Supplemental Material for "First observation of the direct production of the χ_{c1} in e^+e^- annihilation"

Supplemental Material for "First observation of the direct production of the X_{c1} in e⁺e⁻ annihilation"
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THE DISTRIBUTION OF -LN(L) IN A LARGER PARAMETER SPACE REGION

Figure 1 shows the distribution of the log-likelihood value $(-\ln(L))$ as a function of Γ_{ee} (x-axis) and ϕ (y-axis) in a larger parameter space region. The red square (0.12 eV, 205.0°) represents the point where the likelihood value is maximum. The orange triangle (0.41 eV, 212.0°) comes from the theoretical calculation in Ref. [14]. The green circles are the parameter points where MC samples are produced.



FIG. 1. The distribution of $-\ln(L)$ in a larger parameter space region.

THE CORRECTION FACTOR

Figure 2 shows the correction factors used for the two-dimensional correction to the distribution of $M_{\mu^+\mu^-}$ and $|\cos\theta_{\mu}|$. The left plot shows the correction factors derived from the $\sqrt{s} = 3.773$ GeV sample and the right plot is from the $\sqrt{s} = 4.178$ GeV sample. Figure 3, Fig. 4, and Fig. 5 show the results from the two-dimensional fits to the $M_{\mu^+\mu^-}$ and $|\cos\theta_{\mu}|$ distributions from the control samples before correction, after correction using the correction factors extracted from data and MC samples at $\sqrt{s} = 3.773$ GeV, and the correction factors from $\sqrt{s} = 4.178$ GeV.



FIG. 2. The correction factors extracted from the $\sqrt{s} = 3.773$ GeV sample (left) and the $\sqrt{s} = 4.178$ GeV sample (right).



FIG. 3. One-dimensional projections of the two-dimensional fit to the $M_{\mu^+\mu^-}$ and $|\cos \theta_{\mu}|$ distributions from the control data samples. The two-dimensional correction is not applied in this fit. The black dots with error bars are the distributions from data, the gray histograms are the irreducible background predicted by the corrected MC simulation. The red curve is the best fit result, the red dotted (blue dashed) curve is the signal (background) contribution.



FIG. 4. One-dimensional projections of the two-dimensional fit to the $M_{\mu^+\mu^-}$ and $|\cos \theta_{\mu}|$ distributions from the control data samples. The two-dimensional correction factor is determined from $\sqrt{s} = 3.773$ GeV sample.



FIG. 5. One-dimensional projections of the two-dimensional fit to the $M_{\mu^+\mu^-}$ and $|\cos \theta_{\mu}|$ distributions from the control data samples. The two-dimensional correction factor is determined from $\sqrt{s} = 4.178$ GeV sample.

THE 2-DIMENSIONAL FIT METHOD

Figure 6 shows the $|\cos\theta_{\mu}|$ distribution of the signal MC simulation at different center-of-mass energies, compared with the distribution from the irreducible background MC simulation. The signal MC samples are produced with Γ_{ee} and ϕ fixed to the best value determined from this study.



FIG. 6. Comparison of the line-shape of $|\cos\theta_{\mu}|$ from the background simulation (green histogram) and the signal MC simulation (other histograms).

SCATTER PLOT AND CHI DISTRIBUTION OF χ_{c1} SCAN SAMPLES

Figure 7 shows the scatter plots of data (left), MC (middle), and the pull distributions from the two-dimensional fit (right) at χ_{c1} scan samples.



FIG. 7. The scatter plots and pull distributions at $\sqrt{s} = 3.5080, 3.5097, 3.5104$, and 3.5146 GeV.

STATISTICAL TEST FOR THE COMMON FIT

For the χ_{c1} scan samples, statistical tests are performed by using the toy MC samples based on the common fit result under the signal and the null-signal hypotheses. The difference of the log-likelihood values, $t = -\ln L_s + \ln L_{ns}$, is used as a test variable, where the signal hypothesis is given by $(-\ln L_s)$ and the null-signal hypothesis by $(-\ln L_{ns})$. The distributions of t for the four χ_{c1} scan samples are shown in Fig. 8, and the result combining the four samples is shown in Fig. 9.



FIG. 8. Distributions of the test variable t from the toy MC samples based on the common fit result under the signal and null-signal hypotheses at $\sqrt{s} = 3.5080, 3.5097, 3.5104$, and 3.5146 GeV. The red and the blue histograms show the distributions under the signal and null-signal hypotheses, respectively, while the black vertical lines indicate the values from real data.



FIG. 9. Distribution of the test variable t from the toy MC samples using all four χ_{c1} scan samples.