Technical Appendix
Why Does the Schooling Gap Close while the Wage Gap Persists across Country Income Comparisons?

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A Introduction

The supplementary material details the mathematical derivations and computational procedure we have used to derive the results of the main article. Appendices B to F show how the equilibrium labor, production, and consumption choices of firms and households can be expressed as functions of the wage gap and schooling choices. Appendix G discusses the model's calibration strategy. Additionally, it provides the numerical and programming details of our implementation that can be used to easily replicate the results of the main text.

A.1 Notation

To facilitate the reader, we summarize the conventions we have used in our calculations. Superscripts are used to denote genders with m standing for *male* and f for *female*. Each variable can have up to two subscripts. The first subscript indicates the sector of a variable, which can take the values A for *agriculture*, M for *manufacturing*, and S for *services*. The second subscript denotes the production technology taking the values h for *traditional* and r for *modern* production technology.

Many expressions involve expressions of ratios between female and male variables. To simplify notation, all such ratios will be expressed using a single symbol instead of fractions. Specifically, the female to male ratio of any female and male arbitrary variables x^f, x^m is denoted as

$$\tilde{x} = \frac{x^f}{x^m}.$$

Similar shorthand notation is used for female to male ratios of variables mapped via functions. For any function f and x^f, x^m as before, we denote

$$\tilde{f} = \frac{f(x^f)}{f(x^m)}.$$

We use the function $\delta(t) = T - t$ to symbolize the remaining time horizon as of time t, and the function $d(t) = \int_t^T \mathrm{e}^{-\rho s} \mathrm{d}s$ for the discounter corresponding to time t.

B Firm Production

We begin with the derivation of the solution of a representative firm's maximization problem. For each sector $i \in \{A, M, S\}$, the firm solves

$$\max_{L_{ir}^f, L_{ir}^m} p_{ir} Z_{ir} L_{ir} - w^m \delta(s^m) H(s^m) L_{ir}^m - w^f \delta(s^f) H(s^f) L_{ir}^f$$

$$\text{s.t.}$$

$$L_{ir} = \left(\xi_{ir}^f \left(H(s^f) \delta(s^f) L_{ir}^f \right)^{\frac{\eta-1}{\eta}} + \xi_{ir}^m \left(H(s^m) \delta(s^m) L_{ir}^m \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

The labor production shares sum up to one, i.e. $\xi_{ir}^f + \xi_{ir}^m = 1$. We directly impose the market-clearing conditions and use c_{ir} and L_{ir} to symbolize output and labor demand; namely the same symbols we use for the household side in appendix C to symbolize consumption and labor supply.

The first order conditions with respect to L_{ir}^f and L_{ir}^m are

$$L_{ir}^f : w^f = p_{ir} Z_{ir} \xi_{ir}^f \left(\delta(s^f) H(s^f) \right)^{-\frac{1}{\eta}} \left(L_{ir}^f \right)^{-\frac{1}{\eta}} L_{ir}^{\frac{1}{\eta}} \tag{B.1}$$

$$L_{ir}^m : w^m = p_{ir} Z_{ir} \xi_{ir}^m \left(\delta(s^m) H(s^m) \right)^{-\frac{1}{\eta}} \left(L_{ir}^m \right)^{-\frac{1}{\eta}} L_{ir}^{\frac{1}{\eta}}. \tag{B.2}$$

Combining eqs. (B.1) and (B.2) gives

$$\frac{L_{ir}^m}{L_{ir}^f} = \left(\frac{\xi_{ir}^f}{\xi_{ir}^m} \right)^{-\eta} \left(\frac{w^f}{w^m} \right)^{\eta} \frac{\delta(s^f) H(s^f)}{\delta(s^m) H(s^m)} = \tilde{\xi}_{ir}^{-\eta} \tilde{w}^{\eta} \tilde{\delta} \tilde{H}. \tag{B.3}$$

The female wage bill in sector i is given by

$$I_{ir}^f = \frac{w^f \delta(s^f) H(s^f) L_{ir}^f}{w^f \delta(s^f) H(s^f) L_{ir}^f + w^m \delta(s^m) H(s^m) L_{ir}^m}.$$

Using eq. (B.3) to replace L_{ir}^m / L_{ir}^f in the female wage bill share gives

$$I_{ir}^f = \frac{1}{1 + \tilde{\xi}_{ir}^{-\eta} \tilde{w}^{\eta-1}}. \tag{B.4}$$

The male wage bill share can be analogously expressed as

$$I_{ir}^{m} = 1 - \frac{1}{1+\tilde{\xi}_{ir}^{-\eta}\tilde{w}^{\eta-1}} = \frac{1}{1+\tilde{\xi}_{ir}^{\eta}\tilde{w}^{1-\eta}}. \tag{B.5}$$

The wage bill shares in all sectors are independent of labor units and schooling years. Wage bill shares are determined by wage ratio $\tilde{w}$ and the labor production share ratio in sector i, $\tilde{\xi}_{ir}$.

At the profit maximizing allocation, the total effective labor units used in the production of i can be expressed as a function of female effective labor units, namely

$$\begin{aligned} L_{ir} &= \left(\xi_{ir}^{f}\left(\delta(s^f)H(s^f)L_{ir}^{f}\right)^{\frac{\eta-1}{\eta}} + \xi_{ir}^{m}\left(\delta(s^m)H(s^m)L_{ir}^{m}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\ &= \left(\xi_{ir}^{f}\left(\delta(s^f)H(s^f)L_{ir}^{f}\right)^{\frac{\eta-1}{\eta}} + \xi_{ir}^{m}\left(\delta(s^m)H(s^m)\tilde{\xi}_{ir}^{-\eta}\tilde{w}^{\eta}\tilde{\delta}\tilde{H}L_{ir}^{f}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \\ &= \delta(s^f)H(s^f)\left(\xi_{ir}^{f}(1+\tilde{\xi}_{ir}^{-\eta}\tilde{w}^{\eta-1})\right)^{\frac{\eta}{\eta-1}} L_{ir}^{f}. \end{aligned}$$

Combining with eq. (B.4) yields

$$L_{ir} = \left(\frac{\xi_{ir}^{f}}{I_{ir}^{f}}\right)^{\frac{\eta}{\eta-1}} \delta(s^f)H(s^f)L_{ir}^{f}, \tag{B.6}$$

while the analogous expression in terms of male effective labor units is

$$L_{ir} = \left(\frac{\xi_{ir}^{m}}{I_{ir}^{m}}\right)^{\frac{\eta}{\eta-1}} \delta(s^m)H(s^m)L_{ir}^{m}. \tag{B.7}$$

Equation (B.6) can be used to simplify eq. (B.1) to

$$\begin{aligned} w^f &= p_{ir}Z_{ir}\xi_{ir}^{f}\left(\delta(s^f)H(s^f)L_{ir}^{f}\right)^{-\frac{1}{\eta}}\left(\delta(s^f)H(s^f)L_{ir}^{f}\left(\xi_{ir}^{f}\right)^{\frac{\eta}{\eta-1}}\left(I_{ir}^{f}\right)^{\frac{\eta}{1-\eta}}\right)^{\frac{1}{\eta}} \\ &= p_{ir}Z_{ir}\left(\xi_{ir}^{f}\right)^{\frac{\eta}{\eta-1}}\left(I_{ir}^{f}\right)^{\frac{1}{1-\eta}}. \end{aligned} \tag{B.8}$$

With free labor mobility across sectors, wages equalize for each gender in equilibrium. We can combine eq. (B.8) for two sectors i, j to get

$$p_{jr}Z_{jr}\left(\xi_{jr}^{f}\right)^{\frac{\eta}{\eta-1}}\left(I_{jr}^{f}\right)^{\frac{1}{1-\eta}} = w^f = p_{ir}Z_{ir}\left(\xi_{ir}^{f}\right)^{\frac{\eta}{\eta-1}}\left(I_{ir}^{f}\right)^{\frac{1}{1-\eta}}.$$

Therefore, the relative prices of outputs produced using the modern production technology are given by

$$\frac{p_{ir}}{p_{jr}} = \frac{Z_{jr}}{Z_{ir}} \left(\frac{\xi^f_{jr}}{\xi^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ir}}{I^f_{jr}}\right)^{\frac{1}{\eta-1}}. \tag{B.9}$$

C Household Decisions

The optimization problem for the representative couple is

$$\max_{\{s^g,\ell^g,c_{ir},L^g_{ih}\}_{g,i}} \int_{t=0}^{T} \mathrm{e}^{-\rho t} \left(\log(c-\bar{c}) + \varphi\log(\ell) - \beta^f \mathbb{1}_{t\le s^f} - \beta^m \mathbb{1}_{t\le s^m}\right) \mathrm{d}t \tag{C.1}$$

s.t.

$$c = \left(\sum_{i\in\{A,M,S\}} \omega_i c_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{C.2}$$

$$c_i = \left(\psi_i(c_{ir})^{\frac{\sigma-1}{\sigma}} + (1-\psi_i)(c_{ih})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{C.3}$$

$$c_{ih} = Z_{ih}\left(\xi^f_{ih}(L^f_{ih})^{\frac{\eta-1}{\eta}} + \xi^m_{ih}(L^m_{ih})^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{C.4}$$

$$\ell = \left(\xi^f_l(\ell^f)^{\frac{\eta_l-1}{\eta_l}} + \xi^m_l(\ell^m)^{\frac{\eta_l-1}{\eta_l}}\right)^{\frac{\eta_l}{\eta_l-1}} \tag{C.5}$$

$$H = \exp\left(\frac{\zeta}{1-\nu}(s^g)^{1-\nu}\right), \qquad g = m, f \tag{C.6}$$

$$\int_{t=0}^{T} \mathrm{e}^{-\rho t}\left(\sum_i p_{ir}c_{ir}\right)\mathrm{d}t = \sum_g \int_{t=s^g}^{T} \mathrm{e}^{-\rho t}\left(w^g H(s^g)\left(L^g - \ell^g - \sum_i L^g_{ih}\right)\right)\mathrm{d}t. \tag{C.7}$$

The first order conditions are

$$c_{ir} : \frac{\partial U}{\partial c_{ir}} = \lambda d(0) p_{ir}, \qquad i\in\{A,M,S\} \tag{C.8}$$

$$L^g_{ih} : \frac{\partial U}{\partial c_{ih}}\frac{\partial c_{ih}}{\partial L^g_{ih}} = \lambda w^g d(s^g) H(s^g), \qquad i\in\{A,M,S\} \tag{C.9}$$

$$\ell^g : \frac{\partial U}{\partial \ell^g} = \lambda w^g d(s^g) H(s^g), \qquad g = f, m \tag{C.10}$$

$$s^g : -\beta^g \mathrm{e}^{-\rho s^g} = -\lambda w^g M^g \left(d(s^g)H'(s^g) + d'(s^g)H(s^g)\right), \qquad g = f, m \tag{C.11}$$

where

$$M^g = L^g - \sum_{i\in\{A,M,S\}} L^g_{ih} - \ell^g \tag{C.12}$$

denotes the hours of work in the modern production technology of gender g. Since labor markets clear in equilibrium allocations, we simplify by using the same symbols for labor choices of households and firms for the modern production technology.

C.1 Relative Prices

The consumption of the commodities and services produced with the traditional production technologies is not priced in the market. In equilibrium, we can combine the firm conditions obtained in appendix B with the substitution conditions between traditionally and modernly produced consumption induced by the household's decision problem to calculate implicit relative prices for traditionally produced output.

C.1.1 Relative prices of modern and traditional production outputs

For any sector $i \in \{A, M, S\}$ combining eq. (C.9) for females and males gives

$$\frac{\frac{\partial c_{ih}}{\partial L^f_{ih}}}{\frac{\partial c_{ih}}{\partial L^m_{ih}}} = \frac{w^f}{w^m}\frac{d(s^f)}{d(s^m)}\frac{H(s^f)}{H(s^m)} = \tilde{w}\tilde{d}\tilde{H}. \tag{C.13}$$

Since

$$\frac{\partial c_{ih}}{\partial L^f_{ih}} = Z_{ih}\xi^f_{ih}\left(L^f_{ih}\right)^{-\frac{1}{\eta}} L_{ih}^{\frac{1}{\eta}} \tag{C.14}$$

$$\frac{\partial c_{ih}}{\partial L^m_{ih}} = Z_{ih}\xi^m_{ih}\left(L^m_{ih}\right)^{-\frac{1}{\eta}} L_{ih}^{\frac{1}{\eta}}, \tag{C.15}$$

we get male labor hours in traditional production as a function of female hours

$$L^m_{ih} = \tilde{\xi}^{-\eta}_{ih}(\tilde{w}\tilde{d}\tilde{H})^\eta L^f_{ih}. \tag{C.16}$$

Substituting L^m_{ih} with the right-hand side expression of eq. (C.16) in the traditional production aggregator (i.e., eq. (C.4)) gives

$$L_{ih} = \left(\xi^f_{ih}\left(L^f_{ih}\right)^{\frac{\eta-1}{\eta}} + \xi^m_{ih}\left(\tilde{\xi}^{-\eta}_{ih}(\tilde{w}\tilde{d}\tilde{H})^\eta L^f_{ih}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}},$$

or

$$\frac{L_{ih}}{L^f_{ih}} = \left(\xi^f_{ih}\left(1+\tilde{\xi}^{-\eta}_{ih}(\tilde{w}\tilde{d}\tilde{H})^{\eta-1}\right)\right)^{\frac{\eta}{\eta-1}}.$$

To simplify the last expression, which is useful for the following calculations, we define an expression for the labor allocated to traditional production that is analogous to the wage bills of eqs. (B.4) and (B.5). Thus, let

$$I^f_{ih} := \frac{w^f d(s^f)H(s^f)L^f_{ih}}{w^f d(s^f)H(s^f)L^f_{ih} + w_m d(s^m)H(s^m)L^m_{ih}}. \tag{C.17}$$

Using eq. (C.16) to replace male by female hours in the above denominator yields

$$I^f_{ih} = \frac{w^f d(s^f)H(s^f)L^f_{ih}}{w^f d(s^f)H(s^f)L^f_{ih}\left(1+\tilde{w}\tilde{\xi}^{-\eta}_{ih}(\tilde{w}\tilde{d}\tilde{H})^{\eta}\tilde{d}\tilde{H}\right)},$$

so that

$$I^f_{ih} = \frac{1}{1+\tilde{\xi}^{-\eta}_{ih}\left(\tilde{w}\tilde{d}\tilde{H}\right)^{\eta-1}}. \tag{C.18}$$

The corresponding male expression is

$$I^m_{ih} = 1 - I^f_{ih} = \frac{1}{1+\tilde{\xi}^{\eta}_{ih}\left(\tilde{w}\tilde{d}\tilde{H}\right)^{1-\eta}}. \tag{C.19}$$

With the above definitions, we can express eq. (C.17) as

$$\frac{L_{ih}}{L^f_{ih}} = \left(\frac{\xi^f_{ih}}{I^f_{ih}}\right)^{\frac{\eta}{\eta-1}}. \tag{C.20}$$

Analogously to the optimality conditions for the modernly produced consumption c_{ir} in eq. (C.8), we require that the implicit prices of the traditionally produced output satisfy

$$\frac{\partial U}{\partial c_{ih}} = d(0)p_{ih}\lambda. \tag{C.21}$$

We can solve eq. (C.9) for the shadow price λ, i.e.,

$$\lambda = \frac{\partial U}{\partial c_{ih}} \frac{\partial c_{ih}}{\partial L_{ih}^f} \frac{1}{d(s^f)H(s^f)w^f},$$

and eliminate it using eq. (C.21) to get

$$\begin{aligned} \frac{\partial U}{\partial c_{ih}} \frac{1}{p_{ih} d(0)} &= \frac{\partial U}{\partial c_{ih}} \frac{\partial c_{ih}}{\partial L_{ih}^f} \frac{1}{d(s^f)H(s^f)w^f} \\ p_{ih} &= w^f \frac{d(s^f)H(s^f)}{d(0)} \left(\frac{\partial c_{ih}}{\partial L_{ih}^f} \right)^{-1}. \end{aligned} \tag{C.22}$$

Further, by eqs. (C.14) and (C.20), we have

$$\begin{aligned} \frac{\partial c_{ih}}{\partial L_{ih}^f} &= Z_{ih} \xi_{ih}^f \left(L_{ih}^f \right)^{-\frac{1}{\eta}} \left(\left(\frac{\xi_{ih}^f}{I_{ih}^f} \right)^{\frac{\eta}{\eta-1}} L_{ih}^f \right)^{\frac{1}{\eta}} \\ &= Z_{ih} \left(\xi_{ih}^f \right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{I_{ih}^f} \right)^{\frac{1}{\eta-1}} \end{aligned} \tag{C.23}$$

Using eq. (C.23), we can eliminate the derivative in eq. (C.22) to get

$$p_{ih} = w^f \frac{d(s^f)H(s^f)}{d(0)} \left(Z_{ih} \left(\xi_{ih}^f \right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{I_{ih}^f} \right)^{\frac{1}{\eta-1}} \right)^{-1},$$

or

$$w^f = p_{ih} Z_{ih} \left(\xi_{ih}^f \right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{I_{ih}^f} \right)^{\frac{1}{\eta-1}} \frac{d(0)}{d(s^f)H(s^f)}. \tag{C.24}$$

We combine eqs. (B.8) and (C.24), to obtain an expression for the relative price of modernly produced outputs (jr) with respect to the implicit price of traditionally produced outputs (ih). Specifically, for any combination of $i, j \in \{A, M, S\}$, we have

$$p_{ih} Z_{ih} \left(\xi_{ih}^f \right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{I_{ih}^f} \right)^{\frac{1}{\eta-1}} \frac{d(0)}{d(s^f)H(s^f)} = p_{jr} Z_{jr} \left(\xi_{jr}^f \right)^{\frac{\eta}{\eta-1}} \left(I_{jr}^f \right)^{\frac{1}{1-\eta}},$$

and, hence,

$$\frac{p_{ih}}{p_{jr}} = \frac{Z_{jr}}{Z_{ih}} \left(\frac{\xi^f_{jr}}{\xi^f_{ih}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ih}}{I^f_{jr}} \right)^{\frac{1}{\eta-1}} \frac{d(s^f)H(s^f)}{d(0)}. \tag{C.25}$$

C.1.2 Relative prices of traditional production outputs

We interject the price of any modernly produced output and use eq. (C.25) to obtain an expression for the relative prices of traditionally produced outputs. In particular, we have

$$\begin{aligned} \frac{p_{ih}}{p_{jh}} &= \frac{p_{ih}}{p_{kr}} \frac{p_{kr}}{p_{jh}} \\ &= \frac{Z_{kr}}{Z_{ih}} \left(\frac{\xi^f_{kr}}{\xi^f_{ih}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ih}}{I^f_{kr}} \right)^{\frac{1}{\eta-1}} \frac{d(s^f)H(s^f)}{d(0)} \frac{Z_{jh}}{Z_{kr}} \left(\frac{\xi^f_{jh}}{\xi^f_{kr}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{kr}}{I^f_{jh}} \right)^{\frac{1}{\eta-1}} \frac{d(0)}{d(s^f)H(s^f)}, \end{aligned}$$

which gives

$$\frac{p_{ih}}{p_{jh}} = \frac{Z_{jh}}{Z_{ih}} \left(\frac{\xi^f_{jh}}{\xi^f_{ih}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ih}}{I^f_{jh}} \right)^{\frac{1}{\eta-1}}. \tag{C.26}$$

Since human capital does not affect productivity in the traditional production technology, the expression of relative prices in eq. (C.26) is independent of the schooling choices s^f and s^m.

C.1.3 Relative prices of modern production outputs

For prices of modernly produced outputs, we calculate in a similar fashion

$$\frac{p_{ir}}{p_{jr}} = \frac{p_{ir}}{p_{kh}} \frac{p_{kh}}{p_{jr}} = \frac{Z_{jr}}{Z_{ir}} \left(\frac{\xi^f_{jr}}{\xi^f_{ir}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ir}}{I^f_{jr}} \right)^{\frac{1}{\eta-1}}. \tag{C.27}$$

Since human capital affects productivity in the modern production technology for both sectors, the effect is canceled out in the ratios and the expression of relative prices in eq. (C.26) is independent of the schooling choices s^f and s^m.

C.1.4 Relative prices of traditional production outputs and leisure

Combining the first order conditions with respect to ℓ^f and ℓ^m, i.e., eq. (C.5) for $g = f, m$, we have

$$\tilde{w}\tilde{d}\tilde{H} = \frac{w^f d(s^f)H(s^f)}{w^m d(s^m)H(s^m)} = \frac{\frac{\partial U}{\partial \ell^f}}{\frac{\partial U}{\partial \ell^m}} = \frac{\xi_l^f}{\xi_l^m}\left(\frac{\ell^m}{\ell^f}\right)^{\frac{1}{\eta_l}}, \tag{C.28}$$

or, equivalently,

$$\frac{\ell^m}{\ell^f} = \left(\tilde{\xi}_l^f\right)^{-\eta_l}(\tilde{w}\tilde{d}\tilde{H})^{\eta_l}. \tag{C.29}$$

We use the last expression to substitute the male leisure time in eq. (C.5) and rewrite aggregate leisure ℓ as

$$\ell = \left(\xi_l^f(\ell^f)^{\frac{\eta_l-1}{\eta_l}} + \xi_l^m\left(\left(\tilde{\xi}_l^f\right)^{-\eta_l}(\tilde{w}\tilde{d}\tilde{H})^{\eta_l}\ell^f\right)^{\frac{\eta_l-1}{\eta_l}}\right)^{\frac{\eta_l}{\eta_l-1}}. \tag{C.30}$$

To simplify the last expression, we define

$$I_l^f := \frac{w^f d(s^f)H(s^f)\ell^f}{w^f d(s^f)H(s^f)\ell^f + w^m d(s^m)H(s^m)\ell^m}, \tag{C.31}$$

which can be equivalently written as

$$I_l^f = \frac{1}{1 + \left(\tilde{\xi}_l^f\right)^{-\eta_l}(\tilde{w}\tilde{d}\tilde{H})^{\eta_l-1}}. \tag{C.32}$$

We can, then, shorten eq. (C.30) to

$$\ell = \ell^f\left(\frac{\xi_l^f}{I_l^f}\right)^{\frac{\eta_l}{\eta_l-1}}. \tag{C.33}$$

We obtain an expression for the implicit price of leisure following the approach with which we obtained expressions for the implicit prices of traditionally produced outputs. We start from

$$\frac{\partial U}{\partial \ell} = \lambda d(0)p_l, \tag{C.34}$$

rearrange, and use the derivative of eq. (C.10) to get

$$
\begin{aligned}
p_l &= \frac{\partial U}{\partial \ell} \frac{1}{\lambda d(0)} \\
&= \lambda w^f d(s^f) H(s^f) \left(\frac{\partial \ell}{\partial \ell^f} \right)^{-1} \frac{1}{\lambda d(0)} \\
&= w^f \frac{d(s^f) H(s^f)}{d(0)} \left(\frac{\partial \ell}{\partial \ell^f} \right)^{-1} .
\end{aligned} \tag{C.35}
$$

Using eq. (C.33), the derivative of eq. (C.5) with respect to female leisure hours is equal to

$$
\frac{\partial \ell}{\partial \ell^f} = \left(\xi_l^f \right)^{\frac{\eta_l}{\eta_l - 1}} \left(\frac{1}{I_l^f} \right)^{\frac{1}{\eta_l - 1}} , \tag{C.36}
$$

which can, then, be used to rewrite eq. (C.35) as

$$
w^f = p_l \left(\xi_l^f \right)^{\frac{\eta_l}{\eta_l - 1}} \left(\frac{1}{I_l^f} \right)^{\frac{1}{\eta_l - 1}} \frac{d(0)}{d(s^f) H(s^f)} \tag{C.37}
$$

Combining eqs. (C.24) and (C.37) we find that the price of leisure relative to the price of a traditionally produced output i is

$$
\frac{p_l}{p_{ih}} = Z_{ih} \left(\xi_{ih}^f \right)^{\frac{\eta}{\eta - 1}} \left(\frac{1}{\xi_l^f} \right)^{\frac{\eta_l}{\eta_l - 1}} \left(\frac{1}{I_{ih}^f} \right)^{\frac{1}{\eta - 1}} \left(I_l^f \right)^{\frac{1}{\eta_l - 1}} . \tag{C.38}
$$

C.1.5 Relative prices of modern production outputs and leisure

For the price ratio of leisure and modern production outputs, we have

$$
\frac{p_l}{p_{ir}} = \frac{p_l}{p_{ih}} \frac{p_{ih}}{p_{ir}} = Z_{ir} \left(\xi_{ir}^f \right)^{\frac{\eta}{\eta - 1}} \left(\frac{1}{\xi_l^f} \right)^{\frac{\eta_l}{\eta_l - 1}} \left(\frac{1}{I_{ir}^f} \right)^{\frac{1}{\eta - 1}} \left(I_l^f \right)^{\frac{1}{\eta_l - 1}} \frac{d(s^f) H(s^f)}{d(0)} .
$$

C.2 Marginal Rates of Substitution and Equilibrium Expenditures

There are three classes of marginal rates of substitution that are important in household decisions. These are exhausted by the combinations of modern output, traditional output, and leisure.

C.2.1 Marginal rate of substitution between traditional and modern output

For any sector i, we can calculate the marginal rate of substitution between traditionally and modernly produced outputs as

$$\text{MRS}_{irih} = \frac{\frac{\partial U}{\partial c_{ir}}}{\frac{\partial U}{\partial c_{ih}}} = \frac{\frac{\partial U}{\partial c}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ir}}}{\frac{\partial U}{\partial c}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ih}}} = \frac{\frac{\partial c_i}{\partial c_{ir}}}{\frac{\partial c_i}{\partial c_{ih}}} = \frac{\psi_i c_{ir}^{-\frac{1}{\sigma}} c_i^{\frac{1}{\sigma}}}{(1-\psi_i) c_{ih}^{-\frac{1}{\sigma}} c_i^{\frac{1}{\sigma}}} = \frac{\psi_i}{1-\psi_i}\left(\frac{c_{ir}}{c_{ih}}\right)^{-\frac{1}{\sigma}}.$$

In equilibrium, the MRS is equal to the relative prices. This can also be explicitly derived here by combining eqs. (C.8), (C.9) and (C.22). Thus, we get

$$\frac{\psi_i}{1-\psi_i}\left(\frac{c_{ir}}{c_{ih}}\right)^{-\frac{1}{\sigma}} = \frac{p_{ih}}{p_{ir}},$$

or, equivalently,

$$\frac{c_{ir}}{c_{ih}} = \left(\frac{p_{ih}}{p_{ir}}\right)^{\sigma}\left(\frac{\psi_i}{1-\psi_i}\right)^{\sigma}. \tag{C.39}$$

C.2.2 Relative expenditures of modern and traditional production for fixed-output type

We can use the equilibrium condition in eq. (C.39) and the results of appendix C.1 to obtain expressions of relative expenditures between modernly and traditionally produced outputs. Specifically,

$$E_{irih} = \frac{p_{ir}c_{ir}}{p_{ih}c_{ih}} = \left(\frac{p_{ih}}{p_{ir}}\right)^{\sigma-1}\left(\frac{\psi_i}{1-\psi_i}\right)^{\sigma}, \tag{C.40}$$

and from eq. (C.25), we get

$$E_{irih} = \left(\frac{Z_{ir}}{Z_{ih}}\left(\frac{\xi_{ir}^f}{\xi_{ih}^f}\right)^{\frac{\eta}{\eta-1}}\left(\frac{I_{ih}^f}{I_{ir}^f}\right)^{\frac{1}{\eta-1}}\frac{d(s^f)H(s^f)}{d(0)}\right)^{\sigma-1}\left(\frac{\psi_i}{1-\psi_i}\right)^{\sigma}.$$

For brevity, define

$$Z_{irih} := \frac{Z_{ir}}{Z_{ih}}\left(\frac{\psi_i}{1-\psi_i}\right)^{\frac{\sigma}{\sigma-1}},$$

so we can write the relative expenditure of modernly to traditionally produced output i as

$$E_{irih} = Z_{irih}^{\sigma-1} \left(\left(\frac{\xi_{ir}^{f}}{\xi_{ih}^{f}} \right)^{\frac{\eta}{\eta-1}} \left(\frac{I_{ih}^{f}}{I_{ir}^{f}} \right)^{\frac{1}{\eta-1}} \frac{d(s^f)H(s^f)}{d(0)} \right)^{\sigma-1} . \tag{C.41}$$

C.2.3 Marginal rate of substitution between distinct modern outputs

We can calculate the MRS between two distinct modernly produced outputs with some more intermediate steps. For any sector i, we start from eq. (C.3) and use eq. (C.39) to get

$$\begin{aligned} c_i &= \left(\psi_i c_{ir}^{\frac{\sigma-1}{\sigma}} + (1-\psi_i) c_{ih}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\ &= \left(\psi_i c_{ir}^{\frac{\sigma-1}{\sigma}} + (1-\psi_i) c_{ir}^{\frac{\sigma-1}{\sigma}} \left(\frac{p_{ir}}{p_{ih}} \frac{1-\psi_i}{\psi_i} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} . \end{aligned}$$

or

$$\frac{c_i}{c_{ir}} = \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + \left(\frac{1-\psi_i}{\psi_i} \right)^{\sigma} \left(\frac{p_{ir}}{p_{ih}} \right)^{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}} = \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} . \tag{C.42}$$

Taking the ratio of eq. (C.42) for two distinct sectors gives

$$\frac{c_i}{c_j} = \left(\frac{\psi_i}{\psi_j} \right)^{\frac{\sigma}{\sigma-1}} \left(\frac{1+E_{ihir}}{1+E_{jhjr}} \right)^{\frac{\sigma}{\sigma-1}} \frac{c_{ir}}{c_{jr}} . \tag{C.43}$$

Hence,

$$\begin{aligned} \text{MRS}_{irjr} = \frac{\frac{\partial U}{\partial c}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ir}}}{\frac{\partial U}{\partial c}\frac{\partial c}{\partial c_j}\frac{\partial c_j}{\partial c_{jr}}} &= \frac{\omega_i}{\omega_j} \frac{c_i^{-\frac{1}{\varepsilon}} \psi_i c_{ir}^{-\frac{1}{\sigma}} c_i^{\frac{1}{\sigma}}}{c_j^{-\frac{1}{\varepsilon}} \psi_j c_{jr}^{-\frac{1}{\sigma}} c_j^{\frac{1}{\sigma}}} \\ &= \frac{\omega_i}{\omega_j} \frac{\psi_i}{\psi_j} \left(\frac{c_j}{c_i} \right)^{\frac{1}{\varepsilon}} \left(\frac{c_{jr}}{c_j} \right)^{\frac{1}{\sigma}} \left(\frac{c_i}{c_{ir}} \right)^{\frac{1}{\sigma}} \\ &= \frac{\omega_i}{\omega_j} \frac{\psi_i}{\psi_j} \left(\frac{c_j}{c_i} \right)^{\frac{\sigma-\varepsilon}{\varepsilon\sigma}} \left(\frac{c_{jr}}{c_{ir}} \right)^{\frac{1}{\sigma}} \\ &= \frac{\omega_i}{\omega_j} \left(\frac{\psi_j}{\psi_i} \right)^{\frac{\sigma(1-\varepsilon)}{\varepsilon(\sigma-1)}} \left(\frac{1+E_{jhjr}}{1+E_{ihir}} \right)^{\frac{\sigma-\varepsilon}{\varepsilon(\sigma-1)}} \left(\frac{c_{jr}}{c_{ir}} \right)^{\frac{1}{\varepsilon}} . \end{aligned}$$

Finally, in equilibrium, we have

$$\frac{p_{ir}}{p_{jr}} = \frac{\omega_i}{\omega_j} \left(\frac{\psi_j}{\psi_i} \right)^{\frac{\sigma(1-\varepsilon)}{\varepsilon(\sigma-1)}} \left(\frac{1+E_{jhjr}}{1+E_{ihir}} \right)^{\frac{\sigma-\varepsilon}{\varepsilon(\sigma-1)}} \left(\frac{c_{jr}}{c_{ir}} \right)^{\frac{1}{\varepsilon}} . \tag{C.44}$$

C.2.4 Relative expenditures of distinct modern outputs

Using eq. (C.44), we have

$$E_{irjr} = \frac{p_{ir}c_{ir}}{p_{jr}c_{jr}} = \left(\frac{p_{jr}}{p_{ir}}\right)^{\varepsilon-1} \left(\frac{\omega_i}{\omega_j}\right)^{\varepsilon} \left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma(1-\varepsilon)}{\sigma-1}} \left(\frac{1+E_{ihir}}{1+E_{jhjr}}\right)^{\frac{\sigma-\varepsilon}{\sigma-1}}. \tag{C.45}$$

and substituting the relative prices from eq. (B.9) gives

$$E_{irjr} = \left(\frac{Z_{ir}}{Z_{jr}}\left(\frac{\xi^f_{ir}}{\xi^f_{jr}}\right)^{\frac{\eta}{\eta-1}}\left(\frac{I^f_{ir}}{I^f_{jr}}\right)^{\frac{1}{1-\eta}}\right)^{\varepsilon-1} \left(\frac{\omega_i}{\omega_j}\right)^{\varepsilon} \left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma(1-\varepsilon)}{\sigma-1}} \left(\frac{1+E_{ihir}}{1+E_{jhjr}}\right)^{\frac{\sigma-\varepsilon}{\sigma-1}}. \tag{C.46}$$

Defining

$$Z_{irjr} := \frac{Z_{ir}}{Z_{jr}}\left(\frac{\omega_i}{\omega_j}\right)^{\frac{\varepsilon}{\varepsilon-1}} \left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma}{1-\sigma}} \tag{C.47}$$

and substituting to eq. (C.46) results in

$$E_{irjr} = Z_{irjr}^{\varepsilon-1}\left(\frac{\xi^f_{ir}}{\xi^f_{jr}}\right)^{\frac{\eta(\varepsilon-1)}{\eta-1}}\left(\frac{I^f_{ir}}{I^f_{jr}}\right)^{\frac{\varepsilon-1}{1-\eta}}\left(\frac{1+E_{ihir}}{1+E_{jhjr}}\right)^{\frac{\sigma-\varepsilon}{\sigma-1}}. \tag{C.48}$$

C.2.5 Relative expenditures of distinct outputs and production types

These can be calculated indirectly by

$$E_{irjh} = E_{irjr}E_{jrjh}. \tag{C.49}$$

C.2.6 Marginal rate of substitution between modern outputs and leisure

For calculating the marginal rate of substitution for modernly produced outputs and leisure, we start from eq. (C.2)

$$\left(\frac{c}{c_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \omega_i \sum_j \frac{\omega_j}{\omega_i}\left(\frac{c_j}{c_i}\right)^{\frac{\varepsilon-1}{\varepsilon}},$$

so, from eq. (C.43), we get

$$\left(\frac{c}{c_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \omega_i \sum_j \frac{\omega_j}{\omega_i}\left(\left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1+E_{jhjr}}{1+E_{ihir}}\right)^{\frac{\sigma}{\sigma-1}}\frac{c_{jr}}{c_{ir}}\right)^{\frac{\varepsilon-1}{\varepsilon}},$$

and, from eq. (C.44), we conclude

$$\left(\frac{c}{c_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \omega_i \sum_j \frac{\omega_j}{\omega_i} \left(\left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma\varepsilon}{\sigma-1}} \left(\frac{1+E_{jhjr}}{1+E_{ihir}}\right)^{\frac{\varepsilon}{\sigma-1}} \left(\frac{p_{ir}}{p_{jr}}\right)^{\varepsilon} \left(\frac{\omega_i}{\omega_j}\right)^{-\varepsilon} \right)^{\frac{\varepsilon-1}{\varepsilon}}$$
$$= \omega_i \sum_j \left(\frac{\omega_j}{\omega_i}\right)^{\varepsilon} \left(\frac{\psi_j}{\psi_i}\right)^{\frac{\sigma(\varepsilon-1)}{\sigma-1}} \left(\frac{1+E_{jhjr}}{1+E_{ihir}}\right)^{\frac{\varepsilon-1}{\sigma-1}} \left(\frac{p_{ir}}{p_{jr}}\right)^{\varepsilon-1}. \tag{C.50}$$

From eq. (C.45), relative prices can be substituted out in the last expression. We, then, conclude

$$\left(\frac{c}{c_i}\right)^{\frac{\varepsilon-1}{\varepsilon}} = \omega_i \sum_j \left(\frac{1+E_{jhjr}}{1+E_{ihir}}\right)^{\frac{\varepsilon-1}{\sigma-1}} \frac{1}{E_{irjr}} \left(\frac{1+E_{jhjr}}{1+E_{ihir}}\right)^{\frac{\sigma-\varepsilon}{\sigma-1}}$$
$$= \omega_i \sum_j \frac{1+E_{jhjr}}{1+E_{ihir}} E_{jrir}. \tag{C.51}$$

Therefore, from eqs. (C.42) and (C.51),

$$\begin{aligned}\mathrm{MRS}_{irl} &= \frac{\frac{\partial U}{\partial c}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ir}}}{\frac{\partial U}{\partial \ell}} = \frac{\ell}{\varphi}\frac{1}{c-\bar{c}}\omega_i\psi_i \left(\frac{c}{c_i}\right)^{\frac{1}{\varepsilon}} \left(\frac{c_i}{c_{ir}}\right)^{\frac{1}{\sigma}} \\ &= \frac{\ell}{\varphi c_{ir}}\frac{c}{c-\bar{c}}\omega_i\psi_i \left(\frac{c}{c_i}\right)^{\frac{1-\varepsilon}{\varepsilon}} \left(\frac{c_i}{c_{ir}}\right)^{\frac{1-\sigma}{\sigma}} \\ &= \frac{\ell}{\varphi c_{ir}}\frac{c}{c-\bar{c}}\omega_i\psi_i \left(\omega_i \sum_j \frac{1+E_{jhjr}}{1+E_{ihir}} E_{jrir}\right)^{-1} \left(\psi_i^{\frac{\sigma}{\sigma-1}} (1+E_{ihir})^{\frac{\sigma}{\sigma-1}}\right)^{\frac{1-\sigma}{\sigma}} \\ &= \frac{\ell}{\varphi c_{ir}}\frac{c}{c-\bar{c}} \left(\sum_j \frac{1+E_{jhjr}}{1+E_{ihir}} E_{jrir}\right)^{-1} (1+E_{ihir})^{-1}.\end{aligned}$$

It will be useful for the forthcoming calculations to denote as

$$E_i := \left(\sum_j \frac{1+E_{jhjr}}{1+E_{ihir}} E_{jrir}\right)^{-1} = \frac{p_{ir}c_{ir} + p_{ih}c_{ih}}{\sum_j p_{jr}c_{jr} + p_{jh}c_{jh}}, \tag{C.52}$$

the share of expenditure from modern and traditional production of output i in total expenditure. Then

$$\mathrm{MRS}_{irl} = \frac{\ell}{\varphi c_{ir}}\frac{c}{c-\bar{c}}\frac{E_i}{1+E_{ihir}}. \tag{C.53}$$

C.2.7 Relative expenditures of modern output types and leisure

Relative expenditures of modern output and leisure is calculated by equating eq. (C.53) with relative prices, i.e.,

$$\frac{p_{ir}}{p_l} = \frac{\ell}{\varphi c_{ir}} \frac{c}{c - \bar{c}} \frac{E_i}{1 + E_{ihir}},$$

hence,

$$E_{irl} = \frac{p_{ir} c_{ir}}{p_l \ell} = \frac{1}{\varphi} \frac{c}{c - \bar{c}} \frac{E_i}{1 + E_{ihir}}. \tag{C.54}$$

C.2.8 Relative expenditures between traditional output types and leisure

These can be indirectly obtained by

$$E_{lih} = E_{lir} E_{irih}.$$

D Constraints and Equilibrium

In this section we combine the market clearing conditions, aw well as, the time and budget constraints into a single equation that the female to male wage ratio has to satisfy in equilibrium.

D.1 Labor Hours Allocations

From the market clearing conditions, demand for modern and traditional production output equals supply from modern and traditional production. From eq. (B.6),

$$\delta(0) c_{ir} = Z_{ir} \left(\frac{\xi^f_{ir}}{I^f_{ir}} \right)^{\frac{\eta}{\eta - 1}} \delta(s^f) H(s^f) L^f_{ir}, \tag{D.1}$$

and, from eq. (C.4),

$$c_{ih} = Z_{ih} \left(\frac{\xi^f_{ih}}{I^f_{ih}} \right)^{\frac{\eta}{\eta - 1}} L^f_{ih}, \tag{D.2}$$

for any $i = A, M, S$. Additionally, the time constraint,

$$L^g_{Ar} + L^g_{Mr} + L^g_{Sr} + L^g_{Ah} + L^g_{Mh} + L^g_{Sh} + \ell^g = L, \tag{D.3}$$

holds for any equilibrium allocation.

D.1.1 Relative female labor hours in modern production outputs

Using eqs. (C.27) and (D.1), for any two distinct modern production in sectors $i, j = A, M, S$, we have

$$E_{irjr} = \frac{p_{ir}c_{ir}}{p_{jr}c_{jr}} = \frac{Z_{jr}}{Z_{ir}} \left(\frac{\xi^f_{jr}}{\xi^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{ir}}{I^f_{jr}}\right)^{\frac{1}{\eta-1}} \frac{Z_{ir}}{Z_{jr}} \left(\frac{\xi^f_{ir}}{\xi^f_{jr}}\right)^{\frac{\eta}{\eta-1}} \left(\frac{I^f_{jr}}{I^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \frac{L^f_{ir}}{L^f_{jr}}, \tag{D.4}$$

hence,

$$\frac{L^f_{ir}}{L^f_{jr}} = E_{irjr} \frac{I^f_{ir}}{I^f_{jr}}. \tag{D.5}$$

D.1.2 Relative female labor hours between modern and traditional production outputs

Using eqs. (C.25), (D.1) and (D.2), we calculate

$$E_{irjh} = \frac{p_{ir}c_{ir}}{p_{jh}c_{jh}} = \frac{I^f_{jh}}{I^f_{ir}} \frac{d(0)}{d(s^f)} \frac{\delta(s^f)}{\delta(0)} \frac{L^f_{ir}}{L^f_{jh}},$$

so that relative labor supply becomes

$$\frac{L^f_{ir}}{L^f_{jh}} = E_{irjh} \frac{I^f_{ir}}{I^f_{jh}} \frac{d(s^f)}{d(0)} \frac{\delta(0)}{\delta(s^f)}. \tag{D.6}$$

D.1.3 Relative female labor hours between leisure and traditional production outputs

From eqs. (C.25) and (C.38), we have

$$E_{lih} = \frac{p_l \ell}{p_{ih}c_{ih}} = \frac{I^f_{ih}}{I^f_l} \frac{\ell^f}{L^f_{ih}}.$$

We, thus, have

$$\frac{\ell^f}{L^f_{ih}} = E_{lih} \frac{I^f_l}{I^f_{ih}} \tag{D.7}$$

D.1.4 Relative female labor hours in traditional production outputs

For the relative labor hours in traditional production, we have

$$\frac{L^f_{ih}}{L^f_{jh}} = \frac{L^f_{ih}}{L^f_{kr}}\frac{L^f_{kr}}{L^f_{jh}} = E_{ihjh}\frac{I^f_{ih}}{I^f_{jh}}. \tag{D.8}$$

D.1.5 Female labor shares

We can rewrite the expressions in eqs. (D.5) to (D.8) to substitute for female labor supply in modern production sectors in eq. (D.3). First, for any combination of $s, q \in \{h, r\}$, define the female labor ratios

$$R^f_{isjq} = E_{isjq}\frac{I^f_{is}}{I^f_{jq}}\left(\frac{d(s^f)}{d(0)}\frac{\delta(0)}{\delta(s^f)}\right)^{\mathbb{1}_{s=r}-\mathbb{1}_{q=r}}, \tag{D.9}$$

the female leisure to labor ratios

$$R^f_{ljq} = E_{ljq}\frac{I^f_{l}}{I^f_{jq}}\left(\frac{d(s^f)}{d(0)}\frac{\delta(0)}{\delta(s^f)}\right)^{-\mathbb{1}_{q=r}}, \tag{D.10}$$

and the aggregate female labor ratio

$$R^f_{jq} = \sum_{i,s} R_{isjq} = \sum_{i,s} E_{isjq}\frac{I^f_{is}}{I^f_{jq}}\left(\frac{d(s^f)}{d(0)}\frac{\delta(0)}{\delta(s^f)}\right)^{\mathbb{1}_{s=r}-\mathbb{1}_{q=r}}. \tag{D.11}$$

Then, we have

$$\begin{aligned}\frac{L}{L^f_{jq}} &= \frac{L^f_{Ar}}{L^f_{jq}} + \frac{L^f_{Mr}}{L^f_{jq}} + \frac{L^f_{Sr}}{L^f_{jq}} + \frac{L^f_{Ah}}{L^f_{jq}} + \frac{L^f_{Mh}}{L^f_{jq}} + \frac{L^f_{Sh}}{L^f_{jq}} + \frac{\ell^f}{L^f_{jq}}\\ &= \sum_i R^f_{irjq} + \sum_i R^f_{ihjq} + R^f_{ljq}\\ &= R^f_{jq} + R^f_{ljq}. \end{aligned}\tag{D.12}$$

Finally, eq. (D.12) is rewritten as

$$\frac{L^f_{jq}}{L} = \frac{1}{R^f_{jq} + R^f_{ljq}}. \tag{D.13}$$

D.1.6 Male labor shares

The male labor shares can be calculated using the female labor shares via interjection. Specifically, we have

$$\frac{L^m_{is}}{L^m_{jq}} = \frac{L^m_{is}}{L^f_{is}}\frac{L^f_{is}}{L^f_{jq}}\frac{L^f_{jq}}{L^m_{jq}}.$$

Thus, from eqs. (B.3), (C.16), (C.29) and (D.9) to (D.11), we see that the male labor ratios are

$$R^m_{isjq} = R^f_{isjq}\left(\frac{\tilde{w}\tilde{\delta}^{\frac{1}{\eta}}\tilde{H}^{\frac{1}{\eta}}}{\tilde{\xi}_{is}}\right)^{\eta\mathbb{1}_{s=r}}\left(\frac{\tilde{w}\tilde{\delta}^{\frac{1}{\eta}}\tilde{H}^{\frac{1}{\eta}}}{\tilde{\xi}_{jq}}\right)^{-\eta\mathbb{1}_{q=r}}\left(\frac{\tilde{w}\tilde{d}\tilde{H}}{\tilde{\xi}_{is}}\right)^{\eta\mathbb{1}_{s=h}}\left(\frac{\tilde{w}\tilde{d}\tilde{H}}{\tilde{\xi}_{jq}}\right)^{-\eta\mathbb{1}_{q=h}}, \tag{D.14}$$

the male leisure to labor ratios are

$$R^m_{ljq} = R^f_{ljq}\left(\frac{\tilde{w}\tilde{d}\tilde{H}}{\tilde{\xi}_l}\right)^{\eta_l}\left(\frac{\tilde{w}\tilde{\delta}^{\frac{1}{\eta}}\tilde{H}^{\frac{1}{\eta}}}{\tilde{\xi}_{jq}}\right)^{-\eta\mathbb{1}_{q=r}}\left(\frac{\tilde{w}\tilde{d}\tilde{H}}{\tilde{\xi}_{jq}}\right)^{-\eta\mathbb{1}_{q=h}}, \tag{D.15}$$

and the aggregate male labor ratio is

$$R^m_{jq} = \sum_{i,s} R^m_{isjq}. \tag{D.16}$$

As in the female case, we conclude

$$\begin{aligned}\frac{L}{L^m_{iq}} &= \frac{L^m_{Ar}}{L^m_{iq}} + \frac{L^m_{Mr}}{L^m_{iq}} + \frac{L^m_{Sr}}{L^m_{iq}} + \frac{L^m_{Ah}}{L^m_{iq}} + \frac{L^m_{Mh}}{L^m_{iq}} + \frac{L^m_{Sh}}{L^m_{iq}} + \frac{\ell^m}{L^m_{iq}}\\ &= \sum_j R^m_{jriq} + \sum_j R^m_{jhiq} + R^m_{liq}\\ &= R^m_{iq} + R^m_{liq}, \end{aligned} \tag{D.17}$$

and

$$\frac{L^m_{iq}}{L} = \frac{1}{R^m_{iq} + R^m_{liq}}. \tag{D.18}$$

D.2 The Budget Constraint

Define the ratio of female in total household earnings by.

$$I_L^f = \frac{w^f d(s^f) H(s^f) L}{w^f H(s^f) d(s^f) L + w^m d(s^m) H(s^m) L} = \frac{1}{1 + \left(\tilde{w}\tilde{d}\tilde{H}\right)^{-1}} \tag{D.19}$$

We start by rewriting the budget constraint in eq. (C.7) as

$$\frac{1}{I_L^f} = \sum_g \frac{w^g d(s^g) H(s^g)}{w^f d(s^f) H(s^f)} = \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f) L} + \sum_g \frac{w^g d(s^g) H(s^g)}{w^f d(s^f) H(s^f) L} \left(L_{jh}^g + \frac{1}{3} \ell^g \right) \right).$$

From which we get

$$\begin{aligned}
\frac{L}{I_L^f} &= \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f)} + \sum_g \frac{w^g d(s^g) H(s^g)}{w^f d(s^f) H(s^f)} \left(L_{jh}^g + \frac{1}{3} \ell^g \right) \right) \\
&= \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f)} + L_{jh}^f \left(1 + \frac{L_{jh}^m}{\tilde{d}\tilde{w}\tilde{H} L_{jh}^f} \right) + \frac{\ell^f}{3} \left(1 + \frac{L_l^m}{\tilde{d}\tilde{w}\tilde{H} \ell^f} \right) \right) \\
&= \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f)} + L_{jh}^f \left(1 + \frac{\left(\tilde{w}\tilde{d}\tilde{H}\right)^{\eta - 1}}{\left(\tilde{\xi}_{jh}\right)^{\eta}} \right) + \frac{\ell^f}{3} \left(1 + \frac{\left(\tilde{w}\tilde{d}\tilde{H}\right)^{\eta_l - 1}}{\left(\tilde{\xi}_{jh}\right)^{\eta_l}} \right) \right) \\
&= \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f)} + \frac{L_{jh}^f}{I_{jh}^f} + \frac{1}{3} \frac{\ell^f}{I_l^f} \right) \\
&= \sum_j \left(\frac{d(0) p_{jr} c_{jr}}{w^f d(s^f) H(s^f)} + \frac{L_{jh}^f}{I_{jh}^f} \right) + \frac{\ell^f}{I_l^f}.
\end{aligned}$$

Hence, by eqs. (C.24) and (C.37), we have

$$\begin{aligned}
\frac{w^f d(s^f) H(s^f) L}{d(0) I_L^f} &= \sum_j \left(p_{jr} c_{jr} + \frac{w^f d(s^f) H(s^f)}{d(0)} \frac{L_{jh}^f}{I_{jh}^f} \right) + \frac{w^f d(s^f) H(s^f)}{d(0)} \frac{\ell^f}{I_l^f} \\
&= \sum_j \left(p_{jr} c_{jr} + p_{jh} Z_{jh} \left(\xi_{jh}^f \right)^{\frac{\eta}{\eta - 1}} \left(\frac{1}{I_{jh}^f} \right)^{\frac{1}{\eta - 1}} \frac{L_{jh}^f}{I_{jh}^f} \right) + p_l \left(\xi_l^f \right)^{\frac{\eta_l}{\eta_l - 1}} \left(\frac{1}{I_l^f} \right)^{\frac{1}{\eta_l - 1}} \frac{\ell^f}{I_l^f} \\
&= \sum_j \left(p_{jr} c_{jr} + p_{jh} Z_{jh} \left(\frac{\xi_{jh}^f}{I_{jh}^f} \right)^{\frac{\eta}{\eta - 1}} L_{jh}^f \right) + p_l \left(\frac{\xi_l^f}{I_l^f} \right)^{\frac{\eta_l}{\eta_l - 1}} \ell^f.
\end{aligned}$$

Further, from eqs. (C.33) and (D.2), we obtain

$$\frac{w^f d(s^f) H(s^f) L}{d(0) I_L^f} = \sum_j \left(p_{jr} c_{jr} + p_{jh} c_{jh}\right) + p_l \ell. \tag{D.20}$$

The goal is to derive a condition including relative expenditures. Dividing eq. (D.20) by $p_{ih} c_{ih}$, we have

$$\frac{w^f d(s^f) H(s^f) L}{d(0) I_L^f p_{ih} c_{ih}} = \sum_j \left(\frac{p_{jr} c_{jr}}{p_{ih} c_{ih}} + \frac{p_{jh} c_{jh}}{p_{ih} c_{ih}}\right) + \frac{p_l \ell}{p_{ih} c_{ih}} = \sum_j \left(E_{jrih} + E_{jhih}\right) + E_{lih}.$$

Thus,

$$\frac{p_{ih} c_{ih} I_L^f}{L} = \frac{d(s^f) w^f H(s^f)}{d(0) \sum_j \left(E_{jrih} + E_{jhih}\right) + E_{lih}}.$$

From eqs. (C.24) and (D.2), we have

$$\begin{aligned}
\frac{d(s^f) w^f H(s^f) L^f}{d(0) \sum_j \left(E_{jrih} + E_{jhih}\right) + E_{lih}} &= p_{ih} c_{ih} I_L^f \\
&= \frac{w^f}{Z_{ih} \left(\xi_{ih}^f\right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{I_{ih}^f}\right)^{\frac{1}{\eta-1}} \frac{d(0)}{d(s^f) H(s^f)}} Z_{ih} \left(\frac{\xi_{ih}^f}{I_{ih}^f}\right)^{\frac{\eta}{\eta-1}} L_{ih}^f I_L^f \\
&= \frac{d(s^f) w^f H(s^f)}{d(0)} \frac{I_L^f}{I_{ih}^f} L_{ih}^f
\end{aligned}$$

Therefore,

$$\frac{L_{ih}^f}{L^f} = \frac{I_{ih}^f}{\sum_j \left(E_{jrih} + E_{jhih}\right) I_L^f + E_{lih} I_L^f}. \tag{D.21}$$

D.3 The Wage Gap Equation

We combine eq. (D.21) with eq. (D.13) to arrive at a condition that the wage ratio $\tilde{w}$ and years of schooling s^f, s^m must satisfy

$$\frac{1}{R_{ih}^f + R_{lih}^f} = \frac{I_{ih}^f}{\sum_j \left(E_{jrih} + E_{jhih}\right) I_L^f + E_{lih} I_L^f}, \tag{D.22}$$

which simplifies to

$$R_{ih}^f + R_{lih}^f = \frac{I_L^f}{I_{ih}^f}\left(\sum_j (E_{jrih} + E_{jhih}) + E_{lih}\right). \tag{D.23}$$

E Schooling Equations

We use the first order conditions with respect to female schooling and female hours in traditional production to derive a condition on the optimal level of (female) schooling years. Define

$$W(s^g) = -\beta^g \mathrm{e}^{-\rho s^g} \qquad (g \in f, m)$$

and

$$G(s) = \frac{H'(s)}{H(s)} + \frac{d'(s)}{d(s)}.$$

By eqs. (C.9) and (C.11), we have

$$W(s^f) = -M^f\left(\frac{H'(s^f)}{H(s^f)} + \frac{d'(s^f)}{d(s^f)}\right)\delta(0)\frac{1}{c-\bar{c}}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ih}}\frac{\partial c_{ih}}{\delta L_{ih}^f}$$

Then,

$$\begin{aligned}
W(s^f) &= -M^f G(s^f)\delta(0)\frac{1}{c-\bar{c}}\frac{\partial c}{\partial c_i}\frac{\partial c_i}{\partial c_{ih}}\frac{\partial c_{ih}}{\delta L_{ih}^f} \\
&= -M^f G(s^f)\delta(0)\frac{1}{c-\bar{c}}\omega_i\left(\frac{c}{c_i}\right)^{\frac{1}{\epsilon}}(1-\psi_i)\left(\frac{c_i}{c_{ih}}\right)^{\frac{1}{\sigma}} Z_{ih}\xi_{ih}^f\left(\frac{L_{ih}}{L_{ih}^f}\right)^{\frac{1}{\eta}} \\
&= -M^f G(s^f)\delta(0)\frac{c}{c-\bar{c}}\omega_i\left(\frac{c}{c_i}\right)^{\frac{1}{\epsilon}-1}(1-\psi_i)\left(\frac{c_i}{c_{ih}}\right)^{\frac{1}{\sigma}-1}\frac{Z_{ih}}{c_{ih}}\xi_{ih}^f\left(\frac{L_{ih}}{L_{ih}^f}\right)^{\frac{1}{\eta}}.
\end{aligned}$$

Since $L_{ih} = c_{ih}/Z_{ih}$,

$$W(s^f) = -M^f G(s^f)\delta(0)\frac{c}{c-\bar{c}}\omega_i\left(\frac{c}{c_i}\right)^{\frac{1}{\epsilon}-1}(1-\psi_i)\left(\frac{c_i}{c_{ih}}\right)^{\frac{1}{\sigma}-1}\left(\frac{c_{ih}}{Z_{ih}L_{ih}^f}\right)^{\frac{1}{\eta}-1}\frac{\xi_{ih}^f}{L_{ih}^f}.$$

From eq. (C.51), we have

$$W(s^f) = -M^f G(s^f)\delta(0)\frac{c}{c-\bar{c}}E_i(1-\psi_i)\left(\frac{c_i}{c_{ih}}\right)^{\frac{1}{\sigma}-1}\left(\frac{c_{ih}}{Z_{ih}L^f_{ih}}\right)^{\frac{1}{\eta}-1}\frac{\xi^f_{ih}}{L^f_{ih}}.$$

Similar to eq. (C.42), one calculates

$$c_i = (1-\psi_i)^{\frac{\sigma}{\sigma-1}}(1+E_{irih})^{\frac{\sigma}{\sigma-1}}c_{ih}, \tag{E.1}$$

therefore,

$$W(s^f) = -M^f G(s^f)\delta(0)\frac{c}{c-\bar{c}}\frac{E_i}{1+E_{irih}}\left(\frac{c_{ih}}{Z_{ih}L^f_{ih}}\right)^{\frac{1}{\eta}-1}\frac{\xi^f_{ih}}{L^f_{ih}}.$$

Lastly, by eq. (D.2), the female schooling condition takes the form

$$W(s^f) = -\frac{M^f}{L^f_{ih}}G(s^f)\delta(0)\frac{c}{c-\bar{c}}\frac{E_i}{1+E_{irih}}I^f_{ih}. \tag{E.2}$$

A similar calculation on the male side gives

$$W(s^m) = -\frac{M^m}{L^m_{ih}}G(s^m)\delta(0)\frac{c}{c-\bar{c}}\frac{E_i}{1+E_{irih}}I^m_{ih}, \tag{E.3}$$

which can be rewritten as

$$W(s^m) = -\frac{M^m}{L^m_{ih}}G(s^m)\delta(0)\frac{c}{c-\bar{c}}\frac{E_i}{1+E_{irih}}I^f_{ih}\tilde{\xi}^{-\eta}_{ih}\left(\tilde{w}\tilde{d}\tilde{H}\right)^{\eta-1}.$$

Combining the schooling equations for females and males gives

$$\frac{W(s^f)}{W(s^m)} = \frac{M^f}{M^m}\frac{G(s^f)}{G(s^m)}\tilde{w}\tilde{d}\tilde{H}, \tag{E.4}$$

or in female-to-male ratio notation

$$\tilde{W} = \tilde{M}\tilde{G}\tilde{w}\tilde{d}\tilde{H}. \tag{E.5}$$

F Subsistence and Income Effect

Let γ be the share of subsistence in total consumption, namely

$$\gamma = \frac{\bar{c}}{c}.$$

We can express c as a function of L^f_{ih} by combining eqs. (C.42), (C.51), (D.6) and (D.9). We have

$$\begin{aligned}
c &= \omega_i^{\frac{\epsilon}{\epsilon-1}} E_i^{\frac{\epsilon}{1-\epsilon}} c_i \\
&= \omega_i^{\frac{\epsilon}{\epsilon-1}} E_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} c_{ir} \\
&= \omega_i^{\frac{\epsilon}{\epsilon-1}} E_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} Z_{ir} \left(\frac{\xi^f_{ir}}{I^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \delta(s^f) H(s^f) L^f_{ir} \\
&= \omega_i^{\frac{\epsilon}{\epsilon-1}} E_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} Z_{ir} \left(\frac{\xi^f_{ir}}{I^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \delta(s^f) H(s^f) E_{irih} \frac{I^f_{ir}}{I^f_{ih}} \frac{d(s^f)}{d(0)} \frac{\delta(0)}{\delta(s^f)} L^f_{ih} \\
&= \omega_i^{\frac{\epsilon}{\epsilon-1}} E_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{\sigma-1}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} Z_{ir} \left(\frac{\xi^f_{ir}}{I^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \delta(s^f) H(s^f) R^f_{irih} L^f_{ih}
\end{aligned}$$

For brevity, let

$$P^f_{ih} = E_i^{\frac{\epsilon}{1-\epsilon}} \left(1 + E_{ihir}\right)^{\frac{\sigma}{\sigma-1}} Z_{ir} \left(\frac{\xi^f_{ir}}{I^f_{ir}}\right)^{\frac{\eta}{\eta-1}} \delta(s_f) H(s_f) R^f_{irih}. \tag{F.1}$$

Using the last definition and the definition of γ, we get

$$\omega_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{1-\sigma}} \frac{\bar{c}}{\gamma} = P^f_{ih} L^f_{ih}.$$

Let

$$\hat{c}_i = \omega_i^{\frac{\epsilon}{1-\epsilon}} \psi_i^{\frac{\sigma}{1-\sigma}} \bar{c} \tag{F.2}$$

so that

$$\gamma = \frac{\hat{c}_i}{P^f_{ih} L^f_{ih}}. \tag{F.3}$$

For any value of L^f_{ih}, eq. (F.3) can be used to calibrate $\hat{c}_i$ to match γ. How to get L^f_{ih}? From eqs. (C.54), (D.10) and (D.13), we have

$$\begin{aligned}
\frac{L}{L^f_{ih}} &= R^f_{ih} + R^f_{lih} \\
&= R^f_{ih} + E_{lih}\frac{I^f_l}{I^f_{ih}} \\
&= R^f_{ih} + E_{lir}E_{irih}\frac{I^f_l}{I^f_{ih}} \\
&= R^f_{ih} + \varphi\frac{c-\bar{c}}{c}\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}} \\
&= R^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\left(1-\frac{\bar{c}}{c}\right) \\
&= R^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\left(1-\frac{\hat{c}_i}{P^f_{ih}L^f_{ih}}\right),
\end{aligned}$$

This can be analytically solved for L^f_{ih}. Doing so gives

$$L^f_{ih} = \frac{LP^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\hat{c}_i}{P^f_{ih}\left(R^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\right)} \tag{F.4}$$

Having L^f_{ih}, we can calculate the subsistence share predicted by the model, i.e.,

$$\frac{\bar{c}}{c} = \frac{\hat{c}_i}{P^f_{ih}L^f_{ih}} = \frac{\hat{c}_i\left(R^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\right)}{LP^f_{ih} + \varphi\frac{1+E_{irih}}{E_i}\frac{I^f_l}{I^f_{ih}}\hat{c}_i}. \tag{F.5}$$

G Model Calibration

G.1 Procedure and Numerical Details

We calibrate the model multiple times using various combinations of targets and weights, depending on the counterfactual or empirical perspective we want to highlight. Nonetheless, the calibration procedure is the same irrespective of the targets and the weights. We use a nested calibration procedure with two levels, namely an outer and an inner level. The outer level receives as input a set of parameters to be calibrated, calculates the model's predictions for the targeted variables, and minimizes the distance from the observed values of the targets in our data. The model's predictions for the target variables depend on the model solutions obtained at the inner level of the procedure. The inner level takes the calibration input from the output level and numerically approximates model equilibria for the given parameters.

For the minimization problem of the outer level, i.e., minimizing the distance between model predictions and data, we use the *Nelder-Mead*, with upper bounds of both domain and range absolute errors equal to 10^{-4}. In unreported calibrations, we obtained similar results using *BFGS* with faster execution, but at the cost of more execution failures. The $\mathcal{L}^1$ norm is used to calculate distances between model predictions and targets.

For the inner problem, we calculate model equilibria as solutions to the system of eqs. (D.23), (E.2) and (E.3) using Newton's method. For the Newton method, both the range and domain error tolerances are set equal to 10^{-8}. Our algorithm allows for a maximum number of 35 iterations to be used and emits a warning message if the desired accuracy is not achieved by the time the iteration bound is reached. In practice, this upper bound was not binding for any set of good initializing values in our calibrations.

Our algorithm approximates the Jacobian of the system using the symmetric difference quotient (i.e., the secant for small symmetric steps in both directions of each coordinate at the point of differentiation) with adaptive stepping. The algorithm initially attempts to calculate the Jacobian with a step equal to 10^{-10}, and if it fails due to numerical errors, it reduces the step by half and retries. The minimum accepted step is set equal to 10^{-12}, and the calibration process fails if the Jacobian cannot be calculated for any step above this minimum.

A similar adaptive stepping algorithm is employed in Newton's method using a dampening parameter λ in the update of each iteration. The Newton update is performed using the product of the Jacobian with λ. The initial update attempt uses $\lambda = 1$ (i.e., the vanilla Newton step), and if the step leads to points outside the function's domain, λ is divided by 10, and the update is recalculated. If the value of lambda is driven below 10^{-6}, the calibration fails.

We use two distinct approaches to initialize the successive calls to Newton's method during a calibration. The first approach uses the calculated equilibrium from Newton's method (inner problem) of the previous calibration step (outer problem) to initialize the call to Newton's method (inner problem) of the current calibration step (outer problem). The second approach sets fixed initializing values for $\tilde{w}$, s^f, and s^m equal to the values observed in the data. Since the model is continuous in the calibrated parameters, small changes in the calibration step result in small changes in equilibria. Thus, we employ the first approach to reduce the computation time of reaching small neighborhoods surrounding roots. Subsequently, we employ the second, slower approach to fine-tune our calibration exercises.

Parts of the analysis of the main text attribute special emphasis to some aspects of the model (e.g., wage ratio rigidities). To make the corresponding calibration exercises reflect this emphasis, we assign weights in the calculation of distances for the relevant targets. For example, to emphasize the targeting of the wage ratio, we multiply it by 100 to make its scale similar to the schooling targets, which are measured in years. We used weights for four variables in various executions; for the wage ratio ($\times 100$), consumption share ($\times 100$), female ($\times 1/T$), and male schooling years ($\times 1/T$). Our default calibration does not employ any weights.

G.2 Implementation Details

The source files of our implementation are available online under the Expat license.[1] The nested procedure is implemented in `Python` (version 3.8.10). Additionally, we rely on `NumPy` (version 1.22.3) for vector calculations. The calibration procedure can be implemented more efficiently in languages more geared with concurrent programming features. We exchange the efficiency hit with what we perceive to be increased accessibility. Our goal is that our work can be easily replicated by a greater number of researchers. For the outer minimization problem, we use the *Nelder-Mead* implementation of `SciPy` (version 1.3.3).

The code is written using the functional paradigm to minimize the possibility of side effects. The functions of the calibration code follow the derivations of this appendix. References in the documentation of the sources give the corresponding equations used from the appendix's text.

A typical function in our implementation receives a model data structure (i.e., a nested Python dictionary) and up to two indices. The data structure contains all the model parameters, either fixed or calibrated. Each index is either a pair of a sector (A, M, S) and technology (h, r) or a leisure index (l). The typical function output in our implementation returns one of the variables calculated in this appendix as a function of the wage ratio and the female and male schooling years. For example, the call

```
make_female_labor_ratio(model_data, "Ah", "Sr")
```

creates and returns a function $\tilde{w}, s^f, s^m \mapsto R_{AhSr}$ for the parameters given in `model_data`.

For cases in which the calculation of a variable requires the calculation of another variable derived in this appendix, we stack the needed function creation in the definition of the called function. As an example, to calculate R_{AhSr}, the variable E_{AhSr} is required and, hence,

[1]The address of the code repository is https://github.com/pi-kappa-devel/structural-schooling.

the definition of `make_female_labor_ratio` creates a function $\tilde{w}, s^f, s^m \mapsto E_{AhSr}$ using `make_relative_consumption_expenditure`, and evaluates it.