

Supplemental Material for “Study of light scalar mesons through $D_s^+ \rightarrow \pi^0 \pi^0 e^+ \nu_e$ and $K_S^0 K_S^0 e^+ \nu_e$ decays”

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The data samples used in this analysis are collected at $\sqrt{s} = 4.178, 4.189, 4.199, 4.209, 4.219, 4.226$ GeV. For some aspects of the analysis, these samples are organized into three sample groups, 4.178 GeV, 4.189-4.219 GeV, and 4.226 GeV, that were acquired during the same year under consistent running conditions.

The tag D_s^- invariant mass M_{tag} requirement, ST tag yield, and ST efficiency for each tag mode are listed in Table I. The DT efficiencies for $D_s^+ \rightarrow f_0(980)e^+\nu_e$ with $f_0(980) \rightarrow \pi^0\pi^0$, $D_s^+ \rightarrow f_0(500)e^+\nu_e$ with $f_0(500) \rightarrow \pi^0\pi^0$, and $D_s^+ \rightarrow K_S^0 K_S^0 e^+\nu_e$ are listed in Tables II, III, and IV, respectively.

TABLE I. ST yields ($N_{\alpha,i}^{\text{ST}}$) and ST efficiencies ($\epsilon_{\alpha,i}^{\text{ST}}$) for (I) $\sqrt{s} = 4.178$ GeV, (II) 4.189 – 4.219 GeV, and (III) 4.226 GeV. Uncertainties are statistical only. These efficiencies do not include the following intermediate-state branching fractions for $K_S^0 \rightarrow \pi^+\pi^-$, $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $\eta' \rightarrow \pi^+\pi^-\eta$.

Tag mode	M_{tag} (GeV/ c^2)	(I) $N_{\alpha,i}^{\text{ST}}$	(I) $\epsilon_{\alpha,i}^{\text{ST}}$
$D_s^- \rightarrow K_S^0 K^-$	[1.948, 1.991]	31941 ± 312	47.36 ± 0.07
$D_s^- \rightarrow K^+ K^- \pi^-$	[1.950, 1.986]	137240 ± 614	39.47 ± 0.03
$D_s^- \rightarrow K_S^0 K^- \pi^0$	[1.946, 1.987]	11385 ± 529	16.12 ± 0.11
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	[1.947, 1.982]	39306 ± 799	10.50 ± 0.03
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	[1.958, 1.980]	8093 ± 326	20.40 ± 0.12
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	[1.953, 1.983]	15719 ± 289	21.83 ± 0.06
$D_s^- \rightarrow \pi^- \pi^- \pi^+$	[1.952, 1.982]	37977 ± 859	51.43 ± 0.15
$D_s^- \rightarrow \pi^- \eta'$	[1.940, 1.996]	7759 ± 141	19.12 ± 0.06
$D_s^- \rightarrow K^- \pi^+ \pi^-$	[1.953, 1.986]	17423 ± 666	47.46 ± 0.22

Tag mode	(II) $N_{\alpha,i}^{\text{ST}}$	(II) $\epsilon_{\alpha,i}^{\text{ST}}$	(III) $N_{\alpha,i}^{\text{ST}}$	(III) $\epsilon_{\alpha,i}^{\text{ST}}$
$D_s^- \rightarrow K_S^0 K^-$	18559 ± 261	47.26 ± 0.09	6582 ± 160	46.37 ± 0.16
$D_s^- \rightarrow K^+ K^- \pi^-$	81286 ± 505	39.32 ± 0.04	28439 ± 327	38.38 ± 0.07
$D_s^- \rightarrow K_S^0 K^- \pi^0$	6832 ± 457	15.71 ± 0.16	2227 ± 220	15.93 ± 0.29
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	23311 ± 659	10.58 ± 0.05	7785 ± 453	10.39 ± 0.08
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	5269 ± 282	20.19 ± 0.17	1662 ± 217	19.50 ± 0.31
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	8948 ± 231	21.63 ± 0.09	3263 ± 172	21.29 ± 0.15
$D_s^- \rightarrow \pi^- \pi^- \pi^+$	21909 ± 776	50.35 ± 0.22	7511 ± 393	49.32 ± 0.41
$D_s^- \rightarrow \pi^- \eta'$	4428 ± 111	19.00 ± 0.08	1648 ± 74	18.56 ± 0.13
$D_s^- \rightarrow K^- \pi^+ \pi^-$	10175 ± 448	47.19 ± 0.32	4984 ± 458	45.66 ± 0.59

TABLE II. DT efficiencies ($\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$) of each tag mode for the signal process $D_s^+ \rightarrow f_0(980)e^+\nu_e$, $f_0(980) \rightarrow \pi^0\pi^0$ at (I) $\sqrt{s} = 4.178$ GeV, (II) 4.189 – 4.219 GeV, and (III) 4.226 GeV. Uncertainties are statistical only. These efficiencies do not include the following intermediate-state branching fractions for $K_S^0 \rightarrow \pi^+\pi^-$, $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $\eta' \rightarrow \pi^+\pi^-\eta$.

Tag mode	(I) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(II) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(III) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)
$D_s^- \rightarrow K_S^0 K^-$	7.11 ± 0.26	6.64 ± 0.13	6.48 ± 0.25
$D_s^- \rightarrow K^+ K^- \pi^-$	5.61 ± 0.10	5.24 ± 0.05	5.08 ± 0.10
$D_s^- \rightarrow K_S^0 K^- \pi^0$	1.93 ± 0.13	1.93 ± 0.07	1.87 ± 0.14
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	1.43 ± 0.05	1.38 ± 0.02	1.41 ± 0.05
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	2.74 ± 0.20	2.43 ± 0.09	2.38 ± 0.18
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	2.48 ± 0.15	2.41 ± 0.08	2.40 ± 0.14
$D_s^- \rightarrow \pi^- \pi^- \pi^+$	8.31 ± 0.27	7.73 ± 0.13	7.60 ± 0.25
$D_s^- \rightarrow \pi^- \eta'$	2.79 ± 0.20	2.51 ± 0.10	2.18 ± 0.17
$D_s^- \rightarrow K^- \pi^+ \pi^-$	7.05 ± 0.32	6.51 ± 0.15	6.55 ± 0.29

TABLE III. DT efficiencies ($\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$) of each tag mode for the signal process $D_s^+ \rightarrow f_0(500)e^+\nu_e$, $f_0(500) \rightarrow \pi^0\pi^0$ at energy points, (I) $\sqrt{s} = 4.178$ GeV, (II) $4.189 - 4.219$ GeV, and (III) 4.226 GeV. Uncertainties are statistical only. These efficiencies do not include the following intermediate-state branching fractions for $K_S^0 \rightarrow \pi^+\pi^-$, $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $\eta' \rightarrow \pi^+\pi^-\eta$.

Tag mode	(I) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(II) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(III) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)
$D_s^- \rightarrow K_S^0 K^-$	4.34 ± 0.20	4.21 ± 0.10	3.64 ± 0.19
$D_s^- \rightarrow K^+ K^- \pi^-$	3.61 ± 0.08	3.50 ± 0.04	3.30 ± 0.08
$D_s^- \rightarrow K_S^0 K^- \pi^0$	1.24 ± 0.11	1.15 ± 0.05	1.01 ± 0.10
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	0.93 ± 0.04	0.94 ± 0.02	0.90 ± 0.04
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	1.96 ± 0.17	1.70 ± 0.08	1.78 ± 0.16
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	1.65 ± 0.12	1.79 ± 0.06	1.60 ± 0.12
$D_s^- \rightarrow \pi^- \pi^- \pi^+$	5.30 ± 0.22	5.06 ± 0.11	4.47 ± 0.20
$D_s^- \rightarrow \pi^- \eta'$	1.73 ± 0.16	1.54 ± 0.08	1.51 ± 0.15
$D_s^- \rightarrow K^- \pi^+ \pi^-$	4.13 ± 0.24	4.36 ± 0.13	3.96 ± 0.24

TABLE IV. DT efficiencies ($\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$) of each tag mode for the signal process $D_s^+ \rightarrow K_S^0 K_S^0 e^+\nu_e$ at (I) $\sqrt{s} = 4.178$ GeV, (II) $4.189 - 4.219$ GeV, and (III) 4.226 GeV. Uncertainties are statistical only. These efficiencies do not include the following intermediate-state branching fractions for $K_S^0 \rightarrow \pi^+\pi^-$, $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$, and $\eta' \rightarrow \pi^+\pi^-\eta$.

Tag mode	(I) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(II) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)	(III) $\epsilon_{\alpha,\text{sig},i}^{\text{DT}}$ (%)
$D_s^- \rightarrow K_S^0 K^-$	4.87 ± 0.22	4.51 ± 0.10	4.85 ± 0.21
$D_s^- \rightarrow K^+ K^- \pi^-$	3.84 ± 0.08	3.65 ± 0.04	3.78 ± 0.08
$D_s^- \rightarrow K_S^0 K^- \pi^0$	1.41 ± 0.12	1.37 ± 0.06	1.31 ± 0.11
$D_s^- \rightarrow K^+ K^- \pi^- \pi^0$	1.00 ± 0.04	1.03 ± 0.02	0.97 ± 0.04
$D_s^- \rightarrow K_S^0 K^- \pi^- \pi^+$	1.25 ± 0.14	1.04 ± 0.06	1.01 ± 0.12
$D_s^- \rightarrow K_S^0 K^+ \pi^- \pi^-$	1.22 ± 0.10	1.23 ± 0.05	1.30 ± 0.11
$D_s^- \rightarrow \pi^- \pi^- \pi^+$	5.56 ± 0.23	5.09 ± 0.11	4.78 ± 0.21
$D_s^- \rightarrow \pi^- \eta'$	1.42 ± 0.15	1.69 ± 0.08	1.54 ± 0.15
$D_s^- \rightarrow K^- \pi^+ \pi^-$	4.44 ± 0.25	4.32 ± 0.12	4.30 ± 0.25