# Microscopic origin of the valence transition in tetragonal $EuPd_2Si_2$

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(Dated: September 22, 2022)

Under temperature or pressure tuning, tetragonal EuPd<sub>2</sub>Si<sub>2</sub> is known to undergo a valence transition from nearly divalent to nearly trivalent Eu accompanied by a volume reduction. Albeit intensive work, its origin is not yet completely understood. Here, we investigate the mechanism of the valence transition under volume compression by density functional theory calculations (DFT). Our analysis suggests that the transition is a consequence of an enhanced c-f hybridization between localized Eu 4f states and itinerant conduction states (Eu 5d, Pd 4d, and Si 3p) where the interplay of the electronic bandwidth, crystal field environment, Coulomb repulsion, Hund's coupling and spin-orbit coupling plays a crucial role for the transition to happen. The change in the electronic structure is intimately related to the volume reduction where Eu-Pd(Si) bond lengths shorten. In a next step we compare our DFT results to surface-sensitive photoemission data in which the mixed-valent properties of EuPd<sub>2</sub>Si<sub>2</sub> are reflected in a simultaneous observation of divalent and trivalent signals from the Eu 4f shell.

## I. INTRODUCTION

For decades, 4f electron systems have attracted much attention due to the realization of a large variety of interesting phenomena, such as the Kondo effect and emergence of heavy fermion features, quantum criticality, unconventional superconductivity, exotic magnetism, nontrivial topological phases, or valence transitions, to mention a few [1–8]. The latter have been notably investigated in Eu-based systems where Eu can attain two possible valence states; divalent Eu<sup>2+</sup>( $4f^7$ ) and trivalent Eu<sup>3+</sup>( $4f^6$ ). In divalent Eu<sup>2+</sup>, following the Hund's rule in a *LS* description, seven electrons fill the *f* states with a total orbital angular momentum *L*=0 and a spin momentum  $S=\frac{7}{2}$ , giving rise to a total angular momentum  $J=\frac{7}{2}$ . In trivalent Eu<sup>3+</sup>, L=S=3 and J=0.

Upon lowering the temperature a smooth change of the valency from a non-integer value close to a magnetic  $Eu^{2+}$  state to a non-integer value close to nonmagnetic  $Eu^{3+}$  state has been observed, for instance, in tetragonal  $EuPd_2Si_2$  [9],  $EuCu_2Si_2$  [10, 11], and  $EuIr_2Si_2$  [12, 13]. Alternatively, such valence transitions have been also reported under application of pressure in tetragonal antiferromagnetic (AFM)  $EuRh_2Si_2$  [14],  $EuNi_2Ge_2$  [15], and  $EuCo_2Ge_2$  [16]. While the valence transition is expected to be related to structural and chemical bonding changes between localized 4f and the more itinerant s, p and delectrons under temperature or pressure effects, a full understanding of the transition mechanism is still lacking. We focus here on the valence transition in  $EuPd_2Si_2$ .

The valence transition in mixed-valence tetragonal  $EuPd_2Si_2$  was first reported in 1981 in Ref. [9] where a kink in the magnetic susceptibility was observed when lowering the temperature from 200 K to 150 K [9, 17, 18], accompanied by an increase in the Eu average valency [9, 19, 20]. The Curie-Weiss moment of  $EuPd_2Si_2$  at high

temperatures was reported to be 8.04  $\mu_B$  [18], which is very close to the theoretically calculated value of 7.94  $\mu_B$ for the divalent state  $(J = \frac{7}{2})$ . It was shown by means of X-ray absorption spectroscopy measurements that at a temperature of about 200 K EuPd<sub>2</sub>Si<sub>2</sub> enters into a valence crossover regime with the mean Eu valency changing smoothly from 2.3 to 2.8 between 200 and 100 K [19]. Interestingly, within the valence crossover the lattice parameter a=4.24 Å at room temperature reduces below 50 K to a=4.16 Å, whereas c remains unchanged [17, 20, 21] (Fig. 1(a)). The authors of Ref. [22] argued that the valence transition in EuPd<sub>2</sub>Si<sub>2</sub> would also occur under application of an external pressure of about 2 GPa, when a reduces to about 4.16 Å, which is similar to the value of a in the low-temperature regime. Besides, isostructural and isovalent EuPd<sub>2</sub>Ge<sub>2</sub> which has a bigger unit cell volume than EuPd<sub>2</sub>Si<sub>2</sub> doesn't undergo a valence transition at low temperatures but an AFM transition at  $T_{\rm N} \approx 17 \ {\rm K}$  [5]. Actually, the tunability of the Eu valence state by applying chemical and finite hydrostatic pressure was reported in single crystals of  $\operatorname{EuPd}_2(\operatorname{Si}_{1-x}\operatorname{Ge}_x)_2$  [23]. At x=0.2 the long-range AFM order of Eu moments observed below  $T_{\rm N} \approx 47$  K is suppressed by the application of hydrostatic pressure of 0.1 GPa inducing an intermediate valence state in Eu. All these observations point to the important role of the structural changes under temperature or pressure affecting the Eu valency.

In this work we investigate the microscopic origin of the valence transition in EuPd<sub>2</sub>Si<sub>2</sub> by a combination of density functional theory (DFT) based calculations and photoemission spectroscopy measurements and show that the transition is a result of an enhanced hybridization between localized Eu 4f states and itinerant conduction c states (Eu 5d, Pd 4d, and Si 3p). Moreover, we find that the valence transition is dictated by a delicate interplay of electronic bandwidth, Coulomb repulsion, Hund's

coupling and spin-orbit coupling effects upon volume reduction.

The paper is organized as follows. In Section II we describe the methods used for our study. In Section III we present a comparative analysis of structural details in mixed-valence tetragonal Eu compounds  $EuTM_2X_2$  where TM denotes the transition metal ion and X = Si, Ge. In Section IV we present our results on the electronic properties of  $EuPd_2Si_2$  for bulk and slab calculations and compare to experimental photoemission measurements. Finally in Section V we present our conclusions.

### II. METHODS

We performed DFT calculations using the fullpotential all-electron codes WIEN2k [25], and FPLO [26, 27]. The former code considers a linear augmented plane wave basis to solve the Kohn-Sham equations, while the latter is based on a local-orbital minimum-basis. The exchange-correlation functional was treated within the local (spin) density approximation [L(S)DA] in both WIEN2k and FPLO codes. Crystal bulk structures of EuPd<sub>2</sub>Si<sub>2</sub> were fully relaxed in the tetragonal space group I4/mmm within LDA using the *open-core* approximation as implemented in FPLO until forces were smaller than 1 meV/Å. In this approximation the Eu 4f states are



FIG. 1. (a) Crystal structure of tetragonal  $EuPd_2Si_2$ , produced by VESTA [24]. (b) First Brillouin zone of the tetragonal I4/mmm (black, solid) and P4/mmm (blue, dashed) structure with special k points for the band structure. The (001) surface was projected in the slab calculations, which is drawn by a pink rectangle. (c) Fully relaxed lattice parameters of  $EuPd_2Si_2$  at each Eu valency obtained using the *open-core* approximation within LDA.

removed from the valence basis and enter the bulk description as core orbitals, while fixing the mean 4f occupancy n to a given value. The value of n was considered in steps of 0.1 from 6 to 7 in our structural relaxation. Among various n values, we present results for n=6.7 $(Eu^{2.3+})$  as an optimized bulk structure at room temperature, and n=6.2 (Eu<sup>2.8+</sup>) as an optimized structure at low temperature (below 30 K), following the suggested mean valencies from experimental reports in Ref. [9, 19]. The  $12 \times 12 \times 12$  k-mesh was adopted for atomic position relaxations, while a dense k-mesh of  $21 \times 21 \times 21$  was used for accurate total energy calculations to determine the energetically stable structure. All relaxed lattice parameters obtained at each n value in EuPd<sub>2</sub>Si<sub>2</sub> are shown in Fig. 1(c), which agree well with the experimentally observed lattice parameters in dependence of the Eu valence states [17, 20, 21]. Specifically we mention the fully relaxed lattice parameters are a=4.214 Å and c=9.895 Å at n=6.7 (Eu<sup>2.3+</sup>), and a=4.128 Å and c=9.886 Å at n=6.2 (Eu<sup>2.8+</sup>). For surface-sensitive electronic structure calculations, we constructed Eu-terminated  $1 \times 1 \times 4$ slab structures of  $EuPd_2Si_2$  with a vacuum layer of 15 Å using our optimized bulk structure at room temperature and then relaxed the atomic positions of the four layers close to the surface using the *open-core* approximation with n=6.7 for Eu 4f within LDA in FPLO.

All electronic and magnetic structures were calculated with WIEN2k including spin-orbit coupling (SOC) and correlation effects  $(U, J_H)$  so as to deal with the localized nature of Eu 4f orbitals. We fixed U=6 eV and the Hund's coupling  $J_H=1$  eV for the Eu atom. The kmesh sampling was  $17 \times 17 \times 17$  for the bulk states, and  $17 \times 17 \times 1$  for the slab structure. The size of the basis set was determined by the value of  $R_{mt}K_{max}=9.0$  with the muffin-tin radius of 2.5(Eu), 2.4(Pd), 1.95(Si), and 2.25(Ge) in atomic units.

In order to describe divalent states  $(Eu^{2+})$  in DFT, the inclusion of spin degrees of freedom in spin-polarized calculations was taken into account for simulation purposes, even though EuPd<sub>2</sub>Si<sub>2</sub> at room temperature doesn't magnetically order. We assumed ferromagnetic (FM) spin order with an easy axis parallel to the z-axis. In addition, for the comparison of the electronic structures between EuPd<sub>2</sub>Si<sub>2</sub> and EuPd<sub>2</sub>Ge<sub>2</sub>, an A-type AFM spin order where the Eu magnetic ions are ferromagnetically aligned within the *ab* plane and antiferromagnetically aligned between consecutive planes was set. The corresponding magnetic space group is  $P_I 4/mnc$ (No.128.410). For these calculations, we also fully relaxed the  $EuPd_2Ge_2$  crystal bulk structures as we did in EuPd<sub>2</sub>Si<sub>2</sub>. The fully relaxed lattice parameters in  $EuPd_2Ge_2$  at n=7 are a=4.344 Å and c=10.217 Å which are also in good agreement with experiment  $(a_{exp}=4.3764)$ Å and  $c_{exp} = 10.072$  Å) [5].

Single crystals of EuPd<sub>2</sub>Si<sub>2</sub> were grown using the Czochralski method according to the procedure described in [21]. Angular-resolved photoemission measurements (ARPES) on the (001) surface were performed at the  $1^3$ 



FIG. 2. Classification of tetragonal Eu compounds  $EuTM_2X_2$  according to their experimentally reported Eu-TM and Eu-X bond lengths, where TM = Fe, Co, Ni, Cu, Ru, Rh, Pd, Ag, Ir, Au, and X = Si, Ge. Data were obtained from, respectively, TM/X = Ni/Si [30], Co/Si [31], Fe/Si [32], Cu/Si [33], Co/Ge [34], Ir/Si [13], Rh/Si [35], Pd/Si [36], Ir/Ge [37, 38], Ru/Ge [37, 39], Cu/Ge [40, 41], Ni/Ge [32, 34], Rh/Ge [39, 42], Au/Si [43, 44], Ag/Si [43, 45]. Systems with divalent  $Eu^{2+\delta}$  states at low temperatures are shown by red circles, whereas purple triangles denote trivalent  $\mathrm{Eu}^{3-\delta}$  compounds. Green squares indicate compounds which undergo a valence transition by varying temperature. Brown rhombuses indicate bond lengths of relaxed EuPd<sub>2</sub>Si<sub>2</sub> at a given volume with respect to the relaxed  $one(V_o)$  calculated for n=6.7 (see Methods section). Below  $0.93V_o$ , bond lengths become similar to those of trivalent Eu compounds.

ARPES instrument at BESSY II [28]. To prepare a clean surface the samples were cleaved in-situ under ultra-high vacuum conditions at a temperature of 41 K.

# **III. STRUCTURAL DETAILS**

EuPd<sub>2</sub>Si<sub>2</sub> crystallizes in a tetragonal body-centered ThCr<sub>2</sub>Si<sub>2</sub>-type [29] structure with space group I4/mmm (No.139) [17]. It consists of layers of edge-sharing PdSi<sub>4</sub> tetrahedra intercalated between Eu planes, as shown in Fig. 1(a). Eu, Pd and Si are at Wyckoff positions 2a, 4d, and 4e ( $z_{\rm rel}$ =0.3779 for Eu<sup>2.3+</sup> and 0.3818 for Eu<sup>2.8+</sup>) respectively). Similar to Si, Ge in EuPd<sub>2</sub>Ge<sub>2</sub> also sits on the 4e site with  $z_{\rm rel}$ =0.3715 for Eu<sup>2+</sup>.

The valence transition in EuPd<sub>2</sub>Si<sub>2</sub> is accompanied by a volume contraction where the lattice parameter *a* shrinks from a=4.24 Å at room temperature to a=4.16 Å below 50 K. In order to study the relation between volume contraction (and corresponding atomic bond-lengths shortening) with the valence transition, we collected in Fig. 2 crystal information of ThCr<sub>2</sub>Si<sub>2</sub>-type tetragonal Eu-based compounds EuTM<sub>2</sub>X<sub>2</sub> whose valence states are confirmed experimentally. Fig. 2 illustrates two kinds of bond lengths between Eu and the transition metal (TM) ion, and between Eu and the carbon-group (X) ion, respectively. Red circles indicate tetragonal Eu compounds that show magnetic ground states (e.g. with divalent  $\operatorname{Eu}^{2+\delta}$ ), whereas purple triangles are nonmagnetic ( $\operatorname{Eu}^{3-\delta}$ ) compounds. Systems undergoing a valence transition when temperature decreases are marked by green squares. The following compounds undergo a pressure-induced valence transition at pressures of about 1 GPa for EuRh<sub>2</sub>Si<sub>2</sub> [14], 2 GPa for EuNi<sub>2</sub>Ge<sub>2</sub> [15], and 3 GPa for EuCo<sub>2</sub>Ge<sub>2</sub> [16]. Furthermore, recent surface-sensitive photoemission experiments on tetragonal EuIr<sub>2</sub>Si<sub>2</sub>, marked by a green square in Fig. 2, reveal divalent Eu<sup>2+</sup> states in the surface region with two-dimensional ferromagnetic order at low temperatures, while in the bulk Eu is almost trivalent and has a nonmagnetic ground state. [46, 47].

Interestingly, EuCu<sub>2</sub>Si<sub>2</sub>, denoted by a green square in Fig. 2, has been reported to show a valence transition by lowering temperature [10, 11], however, while Ref. [48] reported the appearance of Eu antiferromagnetism at 10 K in single crystals, the authors of Ref. [49] suggested that the appearance of volume contractions and corresponding change of Eu valence states at low temperatures originate from the crystallization method and crystal defects. These authors confirmed the presence of trivalent states on a single crystal EuCu<sub>2</sub>Si<sub>2</sub> at low temperatures. Actually, the single crystal from Ref. [48] has a larger volume by about 3 % compared to the samples showing trivalent states [49]. Recently, it was found experimentally that bond lengths and valence transition temperature in  $EuPd_2Si_2$  can also change depending on the amount of disorder in the Pd-Si layer [21].

Summarizing the above observations, the Eu valence state in these tetragonal Eu-based compounds is intimately related to the value of Eu-TM and Eu-X bond lengths in the systems. In what follows we concentrate on EuPd<sub>2</sub>Si<sub>2</sub> and analyze via *ab initio* DFT the Eu valence transition under volume reduction.

# IV. ELECTRONIC STRUCTURE

#### A. Calculations for bulk EuPd<sub>2</sub>Si<sub>2</sub>

We start by examining the description of Eu 4f states in the context of competing Coulomb interaction, crystal field environment and spin-orbit coupling at ambient pressure. As it is known from atomic physics [50], the description of the electronic structure of an atom depends on the hierarchy of the involved interactions. In the case that the Coulomb interaction between the electrons is stronger than the spin-orbit interactions in each of them, then the total angular momentum  $L = \sum_i l_i$  and total spin  $S = \sum_i s_i$  of the electron system, where  $l_i$  and  $s_i$ are, respectively, the angular momentum and spin of each individual electron, couple to a total J. This situation corresponds to the LS description. If, however, the individual electron coupling via the spin-orbit interaction is stronger than the Coulomb interaction between electrons  $U_{ee}$ , then the individual total momenta  $j_i = l_i + s_i$ , couple to a total  $J = \sum_{i} j_i$  which corresponds to a jjdescription. Usually the former description is valid for light atoms, while the latter is more appropriate for heavy atoms. In 4f systems  $U_{ee}$  is usually larger than the SOC constant  $\xi_{SOC}$  and the LS coupling scheme may be more appropriate [51]. Actually, in a LS description  $Eu^{2+}$  $(4f^7)$  has S = 7/2, L = 0 and a total J = 7/2. Alternatively, in the jj description  $j_i$  with i = 1, 2, ..., 7 can take values 5/2 and 7/2 and the total J in the ground state is then J = 7/2 as well. Analogously Eu<sup>3+</sup> (4f<sup>6</sup>) in a LS description has S = L = 3 and J = 0. Considering the jj description,  $j_i$ , i = 1, 2, ..., 6 it results in J = 0. The differences between the two schemes is perceived when investigating the magnetic moments [51]  $M = g_J \mu_B J$ where the Landé factor  $g_J$  corresponds to the electron gyromagnetic factor  $g_J = 2$  in the LS scheme, while in the jj coupling  $g_J = 8/7$  for Eu<sup>2+</sup>. These nuances in the description are important when comparing the calculated magnetic properties to experiment.

Now we analyze the limiting case of the relaxed structure obtained for n = 6.7 (see Methods section) corresponding to EuPd<sub>2</sub>Si<sub>2</sub> at ambient pressure and ambient temperature [19]. Fig. 3(a) shows the LSDA+SOC+Ucalculated electronic structure and corresponding orbital projected densities of states (DOSs) for EuPd<sub>2</sub>Si<sub>2</sub> where we assumed a FM configuration for Eu. Due to the half-filled shell ( $4f^7$ ), the fully occupied 4f bands in the majority spin channel (colored pink in Fig. 3(a)) are centered around -1.0 eV below the Fermi level with a bandwidth of about 1 eV. Note that the energy of the calculated 4f states corresponds to the energy position of the Eu<sup>2+</sup> final-state multiplet seen in photoemission, Fig. 5(b).

Furthermore, in this energy range, itinerant conduction states of Pd 4d, Si 3p, and Eu 5d are present, which hybridize with the half-filled Eu 4f bands. Specifically, there is a hole pocket with dominant Eu 5d character at the  $\Gamma$  point, as well as electron pockets with dominant Pd 4d and Si 3p characters at around the X and P points. The calculated total spin moment  $(M_S)$  is 6.95  $\mu_B/\text{Eu}$ with a negligible angular moment of  $-0.024 \ \mu_B/\text{Eu}$ , which is in good agreement with the values of  $S = \frac{7}{2}$ and L = 0 expected from the Hund's rule in the divalent  $Eu^{2+}$  state in the LS scheme. Specifically, the occupation numbers of each orbital are 6.716 for Eu 4f, 8.016 for Pd 4d, 0.815 for Si 3p, and 0.331 for Eu 5d orbitals. In contrast to EuPd<sub>2</sub>Si<sub>2</sub>, EuPd<sub>2</sub>Ge<sub>2</sub> doesn't undergo a valence transition at low temperatures but an AFM transition at  $T_{\rm N} = 17 \, {\rm K}$  [5]. This is directly related to the fact that Eu-Pd / Eu-Ge bond lengths in EuPd<sub>2</sub>Ge<sub>2</sub> are longer than Eu-Pd / Eu-Si bond lengths in EuPd<sub>2</sub>Si<sub>2</sub> and, following Fig. 2,  $Eu^{2+}$  states are expected. Our relaxed bond lengths of Eu-Pd /Eu-Ge in EuPd<sub>2</sub>Ge<sub>2</sub> differ by 3.2 / 3.9% from those of Eu-Pd / Eu-Si in EuPd<sub>2</sub>Si<sub>2</sub>. Figure 3(d) illustrates band structures and Eu 4f orbital projected DOS of A-AFM EuPd<sub>2</sub>Si<sub>2</sub> (black) and EuPd<sub>2</sub>Ge<sub>2</sub> (cyan) within LSDA+SOC+U. Due to the longer bond

lengths, 4f states in EuPd<sub>2</sub>Ge<sub>2</sub> near the Fermi level are less dispersive and have a smaller bandwidth than those of EuPd<sub>2</sub>Si<sub>2</sub>.

We consider now the relaxed EuPd<sub>2</sub>Si<sub>2</sub> structure at n=6.2 (see Methods section). To obtain the electronic structure we performed calculations within LSDA+SOC+U ( $U = 6 \text{ eV}, J_{\text{H}} = 1 \text{ eV}$ ), with zero initial magnetization for the Eu atom. Both combined effects, SOC and correlation (U) shift down the occupied sixfold  $|j = \frac{5}{2}\rangle$  states to about 6.4 eV below the Fermi level and show almost no dispersion, whereas the empty eightfold  $|j = \frac{7}{2}\rangle$  states are located around 1.6 V above the Fermi level, not shown here. With the choice of Uand  $J_{\rm H}$  values above, the energy position of the occupied  $|j = \frac{5}{2}\rangle$  states coincides with the position of the trivalent Eu states in photoemission measurements (compare to Fig. 5(b)). Note however that in Fig. 5(b), due to the multiplet nature of trivalent Eu, two separated main peaks are observed at about 7 eV below the Fermi level. This feature is not captured in the DFT calculations, due to the limitations of the method to describe manybody multiplet states.

# B. Role of volume reduction on the hybridization

We analyze now the effect of volume reduction on the valence transition from divalent Eu to trivalent Eu in  $EuPd_2Si_2$ . As mentioned above, this system undergoes a valence transition by lowering temperature or by increasing pressure [9, 22]. Both effects reduce the volume in  $EuPd_2Si_2$ , implying a shortening of the Eu-Pd and Eu-Si bond lengths.

To investigate how the volume reduction affects the electronic structure and the valence state of EuPd<sub>2</sub>Si<sub>2</sub>, we obtained a few bulk EuPd<sub>2</sub>Si<sub>2</sub> structures with a smaller volume than our fully relaxed one  $(V_o)$  at n = 6.7by relaxing them using the *open-core* approximation as implemented in FPLO within LDA. Our fully relaxed lattice parameters at n=6.7 (Eu<sup>2.3+</sup>), which correspond to the room temperature bulk structure at ambient pressure are a=4.214 and c=9.895 Å as given in the Methods section with bond lengths of Eu-Pd=3.25 Å and Eu-Si=3.22 Å. Note that the Eu-Si bond length is in good agreement with our value determined in [21]. These bond lengths are marked by a brown rhombus with  $V_o$  in Fig. 2. These values are slightly larger than the reported ones in the literature [36]. Also plotted in Fig. 2 are the results for relaxed structures at a volume of  $0.97V_o$ ,  $0.93V_o$ , and  $0.88V_o$ . We note that the lattice parameter a for the crystal structure relaxed at  $0.97 V_o$  is similar to the experimentally reported one at a temperature below 30 K and 0 GPa when the system has undergone the valence transition and has experienced a volume contraction. The values of bond lengths are in the same range as those for compounds that undergo a valence transition (marked by green squares in Fig. 2).

Fig. 3(a) and (b) show the fat band electronic struc-



FIG. 3. Fat band electronic structures and orbital projected DOSs obtained from LSDA+SOC+U for EuPd<sub>2</sub>Si<sub>2</sub> (a-c) and EuPd<sub>2</sub>Ge<sub>2</sub> (d). (a) FM band structures for EuPd<sub>2</sub>Si<sub>2</sub> at V<sub>o</sub> (n = 6.7 relaxed structure, see Methods section) and (b) at a volume of 0.88V<sub>o</sub>. At V<sub>o</sub> in (a), the occupied spin-up Eu 4f bands in the FM results are located just below the Fermi level. The energy position of the Eu 4f states is in good agreement with photoemission results, as shown in Fig. 5(b). At 0.88V<sub>o</sub>, the c-f hybridization is enhanced leading to a larger bandwidth of 4f bands in (b). The weight of Eu 4f band characters (a-b) was scaled by a factor of 0.25 for clarity. (c) Overlapped band structures and Eu 4f orbital projected DOSs within LSDA+SOC+U for EuPd<sub>2</sub>Si<sub>2</sub> at V<sub>o</sub> (black color) and at 0.88V<sub>o</sub> (light red color). The volume reduction in EuPd<sub>2</sub>Si<sub>2</sub> with shorter bond lengths induces an enhanced c-f hybridization leading to more dispersive bands and a larger bandwidth of Eu 4f. (d) Overlapped band structures and EuPd<sub>2</sub>Ge<sub>2</sub> (cyan color) at V<sub>o</sub> both calculated in the A-AFM state. Longer bond lengths in EuPd<sub>2</sub>Ge<sub>2</sub> lead to more localized Eu 4f states.

ture and orbital projected DOSs of FM  $EuPd_2Si_2$  in LSDA+SOC+U at a volume of  $V_o$  and  $0.88V_o$ , respectively. The majority 4f bands are located just below the Fermi level. Compared to the  $V_o$  results, at  $0.88V_o$  the 4f bandwidth is somewhat larger and more dispersive with an enhanced c-f hybridization. This can be better observed in Fig. 3(c) where we superimposed both electronic structure contributions. Specifically, a reduction of volume leads to a decrease of the occupation number of Eu 4f states, whereas the occupation number in the other orbitals goes up, as shown in Fig. 4 where we display the occupation of Eu 4f, Pd 4d, Si 3p, and Eu 5dstates as a function of volume. This is directly related to a shortening of the Eu-TM and Eu-X bond lengths, (see brown rhombuses in Fig. 2) and an enhanced hybridization between Eu 4f and the itinerant conduction states (Eu 5d, Pd 4d, and Si 3p). Further, under volume

reduction the total spin moment decreases, whereas the orbital moment increases in magnitude. (See Fig. 4(b)) At 0.88V<sub>o</sub>, the orbital moment of Eu becomes  $-0.152 \mu_B/\text{Eu}$  which is a consequence of a slight decrease of the occupation of a mostly  $|3,3\rangle$  state, whereas the occupation of the other states remains nearly unchanged. Furthermore, the orbital moment increases to  $-0.228 \mu_B/\text{Eu}$  at 0.82V<sub>o</sub>. This behavior shows a tendency to follow the Hund's rules for Eu<sup>3+</sup> which is L=S=3 and J=|S-L|=0.

We conclude this section by the observation that an LS scheme is more suitable to describe the electronic structure of EuPd<sub>2</sub>Si<sub>2</sub> with Eu in the nearly divalent state, while in the case of Eu in a nearly trivalent state corresponding to the reduced volume case, the jj coupling scheme seems more appropriate, as discussed in Ref. [51]. The reason for that is the relative changes in the ra-



FIG. 4. (a) Occupation number of each orbital as a function of volume reduction of  $\text{EuPd}_2\text{Si}_2$  within LSDA+SOC+Uwhere  $V_o$  corresponds to the structure relaxed with n = 6.7(see Methods section). When the volume is reduced, the number of occupied Eu 4f states decreases, whereas there is an increase in the other states. (b) Corresponding total spin moment and orbital moment of Eu as a function of volume compression. A decrease of volume results in a decrease of the total spin moment, whereas the orbital moment of Eu increases in magnitude.

tio  $\xi_{SOC}/U_{ee}$  [51] where  $\xi_{SOC}$  is the spin-orbit coupling strength and  $U_{ee}$  the Coulomb interaction. This ratio increases at reduced volumes, as happens while lowering temperature, due to an increased c-f hybridization.

## C. Photoemission from the 4f shell

Recently, some of us [46] reported two-dimensional ferromagnetism at a temperature below 48 K in a single Eu layer located below the iridium-silicide surface of tetragonal  $EuIr_2Si_2$ , which shows a temperature-driven valence crossover (marked by a green square in Fig. 2), whereas bulk regions display no magnetism due to the presence of  $Eu^{3+}$ . In a later experiment, surface ferromagnetism was observed at the Eu-terminated surface of  $EuIr_2Si_2$  as well [47]. Fig. 5(b) shows an ARPES spectrum acquired from the (001) surface of a freshly cleaved EuPd<sub>2</sub>Si<sub>2</sub> single crystal at a temperature of 41 K. Although the compound was first synthesized decades ago, only recently large single crystals are available [21], enabling ARPES measurements. On the right-hand side the angle-integrated spectrum is plotted. To maximally enhance the emission from the 4f shell over contributions from the valence band we used a photon energy of 145 eV which corresponds to the maximum of the  $4d \rightarrow 4f$  Fano-Beutler resonance of Eu<sup>3+</sup>. Note that at the given photon energy the 4f emission of  $Eu^{2+}$  is resonantly enhanced too. In the spectrum, three dominating non-dispersive 4f features are present: (1) The straight line at the Fermi level represents the  $4f^7 \rightarrow 4f^6$  final-state multiplet of Eu<sup>2+</sup> in bulk-like layers: (2) The most intense line at a slightly higher binding energy of about 1 eV is the surface-core-level shifted 4f emission of Eu<sup>2+</sup> at the surface; (3) The broad structure consisting of several lines between 6 and 10 eV forms the  $4f^6 \rightarrow 4f^5$  final-state multiplet of  $Eu^{3+}$  in bulk-like layers. The simultaneous observation of both the  $Eu^{2+}$  and  $Eu^{3+}$  final-state multiplets reflects the mixed-valent properties of Eu in this compound.

#### D. Slab calculations

To understand the observations from surface-sensitive ARPES measurements presented in the previous section, we constructed a  $1 \times 1 \times 4$  Eu-terminated slab geometry as described in the Methods section. Due to the symmetric geometry, where the space group of this slab structure is P4/mmm, there are five inequivalent Eu atoms in the unit cell. Numbering these Eu atoms in relation to proximity to the surface, they are represented by Eu1 through Eu5 with Eu1 at the surface and Eu5 situated furthest from the surface *i.e.* located at the center of the unit cell. For the calculation of the electronic structure, we considered FM configurations of these Eu atoms as in our bulk calculations. Figure 5(a) illustrates the (001)-projected band structure and Eu 4f orbital resolved DOS of the slab structure described above for the FM spin configuration in LSDA+SOC+U. All nearly divalent Eu 4f states appear at around 1 eV below the Fermi level. Interestingly, 4f states of Eu2, Eu3, Eu4, Eu5 states belonging to the bulk are much closer to the Fermi level than those of Eu1 at the surface. In this case, the on-site energy of the 4f states was calculated as -1.31 eV for Eu1 and about -0.8 eV for the rest of Eu. This result is in good agreement with our PES results, shown in Fig. 5(b), where a peak from  $Eu^{2+}(surface)$  is farther from the Fermi level than that from bulk. We would like to stress that these separated 4f states are a result of the slab geometry and the fact that the bond lengths of Eu-Pd(Eu-Si) are different between Eu1 and Eu2, Eu3, Eu4, Eu5

# V. CONCLUSIONS

In this work we have investigated the mechanism of the valence transition in the testbed system  $EuPd_2Si_2$  which is known to undergo a valence transition from nearly divalent Eu to nearly trivalent Eu upon lowering the tem-



FIG. 5. (a) LSDA+SOC+U electronic band structure and 4f orbital projected DOS of the Eu-terminated  $1 \times 1 \times 4$  slab structure with a spin configuration where all Eu atoms were set to be of FM order. (b) ARPES spectrum acquired with  $h\nu = 145$  eV at a temperature of 41 K from the (001) surface of EuPd<sub>2</sub>Si<sub>2</sub> for a mixture of Si- and Eu-terminated areas, that shows the Eu 4f emission. On the right the angle-integrated spectrum is given.

perature. By making use of the observation that the valence transition is accompanied by a volume contraction, we studied the valence transition by a combination of (i) density functional theory calculations where we considered volume contracted structures, and (ii) photoemission measurements taken at  $T = 41 \,\mathrm{K}$  with  $h\nu = 145$  eV. Our results show that a decrease of Eu-Pd(Si) bond lengths resulting from a volume contraction induces an enhanced c-f hybridization between localized Eu 4f states and itinerant conduction states (Eu 5d, Pd 4d, and Si 3p). As the bond lengths get shorter the occupation number of the Eu 4f states decreases whereas that of the conduction states increases. We conclude that the redistribution of electronic weight and Eu valence crossover is dictated by a delicate interplay of the electronic bandwidth, crystal field environment, Coulomb repulsion, Hund's coupling and spin-orbit coupling effects.

Further, our DFT  $EuPd_2Si_2$  bulk and Eu-terminated slab results are in good agreement with our surface sensitive photoemission experiments reproducing the presence of divalent Eu states near the Fermi level coming from surface/bulk Eu and of trivalent Eu states at high binding energies developing from bulk Eu.

# VI. ACKNOWLEDGMENTS

We thank Daniel Khomskii, Michael Lang, Igor Mazin and Bernd Wolf for useful discussions. Y.-J. S., K.K., C.K. and R.V. acknowledge support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) for funding through TRR 288 – 422213477 (projects A03, A05). S.S. acknowledges DFG support through Grant No. KR3831/5-1. We thank the Helmholtz-Zentrum Berlin für Materialien und Energie for the allocation of synchrotron radiation beamtime.

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