Measurements of Differential Cross Sections of Inclusive π^0 and K_S^0 Production in e^+e^- Annihilation at Energies from 2.2324 to 3.6710 GeV

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Based on electron positron collision data collected with the BESIII detector operating at the BEPCII storage rings, the differential cross sections of inclusive π^0 and K_S^0 production as a function of hadron momentum, normalized by the total cross section of the $e^+e^- \to$ hadrons process, are measured at six center-of-mass energies from 2.2324 to 3.6710 GeV. Our results with a relative hadron energy coverage from 0.1 to 0.9 significantly deviate from several theoretical calculations based on existing fragmentation functions, especially at lower energies.

Color confinement at long distance is one of the fundamental properties of Quantum Chromodynamics (QCD), which is the underlying theory to describe the strong interactions between quarks and gluons. Because of confinement, quarks and gluons produced in hard scattering will ultimately become colorless hadrons. The

transition from quarks q, antiquarks \bar{q} and gluons g to hadrons h, which occurs at a scale with a small momentum transfer, must be treated non-perturbatively due to the large strong coupling constant α_s [1, 2]. Consequently, the fragmentation function (FF) $D_{q,\bar{q},g}^h(z,\mu_F)$ is used to describe the non-perturbative long distance

behavior associated with the hadronization process [3]. Here, μ_F is the factorization scale defining the transition between the perturbative and non-perturbative regimes and is usually set to be the center-of-mass (c.m.) energy (\sqrt{s}) in e^+e^- experiments. The term $z \equiv 2\sqrt{p_h^2 + M_h^2}/\sqrt{s}$ is the relative hadron energy, where p_h and M_h are the momentum and mass of hadron h, respectively. Single inclusive e^+e^- annihilation, $e^+e^- \to h+X$, where h is an identified hadron and X represents everything else, provides a clean way to study FFs [4]. A typical experimental observable is

$$\frac{1}{\sigma(e^+e^- \to \text{hadrons})} \frac{d\sigma(e^+e^- \to h + X)}{dp_h}, \qquad (1)$$

where $\sigma(e^+e^- \to \text{hadrons})$ is the total cross section for e^+e^- annihilation to all possible hadronic final states. At the leading order, the observable can be interpreted as $\sum_q e_q^2 [D_q^h(z,\sqrt{s}) + D_{\bar{q}}^h(z,\sqrt{s})]$, where e_q is the charge of the quark q. As summarized in Ref. [4], extensive high-precision measurements of this observable have been made in e^+e^- experiments in the c.m. energy range above 10 GeV, but measurements in the region from 3.6 to 5.2 GeV have statistical uncertainties ranging from 20% to 50% [5, 6].

On the other hand, remarkable progress has been made in the experimental study of nucleon structure by taking advantage of the semi-inclusive deep-inelastic scattering (SIDIS) process. Due to its explicit dependence on both the identified initial and final state hadrons, the SIDIS process plays a crucial role in probing specific quark flavors. In particular, the FFs act as the weights used to perform flavor separation in the initial state. For example, precise kaon FFs might be the key to understand the puzzle of the strange quark polarization inside a longitudinally polarized proton [7, 8]. The momentum-transfer Q (equivalent to \sqrt{s} in e^+e^- annihilation) coverage for the existing SIDIS data from fixed-target experiments like JLab, COMPASS and HERMES [9] is from 1 to 10 GeV, where only sparse single inclusive e^+e^- annihilation data exist. Moreover, the proposed Electron-Ion Collider (EIC) and the Electron-Ion Collider in China (EicC) [10, 11] would provide very high precision structure function measurements, which in order to determine precise parton distribution functions (PDFs) for various quark flavors, would put unprecedented requirements on the precision of FFs for the Q value down to 1 GeV and almost complete zcoverage from 0 to 1. Therefore, comprehensive investigations on the single inclusive annihilation with identified pion and kaon final states, especially in the region not well covered by previous data, will provide valuable inputs for the nucleon structure study.

In this Letter, the processes $e^+e^- \to \pi^0/K_S^0 + X$ are studied at six c.m. energies from 2.2324 to 3.6710 GeV, with a z coverage from 0.1 to 0.9. Data sets used in this research were collected with the BESIII detector [12] running at BEPCII. The detector has a geometrical acceptance of 93% of the 4π solid angle. It is based

on a superconducting solenoid magnet with a main drift chamber as a central tracking system, plastic scintillators as a time-of-flight system, CsI crystals as an electromagnetic calorimeter (EMC) and resistive plate chambers as a muon system.

Monte Carlo (MC) simulations based on GEANT4 software [13], which includes the geometric description of the BESIII detector and implements the interactions between the final state particles and the detector, are used to optimize the event selection criteria, estimate the number of residual background events, and determine the correction factors accounting for the efficiency loss and initial state radiation (ISR) effects. inclusive hadronic events are simulated with the LUARLW generator [14–16], where the signal processes $e^+e^- \rightarrow$ $\pi^0/K_S^0 + X$ are contained. Background MC samples of the processes $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-$, and $\gamma\gamma$ are generated by BABAYAGA3.5 [17]. At $\sqrt{s} = 3.6710 \,\mathrm{GeV}$, the $e^+e^- \rightarrow$ $\tau^+\tau^-$ process is simulated via the KKMC [18] generator, in which the decay of τ lepton is modeled by EVTGEN [19]. The background events from the two-photon processes are simulated by the corresponding dedicated MC generators, as detailed in Ref. [16]. In addition, the beamassociated events are estimated by a side-band method.

In this analysis, the inclusive hadronic events are selected first, from which the $e^+e^-\to \pi^0/K_S^0+X$ events are identified by reconstructing a π^0 or K_S^0 meson. We begin by removing the dominant background processes, i.e., the $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow \gamma\gamma$ events. In each remaining event, the good charged hadronic tracks are identified with the same selection criteria described in Ref. [16], and events with at least two good tracks are kept for further analysis. For events with two or three good charged tracks, additional requirements on the charged tracks and the showers in the EMC are implemented to further suppress the QED-related background. Events with more than three good charged tracks are retained directly without any additional requirements. The details of the selection of the inclusive hadronic events are presented in Ref. [16]. The numbers of inclusive hadronic $(N_{\rm had}^{\rm tot})$ events and residual background $(N_{\rm bkg})$ events, as well as the integrated luminosity of each data sample, are summarized in Table I.

TABLE I. The integrated luminosities and the total observed hadronic and residual background events at various c.m. energy points.

$\sqrt{s} \; (\text{GeV})$	\mathcal{L} (pb ⁻¹)	$N_{ m had}^{ m tot}$	$N_{ m bkg}$
2.2324	2.645	83227	2041
2.4000	3.415	96627	2331
2.8000	3.753	83802	2075
3.0500	14.89	283822	7719
3.4000	1.733	32202	843
3.6710	4.628	75253	6461

A π^0 candidate is reconstructed via the decay $\pi^0 \to \gamma\gamma$. Photon candidates are identified using showers in

the EMC. The deposited energy of each shower must be greater than 25 MeV in the barrel region ($|\cos \theta| < 0.80$) and 50 MeV in the end-cap region $(0.86 < |\cos \theta| < 0.92)$. The shower is required to be separated by more than 10 degrees from the closest charged track to eliminate those produced by the charged particles. The difference between the EMC time and the event start time has to be within [0, 700] ns to suppress electronic noise and showers unrelated to the event. All combinations of two photons are used to form π^0 candidates. To suppress background due to the miscombination of photons, requirements are made on the polar angle of one photon in the helicity frame of the π^0 candidate (θ_{γ}) . For π^0 candidates with momentum less than 0.3 GeV/c, $|\cos\theta_{\gamma}|$ is required to be less than 0.8, while for those with momentum larger than 0.3 GeV/c, it must be less than 0.95. All π^0 candidates in an event are counted as separate inclusive π^0 events.

A K_S^0 candidate is formed by combining a pair of oppositely charged tracks. The two tracks are required to satisfy $|\cos \theta| < 0.93$, where θ is the polar angle of the track with respect to the z-axis, which is the symmetry axis of the MDC. Due to the relatively long lifetime of the K_S^0 meson, the distance of closest approach of these charged tracks from the interaction point must be less than 30 cm along and 10 cm perpendicular to the z axis. As a result of the different requirements, the charged track here is not necessarily one of the good chagred tracks identified previously. To increase the number of events, no particle identification is applied. Further, to select K_S^0 signal events, the production and decay vertices are reconstructed, and the decay length between these two vertices is required to be at least twice its uncertainty. All K_S^0 candidates in an event are counted as separate inclusive K_S^0 events.

After imposing the above selection criteria, the residual contributions to the mass spectra of the photon and charged pion pairs, $M(\gamma\gamma)$ and $M(\pi^+\pi^-)$, from lepton pair production, two-photon processes, and beam-associated events are less than 0.1% in both signal processes. The dominant background is caused by the miscombinations of photons and pions, which are reproduced by the inclusive hadronic MC samples and well described by a polynomial.

The $M(\gamma\gamma)$ and $M(\pi^+\pi^-)$ spectra are divided into momentum intervals with a step of $\Delta p_{\pi^0/K_S^0}=0.1~{\rm GeV}/c$. Unbinned maximum likelihood fits are performed on the events in the $M(\gamma\gamma)$ and $M(\pi^+\pi^-)$ momentum intervals to determine the numbers of events, $N_{\pi^0}^{\rm obs}$ and $N_{K_S^0}^{\rm obs}$ versus momentum. For the $e^+e^- \to \pi^0 + X$ candidates, the signal is described by a Crystal Ball function, while the background is parameterized by a second order Chebychev polynomial. For the $e^+e^- \to K_S^0 + X$ process, the signal is modeled by a double Gaussian function, and the background is described by a first order Chebychev polynomial. Figure 1 illustrates the result of the fits of the π^0 candidates and the K_S^0 candidates with $p_{\gamma\gamma/\pi^+\pi^-} \in (0.4, 0.5)~{\rm GeV}/c$ from the

data sample at $\sqrt{s}=2.8000$ GeV. The summary of $N_{\pi^0}^{\rm obs}$ and $N_{K_S^0}^{\rm obs}$ obtained in each momentum range at each c.m. energy is given in the Supplemental Material [23].

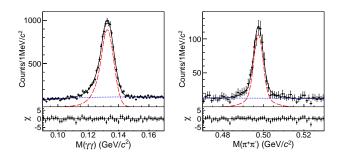


FIG. 1. $M(\gamma\gamma)$ and $M(\pi\pi)$ distributions for π^0 (left) and K_S^0 (right) candidates, respectively, with $p_{\gamma\gamma/\pi^+\pi^-}\in (0.4,0.5)$ GeV/c at $\sqrt{s}=2.8000$ GeV. The fit results are overlaid. The black points with error bars are data. The black solid curves are the sum of fit functions, while the red dashed and blue dotted curves represent the signal and background, respectively. The pull variable χ , defined as the residual between the data and the total fit function, normalized by the uncertainty of the data, is shown beneath the figures.

In practice, the normalized differential cross section for the inclusive production of any identified hadron, namely Eq. (1), is determined with

$$\frac{N_h^{\text{obs}}}{N_{\text{bad}}^{\text{obs}}} \frac{1}{\Delta p_h} f_h, \tag{2}$$

where $N_{\rm had}^{\rm obs}=N_{\rm had}^{\rm tot}-N_{\rm bkg}$ is the net number of observed hadronic events in e^+e^- annihilation at a given c.m. energy, $N_h^{\rm obs}$ is that of the $e^+e^-\to h+X$ events within a specific momentum range Δp_h , and f_h is a correction factor which accounts for the effects caused by the limited detector acceptance, the selection criteria, ISR, and vacuum polarization. Since both $N_{\rm had}^{\rm obs}$ are obtained from the same data sample, the integrated luminosity usually used in the cross section measurement cancels.

In this analysis, the correction factor, f_h , is obtained from the signal MC sample and is given by

$$f_h = \frac{\bar{N}_h^{\text{tru}}(\text{off})}{\bar{N}_{\text{had}}^{\text{tru}}(\text{off})} / \frac{\bar{N}_h^{\text{obs}}(\text{on})}{\bar{N}_{\text{had}}^{\text{obs}}(\text{on})},$$
(3)

where the variable " \bar{N} " denotes the numbers of events determined from the signal MC sample, either at the detector observed level, similar to the experimental data, with superscript "obs" or at the generation level with superscript "tru". The terms "on" and "off" in the parentheses indicate that the corresponding quantities are obtained from signal MC samples with and without simulating the ISR effects, respectively. $\bar{N}_h^{\rm obs}({\rm on})$ is determined by applying the same fit procedure on the

signal MC events as data. Calculated results of f_h in different momentum ranges for π^0 and K_S^0 are presented in the Supplemental Material [23].

The systematic uncertainties of the normalized differential cross section measurements mainly originate from the differences between the signal MC and data samples, the reconstruction efficiencies of the π^0 and K^0_S candidates, the fits to the events in the $M(\gamma\gamma)$ and $\tilde{M}(\pi^+\pi^-)$ bins, and the MC simulation model of the inclusive hadronic events. To estimate the uncertainty caused by the imperfect simulation of various kinematic variables of the signal events, all the selection criteria are varied to be larger or smaller than their nominal values by one time their resolutions, and the maximum changes of the normalized differential cross sections are taken as the systematic uncertainties. The uncertainty in the photon reconstruction is estimated to be 1\% per photon [20], therefore 2% is taken as the systematic uncertainty due to the π^0 reconstruction efficiency. The momentum-dependent systematic uncertainties due to the K_S^0 reconstruction efficiency are obtained by applying a dedicated weight procedure [21], which includes those of the tracking efficiencies of charged tracks and the decay length requirement.

The uncertainties due to the fits on the events in the $M(\gamma\gamma)$ and $M(\pi^+\pi^-)$ intervals are examined by using alternative signal and background shapes. The alternative signal shape of $e^+e^-\to\pi^0+X$ is taken as the shape from the signal MC sample, while that for $e^+e^-\to K_S^0+X$ is chosen as a single Gaussian function. The alternative background shapes are obtained by varying the order of the Chebychev polynomials. The relative differences from the original differential cross sections are taken as the corresponding systematic uncertainties.

The dominant source of systematic uncertainty is the MC simulation model of the inclusive hadronic events. The generation fractions of the exclusive processes containing π^0 and K_S^0 , which make up the inclusive process, directly affect the correction factors f_{π^0} and $f_{K_{\mathcal{S}}^0}$. To address the corresponding uncertainty, the HYBRID generator, which was developed in Ref. [22] and improved in Ref. [16], is used as an alternative MC model to generate the inclusive hadronic events. In the HYBRID generator, much knowledge of the allowed exclusive processes in the c.m. energy region of BESIII has been incorporated, including measured cross sections and production mechanisms. In addition, a different simulation scheme of the ISR process is adopted [16]. The changes of the correction factors f_{π^0} and $f_{K_s^0}$ are assigned as the systematic uncertainties. All these individual systematic uncertainties are regarded as uncorrelated with each other and summed in quadrature.

The normalized differential cross sections for the inclusive π^0 and K_S^0 production in e^+e^- annihilation at the six c.m. energies are tabulated in the Supplemental Material [23] and shown in Fig. 2 and Fig. 3, respectively. Figure 2 also shows various theoretical predictions extrapolated from different FFs determined from

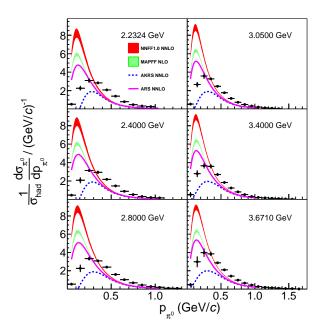


FIG. 2. Normalized differential cross section of the $e^+e^- \to \pi^0 + X$ process. The points with error bars are the measured values. The bands or curves in red, green, blue and magenta denote the NNFF, MAPFF, AKRS, and ARS calculations, respectively, where only the former two cover $\pm 1\sigma$ limits.

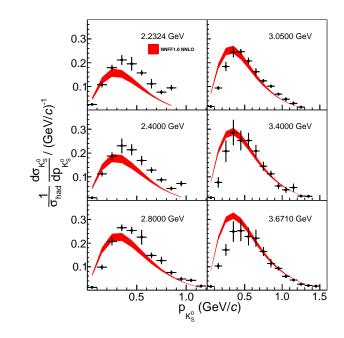


FIG. 3. Normalized differential cross section of the $e^+e^- \to K_S^0 + X$ process. The points with error bars are the measured values. The red band shows the theoretical calculation from NNFF with $\pm 1\sigma$ limits.

existing world data [4]. The FFs are obtained with slightly different assumptions and show the sensitivity of the prediction to assumptions about the behavior at low-z and different \sqrt{s} . ARS [24], AKRS [25] and NNFF1.0 [26] are all obtained from inclusive annihilation data at NNLO accuracy. However, AKRS includes small-z resummation, NNFF1.0 includes hadron mass corrections and all of them have different initial evolution scales and kinematics requirements on the data. The MAPFF1.0 [27] NLO study contains low Q^2 data from the lepton-proton fixed target experiments at HERMES [28] and COMPASS [29]. Figure 3 shows the comparison of the normalized differential cross section of the inclusive K_S^0 production with predictions using NNFF1.0 at NNLO precision. In these comparisons, the agreement degenerates as the c.m. energies decrease. Here, the FFs are further away from the kinematic region of the original data. One possible reason for these discrepancies is that the theoretical calculation at the leading twist may not be sufficient at the BESIII energy scale. It may also be important to consider quark mass and hadron mass correction effects [30], and smallz resummation effects [25]. Another problem may be with the extrapolation of the FFs from existing $e^+e^$ annihilation data at high energy to the low energy scale at BESIII. For instance, the predictions using QCDbackward evolution for initial state parton distribution functions from high to low energies have been found to deviate from the experimental measurements [31]. Both Fig. 2 and Fig. 3 show that BESIII data can be used to improve the fit procedure to determine the FFs at the low energy scale. Also, the difference between the components of direct (primary) and indirect (secondary) processes has to be taken into account as well. The possibility of quarks directly becoming hadrons in the detector or the hadron being detected after the decay of other hadrons is different at various energy scales. The results presented in this Letter will be key to explore these possibilites, as well as help to test collinear perturbative QCD with data at the relatively low energy scale.

In summary, we have measured the normalized differential cross sections of the $e^+e^- \to \pi^0/K_S^0+X$ processes, using data samples collected from $\sqrt{s}=2.2324$ to 3.6710 GeV. The results obtained in this work help to fill the region with $\sqrt{s}<10$ GeV where precision e^+e^- annihilation data have been rarely reported. The results provide broad z coverage from 0.1 to 0.9 with precision of around 3% at $z\sim0.4$. These results provide new

ingredients for FF global data fits, in which almost no single inclusive annihilation data measured in this special energy region has been included.

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