Supplemental material of Precision measurement of $\Sigma^+ \to p\gamma$ decay in $J/\psi \to \Sigma^+\bar{\Sigma}^-$ process

I. FIT RESULT OF ST ANALYSIS

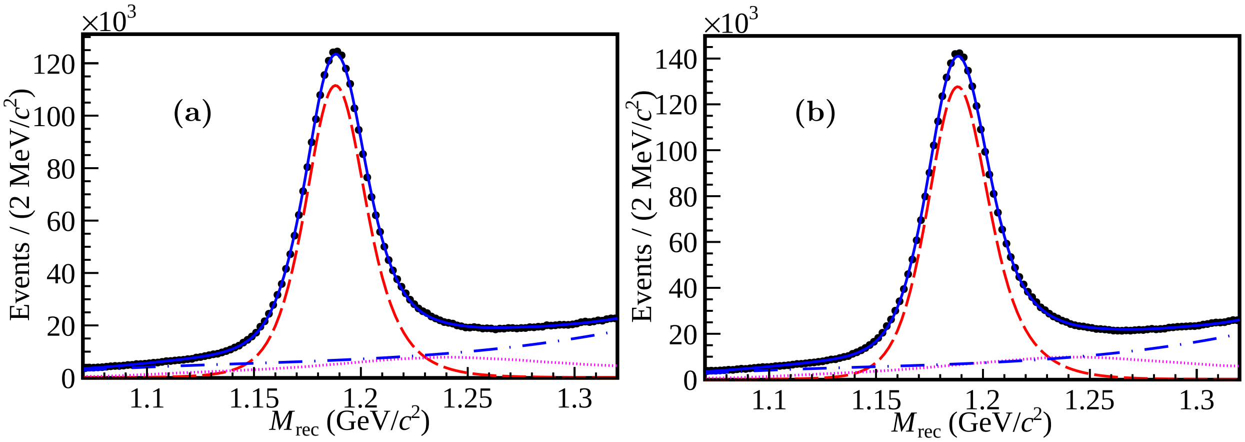

FIG. 1: Fit to the M_{rec} distribution of ST candidates in data with (a) $\bar{\Sigma}^-$ tagging and (b) Σ^+ tagging. Points with error bars represent data. The blue solid curve is the total fit result. The red dashed curve, the magenta dotted curve and the blue dash-dotted curve show the shapes of signal, $J/\psi \to \Delta(1232)^+(\to \text{anything})\bar{\Delta}(1232)^-(\to \bar{p}\pi^0)$ background and other non-peaking background components, respectively.

II. DETAILS ABOUT THE DECAY LENGTH REQUIREMENT IN DT ANALYSIS

The final state particles from the main background process ($\Delta(1232)^+ \to p\pi^0$) are decayed from the interaction point due to the vanish decay length of $\Delta(1232)^+$, while that from the signal process may deviate from the interaction point as Σ^+ has a decay length of 2.404 cm. So a secondary vertex fit is performed on the proton and anti-proton in final state. For events passing secondary vertex fit, Fig. 2 (a) shows the distribution of L/σ_L for $J/\psi \to \Sigma^+\bar{\Sigma}^-$ process, inclusive MC and data. According to the figure-of-merit (FoM) optimization also around signal's momentum region ($0.21 < P_p < 0.24\,\text{GeV}/c$), a event will be abandoned if both of the following requirements are fulfilled (Fig. 2 (b)): the event passes secondary vertex fit; the event has a L/σ_L less than 1.5. Events not passing secondary vertex fit and events passing secondary vertex fit with a L/σ_L larger than 1.5 are perserved for further analysis.

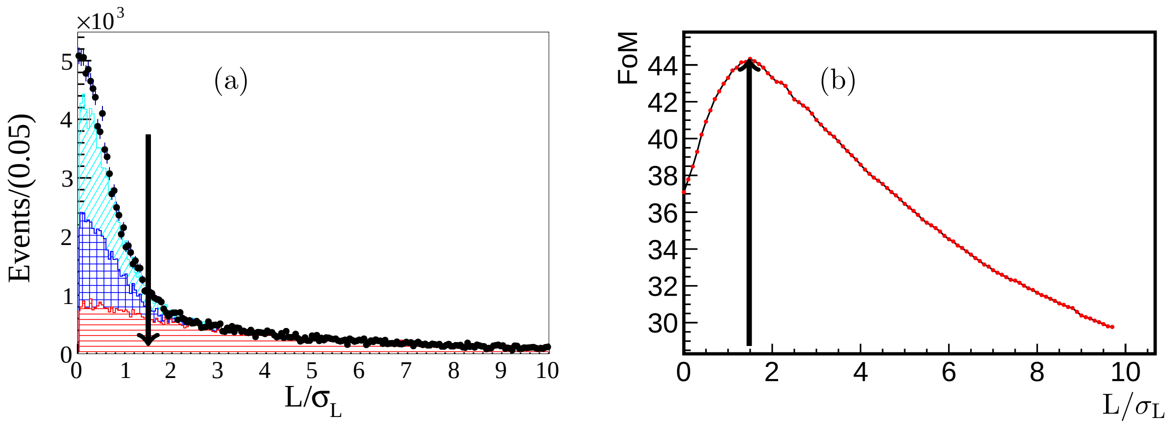

FIG. 2: (a) The L/σ_L distribution of data (black dots), signal (red dashed histogram) and background obtained from inclusive MC (blue and cyan dashed histograms). The black arrow in the plot indicates the cut value. (b) The FoM distribution of L/σ_L and the black arrow indicates the optimized cut.

III. SYSTEMATIC UNCERTAINTIES OF BF MEASUREMENT

The efficiency of tracking and PID for charged tracks and that from detection of photons are studied in terms of momentum (energy) and polar angle by $J/\psi \to p\bar{p}\pi^+\pi^-$ and $J/\psi \to \gamma\mu^+\mu^-$ control samples, and correction factors are extracted and implemented to compensate the differences of the efficiency between data and MC simulation. Therefore, the weighted uncertainties on the correction factors, 0.4 % per charged track including tracking and PID and 0.3 % per photons, are taken as the uncertainties. The uncertainty of the kinematic fit, including $\chi^2_{5C} < 30$ and $\chi^2_{5C} < \chi^2_{p\bar{p}\pi^0\gamma\gamma}$ requirements, is estimated with a control sample of $J/\psi \to \Sigma^+ (\to p\pi^0)\, \bar{\Sigma}^- (\to \bar{p}\pi^0)$, the resultant difference in efficiency between data and MC simulation, 0.9 %, is taken as the uncertainty. The uncertainty associated with the decay length is 0.4 %, estimated with the control sample $J/\psi \to \Sigma^+ (\to p\pi^0)\, \bar{\Sigma}^- (\to \bar{p}\pi^0)$, too. The uncertainties for the ST and DT yields from the fit are estimated by performing the alternative fits. In the alternative fits, the fit range, the parameters of the convolved Gaussian function are varied separately. The polynomial background is also changed from the 2nd order Chebyshev Polynomial to a 3rd order Chebyshev polynomial. The maximum resultant changes in the signal yields are taken as the uncertainties. The uncertainty of the ST yield is 0.4%, estimated by performing an alternative fit with a third order Chebyshev polynomial function for all background components due to the relatively broad distribution of the $J/\psi \to \Delta(1232)^+\bar{\Delta}(1232)^-$ process in the fit range. Other uncertainties associated with the fit range and the shape of the signal are negligible. The overall uncertainty of the DT yield is 1.2 %, which is the quadratic sum of the changes of the DT yields with alternative fits by varying the fit range, using an different polynomial function for background and varying the parameters of the convolved Gaussian function, individually. The uncertainty for the MC model is studied by varying the values of decay parameters α_ψ, $\Delta\Phi$ and $\bar{\alpha}_0$ within $\pm1\sigma$ according to values from Ref. [1], and varying the value of α_γ to that of the PDG in the MC simulation, the resultant change in the detection efficiency, 0.6 %, is taken as the systematic uncertainty. The Contribution of each uncertainty source is summarized in Table I.

IV. SYSTEMATIC UNCERTAINTIES OF α_γ MEASUREMENT

For the event selection criteria induced uncertainty in the α_γ measurement, only the angular dependent requirements are considered. The dependence of the angular distribution on one requirement is investigated using the moments $M_{1,2}$ defined before, which should be zero for the PHSP MC of the signal process. If a requirement changes the distribution

TABLE I: Summary of the systematic uncertainties in the BF measurement

Uncertainty source	BF uncertainty (%)
Tracking and PID	0.4
Photon detection	0.3
$\chi^2_{5C} < 30$	0.8
$\chi^2_{5C} < \chi^2_{p\bar{p}\pi^0\gamma\gamma}$	0.2
Decay length requirement	0.4
ST yield fit	0.4
DT fit range	0.8
DT Signal shape	0.2
DT $\Sigma^+ \to p\pi^0$ background shape	0.5
DT Polynomial background shape	0.8
Signal MC model	0.6
Total	1.8

of these two moments, it's considered as an angular dependent requirement. Specifically speaking, χ^2_{ang} induced by one requirement is defined as:

$$\chi^2_{ang} = \frac{1}{2m}\sum_{i=1}^{m}\frac{(M'_{1,i}-M_{1,i})^2}{\sigma^2_{1,i}} + \frac{(M'_{2,i}-M_{2,i})^2}{\sigma^2_{2,i}}, \tag{1}$$

where $M'_{1,i(2,i)}$ are the moments of the ith $\cos\theta_{\Sigma^+}$ interval with the requirement, $M_{1,i(2,i)}$ are that without the requirement and $\sigma_{1,i(2,i)}$ are the corresponding statistical uncertainties in the ith interval. When calculating the χ^2_{ang} of one event selection criteria, all the other cuts are applied. Requirements with χ^2_{ang} larger than 0.5 are regarded as angular dependent cuts, which is sufficient to distinguish an angular dependent requirement statistically speaking. The χ^2_{ang} value for each requirement is summarized in Table II, where track detection efficiency, decay length and $\chi^2_{\gamma} < \chi^2_{\pi^0}$ requirements are studied as uncertainty sources.

TABLE II: The χ^2_{ang} value for each requirement

Event selection Requirement	χ^2_{ang}
Track selection	10.31
ST selection	0.28
$\chi^2_{5C} < \chi^2_{p\bar{p}\pi^0\gamma\gamma}$	0.16
$\chi^2 < 30$	0.21
Decay length requirement	5.12
$\chi^2_{\gamma} < \chi^2_{\pi^0}$	15.95

The track detection efficiency induced uncertainty is studied by correcting the track detection efficiency of PHSP MC, corresponding to an uncertainty of 0.001. To study the decay length and $\chi^2_{\gamma} < \chi^2_{\pi^0}$ requirements induced uncertainty, the same angular distribution fit is applied to the $J/\psi \to \Sigma^+(\to p\pi^0)\,\bar{\Sigma}^-(\to \bar{p}\pi^0)$ control sample. The decay length requirement and the customized $\chi^2_{\pi^0} < \chi^2_{\gamma}$ requirement are applied to the control sample separately and the induced change on the decay parameter of the $\Sigma^+ \to p\pi^0$ decay is compared between data and MC, which is 0.005 and 0.006 for the two criteria respectively. Table III summarizes the systematic uncertainties associated with the α_γ measurement.

[1] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **125**, 052004 (2020).

TABLE III: Summary of the systematic uncertainties in decay parameter measurement

Source	α_γ uncertainty
Background description	0.004
Fixed decay parameters	0.011
Signal region	0.014
Track detection efficiency	0.001
Decay length requirement	0.005
$\chi^2_\gamma < \chi^2_{\pi^0}$	0.006
Total uncertainty	0.020