The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a],[*], Christian S. Fischer [b],[c], Francesco Giacosa [a],[d]

[a] *Institute of Physics, Jan Kochanowski University, ul. Uniwersytecka 7, P-25-406 Kielce, Poland*
[b] *Institut für Theoretische Physik, Justus-Liebig Universität Gießen, 35392 Gießen, Germany*
[c] *Helmholtz Forschungsakademie Hessen für FAIR (HFHF), GSI Helmholtzzentrum für Schwerionenforschung, Campus Gießen, 35392 Gießen, Germany*
[d] *Institute for Theoretical Physics, Johann Wolfgang Goethe - University, Max von Laue–Str. 1, D-60438 Frankfurt, Germany*

A B S T R A C T

We study the decays of the $J^{PC} = 1^{-+}$ hybrid nonet using a Lagrangian invariant under the flavor symmetry, parity reversal, and charge conjugation. We use the available experimental data, the lattice predictions, and the flavor constraints to evaluate the coupling strengths of the $\pi_1(1600)$ to various two-body mesonic states. Using these coupling constants, we estimate the partial widths of the two-body decays of the hybrid pion, kaon and the isoscalars. We find that the hybrid kaon can be nearly as broad as the $\pi_1(1600)$. Quite remarkably, we find also that the light isoscalar must be significantly narrow while the width of the heavy isoscalar can be matched to the recently observed $\eta_1(1855)$.

1. Introduction

According to the non-relativistic quark model, conventional mesons ($\bar{q}q$ states) can only have specific values for the spin (J), parity (P), and charge conjugation (C) quantum numbers. These values are determined by the spin and angular momentum of the constituent quarks, which are conserved separately in a non-relativistic framework. States with unconventional quantum numbers, *e.g.*, $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$, are termed exotic. These exotic quantum numbers can arise due to various mechanisms like relativity, addition of non-$\bar{q}q$ degrees of freedom (like gluons) or the formation of multiquark bound states. All these may lead to a rich "exotic" spectrum of QCD involving, amongst others, glueballs, four-quark states and quark-gluon hybrids.

Although a number of candidates for exotic states have been observed experimentally, the identification of whole multiplets is far from complete. There is ample evidence for the presence of four- and five-quark states in the heavy quark sector of QCD, see *e.g.* Ref. [1] for a review. In the light quark sector two potential hybrid states, the isovector $\pi_1(1600)$ [2] and the very recently observed isoscalar $\eta_1(1855)$ [3], have been identified as states with quantum numbers $J^{PC} = 1^{-+}$. The observation of the hy-

brid isoscalar has piqued the interests of the community [4–6], since it may provide important guidance for high-quality predictions of masses and decay widths of the missing members of the $J^{PC} = 1^{-+}$-nonet. The members of (light) hybrid nonet are the subject matter of the present study.

First observed in 1998 by the E852 collaboration in the π-p scattering process [7], the mass and width of the $\pi_1(1600)$ have been determined as 1661^{+15}_{-11} MeV and 240 ± 50 MeV [2]. This state has been observed to decay into $b_1\pi$, $\rho\pi$, $\eta'\pi$, and $f_1\pi$ [2] and it can be identified with the light hybrid isovector predicted by models [8]; its partners in the 1^{-+} nonet are expected to have masses less than 2 GeV. An additional predicted decay channel, not yet observed by experiment is the $\eta\pi$ channel. Interestingly, the experiments have reported a second (lighter) resonance with hybrid quantum numbers 1^{-+}, called the $\pi_1(1400)$, that decays into $\eta\pi$ [9–12]. The fact that there are two hybrid isovector states with the same quantum numbers but mutually exclusive decay channels suggests that these two states must be the same, and the difference in the observed masses must be due to interference of background processes [13]. This hypothesis has been extensively studied and corroborated by Ref. [14]. Besides the π_1 states, the search for additional hybrid states represents an ongoing experimental effort with contributions from various collaborations [15–17].

On the theoretical side, continuum methods such as quark models [18–22], bag models [23], QCD sum rules [24,25], functional methods via Dyson-Schwinger, Bethe-Salpeter and Faddeev equations [26–31], light-front quantization models [32] and

* Corresponding author.
E-mail addresses: vanamalishastry@gmail.com (V. Shastry), christian.fischer@theo.physik.uni-giessen.de (C.S. Fischer), fgiacosa@ujk.edu.pl, francescogiacosa@gmail.com (F. Giacosa).

coupled-channel K-matrix analysis [33] have been used to study the properties of hybrid mesons. These are complemented by various lattice studies exploring the exotic side of the QCD spectrum resulting in various qualitative predictions [34–41]. A recent comprehensive study of the hybrid (isovector) mesons on the lattice reported the mass of the $\pi_1(1600)$ to be ~ 1.564 GeV [42] and also the possible ranges for the partial widths of the decays of the $\pi_1(1600)$.

In the present work, we combine in a unique fit, experimental data [2], lattice QCD results [42], as well as constraints from flavor symmetry in order to constraint the mass and the decay widths of the resonance $\pi_1(1600)$. [As a means, we shall construct a Lagrangian invariant under flavor symmetry, parity reversal and charge conjugation, whose coupling constants are fixed by the aforementioned fit.] As a result, we are able to estimate the mass and the decays of the $\pi_1(1600)$ to a better accuracy than the present experimental and lattice results.

Moreover, flavor symmetry implies that hybrid mesons also appear in nonets, just as it happens for the regular quark - antiquark states. Within our approach, we are able to predict the decay properties of the remaining members of the nonet *viz.*, the kaonic and the isoscalar ones. In particular, we find that the light isoscalar state (η_1^L) is quite narrow, but the heavy isoscalar state (η_1^H) can be as broad as ~ 200 MeV. The latter, in fact, is consistent with the recently observed $\eta_1(1855)$ which has a width of $188 \pm 18^{+3}_{-8}$ MeV [3]. This is quite interesting, since the existence of these states could be verified in ongoing and future experiments.

The paper is divided into the following sections: in Sec. 2 we briefly discuss the Lagrangian for π_1 and in Sec. 3 we perform the fit. In Sec. 4 we present our results for other members of the nonet and discuss their possible implications; in Sec. 5 we summarize the paper.

2. The Lagrangian for the π_1 state

In order to describe the decays of the state $\pi_1(1600)$, we write down a simple Lagrangian containing the relevant interaction terms and respecting invariance under flavor symmetry, parity reversal (P) and charge conjugation (C):

$$
\begin{aligned}
\mathcal{L}^{\pi}_{hyb} = {} & g^c_{b_1\pi} \langle \pi_{1,\mu} b_1^{\mu} \pi \rangle + g^d_{b_1\pi} \langle \pi_{1,\mu\nu} b_1^{\mu\nu} \pi \rangle \\
& + g_{f_1\pi} \langle \pi_{1,\mu} f^{\mu\nu}_{1,N} \partial_\nu \pi + \pi_{1,\mu} f^{\mu\nu}_{1,S} \partial_\nu \pi \rangle \\
& + g_{\eta\pi} \langle \pi_{1,\mu} (\eta_N \partial^\mu \pi + \eta_S \partial^\mu \pi) \rangle + g_{\rho\pi} \langle \tilde{\pi}_{1,\mu\nu} \rho^{\mu\nu} \pi \rangle \\
& + g_{\rho\omega} \langle \pi_{1,\mu} (\rho^{\mu\nu} \omega_\nu + \omega^{\mu\nu} \rho_\nu) \rangle.
\end{aligned}
\tag{1}
$$

In the above Lagrangian, the isospin factors have been represented as $\langle \cdots \rangle$, and the subscripts N and S represent non-strange and strange flavor states respectively. The first two terms describe the decay of the π_1 to $b_1\pi$, but the latter is a higher order term that includes derivatives. This has been included to take care of the large ratio of the partial wave amplitudes in the decay [2,43]. The term in the second line of Eq. (1) gives rise to the decay of the π_1 to the axial-vector isoscalars. As we demonstrate later, when extended to the entire nonet, this term describes some of the dominant decays of the kaonic hybrid and the isoscalars. The term describing the $\eta^{(\prime)}\pi$ decay channels is peculiar, since it arises entirely due to the axial anomaly [44]. The last line of Eq. (1) describes the decay of the hybrid into two vector states. These channels, however, turn out to be highly suppressed, similarly to the $\eta\pi$ channel.

In summary, the Lagrangian above can be seen as a tool to summarize the available decay channels, allowing us to write down the corresponding decay widths in each case. The coupling constants, that shall be determined by a fit to data and lattice, contain the (non-trivial) link between the hybrid state π_1 and the ordinary

mesons. Eventual form factors, and other non-perturbative effects are absorbed into the values of the coupling constants.

From Eq. (1), we get the following expressions for the decay widths:

$$
\begin{aligned}
\Gamma_{b_1\pi} = {} & \frac{1}{2} \frac{k_{b_1}}{24\pi m_{\pi_1}^2} \left(\frac{1}{m_{b_1}^2} \left(E_{b_1} g^c_{b_1\pi} + 2g^d_{b_1\pi} m_{b_1}^2 m_{\pi_1} \right)^2 \right. \\
& \left. + 2(2E_{b_1} g^2_{b_1\pi} m_{\pi_1} + g^c_{b_1\pi})^2 \right)
\end{aligned}
\tag{2}
$$

$$
\frac{G_2}{G_0} = \sqrt{2} \frac{g^c_{b_1\pi}(-E_{b_1}+m_{b_1}) + 2g^d_{b_1\pi} m_{\pi_1}(-m_{b_1}^2 + E_{b_1} m_{b_1})}{g^c_{b_1\pi}(E_{b_1}+2m_{b_1}) + 2g^d_{b_1\pi} m_{\pi_1}(m_{b_1}^2 + 2E_{b_1} m_{b_1})}
\tag{3}
$$

$$
\Gamma_{\rho\pi} = g^2_{\rho\pi} \frac{k_\rho^3}{6\pi}
\tag{4}
$$

$$
\Gamma_{K^*K} = g^2_{\rho\pi} \frac{k_{K^*}^3}{12\pi}
\tag{5}
$$

$$
\begin{aligned}
\Gamma_{f_1\pi} = {} & \frac{1}{4} g^2_{f_1\pi} \cos^2\theta_a \frac{k_{f_1}}{24\pi m_{\pi_1}^2} \left(m_{f_1}^2 (2m_{f_1}^2 + m_{\pi_1}^2 - 6m_{\pi_1} E_{f_1}) \right. \\
& \left. + E_{f_1}^2 (2m_{\pi_1}^2 + m_{f_1}^2) \right)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
\Gamma_{f_1'\pi} = {} & \frac{1}{4} g^2_{f_1\pi} \sin^2\theta_a \frac{k_{f_1'}}{24\pi m_{\pi_1}^2} \left(m_{f_1'}^2 (2m_{f_1'}^2 + m_{\pi_1}^2 - 6m_{\pi_1} E_{f_1'}) \right. \\
& \left. + E_{f_1'}^2 (2m_{\pi_1}^2 + m_{f_1'}^2) \right)
\end{aligned}
\tag{7}
$$

$$
\Gamma_{\eta\pi} = \frac{1}{4} \left(\sqrt{\frac{2}{3}} \cos\theta_p + \frac{1}{\sqrt{3}} \sin\theta_p \right)^2 g_\eta^2 \frac{k_\eta^3}{24\pi m_{\pi_1}^2}
\tag{8}
$$

$$
\Gamma_{\eta'\pi} = \frac{1}{4} \left(\sqrt{\frac{2}{3}} \sin\theta_p - \frac{1}{\sqrt{3}} \cos\theta_p \right)^2 g_\eta^2 \frac{k_{\eta'}^3}{24\pi m_{\pi_1}^2}
\tag{9}
$$

$$
\Gamma_{\rho\omega} = \frac{\cos^2\theta_v}{4} g^2_{\rho\omega} \frac{k_\rho^3}{24\pi} \left(\frac{\left(-8E_\rho m_{\pi_1} + 4m_\rho^2 + 4E_\rho^2 + 3m_{\pi_1}^2 \right)}{m_\rho^2 m_\omega^2} \right),
\tag{10}
$$

where m_{π_1} is the mass of the parent, G_2 and G_0 are the amplitudes of the $\ell = 0, 2$ partial waves, k_x and g_x are the 3-momenta carried by the decay products and the coupling constants in the channel x respectively. In the expressions above, the notations f_1 and f_1' represent $f_1(1285)$ and $f_1'(1420)$. Further, θ_p is the η-η' mixing angle, θ_v is the $\omega - \phi$ mixing angle, and θ_a is the f_1-f_1' mixing angle in the strange-nonstrange basis. The values of the masses of the final states and, wherever applicable, the mixing angles are listed in Table 1.

3. Combined fit for $\pi_1(1600)$

The available experimental data on the properties of $\pi_1(1600)$, used in our fit, are:

1. The mass of the hybrid: $m_{\pi_1} = 1661^{+15}_{-11}$ MeV [2]. [For the different experimental results considered by the PDG to arrive at this value, see Refs. [49–52]]. We take the uncertainty in the mass to be 15 MeV.
2. The decay width: $\Gamma_{\text{tot}} = 240 \pm 50$ MeV [2].
3. The ratio of the branching ratios of the $b_1\pi$ channel in the D-wave and S-wave: $\frac{BR(\pi_1 \to b_1\pi)_D}{BR(\pi_1 \to b_1\pi)_S} = 0.3 \pm 0.1$ [2,53]. This ratio is equal to the square of the ratio of the corresponding partial wave amplitudes (PWAs). Hence, the D/S-ratio for the $b_1\pi$ channel is

Table 1

The values of the masses and mixing angles of the decay products used in the fit. All values taken from the PDG [2] unless otherwise noted.

Meson	Mass (GeV)	Meson	Mass (GeV)	Meson	Mass (GeV)	States	Mixing angle
π	0.135	K	0.494	η	0.548	$\eta - \eta'$	$-44.5°$ [45]
η'	0.958	ρ	0.775	ω	0.782	$\phi - \omega$	$-3°$
K^*	0.892	ϕ	1.020	a_1	1.23	$f_1 - f_1'$	$24°$ [46]
$K_1(1270)$	1.253	f_1	1.285	f_1'	1.426	$h_1 - h_1'$	$25°$ [43]
b_1	1.23	$K_1(1400)$	1.403	$h_1(1170)$	1.17	$K_1^A - K_1^B$	$56°$ [47,48]

$$\sqrt{\frac{BR(\pi_1 \to b_1\pi)_D}{BR(\pi_1 \to b_1\pi)_S}} = \pm(0.55 \pm 0.165).$$

The sign of the D/S-ratio is, unfortunately, unknown. Thus, we perform two fits – one for each sign of this ratio. From Eq. (3) we see that the D/S-ratio fixes the magnitude of the ratio of the coupling constants $g_{b_1\pi}^d / g_{b_1\pi}^c$. It should be noted, however, the relative sign of the coupling constants does not reflect that of the partial wave amplitudes.

4. The ratio of the partial widths of the $f_1\pi$ channel to that of the $\eta'\pi$ channel is $\frac{\Gamma_{f_1\pi}}{\Gamma_{\eta'\pi}} = 3.8 \pm 0.78$ [2,52].

The ranges of the partial widths of the decay channels were estimated using lattice methods in Ref. [42]. We use the midpoints of their values and estimate the uncertainty to be 50%. For example, the lattice result for the partial width of the $b_1\pi$ decay channel is 139-529 MeV, then we use the value 334 ± 167 MeV as an input in the fit. The following are the lattice estimates [42]:

1. $\Gamma_{b_1\pi} = 139\text{-}529$ MeV,
2. $\Gamma_{\rho\pi} = 0\text{-}20$ MeV,
3. $\Gamma_{K^*K} = 0\text{-}2$ MeV,
4. $\Gamma_{f_1\pi} = 0\text{-}24$ MeV,
5. $\Gamma_{f_1'\pi} = 0\text{-}2$ MeV,
6. $\Gamma_{\rho\omega} \leq 0.15$ MeV,
7. $\Gamma_{\eta\pi} = 0\text{-}1$ MeV,
8. $\Gamma_{\eta'\pi} = 0\text{-}12$ MeV.

Finally, we can also use the following flavor constraints:

1. The f_1 and the f_1' are isoscalar doublets and arise as a result of the mixing between the corresponding strange and non-strange flavor states. Since the $\pi_1(1600)$ is an isovector, the coupling constants defining the $\pi_1 f_1^{(\prime)}\pi$ interactions differ only in the mixing angle. The decay widths also differ in the 3-momenta carried by the decay products. Thus, from Eq. (6) and Eq. (7), the ratio of the partial decay widths is $\frac{\Gamma_{f_1'\pi}}{\Gamma_{f_1\pi}} = 0.0512$, when $m_{\pi_1} = 1661$ MeV.

2. The above argument also applies to the $\rho\pi$ and K^*K channels, where the difference lies in the 3-momenta and the isospin factors only. We can derive the ratio of the partial widths using Eq. (4) and Eq. (5) as $\frac{\Gamma_{K^*K}}{\Gamma_{\rho\pi}} = 0.178$.

3. The partial widths of the $\eta\pi$ and $\eta'\pi$ channels differ in the 3-momenta, isospin factors, and the mixing angle. Using these arguments, we obtain from Eq. (8) and Eq. (9), $\frac{\Gamma_{\eta'\pi}}{\Gamma_{\eta\pi}} = 12.72$.

4. We assume an arbitrary 30% error in all the inputs coming from the chiral constraints.

The values of the parameters were estimated using a χ^2-fit to the available data and are listed in Table 2. Table 3 lists the partial widths of the various decays of the $\pi_1(1600)$.

It is clear that the most dominant channel for the decay of the $\pi_1(1600)$ is the $b_1(1235)\pi$ channel. As discussed in Ref. [43],

Table 2

The values of the mass of $\pi_1(1600)$ and the coupling constants along with the uncertainties when the D/S-ratio for the $b_1\pi$ decay channel is positive (second column), and negative (third column).

Parameter	Value	
	Set-1 ($D/S > 0$)	Set-2 ($D/S < 0$)
m_{π_1}	1.663 ± 0.01 GeV	1.662 ± 0.01
$g_{b_1\pi}^c$	88 ± 23 GeV	$-(119 \pm 22)$
$g_{b_1\pi}^d$	$-(23.3 \pm 5.60)$ GeV^{-1}	26.7 ± 5.3
$g_{\rho\pi}$	(0.35 ± 0.05) GeV	0.35 ± 0.05
$g_{f_1\pi}$	8.02 ± 0.83 GeV	8.12 ± 0.83
$g_{\rho\omega}$	$-(0.37 \pm 0.07)$	$-(0.38 \pm 0.07)$
$g_{\eta\pi}$	4.91 ± 0.56	4.94 ± 0.55
$\chi^2/$d.o.f	0.35	0.28

Table 3

The partial widths and branching ratios of various decay channels and the total width (parameter Set-1; see text for discussion).

Channel	Width (MeV)	Channel	Width (MeV)
$\Gamma_{b_1\pi}$	220 ± 34	$\Gamma_{f_1\pi}$	16.2 ± 3.1
$\Gamma_{\rho\pi}$	7.1 ± 1.8	$\Gamma_{f_1'\pi}$	0.83 ± 0.16
Γ_{K^*K}	1.2 ± 0.3	$\Gamma_{\eta\pi}$	0.37 ± 0.08
$\Gamma_{\rho\omega}$	0.08 ± 0.03	$\Gamma_{\eta'\pi}$	4.6 ± 1.0
		Γ_{tot}	250 ± 34

derivative interactions are needed to explain the large D/S ratio for the $\pi_1(1600) \to b_1(1235)\pi$ decay mentioned in the PDG [2,53]. The difference in the sign of the D/S ratio does not affect the results of the fit or the values of the parameters. However, the kaonic and the isoscalar hybrid states (see discussions below) show strong sensitivities to this sign. Thus, from a theoretical point of view, it would be crucial to know the phase difference between the S-wave and the D-wave in the $\pi_1(1600) \to b_1\pi$ decay. On the other hand, a knowledge of the total width of the kaons can hint at the correct sign of the D/S-ratio.

The other decays are smaller than 20 MeV, but $f_1\pi$, $\rho\pi$, and $\eta'\pi$ are however not negligible. The remaining channels *viz.*, K^*K, $f_1'\pi$, and $\rho\omega$, are largely suppressed. We also note that the partial width of the $\eta\pi$ decay channel is nearly one order of magnitude smaller than the $\eta'\pi$ one. This results from the specific form of the factor dependent on the mixing angle (see, Eq. (8) and Eq. (9)) which suppresses the contributions of the $\eta\pi$ channel by a factor of ~ 30 compared to that of the $\eta'\pi$.

An important remark is in order: the smallness of the $\eta\pi$ channel may imply that the background effects, such as final state interactions, may influence the $\eta\pi$ channel more than the $\eta'\pi$ channel. This fact can explain the observed $\pi_1(1400)$ resonance with mass ~ 1350 MeV [13,14] as the very same state $\pi_1(1600)$,

Table 4

The masses of the kaons and the isoscalars, and isoscalar mixing angle for the three scenarios discussed in the text.

	m_{K_1} (GeV)	$m_{\eta_1^L}$ (GeV)	$m_{\eta_1^H}$ (GeV)	θ_h	δ_S^{hyb} (GeV^2)
Scenario-1	1.707	1.542	1.855	36.7°	0.151
Scenario-2	1.761	1.661	1.855	0°	0.341
Scenario-3	1.754	1.646	1.855	15°	0.317

Table 5

The masses and widths of the kaons and the isoscalars in the three scenarios discussed in the text. The most probable scenario is highlighted in bold.

State	Scenario-1		Scenario-2		Scenario-3		Parameter
	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	Set
K_1^{hyb}	1706	140 ± 45	**1761**	**312 ± 97**	1754	286 ± 88	1
		73 ± 29		**170 ± 65**		155 ± 59	2
η_1^L	1543	21 ± 4	**1661**	**81 ± 15**	1646	69 ± 13	1
		22 ± 4		**83 ± 16**		71 ± 13	2
η_1^H	1855	607 ± 159	**1855**	**259 ± 92**	1855	411 ± 130	1
		249 ± 80		**157 ± 68**		192 ± 80	2

whose peak is shifted [14]. A detailed study of this effect is left for the future.

4. Predictions for the other hybrid members

With the parameters determined above, we use flavor symmetry to predict the magnitude of the decays of the remaining members of the hybrid nonet - the kaonic and the isoscalar ones.

4.1. The hybrid isoscalars: η_1^L and η_1^H

Referring to the isoscalars, the flavor states are the strange ($|\bar{s}s\rangle_h$) and the non-strange ($|\bar{n}n\rangle_h$) states. As per our model, the non-strange state is expected to have the same mass as that of the isovector. The strange state, however, is expected to get an additional mass proportional to the contributions from the strange constituent quarks (hereafter, strangeness contribution) [44]. The masses of these states are,

$$m_{\eta_{1,N}}^2 = m_{\pi_1}^2 \tag{11}$$

$$m_{\eta_{1,S}}^2 = m_{\pi_1}^2 + 2\delta_S^{hyb} \tag{12}$$

where, δ_S^{hyb} is the strangeness contribution, and the subscripts N and S refer to non-strange and strange states, respectively. The mixing between these two states lead to the physical hybrid isoscalar mesons η_1^L and η_1^H (L = light, H = heavy) given by:

$$\begin{pmatrix} |\eta_1^L\rangle \\ |\eta_1^H\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_h & \sin\theta_h \\ -\sin\theta_h & \cos\theta_h \end{pmatrix} \begin{pmatrix} |\bar{n}n\rangle_h \\ |\bar{s}s\rangle_h \end{pmatrix} \tag{13}$$

where θ_h is the mixing angle. Combining the Eq. (11)-(13), we get the masses of the η_1^L and η_1^H as

$$m_{\eta_1^L}^2 = m_{\pi_1}^2 + \delta_S^{hyb}(1 - \sec(2\theta_h)) \tag{14}$$

$$m_{\eta_1^H}^2 = m_{\pi_1}^2 + \delta_S^{hyb}(1 + \sec(2\theta_h)) . \tag{15}$$

Note, since the hybrid isoscalars belong to a so-called homochiral multiplet ([54]) we do not expect them to mix significantly. Accordingly, the light isoscalar is expected to have a mass similar to (or lower than) the $\pi_1(1600)$, and the heavy one can be identified with the very recently discovered resonance $\eta_1(1855)$.

Applying the recently reported data [3] to the analysis done in Ref. [44], we find that the observed mass of the $\eta_1(1855)$ can be explained in three possible ways:

1. Scenario-1: The strangeness contribution to the hybrid $\eta_{1,S}$ state is the same as the ones to the conventional states, *i.e.,* $\delta_S^{hyb} = 0.151$ GeV^2 [44,55] (see, Table 4). This would imply that for the η_1^H to be identified with the $\eta_1(1855)$, a large mixing angle ($\theta_h \sim 36.7°$) would be needed, contrary to the homochiral nature of the states [44,54]. The total width of the η_1^H would then be 607 ± 159 MeV (249 ± 80 MeV) for the parameter set-1 (set-2) (see, Table 5). Both these values are significantly different from the experimentally measured value of $188 \pm 18^{+3}_{-8}$ MeV [3].

2. Scenario-2: The strangeness contribution provides bulk of the extra mass of the $\eta_1(1855)$. In this scenario, the mixing angle vanishes ($\theta_h = 0$), thus $\delta_S^{hyb} = 0.341$ GeV^2. The kaon turns out to have a mass of 1761 MeV, while the mass of the light isoscalar amounts to 1661 MeV. This case, however, leads to a total width of 259 ± 92 MeV (157 ± 68 MeV) for the η_1^H, which are in agreement with the experimental observation at 1σ level.

3. Scenario-3: The mass of the $\eta_1(1855)$ is a consequence of the combination of the above two scenarios, *i.e.* the strangeness contribution accounts for some part of the extra mass and a non-zero (but small) mixing angle provides for the rest. For our purpose, we take $\theta_h = 15°$, giving $\delta_S^{hyb} = 0.317$ GeV^2. The masses of the kaon and the light isoscalar would fall between the estimates of the previous two cases - at 1754 MeV and 1646 MeV respectively. This leads to a total width of 411 ± 130 MeV (192 ± 80 MeV) for the parameter set-1 (set-2).

The partial widths of the various decays of the isoscalar hybrids corresponding to scenario-2 are given in Table 6. The η_1^L has four open decay channels: $a_1(1260)\pi$, K^*K, $\eta\eta'$, and the $\rho\rho$. Of these, the $a_1(1260)\pi$ channel is the dominant one. The total width of this isoscalar is ~ 81 MeV, which is small compared to its siblings. It should be noted that the $K_1(1270/1400)K$, $f_1^{(\prime)}\eta^{(\prime)}$ channels are forbidden at tree-level as they are sub-threshold. These channels could eventually contribute a significant amount to the width of the η_1^L, leading to a somewhat broader state if the finite width of the states is taken into account (see below).

Table 6

The partial widths and branching ratios of various decay channels and the total width of the η_1^L (left) and the $\eta_1(1855)$ (right) for $\theta_h = 15°$. This corresponds to the "Scenario-2" discussed in the text.

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{a_1\pi}$	80 ± 15	82 ± 16	$\Gamma_{K_1(1270)K}$	253 ± 92	151 ± 67
Γ_{K^*K}	0.29 ± 0.075	0.29 ± 0.075	Γ_{K^*K}	1.45 ± 0.37	1.46 ± 0.38
$\Gamma_{\eta'\eta}$	0.41 ± 0.09	0.41 ± 0.09	$\Gamma_{\eta'\eta}$	2.28 ± 0.51	2.31 ± 0.51
$\Gamma_{K_1(1270)K}$	0	0	$\Gamma_{a_1\pi}$	0	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028	0.082 ± 0.029	$\Gamma_{\rho\rho}$	0	0
$\Gamma_{K^*K^*}$	0	0	$\Gamma_{K^*K^*}$	0.075 ± 0.027	0.077 ± 0.028
$\Gamma_{\omega\phi}$	0	0	$\Gamma_{\omega\phi}$	$\sim 10^{-4}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	0	0	$\Gamma_{f_1\eta}$	2.15 ± 0.56	2.21 ± 0.57
Γ_{tot}	81 ± 15	83 ± 16	Γ_{tot}	259 ± 92	157 ± 68

Table 7

The partial widths and branching ratios of various decay channels and the total width for the hybrid kaon $K_1^{hyb}(1750)$. We have assumed the mass of the state to be 1761 MeV [44].

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{K_1(1270)\pi}$	125 ± 42	48 ± 25	$\Gamma_{\rho K}$	2.18 ± 0.56	2.19 ± 0.57
$\Gamma_{K_1(1400)\pi}$	103 ± 45	98 ± 43	$\Gamma_{\omega K}$	0.82 ± 0.21	0.82 ± 0.21
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28	1.37 ± 0.24	$\Gamma_{\phi K}$	0.49 ± 0.12	0.49 ± 0.13
$\Gamma_{\eta K}$	0.29 ± 0.07	0.29 ± 0.07	$\Gamma_{K^*\pi}$	0.67 ± 0.17	0.67 ± 0.17
$\Gamma_{\eta'K}$	2.77 ± 0.62	2.81 ± 0.62	$\Gamma_{K^*\eta}$	0.30 ± 0.08	0.30 ± 0.08
$\Gamma_{\rho K^*}$	0.045 ± 0.016	0.047 ± 0.016	$\Gamma_{\omega K^*}$	0.011 ± 0.004	0.012 ± 0.004
Γ_{a_1K}	11.0 ± 2.32	11.3 ± 2.35	Γ_{b_1K}	64 ± 14	3.11 ± 2.88
			Γ_{tot}	312 ± 97	170 ± 65

On the other hand, η_1^H isoscalar can decay into $K_1(1270)K$, K^*K, $\eta\eta'$, $f_1\eta$, K^*K^*, and the $\omega\phi$ states. The $K_1(1270)K$ is expected to be the dominant decay channel, followed by the $a_1\pi$ channel. With these decay channels, the width of $\eta_1(1855)$ is 259 ± 92 MeV (parameter set-1), which is consistent with the experimental value at 1σ level.

However, the broad nature of the (dominant) a_1 and the presence of sub-threshold channels could eventually modify width for this state, (possibly) requiring a tweaking of the mixing angle.

It is furthermore interesting that the $K_1(1270)K$ is always the dominant channel for the decay of the heavy η_1^H and the decay channel $a_1\pi$ is always dominant for η_1^L, independently of the three scenarios that we have discussed.

The decays of the hybrid isoscalars involve broad states and a large number of sub-threshold channels. To get a full picture of the decays of the hybrid isoscalars, it becomes necessary to perform a spectral integration over the final states. A detailed study in this direction will be attempted in the future.

4.2. The hybrid kaon: $K_1^{hyb}(1750)$

The mass of the kaon is given by [44],

$$m_{K_1}^2 = m_{\pi_1}^2 + \delta_s^{hyb}. \tag{16}$$

Based on the discussion present in the previous subsection, we take the mass of the kaon as 1761 MeV (i.e. Scenario-3). As seen in the Table 7, the kaons are expected to be broad states, similar to the isovectors.

The hybrid kaons have more decay channels available at tree level compared to the $\pi_1(1600)$. One can expect the hybrid kaons to decay into $K_1(1270)\pi$, $K_1(1400)\pi$, a_1K, b_1K, $h_1(1170)K$, ρK, ωK, ϕK, $K^*\pi$, $K^*\eta$, ηK, $\eta'K$, ρK^*, and ωK^*. Of these the axial-kaon channels are expected to be the most dominant channels. The estimated partial widths of these decay channels are given in Table 7. Apart from the axial-kaon decay channels, we also expect the hybrid kaon to decay into the b_1K, a_1K, and the $\eta'K$ channels with significant widths. The ηK channel is nearly one order of magnitude smaller than the $\eta'K$ channel. The vector-vector decay channels (ρK^* and ωK^*) appear to be strongly suppressed.

In the axial-kaonic channels, since the decay thresholds are very close to the mass of the kaon, we expect the $K_1(1270)\pi$, $K_1(1400)\pi$, a_1K, and the b_1K thresholds to distort the line shape of the hybrid kaon significantly. This is true for other channels as well (except for the $K^*\pi$ and the ηK channels), but their partial widths are too small to cause any significant change.

Another interesting observation is that the partial width of the $K_1^{hyb}(1750) \to h_1(1170)K$ decay is sensitive to the $h_1(1170) - h_1'(1415)$ mixing angle. In the calculation of the partial widths listed in Table 7 we have assumed a mixing angle of $\theta_{pv} = +25°$ as derived in [43] (for other possible values, see, *e.g.*, Refs. [48,56]). However, if the mixing angle is reduced to $\theta_{pv} = 0.6°$, as reported by the BESIII collaboration [57], we obtain the partial width to be 15.5 ± 2.8 MeV (14.0 ± 2.5 MeV) for the parameter Set-1 (Set-2). This would also increase the total width of the hybrid kaon to 326 ± 97 MeV (182 ± 64 MeV). On the other hand, if the mixing angle is negative ($\theta_{pv} = -25°$), then the partial width increases to 36 ± 6.5 MeV (33 ± 5.8) MeV, and the total width to 346 ± 99 MeV (201 ± 64 MeV). We thus expect the kaon to have a total width of $300 - 400$ MeV.

Finally, it should be recalled that the hybrid kaon state can, in principle, mix with nearby vector states, such as the excited vector kaons $K^*(1410)$ and $K^*(1680)$ [2]. At present, the results of

$K^*(1410)$ and $K^*(1680)$ fit quite well with being purely quark-antiquark states [58,59] belonging to the nonet of radially excited and orbitally excited vector kaons, hence we neglect this mixing. Further, even though the orbitally excited $\bar{q}q$ and the hybrid kaonic states have the same possible decay channels, their partial widths are "complementary" in the sense that, the dominant decay channels of the $K^*(1680)$ are suppressed for the K_1^{hyb} (for instance, $K^*(1680)$ strongly decays into $K\pi$, ρK, and the $K^*\pi$ channels, which -according to our results- are suppressed for the hybrid kaon). Moreover, because of the difference in their masses, some of the channels available for the decay of the hybrid kaon (i.e., a_1K, b_1K) are sub-threshold for the vector kaon. These features can be used to detect the existence of the hybrid kaon and can be an important test for our model.

Nevertheless, the mixing of vector kaonic states belonging to distinct nonets should be eventually reconsidered when experimental data about this yet putative state will be available.

5. Summary and conclusions

In this work, we have studied the decays of the 1^{-+} hybrid nonet. The experimental and lattice results for the $\pi_1(1600)$ were implemented, together with flavor constraints, in a single fit. As an outcome, both the mass and the decay widths $\pi_1(1600)$ could be re-determined with a better accuracy. For definiteness, a Lagrangian invariant under the flavor symmetry, parity reversal, and charge conjugation was utilized for describing the various decays.

Next, we have estimated the partial widths for the various allowed decay channels and also the total widths of the remaining members of the hybrid nonet (the putative states $K_1^{hyb}(1750)$, η_1^L, η_1^H). We are able to identify the η_1^H with the $\eta_1(1855)$, whereas the light isoscalar η_1^L turns out to be rather narrow.

The quantities estimated in this work can be used as guiding values for the ongoing experimental searches for the missing hybrid states η_1^L and K_1^{hyb} [15] and toward a better understanding of $\pi_1(1600)$ as well as the very recently discovered $\eta_1(1855)$ (for which an independent confirmation is needed). We expect the 1^{-+} kaonic and isoscalar hybrids to be observable in the $K3\pi$, 4π and $2\pi 2K$ channels, respectively. Needless to say, the discovery and assessment of the lightest hybrid nonet would constitute an important step forward in low-energy QCD.

A more complete description of the decays studied in this work can be obtained by expanding the formalism to the sub-threshold decay channels using spectral integration. This can lead to interesting results, as there are quite a few decay channels with thresholds slightly greater than the masses of the hybrids. Using spectral distribution functions that inherently take into account the thresholds (e.g., Ref. [60]), one can study the contributions of these decay channels. This will be attempted in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

All the data used are from the cited references.

Acknowledgements

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