-p)L^2+p\sigma_L^2}{(1-p)^2\left(\Phi^{-1}(1-\psi)\right)^2\chi^2 - p(1-p)}}. \tag{19}$$

In this example, $\varepsilon^* = \infty$, since

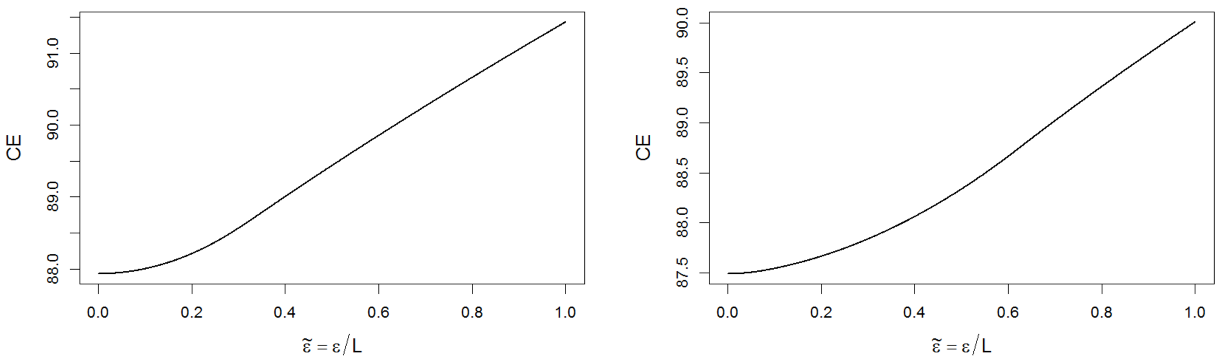
$$\lim_{\varepsilon\to\infty} s'(\varepsilon) = \chi\sqrt{p(1-p)}\Phi^{-1}(1-\psi) = 0.062 < p \tag{20}$$

for $\chi = 0.06$. Therefore, the marginal safety loading is smaller than the decrease in prices, $s'(\varepsilon) < p$ for all levels of parameter uncertainty.

[26] Fröhlich and Weng (2015) derive parameter uncertainty sensitive capital requirements for a Solvency II framework in a similar manner.

[27] A stochastic safety loading will be slightly smaller in case of underestimation than in case of overestimation. Thus, our results will be slightly biased towards a non-welfare improving effect of parameter uncertainty.

Figure 1 (a) depicts the ex ante certainty equivalent for different levels of parameter uncertainty.[28] *It reflects the level of deterministic consumer wealth that yields the same expected utility as the prospect of purchasing insurance after observing uncertain insurance prices. The safety loading and the level of risk aversion are sufficiently small such that the ex ante certainty equivalent is increasing with the level of parameter uncertainty. Hence, prices are small enough, such that consumers can increase welfare by speculating on the insurer underestimating the expected loss. While a larger cost of capital rate increases prices and, thus, reduces social welfare, a high but still reasonable level of* $\chi = 0.1$ *is not sufficient to compensate the insurer for uncertain demand, as Figure 1 (b) shows.*



(a) $\chi = 0.06$. (b) $\chi = 0.1$.

Figure 1: Ex-ante expected utility with safety loading for $\chi = 0.1$ and $\chi = 0.3$, $\psi = 0.005$, $\gamma = 0.8$, $p = 0.2$, $w_0 = 100$, $L = 50$, and $\sigma_L = 10$. The x-axis corresponds to the relative level of parameter uncertainty $\tilde{\varepsilon} = \varepsilon/L$.

This example shows, that a possible capital requirement that takes an insurer's parameter uncertainty into account with reasonable values for cost of capital, is not necessarily sufficient to prevent consumers from benefiting from the firm's uncertainty about expected loss. In contrast, social welfare increases with the insurer's parameter uncertainty. Hence, consumers do not desire firms to gain in knowledge about expected losses. In contrast, there is a transfer of ex ante expected wealth from insurers to consumers that results from demand adapting to under- and overestimation.

The following lemma shows, that in the absence of any safety loading, social welfare is increasing with the insurer's level of parameter uncertainty if risk aversion is small enough. It highlights the fact that only consumer with a small degree of risk aversion can benefit from speculating on an insurer underestimating expected losses, while more risk averse consumers suffer from an increase in price volatility.

[28] Since there is no closed-form solution for expected CRRA utility with normally distributed wealth, we employ a Monte-Carlo procedure with sample size 100,000. In realizations with losses and premium payments exceeding the wealth endowment we set wealth equal to zero.

Lemma 3 (Social welfare without safety loading)**.** *Assume that consumers can purchase any partial coverage $q \in [0,1]$ without safety loading such that the total price for insurance is given by $P(q) = qP = qp\vartheta$. Then, marginal ex ante expected utility is strictly positive for all $\varepsilon > 0$, $\frac{dEU}{d\varepsilon} > 0$, if consumers' risk aversion is sufficiently small.*

Define $\underline{\varepsilon}$ such that $P_+ = p(L + \underline{\varepsilon})$ is equal to the consumers' reservation price. Then, $\frac{dEU}{d\varepsilon} > 0$ for $\varepsilon \geq \underline{\varepsilon}$ regardless of the level of risk aversion.

Proof: See Proof 4 in Appendix B.

Example 3 (CRRA consumers)**.** *We continue Example 2 by considering CRRA-maximizing consumers with endowment wealth $w_0 = 100$. Losses occur with probability $p = 0.2$ and exhibit a normally distributed loss size with expected value $L = 50$ and standard deviation $\sigma_L = 10$.*

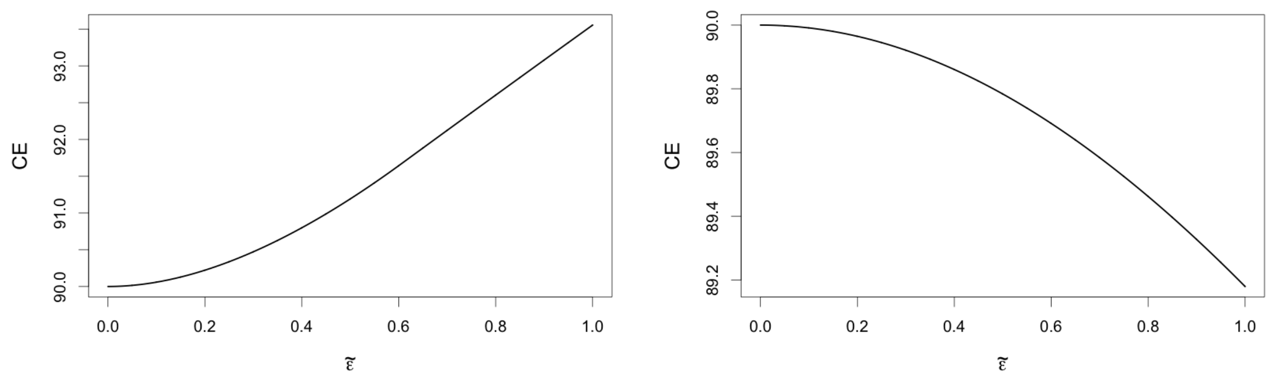
In Figure 2 we study two different levels of risk aversion: $\gamma = 0.8$ and $\gamma = 4$. For $\gamma = 0.8$ in Figure 2 (a), welfare is increasing for any level of parameter uncertainty $\varepsilon > 0$. The reason is that due to a relatively low level of risk aversion, the price elasticity of insurance demand is such that optimal coverage in case of overestimation declines sufficiently fast. This diminishes the a potential reduction in welfare due to overestimation.

If risk aversion increases to $\gamma = 4$, the ex ante certainty equivalent decreases with parameter uncertainty, as Figure 2 (b) shows. In this case, insurance demand is not sufficiently elastic as too risk averse consumers are not willing to give up enough insurance coverage in order to compensate the reduction in utility by overestimated prices. Nevertheless, if $\tilde{\varepsilon}$ is large enough such that the overestimated price, P_+, exceeds the reservation price, EU again increases with parameter uncertainty, as then consumers purchase insurance only in case of underestimation.

4 Safety Loadings in Insurance Markets

The theoretical model in Section 3 implies that insurance companies need to anticipate that demand adjusts to under- and overestimated expected losses; otherwise the prospect of purchasing insurance has a positive net present value for consumers. We have derived the level of the average price increase ('safety loading') that prevents consumers from benefiting ex ante from an insurer's parameter uncertainty.

In the following we examine whether insurance premiums in the US property and casualty insurance market indeed incorporate a safety loading for parameter uncertainty. In this market in particular we expect substantial differences in the level of parameter uncertainty across different lines of business. For example, for losses from rare extreme events like earthquakes or that depend on human behavior, like



(a) $\gamma = 0.8$.

(b) $\gamma = 4$.

Figure 2: Ex-ante certainty equivalent for $w_0 = 100$, $p = 0.2$, $L = 50$ and $\sigma_L = 10$. The x-axis corresponds to the relative level of parameter uncertainty $\tilde{\varepsilon} = \varepsilon/L$.

burglary and theft or fidelity insurance, there is far less information than for frequently occurring events in common insurance lines as automobile accidents or multiple peril insurance.[29]

4.1 Data

We extract premium loadings on actuarially fair insurance premiums from the combined ratio of an insurance company in a particular insurance line. The combined ratio is defined as the ratio between aggregate losses and expenses incurred, and aggregate net premiums earned during one year,

$$CR = \frac{\sum_{k=1}^{N} \tilde{C}_k + E_k}{NP}, \tag{21}$$

where N is the number of contracts, and $\tilde{C}_k$ and E_k the incurred loss and expenses per contract, respectively. If premiums did not incorporate any loading on the actuarially fair premium, $P = \mu = \mathbb{E}[\tilde{C}_k]$, the combined ratio would be approximately equal to one for a sufficiently large number contracts. If the combined ratio is larger than one, $CR > 1$, the company makes an underwriting loss, while a smaller combined ratio, $CR < 1$, indicates that the company makes an underwriting profit.[30]

The law of large numbers implies that the average loss incurred approximates the expected loss, $\sum \tilde{C}_k/N \approx \mu$, if the company writes a sufficiently large number of contracts. We assume that prices equal the sum of the company's estimate for the average loss, $\tilde{\vartheta}$, a safety loading for parameter uncertainty, s, and potentially other loadings, p, e.g. to increase profits. Then, we rewrite the combined ratio such

[29] Although parameter uncertainty might still be large in distinct contract pools in motor insurance, as argued in Section 2.3, in this section we focus on aggregate premiums.

[30] The combined ratio might also be larger than one due to losses being paid by investment returns. However, since investment gains are independent from the parameter uncertainty in different lines of insurance, these do not affect our analysis.

that[31]

$$1 - CR \approx \frac{\tilde{\vartheta} - \mu + s + p}{P}. \quad (22)$$

In line with the previous section, we expect the safety loading, s, to be larger in lines of business with less information. These are in particular insurance lines with less observations of historical losses, either due to a general limitation of available observations, frequently changing parameters and risk characteristics, or less sold insurance contracts. As the estimation error, $\tilde{\vartheta} - \mu$, is unlikely to be correlated across insurance lines, effects of the insurance lines on $1 - CR$ will exclusively be due to differences in the relative premium loading, $\frac{s+p}{P}$.

If parameter uncertainty about losses was large for both insurers and consumers, prices might also include large profit loadings, since consumers would have difficulties to assess the actuarially fair premium.[32] However, this effect crucially depends on the degree of competition. For more competitive markets we expect profit loadings to be smaller than for less competitive markets. One measure for the competitiveness is the number of active companies in a particular line of insurance, $Compet$, (Browne and Hoyt (1995)). The basic idea of this variable is, that, although their market shares might differ, more active companies will on average relate to more competition.

Another measure that describes competition is the Herfindahl-Hirschman (HHI) index, HHI, which measures the degree of market concentration. We compute the normalized HHI index based on the proportion of squared net premiums written by different companies in a particular line of insurance.[33] If one company dominates the market in a particular insurance line, the normalized HHI index tends to 1, and vice versa. A large concentration could on the one hand relate to large market power and, hence, the possibility of large profit loadings. On the other hand large concentration might result from intensive competition driving profit loadings down until only a few companies are left in the market. To control for these two possibilities, we interact concentration with the number of active companies, $Compet$.

The volatility of losses differs substantially between insurance lines. The average standard deviation of quarterly losses incurred across insurance companies and years ranges from approximately 320,000 USD for burglary and theft losses to 211 mil USD for financial guaranty losses. In order to exclusively focus on the effect of a different amount information, we normalize the premium loading, $1 - CR$, by the standard

[31] $1 - CR$ might also be called *underwriting rate of return*. Here, we will, equivalently, refer to $1 - CR$ as premium loadings.

[32] Note the distinction between a safety and profit loading: Safety loadings compensate for negative expected underwriting returns, as in Section 3.2. Profit loadings yield positive expected profits and thus can only be presented in less competitive markets.

[33] Net premiums written is the sum of premiums received in a particular year, less premiums ceded to reinsurance companies, plus premiums for assumed reinsurance. Thus, it represents the ultimate absolute market share of an insurance company.

deviation of direct incurred losses (in million USD), σ, for each insurance line and insurance company.[34]

Annual combined ratios and financial data are from A.M. Best Company for US property and casualty insurers in fiscal years 2005 to 2015. These are filed with the National Association of Insurance Commissioners (NAIC). Firstly, we exclude observations from aggregate and residual lines of business, which are *Aggregate Write Ins*, *International*, *Reinsurance-nonproportional assumed*, *Other Liabilities*, *Other Liabilities by Occurrence*, and *Other Liabilities by Claims*. Secondly, due to large outliers that are likely to result from extreme events and data inconsistencies, we winsorize the observations for the combined ratio and net premiums written by excluding 1% of the largest and smallest observations, respectively. Thirdly, we estimate the standard deviation of incurred losses based on quarterly reports about company and insurance line specific direct incurred losses (in million USD) that are provided by A.M. Best Company from the fourth quarter of 2012 to the second quarter of 2017. We winsorize the observations of loss volatility at the 1% level to limit the impact of estimation errors.

Table 1 summarizes the resulting data sample. For the year 2015 our samples includes 1986 companies, which represents roughly 75% of US property and casualty insurance companies in 2015.[35] Table 5 in Appendix D provides an overview on lines of business included in the sample and descriptions of the insurance coverage provided in these lines.

Statistic	N	Mean	St. Dev.	Min	Max
Combined Ratio	112,302	0.98	0.59	−0.39	5.24
Net Premiums Written (in thd USD)	112,302	46.43	153.84	−0.15	2,154
Competition ($Compet$)	112,302	856	337	20	1,282
Concentration (HHI)	112,302	0.08	0.01	0.07	0.16
Standard dev. of losses ($Vola$, in mil USD)	112,302	4.52	12.68	0.001	169.47

Table 1: Summary statistics of observations discriminated by insurer, insurance line, and year. Loss volatility is not year-specific and estimated based on quarterly observations of direct losses incurred. The sample covers direct US property and casualty insurers from 2005 to 2015 based on data provided by A.M. Best Company.

4.2 Empirical Analysis

We expect that parameter uncertainty increases insurance prices via a safety loading on the actuarially fair price. Then, premium loadings in excess of volatility are larger for lines of insurance that exhibit a larger parameter uncertainty. To test this hypothesis, we control for a company's financial condition as

[34]For example, assume that loadings, $s+p$, are proportional to the volatility of the predictive distribution in case of a normal distribution, $s+p \propto \sqrt{\sigma^2+\sigma^2/n}$. Then, normalization yields $(s+p)/\sigma \propto \sqrt{1+1/n}$, and the resulting variable solely depends on the number of observations n.

[35]The National Association of Insurance Commissioners (2016) reports 2642 property and casualty filers in the US market in 2015.

well as the degree of market competition in order to subtract the effect of potential profit loadings. Then, (relative) premium loadings are given by

$$\begin{aligned}\frac{1-CR_{i,L,t}}{\sigma_{i,L}} =&\beta_L LoB_L + \beta_1 Compet_{L,t} + \beta_2 HHI_{L,t} + \beta_3 Compet_{L,t} * HHI_{L,t} \\ &+ \beta_4 year_t + \beta_5 company_i + \varepsilon_{i,L,t}, \end{aligned} \tag{23}$$

where LoB_L is a dummy variable for each line of business, $Compet_{L,t}$ is the number of active companies and $HHI_{L,t}$ the normalized Herfindahl-Hirschman index for net written premiums in line L in year t. $\sigma_{i,L}$ is the standard deviation of incurred losses for company i in line L.

Table 6 in Appendix D presents the detailed results of the regression. Table 2 depicts a ranking of lines of business according to their premium loading as implied by β_L in Regression (1). Firstly, we observe that loadings range between -4 and 103 units of loss volatility and, thus, are economically significant.[36] As Table 6 in Appendix D reveals, only earthquake and burglary and theft insurance exhibit a loading that is statistically significantly different from zero. However, additional unreported regressions show that differences between insurance lines are highly significant.[37]

Secondly, Table 2 shows that large loadings are present particularly in earthquake, burglary and theft, financial insurance, fidelity, product liability, ocean marine, and surety insurance. Accident and health (A& H) insurance, allied lines, multiple peril, and private passenger physical damage insurance is related to a particularly small loading.[38]

This ranking corresponds to an intuitive assessment of the parameter uncertainty in different lines of insurance. For example, observations are particularly rare for catastrophic events as earthquakes, which are at the top of Table 2. Losses that largely depend on human behavior, as in burglary and theft, fidelity, product liability, or surety insurance, are naturally associated with large uncertainty due to frequently changing and very heterogeneous risk characteristics. Moreover, the financial crisis 2007-08 revealed large parameter uncertainty with regard to pricing financial risks, which explains the large premium loading for mortgage and financial guaranty insurance.

Additionally, the more common an insurance product is, the more observations are available and, thus,

[36] A negative safety loading indicates that accounting for the estimated profit loading leads to a larger than observed price. This case only occurs for *Other Accident & Health* insurance. We interpret it as an estimation error of the profit loading and handle $\beta_L < 0$ as if $\beta_L = 0$.

[37] For example, earthquake as well as burglary and theft insurance exhibit a significantly larger loading in comparison to all other lines; private passenger auto physical damage insurance exhibits a significantly smaller safety loading than earthquake, burglary and theft, fidelity, product liability, inland marine, commercial auto liability, and private passenger auto liability insurance.

[38] A robustness check for Regression (1) without company fixed effects and without normalizing by loss volatility confirms that loadings for earthquake, burglary and theft, fidelity, and surety insurance are among the largest, while loadings for commercial and homeowners' multiple peril, allied lines and private passenger auto physical damage are among the smallest. The estimates for the robustness check can be found in Table 7 in Appendix D.

LoB (Regression (1))	β_L
Earthquake	103.43
BurglaryTheft	71.77
MortgageGuaranty	52.58
FinancialGuaranty	49.36
Fidelity	37.17
ProductsLiab	28.69
OceanMarine	24.45
Surety	23.43
Credit	22.29
Aircraft	21.52
ExcessWorkersCompensation	20.23
Warranty	19.38
BoilerMachinery	18.69
PrivatePassengerAuto_Liab	18.58
MedicalProfLiab	18.21
InlandMarine	14.24
WorkersCompensation	9.56
FarmownersMultiplePeril	9.10
CommericalAuto_Liab	6.99
CommercialMultiplePeril_Liab	6.29
Fire	5.66
GroupAH	5.20
PrivatePassengerAuto_PhysicalDamage	4.24
AlliedLines	3.96
HomeownersMultiplePeril	3.69
CreditAH	0.54
OtherAH	-4.01

Table 2: Ranking of lines of business (LoBs) according to premium loadings based on estimates for β_L in Regression (1).

the smaller is parameter uncertainty. Therefore, it is not surprising that we find some of the insurance lines that exhibit a particularly small coefficient β_L among the largest insurance lines according to aggregate premiums (as multiple peril and private passenger auto physical damage insurance).[39] Vice versa, some of the insurance lines that exhibit a particularly large safety loading are among the smallest insurance lines according to aggregate premiums (as burglary and theft, fidelity, and financial guaranty insurance).

However, very common insurance products might exhibit a small average premium and, thus, are not associated with a particularly large aggregate premium. This might explain why accident and health (A& H) insurance exhibits a small loading but also small aggregate premiums. Nevertheless, differences in aggregate premiums across companies might reflect the number of sold contracts: For example, if an insurance company sold more A& H products than other companies, it is likely to face a smaller level of parameter uncertainty. Thus, we expect the premium loading to be smaller for this company.

An additional regression analysis in Appendix D confirms that particularly large (small) insurance premiums in comparison to other lines and companies relate to particularly small (large) loadings. We focus on extremely large or small aggregate premium loadings in a particular year, as it is most likely that these are caused by different numbers of contracts purchased.[40] In Table 9 in Appendix D we report

[39]Aggregate premiums for each insurance line can be found in Table 8 in Appendix D.

[40]Some evidence for this intuition is provided by comparing the aggregate premiums in Table 8 in Appendix D in our

the results of a logit regression of the premium loading (relative to loss volatility) being among the 25% largest in a particular year on differences in insurance lines. We find that such particularly large (small) aggregate premiums are highly significantly related particularly small (large) relative premium loadings. As we control for concentration, an increase in written premiums does not relate to a larger market share. If, nonetheless, our results would reflect a relation between premiums and profit loading but not safety loading, there is no compelling reason why it should be negative.

In summary, it seems very likely that the differences in premium loadings resulting from Regression (1) result from a safety loading for parameter uncertainty. Possible reasons for differences in premium loadings other than a safety loading for parameter uncertainty include different time horizons in claim settlement. Long-tail business can particularly be found in (auto) liability insurance or workers compensation. In these lines it can take substantial time until claims are settled. Long-tail business might result in larger loadings since the final settlement claims are less certain. In addition, business expenses are likely to be larger in long-tail business, as claims processing takes more time in these lines. However, we do not find these insurance lines to exhibit particularly large loadings in Table 2.

Moreover, insurers might require an upfront loading for moral hazard. Moral hazard describes situations in which individuals behave more risky after purchasing insurance and, thereby, potentially increase losses (Shavell (1979)). However, there is no compelling reason for differences in the risk of moral hazard that would explain the differences between insurance lines found in the regressions.

4.3 Comparison of Empirical and Theoretical Safety Loadings

In the following we compare the empirical safety loading identified in the previous Section 4.2 with the one implied by our model in Section 3.2. For this purpose we make the simplifying assumption that demand is exponential in price, i.e. $q(P) = \min\left(1, e^{a-bP}\right)$. We define by

$$\tilde{s}(\varepsilon) = p\varepsilon \frac{q_- - q_+}{q_- + q_+} = p\varepsilon \left(1 - \frac{2}{1 + e^{2bp\varepsilon}}\right) \tag{24}$$

the safety loading for parameter uncertainty as in Section 3.2 for $q_- \leq 1$. As it is common practice among insurers, we restrict demand by 100%. Thus, if $e^{a-b(p(L-\varepsilon)+\tilde{s})} > 1$, we set $q_- = 1$ and the safety loading

sample with the actually available number of contracts purchased in Germany in different insurance classes as provided by German Insurance Association (GDV) (2016). Both aggregate premiums in our sample and number of contracts reported by German Insurance Association (GDV) (2016) are largest for motor insurance and multiple peril (property) insurance. Those for ocean marine and aircraft (by German Insurance Association (GDV) (2016) reported as *marine and aviation*) and surety, and fidelity insurance are substantially smaller. Due to these observations, we find it very likely that extremely large or small aggregate premiums in a particular line are caused by a particularly large or small number of contracts purchased, respectively.

is given by $\hat{s}$ which is the solution to $\hat{s} = p\varepsilon\left(\frac{2}{1+e^{a-b(p(L+\varepsilon)+\hat{s})}} - 1\right)$.[41] Then, in general the safety loading is given as

$$s(\varepsilon) = \begin{cases} \tilde{s}(\varepsilon), & \text{if } e^{a-b(p(L-\varepsilon)+\tilde{s})} \leq 1, \\ \hat{s}(\varepsilon), & \text{else.} \end{cases} \tag{25}$$

As in the numerical example before, we do not consider any additional background risk. Then, it is $q(pL) = 1$ (as shown by Doherty and Schlesinger (1995)) and, hence, $a = bpL$. We estimate b by minimizing the sum of squared deviations between $q(P)$ and the optimal insurance coverage implied by maximizing expected CRRA utility. Optimal insurance coverage is computed by numerically maximizing expected CRRA utility in a Monte-Carlo simulation with 20,000 loss realizations.

For this purpose, we need to determine the loss distribution for the different insurance lines studied in Section 4.2. Since detailed data about individual losses is not publicly available, we base our estimation on nonnegative quarterly incurred losses for each insurance company and insurance line. For simplicity, we set the probability of loss occurrence equal to $p = 1$. We fit two different severity distributions to observed losses. The first is a normal distribution. The second distribution is a log-normal distribution, that is often considered particularly for catastrophe losses as it exhibits fat tails. For each insurance company and each line business, we yield estimates for each loss distribution, and end up with $1986 \times 27 \times 2 = 107,244$ estimates.

It would take enormous computation time to numerically estimate demand functions for each company-line pair. As companies differ in their exposure to different insurance lines, we cannot average the estimates. To overcome this issue, for each insurance line we employ the parameter estimates for the company with the median exposure (as given by the median value of the average loss incurred). These are reported in Table 3. For both distributions, the resulting loss volatility is $var(\tilde{L}) = \sigma_L^2$ as $p = 1$ and, hence, the relative safety loading per unit of volatility is $s_{rel}(\varepsilon) = \frac{s(\varepsilon)}{(L+s(\varepsilon))\sigma_L}$.[42] As before, we interpret ε as the standard error of the insure's estimator for the expected loss size, i.e. $\varepsilon = \sigma_L/\sqrt{n}$, where n is the number of historical observations associated with the respective standard error.

In the previous Section 4.2 we derive estimates, β_L, for the empirical relative safety loading in a particular insurance line, that are reported in Table 2. Now we compare this empirical safety loading with the theoretical safety loading that is required for an ex ante zero net present value of insurance (Section 3.2). For this purpose, for a given level of wealth, w_0, and level of risk aversion, γ, we now

[41]As $s = p\varepsilon\frac{q_- - q_+}{q_- + q_+} = p\varepsilon\left(1 - 2\frac{q_+}{q_- + q_+}\right)$ is increasing with q_-, $\hat{s}$ is smaller than $\tilde{s}$ and, therefore, it is $e^{a-b(p(L-\varepsilon)+\hat{s})} \leq 1$ if $e^{a-b(p(L-\varepsilon)+\tilde{s})} \leq 1$.

[42]Note that, as before, σ_L is the standard deviation of quarterly losses.

Insurance Line	μ (Normal)	σ (Normal)	μ (Log-Normal)	σ (Log-Normal)
Aircraft	0.41	0.69	-0.62	0.96
AlliedLines	0.26	0.84	-2.28	1.48
BoilerMachinery	0.03	0.12	-4.44	2.09
BurglaryTheft	0.00	0.02	-5.83	2.15
CommercialMultiplePeril_Liab	1.09	2.62	-2.42	2.62
CommericalAuto_Liab	0.86	0.67	-0.16	0.65
Credit	0.27	0.60	-2.24	1.89
CreditAH	0.04	0.11	-3.61	1.40
Earthquake	0.01	0.01	-5.54	1.18
ExcessWorkersCompensation	0.41	0.53	-0.55	1
FarmownersMultiplePeril	0.36	0.52	-1.16	0.60
Fidelity	0.02	0.05	-3.98	0.82
FinancialGuaranty	0.05	0.10	-2.56	1.16
Fire	0.19	0.21	-2.00	0.91
GroupAH	0.24	0.39	-3.05	2.08
HomeownersMultiplePeril	1.51	0.66	0.32	0.44
InlandMarine	0.12	0.10	-2.30	0.82
MedicalProfLiab	0.44	0.40	-1.22	0.99
MortgageGuaranty	0.38	0.54	-1.84	2.15
OceanMarine	0.16	0.33	-3.11	2.25
OtherAH	0.06	0.06	-2.94	0.80
PrivatePassengerAuto_Liab	2.32	0.80	0.78	0.40
PrivatePassengerAuto_PhysicalDamage	1.39	0.78	0.18	0.57
ProductsLiab	0.16	0.33	-2.63	1.58
Surety	0.12	0.37	-1.12	0.95
Warranty	0.29	0.35	-1.48	0.73
WorkersCompensation	1.64	1.89	0.01	1.03

Table 3: Results from fitting of normal and log-normal distributions to empirical observations of quarterly direct losses incurred for each company and insurance lines in million USD. The parameters are estimated by maximum likelihood. For each insurance line we report the parameter value for the company with the median average losses incurred.

compute the minimum number of observations, $n_{\min}$, such that the theoretical safety loading implied by Equation (25) is at least as small as the empirically estimated safety loading, $\beta_L \leq s_{rel}$. For the level of relative risk aversion we assume $\gamma = 0.8$, which is in line with the results of Harrison and Rutström (2008).

Due to the absence of consumer individual data, we examine the theoretical situation of a representative consumer that faces a loss that is distributed as quarterly industry-wide losses. We derive the theoretical safety loading at least needed such that this consumer does not benefit from speculating on the insurer underestimating expected losses. Thus, empirical safety loadings need to be at least as large as this theoretical safety loading as they also incorporate other costs that increase with parameter uncertainty, for example costs for external funding. Based on a Monte-Carlo procedure with losses distributions as given in Table 3, we compute the minimum number of observations, $n_{\min}$, for each insurance line such that the theoretical safety loading is smaller than or equal to the empirically observed. If an insurer is able to base the estimation of expected losses on at least $n_{\min}$ observations, the empirical loading is sufficiently large to prevent nonnegative ex ante expected losses due to uncertain demand.

We compute $n_{\min}$ for different levels of endowment wealth $w_0 = \max\{\Delta\mu_L, \mu_L + \Delta\sigma_L\}$ with $\Delta \in$

$\{2, 3, 4\}$. The level of endowment wealth is chosen in order to account for different sizes of the insurance lines and resulting differences in the average loss μ_L. As consumers are risk averse, we expect price elasticity of demand and thus the theoretical safety loading to increase with deterministic wealth. Therefore, the required number of observations for the empirical safety loading to be sufficiently large, $n_{\min}$, is likely to increase with w_0, which is confirmed by our results.

Table 10 reports $n_{\min}$ across different wealth endowments w_0. As the log-normal distribution exhibits larger tails than the normal distribution, price elasticity of insurance demand and, hence, the theoretical safety loading are smaller. Therefore, a smaller number of observations, $n_{\min}$, than with the normal distribution is needed, as Table 10 shows.

We find that $n_{\min}$ is extremely small for all insurance lines. Indeed, $n_{\min} = 1$ for most insurance lines. This finding suggests that, in practice, safety loadings are large enough to compensate for changes in demand due to under- and overestimation. In fact, safety loadings tend to be much larger than theoretically requires, which can be explained by other costs of parameter uncertainty as costs for external funding or costs for information. With the result from Section 3.3 it follows that social welfare increases with more informed insurers in practice, as sufficiently large safety loadings prevent consumers to benefit from speculating on the insurers underestimating expected losses.

Since a smaller level of risk aversion increases price elasticity of demand and, thereby, the safety loading, in in Table 11 in Appendix D we report the results of a sensitivity analysis with $\gamma = 0.2$. As expected, the theoretical safety and, thus, $n_{\min}$, increase in comparison to the case with $\gamma = 0.8$. Nonetheless, $n_{\min}$ remains at very small levels, which confirms our result that, in practice, parameter uncertainty does not raise the ex ante net present value of insurance, but consumers benefit from more informed firms.

5 Conclusion

This article extends previous work on the theory of consumers and firms in a market with cost uncertainty. We focus exclusively on uncertainty that arises from a firm's parameter uncertainty about expected costs. In this case, a firm might under- or overestimate expected costs and offer a price below or above consumers' belief about expected costs, respectively. We show that, as consumer increase demand in case of underestimation and reduce it in case of overestimation, they might extract a positive net present present value if firms did not increase prices above expected costs. We call the required increase *safety loading.*

We study the insurance market as a prime example for firms (i.e. insurers) that face parameter uncertainty about expected costs (i.e. expected policyholder losses). In this market, an increase in parameter uncertainty on the one hand increases the ex ante volatility of prices and, on the other hand, increases consumers' possible gain from the insurer underestimating expected costs. If prices meet the required safety loading, the first effect dominates social welfare and consumers always benefit from more informed firms. However, if the safety loading is too small, insurers' parameter uncertainty increases social welfare as less risk averse consumers speculate on the insurer underestimating expected losses.

By examining combined ratios in the US property and casualty insurance market, we estimate premium loadings for different insurance lines of business and argue that these reflect a safety loading for parameter uncertainty. A comparison with the theoretically required safety loading suggests that in practice safety loadings are large enough to offset potential positive effects of parameter uncertainty for consumers. In summary, our findings indicate that indeed consumers benefit from more informed firms.

Since the amounts and quality of information and data is rapidly changing, understanding the implications of parameter uncertainty on the supply and demand in markets, particularly for financial products, gains relevance. As we show in this article, the consequences for market equilibria and welfare are far from trivial but crucially depend on consumer preferences and market properties. Therefore, our framework provides a starting point for numerous other studies that might extend, test, or challenge our model.

Appendix

A Parameter Uncertainty in Motor Insurance: Additional Tables

Risk Factor	**# Levels** (original)	**Levels** (used)	**# Levels** (used; baseline)	**# Levels** (used; with regions)
type of vehicle	1005	3 price, 3 engine, 3 age groups	27	27
insurance holder	2	individual or legal person	2	2
date of birth	DATE	...	3	3
registration date	DATE	...	3	3
current financing	3	leasing, credit, self-financing	3	3
type of number plate	3	normal, saisonal, changing	3	3
keeper of the car	5	insurance holder, spouse, partner, child, other	5	5
keeper's ZIP code	28000	...	1	415
expected registration to keeper	DATE	...	1	1
use of vehicle	4	exclusively/mostly private/business	4	4
yearly mileage	100	...	2	2
regular parking space	6	garage, carport, no private parking space,...	6	6
self occupied residential property	4	one-family, multi-family house, condimonium, no proprietary	4	4
drivers	16	policyholder, spouse, partner, children, additional persons	5	5
driver age above 25 years	2	yes/no	2	2
professional occupation	8	...	8	8
employment status	1 or 2	(depends on occupation)	1	1
employer	6 or 9	(depends on employment status)	6	6
potential no-claims bonuses	2	initial contract or take over	2	2
existing contract for spouse/ partner with no-claims bonuses	3	...	3	3
existing contract for parent/ with no-claims bonuses	3	...	3	3
issuance of driver's license	DATE	...	1	1
		TOTAL:	3.6×10^{10}	1.5×10^{10}
		Observations (GDV)	9.2^{6}	9.2^{6}
		Observations/Pool/Year:	2.5^{-4}	6×10^{-7}
		Observations (German Federal Statistical Office)	2585191	2585191
		Observations/Pool/Year:	7.1×10^{-5}	1.7×10^{-7}

Table 4: Risk factors for first time car insurance at HUK Coburg. Source: `https://www.huk.de/tarifrechner.jsp`.

B Proofs

Proof 1 (Lemma 2). *Assume that the unit price equals P_- and P_+ in case of under- and overestimation, respectively. Then, ex ante expected utility is increasing with parameter uncertainty if and only if*

$$-q_- \frac{dP_-}{d\varepsilon} \mathbb{E}\left[u'(w_-)\right] > q_+ \frac{dP_+}{d\varepsilon} \mathbb{E}\left[u'(w_+)\right]. \tag{26}$$

Proof:

Define by $q_- \geq 0$ and $q_+ \geq 0$ optimal consumer demand for a unit price P_- and P_+, respectively. Moreover, define the different possible states of wealth conditional on the realization of the price as

$$w_- = w_0 + q_-(\tilde{C} - P_-) - \tilde{\eta}, \tag{27}$$

$$w_+ = w_0 + q_+(\tilde{C} - P_+) - \tilde{\eta}. \tag{28}$$

Marginal wealth is given as

$$\frac{dw_-}{d\varepsilon} = -\frac{d(q_- P_-)}{dP_-} \frac{dP_-}{d\varepsilon} + \frac{dq_-}{dP_-} \frac{dP_-}{d\varepsilon} \tilde{C} \tag{29}$$

$$\frac{dw_+}{d\varepsilon} = -\frac{d(q_+ P_+)}{dP_+} \frac{dP_+}{d\varepsilon} + \frac{dq_+}{dP_+} \frac{dP_+}{d\varepsilon} \tilde{C}. \tag{30}$$

Ex ante expected utility is given as

$$EU(\varepsilon) = \frac{1}{2} \left(\mathbb{E}[u(w_-)] + \mathbb{E}[u(w_+)]\right) \tag{31}$$

and marginal utility is given as

$$\frac{dEU}{d\varepsilon} = \frac{1}{2}\Bigg(\mathbb{E}\left[u'(w_-)\left(-\frac{d(q_- P_-)}{dP_-} \frac{dP_-}{d\varepsilon} + \frac{dq_-}{dP_-} \frac{dP_-}{d\varepsilon} \tilde{C}\right)\right] \tag{32}$$

$$+ \mathbb{E}\left[u'(w_+)\left(-\frac{d(q_+ P_+)}{dP_+} \frac{dP_+}{d\varepsilon} + \frac{dq_+}{dP_+} \frac{dP_+}{d\varepsilon} \tilde{C}\right)\right]\Bigg). \tag{33}$$

Conditional on price P optimal demand q satisfies

$$\mathbb{E}[u'(w)(\tilde{C} - P)] = 0, \tag{34}$$

or equivalently

$$\mathbb{E}[u'(w)\tilde{C}] = P\mathbb{E}[u'(w)]. \tag{35}$$

Plugging that into marginal utility yields

$$\begin{aligned}
\frac{dEU}{d\varepsilon} &= \frac{1}{2}\Bigg(\mathbb{E}\left[u'(w_-)\left(-\frac{d(q_-P_-)}{dP_-}\frac{dP_-}{d\varepsilon} + \frac{dq_-}{dP_-}\frac{dP_-}{d\varepsilon}P_-\right)\right] && (36)\\
&\quad + \mathbb{E}\left[u'(w_+)\left(-\frac{d(q_+P_+)}{dP_+}\frac{dP_+}{d\varepsilon} + \frac{dq_+}{dP_+}\frac{dP_+}{d\varepsilon}P_+\right)\right]\Bigg) && (37)\\
&= \frac{1}{2}\Bigg(\frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\left(-\frac{dq_-}{dP_-}P_- - q_- + \frac{dq_-}{dP_-}P_-\right)\right] && (38)\\
&\quad + \frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\left(-\frac{dq_+}{dP_+}P_+ - q_+ + \frac{dq_+}{dP_+}P_+\right)\right]\Bigg) && (39)\\
&= \frac{1}{2}\Bigg((-q_-)\frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\right] - q_+\frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\right]\Bigg). && (40)
\end{aligned}$$

Therefore, ex ante marginal expected utility is positive if and only if

$$-q_-\frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\right] > q_+\frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\right]. \tag{41}$$

Proof 2 (Corollary 1). *Assume that prices are given by $P = p\vartheta + s(\varepsilon)$. Define $\underline{\varepsilon} > 0$ and $\bar{\varepsilon} > 0$ as the smallest level of parameter uncertainty such that optimal demand is zero in case of overestimation and underestimation, respectively, and $\varepsilon^* = s'^{-1}(p)$. Then, ex ante expected utility is increasing for $\varepsilon \in [\underline{\varepsilon}, \min\{\bar{\varepsilon}, \varepsilon^*\})$ if the safety loading is convex, and increasing for $\varepsilon \in [\max\{\underline{\varepsilon}, \varepsilon^*\}, \bar{\varepsilon})$ if the safety loading is concave.*

Proof:

Since $P_- < P_+$ we have that $q_- \geq q_+$ and $q_- > 0$ for $\varepsilon = \underline{\varepsilon}$. Thus, $\underline{\varepsilon} < \bar{\varepsilon}$. For $\varepsilon \geq \underline{\varepsilon}$, consumers do not purchase in case of overestimation, $q_+ = 0$, and ex ante marginal expected utility boils down to

$$\frac{dEU}{d\varepsilon} = -\frac{q_-}{2}\frac{dP_-}{d\varepsilon}\mathbb{E}[u'(w_-)] \tag{42}$$

and is positive for $q_- > 0$, and $\frac{dP_-}{d\varepsilon} = -p + s(\varepsilon) < 0$, which is equivalent to $s'(\varepsilon) < p$. $q_- > 0$ is true for $\varepsilon < \bar{\varepsilon}$. If s is convex, we have that $s'(\varepsilon) < p$ for $\varepsilon < s'^{-1}(p) = \varepsilon^*$, thus, marginal ex ante expected utility is positive for $\varepsilon \in [\underline{\varepsilon}, \min\{\bar{\varepsilon}, \varepsilon^*\})$. If s is concave, we have that $s'(\varepsilon) < p$ for $\varepsilon > s'^{-1}(p) = \varepsilon^*$, thus, marginal ex ante expected utility is positive for $\varepsilon \in [\max\{\underline{\varepsilon}, \varepsilon^*\}, \bar{\varepsilon})$.

Proof 3 (Corollary 3). *Assume that prices are given as $P = p\vartheta + s(\varepsilon)$ and that the marginal safety loading is positive in case of perfect information, $s'(0) > 0$. Then, $EU'(0) < 0$ and there exists $\hat{\varepsilon} > 0$ such that ex ante expected utility satisfies $EU(\varepsilon) < EU(0)$ for all $\varepsilon \in (0, \hat{\varepsilon})$.*

Proof:

For $\varepsilon = 0$ we have that $P_- = P_+$, thus, $q_- = q_+$ and $w_+ = w_-$. Thus, $\mathbb{E}[u'(w_+)] = \mathbb{E}[u'(w_-)]$ and ex ante expected utility is decreasing with parameter uncertainty if and only if

$$
\begin{aligned}
& -q_- \frac{dP_-}{d\varepsilon} \mathbb{E}\left[u'(w_-)\right] < q_+ \frac{dP_+}{d\varepsilon} \mathbb{E}\left[u'(w_+)\right] && (43)\\
\Leftrightarrow \quad & -(-p + s'(0)) < p + s'(0) && (44)\\
\Leftrightarrow \quad & -s'(0) < s'(0). && (45)
\end{aligned}
$$

As the $u(w_-)$ and $u(w_+)$ are differentiable at $\varepsilon = 0$, the result follows.

Lemma 4 (Risk aversion and optimal insurance coverage)**.** *Consider an increase in risk aversion that is given by a concave function g, $g' > 0$ and $g'' < 0$ and $g''' \geq 0$, such that new preferences are given as $v = g(u)$. Then, optimal insurance coverage is larger with preferences v than with preferences u, $q_v > q_u$, for any unit price for insurance $P > pL$.*

Proof. Optimal insurance coverage under v maximizes $EU = p\mathbb{E}[v(w_1)] + (1-p)\mathbb{E}[v(w_2)]$ and therefore marginal expected utility is

$$
\begin{aligned}
\frac{dEV}{dq} &= p\mathbb{E}[v'(w_1)(\tilde{L} - P)] + (1-p)v'(w_2)(-P), && (46)\\
&= p\mathbb{E}[g'(u(w_1))u'(w_1)(\tilde{L} - P)] - P(1-p)g'(u(w_2))u'(w_2), && (47)
\end{aligned}
$$

while q_u satisfies

$$
p\mathbb{E}[u'(w_1)(\tilde{L} - P)] = (1-p)Pu'(w_2). \tag{48}
$$

Thus, $\frac{dEV}{dq}$ evaluated at q_u is equal to

$$p\mathbb{E}[g'(u(w_1))u'(w_1)(\tilde{L}-P)] - P(1-p)g'(u(w_2))u'(w_2) \tag{49}$$

$$= p\mathbb{E}[g'(u(w_1))]\mathbb{E}[u'(w_1)(\tilde{L}-P)] + pcov\left(g'(u(w_1)),\, u'(w_1)(\tilde{L}-P)\right) - P(1-p)g'(u(w_2))u'(w_2) \tag{50}$$

$$= \mathbb{E}[g'(u(w_1))](1-p)Pu'(w_2) - g'(u(w_2))P(1-p)u'(w_2) + pcov\left(g'(u(w_1)),\, u'(w_1)(\tilde{L}-P)\right) \tag{51}$$

$$= \left(\mathbb{E}[g'(u(w_1))] - g'(u(w_2))\right)P(1-p)u'(w_2) + pcov\left(g'(u(w_1)),\, u'(w_1)\tilde{L}\right) \tag{52}$$

$$\geq \left(g'(\mathbb{E}[u(w_1)]) - g'(u(w_2))\right)P(1-p)u'(w_2) + pcov\left(g'(u(w_1)),\, u'(w_1)\tilde{L}\right) > 0, \tag{53}$$

since $\mathbb{E}[u(w_1)] < u(\mathbb{E}[w_1]) < u(w_2)$ due to the concavity of u, $\mathbb{E}[u(w_1)] < u(w_2)$ if $P > pL$, and $g'' < 0$.

□

Proof 4 (Lemma 3). *Assume that consumers can purchase any partial coverage $q \in [0,1]$ without safety loading such that the total price for insurance is given by $P(q) = qP = qp\vartheta$. Then, marginal ex ante expected utility is strictly positive for all $\varepsilon > 0$, $\frac{dEU}{d\varepsilon} > 0$, if consumers' risk aversion is sufficiently small.*

Define $\underline{\varepsilon}$ such that $P_+ = p(L+\underline{\varepsilon})$ is equal to the consumers' reservation price. Then, $\frac{dEU}{d\varepsilon} > 0$ for $\varepsilon \geq \underline{\varepsilon}$ regardless of the level of risk aversion.

Proof:

Define by

$$w_{1,+} = w_0 - q_+P_+ - (1-q_+)\tilde{L}, \tag{54}$$

$$w_{2,+} = w_0 - q_+P_+. \tag{55}$$

wealth in case of overestimation if either a loss occurs, or no loss occurs, respectively.

In case of underestimation, the unit insurance premium is smaller than the actuarially fair premium, $P_- = p(L-\varepsilon) < pL$, and thus consumers purchase full insurance coverage, $q_- = 1$, and $\mathbb{E}[u'(w_-)] = u'(w_0 - p(L-\varepsilon))$. Lemma 2 implies that ex ante expected utility is increasing with parameter uncertainty if

$$-q_-\frac{dP_-}{\varepsilon}\mathbb{E}[u'(w_-)] > q_+\frac{dP_+}{d\varepsilon}\mathbb{E}[u'(w_+)]. \tag{56}$$

Marginal prices are $\frac{dP_-}{\varepsilon} = -p$ and $\frac{dP_+}{\varepsilon} = p$. Plugging in the results from above and simplifying yields the *following condition for increasing ex ante expected utility*

$$\frac{u'(w_0 - p(L-\varepsilon))}{\mathbb{E}[u'(w_+)]} > q_+. \tag{57}$$

For $\varepsilon = 0$ consumers purchase full insurance in any case and, thus, the left hand side (LHS) and right hand side (RHS) of Equation (57) are equal. We denote the maximum level of parameter uncertainty such that consumers are willing to purchase a positive insurance coverage in case of overestimation by $\underline{\varepsilon}$.

For $\varepsilon = \underline{\varepsilon}$ we have that $q_+ = 0$ and $q_- = 1$. Hence, in this case (note that $u' > 0$ a.s.)

$$\frac{u'(w_0 - p(L-\varepsilon))}{\mathbb{E}[u'(w_+)]} > q_+ = 0. \tag{58}$$

For $0 < \varepsilon < \underline{\varepsilon}$, $\mathbb{E}[u'(w_+)]$ is increasing with ε as $\frac{d}{d\varepsilon}\mathbb{E}[u'(w_+)] = p\mathbb{E}[u''(w_+)\frac{dw_+}{dP_+}] = p\mathbb{E}[u''(w_+)]\mathbb{E}[\frac{dw_+}{dP_+}] + p\frac{dq_+}{dP_+}cov(u''(w_+), \tilde{C}) > 0$ since $u''' \geq 0$. $u'(w_0 - p(L-\varepsilon))$ decreases with ε. Thus, the overall LHS of Equation (57) is decreasing with ε.

Moreover, $\frac{dq_+}{d\varepsilon} < 0$. The smaller the level of risk aversion, the smaller is q_+ for $\varepsilon > 0$ (see Lemma 4) and the larger is

$$\frac{d}{d\varepsilon}\frac{u'(w_-)}{\mathbb{E}[u'(w_+)]} = (-p)\frac{u''(w_-)\frac{dw_-}{dP_-}\mathbb{E}[u'(w_+)] + \mathbb{E}[u''(w_+)\frac{dw_+}{dP_+}]u'(w_-)}{\mathbb{E}[u'(w_+)]^2} < 0, \tag{59}$$

as $-u''/u'$ increases. For risk neutral consumers we have $u' \equiv 1$ and thus (57) is equivalent to $1 > q_+ = 0$ for any $\varepsilon > 0$.

Therefore, for sufficiently small levels of risk aversion (57) is fulfilled for $0 < \varepsilon < \underline{\varepsilon}$. Then, the result follows with Lemma 1. For $\varepsilon \geq \underline{\varepsilon}$ the RHS of (57) is zero and, hence, the result follows with Lemma 1 for every level of risk aversion.

Proof 5 (Corollary 4). *Define by $\underline{\varepsilon} > 0$ and $\bar{\varepsilon} > 0$ the smallest level of parameter uncertainty such that consumers do not purchase any insurance coverage in case of overestimation and underestimation, respectively, and $\varepsilon^* = s'^{-1}(p)$. Let $s'(0) > 0$ and define*

$$\hat{\varepsilon} = \min\left\{\varepsilon > 0 : -q_- \frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\right] = q_+ \frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\right] > 0\right\} \tag{60}$$

and $\min(\emptyset) = \infty$.

a) Either $\hat{\varepsilon} < \underline{\varepsilon}$ or $\hat{\varepsilon} = \infty$.

b) If risk aversion is sufficiently small and the safety loading is convex, ex ante expected utility is increasing *for* $\varepsilon \in (\hat{\varepsilon}, \min\{\bar{\varepsilon}, \varepsilon^*\})$. *If the safety loading is concave, ex ante expected utility is increasing for* $\varepsilon \in (\max\{\varepsilon^*, \hat{\varepsilon}\}, \bar{\varepsilon})$.

c) If $s(0) = 0$ *and* $s'(\varepsilon) \leq \gamma < \infty$ *for all* ε*, then* $\lim_{\gamma \to 0} \hat{\varepsilon} \to 0$.

d) If risk aversion is sufficiently small, ex ante expected utility is increasing in ε *for* $\varepsilon \in \{\varepsilon > \hat{\varepsilon} : s'(\varepsilon) < p \text{ and } s(\varepsilon) \leq p\varepsilon\}$.

Proof:

1. *For* $\varepsilon \geq \underline{\varepsilon}$ *it is* $q_+ = 0$*. Hence,* $\hat{\varepsilon} < \underline{\varepsilon}$ *or* $\hat{\varepsilon} = \min(\emptyset) = \infty$.

2. *According to Lemma 2, ex ante expected utility is increasing if and only if*

$$-q_- \frac{dP_-}{d\varepsilon} \mathbb{E}\left[u'(w_-)\right] > q_+ \frac{dP_+}{d\varepsilon} \mathbb{E}\left[u'(w_+)\right], \tag{61}$$

For $\varepsilon \geq \bar{\varepsilon}$*, consumers do not purchase any insurance coverage and, thus, marginal ex ante expected utility is equal to zero. If* $\varepsilon < \bar{\varepsilon}$*, it is* $q_- > 0$ *and ex ante expected utility is increasing if and only if*

$$\frac{p - s'(\varepsilon)}{p + s'(\varepsilon)} \frac{\mathbb{E}\left[u'(w_-)\right]}{\mathbb{E}\left[u'(w_+)\right]} > \frac{q_+}{q_-}, \tag{62}$$

since $\frac{dP_-}{d\varepsilon} = -p + s'(\varepsilon)$ *and* $\frac{dP_+}{d\varepsilon} = p + s'(\varepsilon)$.

Assume that $s'(\varepsilon) < p$*. Then,* $\frac{dP_-}{d\varepsilon} < 0$ *and* $\frac{dP_+}{d\varepsilon} > 0$ *and expected wealth increases in case of underestimation,* w_-*, and decreases in case of overestimation,* w_+*. Since* $u''' \geq 0$*,* $\mathbb{E}[u'(w_+)]$ *is increasing with* ε *(see Proof 3) und* $\mathbb{E}[u'(w_-)]$ *is decreasing with* ε *as* $\frac{d}{d\varepsilon}\mathbb{E}[u'(w_-)] = (-p+s'(\varepsilon))\mathbb{E}[u''(w_-)\frac{dw_-}{dP_-}] = (-p+s'(\varepsilon))\mathbb{E}[u''(w_-)]\mathbb{E}[\frac{dw_-}{dP_-}] + (-p+s'(\varepsilon))\frac{dq_-}{dP_-} cov(u''(w_-), \tilde{C}) < 0$.

Assume that s *is convex, i.e.* $s'' > 0$*. Then, the LHS and RHS of Equation (62) are decreasing with* ε*. Analogously to Proof 3, if risk aversion is sufficiently low,* q_+ *is sufficiently small such that (62) is fulfilled. For convex* s*, it is* $s'(\varepsilon) < p \Leftrightarrow \varepsilon \leq s'^{-1}(p) = \varepsilon^*$*. Therefore, it is necessary that* $\varepsilon < \min\{\varepsilon^*, \bar{\varepsilon}\}$*. As* $LHS < RHS$ *for* $\varepsilon = 0$ *(see Proof 3) and* $LHS = RHS$ *for* $\hat{\varepsilon}$*, it is necessary as well that* $\varepsilon > \hat{\varepsilon}$ *to satisfy (62).*

Assume that s *is concave, i.e.* $s'' < 0$*. Then,* $s'(\varepsilon) < p \Leftrightarrow \varepsilon \geq s'^{-1}(p) = \varepsilon^*$ *and* $\frac{q_+(p-s'(\varepsilon))}{q_-(p+s'(\varepsilon))}$ *is decreasing in* ε*. Analogously to above, (62) is satisfied if risk aversion is sufficiently low and* $\varepsilon \in (\max\{\varepsilon^*, \hat{\varepsilon}\}, \bar{\varepsilon})$.

3. *Assume that* $s'(\varepsilon) \leq \gamma < \infty$ *for all* $\varepsilon > 0$ *and* $s(0) = 0$*. We have that* $\lim_{\gamma \to 0} P_- = p(L - \varepsilon)$ *and* $\lim_{\gamma \to 0} P_+ = p(L + \varepsilon)$ *for* $\varepsilon > 0$*. Thus,* $\lim_{\gamma \to 0} q_- = 1$ *and* $\lim_{\gamma \to 0} q_+ = \hat{q}_+ < 1$*, where*

$\hat{q}_+$ is the optimal coverage for the unit premium $p(L+\varepsilon)$. Moreover, we have that $\lim_{\gamma\to 0}\frac{dP_+}{d\varepsilon} = -\lim_{\gamma\to 0}\frac{dP_-}{d\varepsilon} = p$. Thus,

$$\lim_{\gamma\to 0} -q_- \frac{dP_-}{d\varepsilon}\frac{p\mathbb{E}[u'(w_{1,-})\tilde{L}]}{P_-} = \lim_{\gamma\to 0}(p - s'(\varepsilon))\frac{q_-}{P_-}p\mathbb{E}[u'(w_{1,-})\tilde{L}] = \frac{p^2\mathbb{E}[u'(w_{1,-})\tilde{L}]}{p(L-\varepsilon)} \tag{63}$$

$$\lim_{\gamma\to 0} q_+ \frac{dP_+}{d\varepsilon}\frac{p\mathbb{E}[u'(w_{1,+})\tilde{L}]}{P_+} = \lim_{\gamma\to 0} q_+(p + s'(\varepsilon))\frac{p\mathbb{E}[u'(w_{1,+})\tilde{L}]}{P_+} = \hat{q}_+\frac{p^2\mathbb{E}[u'(w_{1,+})\tilde{L}]}{p(L+\varepsilon)} \tag{64}$$

and, therefore, $\lim_{\gamma\to 0} -q_-\frac{dP_-}{d\varepsilon}\mathbb{E}\left[u'(w_-)\right] = \lim_{\gamma\to 0} q_+\frac{dP_+}{d\varepsilon}\mathbb{E}\left[u'(w_+)\right] > 0$ for $\varepsilon = 0$.

4. *Consumers purchase full insurance coverage in case of underestimation, $q_- = 1$, if $s(\varepsilon) \leq p\varepsilon$, since then the premium is smaller than or equal to the actuarially fair price, $p(L-\varepsilon) + s(\varepsilon) \leq p(L-\varepsilon) + p\varepsilon \leq pL$. Therefore, $\bar{\varepsilon} = \infty$. Then, the result follows from b).*

C Search Costs and Parameter Uncertainty

We have referred to search costs as an important reason for a market displaying a distribution of prices instead of one price (Diamond (1971)). This is a necessary condition for prices being affected by parameter uncertainty. Therefore, it is important to examine the effect of parameter uncertainty in a market with search costs. In the this section we provide a brief intuition about the interaction between search costs, price dispersion, and parameter uncertainty.

Assume two insurance companies, A and B, that offer the same insurance product and share the same level of parameter uncertainty, ε. We denote by $\pi_{\overline{AB}}$ the probability that company A and B overestimate the expected loss size, by $\pi_{\overline{A}B}$ that company A overestimates and B underestimates the expected loss size, etc. As before, we assume that $\pi_{\overline{A}} = \pi_{\overline{B}} = \frac{1}{2}$, i.e. the unconditional probability of insurer A or B to overestimate equals the probability to underestimate. Consumers suffer costs, $c > 0$, when searching for the lowest price.

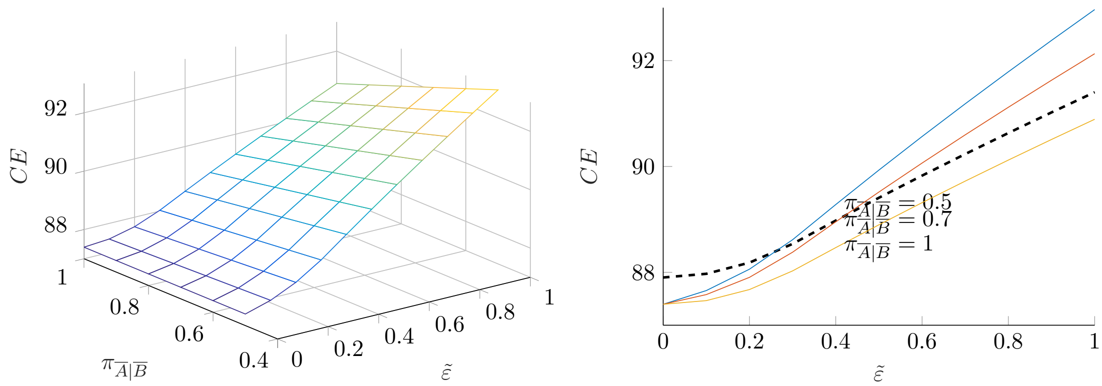
When engaging in search, consumers always find the lowest price in the market. Thus, they face an overestimated price only with probability $\pi_{\overline{AB}}$, i.e. if both insurers overestimate. Ex-ante expected utility is then given as

$$EU = \pi_{\overline{AB}}\mathbb{E}\left[u(w_+)\right] + \left(1 - \pi_{\overline{AB}}\right)\mathbb{E}\left[u(w_-)\right]. \tag{65}$$

In comparison to the case with only one insurer, it is now less likely that the consumer is confronted with an overestimated price. Thus, consumers benefit more from parameter uncertainty. Figure 3 depicts the ex ante certainty equivalent for different levels of the conditional probability that insurer A overestimates conditional on insurer B overestimating, i.e. $\pi_{\overline{A}|\overline{B}} = \frac{\pi_{\overline{AB}}}{\pi_{\overline{B}}} = 2\pi_{\overline{AB}}$. We assume that the safety loading results from a capital requirement as in Example 2 with a cost of capital rate $\chi = 0.06$. The more independent the estimation errors of insurers A and B are, the smaller is $\pi_{\overline{A}|\overline{B}}$. For $\pi_{\overline{A}|\overline{B}} = \frac{1}{2}$ the errors are completely independent, while for $\pi_{\overline{A}|\overline{B}} = 1$ there is perfect dependence. The less dependent the estimation errors are, the less likely it is that the consumers must pay an overestimated price. Thus, the smaller $\pi_{\overline{A}|\overline{B}}$, the larger is the ex ante certainty equivalent.

Consumers engage in search for the lowest price if the ex ante certainty equivalent with price search is not below the one without price search. We assume that consumers purchase randomly from the two insurers if they not engage in search. Since both insurers face the same level of parameter uncertainty, the ex ante certainty equivalent without search is then equal to the ex ante certainty equivalent in the market with only one insurer. It is given by the dashed line in Figure 3 (b).

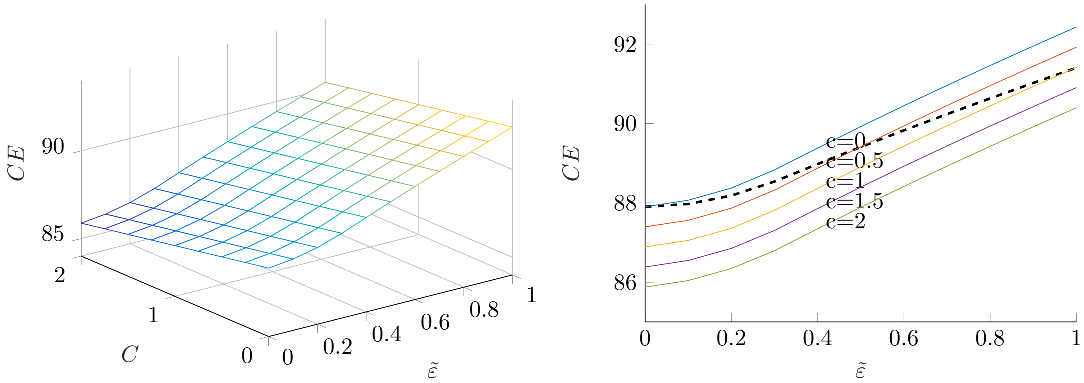
Figure 3 shows that the optimality of price search depends on the level of parameter uncertainty. If parameter uncertainty is small, both insurers offer the same price and, thus, costly price search is not optimal. However, with increasing levels of parameter uncertainty, it is more likely that consumers find a smaller than average price and thus they engage in price search if estimation errors are sufficiently uncorrelated.



(a) 3-dimensional view. (b) Contourplot.

Figure 3: Ex-ante certainty equivalent for different levels of the conditional probability that insurer A overestimates conditional on insurer B overestimating, $\pi_{\overline{A}|\overline{B}}$, in a market with two insurers A and B that share the same relative level of parameter uncertainty, $\tilde{\varepsilon}$, with search costs $c = 0.5$, safety loadings for $\chi = 0.06$, $\psi = 0.005$, $\gamma = 0.8$, $p = 0.2$, $w_0 = 100$, $L = 50$, and $\sigma_L = 10$. The dashed line represents the ex ante certainty equivalent if consumers do not engage in search for the lowest price.

Figure 4 shows the impact of search costs on the ex ante certainty equivalent. In a similar manner as a fixed loading on insurance prices, larger search costs are related to a downward shift in the ex ante certainty equivalent. Intuitively, it is optimal for consumers to engage in searching for the lowest price as long as the certainty equivalent with search is not smaller than the certainty equivalent without search. Figure 4 indicates that the willingness to engage in price search increases with the level of parameter uncertainty. For example, consumers accept search costs at level $c = 0.5$ only for parameter uncertainty levels of $\tilde{\varepsilon} \geq 0.3$ but not for $\tilde{\varepsilon} < 0.3$. Intuitively, a larger level of parameter uncertainty increases the benefits of price search and thus consumers' willingness to pay.



(a) 3-dimensional view.

(b) Contourplot.

Figure 4: Ex-ante certainty equivalent for different levels of search costs, c, in a market with two insurers A and B that share the same level of parameter uncertainty, $\tilde{\varepsilon}$, with the conditional probability that insurer A overestimates conditional on insurer B overestimating equal to $\pi_{\overline{A}|\overline{B}} = 0.75$, and safety loadings for $\chi = 0.06$, $\psi = 0.005$, $\gamma = 0.8$, $p = 0.2$, $w_0 = 100$, $L = 50$, and $\sigma_L = 10$. The dashed line represents the ex ante certainty equivalent if consumers do not engage in search for the lowest price.

D Safety Loadings in Insurance Markets: Additional Tables

Line of Business	Description
Aggregate Write-ins	Coverages not generally described by other lines (e.g., Involuntary Unemployment Insurance).
Aircraft	Coverage for aircraft (hull) and their contents; aircraft owner's and aircraft manufacturers liability to passengers, airports and other third parties.
Allied lines	Coverages which are generally written with property insurance, e.g., glass, tornado, windstorm and hail; sprinkler and water damage; explosion, riot, and civil commotion; growing crops; flood; rain; and damage from aircraft and vehicle, etc.
Auto liability	Coverage that protects the insured against financial loss because of legal liability for motor vehicle related injuries (bodily injury and medical payments) or damage to the property of others caused by accidents arising out of ownership, maintenance or use of a motor vehicle (including recreational vehicles such as motor homes). Commercial is defined as all motor vehicle policies that include vehicles that are used primarily in connection with business, commercial establishments, activity, employment, or activities carried on for gain or profit.
Boiler & Machinery	Coverage for the failure of boilers, machinery and electrical equipment. Benefits include property of the insured, which has been directly damaged by the accident, costs of temporary repairs and expediting expenses, as well as liability for damage to the property of others.
Burglary & Theft	Coverage for property taken or destroyed by breaking and entering the insured's premises, burglary or theft, forgery or counterfeiting, fraud, kidnap and ransom, and off-premises exposure.
Commercial Auto Liability	Bodily Injury, Property Damage, Uninsured Motorist and Underinsured Motorist Coverages
Commercial Auto Physical Damage	Any motor vehicle insurance coverage (including collision, vandalism, fire and theft) that insures against material damage to the insured's vehicle. Commercial is defined as all motor vehicle policies that include vehicles that are used in connection with business, commercial establishments, activity, employment, or activities carried on for gain or profit.
Commercial multiple peril (Liability)	All business covering the liability portion of Multiple Peril policies.
Commercial multiple peril (non-Liability)	All business covering the fire and allied portion of Multiple Peril policies.
Credit	Coverage purchased by consumers, manufacturers, merchants, educational institutions, or other providers of goods and services extending credit, for indemnification of losses or damages resulting from the nonpayment of debts owed to/from them for goods or services provided in the normal course of their business.
Credit A&H	Coverage provided to or offered to borrowers in connection with a consumer credit transaction where the proceeds are used to repay a debt or an installment loan in the event the consumer is disabled as the result of an accident, including business not exceeding 120 months duration.

Earthquake	Property coverages for losses resulting from a sudden trembling or shaking of the earth, including that caused by volcanic eruption. Excluded are losses resulting from fire, explosion, flood or tidal wave following the covered event.
Excess Worker's Compensation	Indemnification coverage provided to self-insured employers on an excess of loss basis.
Farmowners multiple peril	A package policy for farming and ranching risks, similar to a homeowners policy, that has been adopted for farms and ranches and includes both property and liability coverages for personal and business losses. Coverages include farm dwellings and their contents, barns, stables, other farm structures and farm inland marine, such as mobile equipment and livestock.
Federal Flood	Coverage provided by the Federal Insurance Administration (FIA) of the Federal Emergency Management Agency (FEMA) through insurers participating in the National Flood Insurance Program's (NFIP) Write Your Own (WYO) program. Coverage is subject to the terms and conditions provided in the Financial Assistance/Subsidy Arrangement between the reporting entity and the FIA.
Fidelity	A bond covering an employer's loss resulting from an employee's dishonest act (e.g., loss of cash, securities, valuables, etc.).
Financial Guaranty	A surety bond, insurance policy, or when issued by an insurer, an indemnity contract and any guaranty similar to the foregoing types, under which loss is payable upon proof of occurrence of financial loss to an insured claimant, obligee or indemnitee as a result of failure to perform a financial obligation (seeFinancial Guaranty Insurance Model Act).
Fire	Coverage protecting the insured against the loss to real or personal property from damage caused by the peril of fire or lightning, including business interruption, loss of rents, etc.
Group A&H	Coverage written on a group basis (e.g., employees of a single employer and their dependents) that pays scheduled benefits or medical expenses caused by disease, accidental injury or accidental death. Excludes amounts attributable to uninsured accidents and health plans and the uninsured portion of partially insured accident and health plans.
Homeowners multiple peril	A package policy combining broad property coverage for the personal property and/or structure with broad personal liability coverage. Coverage applicable to the dwelling, appurtenant structures, unscheduled personal property and additional living expense are typical. Includes mobile homes at a fixed location
Inland Marine	Coverage for property that may be in transit, held by a bailee, at a fixed location, a movable good that is often at different locations (e.g., off road constructions equipment), or scheduled property (e.g., Homeowners Personal Property Floater) including items such as live animals, property with antique or collector's value, etc. This line also includes instrumentalities of transportation and communication, such as bridges, tunnels, piers, wharves, docks, pipelines, power and phone lines, and radio and television towers.
International	Includes all business transacted outside the U.S. and its territories and possessions where the appropriate line of business is not determinable.

Medical Professional Liability	Insurance coverage protecting a licensed health care provider or health care facility against legal liability resulting from the death or injury of any person due to the insured?s misconduct, negligence, or incompetence in rendering professional services. Medical Professional Liability is also known as Medical Malpractice.
Mortgage Guaranty	Insurance that indemnifies a lender from loss if a borrower fails to meet required mortgage payments.
Multiple Peril	A contract for a commercial enterprise, which packages two or more insurance coverages protecting an enterprise from various property and liability, risk exposures. Frequently includes fire, allied lines, various other coverages (e.g., difference in conditions) and liability coverage (such coverages would be included in other annual statement lines, if written individually). Include multi?peril policies (other than farmowners, homeowners and automobile policies) that include coverage for liability other than auto.
Multiple Peril Crop	Insurance protection that is subsidized or reinsured by the Federal Crop Insurance Corporation for protection against losses due to damage, decreases in revenues and or gross margins from crop, livestock and other agricultural?related production from unfavorable weather conditions, drought, wind, frost, fire or lightning, flood, hail, insect infestation, disease or other yield-reducing conditions or perils.
Ocean Marine	Coverage for ocean and inland water transportation exposures; goods or cargoes; ships or hulls; earnings; and liability.
Other A&H	Accident and health coverages not otherwise properly classified as Group Accident and Health or Credit Accident and Health (e.g., collectively renewable and individual non-cancelable, guaranteed renewable, non-renewable for stated reasons only, etc.). Include all Medicare Part D Prescription Drug Coverage, whether sold on a stand-alone basis or through a Medicare Advantage product and whether sold directly to an individual or through a group.
Private Crop	Private market coverage for crop insurance and agricultural-related protection, such as hail and fire, and is not reinsured by the Federal Crop Insurance Corporation.
Private Flood	Private market coverage (primary standalone, first dollar policies that cover the flood peril and excess flood) for flood insurance that is not offered through the National Flood Insurance Program.
Private Passenger Auto Liability	Bodily Injury, Property Damage, Uninsured Motorist and Underinsured Motorist Coverages
Private Passenger Auto Physical Damage	Any motor vehicle insurance coverage (including collision, vandalism, fire and theft) that insures against material damage to the insured's vehicle. Commercial is defined as all motor vehicle policies that include vehicles that are used in connection with business, commercial establishments, activity, employment, or activities carried on for gain or profit.
Products Liability	Insurance coverage protecting the manufacturer, distributor, seller, or lessor of a product against legal liability resulting from a defective condition causing personal injury, or damage, to any individual or entity, associated with the use of the product

Reinsurance-Nonproportional Assumed	Proportional assumed reinsurance is allocated to and reported in the appropriate lines of business and excluded from the reinsurance lines of business. For assumed reinsurance contracts that afford proportional and nonproportional reinsurance, the business is allocated to its component parts and reported in the appropriate lines of business.
Surety	A three-party agreement where the insurer agrees to pay a second party (the obligee) or make complete an obligation in response to the default, acts, or omissions of a third party (the principal).
Warranty	Coverage that protects against manufacturer's defects past the normal warranty period and for repair after breakdown to return a product to its originally intended use. Warranty insurance generally protects consumers from financial loss caused by the seller's failure to rectify or compensate for defective or incomplete work and cost of parts and labor necessary to restore a product's usefulness. Includes but is not limited to coverage for all obligations and liabilities incurred by a service contract provider, mechanical breakdown insurance and service contracts written by insurers.
Worker's Compensation	Insurance that covers an employer's liability for injuries, disability or death to persons in their employment, without regard to fault, as prescribed by state or Federal workers' compensation laws and other statutes. Includes employer's liability coverage against the (as distinguished from the liability imposed by Workers' Compensation Laws). Excludes excess workers'compensation.common law liability for injuries to employees.

Table 5: Lines of business included in the data sample as provided by A.M. Best Company for US property and casualty insurers in fiscal years 2005 to 2015. Descriptions are taken and adopted from Donovan (2015).

	Dependent variable:
	$(1 - CR)/\sigma$
Aircraft	21.516 (33.013)
AlliedLines	3.958 (42.593)
BoilerMachinery	18.695 (35.459)
BurglaryTheft	71.774* (39.306)
CommercialMultiplePeril_Liab	6.291 (39.028)
CommericalAuto_Liab	6.987 (40.868)
Credit	22.288 (36.923)
CreditAH	0.542 (38.460)
Earthquake	103.434*** (33.694)
ExcessWorkersCompensation	20.233 (36.610)
FarmownersMultiplePeril	9.099 (35.172)
Fidelity	37.174 (38.779)
FinancialGuaranty	49.361 (43.721)
Fire	5.657 (43.850)
GroupAH	5.198 (32.234)
HomeownersMultiplePeril	3.691 (41.766)
InlandMarine	14.238 (43.447)
MedicalProfLiab	18.206 (33.418)
MortgageGuaranty	52.582 (44.596)
OceanMarine	24.454 (34.435)
OtherAH	−4.009 (33.792)
PrivatePassengerAuto_Liab	18.584 (41.023)
PrivatePassengerAuto_PhysicalDamage	4.240 (40.882)
ProductsLiab	28.689 (36.921)
Surety	23.429 (37.605)
Warranty	19.379 (37.142)
WorkersCompensation	9.556 (38.473)
Competition	0.005 (0.053)
HHI	−273.955 (394.748)
Competition:HHI	−0.029 (0.603)
Company Fixed Effects	*Y*
Year Fixed Effects	*Y*
Observations	112,302
R^2	0.107
Adjusted R^2	0.088
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 6: Loadings relative to loss volatility implied by the combined ratio are $(1 - CR)/\sigma$. This table reports the OLS coefficients and standard errors for different lines of business, concentration (normalized HHI index), and competition (number of active companies in a specific line of business). Robust standard errors are clustered by year and reported in parentheses. The sample covers direct US property and casualty insurers from 2005 to 2015 based on data provided by A.M. Best Company.

	Dependent variable:		
	$(1 - CR)/\sigma$	$1 - CR$	
	(1)	(2)	(3)
Aircraft	20.861 (33.013)	−0.148 (0.436)	−0.189 (0.472)
AlliedLines	22.249 (42.593)	−0.348 (0.482)	−0.516 (0.483)
BoilerMachinery	25.263 (35.459)	0.067 (0.453)	−0.022 (0.477)
BurglaryTheft	78.412** (39.306)	0.148 (0.451)	0.052 (0.468)
CommercialMultiplePeril_Liab	22.201 (39.028)	−0.192 (0.462)	−0.343 (0.468)
CommericalAuto_Liab	24.256 (40.868)	−0.185 (0.468)	−0.350 (0.469)
Credit	22.577 (36.923)	−0.009 (0.495)	−0.059 (0.524)
CreditAH	6.446 (38.460)	0.057 (0.519)	−0.110 (0.557)
Earthquake	113.600*** (33.694)	0.412 (0.431)	0.298 (0.448)
ExcessWorkersCompensation	21.737 (36.610)	−0.186 (0.495)	−0.245 (0.521)
FarmownersMultiplePeril	24.921 (35.172)	−0.096 (0.437)	−0.245 (0.458)
Fidelity	46.523 (38.779)	0.073 (0.469)	−0.027 (0.495)
FinancialGuaranty	35.000 (43.721)	−0.369 (0.590)	−0.027 (0.636)
Fire	24.989 (43.850)	−0.150 (0.483)	−0.329 (0.479)
GroupAH	11.156 (32.234)	−0.089 (0.429)	−0.184 (0.453)
HomeownersMultiplePeril	23.063 (41.766)	−0.254 (0.469)	−0.442 (0.468)
InlandMarine	33.569 (43.447)	−0.062 (0.492)	−0.239 (0.490)
MedicalProfLiab	20.525 (33.418)	−0.067 (0.417)	−0.111 (0.434)
MortgageGuaranty	−5.163 (44.596)	−0.354 (0.522)	0.004 (0.543)
OceanMarine	29.904 (34.435)	−0.049 (0.438)	−0.122 (0.464)
OtherAH	3.015 (33.792)	−0.111 (0.471)	−0.211 (0.499)
PrivatePassengerAuto_Liab	24.997 (41.023)	−0.350 (0.486)	−0.475 (0.490)
PrivatePassengerAuto_PhysicalDamage	20.952 (40.882)	−0.206 (0.477)	−0.363 (0.479)
ProductsLiab	38.457 (36.921)	−0.214 (0.440)	−0.331 (0.459)
Surety	33.511 (37.605)	0.059 (0.458)	−0.036 (0.473)
Warranty	24.592 (37.142)	−0.107 (0.483)	−0.202 (0.519)
WorkersCompensation	22.709 (38.473)	−0.192 (0.456)	−0.331 (0.462)
Competition	0.005 (0.053)	0.001 (0.001)	0.001 (0.001)
HHI	−221.078 (394.748)	0.876 (5.230)	−0.384 (5.136)
Competition:HHI	−0.113 (0.603)	−0.006 (0.010)	−0.004 (0.010)
Company Fixed Effects	–	–	Y
Year Fixed Effects	Y	Y	Y
Observations	112,302	112,302	112,302
R^2	0.031	0.058	0.148
Adjusted R^2	0.031	0.058	0.130

Note: *p<0.1; **p<0.05; ***p<0.01

Table 7: Loadings implied by the combined ratio are $1 - CR$. σ is the volatility of quarterly losses incurred in a specific line of business of a specific insurance company. This table reports the OLS coefficients and standard errors for different lines of business, concentration (normalized HHI index), and competition (number of active companies in a specific line of business). Robust standard errors are clustered by year and reported in parentheses. The sample covers direct US property and casualty insurers from 2005 to 2015 based on data provided by A.M. Best Company.

Line of Business	Average Aggregate Premium
CreditAH	170254

BurglaryTheft	1221543
FinancialGuaranty	2003131
Aircraft	5096009
Warranty	5124410
Credit	5244933
ExcessWorkersCompensation	5741474
Fidelity	6371146
BoilerMachinery	8406989
Earthquake	10437466
ProductsLiab	13519200
OceanMarine	14763895
OtherAH	16047616
FarmownersMultiplePeril	18261590
MortgageGuaranty	22631546
GroupAH	23367889
Surety	26857473
MedicalProfLiab	40110401
AlliedLines	45995391
Fire	57435028
InlandMarine	59258296
CommercialMultiplePeril_Liab	67921940
CommericalAuto_Liab	104684338
WorkersCompensation	248056522
PrivatePassengerAuto_PhysicalDamage	381764513
PrivatePassengerAuto_Liab	388261116
HomeownersMultiplePeril	395222210

Table 8: Aggregate premiums written in USD per insurance line averaged across years 2005 to 2015. The sample covers direct US property and casualty insurers from 2005 to 2015 and is based on data from A.M. Best Company after excluding outliers as well as aggregate and residual lines of business.

	Dependent variable:			
	Large relative Loading			
	(3)	(4)	(5)	(6)
Small Premium	1.826***	1.880***		
	(0.016)	(0.021)		
Large Premium			−2.924***	−2.898***
			(0.039)	(0.046)
Competition	0.003***	0.006***	0.004***	0.006***
	(0.0004)	(0.0004)	(0.0004)	(0.0004)
HHI	21.630***	34.492***	41.477***	49.370***
	(2.483)	(3.276)	(2.515)	(3.214)
Competition:HHI	−0.048***	−0.093***	−0.060***	−0.093***
	(0.004)	(0.006)	(0.005)	(0.006)
Constant	−3.176***	−4.057***	−3.788***	−4.223***
	(0.213)	(0.384)	(0.216)	(0.390)
Company Fixed Effects	–	*Y*	–	*Y*
Year Fixed Effects	*Y*	*Y*	*Y*	*Y*
Observations	112,302	112,302	112,302	112,302
Akaike Inf. Crit.	110,156.700	98,470.120	110,845.900	99,784.150
Note:			*p<0.1; **p<0.05; ***p<0.01	

Table 9: This table reports the coefficients and standard errors of logit regressions on an observation exhibiting a large loading (among the 25% largest premium loadings relative to loss volatility in a particular year): $f(P(\text{large loading})_{i,L,t}) = \mathbb{1}\{\text{large/small premium}\}_{i,L,t} + \beta_1 Compet_{L,t} + \beta_2 Conc_{L,t} + \beta_3 Compet_{L,t} * Conc_{L,t} + \beta_4 year_t + \beta_5 company_i + \varepsilon_{i,L,t}$, where we focus on the 25% smallest (largest) premiums and premium loadings across the entire sample in a particular year, and f is the logit link-function. The marginal effects are a dummy for large (small) aggregate premium written (among the 25% largest (smallest) written premiums in a particular year), changes in concentration (normalized HHI index), and competition (number of active companies in a specific line of business). The sample covers direct US property and casualty insurers from 2005 to 2015 based on data provided by A.M. Best Company.

Insurance Line	β_L	$\overline{n_{\min}}$	$\min n_{\min}$	$\max n_{\min}$	$\overline{n_{\min}}$	$\min n_{\min}$	$\max n_{\min}$
Loss Distribution:		N	N	N	LN	LN	LN
Earthquake	103.43	1	1	1	1	1	1
BurglaryTheft	71.77	1	1	1	1	1	1
MortgageGuaranty	52.58	1	1	1	1	1	1
FinancialGuaranty	49.36	1	1	1	1	1	1
Fidelity	37.17	1	1	1	1	1	1
ProductsLiab	28.69	1	1	1	1	1	1
OceanMarine	24.45	1	1	1	1	1	1
Surety	23.43	1	1	1	1	1	1
Credit	22.29	1	1	1	1	1	1
Aircraft	21.52	1	1	1	1	1	1
ExcessWorkersCompensation	20.23	1	1	1	1	1	1
Warranty	19.38	1	1	1	1	1	1
BoilerMachinery	18.69	1	1	1	1	1	1
PrivatePassengerAuto_Liab	18.58	1	1	1	1	1	1
MedicalProfLiab	18.21	1	1	1	1	1	1
InlandMarine	14.24	1	1	1	1	1	1
WorkersCompensation	9.56	1	1	1	1	1	1
FarmownersMultiplePeril	9.10	1	1	1	1	1	1
CommericalAuto_Liab	6.99	1	1	1	1	1	1
CommercialMultiplePeril_Liab	6.29	1	1	1	1	1	1
Fire	5.66	1	1	1	1	1	1
GroupAH	5.20	1	1	1	1	1	1
PrivatePassengerAuto_PhysicalDamage	4.24	1	1	1	1	1	1
AlliedLines	3.96	1	1	1	1	1	1
HomeownersMultiplePeril	3.69	1	1	1	1	1	1
CreditAH	0.54	129	56	195	60	51	74
OtherAH	0.00	3217	1500	4800	1967	1200	2650

Table 10: Minimum number of observations, $n_{\min}$, such that the average relative empirical safety loading, $\bar{\beta}_L$, is smaller or equal to the theoretical safety loading $s_{rel}(\varepsilon(n))$. β_L results from Regression (1). s results from fitting $q = e^{a-bP}$ to the optimal insurance demand for a consumer with a coefficient of relative risk aversion $\gamma = 0.8$ that faces the risk of a loss $\tilde{L}$. Loss distributions are fitted to empirical observations as reported in Table 3 for either the normal (N) or log-normal (LN) distribution. We compute $n_{\min}$ for different levels of the consumer's wealth endowment, $w_0 = \max\{\Delta\mu_L,\, \mu_L + \Delta\sigma_L\}$ with $\Delta \in \{2, 3, 4\}$, and report the average, minimum, and maximum value for $n_{\min}$.

Insurance Line	β_L	$\overline{n_{\min}}$	$\min n_{\min}$	$\max n_{\min}$	$\overline{n_{\min}}$	$\min n_{\min}$	$\max n_{\min}$
Loss Distribution:		N	N	N	LN	LN	LN
Earthquake	103.43	1	1	1	1	1	1
BurglaryTheft	71.77	1	1	1	1	1	1
MortgageGuaranty	52.58	1	1	1	1	1	1
FinancialGuaranty	49.36	1	1	1	1	1	1
Fidelity	37.17	1	1	1	1	1	1
ProductsLiab	28.69	1	1	1	1	1	1
OceanMarine	24.45	1	1	1	1	1	1
Surety	23.43	1	1	1	1	1	1
Credit	22.29	1	1	1	1	1	1
Aircraft	21.52	1	1	1	1	1	1
ExcessWorkersCompensation	20.23	1	1	1	1	1	1
Warranty	19.38	1	1	1	1	1	1
BoilerMachinery	18.69	1	1	1	1	1	1
PrivatePassengerAuto_Liab	18.58	1	1	1	1	1	1
MedicalProfLiab	18.21	1	1	1	1	1	1
InlandMarine	14.24	1	1	1	1	1	1
WorkersCompensation	9.56	1	1	1	1	1	1
FarmownersMultiplePeril	9.10	1	1	1	1	1	1
CommericalAuto_Liab	6.99	1	1	1	1	1	1
CommercialMultiplePeril_Liab	6.29	1	1	1	1	1	1
Fire	5.66	1	1	1	1	1	1
GroupAH	5.20	1	1	1	1	1	1
PrivatePassengerAuto_PhysicalDamage	4.24	1	1	1	1	1	1
AlliedLines	3.96	1	1	1	1	1	1
HomeownersMultiplePeril	3.69	1	1	1	1	1	1
CreditAH	0.54	682	440	955	530	530	530
OtherAH	0	10833	10500	Inf	Inf	Inf	Inf

Table 11: Minimum number of observations, $n_{\min}$, such that the average relative empirical safety loading, $\bar{\beta}_L$, is smaller or equal to the theoretical safety loading $s_{rel}(\varepsilon(n))$. $\bar{\beta}_L$ results from Regression (1). s results from fitting $q = e^{a-bP}$ to the optimal insurance demand for a consumer with a coefficient of relative risk aversion $\gamma = 0.2$ that faces the risk of a loss $\tilde{L}$. Loss distributions are fitted to empirical observations as reported in Table 3. We compute $n_{\min}$ for different levels of the consumer's wealth endowment, $w_0 = \max\{\Delta\mu_L,\, \mu_L + \Delta\sigma_L\}$ with $\Delta \in \{2, 3, 4\}$, and report the average, minimum, and maximum value for $n_{\min}$.

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