Identifying Scenarios for the Own Risk and Solvency Assessment of Insurance Companies*

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Abstract

Most insurers in the European Union determine their regulatory capital requirements based on the standard formula of Solvency II. However, there is evidence that the standard formula inaccurately reflects insurers' risk situation and may provide misleading steering incentives. In the second pillar, Solvency II requires insurers to perform a so-called "Own Risk and Solvency Assessment" (ORSA). In their ORSA, insurers must establish their own risk measurement approaches, including those based on scenarios, in order to derive suitable risk assessments and address shortcomings of the standard formula. The idea of this paper is to identify scenarios in such a way that the standard formula in connection with the ORSA provides a reliable basis for risk management decisions. Using an innovative method for scenario identification, our approach allows for a simple but relatively precise assessment of marginal and even non-marginal portfolio changes. We numerically evaluate the proposed approach in the context of market risk employing an internal model from the academic literature and the Solvency Capital Requirement (SCR) calculation under Solvency II.

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1 Introduction

Even in modern regulation systems such as Solvency II, the majority of insurance companies determine their capital requirements using a pre-specified standard formula rather than a self-developed internal risk capital model.[1] However, various academic papers show that the risk landscape may not be realistically depicted on the basis of such a standardized approach, which can lead to incorrect steering incentives when portfolio decisions are made. A relatively well known deficiency of the Solvency II standard formula is that it disregards credit risks of any government counterparties in the European Economic Area (EEA) or the Organisation for Economic Co-operation and Development (OECD).[2] Gatzert and Martin (2012), for instance, identify this problem and show that there is a gap between the results of an internal risk model and those of the standard formula in the context of market risks. Becker and Ivashina (2015) find empirical evidence that regulatory constraints on insurance companies investing in the corporate bond market can lead to portfolio distortions. Chen et al. (2019) find that relying on the square root rule for the calculation of risk-based capital (RBC) may provide a misleading view of changing diversification effects. In the case of equity risk, Fischer and Schlütter (2015) show that insurers' asset selection is highly sensitive to parameters of the standard formula and can end up too risky or conservative. Braun et al. (2017) find that the standard formula is likely to have an adverse impact on life insurers' asset allocation, as it hinders insurers in selecting an efficient investment portfolio. Furthermore, Pfeifer and Strassburger (2008) provide evidence that for several classes of distributions, the standard formula underestimates the true Solvency Capital Requirements (SCR). A more general criticism regarding the regulatory framework is formulated by Scherer and Stahl (2021), stating that the Solvency II standard formula "lacks sound economic and mathematical reasoning". A mitigation of the standard formula's deficiencies could potentially be offered by second pillar requirements. These requirements aim to improve insurers' Enterprise Risk Management and to this end require — in addition to the standard formula — stress tests and scenario analyses.[3] These approaches aim to extend the regulatory risk mea-

[1] For instance, the European Insurance and Occupational Pensions Authority (EIOPA) states in their annual Insurance Statistics report regarding the own funds, cf. EIOPA (2019), that 2,470 of the 2,658 insurers evaluated base their risk calculations on the standard formula provided by Solvency II.

[2] Cf. BaFin (2016).

[3] For instance, Schlütter (2021) identifies deficiencies in the measurement of interest rate risk under the Solvency II regulation and derives correlated scenarios to find an appropriate adjustment.

surement “in order to provide an adequate basis for the assessment of the overall solvency needs”, cf. EIOPA (2015), Guideline 7. Similarly, the requirements for the Own Risk and Solvency Assessment (ORSA) of an insurer expect to “also take into account risks that are not or not adequately included in the standard formula, and must develop a suitable assessment procedure for them”, cf. BaFin (2016).

In general, stress scenarios are a tool to “help decision makers understand better the level of resilience of the organization”, cf. Albrecher et al. (2018, Chapter 5.5). In the literature, there are several different suggestions on how to identify (reverse) stress scenarios by stressing the underlying distribution of risk drivers: for example, Korn and Müller (2021) apply the worst-case scenario in the setting of a portfolio optimization. Pesenti et al. (2019) employ the Kullback-Leibler divergence measure, Makam et al. (2021) use the χ^2 divergence considering a discrete sample to derive stress scenarios. Breuer and Csiszár (2013) suggest identifying stress scenarios by using a relative entropy measure to ensure plausibility. However, they do not address the ORSA requirements explicitly to quantify scenarios that are not captured by the standard formula. A different idea more in line with the ORSA requirements is provided by McNeil and Smith (2012), proposing the so-called “least solvent likely event” (LSLE), a deterministic scenario that allows for a reliable evaluation of risk resulting from a given portfolio.[4]

This paper proposes a new approach to identify ORSA scenarios. We assume that the insurer's strategy can be expressed by an exposure vector $u \in R^n$. The entries of u can represent, for example, investments in asset categories or sizes of the insurer's lines of business. Corresponding to strategy u, the function $f^{\text{true}}(u)$ provides the capital requirement which satisfies the safety level defined in pillar 1 (i.e. the 99.5% Value-at-Risk). $f^{\text{true}}(u)$ can be considered as the outcome of a perfect internal model. Function $f^{\text{SF}}(u)$ presents the capital requirement according to the standard formula. Our target is to identify scenarios which approximate the residual

$$f^{\text{true}}(u) - f^{\text{SF}}(u) \tag{1}$$

[4]Notably, McNeil and Smith (2012, Corollary 4.4) show that their suggestion coincides with the so-called "gradient scenario", which is often also referred to as Euler allocation, cf. for instance Tasche (2008).

To this end, we consider $g_m(u)$ as the risk measurement of strategy u based on m scenarios. If $g_m(u)$ approximates the residual in (1), then the standard formula in connection with ORSA scenarios provides an approximation of the true risk:

$$f^{\text{true}}(u) \approx f^{\text{SF}}(u) + g_m(u) \tag{2}$$

Our technical basis to derive scenarios for function $g_m(u)$ are so-called "orthogonal convexity scenarios" (OCS) as proposed by Aigner and Schlütter (2022). The aim of OCS is to translate the risk measurement of a portfolio into a small number of multivariate realization vectors. In contrast to the aforementioned stress scenario literature, OCS therefore do not change the risk distribution.By construction, OCS are orthogonal in the sense of sensitivity-implied tail correlations, as proposed by Paulusch and Schlütter (2022), and hence their use is not limited to elliptical distributions. Additionally, by employing deterministic scenarios, a combination with the standard formula leads to a deterministic risk measurement in the right–hand side of Eq. (2). We show that such a combination with only a single OCS can then be set to reflect the outcome of the internal model for the initial portfolio as well as all first-order derivatives. The approach thus coincides with the well known Euler capital allocation.[5] When additional scenarios are taken into account, second-order derivatives are also reflected correctly. By approximating the internal model not only in a linear way, the scenarios reflect how portfolio-wide diversification effects alter when the portfolio volumes are changed. Deriving scenarios for a representative insurer could thereby provide a regulatory authority with a tool that can be handed out to insurance companies as an addition to the standard formula.

Numerically, the suggestion is evaluated based on Gatzert and Martin (2012) and Eckert et al. (2016) dealing with market risks. In the latter, the authors employ an internal model which comprises the risks of three sub-modules, the outcome of which is considered as the true portfolio risk. A difference in risk capital between the internal model and the standard formula is found, providing a good starting point for the scenario-based extension of the regulatory approach.

This article contributes to the literature in three ways: Firstly, it highlights the shortcomings of a standardized approach to risk measurement. Secondly, a practical idea is presented on how to formalize and implement the suggestions of the ORSA. Thirdly, empirical indications are given as to what meaningful scenarios in the sense of the ORSA

[5] Cf. for instance Tasche (2008), Buch et al. (2011), or Guo et al. (2021).

might look like for an example company.

The remainder of this article is structured as follows. section 2 presents the ideas of Aigner and Schlütter (2022) for the determination of scenarios. Section 2 summarizes the suggestion of Solvency II for the measurement of market risk as well as an internal model counterpart given by Eckert et al. (2016) and Gatzert and Martin (2012). Section 3 numerically evaluates the goodness of the suggested extension of the standard formula. Section 4 provides concluding remarks about the concrete practical usefulness of the proposed approach.

2 Orthogonal convexity scenarios for the ORSA

We suppose that a company's risk can be specified by inspecting a random vector

$$\mathbf{X} = (X_1, ..., X_n)^{\mathrm{T}}$$

with X_i modeling the losses (or gains in case of negative values) of the ith risk driver. The X_i could be losses resulting from risks of the various submodules in the standard formula, but they could also be defined at a more granular level. For example, in section 2, we will consider the X_i to reflect losses from single equity and bond investments. It is further assumed that the insurer can change its portfolio by linearly scaling the X_i. To this end, we introduce an exposure vector $u \in \mathbb{R}^n$ representing the portfolio volumes. The true risk function can then be defined, in line with Solvency II, as

$$\begin{aligned} f^{\text{true}} : \mathbb{R}^n &\to \mathbb{R} \\ u &\mapsto \varrho(\mathbf{X}^{\mathrm{T}} u) - \mathbb{E}(\mathbf{X}^{\mathrm{T}} u) \end{aligned} \tag{3}$$

with ϱ being a risk measure. For the identification of scenarios later on, it is sufficient to assume that ϱ is homogeneous of degree one and law-invariant. Paulusch (2017) ensures that the common risk measure under Solvency II, the 99.5% Value-at-Risk, fulfills this property. Notably, our method could also be applied in connection with the Expected Shortfall, for example, cf. also Paulusch (2017). The function f^{true} could then be derived from a stochastic model or a "perfect" internal model.

Secondly, we will inspect the regulatory standard formula that measures the risk of portfolio u in terms of

$$f^{\mathrm{SF}} : \mathbb{R}^n \to \mathbb{R}, u \mapsto f^{\mathrm{SF}}(u) \tag{4}$$

which is explicitly specified in the ORSA, cf. EIOPA (2015).[6] We assume that both functions, $f^{\mathrm{true}}(u)$ and $f^{\mathrm{SF}}(u)$, are twice continuously differentiable in a neighborhood of an initial portfolio u_{initial}. The central concern that we would like to tackle is that $f^{\mathrm{true}}(u)$ and $f^{\mathrm{SF}}(u)$ may deviate, and we will thus inspect and approximate the residual

$$f^{\mathrm{diff}}(u) = f^{\mathrm{true}}(u) - f^{\mathrm{SF}}(u) \tag{5}$$

Here, situations that are not captured by the standard formula can be considered. In order to approximate $f^{\mathrm{diff}}(u)$ relying on deterministic scenarios also taking into account tail dependencies, the approach presented by Aigner and Schlütter (2022) will be used.[7] The authors suggest employing so-called "orthogonal convexity scenarios" (OCS) resulting in a scenario-based risk-measurement function

$$\begin{aligned} g_m &: \mathbb{R}^n \to \mathbb{R} \\ u &\mapsto \sqrt{\sum_{i=1}^{m} \left(\left(x_i^{\mathrm{OCS}}\right)^{\mathrm{T}} u \right)^2} \end{aligned} \tag{6}$$

with $1 \leq m \leq n$ a pre-specified number of scenarios that should be considered and x_i^{OCS}, $i = 1, \ldots, m$, denoting the OCS.[8] The latter are defined as

$$x_i^{\mathrm{OCS}} = \frac{w_i^{\mathrm{T}} H}{\sqrt{2 \cdot w_i^{\mathrm{T}} H w_i}} \tag{7}$$

with weightvectors $w_1, \ldots, w_m \in \mathbb{R}^n$ and H the Hessian matrix of $\left(f^{\mathrm{diff}}\right)^2$.[9] The weightvectors have to be specified by the user of the method and are supposed to be selected orthogonally in the sense of the bilinearform

$$\langle w_i, w_j \rangle_H = w_i^{\mathrm{T}} H w_j \tag{8}$$

[6]Although this paper focuses on the standard formula for insurance companies, the methodology could also be adapted to other models which are to be compared to a benchmark model.

[7]There is some literature dealing with tail dependencies such as Mittnik (2014) and Paulusch and Schlütter (2022).

[8]These scenarios can be understood as realizations of the risk vector $\mathbf{X}$ in Eq. (9).

[9]Appendix A.2 highlights how to calibrate the Hessian matrix.

with H as before. Appendix A.3 provides some guidance on how to identify the necessary weightvectors. Explicitly, it can be ensured that

$$x_1^{\mathrm{OCS}} = \nabla_u f^{\mathrm{diff}}\left(u_{\mathrm{initial}}\right)$$

such that the first scenario coincides with the Euler allocation of f^{diff}. The OCS approach can also be interpreted as an extension of the "Least solvent likely event" (LSLE) introduced by McNeil and Smith (2012) by including additional scenarios that can capture the convexity of the approximated risk measurement.

By basing the identification of the scenarios on the Hessian matrix, the function g_m is capable of capturing non-linear dependencies and heavy tails in the portfolio as outlined by Paulusch and Schlütter (2022).[10] When now approximating $f^{\mathrm{true}}(u)$ by the sum $f^{\mathrm{SF}}(u) + g_m(u)$, it fulfills the properties summarized in Proposition 1.

Proposition 1. *Let* $1 \leq m \leq n$, $f^{true}(u)$ *and* $f^{SF}(u)$ *as before and* $f^{true}\left(u_{initial}\right) > f^{SF}\left(u_{initial}\right)$. *Then, it holds:*

1) $f^{true}\left(u_{initial}\right) = f^{SF}(u_{initial}) + g_m(u_{initial})$

2) For all $v \in \mathbb{R}^n$, *it is*

$$\frac{\partial}{\partial h} f^{true}\left(u_{initial} + hv\right)|_{h=0} = \frac{\partial}{\partial h}\left(f^{SF}\left(u_{initial} + hv\right) + g_m\left(u_{initial} + hv\right)\right)|_{h=0}$$

3) For $v_1, v_2 \in \operatorname{span}\{w_1, \ldots, w_m\}$, *it is*

$$\frac{\partial^2}{\partial h_1 \partial h_2} f^{true}\left(u_{initial} + h_1 v_1 + h_2 v_2\right)|_{h_1=h_2=0} =$$
$$= \frac{\partial^2}{\partial h_1 \partial h_2}\left(f^{SF}\left(u_{initial} + h_1 v_1 + h_2 v_2\right) + g_m\left(u_{initial} + h_1 v_1 + h_2 v_2\right)\right)|_{h_1=h_2=0}$$

Therein, g_m *is as in Eq.* (6) *in connection with* x_i^{OCS}*'s as in* (7).

The proof is presented in Appendix A.1. From Proposition 1, we see that an extension of the standard formula adding risk resulting from OCS indeed allows an approximation of the true risk measurement function in the sense of first and second order sensitivities. To derive ORSA scenarios in the sense of section 2, we have to calibrate the two risk

[10] In the literature, it is often suggested to identify scenarios on the basis of the covariance matrix such that the scenarios can be found through the application of Principal Component Analysis (PCA), cf. for instance Hull (2018). Aigner and Schlütter (2022) highlight that relying on such a linear measure may lead to a misinterpretation of the company's risk situation.

measurements based on the true risk which will be represented by an internal model and the one based on the regulatory requirements. This is done in the following two subsections. Specifically, we restrict the analysis to a market risk setting including equity, interest rate and spread risk, and set up a specific portfolio.

2.1 Specification of f^{true}

To meet the requirements of the regulatory authority in Europe, we employ as risk measure ϱ in (3) the VaR to a confidence level of 99.5% such that the true risk is given by

$$\begin{aligned} f^{\text{true}} &: \mathbb{R}^n \to \mathbb{R} \\ &u \mapsto \mathrm{VaR}_{0.995}\left(\mathbf{X}^{\mathrm{T}} u\right) - \mathbb{E}\left(\mathbf{X}^{\mathrm{T}} u\right) \end{aligned}$$

with a random risk vector

$$\mathbf{X} = (X_1, \ldots, X_n)^{\mathrm{T}} \tag{9}$$

consisting of $n = n_B + n_S$ risk drivers comprising $n_B \in \mathbb{N}$ bond and $n_S \in \mathbb{N}$ stock investments. Without loss of generality, we assume that $X_1, \ldots, X_{n_B}$ reflect the losses/gains resulting from bonds and $X_{n_B+1}, \ldots, X_{n_B+n_S}$ those from stock investments. For f^{true} to be well-defined, we then have to specify $\mathbf{X}$. For notational reasons, we rewrite the risk vector $\mathbf{X} = \left(X^B, X^S\right)^{\mathrm{T}}$ to represent losses/gains resulting from bonds and stocks separately after one year. In order to determine these stochastic vectors

$$\begin{aligned} X^B &= (X_1^B, \ldots, X_{n_B}^B)^{\mathrm{T}} \\ X^S &= (X_1^S, \ldots, X_{n_S}^S)^{\mathrm{T}} \end{aligned}$$

we follow Eckert et al. (2016), who suggest modeling stock investments in combination with a reduced form credit risk model for defaultable bond exposure. Therefore, they let the stochastic default intensity (hazard rate) $h(t)$ follow a Vasicek (1977) process. The resulting model in Eckert et al. (2016) can be written as

$$
\begin{aligned}
dr(t) &= \kappa \cdot (\theta - r(t))\, dt + \zeta dW_r(t) \\
dh_1(t) &= \chi_1 \cdot (o_1 - h_1(t))\, dt + \Gamma_1 dW_{h_1}(t) \\
&\dots \\
dh_{n_B}(t) &= \chi_{n_B} \cdot (o_{n_B} - h_{n_B}(t))\, dt + \Gamma_{n_B} dW_{h_{n_B}}(t) \\
dS_1(t) &= \mu_1 S_1(t) dt + \sigma_1 \cdot S_1(t) dW_{S_1}(t) \\
&\dots \\
dS_{n_S}(t) &= \mu_{n_S} S_{n_S}(t) dt + \sigma_{n_S} \cdot S_{n_S}(t) dW_{S_{n_S}}(t)
\end{aligned}
$$

where $W(t) = \left(W_r(t), W_{h_1}(t), \dots, W_{h_{n_B}}, W_{S_1}, \dots, W_{S_{n_S}}\right)^T$ is a standard Brownian motion with a symmetric correlation matrix R_{IM} implying that the valuation of market risk takes into account interest rate risk, credit risk, equity risk as well as dependencies between them.[11] Herein, κ and χ_i define the speeds of mean reversion, θ and o_i the long-term means, and ζ and Γ_i the volatilities of the processes for $i = 1, \dots, n_B$. Additionally, stock investments are assumed to follow geometric Brownian motions.[12] Here, there are closed form solutions for the pricing of stock investments given by

$$S_i(t) = S_i(0) \cdot \exp\left(\mu_{S_i} - \frac{\sigma_{S_i}^2}{2} \cdot t + \sigma_{S_i} \cdot \sqrt{t} W_{S_i}\right)$$

for $i = n_B + 1, \dots, n_B + n_S$, cf. for instance Gatzert and Martin (2012, Eq. (7)), modeling the stock value at time t. Since we are interested in the stochastic losses of each stock investment, we set

$$\left(X^S\right)_i := -\left(S_i(1) - S_i(0)\right) \tag{10}$$

for $i = 1, \dots, n_S$.[13] Notably, we will set $S_i(0) = 1$ later on.

For the evaluation of bond investments, we have to take into account spread and interest rate risk. As our model states, we assume the interest rate process to follow a Vasicek (1977) process

$$dr(t) = \kappa \cdot (\theta - r(t))\, dt + \zeta dW_r(t)$$

[11]For more details on the model, cf. Eckert et al. (2016).

[12]The selection of geometric Brownian motions for modeling stocks is quite common in the literature, cf. for instance Islam and Nguyen (2020) or Graf and Korn (2020).

[13]In Eq. (10), negative values are reported such that positive values represent losses later on.

allowing us to determine the price of a non-defaultable zero coupon bond with maturity T as[14]

$$
\begin{aligned}
&P(t,T) = \exp(-M_r(t,T) + 0.5 \cdot V_r^2(t,T)), \text{ with}\\
&M_r(t,T) = r(t) \cdot \frac{1-\exp(-\kappa \cdot (T-t))}{\kappa} + \theta \cdot \left((T-t) - \frac{1-\exp(-\kappa \cdot (T-t))}{\kappa}\right)\\
&V_r^2(t,T) = \frac{\zeta^2}{\kappa^2}\left((T-t) - 2 \cdot \frac{1-\exp(-\kappa(T-t))}{\kappa} + \frac{1-\exp(-2\kappa(T-t))}{2\kappa}\right)
\end{aligned}
$$

Furthermore, in line with Eckert et al. (2016) we follow Duffie and Singleton (1999) to account for credit risk in the valuation of defaultable bonds. Therefore, default events for a given bond i are modeled by a Cox process with a stochastic hazard rate $h_i(t)$. Moreover, the model of Duffie and Singleton (1999) allows us to take into account dependencies between credit spread and interest rate by using correlated Brownian motions $W_r(t), W_{h_i}(t),\ i = 1, \ldots, n_B$. Additionally, a recovery of the market value is assumed such that in the case of a default at time τ, each bond pays a fraction of its value before the default

$$\delta_R(\tau) \cdot P^{RMV}(\tau-, T)$$

with $\delta_R(t)$ denoting the recovery rate,[15] $P^{RMV}(t-,T) = \lim_{s \nearrow \tau} P^{RMV}(s,T)$ and $P^{RMV}(t,T)$ the pre-default price at time $t < \tau$ of a recovery of market value (RMV) defaultable bond with maturity T. Then, the price of a defaultable bond is

$$P^{RMV}(t,T) = E^{\mathbb{Q}}\left(\exp\left(-\int_t^T (r(u) + s_i(u))\, du\right)\right)$$

with the credit spread $s_i(t) = (1 + \delta_R(t)) \cdot h_i(t)$. Since the hazard rates are assumed to follow a Vasicek (1977) process, the spread risks $s_i(t)$ follow —according to Itô's Lemma— again a Vasicek (1977) process given by

$$ds_i(t) = \chi_i \cdot (\hat{o}_i - s_i(t))dt + \hat{\Gamma}_i dW_{h_j}$$

[14]These formulas are presented for example in Eckert et al. (2016) and Schönbucher (2003).

[15]In general, the recovery rate could be stochastic, but for simplicity, we assume a constant recovery rate later on.

with $\hat{o}_i = (1-\delta_R)\cdot o_i$, $\hat{\Gamma}_i = (1-\delta_R)\Gamma_i$ and a constant recovery rate $\delta_R(t) = \delta_R$. Then, Eckert et al. (2016) provide the following closed form solutions for the price of a RMV defaultable bond as

$$
\begin{aligned}
P_i^{RMV}(t,T) &= P(t,T)\exp\left(-M_{s_i}(t,T) + 0.5\cdot V_{s_i}^2 + C_i(t,T)\right), \text{ with} \\
M_{s_i}(t,T) &= s_i(t)\cdot\frac{1-\exp(-\chi_i(T-t))}{\chi_i} + \hat{o}_i\cdot\left((T-t) - \frac{1-\exp(-\chi_i(T-t))}{\chi_i}\right) \\
V_{s_i}^2(t,T) &= \frac{\hat{\Gamma}_i^2}{\chi_i^2}\left((T-t) - 2\cdot\frac{1-\exp(-\chi_i(T-t))}{\chi_i} + \frac{1-\exp(-2\chi_i(T-t))}{2\chi_i}\right), \text{ and} \\
C_i(t,T) &= \rho_{r,h_i}\frac{\zeta\hat{\Gamma}_i}{\kappa\chi_i}\left((T-t) - \frac{1-\exp(-\kappa(T-t))}{\kappa} - \frac{1-\exp(-\chi(T-t))}{\chi_i}\right. \\
&\qquad \left. + \frac{1-\exp(-(\kappa+\chi_i))(T-t)}{\kappa+\chi_i}\right)
\end{aligned}
$$

where ρ_{r,h_i} reflects the correlation of the standard Brownian motions $W_r(t)$ and $W_{h_i}(t)$. The price of a defaultable bond i with hazard rate h_i and maturity T_i at time t is then derived as

$$B_i(t) = \sum_{h=t+1}^{T_i} CF_i(h)\cdot P_i^{RMV}(t,h)$$

with cash flows

$$CF_i(t) = \begin{cases} c_i(t)\cdot FV_i, & \text{if } (t<T_i)\wedge\left(\tau_i^B > t\right) \\ (1+c_i(t))\cdot FV_i, & \text{if } (t=T_i)\wedge\left(\tau_i^B > t\right) \\ \delta_R\cdot B_i(t-1), & \text{if } t=\tau_i^B \\ 0, & \text{else} \end{cases} \tag{11}$$

depending on the time of default τ_i^B, coupon $c_i(t)$ and time t. Herein, the face values FV_i are scaled such that $B_i(0) = 1$ for all $i = 1,\dots,n_B$. Since we are again interested in potential losses/gains after one year, we set

$$\left(X^B\right)_i = -(B_i(1) - B_i(0)) \tag{12}$$

with $i = 1,\dots,n_B$. In case of a mixed portfolio consisting of stocks and bonds, the market value risk vector is then

$$\mathbf{X} = \left((X^B)^{\mathrm{T}}, (X^S)^{\mathrm{T}}\right)^{\mathrm{T}}$$

with X_B and X_S as in (12) and (10) respectively. Here, the risk measure function in (3) is well-defined. By selecting the face value as in (11), the cash flows are adjusted in a way that ensures that $\left(\mathbf{X}^T u\right)_i$ represents a loss with a value of u_i at $t = 0$. The portfolio volumes can be steered by adjusting the u_i.

2.2 Specification of $f^{\mathbf{SF}}$

The Solvency II framework suggests a module structure making it necessary to calculate *equity*, *interest rate* and *spread* risks separately on a sub-module basis, cf. Figure 1. The risks are then aggregated towards the market module by the so-called square-root formula denoted by f^{SF}. In particular, we calculate three different values: Mkt_{eq}, Mkt_{int} and

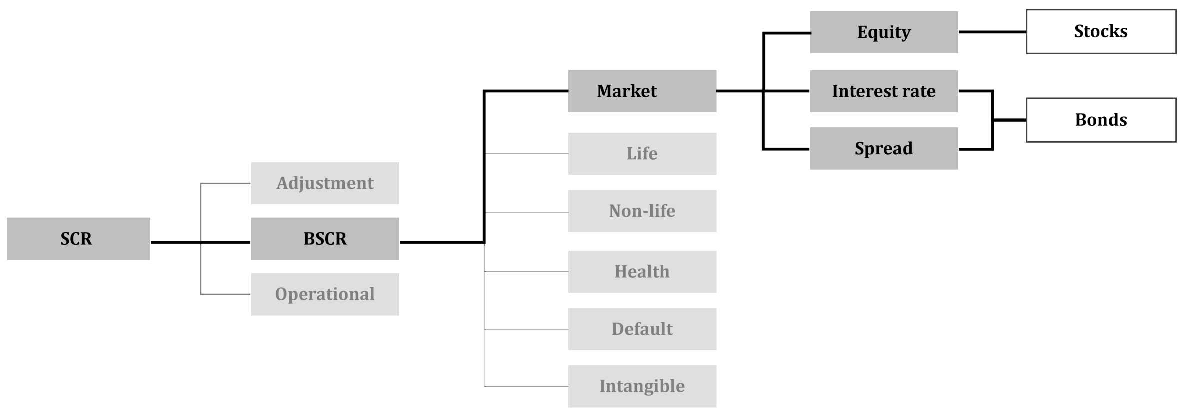

Figure 1: Structure for the SCR calculation in the style of Gatzert and Martin (2012); only sub-modules that are taken into account later on are presented.

Mkt_{sp} representing the capital requirement of the sub-modules respectively. Liquidity, concentration, property and currency risk will be excluded in the following analysis for the sake of simplicity. In this section, we mainly adopt the notation of Gatzert and Martin (2012). For notational reasons, we set the exposure vector as $u = \left(\left(u^B\right)^{\mathrm{T}}, \left(u^S\right)^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{R}^n$ with

$$u^B = (u_1^B, \ldots, u_{n_b}^B)^{\mathrm{T}} \in \mathbb{R}^{n_B}$$
$$u^S = (u_1^S, \ldots, u_{n_s}^S)^{\mathrm{T}} \in \mathbb{R}^{n_S}$$

distinguishing between exposures referring to bond investments, $u^B \in \mathbb{R}^{n_B}$, and those referring to stocks, $u^S \in \mathbb{R}^{n_S}$.

2.2.1 Interest rate risk

First, the risk resulting from a change of the term structure is determined within the *interest rate risk* sub-module. For this purpose, we calculate the present value (PV) of all interest-rate-sensitive exposures—namely u^B—by discounting their cash flows (CF) using the risk-free interest rate structure $r_f(t)$ which is published monthly by EIOPA, cf. Table 7. Specifically, we have to calculate for $i = 1, \ldots, n_B$

$$PV_i^{int} = \sum_{t=1}^{T_i} \frac{\overline{CF}_i(t)}{(1 + r_f(t))^t} \tag{13}$$

with T_i the maturity and $\overline{CF}_i(t)$ the cash flow of investment i at time t. For face values FV_i as discussed in the section before and a coupon payment $c_i(t)$ at time t, the cash flow for bond i is given by

$$\overline{CF}_i(t) = \begin{cases} c_i(t) \cdot FV_i, & \text{if } t < T_i \\ (1 + c_i(t)) \cdot FV_i, & \text{if } t = T_i \end{cases} \tag{14}$$

for $i = 1, \ldots, n_B$. The upward shocked present values are then calculated as

$$\left(PV_{up}^{int}\right)_i = \sum_{t=1}^{T_i} \frac{\overline{CF}_i(t)}{\left(1 + \max\left(r_f(t) \cdot (1 + s^{up}(t))\,, 0.01\right)\right)^t} \tag{15}$$

where the maximum in the denominator is in line with European Commission (2015, Article 166) and ensures that there is at least a shock of one percent. Furthermore, for the downward shock we calculate

$$\left(PV_{down}^{int}\right)_i = \sum_{t=1}^{T_i} \frac{\overline{CF}_i(t)}{\left(1 + \max\left(r_f(t) \cdot (1 + s^{down}(t), 0)\right)\right)^t} \tag{16}$$

with the maximum in the denominator accounting for the current low-level interest rate environment. In both cases, it is again $i = 1, \ldots, n_B$. The shocks $s^{up}(t)$, $s^{down}(t)$ are provided by European Commission (2015, Article 166 and 167) and shown in Table 7. Interpreting the results in Eq. (13), (15) and (16) as vectors in $\mathbb{R}^{n_B}$, allows us to determine the overall risk of the sub-module *interest rate* as

$$Mkt_{int}(u^B) = \max\left(\left(PV^{int} - PV_{int}^{up}\right)^{\mathrm{T}} u^B, \left(PV_{int} - PV_{int}^{down}\right)^{\mathrm{T}} u^B\right)$$

depending on the part of the exposure vector reflecting bond investments.

2.2.2 Spread risk

Changes in the credit spread on exposures are considered in the rating-based risk sub-module of the Solvency II standard approach. The risk consists of three uncorrelated groups: the SCR of bonds Mkt_{sp}^{bonds}, of securization positions $Mkt_{sp}^{securization}$ and of credit derivatives Mkt_{sp}^{cd} which are then easily added up to the total risk of the sub-module *spread*

$$Mkt_{sp} = Mkt_{sp}^{bonds} + Mkt_{sp}^{securization} + Mkt_{sp}^{cd}$$

For simplicity, we will restrict our analysis only to bond assets, ignoring securization positions and credit derivatives. The SCR calculation for spread risk then takes into account the current value $MV_{sp,i}(0) = u_i^B$ of bond $i = 1, \dots, n_B$. The stress referring to each bond depends on shocks that can be specified by including their ratings, which are publicly available, and their durations. In order to determine the latter, we rely on the Macaulay (1938) duration with a floor of one, as suggested by European Commission (2015, Article 176). Given that there is only one coupon period per year, it can be determined as

$$duration_i = \min\left(\frac{\sum_{t=1}^{T} t \cdot \overline{CF}_i(t) \cdot (1 + r_f(t))^{-t}}{\sum_{t=1}^{T} \overline{CF}_i(t) \cdot (1 + r_f(t))^{-t}} \cdot \frac{1}{1 + r_{YtM}}, 1\right)$$

for $i = 1, \dots, n_B$ with r_f and $\overline{CF}_i(t)$ the cash flow as before. Furthermore, we need to specify the yield to maturity r_{YtM} which is obtained by solving

$$PV_i^{int} = \sum_{t=1}^{T_i} \overline{CF}_i(t) \cdot (1 + r_{YtM})^{-t}$$

where PV_i^{int} is calculated as in (13) and T_i as before.[16] Given the rating and the duration of each bond, European Commission (2015, Article 176) further outlines how to specify the stresses $stress_i$ for $i = 1, \dots, n_B$ explicitly, cf. Tables 8 and 9. Here, the SCR of the spread risk sub-module (in the simplified version only including bonds) is calculated as

$$Mkt_{sp}\left(u^B\right) = \max\left(\sum_{i=1}^{n_B} u_i^B \cdot stress_i, 0\right) = \max\left(stress^{\mathrm{T}} u^B, 0\right)$$

[16] For the numerical calculation, there are several common approaches, such as the Newton-Raphson method which we will employ later on.

It should be noted that for bonds issued by governments belonging to the EEA or the OECD, the stress, according to BaFin (2016), is always zero percent, and hence the Mkt_{sp} is also zero when the exposure is a respective bond.

2.2.3 Equity risk

In order to calculate the capital requirements resulting from equity risk, we first have to cluster the n_S stock assets within our portfolio in (9) into "global" and "other". The class "global" comprises all exposures transacted in countries that are members of the European Economic Area (EEA) or the Organisation for Economic Co-operation and Development (OECD), cf. CEIOPS (2010) and European Commission (2015). Without loss of generality, we assume that the first $k_{global} \in \mathbb{N}$ entries of u represent the exposure to "global" investments and the rest $k_{other} = n_S - k_{global}$ the exposure to "other" investments. "Global" stocks are easily multiplied with $shock_{global} = 0.3$ and "other" investments are assumed to have a higher risk and therefore are assigned a shock of $shock_{other} = 0.4$.[17] With these specifications, one can directly calculate the market values of both classes by summing up the market values at time $t = 0$, denoted by $MV_{eq,i}(0) = u_i^S$, $i = 1, \ldots, n_S$ for all assets in the respective class and multiplying it with the respective shock parameter

$$Mkt_{eq,\ global}(u^S) = \max\left(0.3 \cdot \sum_{i=1}^{k_{global}} u_i^S, 0\right)$$

$$Mkt_{eq,\ other}(u^S) = \max\left(0.4 \cdot \sum_{i=k_{global}+1}^{n_S} u_i^S, 0\right)$$

with $k_{global} + k_{other} = n_S$.[18] In order to aggregate the classes with respect to diversification effects, EIOPA (2011) recommends the aggregation via the square-root formula

$$Mkt_{eq}(u^S) = \sqrt{x^{\mathrm{T}} R_{eq} x}$$

[17] The stresses are adjusted here to avoid pro-cyclical effects of adverse capital market developments, cf. Gatzert and Martin (2012) for more details.

[18] For further details on the specification of the shocks including strategic participation, adjustments etc. cf. Gatzert and Martin (2012). These are excluded in this paper for the sake of tractability, but the process in the simulation study later on would work equivalently.

with $x = \left(Mkt_{eq,global}\left(u^S\right), Mkt_{eq,other}\left(u^S\right)\right)^{\mathrm{T}}$ and the correlation matrix

$$R_{\mathrm{eq}} = \begin{array}{c} \begin{array}{cc} \textit{Global} & \textit{Other} \end{array} \\ \begin{pmatrix} 1 & 0.75 \\ 0.75 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \textit{Global} \\ \textit{Other} \end{array}$$

Notably, a portfolio only consisting of stocks is just exhibited to the equity sub-module, and the solvency capital requirement (SCR) calculation of the market risk module ends with this sub-module, cf. Figure 1.

2.2.4 Aggregation of the sub-modules

Following European Commission (2015), the three sub-modules, *equity*, *interest rate* and *spread* are assumed to be correlated by

$$R_{\mathrm{SF}} = \begin{array}{c} \begin{array}{ccc} \textit{Interest rate} & \textit{Equity} & \textit{Spread} \end{array} \\ \begin{pmatrix} 1 & A & A \\ A & 1 & 0.75 \\ A & 0.75 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ \textit{Interest rate} \\ \textit{Equity} \\ \textit{Spread} \end{array}$$

where the correlation parameter A is conditional on the result of Mkt_{int} as

$$A = \begin{cases} 0.5, & \text{if } Mkt_{int}(u^B) = \left(PV_{int} - PV_{int}^{down}\right)^{\mathrm{T}} u^B \\ 0, & \text{if } Mkt_{int}(u^B) = \left(PV_{int} - PV_{int}^{up}\right)^{\mathrm{T}} u^B \end{cases}$$

Then, the overall market risk of a portfolio $u \in \mathbb{N}^{n_S+n_B}$ can be calculated with a square-root formula resulting in a specification of the function in (4) as

$$\begin{aligned} f^{\mathrm{SF}} &: \mathbb{R}^n \to \mathbb{R} \\ u &\mapsto \sqrt{x^{\mathrm{T}} R_{SF} x} \end{aligned} \tag{17}$$

with $x = \left(Mkt_{int}(u^B), Mkt_{eq}(u^S), Mkt_{sp}(u^B)\right)^{\mathrm{T}} \in \mathbb{R}^n$ and $n = n_B + n_S$. It is shown in Appendix A.5 that f^{SF} is indeed homogeneous of degree one, a necessary property for the application of OCS in the latter.

2.3 Portfolio set-up

Let us now construct a (theoretical) portfolio, the market risk of which will be determined on the basis of the approaches presented in the last two sections. We assume an investment portfolio consisting of $n_B = 5$ bond investments, cf. Table 1, and $n_S = 2$ stocks, cf. Table 2. Furthermore, we assume that we are equally invested in each of those positions such that we set the initial portfolio as $u_{\text{initial}} = \mathbb{1}_7$ – with $u_1, \ldots, u_5$ presenting the bond and u_6, u_7 the stock investments.[19] Since there is a strict distinction between *spread* and *default* risk in the regulatory requirements, we assume that none of the bonds default until their maturity. We exclude *default* risk in the internal model here, such that the numerical analysis is indeed in line with the solvency calculation of the market risk module.

The SCR based on the standard formula can now directly be derived on the basis of the given specifications. Notably, the spread risk for the governmental bonds is set to zero, since Germany and Spain are both part of the EEA.

For the internal model, we take into account correlated standard Wiener processes with

B_i	Type$_i$	rating$_i$	maturity$_i$	Coupon (in %) c_i	χ_i	o_i	Γ_i
1	Corporate	AA	16	8.00	0.0392	0.0269	0.0004
2	Corporate	A	12	2.95	0.0180	0.0240	0.0009
3	Corporate	BBB	11	5.75	0.0373	0.0453	0.0027
4	Government	BB	10	1.75	0.2201	0.5670	0.2299
5	Government	A	10	0.50	0.0139	-0.0070	0.0022

Table 1: Specifications of the bonds that are taken into account. B_1: Deutsche Bank AG, B_2: Commerzbank AG, B_3: E.ON SE, B_4: Greece and B_5: Spain. The parameters χ_i, o_i and Γ_i refer to the Vasicek processes and are estimated on the basis of spread data between 09/2011 and 09/2021.

the correlation matrix R_{IM} as discussed in section 2.1. The entries of R_{IM} have been estimated relying on monthly data between 09/2011 and 09/2021 and are presented in

[19] The choice of the initial portfolio is arbitrary thanks to homogeneity of f^{true} and f^{SF} as long as $f^{\text{true}}(u_{\text{initial}}) > f^{\mathrm{SF}}(u_{\text{initial}})$.

Table 3. The stocks are then modeled by Monte Carlo simulations with 5,000,000 paths following geometric Brownian motions with the parameters as in Table 2. Here, S_1, namely Euro Stoxx, represents the investment in a "global" asset and S_2, Shanghai Stock Exchange (SSE) composite index, in an "other" asset. Both potential classes in the equity risk sub-module are thus covered.

The interest rate is modeled on the basis of a Vasicek (1977) process with a long–term

S_i	Index_i	category_i	μ_i	σ_i
1	Euro Stoxx	Global	0.0750	0.1611
2	Shanghai SE composite index	Other	0.0632	0.2091

Table 2: Annualized parameters for the specification of the geometric Brownian motions representing stock investments. The values are estimated on monthly data between 09/2011 and 09/2021.

mean $\theta = -0.0225$, the speed of mean reversion of $\kappa = 0.0046$ and a drift of $\sigma = 0.0015$. The parameters have been estimated on the basis of monthly EURIBOR data between 09/2011 and 09/2021 and relying on Maximum Likelihood estimation, cf. for instance Fergusson and Platen (2015). Furthermore, the initial value is set to $r(0) = \theta$.

As possible bond investments, a mixture of corporate and government bonds is considered. Their specifications are presented in Table 1. Employing again Maximum Likelihood estimation, the hazard processes can be fitted with the parameters presented in Table 1. Additionally, the recovery rate is set constant to $\delta_R = 0.61$.[20]

With these specifications, we can now define

	r	h_1	h_2	h_3	h_4	h_5	S_1	S_2
r	1							
h_1	0.1864	1						
h_2	0.1025	0.4518	1					
h_3	0.0848	0.2443	0.3165	1				
h_4	-0.0186	0.0310	-0.0846	-0.0147	1			
h_5	0.1617	0.4263	0.6409	0.5236	-0.0832	1		
S_1	-0.0865	-0.0213	0.0916	-0.2930	-0.0095	-0.1998	1	
S_2	-0.2118	0.0629	-0.1576	-0.1366	0.0789	-0.1818	0.3365	1

Table 3: The entries of correlation matrix R_{IM}. The values represent the correlation between stocks, bonds and interest rate r based on monthly data from 09/2011 to 09/2021.

[20] Eckert et al. (2016) note a high sensitivity of the model to changes of δ_R, but for our purpose a change in the recovery rate would lead to similar results.

$$f^{\mathrm{diff}}(u) = f^{\mathrm{true}}(u) - f^{\mathrm{SF}}(u)$$

We can now identify OCS as in Eq. (7) such that $g_m(u)$ as in Eq. (6) approximates f^{diff}. To this end, Appendix A.2 and Appendix A.3 provide the necessary technical details specifying the Hessian matrix H of $\left(f^{\mathrm{diff}}\right)^2$ and the weightvectors w_i, $i = 1, \ldots, m$, which are crucial for the definition.

3 Results

Let us put ourselves in the situation of an investor who has seven units to invest in the portfolio specified in section 2.3. Table 4 shows what the risk capital looks like when investing all units separately in each asset and when investing in an equally weighted portfolio $u_{\mathrm{initial}} = \mathbb{1}_7$. We observe that there is a severe gap between the two approaches

u	$f^{\mathrm{SF}}(u)$	$f^{\mathrm{true}}(u)$	Relative difference
$(7,0,0,0,0,0,0)^{\mathrm{T}}$	0.933	0.931	0.21%
$(0,7,0,0,0,0,0)^{\mathrm{T}}$	1.008	0.848	18.87%
$(0,0,7,0,0,0,0)^{\mathrm{T}}$	1.469	0.806	82.26%
$(0,0,0,7,0,0,0)^{\mathrm{T}}$	3.367	6.697	−49.72%
$(0,0,0,0,7,0,0)^{\mathrm{T}}$	0.602	0.887	−32.13%
$(0,0,0,0,0,7,0)^{\mathrm{T}}$	2.1	2.626	−20.03%
$(0,0,0,0,0,0,7)^{\mathrm{T}}$	2.8	3.198	−12.45%
u_{initial}	1.300	1.478	−12.04%

Table 4: The SCRs based on f^{true} investing seven units separately in each of the seven assets and in an equally weighted portfolio are presented.

when investing in assets separately. And even including diversification effects, we observe a severe underestimation of the true risk by

$$\frac{f^{\mathrm{SF}}\left(u_{\mathrm{initial}}\right)}{f^{\mathrm{true}}\left(u_{\mathrm{initial}}\right)} - 1 = \frac{1.300}{1.478} - 1 = -12.04\%$$

Such an underestimation may lead to a capital buffer too low to cover investment risks, as outlined for instance by Asadi and Al Janabi (2020).

Inspecting a common tool of capital allocation, the Euler allocation, of both measurements at u_{initial}, cf. Table 5, also provides evidence that there is a severe gap between the two approaches. Notably, the entries of the gradient add up to the capital requirement for u_{initial} in both cases, cf. Tasche (2008). We can thus conclude that changing

i	$\nabla_u f_i^{\text{SF}}$	$\nabla_u f_i^{\text{true}}$	Relative difference
1	0.126	0.068	85.29%
2	0.136	0.062	122.95%
3	0.190	0.058	229.34%
4	0.316	0.756	−58.20%
5	0.056	0.064	−11.11%
6	0.201	0.203	−1.96%
7	0.277	0.267	3.74%
$\sum$	1.300	1.478	−12.04%

Table 5: Euler allocations of f^{true} and f^{SF} and their relative difference. ∇_u represents the gradient.

diversification effects are not captured when basing risk measurement on the square-root formula, since the slope of the two functions in u_{initial} strongly differ. This observation is in line with Chen et al. (2019), who find empirical evidence that the standard formula does not reflect changing diversification effects correctly.

To approximate then the difference $f^{\text{diff}}(u) = f^{\text{true}}(u) - f^{\text{SF}}(u)$ the OCS provided in Table 6 are determined as suggested in section 2 such that we obtain an approximation $f^{\text{SF}}(u) + g_m(u)$ of $f^{\text{true}}(u)$ in the sense of Proposition 1.[21] In a first step, we use only a

i	Asset$_i$	$x_{1_i}^{\text{OCS}}$	$x_{2_i}^{\text{OCS}}$	$x_{3_i}^{\text{OCS}}$	$x_{4_i}^{\text{OCS}}$	$x_{5_i}^{\text{OCS}}$	$x_{6_i}^{\text{OCS}}$	$x_{7_i}^{\text{OCS}}$
1	B_1	-0.057	-0.001	0.027	-0.034	0.013	0.012	0.004
2	B_2	-0.075	-0.001	0.026	-0.031	0.011	0.008	0.002
3	B_3	-0.130	0.002	0.044	-0.016	-0.011	-0.008	-0.002
4	B_4	0.440	-0.002	-0.020	0.113	-0.088	-0.003	-0.016
5	B_5	0.006	-0.002	-0.004	-0.048	0.040	-0.006	0.004
6	S_1	0.005	0.071	-0.038	0.007	0.006	-0.005	0.071
7	S_2	-0.011	-0.069	-0.034	0.008	0.029	0.002	-0.063

Table 6: Orthogonal convexity scenarios for the definition of the approximation function $g_m(u)$ as in section 2.

single scenario. That scenario x_1^{OCS} mainly reflects losses in bond investment B_4 and is equal to the Euler allocation of f^{diff} such that

$$\sum_{i=1}^{7} x_{1_i}^{\text{OCS}} = 0.178 = f^{\text{diff}}(u_{\text{initial}}) = f^{\text{true}}(u_{\text{initial}}) - f^{\text{SF}}(u_{\text{initial}})$$

[21] Notably, for the determination it has been set $w_1 = u_{\text{initial}}$, $w_2 = (0,0,0,0,0,1,-1)^{\text{T}}$ and $w_3 = (0.2, 0.2, 0.2, 0.2, 0.2, 0.5, -0.5)^{\text{T}}$ since those are the business decisions we want to evaluate later on. Appendix A.3 sketches how Aigner and Schlütter (2022) select the OCS without pre-given decisions considered as well, and the following calculations could be conducted in the same way.

Including this additional scenario comes with two advantages: Firstly, we ensure that the risk resulting from the initial portfolio u_{initial} is estimated precisely. Secondly, first order sensitivities of the true risk are met by our approximation. Employing further scenarios then allows us to fit a quadratic approximation of f^{true} in u_{initial} even allowing us to meet second order sensitivities of the true risk landscape.

For illustration, let us numerically evaluate the goodness of the approximation. Therefore, we shift the portfolio in the direction

$$u_{\text{new}}(h) = u_{\text{initial}} + h \cdot (0,0,0,0,0,1,-1)^{\mathrm{T}} \tag{18}$$

for $h \in \mathbb{R}$. This new portfolio represents a shift between stock investments keeping the exposure to the five bond investments constant. For positive values of h, we shift our portfolio from the "other" investment, S_2, in the direction of the "global" one, S_1. For negative h the shift is opposite, from "global" to "other".

The resulting SCRs based on the different approaches are presented in Figure 2 for $h \in [-1,1]$. There, we see that the overall capital requirement according to the standard formula (red curve) always underestimates the true risk (black curve). Furthermore, we see that it strictly decreases when shifting the portfolio away from S_2 in the direction of S_1, which is reflected by the consistently negative slope. However, the true risk actually increases if the portfolio is shifted "too far" away from S_2 (positive h) due to changing diversification effects that are not captured by the standard formula. Generally, we can say that first and second order sensitivities in $u_{\text{initial}} = \mathbb{1}_7$ (for $h = 0$) are not reflected correctly.

By extending the standard formula with only $m = 1$ OCS (orange curve), we can adjust the risk measurement function in a sense that the true risk in u_{initial} is reflected correctly and that even first order sensitivities of f^{true} in all directions are met. The latter is due to the selection of the Euler allocation of f^{diff} as first scenario. Such an extension is a good starting point, but also does not take into account changing diversification effects, since it still has a consistently negative slope in line with the standard formula.

Including $m = 2$ scenarios (blue curve) allows a fit with a quadratic approximation function that also overcomes this second problem. We observe that now—additional to the properties of the extension with one scenario—even second order sensitivities are met in a neighbourhood of u_{initial}. We thus obtain a risk measurement function that is also capable of evaluating non-marginal portfolio shifts.

In order to inspect the goodness of the extension more granularly, let us inspect a more

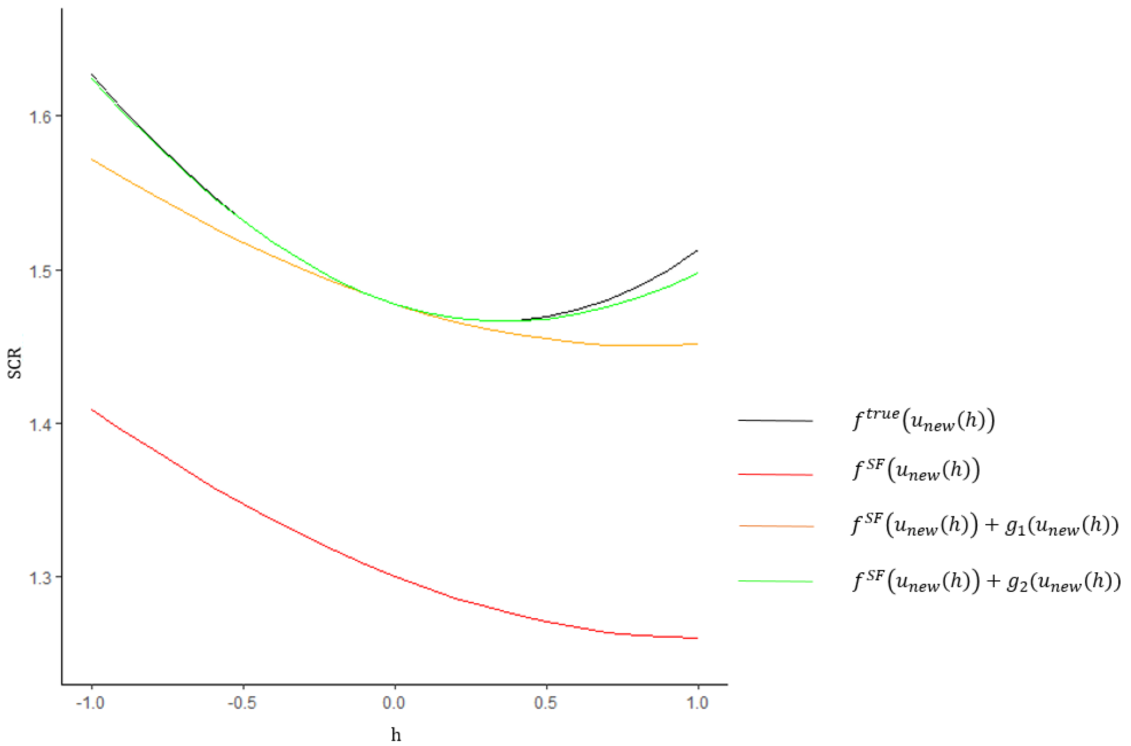

Figure 2: SCRs on the basis of the different risk measurement functions evaluating $u_{\text{new}}(h)$ as in Eq. (18).

complex shift impacting all portfolio positions simultaneously

$$u_{\text{new}}(h_1, h_2) = u_{\text{initial}} + h_1 \cdot \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, -\frac{1}{2}, -\frac{1}{2}\right)^{\mathrm{T}} + h_2 \cdot (0, 0, 0, 0, 0, 1, -1)^{\mathrm{T}} \tag{19}$$

for $h_1, h_2 \in \mathbb{R}$. If we set there $h_1 = 0$, we would again inspect a portfolio shift in the sense of Eq. (18). Setting $h_2 = 0$ would then reflect a shift away from stock into bond investments for positive h and vice versa for negative ones. Notably, the directions have been chosen such that the overall investment sum does not change. Comparing the resulting SCRs on the basis of OCS leads to the results presented in Figure 3 for $h_1, h_2 \in [-1, 1]$.

There, the left part shows the relative error of the standard formula extended by $m = 1$ scenario. It should be noted that we thereby obtain a reasonable approximation of f^{true} if we only evaluate marginal portfolio changes. When h_2 is de-/increased too far, the underlying curvature of the true risk measurement function cannot be reflected any more. That misestimation is represented by the relative errors of $f^{\text{SF}}(u_{new}(h_1, h_2)) + g_1(u_{new}(h_1, h_2))$ (red parts in the figure).

By including a second scenario, we are still able to approximate the true risk in a

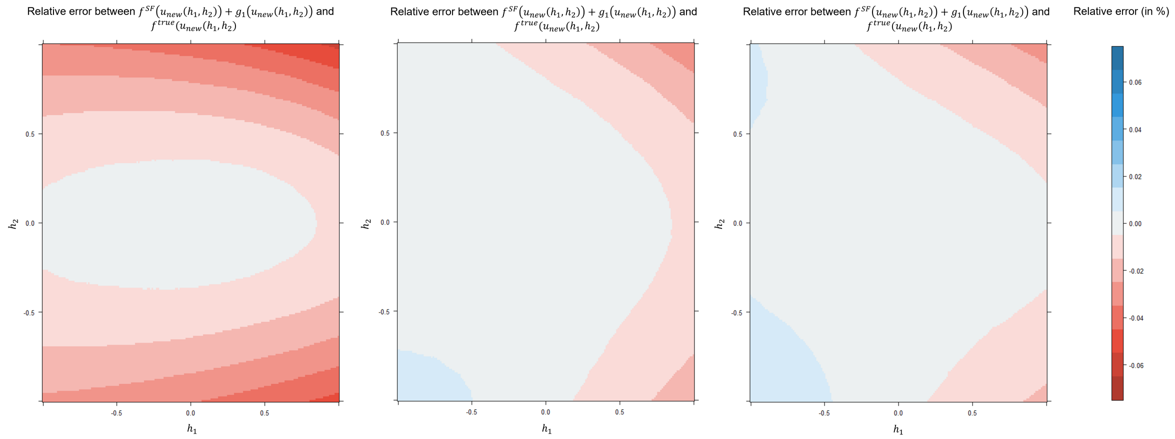

Figure 3: Relative errors measuring risks of portfolios $u_{\text{new}}(h_1, h_2)$ as in (19) on the basis of the standard formula and its extension via orthogonal scenarios for $m = 1, 2$ and 3 scenarios. All results are compared to $f^{\text{true}}\left(u_{\text{new}}\left(h_1, h_2\right)\right)$.

neighborhood of u_{initial}, but changing diversification effects are also considered. In the middle part of Figure 3, we observe that the gray parts—reflecting a relative error of about 0%—are not linear any more, but are instead spread in all directions. At the same time, we have to accept that a slight overestimation of the true risk may occur (blue parts), which is due to the fact that we extend the standard formula by a strictly positive function g. Numerically, including a second and third scenario reduces the absolute amount of the relative error from 5.86% (left) to 3.39% (middle). Notably, the inclusion of the third scenario (right part) does not have a great impact on the maximal relative error (3.00%), but indeed the neighbourhoodregion in which the approximation meets the true risk (gray parts in the right) can be widened. The inclusion of even more scenarios here also seems reasonable.

4 Conclusion

This paper suggests “orthogonal convexity scenarios” (OCS) to address the requirement for the Own Risk and Solvency Assessment (ORSA) of an insurer employing scenario analyses. Explicitly, it is expected to ”also take into account risks that are not or not adequately included in the standard formula, and [...] develop a suitable assessment procedure for them” (BaFin (2016)). We show that the OCS allow the derivation of a deterministic extension of the standard formula such that the overall risk is in line with the true portfolio risk. The approach is applied in the context of market risks comprising interest rate, spread and equity risks. For the set-up of the standard formula, we follow the regulatory requirements and the the approaches from the literature. Additionally, we use an internal model which is supposed to represent the true risk resulting from an asset portfolio. Notably, the true risk is generally unknown in practice, at which point an approximation becomes necessary. We find that extending the standard formula by OCS provides a reasonable approximation of the true risk in the sense of first and second order sensitivities. The latter property allows the evaluation of even non-marginal portfolio shifts, since changing diversification effects are considered by the resulting risk measurement. Although the approximation is only local, the examples in Paulusch and Schlütter (2022) indicate that the methodology is useful for other portfolios in addition to the calibration portfolio. The suggested approach can thus be seen as an answer to the question of how to select scenarios in the ORSA to measure risks that are not captured by the standard formula. Notably, since only deterministic scenarios are taken into account, we provide an easy tool for communicating the difference between an internal model and the standard formula to decision-makers, which has been identified as one of the fundamental aims of stress scenarios, cf. Albrecher et al. (2018, Chapter 5.5).

A Appendix

A.1 Proof of Proposition 1

Let u_{initial} be arbitrary and assume $f^{\text{true}}(u_{\text{initial}}) > f^{\text{SF}}(u_{\text{initial}})$. Furthermore, assume that f^{true} and f^{SF} are homogeneous of degree one and twice continuously differentiable in u_{initial}. Then

$$f^{\text{diff}}(u) := f^{\text{true}}(u) - f^{\text{SF}}(u)$$

directly fulfills these properties as well. According to Aigner and Schlütter (2022, Theorem 1) it then holds for the approximation function g as specified in section 2

$$\begin{aligned} f^{\text{diff}}\left(u_{\text{initial}}\right) &= g_m\left(u_{\text{initial}}\right) \\ \frac{\partial}{\partial h} f^{\text{diff}}\left(u_{\text{initial}} + hv\right)|_{h=0} &= \frac{\partial}{\partial h}\left(g_m\left(u_{\text{initial}} + hv\right)|_{h=0}\right) \\ \frac{\partial^2}{\partial h_1 \partial h_2} f^{\text{diff}}\left(u_{\text{initial}} + h_1 v_1 + h_2 v_2\right)|_{h_1=h_2=0} &= \frac{\partial^2}{\partial h_1 \partial h_2} g_m\left(u_{\text{initial}} + h_1 v_1 + h_2 v_2\right)|_{h_1=h_2=0} \end{aligned}$$

for all $v \in \mathbb{R}^n$ and $v_1, v_2 \in \operatorname{span}\{w_1, \ldots, w_m\}$. Thereby, Proposition 1 follows.

A.2 Estimation of the Hessian matrix

For the identification of OCS in Eq. (7), we have to estimate the Hessian matrix of $\left(f^{\text{diff}}\right)^2$. Employing the chain rule leads to

$$H = 2 \cdot \left(H_{f^{\text{true}}} - H_{f^{\text{SF}}}\right) \cdot \left(f^{\text{diff}}\left(u_{\text{initial}}\right)\right) + 2 \cdot \left(\nabla_u f^{\text{diff}}\left(u_{\text{initial}}\right)\right)^{\text{T}} \cdot \nabla_u f^{\text{diff}}\left(u_{\text{initial}}\right) \quad (20)$$

with $H_{f^{\text{true}}}$ and $H_{f^{\text{SF}}}$ the Hessian matrices of f^{SF} and f^{true} respectively, and $\nabla_u f^{\text{diff}}$ the gradient of f^{diff} all evaluated at u_{initial}. Furthermore, the gradient simplifies to

$$\nabla_u f^{\text{diff}}\left(u_{\text{initial}}\right) = \nabla_u f^{\text{true}}\left(u_{\text{initial}}\right) - \nabla_u f^{\text{SF}}\left(u_{\text{initial}}\right)$$

It is then necessary to estimate the single parts of Eq. (20). On the one hand, $H_{f^{\text{SF}}}$ and $\nabla_u f^{\text{SF}}\left(u_{\text{initial}}\right)$ can be determined easily by numerical derivation providing a reasonable result, since f^{SF} is deterministic. The identification of $H_{f^{\text{true}}}$ and $\nabla_u f^{\text{true}}\left(u_{\text{initial}}\right)$, on the other hand, is more challenging. Monte Carlo simulations can be applied in line with Gourieroux et al. (2000) and Tasche (2009) who suggest Kernel estimators leading to a consistent estimation of $H_{f^{\text{true}}}$ and $\nabla_u f^{\text{true}}\left(u_{\text{initial}}\right)$. Thanks to Slutsky's Theorem, (cf.

Casella and Berger, 2002, p. 239 f.), we can then consistently estimate H in (20), since it is a composition of consistent estimations.

A.3 Selection of weightvectors

For the identification of the necessary weightvectors $w_1, \ldots, w_m$ in Eq. (7), the user of the methodology could follow Appendix B in Aigner and Schlütter (2022) and set $w_1 = u_{\text{initial}}$. For the identification of $w_2, \ldots, w_m$, the authors define a matrix $M \in \mathbb{R}^{n \times \tilde{n}}$ with $\tilde{n} = \text{rank}(H)$, such that

$$(Mv)^{\mathrm{T}} \cdot H \cdot (Mv) = 0$$

for all $v \in \mathbb{R}^{\tilde{n}-m}$ and the columns of M denoted as M_i also fulfill $(M_i)^{\mathrm{T}} HM_j = 0$, for $i \neq j$. Further, they define the diagonal matrix $\Lambda = M^{\mathrm{T}}HM$ and denote by $\Lambda^{-0.5}$ the diagonal matrix with entries $\lambda_1^{-0.5}, \ldots, \lambda_{\tilde{n}-m}^{-0.5}$. The weightvectors can then be determined as

$$w_{m+1} = \frac{1}{\sqrt{s^{\mathrm{T}}\Lambda^{-0.5}M^{\mathrm{T}}M\Lambda^{-0.5}s}} \cdot M\Lambda^{-0.5}s$$

where s is the eigenvector of $\Lambda^{-0.5}M^{\mathrm{T}}M\Lambda^{-0.5}$ which refers to the smallest eigenvalue.

A.4 Parameters for regulatory capital requirement

t	Risk-free interest rate $r_f(t)$	Relative change $s^{up}(t)$	Relative change $s^{down}(t)$
1	-0.00612	0.7	-0.75
2	-0.00594	0.7	-0.65
3	-0.00556	0.64	-0.56
4	-0.00513	0.59	-0.5
5	-0.00462	0.55	-0.46
6	-0.00353	0.52	-0.42
7	-0.00293	0.49	-0.39
8	-0.00232	0.47	-0.36
9	-0.00172	0.44	-0.33
10	-0.00114	0.43	-0.31
...	...	...	...

Table 7: Risk-free interest rate r_f structure provided by EIOPA. The data are from 08/2021. Additionally, upward and downward shocks for the interest rate module are presented, cf. European Commission (2015).

Duration (dur_i) in years	$stress_i$
$dur_i \leq 5$	$3\% \cdot dur_i$
$5 < dur_i \leq 10$	$15\% + 1.7\% \cdot (dur_i - 5)$
$10 < dur_i \leq 20$	$23.5\% + 1.2\% \cdot (dur_i - 10)$
$dur_i > 20$	$\min(35\% + 0.5\% \cdot (dur_i - 20), 1)$

Table 8: Parameters for the determination of $stress_i$ within the spread sub-module under Solvency II regulation for bonds unrated by nominated ECAI are reported.

Rating		AAA		AA		A		BBB		BB		B	
Duration (dur_i) (in years)	$stress_i$	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
$dur_i \leq 5$	$b_i \cdot dur_i$	-	0.9	-	1.1	-	1.4	-	2.5	-	4.5	-	7.5
$5 < dur_i \leq 10$	$a_i + b_i \cdot (dur_i - 5)$	4.5	0.5	5.5	0.6	7.0	0.7	12.5	1.5	22.5	2.5	37.5	4.2
$10 < dur_i \leq 15$	$a_i + b_i \cdot (dur_i - 10)$	7.0	0.5	8.4	0.5	10.5	0.5	20.0	1.0	35.0	1.8	58.5	0.5
$15 < dur_i \leq 20$	$a_i + b_i \cdot (dur_i - 15)$	9.5	0.5	10.9	0.5	13.0	0.5	25	1.0	44.0	0.5	61.0	0.5
$dur_i > 20$	$\min(a_i + b_i \cdot (dur_i - 20), 1)$	12.0	0.5	13.4	0.5	15.5	0.5	30	0.5	46.5	0.5	63.5	0.5

Table 9: Parameters for the determination of $stress_i$ within the spread sub-module under Solvency II regulation for bonds rated by a nominated External Credit Assessment Institution (ECAI). All values for a_i and b_i are in percent.

A.5 Homogeneity of degree one of $f^{\mathbf{SF}}$

We need to show that f^{SF} in (17) is homogeneous of degree one. Therefore, we let every figure as in section 2.2. We can calculate for$\lambda \in \mathbb{R}$ and $u = \left(\left(u^B\right)^{\mathrm{T}}, \left(u^S\right)^{\mathrm{T}}\right)^{\mathrm{T}} \in \mathbb{R}^n$ as in section 2.2

$$\begin{aligned} f^{\text{SF}}(\lambda \cdot u) &= f^{\text{SF}}(\lambda \cdot \left(\left(u^B\right)^{\mathrm{T}}, \left(u^S\right)^{\mathrm{T}}\right) = \sqrt{x_\lambda^{\mathrm{T}} R_{SF} x_\lambda} \overset{(*)}{=} \sqrt{(\lambda \cdot x)^{\mathrm{T}} R_{SF} (\lambda \cdot x)} \\ &= \lambda \cdot \sqrt{x^{\mathrm{T}} R_{SF} x} = \lambda \cdot f^{\text{SF}}(u) \end{aligned}$$

with

$$\begin{aligned} x_\lambda &= \left(Mkt_{int}\left(\lambda \cdot u^B\right), Mkt_{eq}\left(\lambda \cdot u^S\right), Mkt_{sp}\left(\lambda \cdot u^B\right)\right)^{\mathrm{T}} \\ x &= \left(Mkt_{int}\left(u^B\right), Mkt_{eq}\left(u^S\right), Mkt_{sp}\left(u^B\right)\right)^{\mathrm{T}} \end{aligned}$$

The equality $(*)$ holds if and only if Mkt_{int}, Mkt_{eq} and Mkt_{sp} are also homogeneous of degree one. Firstly, it is

$$\begin{aligned} Mkt_{int}\left(\lambda \cdot u^B\right) &= \max\left(\left(PV^{int} - PV_{int}^{up}\right)^{\mathrm{T}}\left(\lambda \cdot u^B\right), \left(PV_{int} - PV_{int}^{down}\right)^{\mathrm{T}}\left(\lambda \cdot u^B\right)\right) \\ &= \lambda \cdot \max\left(\left(PV^{int} - PV_{int}^{up}\right)^{\mathrm{T}} u^B, \left(PV_{int} - PV_{int}^{down}\right)^{\mathrm{T}} u^B\right) = \lambda Mkt_{int}\left(u^B\right) \end{aligned}$$

Secondly, it holds

$$Mkt_{eq}\left(\lambda \cdot u^S\right) = \sqrt{\tilde{x}_\lambda^{\mathrm{T}} R_{eq} \tilde{x}_\lambda} \overset{(**)}{=} \sqrt{(\lambda \cdot \tilde{x})^{\mathrm{T}} R_{eq}\left(\lambda \cdot \tilde{x}\right)} = \lambda \cdot Mkt_{eq}\left(u^S\right)$$

for $\tilde{x}_\lambda = \left(Mkt_{eq,global}\left(\lambda \cdot u^S\right), Mkt_{eq,other}\left(\lambda \cdot u^S\right)\right)^{\mathrm{T}}$ and
$\tilde{x} = \left(Mkt_{eq,global}\left(u^S\right), Mkt_{eq,other}\left(u^S\right)\right)^{\mathrm{T}}$. To see $(**)$, we have to observe that

$$\begin{aligned} Mkt_{eq,\ global}\left(\lambda \cdot u^S\right) &= \max\left(0.3 \cdot \lambda \cdot \sum_{i=1}^{k_{global}} u_i^S, 0\right) = \lambda Mkt_{eq,\ global}\left(u^S\right) \\ Mkt_{eq,\ other}\left(\lambda \cdot u^S\right) &= \max\left(0.4 \cdot \lambda \cdot \sum_{i=k_{global}+1}^{n_S} u_i^S, 0\right) = \lambda \cdot Mkt_{eq,\ other}\left(u^S\right) \end{aligned}$$

Thirdly, we can calculate

$$Mkt_{sp}\left(\lambda \cdot u^B\right) = \max\left(stress^{\mathrm{T}}\left(\lambda \cdot u_i^B\right), 0\right) = \lambda \cdot \max\left(stress^{\mathrm{T}} u^B, 0\right) = \lambda \cdot Mkt_{sp}\left(\lambda \cdot u^B\right)$$

f^{SF} thus is homogeneous of degree one.

A.6 Robustness check

In order to check the robustness of the presented approach to changes in the underlying f^{true}, the portfolio set-up is to be changed to coincide with the parameters presented in Eckert et al. (2016) and Gatzert and Martin (2012). On this basis, we can also consider an investment portfolio consisting of $n_B = 5$ bond investments, cf. Table 10, that are in line with Eckert et al. (2016) and $n_S = 2$ stocks, cf. Table 11. Here, the SCR employing the standard formula can be directly calculated. For the internal model, we again take

B_i	Type$_i$	rating$_i$	maturity$_i$	Coupon (in %) c_i
1	Corporate	AA	10	2.950
2	Corporate	A	10	4.550
3	Corporate	BBB	11	3.500
4	Government	A	15	3.000
5	Government	BB	11	5.125

Table 10: Specifications of the bonds that are taken into account. The parameters are taken from Eckert et al. (2016). B_1: Colgate-Palmolive Company, B_2: Woolworth LTD, B_3: Areva SA, B_4: Poland Republic of (Government) and B_5: Turkey Republic of (Government).

into account correlated standard Wiener processes with correlation matrix R_{IM} as discussed in section 2.1. The entries of R_{IM} are presented in Table 12 and are again taken from Eckert et al. (2016). The stocks are then modeled as before following geometric Brownian motions with the parameters as in Table 11. Here, S_1, namely MSCI World, represents the investment in a "global" asset and S_2, India BSE 100, in an "other" asset. Again, we cover both potential classes in the equity risk sub-module. The parameters

S_i	Index$_i$	rating$_i$	μ_i	σ_i
1	MSCI World	Global	0.0509	0.1574
2	India BSE 100	Other	0.1043	0.3309

Table 11: The parameters for the specification of the geometric Brownian motions representing stock investments. The numbers are taken from Gatzert and Martin (2012) who estimated them based on monthly data from 01/1988 to 07/2011.

of the interest rate Vasicek (1977) process are given by $\kappa = 0.0953$, $\theta = 0.0437$ and $\zeta = 0.0069$, cf. Eckert et al. (2016).[22] Furthermore, the initial value is set to $r(0) = \theta$.

[22]These are based on the monthly "EURIBOR" data from 01/1999 to 12/2008.

The specifications of bond investments are presented in Table 10. Eckert et al. (2016) follow then Liang et al. (2011), assuming that bonds in the same rating class have the same parameters for their hazard rate Vasicek (1977) process which are shown in Table 13. Additionally, the recovery rate is set constant to $\delta_R = 0.61$ as before.

We evaluate the goodness of the approximation of $f^{\text{true}}(u)$ by $f^{\text{SF}}(u) + g_m(u)$ as sug-

i	r	h_{AA}	h_A	h_{BBB}	h_{BB}	S_1	S_2
r	1						
h_{AA}	0.3	1					
h_A	0.3	0.3	1				
h_{BBB}	0.3	0.3	0.3	1			
h_{BB}	0.3	0.3	0.3	0.3	1		
S_1	-0.26	0	0	0	0	1	
S_2	-0.21	0	0	0	0	0.26	1

Table 12: The entries of correlation matrix R_{IM}. The values are taken from Gatzert and Martin (2012) and Eckert et al. (2016) who have estimated the correlation between stocks and interest rate r based on monthly data from 01/1988 to 07/2011. The correlations between bond classes h_i and interest rate r are originally from Liang et al. (2011).

Rating	χ	o	Γ
AA	0.9581	0.0072	0.0181
A	0.7553	0.0141	0.0126
BBB	0.5865	0.0258	0.0113
BB	0.4406	0.0781	0.0454

Table 13: Parameters for the Vasicek (1977) processes modeling the hazard rates depending on the bond rating. The values are again taken from Eckert et al. (2016).

gested in section 2 for investing seven units into an equally weighted portfolio specified as $u_{\text{initial}} = (1, \ldots, 1)^{\text{T}} \in \mathbb{R}^n$. f^{true} is evaluated on the basis of a Monte Carlo simulation with 5,000,000 paths and it results in capital requirements of $f^{\text{true}}(u_{\text{initial}}) = 1.525$ and $f^{\text{SF}}(u_{\text{initial}}) = 1.309$ resulting in a relative error of -14.20%. Following the procedure described in section 2, seven scenarios can be identified which are presented in Table 14. Evaluating the goodness of the performance, we again take into account the shift presented in Eq. (18). Figure 4 presents the outcomes for $h \in [-1, 1]$. There, we obtain a similar result as in section 3: The standard formula is not capable of approximating the true risk reasonably underestimating the portfolio risk, and sensitivities are also not reflected correctly. An extension by $m = 1$ orthogonal convexity scenario overcomes one of

i	Asset_i	x_{1i}^{OC}	x_{2i}^{OC}	x_{3i}^{OC}	x_{4i}^{OC}	x_{5i}^{OC}	x_{6i}^{OC}	x_{7i}^{OC}
1	B_1	0.0119	0.0115	-0.0147	0.0033	-0.0127	-0.0127	0.0056
2	B_2	-0.0044	0.0121	-0.0155	0.0021	-0.0137	-0.0137	0.0004
3	B_3	-0.0718	0.0174	-0.0239	-0.0010	-0.0213	-0.0213	0.0042
4	B_4	0.0557	0.0039	0.0009	0.0119	-0.0001	-0.0001	-0.0005
5	B_5	0.0534	0.0032	0.0018	-0.0163	-0.0072	-0.0072	-0.0005
6	S_1	-0.0100	0.0364	0.0424	0.0018	-0.0266	-0.0266	-0.0104
7	S_2	0.1814	-0.0845	0.0090	-0.0018	0.0815	0.0815	0.0013

Table 14: Scenarios for the definition of the approximation function $g_m(u)$ in the setting of Eckert et al. (2016).

these problems reflecting the true risk at u_{initial} and also first order sensitivities correctly. Extending the regulatory risk calculation by $m = 2$ orthogonal convexity scenarios even allows for an inspection of non-marginal portfolio changes by reflecting the second order sensitivities of f^{true} at least in some subspace. With these observations, we have seen

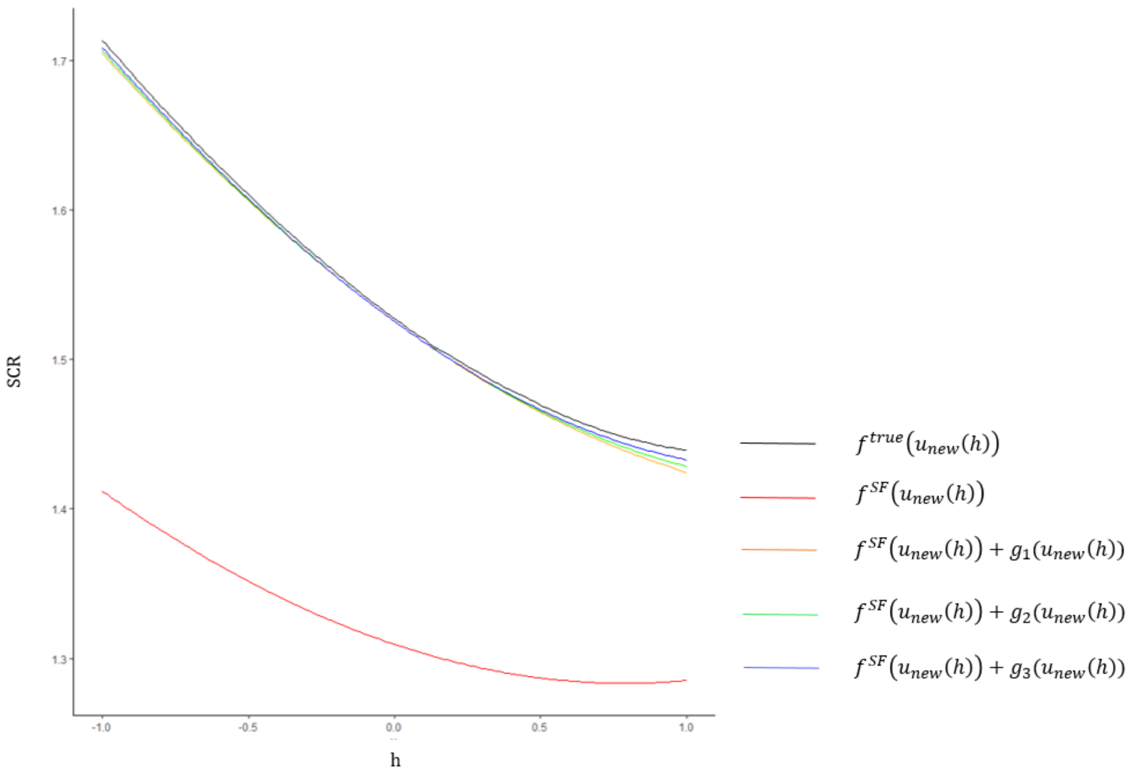

Figure 4: SCRs on the basis of the different risk measurement functions evaluating $u_{\text{new}}(h)$ as in Eq. (18) with the parameters in the internal model as reported in Eckert et al. (2016).

that the presented approach is robust to changes of the underlying true risk measurement function, making it applicable for a wide range of risk functions.

References

Aigner, P., Schlütter, S., 2022. Enhancing gradient capital allocation with orthogonal convexity scenarios. Working Paper , 1–5.

Albrecher, H., Bauer, D., Embrechts, P., Filipović, D., Koch-Medina, P., Korn, R., Loisel, S., Pelsser, A., Schiller, F., Schmeiser, H., et al., 2018. Asset-liability management for long-term insurance business. European Actuarial Journal 8, 9–25.

Asadi, S., Al Janabi, M.A., 2020. Measuring market and credit risk under solvency ii: Evaluation of the standard technique versus internal models for stock and bond markets. European Actuarial Journal 10, 425–456.

Becker, B., Ivashina, V., 2015. Reaching for yield in the bond market. The Journal of Finance 70, 1863–1902.

Braun, A., Schmeiser, H., Schreiber, F., 2017. Portfolio optimization under Solvency II: Implicit constraints imposed by the market risk standard formula. Journal of Risk and Insurance 84, 177–207.

Breuer, T., Csiszár, I., 2013. Systematic stress tests with entropic plausibility constraints. Journal of Banking & Finance 37, 1552–1559.

Buch, A., Dorfleitner, G., Wimmer, M., 2011. Risk capital allocation for RORAC optimization. Journal of Banking & Finance 35, 3001–3009.

Bundesanstalt für Finanzdienstleistungsaufsicht (BaFin), 2016. Governmentbonds: Treatment of risk under solvency ii. URL: `https://www.bafin.de/dok/8136732`.

Casella, G., Berger, R., 2002. Statistical Inference, 2nd Edition. Duxbury.

Chen, T., Goh, J.R., Kamiya, S., Lou, P., 2019. Marginal cost of risk-based capital and risk-taking. Journal of Banking & Finance 103, 130–145.

Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS), 2010. Quantitative impact study 5 - questions & answers. URL: `https://eiopa.europa.eu`.

Duffie, D., Singleton, K.J., 1999. Modeling term structures of defaultable bonds. The Review of Financial Studies 12, 687–720.

Eckert, J., Gatzert, N., Martin, M., 2016. Valuation and risk assessment of participating life insurance in the presence of credit risk. Insurance: Mathematics and Economics 71, 382–393.

European Commission, 2015. Commission Delegated Regulation (EU) 2015/35 of 10 October 2014 supplementing Directive 2009/138/EC of the European Parliament and of the Council on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) .

European Insurance and Occupational Pensions Authority (EIOPA), 2011. Report on the fifth Quantitative Impact Study (QIS5) for Solvency II. URL: https://www.eiopa.europa.eu/content/guidelines-own-risk-solvency-assessment-orsa_en.

European Insurance and Occupational Pensions Authority (EIOPA), 2015. Guidelines on own risk and solvency assessment. URL: https://www.eiopa.europa.eu/document-library/guidelines/guidelines-own-risk-solvency-assessment-orsa_en_en.

European Insurance and Occupational Pensions Authority (EIOPA), 2019. Insurance Statistics. URL: https://www.eiopa.europa.eu/tools-and-data/insurance-statistics_en.

Fergusson, K., Platen, E., 2015. Application of maximum likelihood estimation to stochastic short rate models. Annals of Financial Economics 10, 1550009.

Fischer, K., Schlütter, S., 2015. Optimal investment strategies for insurance companies when capital requirements are imposed by a standard formula. Geneva Risk and Insurance Review 40, 15–40.

Gatzert, N., Martin, M., 2012. Quantifying credit and market risk under Solvency II: Standard approach versus internal model. Insurance: Mathematics and Economics 51, 649–666.

Gourieroux, C., Laurent, J.P., Scaillet, O., 2000. Sensitivity analysis of values at risk. Journal of Empirical Finance 7, 225–245.

Graf, S., Korn, R., 2020. A guide to monte carlo simulation concepts for assessment of risk-return profiles for regulatory purposes. European Actuarial Journal 10, 273–293.

Guo, Q., Bauer, D., Zanjani, G., 2021. Capital allocation techniques: Review and comparison. Variance 14.

Hull, J.C., 2018. Risk Management and Financial Institutions. Wiley Finance Series. 5th Edition.

Islam, M.R., Nguyen, N., 2020. Comparison of financial models for stock price prediction. Journal of Risk and Financial Management 13, 181.

Korn, R., Müller, L., 2021. Optimal portfolios in the presence of stress scenarios a worst-case approach. Mathematics and Financial Economics , 1–33.

Liang, J., Ma, J.M., Wang, T., Ji, Q., 2011. Valuation of portfolio credit derivatives with default intensities using the vasicek model. Asia-Pacific Financial Markets 18, 33–54.

Macaulay, F.R., 1938. Front matter to" some theoretical problems suggested by the movements of interest rates, bond yields and stock prices in the united states since 1856", in: Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856. NBER, pp. 15–6.

Makam, V.D., Millossovich, P., Tsanakas, A., 2021. Sensitivity analysis with $\chi 2$-divergences. Insurance: Mathematics and Economics 100, 372–383.

McNeil, A.J., Smith, A.D., 2012. Multivariate stress scenarios and solvency. Insurance: Mathematics and Economics 50, 299–308.

Mittnik, S., 2014. VaR-implied tail-correlation matrices. Economics Letters 122, 69–73.

Paulusch, J., 2017. The Solvency II standard formula, linear geometry, and diversification. Journal of Risk and Financial Management 10, 11.

Paulusch, J., Schlütter, S., 2022. Sensitivity-implied tail-correlation matrices. Journal of Banking & Finance 134, 106333.

Pesenti, S.M., Millossovich, P., Tsanakas, A., 2019. Reverse sensitivity testing: What does it take to break the model? European Journal of Operational Research 274, 654–670.

Pfeifer, D., Strassburger, D., 2008. Solvency II: stability problems with the SCR aggregation formula. Scandinavian Actuarial Journal 1, 61–77.

Scherer, M., Stahl, G., 2021. The standard formula of solvency ii: a critical discussion. European Actuarial Journal 11, 3–20.

Schlütter, S., 2021. Scenario-based measurement of interest rate risks. The Journal of Risk Finance 22 No. 1, 56–77.

Schönbucher, P.J., 2003. Credit derivatives pricing models: models, pricing and implementation. John Wiley & Sons.

Tasche, D., 2008. Capital allocation to business units and sub-portfolios: The Euler principle, in: Resti, A. (Ed.), Pillar II in the New Basel Accord: The Challenge of Economic Capital. Risk Books, London, pp. 423–453.

Tasche, D., 2009. Capital allocation for credit portfolios with kernel estimators. Quantitative Finance 9, 581–595.

Vasicek, O., 1977. An equilibrium characterization of the term structure. Journal of Financial Economics 5, 177–188.