



Duals of lattice Abelian models with static determinant at finite density



O. Borisenko^a, V. Chelnokov^{b,1}, S. Voloshyn^a, P. Yefanov^{c,*}

^a N.N. Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, 03143 Kyiv, Ukraine

^b Institut für Theoretische Physik, Goethe-Universität Frankfurt, 60438 Frankfurt am Main, Germany

^c Department of Quantum Field Theory, Taras Shevchenko National University of Kyiv, 03022 Kyiv, Ukraine

ARTICLE INFO

Article history:

Received 15 December 2021

Accepted 26 February 2022

Available online 2 March 2022

Editor: N. Lambert

ABSTRACT

Dual formulations of Abelian $U(1)$ and $Z(N)$ LGT with a static fermion determinant are constructed at finite temperatures and non-zero chemical potential. The dual form is valid for a broad class of lattice gauge actions, for arbitrary number of fermion flavors and in any dimension. The distinguished feature of the dual formulation is that the dual Boltzmann weight is strictly positive. This allows to gain reliable results at finite density via the Monte-Carlo simulations. As a byproduct of the dual representation we outline an exact solution for the partition function of the $(1+1)$ -dimensional theory and reveal an existence of a phase with oscillating correlations.

© 2022 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

There are many approaches designed to solve fully or partially the sign problem in QCD at finite chemical potential. One of such approaches is based on the dual representation for the partition function and physical observables. The main idea is to perform an integration over original (gauge and fermion) degrees of freedom and to present the resulting weight in a positive form suitable for numerical simulations. A certain progress along this line of investigations has been achieved during last decade and can be briefly summarized as follows. The dual models with positive Boltzmann weights have been obtained and studied in Refs. [1–5]. The calculations have been performed in the region of vanishing spatial gauge coupling constant and in the static approximation for the quark determinant (or at large quark masses). In the strong coupling limit the $SU(N)$ LGT can be mapped onto monomer-dimer and closed baryon loop model [6]. This dual representation has a soft sign problem and can be studied numerically. The positivity of the Boltzmann weight was also proven in the strong coupling limit of the scalar QCD with one, two or three scalar flavors [7]. Beyond the strong coupling regime the dual formulation of $Z(3)$ gauge-Higgs model is also positive [8,9] and suitable for Monte-Carlo simulations. Attempts to extend these results to full lattice

QCD with the staggered fermions using different schemes of computations have not been so successful so far [10–12]. Important result for the present paper was proven in Ref. [13]: the dual form of the massless two-dimensional $U(1)$ LGT with one or two flavors of staggered fermions is free of the sign problem and can be simulated with the help of a worm algorithm. Generalizing this result to a non-vanishing fermion mass proved to be a non-trivial task, and no solution has been found up to date.

In this paper we extend results of Refs. [1–5] on the dual formulation to the case of arbitrary spatial gauge coupling for Abelian LGTs. The crucial simplification in dealing with Abelian models is the known exact and positive dual form of any $U(1)$ and $Z(N)$ pure gauge theory in any dimension. The purpose of this paper is to derive a positive dual formulation of Abelian LGTs with the full pure gauge action and arbitrary number of the staggered or Wilson flavors taken in the static approximation for the fermion determinants. As an application we discuss the possible updates of the dual Boltzmann weight appropriate for the Monte-Carlo simulations. Another direction we explore here is the solution of $(1+1)$ -dimensional theory based on the dual representation. In particular, we calculate the eigenvalues of the corresponding transfer matrix and reveal the existence of an oscillating phase at finite density in all $Z(N)$ models with one or two fermion flavors.

Our notations and conventions are as follows. We work on an anisotropic periodic $(d+1)$ -dimensional lattice $\Lambda = L^d \times N_t$ with spatial extension L and temporal extension N_t . The lattice sites are denoted as $\vec{x} = (t, x)$ with $x = (x_1, \dots, x_d)$, links in the temporal (spatial) direction are denoted as l_t (l_s) and plaquettes as p_t (p_s). The pure gauge action is of the form

* Corresponding author.

E-mail addresses: oleg@bitp.kiev.ua (O. Borisenko), chelnokov@itp.uni-frankfurt.de (V. Chelnokov), billy.sunburn@gmail.com (S. Voloshyn), paul.yefan@gmail.com (P. Yefanov).

¹ On leave from BITP.

$$S_g(w_p) = \beta_t \sum_{p_t} S(w_{p_t}) + \beta_s \sum_{p_s} S(w_{p_s}), \quad (1)$$

where anisotropic coupling constants are related by $\beta_s = \beta_t \xi^2$ with $\xi = \frac{a_t}{a_s}$. a_t (a_s) is lattice spacing in the temporal (spatial) direction. $\beta = a_t N_t$ is an inverse temperature. The partition functions of $Z(N)$ and $U(1)$ LGTs are given by

$$Z_\Lambda = \sum_{\{s_l\}=0}^{N-1} e^{S_g(s_p)} \prod_{f=1}^{N_f} \text{Det} \mathcal{M}_{\vec{x}, \vec{x}'}^f, \quad (2)$$

$$Z_\Lambda = \int_0^{2\pi} \prod_l \frac{d\phi_l}{2\pi} e^{S_g(\phi_p)} \prod_{f=1}^{N_f} \text{Det} \mathcal{M}_{\vec{x}, \vec{x}'}^f, \quad (3)$$

where s_p and ϕ_p are the standard plaquette angles. In the static approximation valid for large masses and/or for $\xi \ll 1$ the fermion determinant can be approximated as

$$\text{Det} \mathcal{M}_{\vec{x}, \vec{x}'}^f \approx \prod_x A_f \left[1 + h_+^f W(x) \right]^g \left[1 + h_-^f W^\dagger(x) \right]^g. \quad (4)$$

$g = 1(2)$ for the staggered (Wilson) fermions, $W(x) = \prod_{t=1}^{N_t} U_0(t, x)$ is the Polyakov loop. The constants appearing on the right-hand side of (4) are given by

$$A_f = e^{2N_t \text{arcsinh} m_f}, \quad h_\pm^f = e^{-(\text{arcsinh} m_f \mp \mu_f) N_t} \quad (5)$$

for the staggered fermions and

$$A_f = (2\kappa_f)^{4N_t}, \quad h_\pm^f = (2\kappa_f e^{\pm \mu_f})^{N_t}, \quad (6)$$

$$\kappa_f = \frac{1}{2m_f + 2d + 2 \cosh \mu_f}$$

for the Wilson fermions. In this paper we consider a class of ferromagnetic pure gauge actions S_g whose Boltzmann weight can be expanded as

$$e^{S_g(\omega)} = \prod_p \sum_{r=-\infty}^{\infty} C_r e^{ir\omega} \quad (7)$$

with positive coefficients C_r . E.g., for the standard Wilson action one has $C_r = I_r(\beta)$, where $I_r(\beta)$ is the modified Bessel function.

2. Dual representation

The Boltzmann weight of the models (2) and (3) is complex due to the fermion contribution (4). It is straightforward to get a positive expression for this weight by integrating out explicitly all gauge degrees of freedom and rewriting the theory in terms of fermion and plaquette occupation numbers. In order to perform such integration, the static determinant with N_f fermion flavors is presented as

$$\sum_{\substack{k_1(x)=0 \\ k'_1(x)=0}}^1 \dots \sum_{\substack{k_{N_f}(x)=0 \\ k'_{N_f}(x)=0}}^1 \prod_x \prod_{f=1}^{N_f} A_f (h_+^f)^{k_f(x)} (h_-^f)^{k'_f(x)} (W(x))^{k_f(x)-k'_f(x)}. \quad (8)$$

Combining this representation with the expansion (7) one can integrate over gauge fields to obtain, e.g. for the staggered fermions

$$Z = \sum_{\{r(p)\}=-\infty}^{\infty} \sum_{\substack{k_1(x)=0 \\ k'_1(x)=0}}^1 \dots \sum_{\substack{k_{N_f}(x)=0 \\ k'_{N_f}(x)=0}}^1 \prod_{p_s} C_{r(p_s)}(\beta_s) \prod_{p_t} C_{r(p_t)}(\beta_t) \\ \times \prod_x \prod_{f=1}^{N_f} A_f (h_+^f)^{k_f(x)} (h_-^f)^{k'_f(x)} \prod_{l_s} \delta_G \left(\sum_{p \in l_s} \tilde{r}(p) \right) \\ \times \prod_{l_t} \delta_G \left(\sum_{p \in l_t} \tilde{r}(p) + \sum_{f=1}^{N_f} (k_f(x) - k'_f(x)) \right). \quad (9)$$

In case of N_f degenerate flavors the last expression simplifies to

$$Z = A g^{N_f L^d} \sum_{\{r(p)\}=-\infty}^{\infty} \sum_{\substack{k(x)=0 \\ k'(x)=0}}^{g N_f} \\ \times \prod_{p_s} C_{r(p_s)}(\beta_s) \prod_{p_t} C_{r(p_t)}(\beta_t) \prod_{l_s} \delta_G \left(\sum_{p \in l_s} \tilde{r}(p) \right) \\ \prod_{l_t} \delta_G \left(\sum_{p \in l_t} \tilde{r}(p) + k(x) - k'(x) \right) \\ \times \prod_x \binom{g N_f}{k(x)} \binom{g N_f}{k'(x)} h_+^{k(x)} h_-^{k'(x)}. \quad (10)$$

In the last equations $\delta_G(x)$ means the delta-function on the group $G = Z(N), U(1)$. Thus, the partition function is expressed in terms of fermion numbers $k_f(x), k'_f(x)$ and plaquette occupation numbers $r(p)$. Both numbers are subject to constraints expressed via group delta-functions. The constraint on the spatial links l_s is precisely the same as in the pure gauge theory due to the absence of spatial gauge fields in the fermion determinant. The constraint on the temporal links l_t is modified due to a contribution of the Polyakov loops arising from the determinant. Note, fermion numbers $k_f(x), k'_f(x)$ do not depend on the temporal coordinate t , i.e. they are equal for all time-like links with coordinates $l_t = (t, x; 0)$ at fixed x . We have also used the following convention: $\tilde{r}(p) = r(p)$ if a given link l_s or l_t points in a positive direction when going around plaquette p and $\tilde{r}(p) = -r(p)$, otherwise.

As follows from the explicit representation of the group delta function $\delta_G(x)$ the dependence on μ drops out both from the partition function and from all invariant observables for $U(1)$ theory with one fermion flavor. To get a non-trivial dependence one has to consider a theory with $N_f \geq 2$ as in [13]. For $Z(N)$ model the dependence on chemical potential is non-trivial for any number of flavors.

It is straightforward to get dual representations for the most important observables. Taking into account Eq. (5) one obtains for the staggered fermions the particle density of f th flavor

$$B_f = \frac{1}{L^d N_t} \frac{\partial \ln Z}{\partial \mu_f} = \frac{1}{L^d} \left\langle \sum_x (k_f(x) - k'_f(x)) \right\rangle \quad (11)$$

and the fermion condensate of f th flavor

$$\sigma_f = \frac{1}{L^d N_t} \frac{\partial \ln Z}{\partial m_f} = \frac{1}{\sqrt{1 + m_f^2} L^d} \left\langle \sum_x (2 - k_f(x) - k'_f(x)) \right\rangle. \quad (12)$$

Extension to the Wilson fermions is trivial. Plaquette expectation value is

$$P(p) = \frac{1}{2} \left\langle \frac{I_{r(p)-1}(b) + I_{r(p)+1}(b)}{I_{r(p)}(b)} \right\rangle, \quad (13)$$

where $b = \beta_s(\beta_t)$ stays for the spatial (temporal) plaquette. Expectation value of the pure gauge action becomes

$$\langle S_g \rangle = \frac{1}{L^d N_t} \left(\beta_s \sum_{p_s} P(p_s) + \beta_t \sum_{p_t} P(p_t) \right). \quad (14)$$

Correlation functions of the Polyakov loops can be calculated as a ratio of the partition functions

$$\langle W(x)W^*(y) \rangle = \frac{Z(\eta_x, \bar{\eta}_y)}{Z}. \quad (15)$$

The partition function $Z(\eta_x, \bar{\eta}_y)$ coincides with Z up to a modification of the delta's on all temporal links with coordinates $l = (t, x; 0)$ and $l = (t, y; 0)$: arguments of these delta-functions acquire a linear shift by $\eta = -\bar{\eta} = 1$.

The models defined in Eqs. (9) and (10) have explicitly non-negative weights (for $h_+, h_- > 0$), hence they can in principle be studied with numerical Monte-Carlo simulations. The delta functions in the partition function create constraints on the configurations, which have to be preserved by the updates. As a first approach to the numerical simulation we propose a Metropolis algorithm for a $(d + 1)$ model on a lattice with periodic boundary conditions, that attempts following updates:

- For $Z(N)$ models - change of each variable $(k_f(x), k'_f(x), r(p))$ by $\pm N$.
- Change of two k variables at the same x by ± 1 , preserving the sum $\sum_f (k_f(x) - k'_f(x))$.
- Change by ± 1 of two k variables at two neighboring space positions x, y , compensated by the corresponding change at each time-like plaquette between sites x and y .
- Change by ± 1 of $r(p)$ variables on plaquettes forming a unit three dimensional cube.
- Global change by ± 1 of all $r(p)$ variables in $\mu\nu$ direction forming a surface wrapping around the whole lattice.

These updates generate the full set of permitted configurations, though it is possible that two configurations with large weight are connected through configurations with much smaller weight, which would reduce the update algorithm efficiency. A more efficient alternative would be to develop a surface-building worm update algorithm similar to the ones proposed in [8].

Another approach is to get rid of constraints on configurations whenever possible. First, consider the representation (10) for $(2 + 1)$ -dimensional $U(1)$ theory. When $k(x) = k'(x) = 0$ we recover the dual representation for the pure gauge model. The solution of the constraint is well known and reads [14]

$$\tilde{r}(p) = q(x) - q(x + e_\nu). \quad (16)$$

$q(x)$ is a new set of integer variables defined in the sites of the dual lattice. We have neglected some global variables. These global variables are conjugate to global Bianchi identities and do not contribute to thermodynamic limit. Possibility of nonzero k is restored by modifying the conditions for the temporal plaquettes:

$$\tilde{r}(p_s) = q(x) - q(x + e_0), \quad (17)$$

$$\tilde{r}(p_t) = q(x) - q(x + e_n) + \rho_n(x), \quad n = 1, 2. \quad (18)$$

Here $\rho_n(x)$ are new integer variables defined on the dual links (x, n) and depending only on spatial coordinates. Substituting Eqs. (17), (18) into our constraints we see that the constraints on

l_s are satisfied, while the constraints on l_t appear only at one fixed time slice and read

$$\delta_G(\rho(p) - k(p) + k'(p)), \quad (19)$$

$$\rho(p) = \rho_1(x) + \rho_2(x + e_1) - \rho_1(x + e_2) - \rho_2(x).$$

Four links entering this constraint form a dual plaquette p . Since now each of the variables $k(p), k'(p)$ appear just in one delta function, and the terms in partition function that depend on k do not mix at different plaquettes, we can calculate the sum over $k(p)$ and $k'(p)$ at each plaquette to remove the last set of deltas. This leads to the following dual form of the partition function

$$Z = A g^{N_f L^d} \sum_{\{q(x)\}=-\infty}^{\infty} \prod_{l_t} C_{q(x)-q(x+e_0)}(\beta_s) \times \sum_{\rho_n(x)=-\infty}^{\infty} \prod_{l_s} C_{q(x)-q(x+e_n)+\rho_n(x)}(\beta_t) \prod_p K_{\rho(p)}, \quad (20)$$

$$K_{\rho} = \left(\frac{h_+}{h_-} \right)^{\frac{\rho}{2}} \frac{(g N_f)!}{(g N_f + \rho)!} g^{N_f \rho} \left(\frac{1 + h_+ h_-}{1 - h_+ h_-} \right), \quad (21)$$

where $P_n^\rho(x)$ is the associated Legendre function. Product \prod_p runs over all space-like plaquettes of the dual lattice at a fixed time slice. To simulate the model (20) one can precompute K_{ρ} for $-g N_f \leq \rho \leq g N_f$ and then perform Metropolis updates by ± 1 on each variable $q(x), \rho_n(x)$.

For N_f non-degenerate flavors, Eq. (9), the representation (20) remains valid. The only change is the expression for K_{ρ} which becomes more complicated. An extension to $d = 3$ theory can be accomplished in a similar way if one uses the solution of the constraint for the pure gauge model following [14]. Finally, the $Z(N)$ case is recovered by treating each solution as an equality modulo N , thus leaving a degree of freedom for the difference of left and right parts divided by N .

3. (1 + 1)-dimensional theory

As an application, let us consider the dual formulation in $(1 + 1)$ -dimensions. Due to deltas on spatial links l_s all plaquette numbers at fixed position x are equal and can be identified with a link variable $r(l)$ of a one-dimensional lattice. All deltas on temporal links with a fixed coordinate x become also equal and can be associated with a site x of the same one-dimensional lattice. The $U(1)$ partition function (9) gets the form $(\beta_s = \beta_t = \beta)$

$$Z = \sum_{\{r(l)\}=-\infty}^{\infty} \sum_{\substack{k_1(x)=0 \\ k'_1(x)=0}}^1 \dots \sum_{\substack{k_{N_f}(x)=0 \\ k'_{N_f}(x)=0}}^1 \prod_l C_{r(l)}^{N_t}(\beta) \times \prod_x \prod_{f=1}^{N_f} A_f (h_+^f)^{k_f(x)} (h_-^f)^{k'_f(x)} \times \prod_x \delta_G \left(r(l) - r(l-1) + \sum_{f=1}^{N_f} (k_f(x) - k'_f(x)) \right). \quad (22)$$

For $Z(N)$ model one has to make the following replacement in the last expression

$$\sum_{r=-\infty}^{\infty} \rightarrow \sum_{r=0}^N \sum_{q=-\infty}^{\infty}, \quad C_r(\beta) \rightarrow C_{r+qN}(\beta). \quad (23)$$

This partition function can be evaluated as

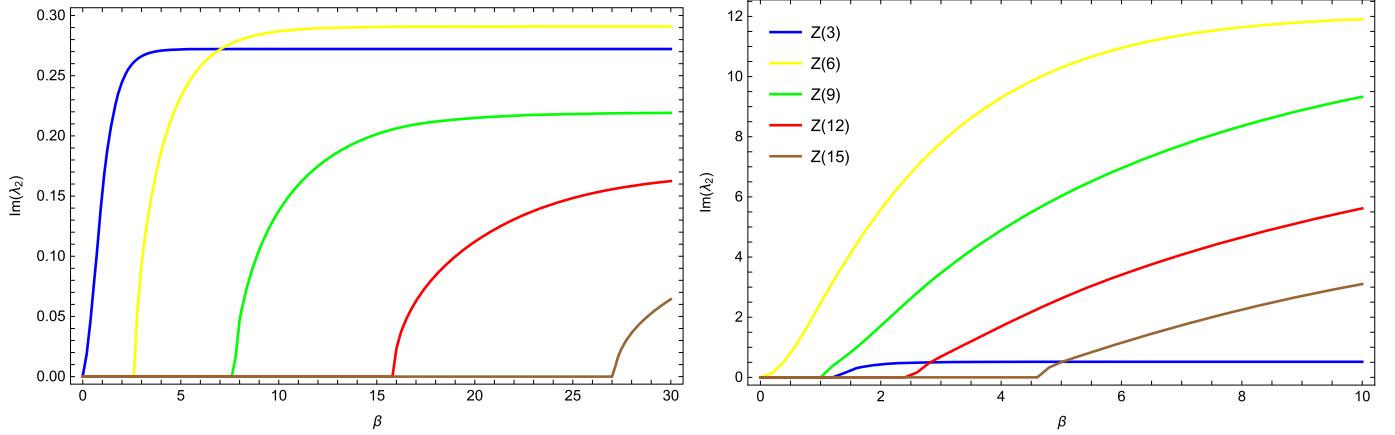


Fig. 1. Plots of the imaginary part of the 2nd eigenvalue of the transfer matrix of $Z(N)$ model with two flavors of staggered fermions as a function of the coupling constant. Left panel: $m_1 = 3, m_2 = 1, \mu_1 = -0.16, \mu_2 = 0.45$. Right panel: $m_1 = 1, m_2 = 0.1, \mu_1 = 2, \mu_2 = 1$.

$$Z = C_0^{LN_t}(\beta) \prod_{f=1}^{N_f} A_f^L \sum_{i=0} \lambda_i^L, \quad (24)$$

where λ_i are eigenvalues of the following transfer matrix

$$T_{r_1 r_2} = \sqrt{B_{r_1} B_{r_2}} \sum_{\substack{k_1=0 \\ k'_1=0}}^1 \dots \sum_{\substack{k_{N_f}=0 \\ k'_{N_f}=0}}^1 \prod_{f=1}^{N_f} (h_+^f)^{k_f} (h_-^f)^{k'_f}, \quad (25)$$

where $B_r = C_r^{N_t}(\beta)/C_0^{N_t}(\beta)$ and all configurations are subject to constraint $r_1 - r_2 + \sum_{f=1}^{N_f} (k_f - k'_f) = 0 \pmod{N}$. Below we analyze the theory with the Wilson action and two staggered fermion flavors.

When chemical potentials are zero all eigenvalues are real. This leads to a familiar exponential decay of the connected part of the Polyakov loop correlation function. However, when non-zero chemical potentials are introduced, one finds such values of the coupling constant above which the eigenvalues become complex. Moreover, the second and the third eigenvalues are conjugate to each other. Typical examples of such behavior are shown in Fig. 1 for various values of N . This implies the following decay of the two-point correlation function

$$\langle W(0)W^*(R) \rangle_c \approx e^{-m_r R} \cos m_i R. \quad (26)$$

Such an oscillating decay should not come as a surprise. Indeed, in a similar settings it was found in the $(1+1)$ -dimensional $SU(3)$ theory with one flavor [15] and in the two-dimensional $Z(3)$ spin model in a complex magnetic field [16] as well as in the 't Hooft-Veneziano limit of $SU(N)$ Polyakov loop models [17]. In all cases studied we have found the increase of the β value with N above which the oscillating phase appears. We do not know if the values of masses and/or chemical potentials can be re-scaled in a way such that in the limit $N \rightarrow \infty$ the oscillating phase would exist. We have, however studied $U(1)$ model directly in the region $\beta \leq 10$ and various values of masses and chemical potentials. No oscillating phase have been found in this case. We thus think the reasonable conjecture is to assume that the complex spectrum of the eigenvalues does not appear in the $U(1)$ model with two fermion flavors though this issue requires more thorough investigation.

4. Summary

In this paper we have derived the dual representations for $U(1)$ and $Z(N)$ lattice gauge theories in $(d+1)$ -dimension with N_f staggered or Wilson fermion flavors and in the static approximation for the fermion determinant. We presented two essentially different representations: one with a set of constraints on the dual variables, the second one is free of constraints. In both cases the dual weight is positive and suitable for numerical simulations. Even the dual model with constraints can be studied numerically if the proper algorithm is developed. One such possible algorithm was suggested here. As an application of the dual form we have studied $(1+1)$ -dimensional model with two staggered flavors. The model can be solved with the help of the transfer matrix. This solution reveals an existence of a phase in all $Z(N)$ models with an exponential decay of the correlations modulated by an oscillating function. The value of the coupling constant, above which such phase appears, grows with N .

Further possible applications of the dual formulation would be to study 1) the large N_f limit of Abelian models at finite density and 2) the Berezinskii-Kosterlitz-Thouless phase transition in $2+1$ models. The dual formulation of $U(1)$ model turned out to be very efficient in the study of this type of phase transition in a pure gauge model [18]. We think it can be also useful to investigate how the finite-density affects the critical behavior. These problems are currently under investigation.

Probably, the most important question is whether this approach can be extended to the full fermion determinant. On our opinion, combining the present approach with the methods of Ref. [13] one could construct the positive dual weight for $(1+1)$ dimensional Abelian models with non-zero fermion masses. This possibility certainly deserves further investigations.

O. Borisenko acknowledges support from the National Academy of Sciences of Ukraine in frames of the project "Meeting new experimental data on proton-proton, proton-nuclei, nuclei-nuclei interactions at high energies at CERN, BNL, FERMILAB, GSI for theoretical analysis" (No. 0121U112254). V. Chelnokov acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC-TR 211 "Strong-interaction matter under extreme conditions" – project number 315477589 – TRR 211.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] C. Gattringer, Nucl. Phys. B 850 (2011) 242, arXiv:1104.2503 [hep-lat].
- [2] O. Borisenko, V. Chelnokov, S. Voloshyn, Phys. Rev. D 102 (2020) 014502, arXiv:2005.11073 [hep-lat].
- [3] M. Fromm, J. Langelage, S. Lottini, O. Philipsen, J. High Energy Phys. 01 (2012) 042, arXiv:1111.4953 [hep-lat].
- [4] O. Borisenko, V. Chelnokov, E. Mendicelli, A. Papa, Nucl. Phys. B 965 (2021) 115332, arXiv:2011.08285 [hep-lat].
- [5] O. Borisenko, V. Chelnokov, E. Mendicelli, A. Papa, Proceedings of science, PoS (LATTICE2021) (2021) 587, arXiv:2112.00043 [hep-lat].
- [6] F. Karsch, K.H. Mütter, Nucl. Phys. B 313 (1989) 541.
- [7] F. Bruckmann, J. Wellenhofer, Phys. Rev. D 97 (2018) 014501, arXiv:1710.08243 [hep-lat].
- [8] Y. Delgado, C. Gattringer, A. Schmidt, Comput. Phys. Commun. 184 (2013) 1535, arXiv:1211.3436 [hep-lat].
- [9] K. Langfeld, String-like theory as solution to the sign problem of a finite density gauge theory, PoS (Confinement2018) (2018) 049, arXiv:1811.12921 [hep-lat].
- [10] O. Borisenko, V. Chelnokov, S. Voloshyn, EPJ Web Conf. 175 (2018) 11021, arXiv:1712.03064 [hep-lat].
- [11] C. Marchis, C. Gattringer, Phys. Rev. D 97 (2018) 034508, arXiv:1712.07546 [hep-lat].
- [12] G. Gagliardi, W. Unger, Phys. Rev. D 101 (2020) 034509, arXiv:1911.08389 [hep-lat].
- [13] C. Gattringer, T. Kloiber, V. Sazonov, Nucl. Phys. B 897 (2015) 732, arXiv:1502.05479 [hep-lat].
- [14] T. Banks, J. Kogut, R. Myerson, Nucl. Phys. B 121 (1977) 493.
- [15] H. Nishimura, M. Ogilvie, K. Pageni, Phys. Rev. D 93 (2016) 094501, arXiv:1512.09131 [hep-lat].
- [16] O. Akerlund, P. de Forcrand, T. Rindlisbacher, J. High Energy Phys. 10 (2016) 055, arXiv:1602.02925 [hep-lat].
- [17] O. Borisenko, V. Chelnokov, S. Voloshin, Proceedings of science, PoS (LATTICE2021) (2021) 453, arXiv:2111.07103 [hep-lat].
- [18] O. Borisenko, V. Chelnokov, M. Gravina, A. Papa, J. High Energy Phys. 09 (2015) 062, arXiv:1507.00833 [hep-lat].