## Measurements of Normalized Differential Cross Sections of Inclusive $\eta$ Production in $e^+e^-$ Annihilation at Energy from 2.0000 to 3.6710 GeV

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 M Ablikin<sup>+</sup>, M. N. Achases<sup>A,\*</sup>, P. Adlaron<sup>+</sup>, O. Bakins<sup>+</sup>, I. Balossin<sup>2,+</sup>, V. Balossin<sup>2,+</sup>, K. Chen<sup>2,+</sup>, K. Chen<sup>2</sup>

H. Y. Xu<sup>66</sup>, M. Xu<sup>71,58</sup>, Q. J. Xu<sup>16</sup>, Q. N. Xu<sup>30</sup>, W. Xu<sup>1</sup>, W. L. Xu<sup>66</sup>, X. P. Xu<sup>55</sup>, Y. C. Xu<sup>77</sup>, Z. P. Xu<sup>42</sup>, Z. S. Xu<sup>63</sup>, F. Yan<sup>12,g</sup>, L. Yan<sup>21,g</sup>, W. B. Yan<sup>71,58</sup>, W. C. Yan<sup>80</sup>, X. Q. Yan<sup>1</sup>, H. J. Yang<sup>51,f</sup>, H. L. Yang<sup>34</sup>, H. X. Yang<sup>1</sup>, Tao Yang<sup>1</sup>, Y. Yang<sup>12,g</sup>, Y. F. Yang<sup>43</sup>, Y. X. Yang<sup>1,63</sup>, Yifan Yang<sup>1,63</sup>, Z. W. Yang<sup>38,k,l</sup>, Z. P. Yao<sup>50</sup>, M. Ye<sup>1,58</sup>, M. H. Ye<sup>8</sup>, J. H. Yin<sup>1</sup>, Z. Y. You<sup>59</sup>, B. X. Yu<sup>1,58,63</sup>, C. X. Yu<sup>43</sup>, G. Yu<sup>1,63</sup>, J. S. Yu<sup>25,i</sup>, T. Yu<sup>72</sup>, X. D. Yu<sup>46,h</sup>, Y. C. Yu<sup>80</sup>, C. Z. Yuan<sup>1,63</sup>, J. Yuan<sup>34</sup>, L. Yuan<sup>2</sup>, S. C. Yuan<sup>1</sup>, Y. Yuan<sup>1,63</sup>, Y. J. Yuan<sup>45</sup>, Z. Y. Yuan<sup>59</sup>, C. X. Yue<sup>39</sup>, A. A. Zafar<sup>73</sup>, F. R. Zeng<sup>50</sup>, S. H. Zeng<sup>72</sup>, X. Zeng<sup>12,g</sup>, Y. Zeng<sup>25,i</sup>, Y. J. Zeng<sup>59</sup>, X. Y. Zhai<sup>34</sup>, Y. C. Zhai<sup>50</sup>, Y. H. Zhan<sup>59</sup>, A. Q. Zhang<sup>1,63</sup>, B. L. Zhang<sup>1,63</sup>,
 B. X. Zhang<sup>1</sup>, D. H. Zhang<sup>43</sup>, G. Y. Zhang<sup>1,68</sup>, J. Zhang<sup>80</sup>, J. Zhang<sup>59</sup>, J. J. Zhang<sup>52</sup>, J. L. Zhang<sup>20</sup>, J. Q. Zhang<sup>41</sup>, J. S. Zhang<sup>12,g</sup>, J. W. Zhang<sup>1,58,63</sup>, H. Y. Zhang<sup>1,68</sup>, J. J. Yang<sup>80</sup>, J. Zhang<sup>59</sup>, J. J. Zhang<sup>52</sup>, J. L. Zhang<sup>63</sup>, L. M. Zhang<sup>61</sup>, Lei Zhang<sup>1,2,g</sup>, J. W. Zhang<sup>1,58,63</sup>, H. Y. Zhang<sup>34,k,l</sup>, J. Y. Zhang<sup>80</sup>, J. Zhang<sup>59</sup>, J. J. Zhang<sup>63</sup>, L. M. Zhang<sup>61</sup>, Lei Zhang<sup>1,63</sup>, Q. Y. Zhang<sup>34</sup>, R. Y Zhang<sup>38,k,l</sup>, Shuihan Zhang<sup>1,63</sup>, Shulei Zhang<sup>25,i</sup>, X. D. Zhang<sup>45</sup>, X. M. Zhang<sup>1,58,63</sup>, H. R. Zhang<sup>71,58</sup>, Y. T. Zhang<sup>38,k,l</sup>, Shuihan Zhang<sup>1,63</sup>, Shulei Zhang<sup>25,i</sup>, X. D. Zhang<sup>45</sup>, X. M. Zhang<sup>1,58,63</sup>, J. Z. L. Zhang<sup>34</sup>, R. Y Zhang<sup>34</sup>, S. Y. Zhang<sup>45</sup>, S. J. Zhang<sup>63</sup>, L. M. Zhang<sup>61</sup>, Lei Zhang<sup>1,58,63</sup>, Lei Zhang<sup>72</sup>, Y. T. Zhang<sup>80</sup>, Y. H. Zhang<sup>1,58</sup>, Y. M. Zhang<sup>39</sup>, Yan Zhang<sup>71,58</sup>, Yao Zhang<sup>1</sup>, Z. D. Zhang<sup>1</sup>, Z. H. Zhang<sup>1,5</sup>, A. Zhang<sup>34</sup>, Z. Y. Zhang<sup>56</sup>, Z. Y. Zhang<sup>43</sup>, Z. Z. Zhand<sup>45</sup>, G. Zhao<sup>1</sup>, J. Y. Zhao<sup>1,58</sup>, Lei Zhan<sup>1,58</sup>, Lei Zhan<sup>1,58</sup>, A. Zheng<sup>72</sup>, Y. T. Zhang<sup>80</sup>, Y. H. Zhang<sup>43</sup>, J. P. Zheng<sup>1,58</sup>, W. J. Zhao<sup>1,58</sup>, Y. J. Zhan<sup>3,5</sup>, Y. Zhan<sup>3,58</sup>, Lei Zhan<sup>59</sup>, H. Zhong<sup>59</sup>, H. Zhou<sup>56</sup>, B. Zhen

(BESIII Collaboration)

<sup>1</sup> Institute of High Energy Physics, Beijing 100049, People's Republic of China

<sup>2</sup> Beihang University, Beijing 100191, People's Republic of China

<sup>3</sup> Bochum Ruhr-University, D-44780 Bochum, Germany

<sup>4</sup> Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia

<sup>5</sup> Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA

<sup>6</sup> Central China Normal University, Wuhan 430079, People's Republic of China

<sup>7</sup> Central South University, Changsha 410083, People's Republic of China

<sup>8</sup> China Center of Advanced Science and Technology, Beijing 100190, People's Republic of China

<sup>9</sup> China University of Geosciences, Wuhan 430074, People's Republic of China

<sup>10</sup> Chung-Ang University, Seoul, 06974, Republic of Korea

<sup>11</sup> COMSATS University Islamabad, Lahore Campus, Defence Road, Off Raiwind Road, 54000 Lahore, Pakistan <sup>12</sup> Fudan University, Shanghai 200433, People's Republic of China

<sup>13</sup> GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany

<sup>14</sup> Guangxi Normal University, Guilin 541004, People's Republic of China

<sup>15</sup> Guangxi University, Nanning 530004, People's Republic of China

<sup>16</sup> Hangzhou Normal University, Hangzhou 310036, People's Republic of China

<sup>17</sup> Hebei University, Baoding 071002, People's Republic of China

<sup>18</sup> Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany

<sup>19</sup> Henan Normal University, Xinxiang 453007, People's Republic of China

<sup>20</sup> Henan University, Kaifeng 475004, People's Republic of China

<sup>21</sup> Henan University of Science and Technology, Luoyang 471003, People's Republic of China

<sup>22</sup> Henan University of Technology, Zhengzhou 450001, People's Republic of China

<sup>23</sup> Huangshan College, Huangshan 245000, People's Republic of China

<sup>24</sup> Hunan Normal University, Changsha 410081, People's Republic of China

<sup>25</sup> Hunan University, Changsha 410082, People's Republic of China

<sup>26</sup> Indian Institute of Technology Madras, Chennai 600036, India

<sup>27</sup> Indiana University, Bloomington, Indiana 47405, USA

<sup>28</sup> INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN

Sezione di Perugia, I-06100, Perugia, Italy: (C)University of Perugia, I-06100, Perugia, Italy

<sup>29</sup> INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara,

Italy

<sup>30</sup> Inner Mongolia University, Hohhot 010021, People's Republic of China

<sup>31</sup> Institute of Modern Physics, Lanzhou 730000, People's Republic of China

<sup>32</sup> Institute of Physics and Technology, Peace Avenue 54B, Ulaanbaatar 13330, Mongolia

<sup>33</sup> Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 7D, Arica 1000000, Chile

<sup>34</sup> Jilin University, Changchun 130012, People's Republic of China

<sup>35</sup> Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany

<sup>36</sup> Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

<sup>37</sup> Justus-Liebiq-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany

<sup>38</sup> Lanzhou University, Lanzhou 730000, People's Republic of China

Liaoning Normal University, Dalian 116029, People's Republic of China

<sup>40</sup> Liaoning University, Shenyang 110036, People's Republic of China

<sup>41</sup> Nanjing Normal University, Nanjing 210023, People's Republic of China

<sup>42</sup> Nanjing University, Nanjing 210093, People's Republic of China

<sup>43</sup> Nankai University, Tianjin 300071, People's Republic of China

<sup>44</sup> National Centre for Nuclear Research, Warsaw 02-093, Poland

<sup>45</sup> North China Electric Power University, Beijing 102206, People's Republic of China

<sup>46</sup> Peking University, Beijing 100871, People's Republic of China

<sup>47</sup> Qufu Normal University, Qufu 273165, People's Republic of China

<sup>48</sup> Renmin University of China, Beijing 100872, People's Republic of China

<sup>49</sup> Shandong Normal University, Jinan 250014, People's Republic of China

<sup>50</sup> Shandong University, Jinan 250100, People's Republic of China

<sup>51</sup> Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

<sup>2</sup> Shanxi Normal University, Linfen 041004, People's Republic of China

<sup>53</sup> Shanxi University, Taiyuan 030006, People's Republic of China

<sup>54</sup> Sichuan University, Chengdu 610064, People's Republic of China

<sup>55</sup> Soochow University, Suzhou 215006, People's Republic of China

<sup>56</sup> South China Normal University, Guangzhou 510006, People's Republic of China

<sup>57</sup> Southeast University, Nanjing 211100, People's Republic of China

<sup>58</sup> State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People's Republic of China

<sup>59</sup> Sun Yat-Sen University, Guangzhou 510275, People's Republic of China

<sup>60</sup> Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand

<sup>61</sup> Tsinghua University, Beijing 100084, People's Republic of China

<sup>62</sup> Turkish Accelerator Center Particle Factory Group, (A)Istinye University, 34010, Istanbul, Turkey; (B)Near East

University, Nicosia, North Cyprus, 99138, Mersin 10, Turkey

<sup>63</sup> University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

<sup>64</sup> University of Groningen, NL-9747 AA Groningen, The Netherlands

<sup>65</sup> University of Hawaii, Honolulu, Hawaii 96822, USA

<sup>66</sup> University of Jinan, Jinan 250022, People's Republic of China

<sup>67</sup> University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom

<sup>68</sup> University of Muenster, Wilhelm-Klemm-Strasse 9, 48149 Muenster, Germany

<sup>69</sup> University of Oxford, Keble Road, Oxford OX13RH, United Kingdom

<sup>70</sup> University of Science and Technology Liaoning, Anshan 114051, People's Republic of China

<sup>71</sup> University of Science and Technology of China, Hefei 230026, People's Republic of China

<sup>72</sup> University of South China, Hengyang 421001, People's Republic of China

<sup>3</sup> University of the Punjab, Lahore-54590, Pakistan

<sup>74</sup> University of Turin and INFN, (A) University of Turin, I-10125, Turin, Italy; (B) University of Eastern Piedmont, I-15121,

Alessandria, Italy; (C)INFN, I-10125, Turin, Italy

<sup>75</sup> Uppsala University, Box 516, SE-75120 Uppsala, Sweden

<sup>76</sup> Wuhan University, Wuhan 430072, People's Republic of China

<sup>77</sup> Yantai University, Yantai 264005, People's Republic of China

<sup>78</sup> Yunnan University, Kunming 650500, People's Republic of China

<sup>79</sup> Zhejiang University, Hangzhou 310027, People's Republic of China

<sup>80</sup> Zhengzhou University, Zhengzhou 450001, People's Republic of China

<sup>a</sup> Deceased

<sup>b</sup> Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia

<sup>c</sup> Also at the Novosibirsk State University, Novosibirsk, 630090, Russia

<sup>d</sup> Also at the NRC "Kurchatov Institute", PNPI, 188300, Gatchina, Russia

<sup>e</sup> Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany

<sup>f</sup> Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory

for Particle Physics and Cosmology: Institute of Nuclear and Particle Physics, Shanghai 200240, People's Republic of China

<sup>g</sup> Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan

University, Shanghai 200443, People's Republic of China

<sup>h</sup> Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People's Republic of

China

<sup>i</sup> Also at School of Physics and Electronics, Hunan University, Changsha 410082, China

<sup>j</sup> Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal

 $University,\ Guangzhou\ 510006,\ China$ 

<sup>k</sup> Also at MOE Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, People's Republic of China

Also at Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, People's Republic of China <sup>m</sup> Also at the Department of Mathematical Sciences, IBA, Karachi 75270, Pakistan

<sup>n</sup> Also at Ecole Polytechnique Federale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland

<sup>o</sup> Also at Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany

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Using data samples collected with the BESIII detector operating at the BEPCII storage ring, the cross section of the inclusive process  $e^+e^- \rightarrow \eta + X$ , normalized by the total cross section of  $e^+e^- \rightarrow \eta + X$ 

hadrons, is measured at eight center-of-mass energy points from 2.0000 GeV to 3.6710 GeV. These are the first measurements with momentum dependence in this energy region. Our measurement shows a significant discrepancy compared to the existing fragmentation functions. To address this discrepancy, a new QCD analysis is performed at the next-to-next-to-leading order with hadron mass corrections and higher twist effects, which can explain both the established high-energy data and our measurements reasonably well.

Fragmentation Functions (FFs) describing the hadronization of color-carrying partons into colorneutral particles are a key non-perturbative ingredient of the Quantum Chromodynamics (QCD) factorized cross-section, which separate the perturbative hard part of the cross section from the non-perturbative part. Gaining precise knowledge of various FFs will help us to understand the mechanism of hadron production, hence improving our understanding of the color confinement property of QCD at long distance. In addition, FFs play indispensable roles in constraining the proton spin configuration and the nuclear parton distribution functions (PDFs), and probing the transport properties of the hot and dense QCD medium created in relativistic heavy-ion collisions [1-3].

Different from PDFs, FFs provide us with other rich ingredients to explore the non-perturbative aspects of QCD, due to the various hadrons produced in fixed target and collider experiments, while PDFs are mainly limited to protons [1]. In the state-of-the-art measurements of FFs, most of the efforts are devoted to pions and kaons owing to their more abundant production yields [4]. Currently, there is still a lack of measurements of  $\eta$  mesons. Compared to pions and kaons,  $\eta$  mesons are expected to provide additional information about the hadronization process as their wave function contains all light quarks and antiquarks [5-7]. In addition, due to the universality property of FFs, one can relate the  $\eta$ production in  $e^+e^-$  collisions, pp collisions, and semiinclusive deep inelastic scatterings (SIDIS) [8, 9]. The QCD analysis has shown good agreement with  $e^+e^$ and proton-proton collisions so far by including the only available  $e^+e^-$  data at a center-of-mass (c.m.) energy  $(\sqrt{s})$  above 9 GeV [7]. This work tests the validity of a factorized QCD framework at low energy region.

It is well known that the single inclusive  $e^+e^-$  annihilation,  $e^+e^- \rightarrow h + X$ , where h is an identified hadron under investigation and X represents everything else, provides an effective way to study collinear FFs [4]. A widely measured experimental observable is

$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})} \frac{\mathrm{d}\sigma(e^+e^- \to h + X)}{\mathrm{d}p_h}, \qquad (1)$$

where  $\sigma(e^+e^- \to \text{hadrons})$  represents the total cross section for  $e^+e^-$  annihilation to all possible hadronic final states (referred to as inclusive hadronic events hereafter), and  $p_h$  denotes the momentum of the identified hadron h. The observable can be interpreted, in terms of the leading order of  $\alpha_s$ , as  $\sum_q e_q^2 [D_q^h(z,\sqrt{s}) + D_{\bar{q}}^h(z,\sqrt{s})]$ , where  $e_q$ is the fractional charge of the quark q, and  $D_{q/\bar{q}}^h(z,\sqrt{s})$  is the FF of quark q or antiquark  $\bar{q}$  at c.m. energy  $\sqrt{s}$ . The variable  $z \equiv 2\sqrt{p_h^2 c^2 + M_h^2 c^4}/\sqrt{s}$  denotes the relative energy of the produced hadron h with mass  $M_h$ .

This Letter, for the first time, reports a measurement of the process  $e^+e^- \rightarrow \eta + X$  at eight c.m. energy points from 2.0000 to 3.6710 GeV, with a z coverage from 0.3 to 0.9. Such a special energy coverage is expected to be very sensitive to the initial parametrization of FFs, and can test the convergence of fixed order perturbative QCD (pQCD) calculations based on leading twist factorization involving final state hadron production [10-In addition, a comprehensive QCD analysis is 12]. performed by involving both experimental measurements and theoretical calculations of  $\eta$  production in  $e^+e^$ annihilation. In the analysis, the data reported in this Letter are implemented into the global study of  $\eta$  meson FFs, after considering the highest precision pQCD calculation at next-to-next-to-leading order (NNLO) [13], the mass corrections [14, 15], and the possible contributions from higher twist [11, 12]. The analysis not only indicates the importance of the BESIII measurement in exploring the  $\eta$  meson hadronization mechanism, but also serves as a testing ground for the validity of QCD factorization at leading twist for inclusive hadron production and their associated fixed order pQCD calculations.

The data sets used in this Letter were collected with the BESIII detector [16] running at BEPCII [17]. Experimentally, the normalized differential cross section characterizing the inclusive production of identified hadron h, as described in Eq. (1), can be determined with

$$\frac{N_h^{\text{obs}}}{N_{\text{had}}^{\text{obs}}} \frac{1}{\Delta p_h} f_h, \qquad (2)$$

where  $N_{\text{had}}^{\text{obs}}$  represents the number of observed hadronic events in the  $e^+e^-$  annihilation at a given c.m. energy, and  $N_h^{\text{obs}}$  denotes the number of  $e^+e^- \rightarrow h + X$  events within a specific momentum range  $\Delta p_h$ . The factor  $f_h$ , described later in detail, is a correction factor accounting for the global detection efficiency and the initial state radiation (ISR) effects. Since both  $N_{\text{had}}^{\text{obs}}$  and  $N_h^{\text{obs}}$  are obtained from the same data sample, the integrated luminosity used in the cross section measurement cancels.

For the first step of this analysis, the hadronic events are identified using the same selection criteria as described in Ref. [18]. The Bhabha and  $e^+e^- \rightarrow \gamma\gamma$  events are removed by applying dedicated requirements on the electromagnetic calorimeter (EMC) shower information. Subsequently, a series of criteria are applied to select good charged tracks. Events with less than two good charged tracks are removed to suppress background

processes. For events with two or three good charged tracks, further requirements are employed to suppress the QED-related backgrounds. Events with more than three good charged tracks are regarded as hadronic events directly. Details of the selection of the inclusive hadronic events can be found in Ref. [18].

Despite comprehensive selection criteria being applied to identify the inclusive hadronic events, there are still residual background events in the data. The numbers of the QED-related backgrounds are estimated by analyzing the corresponding Monte Carlo (MC) simulation samples. The GEANT4-based [19] programs are used to produce these MC samples, where the geometric description of the BESIII detector [20] and the interaction between secondary particles and the detector material are included. The BABAYAGA3.5 [21] package is used to generate the  $e^+e^- \rightarrow e^+e^-$ ,  $\mu^+\mu^-$ , and  $\gamma\gamma$ processes, while KKMC [22] and EVTGEN [23] are utilized for the  $e^+e^- \rightarrow \tau^+\tau^-$  process. The two-photon processes are simulated by dedicated MC generators [18]. The beam-associated background events are estimated using a sideband method [18]. Table I summarizes the integrated luminosities, the number of total selected hadronic events  $(N_{\text{had}}^{\text{tot}})$  and the total remaining backgrounds  $(N_{\text{bkg}})$  at each c.m. energy, where  $N_{\text{had}}^{\text{obs}} = N_{\text{had}}^{\text{tot}} - N_{\text{bkg}}$ .

TABLE I. The integrated luminosities, the numbers of total selected hadronic and residual background events at eight c.m. energy points.

$\sqrt{s}$ (GeV)	$\mathcal{L} (\mathrm{pb}^{-1})$	$N_{ m had}^{ m tot}$	$N_{\rm bkg}$
2.0000	10.074	$\frac{1100}{350298 \pm 591}$	$8722 \pm 93$
2.2000	13.699	$445019 \pm 666$	$10737 \pm 103$
2.3960	66.869	$1869906\pm1365$	$47550 \pm 218$
2.6444	33.722	$817528 \pm 902$	$21042 \pm 145$
2.9000	105.253	$2197328\pm1478$	$56841\pm238$
3.0500	14.893	$283822 \pm 531$	$7719\pm87$
3.5000	3.633	$62670 \pm 249$	$1691\pm41$
3.6710	4.628	$75253 \pm 273$	$6461\pm80$

From the selected inclusive hadronic events, the  $\eta$ candidates are reconstructed via the  $\eta \rightarrow \gamma \gamma$  decay. Photons are required to have a deposited energy in the EMC of more than 25 MeV in the barrel region  $(|\cos \theta| < 0.80)$ , and more than 50 MeV in the end cap region (0.86 <  $|\cos \theta| < 0.92$ ), where  $\theta$  is defined with respect to the z-axis, which is the symmetry axis of the multi-layer drift chamber. To exclude showers that originate from charged tracks, the angle subtended by the EMC shower and the position of the closest charged track at the EMC must be greater than 10 degrees as measured from the interaction point. To suppress electronic noise and showers unrelated to the event, the difference between the EMC time and the event start time is required to be within [0, 700] ns. Since the yield of  $\pi^0$  mesons is much higher comparing to  $\eta$  in the inclusive hadronic events [24], the photons originating from  $\pi^0$  are excluded when reconstructing  $\eta$ . If a pair of photons in one event has an invariant mass within

(115, 155) MeV/ $c^2$ , which covers five times the  $\pi^0$  mass resolution, both photons in that pair are discarded. A test using the control sample  $J/\psi \rightarrow K^+ K^- \pi^+ \pi^- \eta$ indicates that the exclusion of the  $\pi^0$  photons does not introduce any bias to the invariant-mass distribution of  $\eta$ . Moreover, a check using the MC sample of the inclusive hadronic process demonstrates that the exclusion does not induce any peaking background in the  $\eta$  invariantmass distribution. The remaining photons after veto of  $\pi^0$  are paired to reconstruct the  $\eta$  candidates. To suppress background due to photon mis-combinations, the helicity variable of the  $\eta$  candidate, defined as  $|E_{\gamma 1} E_{\gamma 2}|/p_{\gamma \gamma}$  where  $E_{\gamma 1,2}$  represent the deposited energies of photons and  $p_{\gamma\gamma}$  denotes the momentum of  $\eta$  candidate, is required to be less than 0.8. In this analysis, the inclusive hadronic events are simulated with the LUARLW generator [18, 25, 26], in which among others the signal processes  $e^+e^- \rightarrow \eta + X$  is contained.

The  $\eta$  candidates are divided into different momentum intervals with the bin width  $\Delta p_{\eta} = 0.1 \text{ GeV}/c$ , approximately 6 times the momentum resolution of  $\eta$ . The yield of  $\eta$  mesons in each momentum bin is determined by an unbinned maximum likelihood fit. In the fit, the signal is described by the signal shape of  $\eta$ , which is extracted from the LUARLW MC sample, convolved with a Gaussian function corresponding to the difference between data and MC. The background is modeled by a second-order polynomial, except for a few momentum bins where a third-order polynomial is used due to a higher mis-combination background level. Figure 1 shows the fit result of  $\eta$  candidates with  $p_{\gamma\gamma} \in (0.4, 0.5) \text{ GeV}/c$ at  $\sqrt{s} = 2.9000$  GeV, where the higher background at the low mass region is caused by the mis-combinations involving the low energy photons. To extract the signal shape of  $\eta$  from the LUARLW MC sample, the truth-level  $\eta$  mesons decaying to two photons are matched to the reconstructed  $\eta$  candidates according to the momentum direction. The reconstructed  $\eta$  candidate which has the closest momentum direction relative to the truth-level  $\eta$  is regarded as matched. In addition, the angle of the momenta between truth-level  $\eta$  and its matched  $\eta$ candidate is required to be less than 25 degrees. The matched  $\eta$  candidates make up the signal sample of  $\eta$  and their invariant-mass distributions are used as the signal shapes in the fitting procedure. The obtained  $N_n^{\text{obs}}$  in each momentum range of  $\eta$  at various c.m. energies are summarized in the Supplemental Material [27].

The correction factor  $f_h$ , which scales the observable quantity  $N_h^{\rm obs}/N_{\rm had}^{\rm obs}$  to determine the observable given in Eq. (1) in each momentum bin, is extracted from the inclusive hadronic MC sample and is expressed as:

$$f_h = \frac{N_h^{\rm tru}(\text{off})}{\bar{N}_{\rm had}^{\rm tru}(\text{off})} / \frac{N_h^{\rm obs}(\text{on})}{\bar{N}_{\rm had}^{\rm obs}(\text{on})}.$$
 (3)

Here,  $\bar{N}$  denotes the number of events determined from the inclusive hadronic MC sample, either at observable level, similar to the experimental data, with superscript "obs" or at truth level with superscript "tru". The



FIG. 1. The  $M(\gamma\gamma)$  distributions for  $\eta$  candidates, with  $p_{\gamma\gamma} \in (0.4, 0.5) \text{ GeV}/c$  at  $\sqrt{s} = 2.9000 \text{ GeV}$ . The fit results are overlaid. The black points with error bars are data. The red solid curve is the sum of fit functions, while the green solid and blue dashed curves represent the signal and background, respectively. The pull variable  $\chi$ , defined as the residual between data and total fit function, normalized by the uncertainty of the data, is shown on the bottom of the figure.

terms "on" and "off" in the parentheses indicate that the corresponding quantities are extracted from the inclusive hadronic MC sample with or without simulating the ISR process, respectively. In this Letter,  $\bar{N}_{\eta}^{\rm obs}({\rm on})$  is determined with a similar fit to the  $M(\gamma\gamma)$  distribution of the  $\eta$  candidates selected from the inclusive hadronic MC sample.

Extensive comparisons between the LUARLW generated MC events and the experimental data show that the LUARLW model can reasonably reproduce the multiplicity and kinematic quantity of the  $\eta$  mesons [27]. In addition, good agreements are observed in terms of the invariant mass spectra of  $\eta$  in different momentum bins between experimental data and the inclusive hadronic MC sample. Thus, the correction applied in this analysis is valid. The calculated results of  $f_{\eta}$  in the different  $\eta$  momentum ranges are presented in the Supplemental Material [27].

The systematic uncertainties of the normalized differential cross section are mainly caused by the residual deviations between the signal MC and data samples, the reconstruction efficiency of the  $\eta$  candidates, the fit scheme of the  $M(\gamma\gamma)$  spectrum, and the simulation model of the inclusive hadronic events.

The approach described in Ref. [24] is applied here to estimate the uncertainty caused by the imperfect simulation of signal events. Systematic uncertainties of the differential cross section introduced by the determination of  $N_{\rm bkg}$  are found to be negligible.

For the uncertainty of reconstructing the  $\eta$  candidates, several factors are considered, including the identification of photons, the exclusion of the  $\pi^0$  photons, and the helicity requirement. The uncertainty in the photon identification is estimated to be 1% per photon [52], resulting in 2% uncertainty for each  $\eta$  meson. The uncertainty due to the exclusion of the  $\pi^0$  photons is evaluated by varying the nominal invariant-mass range of  $\pi^0$  to (111, 159) MeV/ $c^2$ . The differences of the differential cross section are found to be negligible (less than 1%). To estimate the uncertainty due to the  $\eta$ helicity requirement, the  $\eta$  helicity distributions of the  $J/\psi \to K^+ K^- \pi^+ \pi^- \eta$  events are compared between data and MC simulation. The average relative difference of 2% is taken as the uncertainty.

To evaluate the uncertainty of the fit scheme, different signal and background description functions are applied to fit the  $M(\gamma\gamma)$  spectrum. For the signal, the Crystal Ball function [53] is used as an alternative model. Moreover, the different requirements of the match angle, which are 20 and 30 degrees, are utilized to extract the alternative signal shapes from the signal MC sample. For the background, the alternative models are obtained by varying the order of the Chebychev polynomial. All the resulting relative differences in the differential cross sections are combined in quadrature, and taken as the systematic uncertainty.

In this Letter, the dominant systematic uncertainty is introduced by the MC simulation model of the inclusive hadronic events. According to Eq. (3), the generation fractions of the exclusive processes containing the  $\eta$  mesons, which make up the inclusive process  $e^+e^- \rightarrow \eta + X$ , directly affect the correction factors  $f_{\eta}$ . To address the corresponding uncertainty, the HYBRID generator, which was developed in Ref. [54] and improved in Ref. [18], is used as an alternative model to reproduce the inclusive hadronic events. The discrepancies observed in the correction factors  $f_{\eta}$  relative to the nominal ones are regarded as systematic uncertainties.

All these individual systematic uncertainties are regarded as uncorrelated with each other therefore are summed in quadrature. The normalized differential cross sections for the inclusive  $\eta$  production in  $e^+e^-$  annihilation at the eight c.m. energy points are tabulated in the Supplemental Material [27] and shown in Fig. 2.

The blue dotted curve in Fig. 2 represents a theoretical prediction performed using the  $\eta$  FF from the Aidala-Ellinghaus-Sassot-Seele-Stratmann (AESSS) parametrization at the next-to-leading order [7]. The AESSS FFs are extracted using data of  $\eta$  production in  $e^+e^-$  annihilation with  $\sqrt{s} \approx 10, 30, \text{ and } 90 \text{ GeV}, \text{ and}$ pp collisions with  $\sqrt{s} \approx 200$  GeV whose energy scales are higher than the typical BESIII c.m. energies. The AESSS study is based on the well-established de Florian-Sassot-Stratmann (DSS) framework [55, 56] for FF extractions. It consists of a pure next-to-leading order analysis, based on leading-twist pQCD factorization theorems for  $e^+e^$ annihilation and pp collision processes, where the mass of the  $\eta$ -meson is considered to be negligible and set to be zero. The fit in Ref. [7] reveals good agreement among data sets taken at different energy scales. However, according to Fig. 2, the AESSS fit cannot describe the previous data and the BESIII data at the same time.

The gray line in Fig. 2 shows the calculation with a



FIG. 2. Normalized differential cross sections of the  $e^+e^- \rightarrow \eta + X$  process. The points with error bars are the measured values. The blue dotted curves denote the predictions by using the AESSS FFs, while the curves in gray denote the calculations by using the newly extracted FFs from our fit based on the available  $\eta$  production data in  $e^+e^-$  annihilation and our BESIII data. Notice that an upper cut of z < 0.95, where theoretical curves stop, is employed in our global analysis to avoid large enhancement from threshold logarithms  $\propto \log(1-z)$ . The normalized differential cross section in terms of z is shown in the Supplemental Material [27].

new extraction of  $\eta$  FFs [27, 57] based on the available n production data in  $e^+e^-$  annihilation experiments. namely the data sets included in Ref. [7], except for the un-published BaBar data, and the BESIII data presented in this Letter. The ratio of  $\chi^2/N_{d.o.f.}$  for this fit is 1.52 [27, 57] which is comparable to that of the AESSS fit  $(1.91 \ [7])$  where only the existing  $e^+e^-$  annihilation data are considered. For the first time, data at  $\sqrt{s} < 5$  GeV are included in such a QCD-based analysis where the analysis framework is extended to NNLO accuracy and the hadron mass corrections and higher twist contributions are considered [27, 57]. The inclusion of BESIII data in the original AESSS framework, namely a refit, leads to a significantly large  $\chi^2/N_{d.o.f.}$  (12.79) which confirms the disagreement between the AESSS fit and the BESIII data. Each of the three major effects considered in the new fit plays a fundamental role in achieving the good agreement as shown in Fig. 2. Hadron mass corrections are well known to be an important effect in the fit of FFs of heavier hadron species, e.g., see discussions in Refs. [14, 58, 59]. The extension to NNLO has an important effect on the shape of the observable in the lower-z region, as one can for example explicitly see in the case of the NNLO parton-to-pion FF fit for  $e^+e^-$  production in Fig. 4 of Ref. [60]. That analysis highlighted the importance of the higher accuracy framework in order to perform a reasonable fit including Belle and BaBar data sets which have c.m. energy around 10.5 GeV. At last, higher twist effects are commonly introduced in PDF analysis when incorporating low energy data, such as JLab data in Ref. [61]. They are taken into account as extra fit parameters that parameterize an additional  $1/Q^2$  dependence to the leading twist expression of the observable. In this analysis, both  $1/Q^2$  and  $1/Q^4$  dependence have to be introduced in order to obtain a good fit. The analysis is summarized in Supplemental Material [27] and detailed in Ref. [57].

In summary, we have measured the normalized differential cross sections of the  $e^+e^- \rightarrow \eta + X$  processes, using data samples collected from  $\sqrt{s} = 2.0000$  to  $3.6710~{\rm GeV}.$  The results obtained in this work fill this particular energy region where no such kind of measurements have been reported before. A QCD-based analysis shows that in order to explain both high and low-energy data, one needs to consider higher-order contributions as well as higher twist effects. It would be interesting to check if a more flexible approach, such as NNFF [62] or MAPFF [63], could describe the data over the full energy range. These new results in the relatively low energy region provide special ingredients for FF studies, moreover, they will help to enhance our understanding of the QCD factorization theorem at the leading twist and beyond.

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