Supplementary material

In addition to the results presented in the main body of the paper, we provide a supplementary collection of figures on the fits used in determining the double-tag yields in the two tag modes, and a table summarizing the systematic uncertainties of the partial BFs of $D_s^+ \to \pi^+\pi^+\pi^- X$ in intervals of $M(\pi^+\pi^+\pi^-)$.

Table 1: Summary of all different sources of systematic uncertainties for partial branching fractions of $D_s^+ \to \pi^+\pi^+\pi^- X$. The uncertainties are summed up in quadrature to make the total systematic uncertainties.

	Relative uncertainty (%) in $M(\pi^+\pi^+\pi^-)$ interval										
Source	1	2	3	4	5	6	7	8	9	10	11
Sig. shape	0.70	1.73	0.49	0.86	1.84	2.62	2.55	2.41	1.55	1.56	0.17
Bkg. shape	0.04	0.42	0.04	0.33	0.68	0.63	0.09	0.20	1.28	0.65	0.15
Track eff.	2.44	1.73	1.34	0.96	0.72	0.50	0.37	0.19	0.01	0.17	0.43
PID eff.	0.77	1.26	0.95	0.87	0.68	0.50	0.23	0.19	0.39	0.71	0.59
K_S^0 bkg.	0.30	0.64	0.58	0.40	0.29	0.24	0.17	0.19	0.19	0.06	0.02
Kaon misID	0.40	0.49	0.42	0.36	0.30	0.23	0.17	0.14	0.19	0.28	0.02
Lepton misID	0.28	0.45	0.37	0.35	0.38	0.34	0.31	0.37	0.44	0.25	0.02
Pion mult.	0.91	1.12	1.11	0.73	0.39	0.19	0.01	0.15	0.33	0.06	0.00
Pion mom.	0.46	1.84	1.26	1.75	1.30	1.08	0.90	0.65	0.14	0.79	2.80
MC size	0.32	0.30	0.39	0.41	0.61	0.51	0.61	0.55	0.67	0.63	1.45
Fit bias	0.90	1.24	0.77	0.08	0.38	0.45	0.93	0.57	0.15	0.81	2.57
Total	3.05	3.86	2.68	2.59	2.74	3.11	2.99	2.68	2.25	2.29	4.14

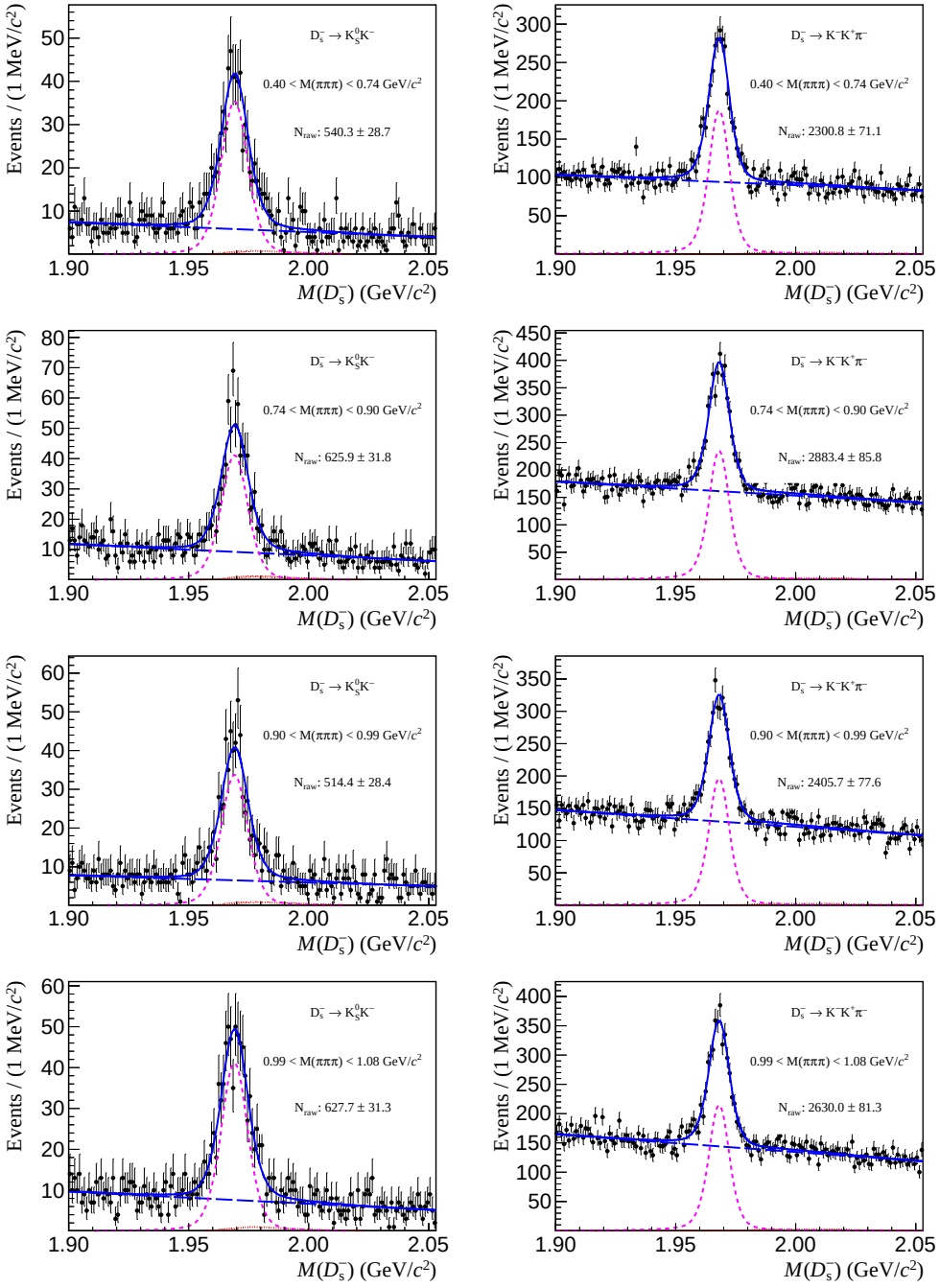

Figure 1: The invariant mass $M(D_s^-)$ distributions for the ST modes $D_s^- \to K_S^0 K^-$ (left) and $D_s^- \to K^- K^+ \pi^-$ (right), part 1/3. Data (black points) are shown overlaid with the fit results including the total (solid blue), signal PDF (dashed magenta), D^- background PDF (dotted red), and combinatorial background PDF (long-dashed blue) components.

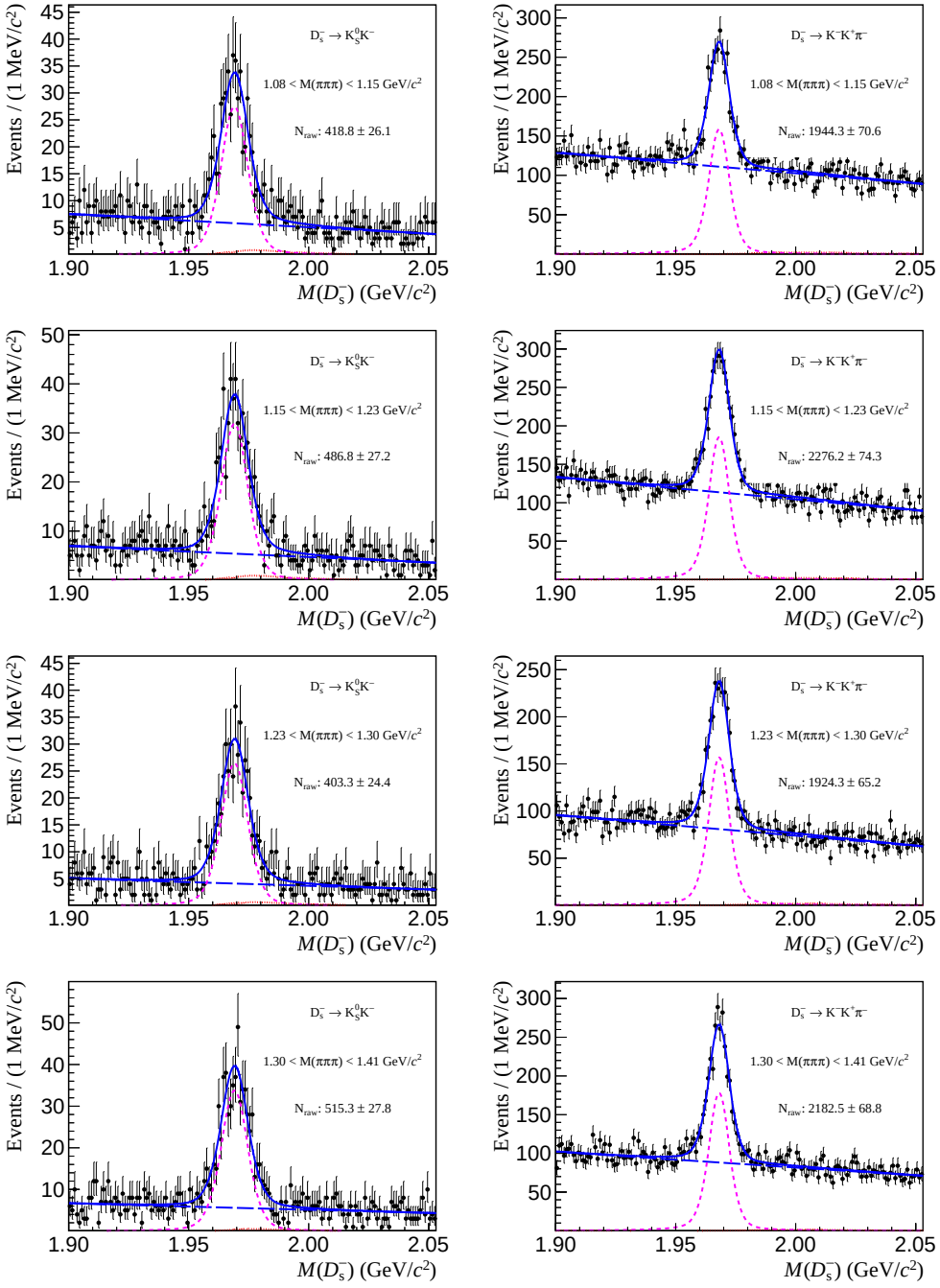

Figure 2: The invariant mass $M(D_s^-)$ distributions for the ST modes $D_s^- \to K_S^0 K^-$ (left) and $D_s^- \to K^- K^+ \pi^-$ (right), part 2/3. Data (black points) are shown overlaid with the fit results including the total (solid blue), signal PDF (dashed magenta), D^- background PDF (dotted red), and combinatorial background PDF (long-dashed blue) components.

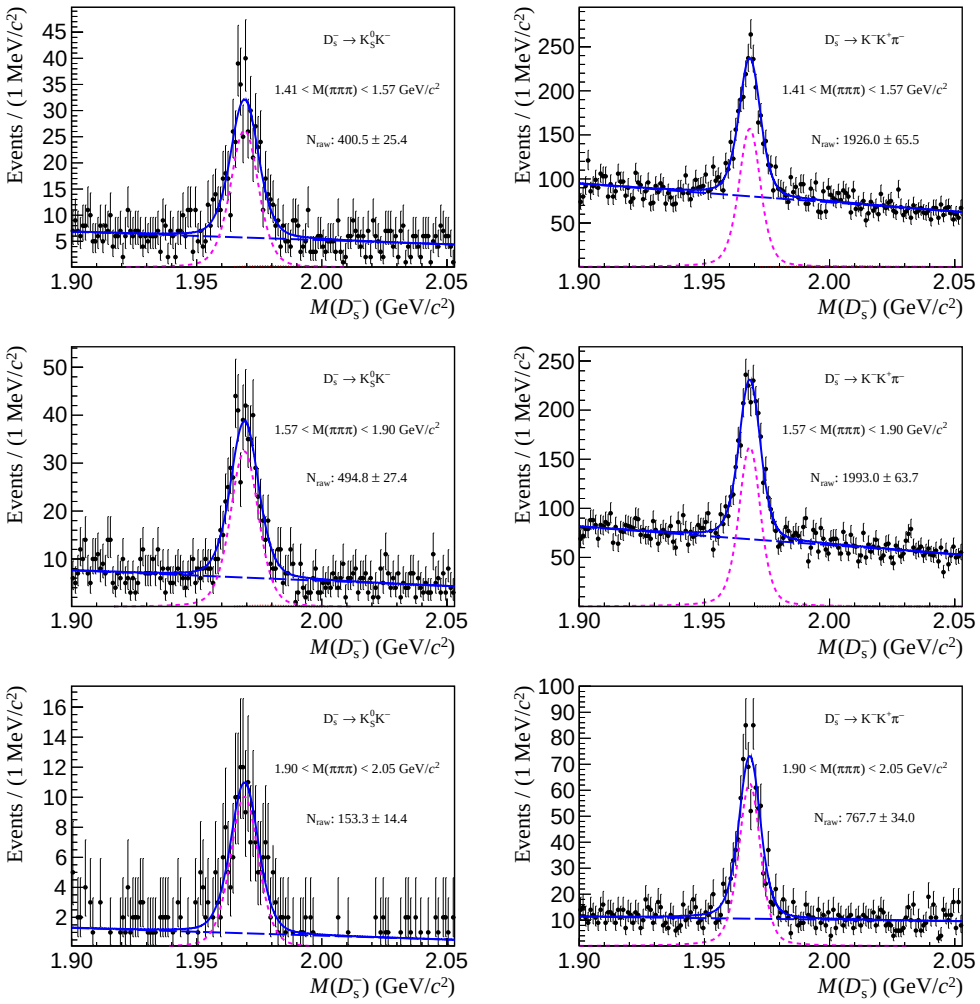

Figure 3: The invariant mass $M(D_s^-)$ distributions for the ST modes $D_s^- \to K_S^0 K^-$ (left) and $D_s^- \to K^- K^+ \pi^-$ (right), part 3/3. Data (black points) are shown overlaid with the fit results including the total (solid blue), signal PDF (dashed magenta), D^- background PDF (dotted red), and combinatorial background PDF (long-dashed blue) components.