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I. FIGURES OF FIT RESULTS FOR ALL SIX DATA SETS

Figures 1-4 show our nominal fits to the all six data sets.

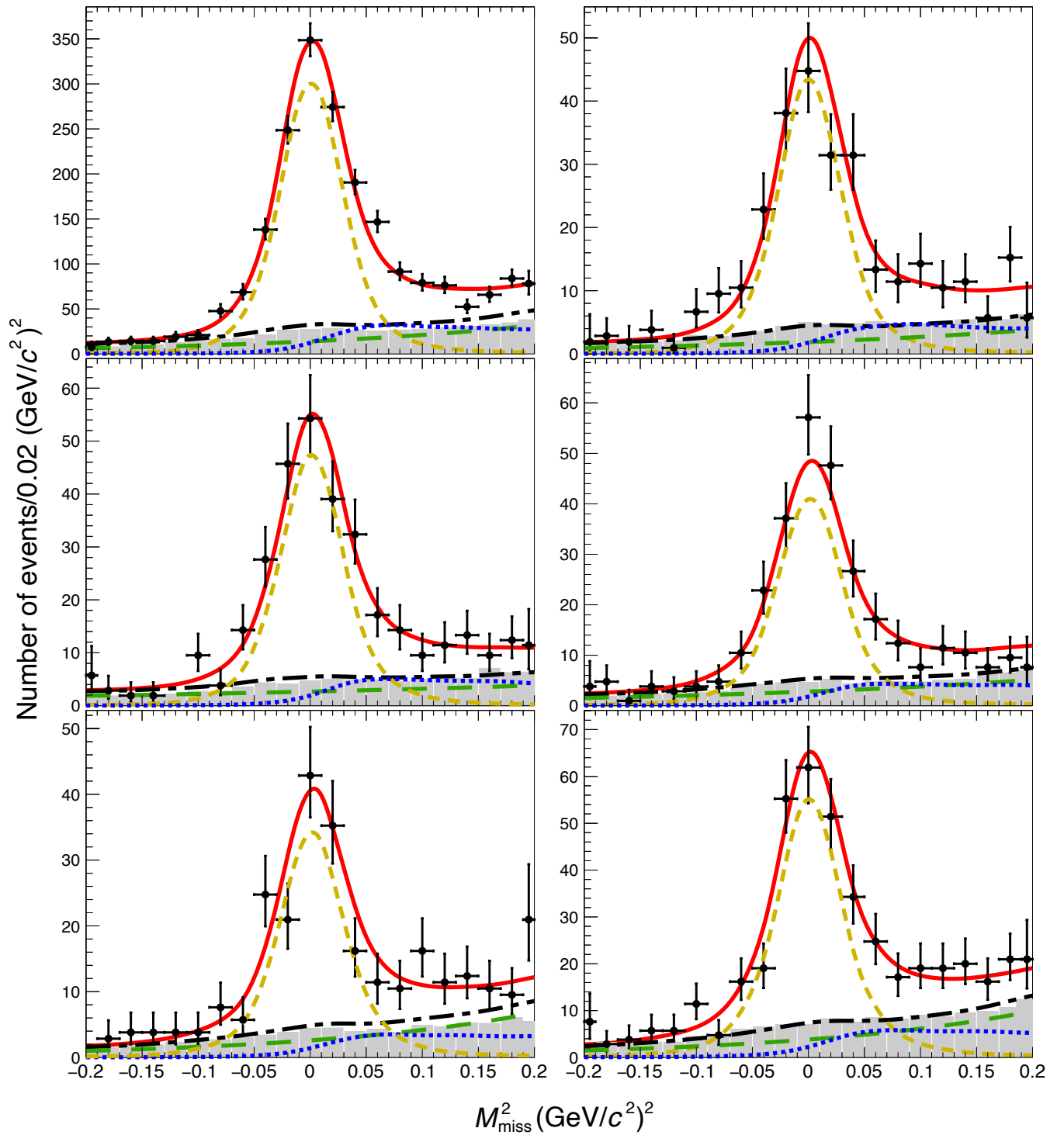


FIG. 1. Projections onto the M_{miss}^2 axis of the two-dimensional fit to 4180 data (top left), 4190 data (top right), 4200 data (middle left), 4210 data (middle right), 4220 data (bottom left), and 4230 data (bottom right) for the μ -like sample. The black points are data, the shaded histograms correspond to the $40\times$ background MC sample scaled to the integrated luminosity of data, and the lines represent the fitted signal and background shapes. The red-solid, orange-dashed, and blue-dotted lines represent the total, $D_s^+ \rightarrow \mu^+ \nu_\mu$, and $D_s^+ \rightarrow \tau^+ \nu_\tau$, while black-dot-dashed and green-long-dashed lines correspond to the total background and the case when both tag and signal sides are misreconstructed, respectively.

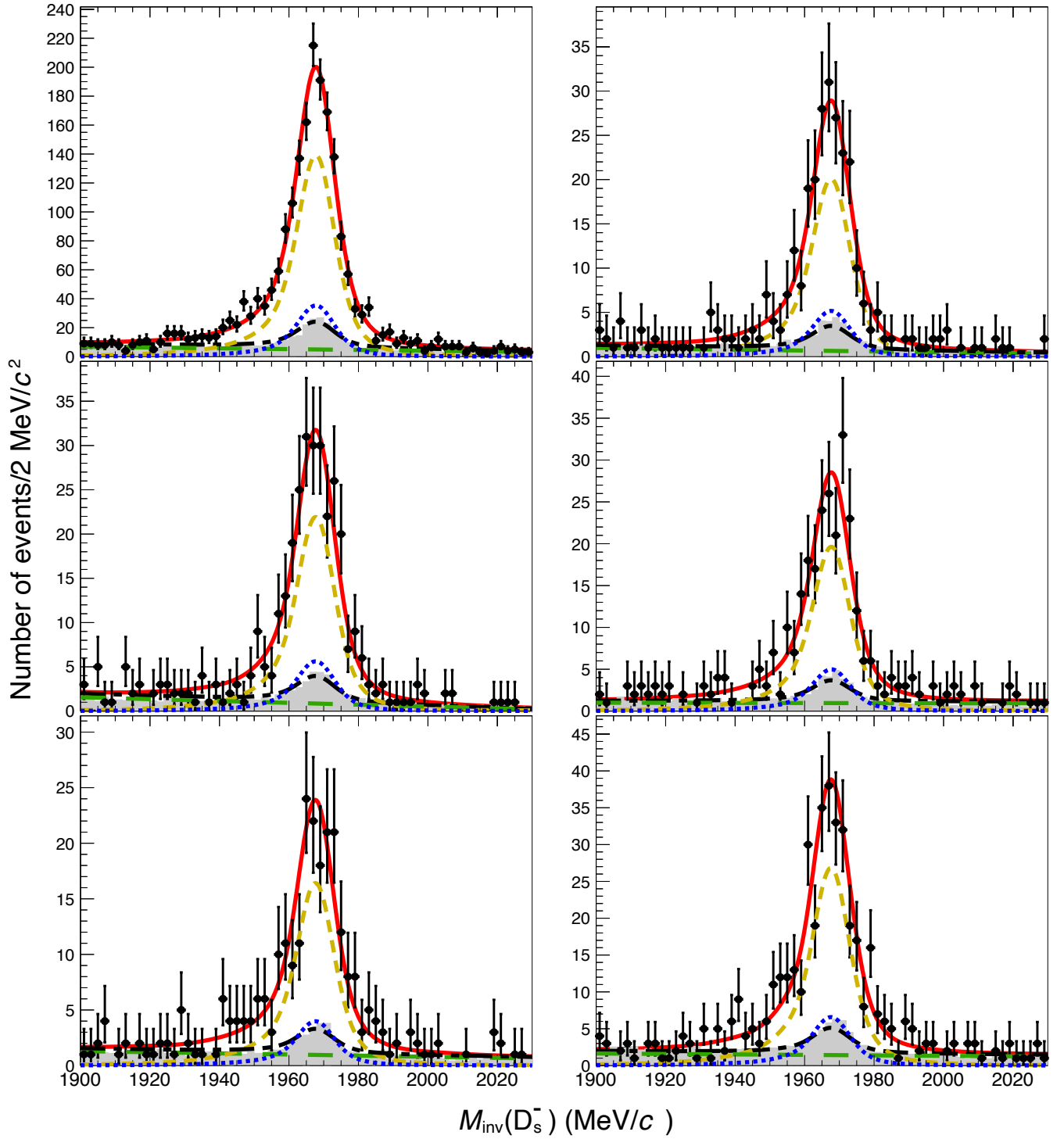


FIG. 2. Projections onto the $M_{\text{inv}}(D_s^-)$ axis of the two-dimensional fit to 4180 data (top left), 4190 data (top right), 4200 data (middle left), 4210 data (middle right), 4220 data (bottom left), and 4230 data (bottom right) for the μ -like sample. The black points are data, the shaded histograms correspond to the $40\times$ background MC sample scaled to the integrated luminosity of data, and the lines represent the fitted signal and background shapes. The red-solid, orange-dashed, and blue-dotted lines represent the total, $D_s^+ \rightarrow \mu^+ \nu_\mu$, and $D_s^+ \rightarrow \tau^+ \nu_\tau$, while black-dot-dashed and green-long-dashed lines correspond to the total background and the case when both tag and signal sides are misreconstructed, respectively.

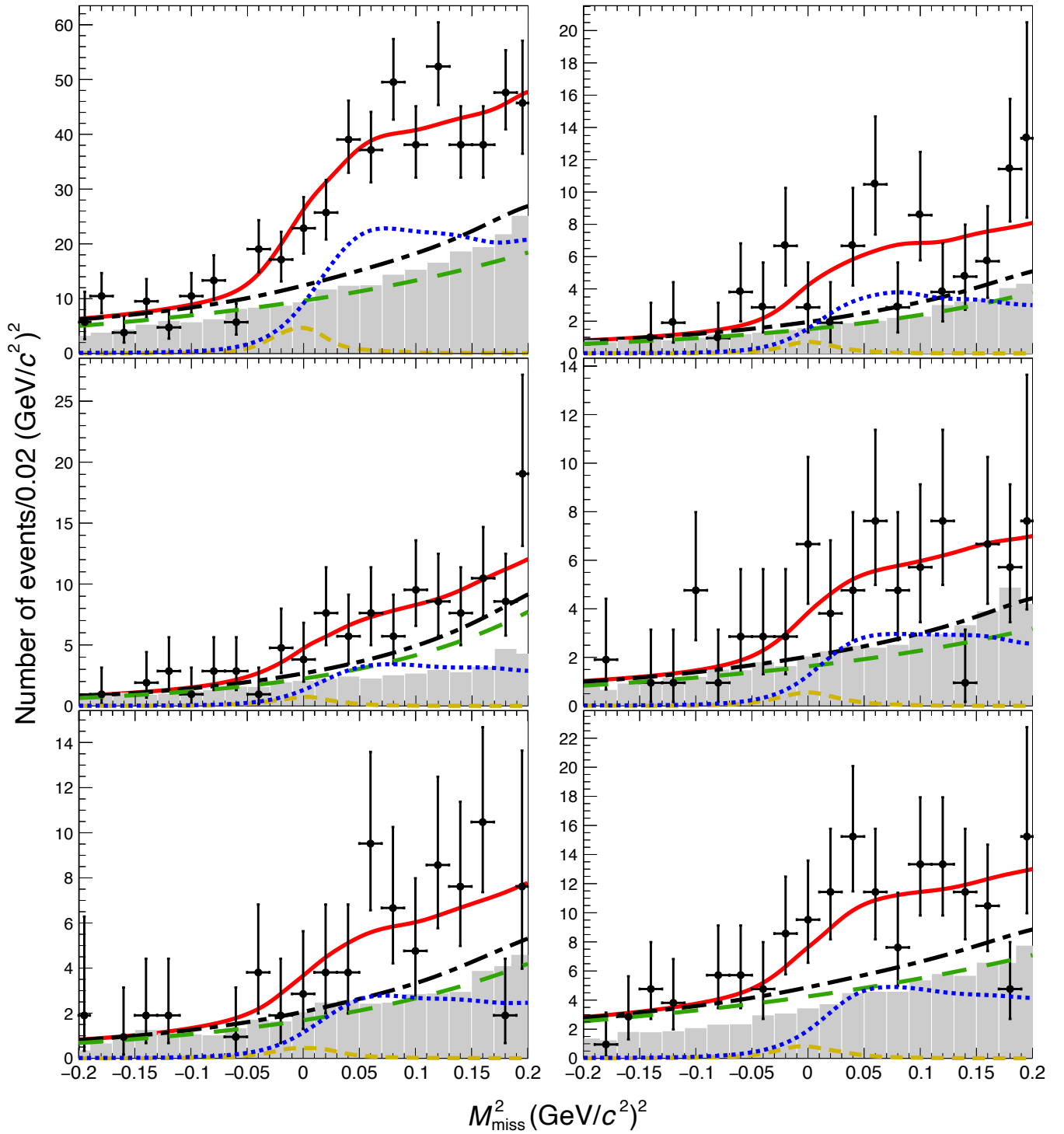


FIG. 3. Projections onto the M_{miss}^2 axis of the two-dimensional fit to 4180 data (top left), 4190 data (top right), 4200 data (middle left), 4210 data (middle right), 4220 data (bottom left), and 4230 data (bottom right) for the π -like sample. The black points are data, the shaded histograms correspond to the $40\times$ background MC sample scaled to the integrated luminosity of data, and the lines represent the fitted signal and background shapes. The red-solid, orange-dashed, and blue-dotted lines represent the total, $D_s^+ \rightarrow \mu^+ \nu_\mu$, and $D_s^+ \rightarrow \tau^+ \nu_\tau$, while black-dot-dashed and green-long-dashed lines correspond to the total background and the case when both tag and signal sides are misreconstructed, respectively.

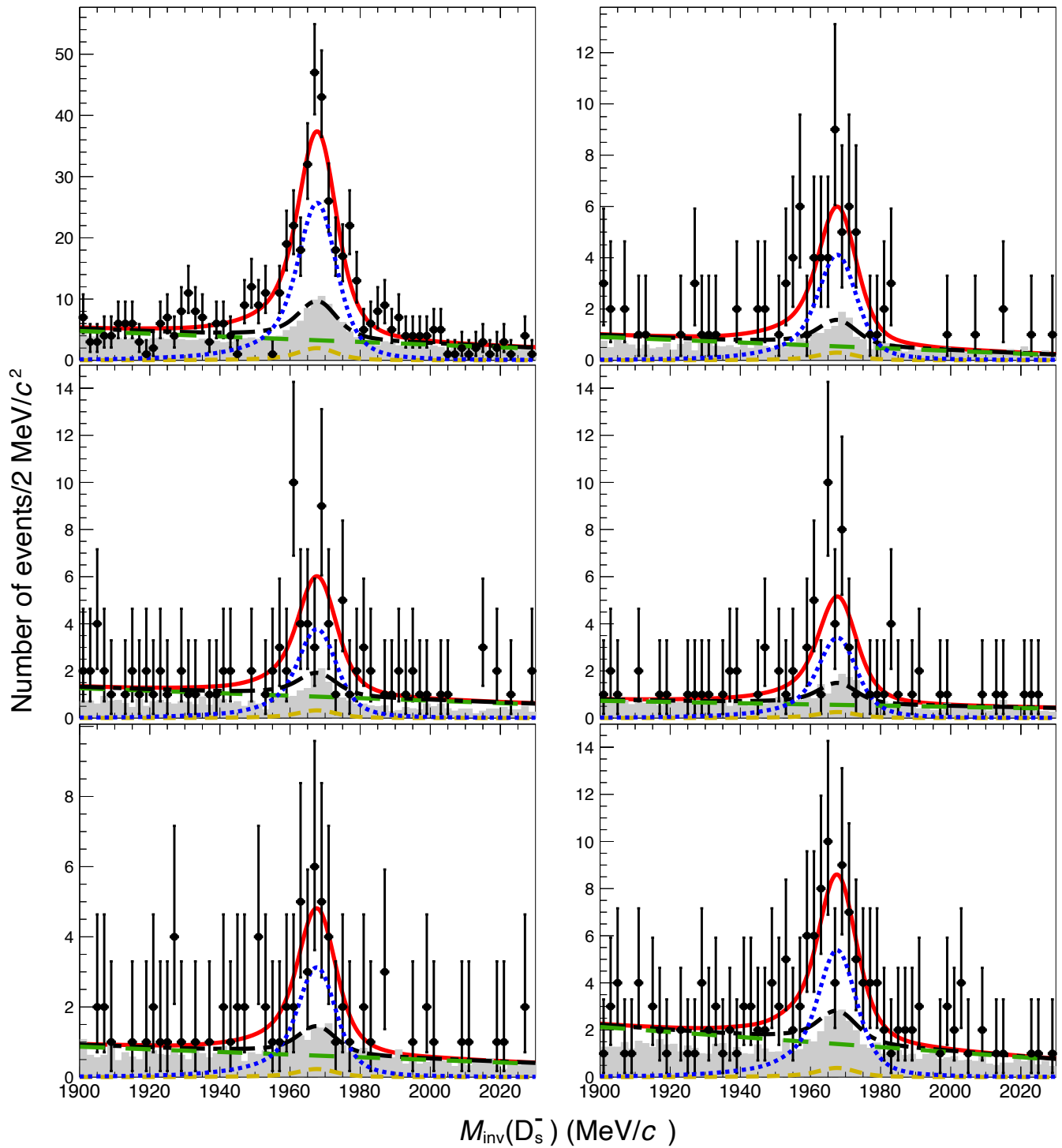


FIG. 4. Projections onto the $M_{\text{inv}}(D_s^-)$ axis of the two-dimensional fit to 4180 data (top left), 4190 data (top right), 4200 data (middle left), 4210 data (middle right), 4220 data (bottom left), and 4230 data (bottom right) for the π -like sample. The black points are data, the shaded histograms correspond to the $40\times$ background MC sample scaled to the integrated luminosity of data, and the lines represent the fitted signal and background shapes. The red-solid, orange-dashed, and blue-dotted lines represent the total, $D_s^+ \rightarrow \mu^+ \nu_\mu$, and $D_s^+ \rightarrow \tau^+ \nu_\tau$, while black-dot-dashed and green-long-dashed lines correspond to the total background and the case when both tag and signal sides are misreconstructed, respectively.