

Supplemental Material for “Absolute measurements of branching fractions of Cabibbo-suppressed hadronic $D^{0(+)}$ decays involving multiple pions”

M. Ablikim¹, M. N. Achasov^{11,b}, P. Adlarson⁷¹, M. Albrecht⁴, R. Aliberti³¹, A. Amoroso^{70A,70C}, M. R. An³⁵, Q. An^{67,53}, X. H. Bai⁶¹, Y. Bai⁵², O. Bakina³², R. Baldini Ferroli^{26A}, I. Balossino^{27A}, Y. Ban^{42,g}, V. Batozskaya^{1,40}, D. Becker³¹, K. Begzsuren²⁹, N. Berger³¹, M. Bertani^{26A}, D. Bettoni^{27A}, F. Bianchi^{70A,70C}, J. Bloms⁶⁴, A. Bortone^{70A,70C}, I. Boyko³², R. A. Briere⁵, A. Brueggemann⁶⁴, H. Cai⁷², X. Cai^{1,53}, A. Calcaterra^{26A}, G. F. Cao^{1,58}, N. Cao^{1,58}, S. A. Cetin^{57A}, J. F. Chang^{1,53}, W. L. Chang^{1,58}, G. Chelkov^{32,a}, C. Chen³⁹, G. Chen¹, H. S. Chen^{1,58}, M. L. Chen^{1,53}, S. J. Chen³⁸, T. Chen¹, X. R. Chen^{28,58}, X. T. Chen¹, Y. B. Chen^{1,53}, Z. J. Chen^{23,h}, W. S. Cheng^{70C}, X. Chu³⁹, G. Cibinetto^{27A}, F. Cossio^{70C}, J. J. Cui⁴⁵, H. L. Dai^{1,53}, J. P. Dai⁷⁴, A. Dbeysi¹⁷, R. E. de Boer⁴, D. Dedovich³², Z. Y. Deng¹, A. Denig³¹, I. Denysenko³², M. Destefanis^{70A,70C}, F. De Mori^{70A,70C}, Y. Ding³⁶, J. Dong^{1,53}, L. Y. Dong^{1,58}, M. Y. Dong^{1,53,58}, X. Dong⁷², S. X. Du⁷⁶, P. Egorov^{32,a}, Y. L. Fan⁷², J. Fang^{1,53}, S. S. Fang^{1,58}, W. X. Fang¹, Y. Fang¹, R. Farinelli^{27A}, L. Fava^{70B,70C}, F. Feldbauer⁴, G. Felici^{26A}, C. Q. Feng^{67,53}, J. H. Feng⁵⁴, K. Fischer⁶⁵, M. Fritsch⁴, C. Fritzsch⁶⁴, C. D. Fu¹, H. Gao⁵⁸, Y. N. Gao^{42,g}, Yang Gao^{67,53}, S. Garbolino^{70C}, I. Garzia^{27A,27B}, P. T. Ge⁷², Z. W. Ge³⁸, C. Geng⁵⁴, E. M. Gersabeck⁶², A. Gilman⁶⁵, K. Goetzen¹², L. Gong³⁶, W. X. Gong^{1,53}, W. Gradl³¹, M. Greco^{70A,70C}, L. M. Gu³⁸, M. H. Gu^{1,53}, Y. T. Gu¹⁴, C. Y. Guan^{1,58}, A. Q. Guo^{28,58}, L. B. Guo³⁷, R. P. Guo⁴⁴, Y. P. Guo^{10,f}, A. Guskov^{32,a}, T. T. Han⁴⁵, W. Y. Han³⁵, X. Q. Hao¹⁸, F. A. Harris⁶⁰, K. K. He⁵⁰, K. L. He^{1,58}, F. H. Heinsius⁴, C. H. Heinz³¹, Y. K. Heng^{1,53,58}, C. Herold⁵⁵, M. Himmelreich^{12,d}, T. Holtmann⁴, G. Y. Hou^{1,58}, Y. R. Hou⁵⁸, Z. L. Hou¹, H. M. Hu^{1,58}, J. F. Hu^{51,i}, T. Hu^{1,53,58}, Y. Hu¹, G. S. Huang^{67,53}, K. X. Huang⁵⁴, L. Q. Huang^{28,58}, L. Q. Huang⁶⁸, X. T. Huang⁴⁵, Y. P. Huang¹, Z. Huang^{42,g}, T. Hussain⁶⁹, N. Hüskens^{25,31}, W. Imoehl²⁵, M. Irshad^{67,53}, J. Jackson²⁵, S. Jaeger⁴, S. Janchiv²⁹, Q. Ji¹, Q. P. Ji¹⁸, X. B. Ji^{1,58}, X. L. Ji^{1,53}, Y. Y. Ji⁴⁵, Z. K. Jia^{67,53}, H. B. Jiang⁴⁵, S. S. Jiang³⁵, X. S. Jiang^{1,53,58}, Y. Jiang⁵⁸, J. B. Jiao⁴⁵, Z. Jiao²¹, S. Jin³⁸, Y. Jin⁶¹, M. Q. Jing^{1,58}, T. Johansson⁷¹, N. Kalantar-Nayestanaki⁵⁹, X. S. Kang³⁶, R. Kappert⁵⁹, M. Kavatsyuk⁵⁹, B. C. Ke⁷⁶, I. K. Keshk⁴, A. Khoukaz⁶⁴, P. Kiese³¹, R. Kiuchi¹, R. Kliemt¹², L. Koch³³, O. B. Kolcu^{57A}, B. Kopf⁴, M. Kuemmel⁴, M. Kuessner⁴, A. Kupsc^{40,71}, W. Kühn³³, J. J. Lane⁶², J. S. Lange³³, P. Larin¹⁷, A. Lavania²⁴, L. Lavezzi^{70A,70C}, Z. H. Lei^{67,53}, H. Leithoff³¹, M. Lellmann³¹, T. Lenz³¹, C. Li⁴³, C. Li³⁹, C. H. Li³⁵, Cheng Li^{67,53}, D. M. Li⁷⁶, F. Li^{1,53}, G. Li¹, H. Li⁴⁷, H. Li^{67,53}, H. B. Li^{1,58}, H. J. Li¹⁸, H. N. Li^{51,i}, J. Q. Li⁴, J. S. Li⁵⁴, J. W. Li⁴⁵, Ke Li¹, L. J. Li¹, L. K. Li¹, Lei Li³, M. H. Li³⁹, P. R. Li^{34,j,k}, S. X. Li¹⁰, S. Y. Li⁵⁶, T. Li⁴⁵, W. D. Li^{1,58}, W. G. Li¹, X. H. Li^{67,53}, X. L. Li⁴⁵, Xiaoyu Li^{1,58}, H. Liang^{67,53}, H. Liang³⁰, H. Liang^{1,58}, Y. F. Liang⁴⁹, Y. T. Liang^{28,58}, G. R. Liao¹³, L. Z. Liao⁴⁵, J. Libby²⁴, A. Limphirat⁵⁵, C. X. Lin⁵⁴, D. X. Lin^{28,58}, T. Lin¹, B. J. Liu¹, C. X. Liu¹, D. Liu^{17,67}, F. H. Liu⁴⁸, Fang Liu¹, Feng Liu⁶, G. M. Liu^{51,i}, H. Liu^{34,j,k}, H. B. Liu¹⁴, H. M. Liu^{1,58}, Huanhuan Liu¹, Huihui Liu¹⁹, J. B. Liu^{67,53}, J. L. Liu⁶⁸, J. Y. Liu^{1,58}, K. Liu¹, K. Y. Liu³⁶, Ke Liu²⁰, L. Liu^{67,53}, Lu Liu³⁹, M. H. Liu^{10,f}, P. L. Liu¹, Q. Liu⁵⁸, S. B. Liu^{67,53}, T. Liu^{10,f}, W. K. Liu³⁹, W. M. Liu^{67,53}, X. Liu^{34,j,k}, Y. Liu^{34,j,k}, Y. B. Liu³⁹, Z. A. Liu^{1,53,58}, Z. Q. Liu⁴⁵, X. C. Lou^{1,53,58}, F. X. Lu⁵⁴, H. J. Lu²¹, J. G. Lu^{1,53}, X. L. Lu¹, Y. Lu⁷, Y. P. Lu^{1,53}, Z. H. Lu¹, C. L. Luo³⁷, M. X. Luo⁷⁵, T. Luo^{10,f}, X. L. Luo^{1,53}, X. R. Lyu⁵⁸, Y. F. Lyu³⁹, F. C. Ma³⁶, H. L. Ma¹, L. L. Ma⁴⁵, M. M. Ma^{1,58}, Q. M. Ma¹, R. Q. Ma^{1,58}, R. T. Ma⁵⁸, X. Y. Ma^{1,53}, Y. Ma^{42,g}, F. E. Maas¹⁷, M. Maggiore^{70A,70C}, S. Maldaner⁴, S. Malde⁶⁵, Q. A. Malik⁶⁹, A. Mangoni^{26B}, Y. J. Mao^{42,g}, Z. P. Mao¹, S. Marcello^{70A,70C}, Z. X. Meng⁶¹, J. G. Messchendorp^{59,12}, G. Mezzadri^{27A}, H. Miao¹, T. J. Min³⁸, R. E. Mitchell²⁵, X. H. Mo^{1,53,58}, N. Yu. Muchnoi^{11,b}, H. Muramatsu⁶³, Y. Nefedov³², F. Nerling^{12,d}, I. B. Nikolaev^{11,b}, Z. Ning^{1,53}, S. Nisar^{9,l}, Y. Niu⁴⁵, S. L. Olsen⁵⁸, Q. Ouyang^{1,53,58}, S. Pacetti^{26B,26C}, X. Pan^{10,f}, Y. Pan⁶², A. Pathak³⁰, M. Pelizaeus⁴, H. P. Peng^{67,53}, K. Peters^{12,d}, J. L. Ping³⁷, R. G. Ping^{1,58}, S. Plura³¹, S. Pogodin³², R. Poling⁶³, V. Prasad^{67,53}, F. Z. Qi¹, H. Qi^{67,53}, H. R. Qi⁵⁶, M. Qi³⁸, T. Y. Qi^{10,f}, S. Qian^{1,53}, W. B. Qian⁵⁸, Z. Qian⁵⁴, C. F. Qiao⁵⁸, J. J. Qin⁶⁸, L. Q. Qin¹³, X. P. Qin^{10,f}, X. S. Qin⁴⁵, Z. H. Qin^{1,53}, J. F. Qiu¹, S. Q. Qu⁵⁶, K. H. Rashid⁶⁹, C. F. Redmer³¹, K. J. Ren³⁵, A. Rivetti^{70C}, V. Rodin⁵⁹, M. Roloff^{70C}, G. Rong^{1,58}, Ch. Rosner¹⁷, S. N. Ruan³⁹, H. S. Sang⁶⁷, A. Sarantsev^{32,c}, Y. Schelhaas³¹, C. Schnier⁴, K. Schoenning⁷¹, M. Scodeggio^{27A,27B}, K. Y. Shan^{10,f}, W. Shan²², X. Y. Shan^{67,53}, J. F. Shangguan⁵⁰, L. G. Shao^{1,58}, M. Shao^{67,53}, C. P. Shen^{10,f}, H. F. Shen^{1,58}, X. Y. Shen^{1,58}, B. A. Shi⁵⁸, H. C. Shi^{67,53}, J. Y. Shi¹, R. S. Shi^{1,58}, X. Shi^{1,53}, X. D. Shi^{67,53}, J. J. Song¹⁸, W. M. Song^{30,1}, Y. X. Song^{42,g}, S. Sosio^{70A,70C}, S. Spataro^{70A,70C}, F. Stieler³¹, K. X. Su⁷², P. P. Su⁵⁰, Y. J. Su⁵⁸, G. X. Sun¹, H. Sun⁵⁸, H. K. Sun¹, J. F. Sun¹⁸, L. Sun⁷², S. S. Sun^{1,58}, T. Sun^{1,58}, W. Y. Sun³⁰, X. Sun^{23,h}, Y. J. Sun^{67,53}, Y. Z. Sun¹, Z. T. Sun⁴⁵, Y. H. Tan⁷², Y. X. Tan^{67,53},

C. J. Tang⁴⁹, G. Y. Tang¹, J. Tang⁵⁴, L. Y. Tao⁶⁸, Q. T. Tao^{23,h}, J. X. Teng^{67,53}, V. Thoren⁷¹, W. H. Tian⁴⁷, Y. Tian^{28,58}, I. Uman^{57B}, B. Wang¹, B. L. Wang⁵⁸, C. W. Wang³⁸, D. Y. Wang^{42,g}, F. Wang⁶⁸, H. J. Wang^{34,j,k}, H. P. Wang^{1,58}, K. Wang^{1,53}, L. L. Wang¹, M. Wang⁴⁵, M. Z. Wang^{42,g}, Meng Wang^{1,58}, S. Wang¹³, S. Wang^{10,f}, T. Wang^{10,f}, T. J. Wang³⁹, W. Wang⁵⁴, W. H. Wang⁷², W. P. Wang^{67,53}, X. Wang^{42,g}, X. F. Wang^{34,j,k}, X. L. Wang^{10,f}, Y. D. Wang⁴¹, Y. F. Wang^{1,53,58}, Y. H. Wang⁴³, Y. Q. Wang¹, Yaqian Wang^{16,1}, Z. Wang^{1,53}, Z. Y. Wang^{1,58}, Ziyi Wang⁵⁸, D. H. Wei¹³, F. Weidner⁶⁴, S. P. Wen¹, D. J. White⁶², U. Wiedner⁴, G. Wilkinson⁶⁵, M. Wolke⁷¹, L. Wollenberg⁴, J. F. Wu^{1,58}, L. H. Wu¹, L. J. Wu^{1,58}, X. Wu^{10,f}, X. H. Wu³⁰, Y. Wu⁶⁷, Y. J. Wu²⁸, Z. Wu^{1,53}, L. Xia^{67,53}, T. Xiang^{42,g}, D. Xiao^{34,j,k}, G. Y. Xiao³⁸, H. Xiao^{10,f}, S. Y. Xiao¹, Y. L. Xiao^{10,f}, Z. J. Xiao³⁷, C. Xie³⁸, X. H. Xie^{42,g}, Y. Xie⁴⁵, Y. G. Xie^{1,53}, Y. H. Xie⁶, Z. P. Xie^{67,53}, T. Y. Xing^{1,58}, C. F. Xu¹, C. J. Xu⁵⁴, G. F. Xu¹, H. Y. Xu⁶¹, Q. J. Xu¹⁵, X. P. Xu⁵⁰, Y. C. Xu⁵⁸, Z. P. Xu³⁸, F. Yan^{10,f}, L. Yan^{10,f}, W. B. Yan^{67,53}, W. C. Yan⁷⁶, H. J. Yang^{46,e}, H. L. Yang³⁰, H. X. Yang¹, L. Yang⁴⁷, S. L. Yang⁵⁸, Tao Yang¹, Y. F. Yang³⁹, Y. X. Yang^{1,58}, Yifan Yang^{1,58}, M. Ye^{1,53}, M. H. Ye⁸, J. H. Yin¹, Z. Y. You⁵⁴, B. X. Yu^{1,53,58}, C. X. Yu³⁹, G. Yu^{1,58}, T. Yu⁶⁸, C. Z. Yuan^{1,58}, L. Yuan², S. C. Yuan¹, X. Q. Yuan¹, Y. Yuan^{1,58}, Z. Y. Yuan⁵⁴, C. X. Yue³⁵, A. A. Zafar⁶⁹, F. R. Zeng⁴⁵, X. Zeng⁶, Y. Zeng^{23,h}, Y. H. Zhan⁵⁴, A. Q. Zhang¹, B. L. Zhang¹, B. X. Zhang¹, D. H. Zhang³⁹, G. Y. Zhang¹⁸, H. Zhang⁶⁷, H. H. Zhang³⁰, H. H. Zhang⁵⁴, H. Y. Zhang^{1,53}, J. L. Zhang⁷³, J. Q. Zhang³⁷, J. W. Zhang^{1,53,58}, J. X. Zhang^{34,j,k}, J. Y. Zhang¹, J. Z. Zhang^{1,58}, Jianyu Zhang^{1,58}, Jiawei Zhang^{1,58}, L. M. Zhang⁵⁶, L. Q. Zhang⁵⁴, Lei Zhang³⁸, P. Zhang¹, Q. Y. Zhang^{35,76}, Shuihan Zhang^{1,58}, Shulei Zhang^{23,h}, X. D. Zhang⁴¹, X. M. Zhang¹, X. Y. Zhang⁴⁵, X. Y. Zhang⁵⁰, Y. Zhang⁶⁵, Y. T. Zhang⁷⁶, Y. H. Zhang^{1,53}, Yan Zhang^{67,53}, Yao Zhang¹, Z. H. Zhang¹, Z. Y. Zhang⁷², Z. Y. Zhang³⁹, G. Zhao¹, J. Zhao³⁵, J. Y. Zhao^{1,58}, J. Z. Zhao^{1,53}, Lei Zhao^{67,53}, Ling Zhao¹, M. G. Zhao³⁹, Q. Zhao¹, S. J. Zhao⁷⁶, Y. B. Zhao^{1,53}, Y. X. Zhao^{28,58}, Z. G. Zhao^{67,53}, A. Zhemchugov^{32,a}, B. Zheng⁶⁸, J. P. Zheng^{1,53}, Y. H. Zheng⁵⁸, B. Zhong³⁷, C. Zhong⁶⁸, X. Zhong⁵⁴, H. Zhou⁴⁵, L. P. Zhou^{1,58}, X. Zhou⁷², X. K. Zhou⁵⁸, X. R. Zhou^{67,53}, X. Y. Zhou³⁵, Y. Z. Zhou^{10,f}, J. Zhu³⁹, K. Zhu¹, K. J. Zhu^{1,53,58}, L. X. Zhu⁵⁸, S. H. Zhu⁶⁶, S. Q. Zhu³⁸, T. J. Zhu⁷³, W. J. Zhu^{10,f}, Y. C. Zhu^{67,53}, Z. A. Zhu^{1,58}, B. S. Zou¹, J. H. Zou¹

(BESIII Collaboration)

¹ Institute of High Energy Physics, Beijing 100049, People's Republic of China

² Beihang University, Beijing 100191, People's Republic of China

³ Beijing Institute of Petrochemical Technology, Beijing 102617, People's Republic of China

⁴ Bochum Ruhr-University, D-44780 Bochum, Germany

⁵ Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA

⁶ Central China Normal University, Wuhan 430079, People's Republic of China

⁷ Central South University, Changsha 410083, People's Republic of China

⁸ China Center of Advanced Science and Technology, Beijing 100190, People's Republic of China

⁹ COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan

¹⁰ Fudan University, Shanghai 200433, People's Republic of China

¹¹ G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia

¹² GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany

¹³ Guangxi Normal University, Guilin 541004, People's Republic of China

¹⁴ Guangxi University, Nanning 530004, People's Republic of China

¹⁵ Hangzhou Normal University, Hangzhou 310036, People's Republic of China

¹⁶ Hebei University, Baoding 071002, People's Republic of China

¹⁷ Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany

¹⁸ Henan Normal University, Xinxiang 453007, People's Republic of China

¹⁹ Henan University of Science and Technology, Luoyang 471003, People's Republic of China

²⁰ Henan University of Technology, Zhengzhou 450001, People's Republic of China

²¹ Huangshan College, Huangshan 245000, People's Republic of China

²² Hunan Normal University, Changsha 410081, People's Republic of China

²³ Hunan University, Changsha 410082, People's Republic of China

²⁴ Indian Institute of Technology Madras, Chennai 600036, India

²⁵ Indiana University, Bloomington, Indiana 47405, USA

²⁶ INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy;

(B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy

²⁷ INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy

²⁸ Institute of Modern Physics, Lanzhou 730000, People's Republic of China

²⁹ Institute of Physics and Technology, Peace Avenue 54B, Ulaanbaatar 13330, Mongolia

³⁰ Jilin University, Changchun 130012, People's Republic of China

³¹ Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany

³² Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

³³ Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany

³⁴ Lanzhou University, Lanzhou 730000, People's Republic of China

³⁵ Liaoning Normal University, Dalian 116029, People's Republic of China

³⁶ Liaoning University, Shenyang 110036, People's Republic of China

³⁷ Nanjing Normal University, Nanjing 210023, People's Republic of China

³⁸ Nanjing University, Nanjing 210093, People's Republic of China

³⁹ Nankai University, Tianjin 300071, People's Republic of China

⁴⁰ National Centre for Nuclear Research, Warsaw 02-093, Poland

⁴¹ North China Electric Power University, Beijing 102206, People's Republic of China

⁴² Peking University, Beijing 100871, People's Republic of China

⁴³ Qufu Normal University, Qufu 273165, People's Republic of China

⁴⁴ Shandong Normal University, Jinan 250014, People's Republic of China

⁴⁵ Shandong University, Jinan 250100, People's Republic of China

⁴⁶ Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

⁴⁷ Shanxi Normal University, Linfen 041004, People's Republic of China

⁴⁸ Shanxi University, Taiyuan 030006, People's Republic of China

⁴⁹ Sichuan University, Chengdu 610064, People's Republic of China

⁵⁰ Soochow University, Suzhou 215006, People's Republic of China

⁵¹ South China Normal University, Guangzhou 510006, People's Republic of China

⁵² Southeast University, Nanjing 211100, People's Republic of China

⁵³ State Key Laboratory of Particle Detection and Electronics,

Beijing 100049, Hefei 230026, People's Republic of China

⁵⁴ Sun Yat-Sen University, Guangzhou 510275, People's Republic of China

⁵⁵ Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand

⁵⁶ Tsinghua University, Beijing 100084, People's Republic of China

⁵⁷ Turkish Accelerator Center Particle Factory Group, (A)Istinye University, 34010, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey

⁵⁸ University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

⁵⁹ University of Groningen, NL-9747 AA Groningen, The Netherlands

⁶⁰ University of Hawaii, Honolulu, Hawaii 96822, USA

⁶¹ University of Jinan, Jinan 250022, People's Republic of China

⁶² University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom

⁶³ University of Minnesota, Minneapolis, Minnesota 55455, USA

⁶⁴ University of Muenster, Wilhelm-Klemm-Strasse 9, 48149 Muenster, Germany

⁶⁵ University of Oxford, Keble Road, Oxford OX13RH, United Kingdom

⁶⁶ University of Science and Technology Liaoning, Anshan 114051, People's Republic of China

⁶⁷ University of Science and Technology of China, Hefei 230026, People's Republic of China

⁶⁸ University of South China, Hengyang 421001, People's Republic of China

⁶⁹ University of the Punjab, Lahore-54590, Pakistan

⁷⁰ University of Turin and INFN, (A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy

⁷¹ Uppsala University, Box 516, SE-75120 Uppsala, Sweden

⁷² Wuhan University, Wuhan 430072, People's Republic of China

⁷³ Xinyang Normal University, Xinyang 464000, People's Republic of China

⁷⁴ Yunnan University, Kunming 650500, People's Republic of China

⁷⁵ Zhejiang University, Hangzhou 310027, People's Republic of China

⁷⁶ Zhengzhou University, Zhengzhou 450001, People's Republic of China

^a Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia

^b Also at the Novosibirsk State University, Novosibirsk, 630090, Russia

^c Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia

^d Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany

^e Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China

^f Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People's Republic of China

^g Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of China

^h Also at School of Physics and Electronics, Hunan University, Changsha 410082, China

ⁱ Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China

^j Also at Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, People's Republic of China

^k Also at Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China

^l Also at the Department of Mathematical Sciences, IBA, Karachi , Pakistan

Figures 1 to 8 show comparisons between data and MC simulations for the distributions of invariant mass spectra of two-, three-, four- or five-body particle combinations, momenta and $\cos\theta$ of daughter particles for the signal DT candidates with more than 100 signal events. The candidates must satisfy additional requirements of $|M_{BC}^{\text{tag(sig)}} - M_D| < 0.006 \text{ GeV}/c^2$ and multiple possible combinations of daughter particles are all plotted when relevant (e.g., all four $\pi^+\pi^-$ combinations for $D^0 \rightarrow 2\pi^+2\pi^-\pi^0$, etc.)

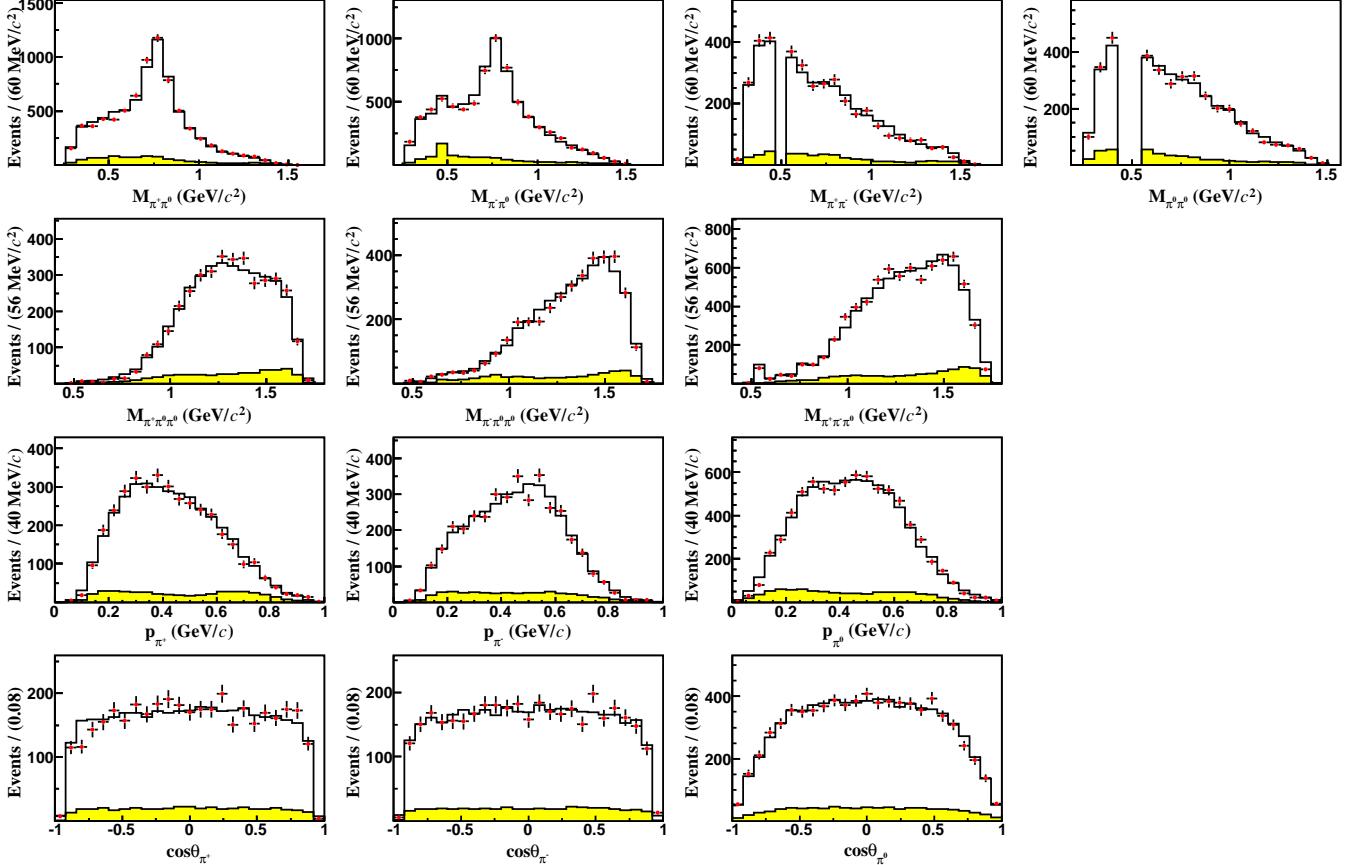


Fig. 1. Comparisons of the distributions of invariant masses of two- or three-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^0 \rightarrow \pi^+\pi^-2\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

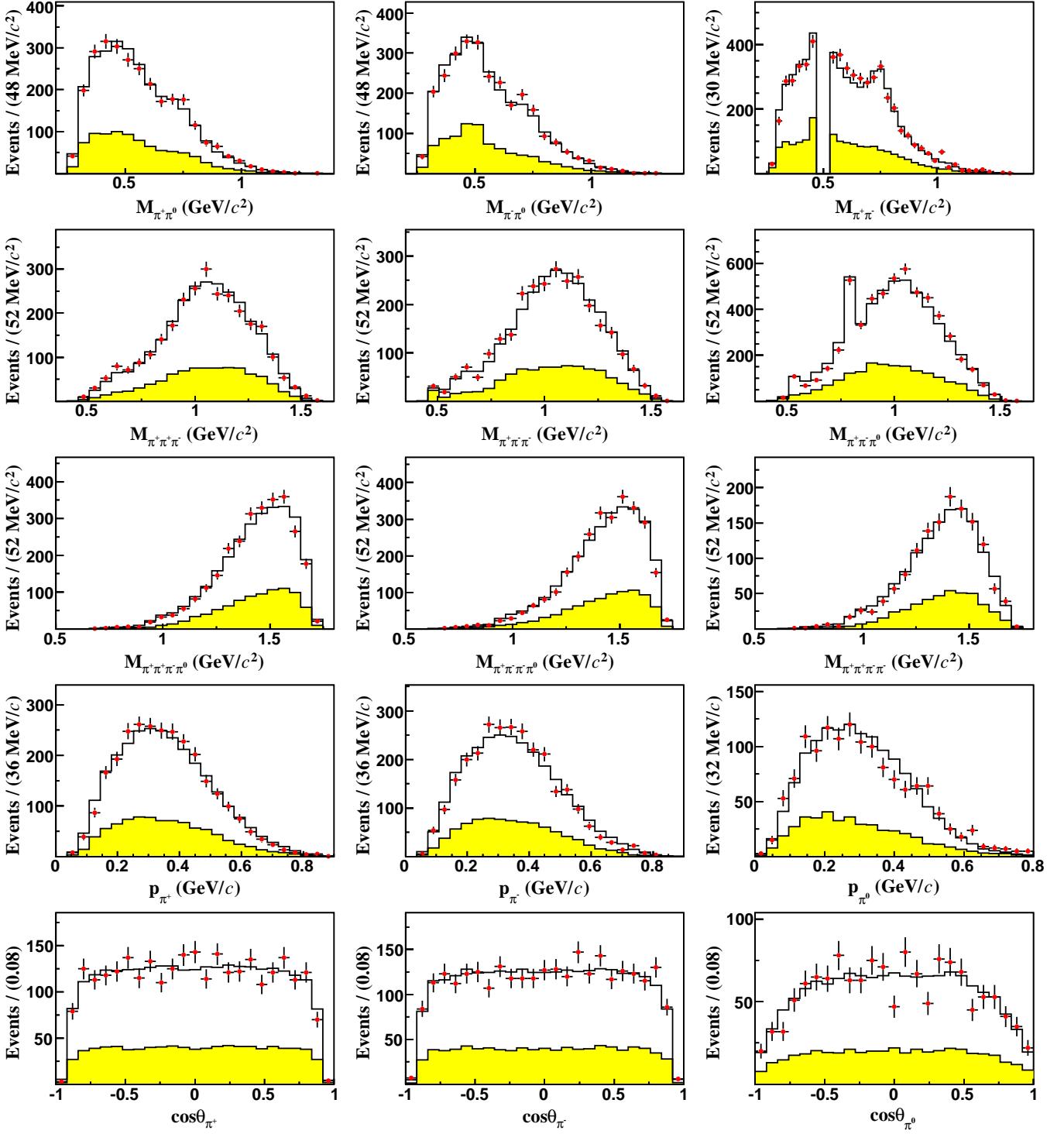


Fig. 2. Comparisons of the distributions of invariant masses of two-, three-, or four-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^0 \rightarrow 2\pi^+ 2\pi^- \pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

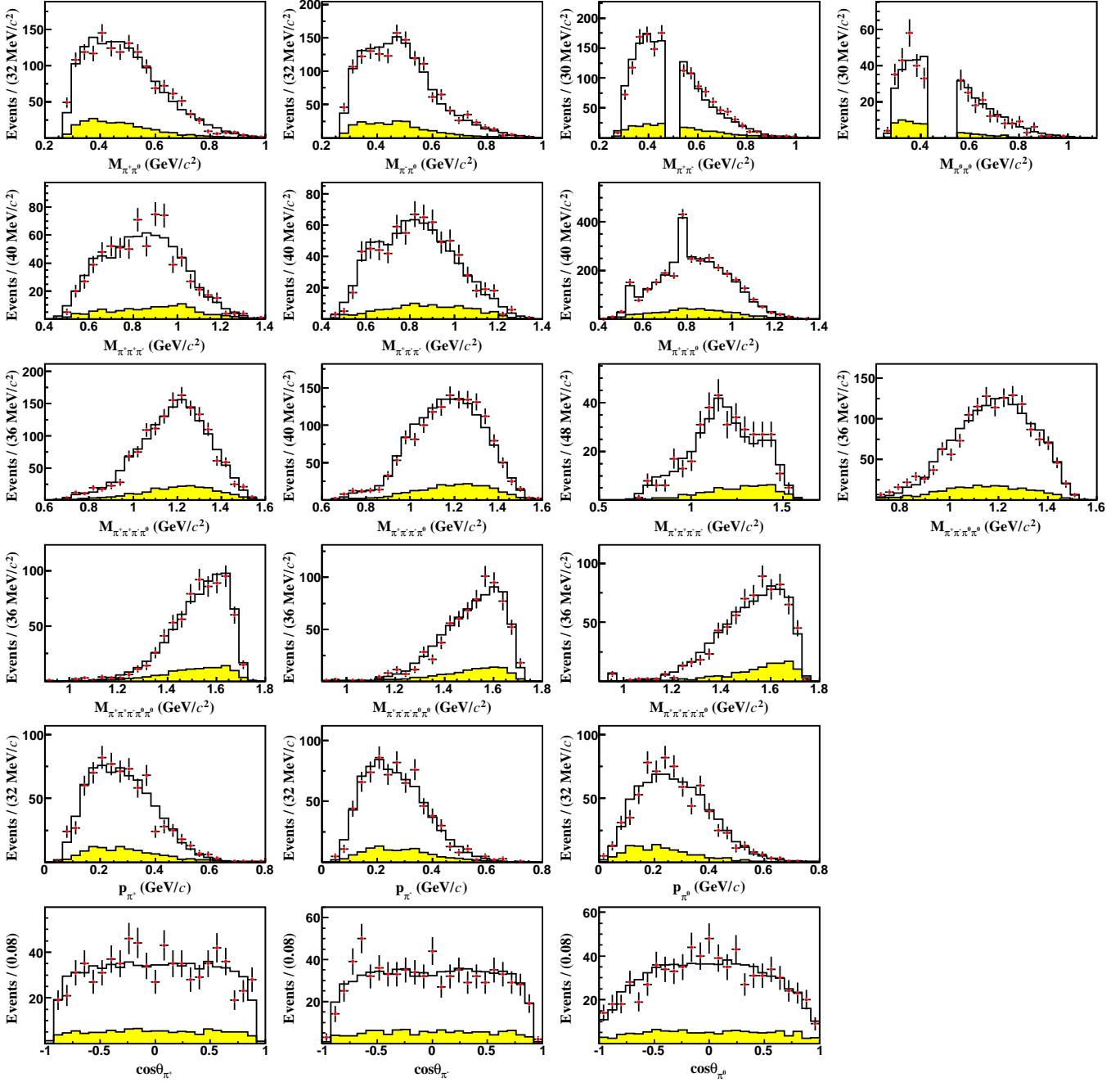


Fig. 3. Comparisons of the distributions of invariant masses of two-, three-, four- or five-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^0 \rightarrow 2\pi^+ 2\pi^- 2\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

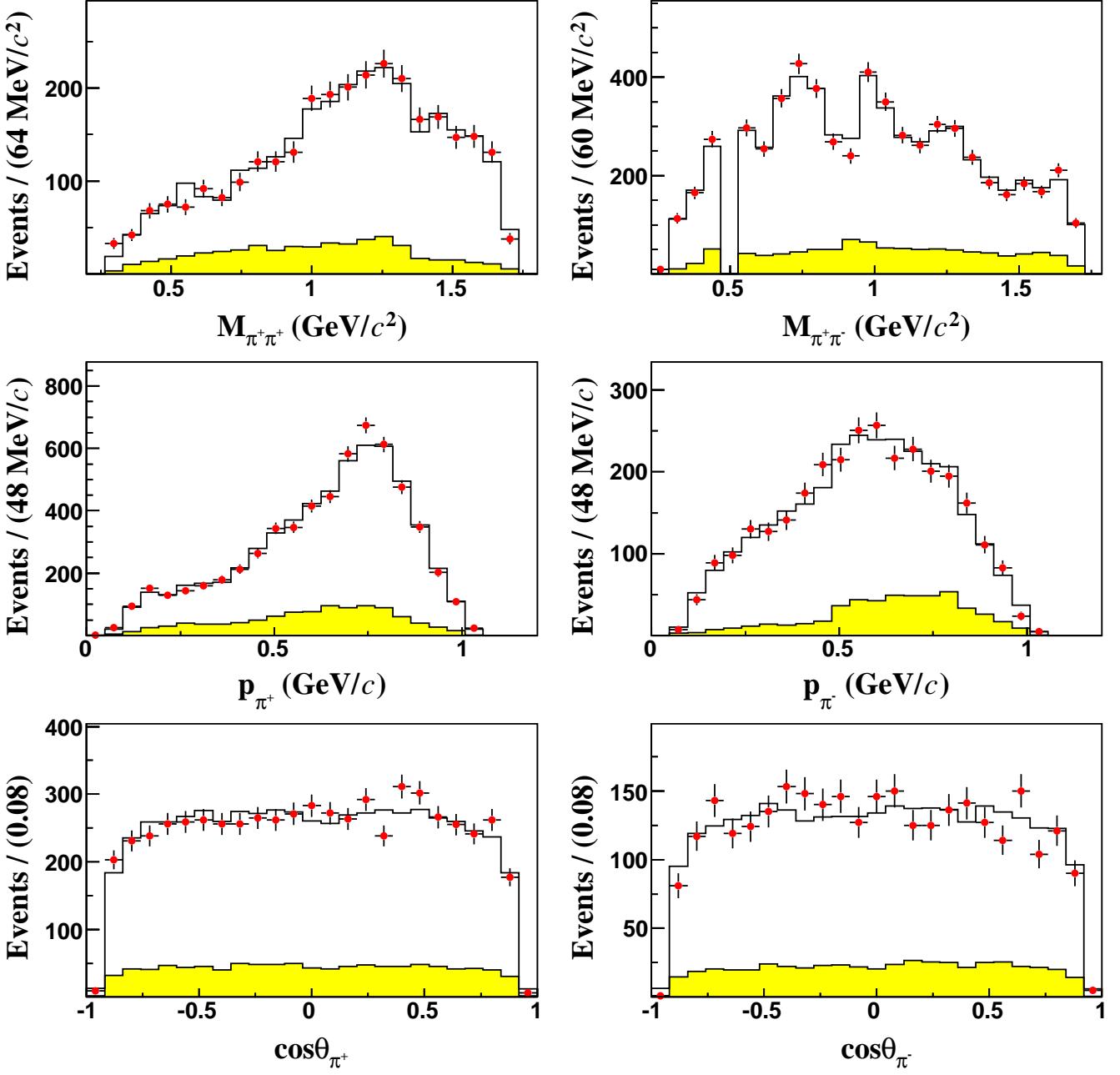


Fig. 4. Comparisons of the distributions of invariant masses of two-body particle combinations, momenta and $\cos \theta$ of daughter particles for the $D^+ \rightarrow 2\pi^+\pi^-$ candidates between data (points with error bars) and the BODY3 signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

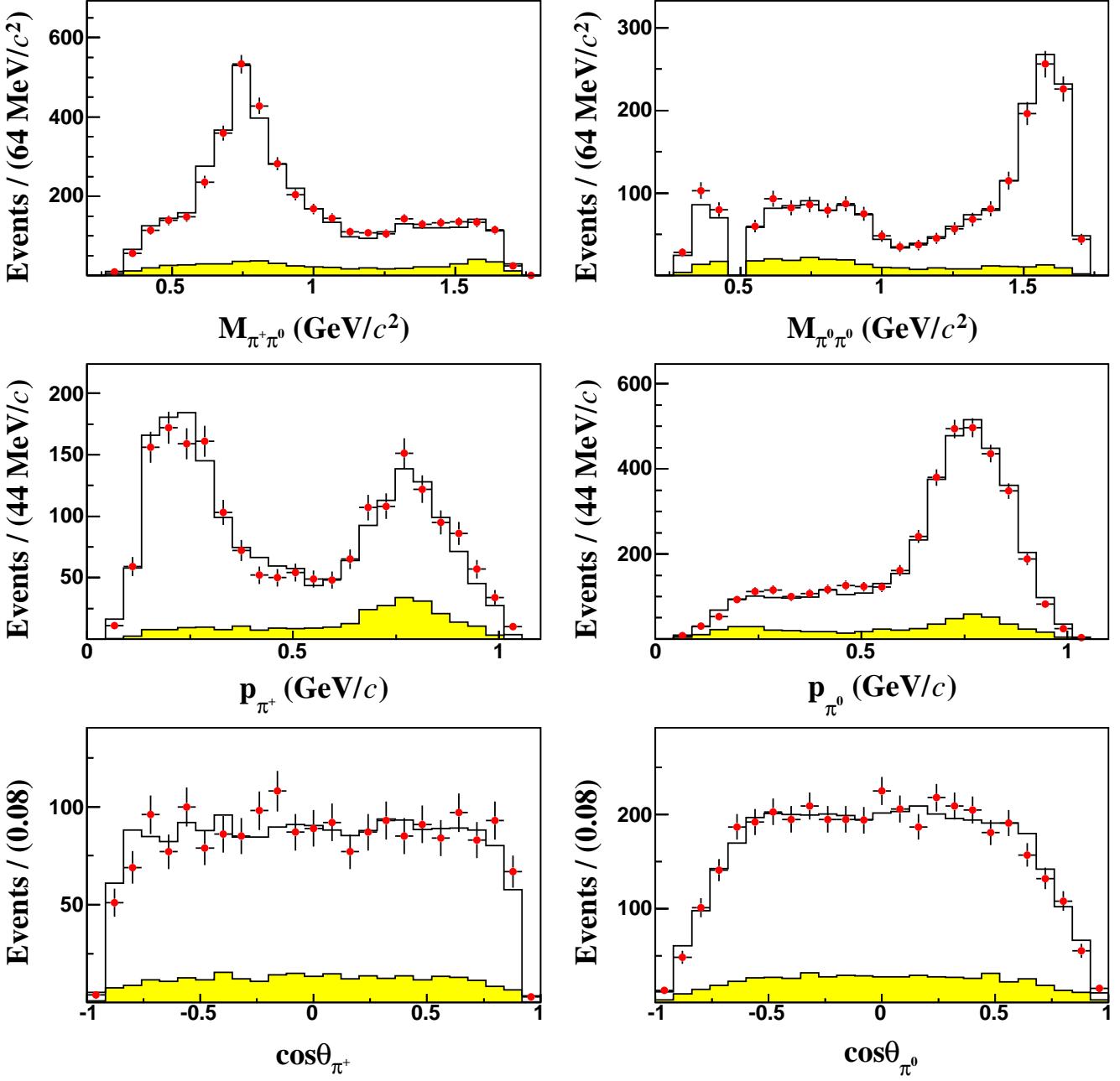


Fig. 5. Comparisons of the distributions of invariant masses of two-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^+ \rightarrow \pi^+ 2\pi^0$ candidates between data (points with error bars) and the BODY3 signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

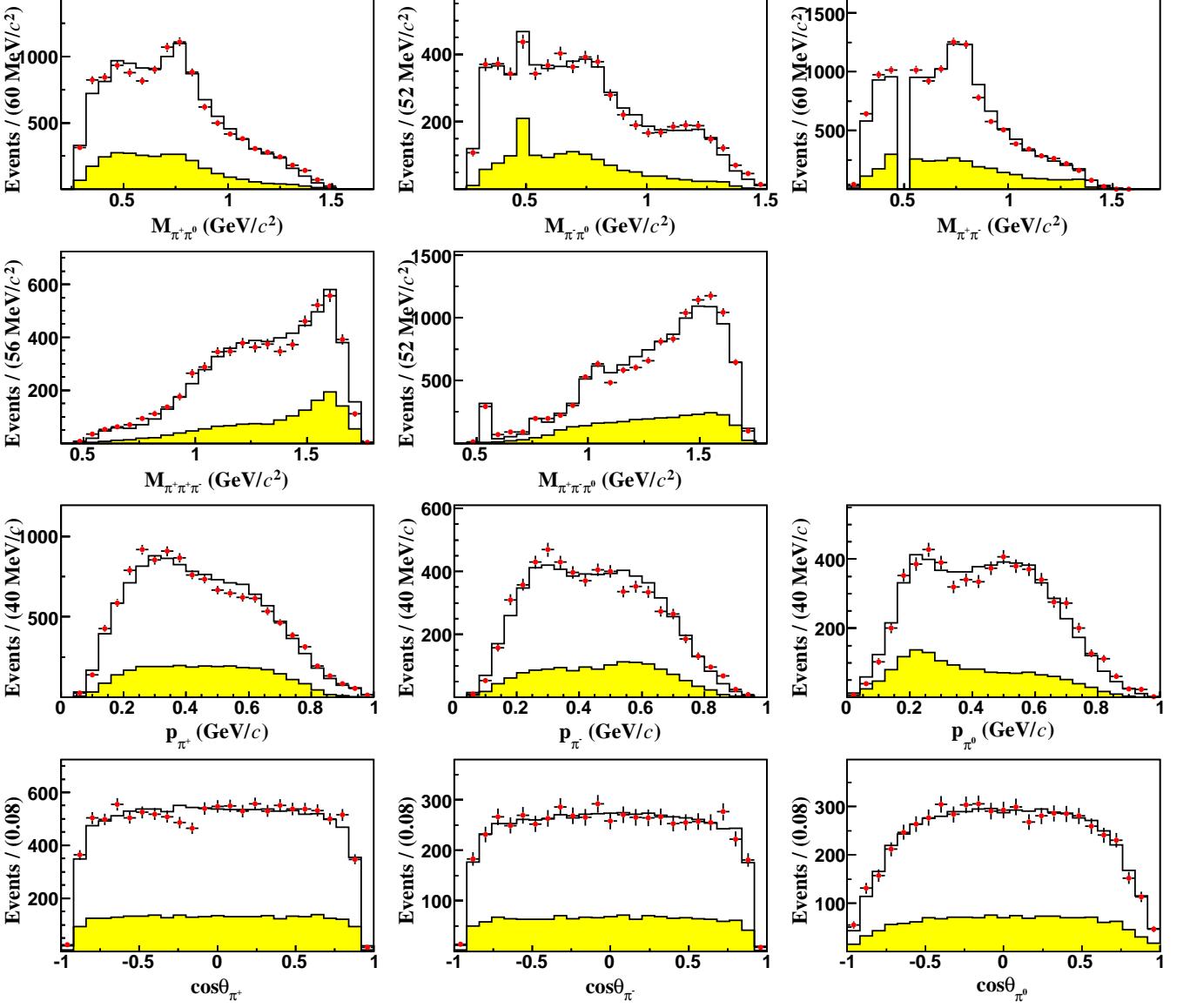


Fig. 6. Comparisons of the distributions of invariant masses of two- or three-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^+ \rightarrow 2\pi^+\pi^-\pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

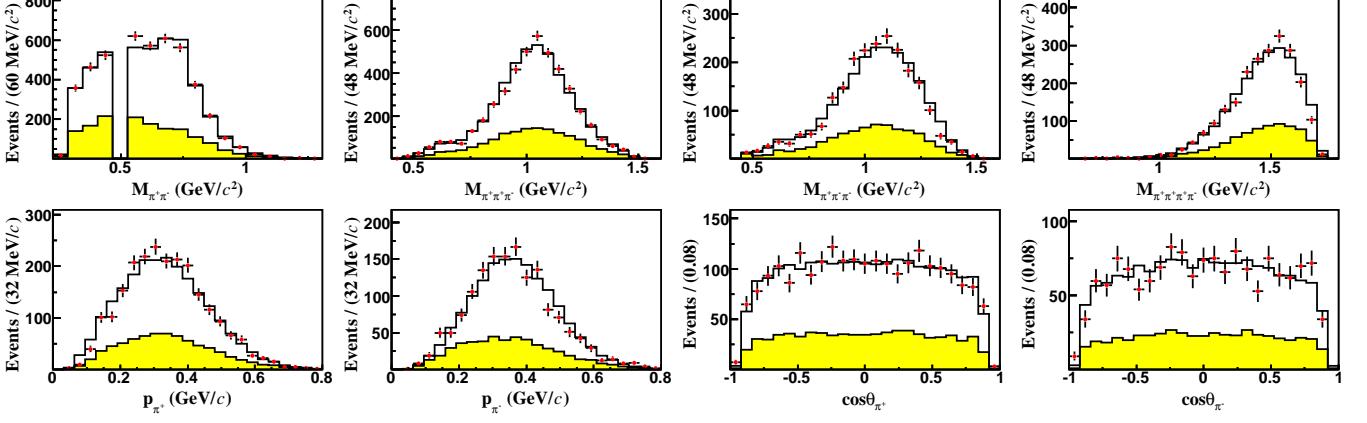


Fig. 7. Comparisons of the distributions of invariant masses of two-, three- or four-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^+ \rightarrow 3\pi^+ 2\pi^-$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).

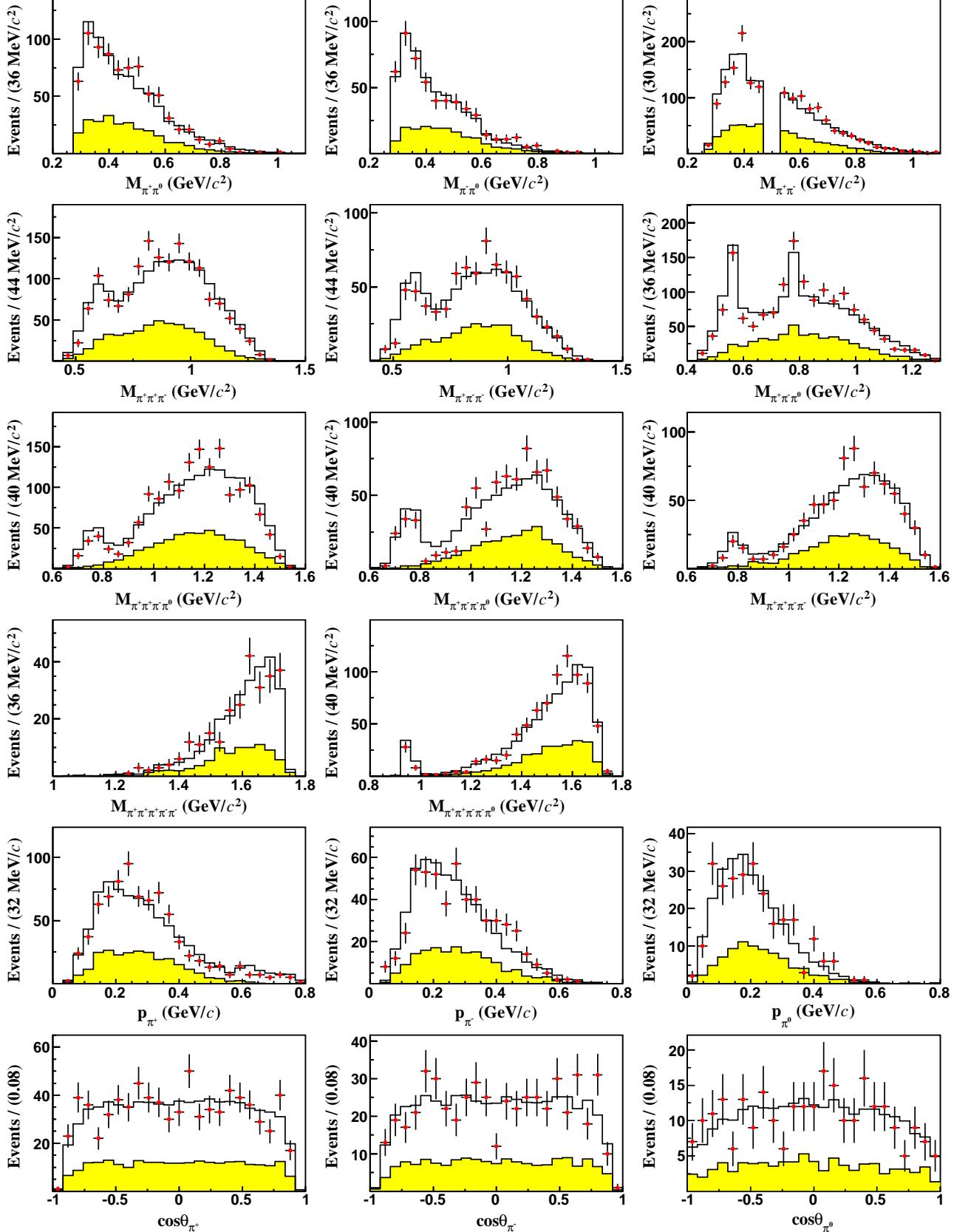


Fig. 8. Comparisons of the distributions of invariant masses of two-, three-, four- or five-body particle combinations, momenta and $\cos\theta$ of daughter particles for the $D^+ \rightarrow 3\pi^+ 2\pi^- \pi^0$ candidates between data (points with error bars) and the mixing signal MC events (black solid line histograms) plus the MC-simulated backgrounds from the inclusive MC sample (yellow filled histograms).