Observation of the $Y(4230)$ and a new structure in $e^+e^- \to K^+K^-J/\psi$

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The cross sections of $e^+e^- \to K^+K^-J/\psi$ at center-of-mass energies from 4.127 to 4.600 GeV are measured based on 15.6 fb^{-1} data collected with the BESIII detector operating at the BEPCII storage ring. Two resonant structures are observed in the line shape of the cross sections. The mass and width of the first structure are measured to be $(4225.3 \pm 2.3 \pm 21.5)$ MeV and $(72.9 \pm 6.1 \pm 30.8)$ MeV, respectively. They are consistent with those of the established $Y(4230)$. The second structure is observed for the first time with a statistical significance greater than 8σ, denoted as $Y(4500)$. Its mass and width are determined to be $(4484.7 \pm 13.3 \pm 24.1)$ MeV and $(111.1 \pm 30.1 \pm 15.2)$ MeV, respectively. The first presented uncertainties are statistical and the second ones are systematic. The product of the electronic partial width with the decay branching fraction $\Gamma(Y(4230) \to e^+e^-)\mathcal{B}(Y(4230) \to K^+K^-J/\psi)$ is reported.

Recent discoveries of charmonium-like states expand our perspective of the hadron spectrum around the τ-charm energy region, and provide excellent laboratories to study perturbative and non-perturbative strong interaction. The dynamic, however, is more complex than the one of conventional mesons due to possible additional dergeees of freedom. Therefore their nature has not yet been established despite many different speculations about them being hybrids, tetra-quarks, molecules,

cusp effects, and so on [1].

Among the exotic states, the $Y(4230)$ state, previously called $Y(4260)$, is the first observed vector charmonium-like state. It was discovered in $e^+e^- \to \pi^+\pi^-J/\psi$ channel by the BaBar Collaboration using initial-state-radiation (ISR) technique [2], and confirmed by CLEO [3] and Belle [4]. In addition, the $Y(4230)$ state was observed in various modes by BESIII, including $e^+e^- \to \pi\pi J/\psi$ [5, 6], $\pi^+\pi^-h_c$ [7], $\pi\pi\psi(2S)$ [8–10], $\omega\chi_{c0}$ [11, 12],

and $\pi^+ D^0 D^{*-} + c.c.$ [13]. It was also found to decay into $X(3872)$ via radiative transition [14] and to $Z_c(3900)$ via pion transition [6]. A better understanding of its internal structure and quark components is crucial, and it will also be helpful to understand the series of the tetra-quark candidates [15].

Two recent measurements on the cross sections of $e^+e^- \to \eta J/\psi$ [16] and $e^+e^- \to \eta' J/\psi$ [17] indicate a considerable strange quark component in $Y(4230)$. This hypothesis was analyzed in Ref. [18]. However, a concrete conclusion is still missing due to the large uncertainty. Thus a measurement of $Y(4230) \to K\bar{K}J/\psi$ is important to clarify the puzzle by comparison with the results of $Y(4230) \to \pi\pi J/\psi$. The first evidence for $Y(4230) \to K^+K^-J/\psi$ was found by CLEO [19]. Later, the cross sections of $e^+e^- \to K^+K^-J/\psi$ at center-of-mass (c.m.) energies between threshold and 6.0 GeV were measured for the first time via ISR process [20] and was updated [21] by Belle. No significant signal was observed in both measurements, and an upper limit $\mathcal{B}(Y(4230) \to K^+K^-J/\psi)\Gamma(Y(4230) \to e^+e^-) < 1.7$ eV at 90% confidence level was obtained. Here and following the natural unit system is adopted, i.e., $\hbar = c = 1$. Recently, BESIII has measured the cross sections of $e^+e^- \to K\bar{K}J/\psi$ at c.m. energies from 4.189 to 4.600 GeV with an integrated luminosity ($\mathcal{L}_{int}$) of 4.7 fb^{-1} [22]. No significant signal of the $Y(4230)$ is observed, but evidence of a structure around $\sqrt{s} = 4.5$ GeV is found. The structure with higher mass is consistent with the calculation by Ref. [23], which suggests a conventional charmonium state $\psi(4500)$ with mass of $4489 - 4529$ MeV by the 5S-4D mixing scheme. It is consistent with a virtual state predicted in Ref. [24] too, where the mass of the heavy-antiheavy hadronic molecule is calculated to be $4483 - 4503$ MeV that is just below the $D_s\bar{D}_{s1}$ threshold. Meanwhile, Ref. [25] predicts an exotic state with $c\bar{c}s\bar{s}$ component in quenched lattice quantum chromodynamics with exact chiral symmetry, and its mass is predicted to be 4450 ± 100 MeV.

In this Letter, we present an updated measurement of the Born cross sections of $e^+e^- \to K^+K^-J/\psi$ at the c.m. energies from 4.127 to 4.600 GeV, using data samples composed of twenty-eight c.m. energy points [26, 27] with $\mathcal{L}_{int} = 15.6$ fb^{-1} [28, 29], collected at the BESIII detector operating at the BEPCII storage ring [30]. In addition, although previously in Ref. [22] a full reconstruction method is applied, considering the low efficiency of kaon reconstruction with low momentum, a new partial reconstruction method is applied improving significantly the efficiency.

The BESIII detector is described in detail in Ref. [30, 31]. The GEANT4-based [32] Monte Carlo (MC) simulation software framework BOOST [33], which consists of detector geometry and its response, is used to produce large simulated event samples. These samples are used to optimize the event selection criteria, determine the detection efficiency, evaluate the ISR correction factor, and estimate background contributions. The signal events are generated at each c.m. energy, where signal events include $e^+e^- \to K^+K^-J/\psi$ (phase space (PHSP) model), $f_0(980)J/\psi$, and $f_2(1270)J/\psi$. Both $f_0(980)$ and $f_2(1270)$ decay into K^+K^-, and J/ψ decays into one lepton pair ($\mu^+\mu^-/e^+e^-$). The simulation includes the beam energy spread and ISR in the e^+e^- annihilations modelled with the generator KKMC [34, 35] and EVTGEN [36, 37], where the angular distributions according to spin and parity are considered by the specific models implemented in the generator. The final state radiation effect associated with leptons is handled by the PHOTOS package [38]. The potential backgrounds are estimated with inclusive MC events generated by KKMC, where the known decay modes are simulated with EVTGEN using branching fractions taken from the Particle Data Group (PDG) [39], and the remaining unknown decays are simulated with LUNDCHARM [40, 41].

A pair of leptons ($e^+e^-/\mu^+\mu^-$) and at least one kaon is required for a signal candidate. For each track, the polar angle θ in the main drift chamber must satisfy $|\cos\theta| < 0.93$, and the point of closest approach to e^+e^- interaction point must be less than 10.0 cm in the beam direction and 1.0 cm in the plane perpendicular to the beam direction. Each charged track with momentum larger than 1.0 GeV in the laboratory frame is assumed to be a lepton. The ratio of the energy deposited in the electromagnetic calorimeter over the momentum of each lepton candidate is required to be greater than 0.8 for electrons and less than 0.8 for muons. For particle identification (PID), the energy loss in the main drift chamber and the time measured with the time-of-flight system are combined to calculate the confidence levels ($C.L.$) with kaon and pion hypotheses, and the confidence level is required to satisfy $C.L.(K) > 0.001$ and $C.L.(K) > C.L.(\pi)$ for each kaon candidate. In order to improve resolution and suppress backgrounds, a vertex fit and a kinematic fit with one constraint on the mass of the missing kaon are performed. If there is more than one kaon track candidate, the selected one has the smallest χ_F^2, defined as the sum of the χ^2 of vertex and kinematic fits and required to be less than 20. To remove radiative Bhabha background, where the radiative photon would convert into an e^+e^- pair, all pairs of oppositely charged tracks must have an opening angle $\cos(\theta_{open}) < 0.98$ for the e^+e^- mode. For the $\mu^+\mu^-$ mode, the penetration depth for one of the muon candidate in the muon counter is required to be greater than 40 cm to remove hadron backgrounds.

After applying aforementioned event selection, the invariant mass distributions of e^+e^- and $\mu^+\mu^-$ at $\sqrt{s} = 4.600$ GeV are shown in Fig. 1. The signal region of J/ψ is defined as [3.061, 3.142] GeV and sideband regions of J/ψ are defined as [2.970, 3.051] and [3.152, 3.233] GeV at $\sqrt{s} = 4.600$ GeV, while these intervals are slightly dif-

ferent for different energies since the mass resolution is energy dependent. The distributions of $M(\ell^+\ell^-)$ at all the energies are presented in Supplemental Material [42], where $\ell^+\ell^-$ is either e^+e^- or $\mu^+\mu^-$. There are no peaking backgrounds based on the study of inclusive MC samples. The yields of signals (N^{obs}) are obtained from background subtraction and the corresponding uncertainties are estimated by the profile likelihood method [43].

To explore potential intermediate states, the invariant mass distributions of K^+K^-, K^-J/ψ, and K^+J/ψ are shown in Fig. 2, with all data samples, PHSP signal MC samples, and the weighted signal MC samples, where PHSP and weighted signal MC samples are both normalized to data. There is no obvious structure in the KJ/ψ invariant mass distributions. The distributions of invariant mass of K^+K^- show signs of mesons $f_0(980)$ and $f_2(1270)$. These distributions roughly match with the theoretical calculation at $\sqrt{s} = 4.23$ and 4.26 GeV [44], while the corresponding predictions are still missing at higher energy regions. We extract the contributions of different components (PHSP, $f_0(980)$, $f_2(1270)$) by fitting to the data samples with large signal yields ($N^{obs} > 55$), then expanding the results to the smaller samples by linear interpolation. The efficiency of event selection (ε) is calculated by the sum of weighted efficiencies of these components.

The Born cross section of $e^+e^- \to K^+K^-J/\psi$ at each energy is obtained by

$$\sigma^B(\sqrt{s}) = \frac{N^{obs}}{\mathcal{L}_{int} \cdot \varepsilon \cdot (1+\delta)_{ISR} \cdot \frac{1}{|1-\Pi|^2} \cdot \mathcal{B}(J/\psi \to \ell^+\ell^-)}, \tag{1}$$

where $\frac{1}{|1-\Pi|^2}$ is the vacuum polarization factor calculated by QED [45], $\mathcal{B}(J/\psi \to \ell^+\ell^-)$ is the branching fraction that is quoted as $(11.93\pm0.05)\%$ [39], $(1+\delta)_{ISR}$ is the radiative correction factor obtained by an iteration method, described in details in Ref. [46]. The results of Born cross sections are listed in Supplemental Material [42]. Compared with the previous BESIII measurements [22], the Born cross sections at the same c.m. energies are consistent and the precision is slightly improved. The observation of two clear structures in the distribution of the cross sections is due to more data samples used.

A maximum likelihood fit is applied to the dressed cross sections ($\sigma^D(\sqrt{s})$, including vacuum polarization effects) of $e^+e^- \to K^+K^-J/\psi$ to determine the parameters of the resonant structures, and the dressed cross sections are shown in Fig. 3. The likelihood is constructed taking the fluctuations of the number of signal events into account. Its definition is described in the Supplemental Material [42]. The fit function is parameterized as a coherent sum of two relativistic Breit-Wigner (BW) functions

$$\sigma^D(\sqrt{s}) = |BW_1(\sqrt{s}) \cdot e^{i\varphi} + BW_2(\sqrt{s})|^2, \tag{2}$$

where φ is the relative phase angle, and

$$BW(\sqrt{s}) = \frac{M}{\sqrt{s}} \cdot \frac{\sqrt{12\pi\Gamma_{ee}\Gamma_{tot}\mathcal{B}}}{s - M^2 + iM\Gamma_{tot}} \cdot \sqrt{\frac{\Phi(\sqrt{s})}{\Phi(M)}}, \tag{3}$$

where M, Γ_{tot}, Γ_{ee}, and $\mathcal{B}$ are the mass, full width, electronic partial width, and branching fraction of corresponding resonance, respectively. $\Phi(\sqrt{s}) = \int\int \frac{1}{(2\pi)^3 32(\sqrt{s})^3} dm_{12}^2 dm_{23}^2$ is the three-body phase space [39], where m_{ij} is the invariant mass of particles i and j. The fitting curve is shown in Fig. 3, and the fit quality is estimated to be $\chi^2/\text{n.d.f} = 37.45 / 21$, where n.d.f is the number of degrees of freedom. The resonance with lower mass is consistent with the previously established $Y(4230)$. Its mass and width are determined to be $M(Y(4230)) = (4225.3 \pm 2.3)$ MeV and $\Gamma(Y(4230)) = (72.9 \pm 6.1)$ MeV. Since there is no observed state corresponding to the resonance with higher mass, we name it as $Y(4500)$. Its mass and width are determined to be $M(Y(4500)) = (4484.7 \pm 13.3)$ MeV, $\Gamma(Y(4500)) = (111.1\pm30.1)$ MeV, respectively. The statistical significances of $Y(4230)$ and $Y(4500)$ have been estimated to be 29σ and 8σ, respectively, via the differences of the likelihood values and the degrees of freedom with and without considering the corresponding resonance by the Wilk's theorem [47]. Two solutions with equal goodness-of-fit qualities are found, which are consistent with the mathematical expectation of multiple solutions [48]. The products of the electron partial width and branching fraction, of the states $Y(4230)$ and $Y(4500)$, are listed in Table I.

The main sources of systematic uncertainties of the cross section measurements are: the integrated luminosity $\mathcal{L}_{int}$, the tracking and PID efficiency, the branching fraction of $J/\psi \to \ell^+\ell^-$, kinematic fit, $(1 + \delta)_{ISR}$, the intermediate structures of K^+K^- system, the resolution of J/ψ, and the requirement on the penetration depth in the muon counter. The $\mathcal{L}_{int}$ is measured with Bhabha events and the uncertainty is found to be 1.0% [28, 29]. The differences between data and MC in the tracking and PID efficiencies are studied by using the process $e^+e^- \to K^+K^-\pi^+\pi^-$, and 2.5% per charged kaon is quoted [22]. For charged leptons, the systematic uncertainty is studied by using the process $\psi(3686) \to \pi^+\pi^- J/\psi(J/\psi \to \ell^+\ell^-)$, and 1.0% uncertainty per lepton is quoted [49]. The uncertainty of the branching fraction of $J/\psi \to \ell^+\ell^-$ is quoted as 0.4% [39]. The uncertainty of kinematic fit is estimated by correcting the corresponding track parameters and the difference between the efficiencies with (default) and without this correction is quoted as the relevant uncertainty [50]. Varied line shape of the input cross sections is constructed by connecting each nearby points with a smooth curve. The difference between the final cross sections with this new line shape and the nominal one is taken as the uncertainty of radiation correction. The uncertainty asso-
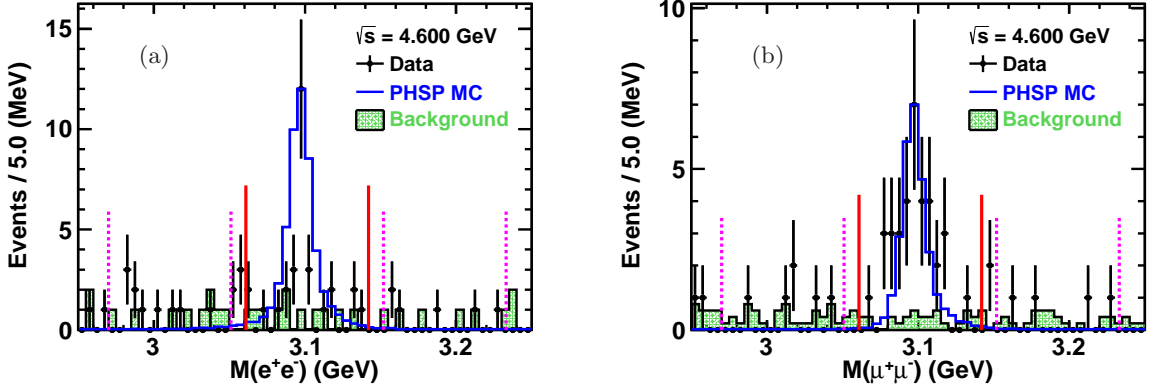
FIG. 1. The invariant mass distributions of the lepton pairs from different J/ψ decay modes at $\sqrt{s} = 4.600$ GeV. (a) is e^+e^- mode, (b) is $\mu^+\mu^-$ mode, where the dots with error bars indicate data, the blue line histograms indicate PHSP signal MC sample, the filled histograms indicate backgrounds from inclusive MC sample, the red solid lines and pink dashed lines identify the J/ψ signal and sideband regions, respectively.
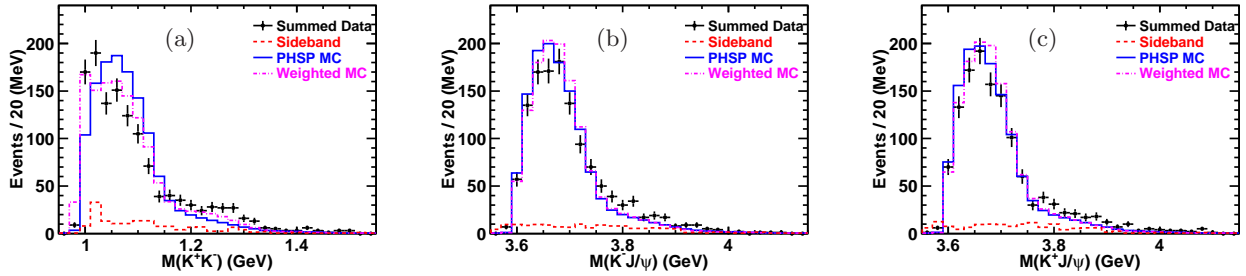

FIG. 2. (a), (b), (c) are distributions of $M(K^+K^-)$, $M(K^-J/\psi)$, and $M(K^+J/\psi)$, respectively. The black dots with error bars indicate data from the J/ψ signal region with all data samples, the red dashed curves indicate data from J/ψ sideband regions, the blue solid curves indicate PHSP signal MC sample, and the yank dashed-dot curves indicated the weighted signal MC sample.
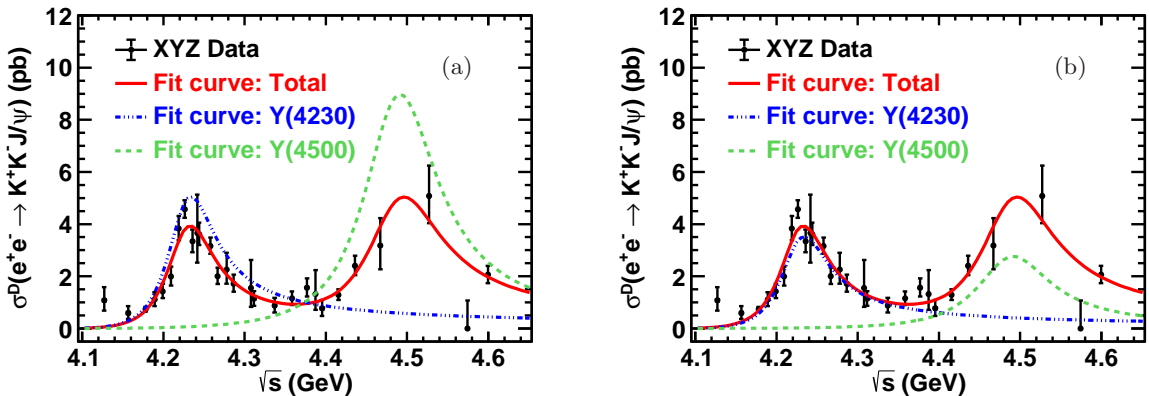

FIG. 3. Dressed cross sections of $e^+e^- \to K^+K^-J/\psi$, indicated by error bars with only statistical uncertainties. They are fitted by a coherent sum of two Breit-Wigner functions indicated by red solid curves, and the blue and pink dashed curves are the amplitudes describing the resonances $Y(4230)$ and $Y(4500)$, respectively. (a) corresponds to solution I, (b) corresponds to solution II.

TABLE I. Fitted parameters of the two resonant structures observed in the cross sections of $e^+e^- \to K^+K^-J/\psi$, where the first uncertainty is statistical and the second one is systematic.

	Parameters	Solution I	Solution II
Y(4230)	M(MeV)	$4225.3 \pm 2.3 \pm 21.5$	
	Γ_{tot}(MeV)	$72.9 \pm 6.1 \pm 30.8$	
	$\Gamma_{ee}\mathcal{B}$(eV)	$0.42 \pm 0.04 \pm 0.15$	$0.29 \pm 0.02 \pm 0.10$
Y(4500)	M(MeV)	$4484.7 \pm 13.3 \pm 24.1$	
	Γ_{tot}(MeV)	$111.1 \pm 30.1 \pm 15.2$	
	$\Gamma_{ee}\mathcal{B}$(eV)	$1.35 \pm 0.14 \pm 0.06$	$0.41 \pm 0.08 \pm 0.13$
phase angle	φ(rad)	$1.72 \pm 0.09 \pm 0.52$	$5.49 \pm 0.35 \pm 0.58$

ciated with the intermediate structures is estimated by weighting the PHSP MC samples according to the observed $M(K^+K^-)$ distribution of data, and the difference between the two efficiencies of the two methods is quoted as the corresponding uncertainty. To estimate the uncertainty according to the difference in the resolutions between data and MC samples, the efficiency is re-obtained by smearing the resolution of the J/ψ signal of the MC sample. The difference is quoted as the associated uncertainty. To consider the uncertainty of the criteria on the penetration depth in the muon counter, the difference between the final results with and without this criteria is adopted. The total uncertainties are calculated by summing all individual items in quadrature. They are energy dependent and vary from 8.6% to 11.1%. All the systematic uncertainties, including the individual and total, are listed in Supplemental Material [42].

The systematic uncertainties for the parameters of resonances mainly come from c.m. energy measurements, the form and parameterization of the fit function, and the systematic uncertainties in the cross section measurements that are discussed before. The c.m. energies were measured with $e^+e^- \to \mu^+\mu^-$ events and the uncertainties are determined correspondingly for different data samples [26, 27]. The associated systematic uncertainty is estimated by varying the c.m. energies during the fit. A three-body PHSP shape for non-resonant component is added to the two coherent BW sum to estimate the uncertainty of the cross section description in the fit, which turns out to be negligible. The uncertainty of the formalism of the full width is estimated by replacing the Γ_{tot} with $\Gamma = \Gamma_{tot}\frac{\Phi(\sqrt{s})}{\Phi(M)}$ in the denominator of Eq. (3), where Γ_{tot} is the nominal width of the resonance. The systematic uncertainties of the two resonance parameters can be divided into two categories. The first one is uncorrelated among the different c.m. energy points including kinematic fit, radiation correction, intermediate structures, resolution of J/ψ, and the requirement on the penetration depth in the muon counter. The associated uncertainty is estimated by considering them while doing the fit and comparing to the results obtained only considering statistical uncertainties of the cross sections. The second category of the systematic uncertainties is corre-

lated and common for all data samples (4.6%), therefore only affects the $\Gamma_{ee}\mathcal{B}$. All of these uncertainties on the parameters of resonances are listed in Supplemental Material [42].

In summary, the Born cross sections of $e^+e^- \to K^+K^-J/\psi$ at c.m. energy from 4.127 to 4.600 GeV are measured with a new partial reconstruction method and larger data samples compared with Ref [22]. Two resonances are observed with high significances. One is consistent to the previous observed $Y(4230)$, and its mass and width are measured to be $M(Y(4230)) = (4225.3 \pm 2.3 \pm 21.5)$ MeV, $\Gamma(Y(4230)) = (72.9 \pm 6.1 \pm 30.8)$ MeV, where the first uncertainties are statistical and the second are systematic. The other one can not be assigned into any experimentally observed resonance, named as $Y(4500)$, and its mass and width are determined to be $M(Y(4500)) = (4484.7 \pm 13.3 \pm 24.1)$ MeV, $\Gamma(Y(4500)) = (111.1 \pm 30.1 \pm 15.2)$ MeV, respectively. The mass of $Y(4500)$ is consistent with the prediction of the 5S-4D mixing scheme [23], the heavy-antiheavy hadronic molecules model [24] and the lattice quantum chromodynamics result for a $(cs\bar{c}\bar{s})$ state [25], while the width is 2σ larger than the prediction of Ref. [23]. More experimental measurements and theoretical studies are needed to reveal its nature.

For the first time, the state $Y(4230)$ has been observed in the $K\bar{K}J/\psi$ mode with the significance larger than 5σ. The product of the electronic partial width and the decay branching fraction is measured to be $\mathcal{B}(Y(4230) \to K^+K^-J/\psi)\Gamma(e^+e^- \to Y(4230)) = (0.29\pm0.02\pm0.10)$ eV or $(0.42 \pm 0.04 \pm 0.15)$ eV, according to different interferences, respectively. The ratio between the branching fractions of the $Y(4230)$ decaying into $K\bar{K}J/\psi$ and $\pi\pi J/\psi$ [5] modes is determined to be

$$0.02 < \frac{\mathcal{B}(Y(4230) \to K^+K^-J/\psi)}{\mathcal{B}(Y(4230) \to \pi^+\pi^-J/\psi)} < 0.26 ,$$

where ranges are determined from the eight combinations of multiple solutions in the two measurements and the large ambiguity is mainly from the multiple solutions in the $\pi\pi J/\psi$ measurement. Author of Ref. [51] predicts the $K\bar{K}$ mode should be suppressed if the $Y(4230)$ is Λ_c baryonium. However, it seems very difficult to draw a

conclusion before determining the physics solution in the measurements, especially for the $\pi\pi J/\psi$ mode.

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Supplemental Material for "Observation of the $Y(4230)$ and a new structure in $e^+e^- \to K^+K^-J/\psi$"

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I. DISTRIBUTIONS OF $M(\ell^+\ell^-)$ AND $M(K^+K^-)$ AT EACH C.M. ENERGY

Figure 1 shows the distributions of the invariant mass of lepton pairs, $M(\ell^+\ell^-)$, for data and PHSP signal MC samples at each c.m. energies. Here the J/ψ signal and sideband regions are indicated with red and blue arrows, respectively. In addition, the distributions of invariant mass of kaons, $M(K^+K^-)$, for data, PHSP signal MC, and weighted MC samples at each c.m. energies are shown in Fig. 2.
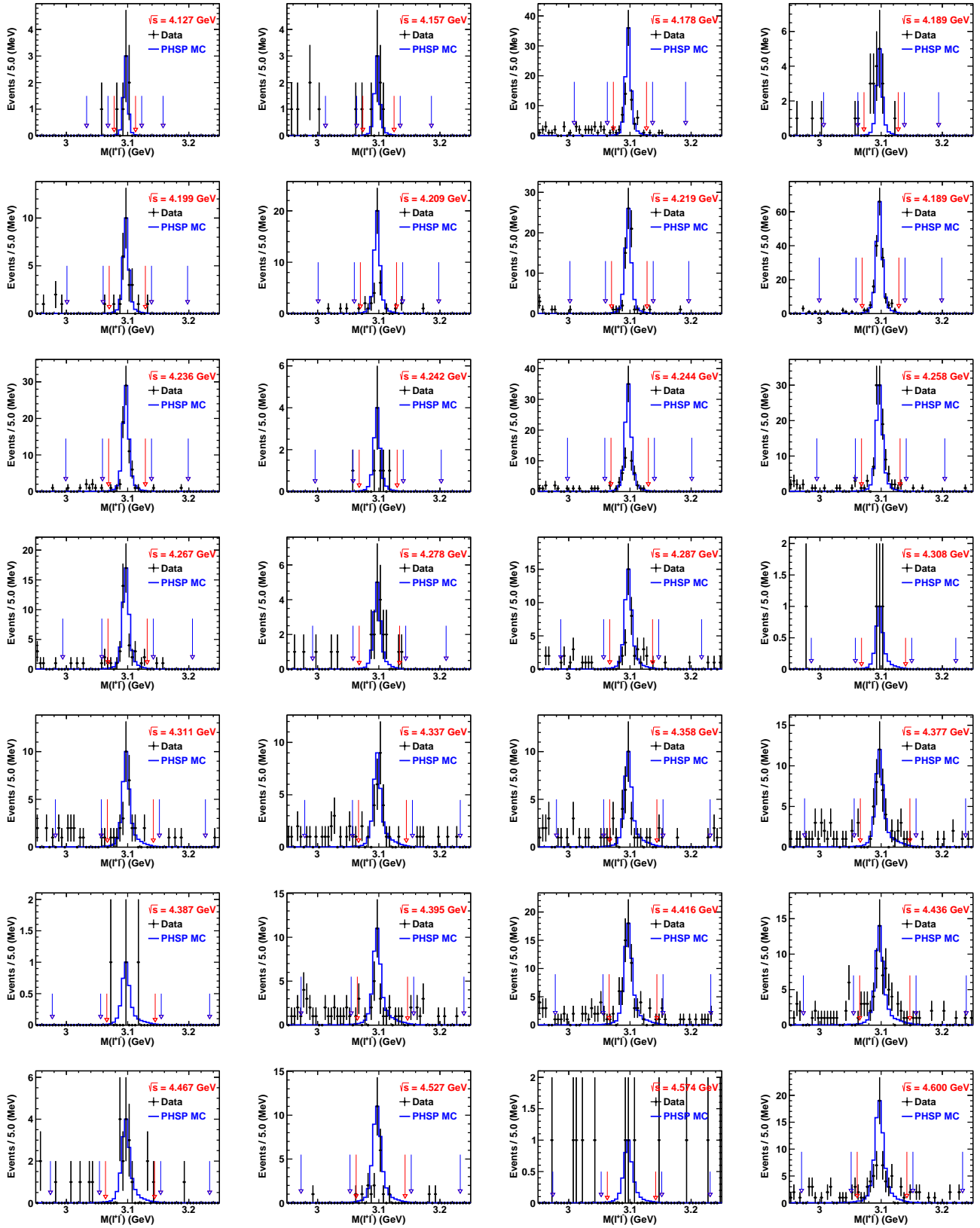
FIG. 1. The distributions of invariant mass of lepton pairs $M(\ell^+\ell^-)$, where the dots with error bars are data, the blue histogram is PHSP signal MC sample (normalized to data.), the red and blue arrows identify the J/ψ signal and sideband regions, respectively.
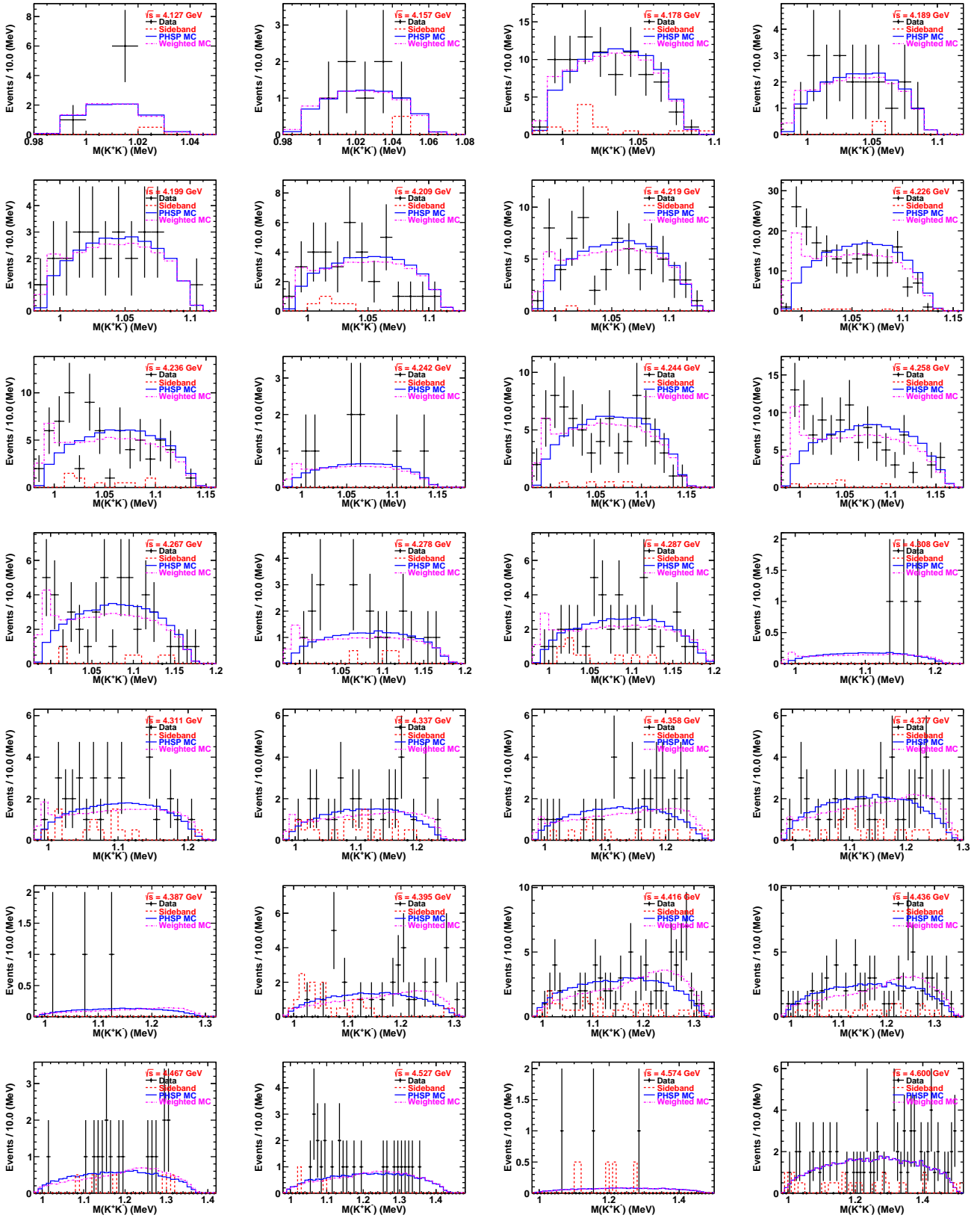
FIG. 2. The distributions of K^+K^- invariant mass of the data, PHSP signal MC and weighted MC samples at each c.m. energy, where the black dots with error bars indicate data from the J/ψ signal region, the red dashed curves indicate data from J/ψ sideband regions, the blue histograms indicate PHSP signal MC sample (normalized to data) and the yank dashed-dot curves indicate the weighted signal MC sample (normalized to data).

II. THE BORN CROSS SECTIONS

The Born cross sections of $e^+e^- \to K^+K^-J/\psi$ and related quantities such as the c.m. energies of data samples, the integrated luminosity, the numbers of observed signal events, the efficiencies, the radiative correction factors, the vacuum polarization factors are listed in Tab. I.

TABLE I. The Born cross sections of $e^+e^- \to K^+K^-J/\psi$ and related quantities. Here $\sqrt{s}$ is the c.m. energy of data samples, $\mathcal{L}_{int}$ is the integrated luminosity, N^{obs} is the number of observed signal events, ε is the efficiency, $(1+\delta)_{ISR}$ is the radiative correction factor, $\frac{1}{|1-\Pi|^2}$ is the vacuum polarization factor, and the σ^B is the Born cross section, the first uncertainties are statistical, and the second systematic.

Data Sample	$\sqrt{s}$(GeV)	$\mathcal{L}_{int}$ (pb^{-1})	N^{obs}	ε	$(1+\delta)_{ISR}$	$\frac{1}{|1-\Pi|^2}$	σ^B(pb)
4130	4.127	401.50	$6.5^{+3.0}_{-2.4}$	0.141	0.896	1.052	$1.02^{+0.47}_{-0.38} \pm 0.09$
4160	4.157	408.70	$6.5^{+3.0}_{-2.4}$	0.263	0.856	1.053	$0.56^{+0.26}_{-0.21} \pm 0.05$
4180	4.178	3194.50	$72.0^{+9.7}_{-9.1}$	0.315	0.820	1.054	$0.69^{+0.09}_{-0.09} \pm 0.06$
4190	4.189	570.10	$18.5^{+4.7}_{-4.1}$	0.335	0.792	1.056	$0.97^{+0.25}_{-0.22} \pm 0.08$
4200	4.199	526.00	$25.0^{+5.2}_{-4.6}$	0.357	0.787	1.056	$1.34^{+0.28}_{-0.25} \pm 0.12$
4210	4.209	572.10	$33.5^{+6.4}_{-5.8}$	0.358	0.762	1.057	$1.70^{+0.33}_{-0.29} \pm 0.15$
4220	4.219	569.20	$69.0^{+8.7}_{-8.1}$	0.379	0.771	1.056	$3.29^{+0.41}_{-0.39} \pm 0.37$
4230	4.226	1100.90	$183.0^{+14.0}_{-13.3}$	0.395	0.771	1.056	$4.33^{+0.33}_{-0.31} \pm 0.38$
4237	4.236	530.30	$65.5^{+8.9}_{-8.3}$	0.391	0.789	1.056	$3.18^{+0.43}_{-0.40} \pm 0.30$
4245	4.242	55.88	$8.0^{+3.2}_{-2.5}$	0.405	0.805	1.055	$3.49^{+1.39}_{-1.09} \pm 0.31$
4246	4.244	538.10	$74.5^{+9.2}_{-8.5}$	0.393	0.812	1.056	$3.44^{+0.43}_{-0.39} \pm 0.30$
4260	4.258	828.40	$107.5^{+10.9}_{-10.3}$	0.396	0.866	1.054	$3.01^{+0.31}_{-0.29} \pm 0.26$
4270	4.267	531.10	$45.0^{+7.4}_{-6.7}$	0.390	0.911	1.053	$1.90^{+0.31}_{-0.28} \pm 0.17$
4280	4.278	175.70	$16.5^{+4.7}_{-4.0}$	0.372	0.935	1.053	$2.15^{+0.61}_{-0.52} \pm 0.19$
4290	4.287	502.40	$35.0^{+6.9}_{-6.3}$	0.363	0.937	1.053	$1.63^{+0.32}_{-0.29} \pm 0.14$
4310	4.308	45.08	$3.0^{+2.1}_{-1.4}$	0.367	0.982	1.052	$1.47^{+1.03}_{-0.69} \pm 0.13$
4315	4.311	501.20	$24.0^{+6.2}_{-5.6}$	0.358	0.985	1.052	$1.08^{+0.28}_{-0.25} \pm 0.10$
4340	4.337	505.00	$18.5^{+6.1}_{-5.6}$	0.364	0.963	1.051	$0.83^{+0.28}_{-0.25} \pm 0.07$
4360	4.358	544.00	$26.5^{+6.3}_{-5.7}$	0.378	0.935	1.051	$1.10^{+0.26}_{-0.24} \pm 0.10$
4380	4.377	522.70	$33.0^{+7.6}_{-7.1}$	0.379	0.891	1.051	$1.49^{+0.34}_{-0.32} \pm 0.13$
4390	4.387	55.57	$3.0^{+2.1}_{-1.4}$	0.397	0.865	1.051	$1.25^{+0.88}_{-0.58} \pm 0.11$
4400	4.395	507.80	$16.0^{+6.6}_{-6.1}$	0.401	0.861	1.051	$0.73^{+0.30}_{-0.28} \pm 0.06$
4420	4.416	1090.70	$57.0^{+9.4}_{-8.8}$	0.417	0.815	1.052	$1.23^{+0.20}_{-0.19} \pm 0.11$
4440	4.436	569.90	$55.5^{+8.9}_{-8.3}$	0.436	0.780	1.054	$2.28^{+0.37}_{-0.34} \pm 0.20$
4470	4.467	111.09	$14.0^{+4.6}_{-4.0}$	0.450	0.738	1.055	$3.02^{+0.99}_{-0.86} \pm 0.26$
4530	4.527	112.12	$23.5^{+5.4}_{-4.8}$	0.460	0.751	1.054	$4.82^{+1.11}_{-0.99} \pm 0.41$
4575	4.574	48.93	$0.0^{+2.3}_{-0.0}$	0.430	0.856	1.054	$0.00^{+1.01}_{-0.00} \pm 0.00$
4600	4.600	586.90	$52.0^{+8.7}_{-8.2}$	0.400	0.899	1.055	$1.96^{+0.33}_{-0.31} \pm 0.17$

III. DEFINITION OF LIKELIHOOD FUNCTION

In the maximum likelihood fit to the dressed cross sections of $e^+e^- \to K^+K^-J/\psi$, the likelihood is constructed as:

$$L(\mu; \vartheta) = \prod_{i}^{6} L_i(\mu_i; \ \vartheta_i) \prod_{j}^{22} L_j(\mu_j; \ \vartheta_j),$$ (1)

where μ_i and μ_j are the numbers of observed signal events, ϑ_i and ϑ_j are the parameters in the likelihood functions, and L_i and L_j are the likelihood functions for the data samples with $\mu \leq 10$ and $\mu > 10$, respectively.

The likelihood functions are defined variously according to the different numbers of observed events. For the data samples wiht $\mu \leq 10$, the likelihood function is described by a Poisson function:

$$L_i(\mu_i; \vartheta_i) = P_i(\mu_i; \vartheta_i) = \frac{1}{\vartheta_i!} \mu_i^{\vartheta_i} e^{-\mu_i},$$ (2)

While for the data samples with $\mu > 10$, the likelihood function is described by an asymmetric Gaussian function:

$$L_j(\mu_j, \ \sigma_{1j}, \ \sigma_{2j}; \ \vartheta_j) = G(\mu_j, \ \sigma_{1j}, \ \sigma_{2j}; \ \vartheta_j)$$

$$= \begin{cases} \frac{1}{\sqrt{2\pi} \ (\sigma_{1j}+\sigma_{2j})} \ e^{-\frac{(\vartheta_j-\mu_j)^2}{2\sigma_{1j}^2}}, & \vartheta_j > \mu_j; \\[3mm] \frac{1}{\sqrt{2\pi} \ (\sigma_{1j}+\sigma_{2j})} \ e^{-\frac{(\vartheta_j-\mu_j)^2}{2\sigma_{2j}^2}}, & \vartheta_j \leq \mu_j; \end{cases}$$ (3)

where σ_1 and σ_2 are the upper and lower statistical uncertainties of μ, respectively.

IV. THE SYSTEMATIC UNCERTAINTIES OF THE BORN CROSS SECTIONS AND THE RESONANCE PARAMETERS

The systematic uncertainties of the Born cross sections are listed in Tab. II and the systematic uncertainties of the resonance parameters are listed in Tab. III. Figure 3 shows dressed cross sections fitting results with different function forms. Generally, there should be four solutions when three coherent amplitudes are used to describe a lineshape of cross sections. However, only two have been found by us. We suppose that the other solutions are very close to the found ones then cannot be separated by the scanning method. And one should notice that the fitting results, with the tentatively additional continuum term, only change slightly.

TABLE II. The systematic uncertainties (in units %) in the measurement of Born cross sections. Here $\sqrt{s}$ is the c.m. energy of data samples, $\mathcal{L}_{int}$ is the integrated luminosity, $\mathcal{B}(J/\psi \to l^+l^-)$ is the branching fraction, $(1+\delta)_{ISR}$ is the radiation correction factor, $M(K^+K^-)$ is the intermediate structures in K^+K^- system, $R(J/\psi)$ is the resolution of J/ψ, MUC is the criteria applied on the penetration depth in the muon counter.

Data Sample	$\sqrt{s}$(GeV)	$\mathcal{L}_{int}$	Tracking and PID	$\mathcal{B}(J/\psi \to l^+l^-)$	kinematic fit	$(1+\delta)_{ISR}$	$M(K^+K^-)$	$R(J/\psi)$	MUC	Total
4130	4.127	1.0	4.5	0.4	0.4	1.2	6.9	0.3	2.1	8.7
4160	4.157	1.0	4.5	0.4	0.7	0.7	6.9	0.3	2.1	8.6
4180	4.178	1.0	4.5	0.4	0.9	2.6	6.9	0.2	2.1	9.0
4190	4.189	1.0	4.5	0.4	1.0	1.0	6.9	0.1	2.1	8.7
4200	4.199	1.0	4.5	0.4	1.1	0.6	6.9	0.3	2.1	8.7
4210	4.209	1.0	4.5	0.4	1.1	1.7	6.9	0.3	2.1	8.8
4220	4.219	1.0	4.5	0.4	1.1	6.9	6.9	0.2	2.1	11.1
4230	4.226	1.0	4.5	0.4	1.1	0.6	6.9	0.4	2.1	8.7
4237	4.236	1.0	4.5	0.4	1.2	3.5	6.9	0.4	2.1	9.3
4245	4.242	1.0	4.5	0.4	1.2	2.4	6.9	0.2	2.1	9.0
4246	4.244	1.0	4.5	0.4	1.2	1.5	6.9	0.2	2.1	8.8
4260	4.258	1.0	4.5	0.4	1.3	1.3	6.9	0.3	2.1	8.8
4270	4.267	1.0	4.5	0.4	1.3	2.7	6.9	0.3	2.1	9.1
4280	4.278	1.0	4.5	0.4	1.3	1.3	6.9	0.4	2.1	8.8
4290	4.287	1.0	4.5	0.4	1.2	1.2	6.9	0.3	2.1	8.7
4310	4.308	1.0	4.5	0.4	1.6	1.3	6.9	0.4	2.1	8.8
4315	4.312	1.0	4.5	0.4	1.6	0.7	6.9	0.4	2.1	8.8
4340	4.337	1.0	4.5	0.4	1.6	1.5	6.9	0.7	2.1	8.9
4360	4.358	1.0	4.5	0.4	1.4	0.5	6.9	0.5	2.1	8.7
4380	4.377	1.0	4.5	0.4	1.4	0.3	6.9	0.6	2.1	8.7
4390	4.387	1.0	4.5	0.4	1.2	1.0	6.9	0.5	2.1	8.7
4400	4.395	1.0	4.5	0.4	1.1	0.9	6.9	0.5	2.1	8.7
4420	4.416	1.0	4.5	0.4	0.9	1.0	6.9	0.7	2.1	8.7
4440	4.436	1.0	4.5	0.4	0.7	1.6	6.9	0.6	2.1	8.8
4470	4.467	1.0	4.5	0.4	0.4	0.2	6.9	0.4	2.1	8.6
4530	4.527	1.0	4.5	0.4	0.3	0.9	6.9	0.6	2.1	8.6
4575	4.574	1.0	4.5	0.4	0.2	0.0	6.9	0.4	2.1	8.6
4600	4.600	1.0	4.5	0.4	0.2	1.2	6.9	0.3	2.1	8.7

TABLE III. The systematic uncertainty in the measurement of the resonance parameters for solution I (solution II), where Non-Resonant indicated a three-body PHSP shape is added in the default fit and considering the coherent between them, the cross section$_1$ is uncorrelated systematic uncertainties from the measurement of cross sections, while cross section$_2$ is correlated.

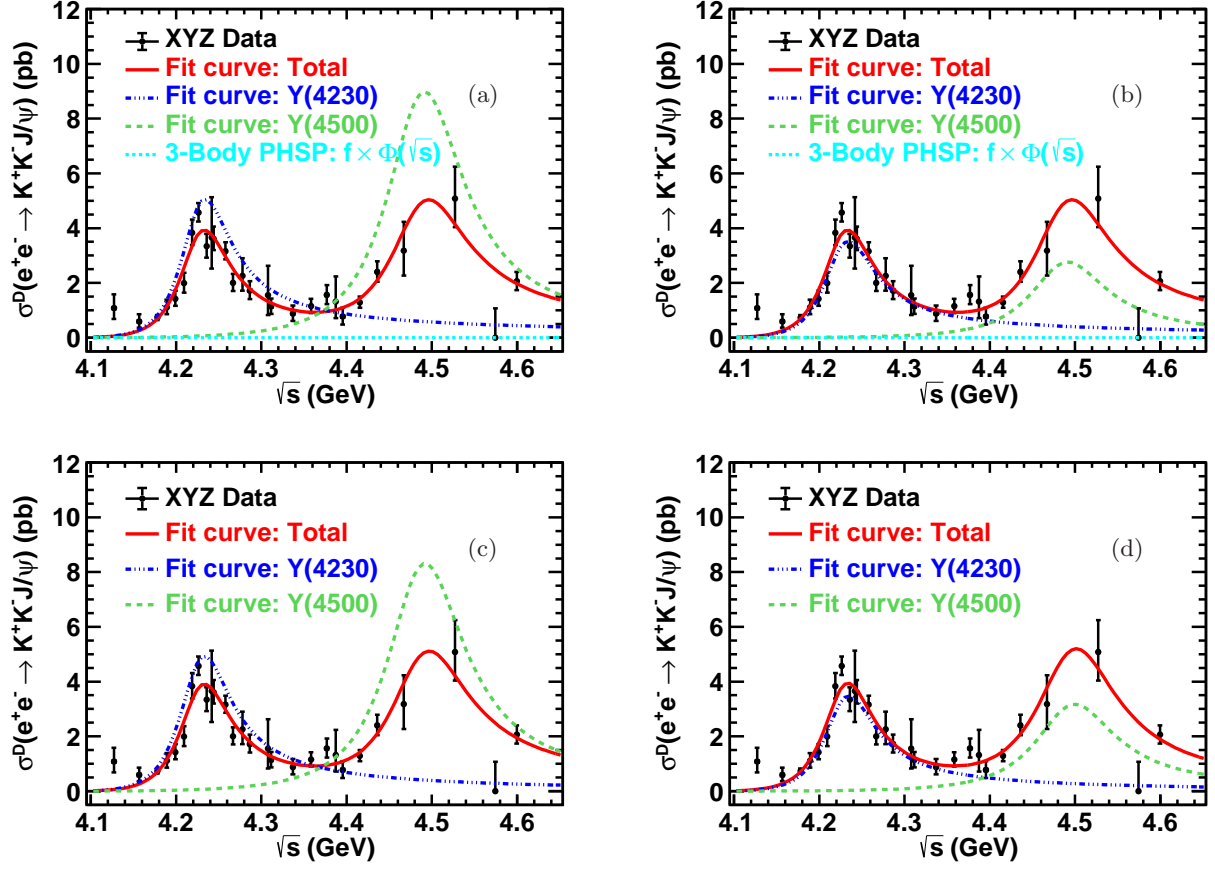| Parameters / Sources	$Y(4230)$			$Y(4500)$			phase angle
	M (MeV)	Γ_{tot} (MeV)	$\Gamma_{ee}\mathcal{B}$ (eV)	M (MeV)	Γ_{tot} (MeV)	$\Gamma_{ee}\mathcal{B}$ (eV)	φ (rad)
c.m. Energy	0.7	0.1	–	1.8	1.4	–	0.00 (0.02)
Non-Resonant	–	–	–	–	–	–	0.13 (0.51)
Γ_{tot}	21.5	30.8	0.15 (0.10)	24.1	15.1	0.02 (0.13)	0.50 (0.28)
cross section$_1$	–	–	– (–)	0.1	0.1	– (–)	–
cross section$_2$	–	–	0.02 (0.01)	–	–	0.06 (0.02)	–
Total	21.5	30.8	0.15 (0.10)	24.1	15.2	0.06 (0.13)	0.52 (0.58)

FIG. 3. The dressed cross sections fitting results with different function forms. (a), (b) are two sets of solutions to the sum of two coherent Breit-Wigner functions and 3-body phase space function fitting, respectively. (c), (d) are two sets of solutions to the two coherent Breit-Wigner functions with an energy-dependent full width fitting, respectively.