Supplemental Material for "Measurements of Weak Decay Asymmetries of $\Lambda_c^+ \to pK_S^0$, $\Lambda\pi^+$, $\Sigma^+\pi^0$, and $\Sigma^0\pi^+$"

(Dated: July 18, 2019)

I. DECAY ASYMMETRY PARAMETER

For the process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to BP$ and $\bar{\Lambda}_c^-$ decaying to anything, where B and P denote a $J^P = \frac{1}{2}^+$ baryon and a pseudoscalar meson, respectively, the amplitude can be constructed under the helicity basis. For the weak non-leptonic decay $\Lambda_c^+ \to BP$, the Lee-Yang variables[1] α_{BP}, β_{BP}, and γ_{BP} are defined with respect to the s-wave and p-wave amplitudes, such as

$$\alpha_{BP} = \frac{2\mathrm{Re}(s^*p)}{|s|^2+|p|^2},\ \beta_{BP} = \frac{2\mathrm{Im}(s^*p)}{|s|^2+|p|^2},\ \gamma_{BP} = \frac{|s|^2-|p|^2}{|s|^2+|p|^2}, \tag{1}$$

and with equality $\alpha_{BP}^2 + \beta_{BP}^2 + \gamma_{BP}^2 = 1$.

We work with helicity amplitudes. For $\Lambda_c^+ \to B(\frac{1}{2}^+)P(0^-)$ decay, we have two helicity amplitudes, $H_{1/2}$ and $H_{-1/2}$. Using relations $s = \frac{1}{\sqrt{2}}(H_{1/2}+H_{-1/2})$, $p = \frac{1}{\sqrt{2}}(H_{1/2}-H_{-1/2})$, we have the asymmetry parameters defined with helicity amplitudes as

$$\begin{aligned}\alpha_{BP} &= |H_{1/2}|^2 - |H_{-1/2}|^2,\ \beta_{BP} = \sqrt{1-\alpha_{BP}^2}\sin\Delta_1^{BP},\\ \gamma_{BP} &= \sqrt{1-\alpha_{BP}^2}\cos\Delta_1^{BP},\end{aligned} \tag{2}$$

here we have taken the normalization $|H_{1/2}|^2 + |H_{-1/2}|^2 = 1$, and Δ_1^{BP} is the phase angle difference between two helicity amplitudes $H_{1/2}$ and $H_{-1/2}$.

If Λ_c^+ and $\bar{\Lambda}_c^-$ decays conserve the CP transformation, we have relations for the $\bar{\Lambda}_c^-$ asymmetry parameters as

$$\bar{\alpha}_{BP} = -\alpha_{BP},\ \bar{\beta}_{BP} = -\beta_{BP},\ \bar{\gamma}_{BP} = \gamma_{BP}. \tag{3}$$

In the context, for the helicity frame of Λ_c^+ production process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, θ_0 is defined as the polar angle of the Λ_c^+ with respect to the e^+ beam axis in the e^+e^- center-of mass (CM) system, as illustrated in Fig. 1.

II. JOINT ANGULAR DISTRIBUTION FOR THE DECAY $\Lambda_c^+ \to pK_S^0$

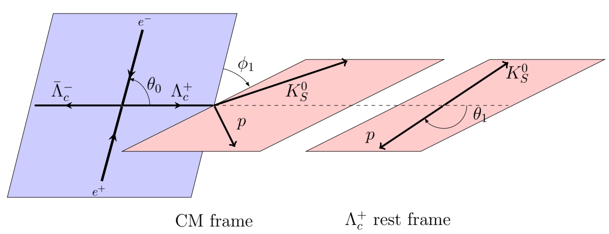

FIG. 1. Definition of the helicity frames for $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to pK_S^0$

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Figure 1 illustrates the definitions of the helicity angles for the process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to pK_S^0$. The Λ_c^+ polar angle θ_0 is defined as the angle between the momenta of the e^+ and Λ_c^+. In the helicity system describing the $\Lambda_c^+ \to pK_S^0$ decay, the angle ϕ_1 is the angle between the $e^+\Lambda_c^+$ plane and pK_S^0 plane, and θ_1 is the polar angle of the p momentum in the rest frame of the Λ_c^+ with respect to the Λ_c^+ momentum in the CM frame. Helicity amplitudes for Λ_c^+ production and decay are given in Tab. I, where λ_1, λ_2, and λ_3 denote the helicities of Λ_c^+, $\bar{\Lambda}_c^-$ and p, respectively. A_{λ_1,λ_2} and B_{λ_3} are the helicity amplitudes.

TABLE I. Definition of decays, helicity angles and amplitudes of $\Lambda_c^+ \to pK_S^0$, where λ_i indicates the helicity for the corresponding hadron.

Level	Decay	Angle	Amplitude
0	$e^+e^- \to \gamma^* \to \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$	θ_0	A_{λ_1,λ_2}
1	$\Lambda_c^+ \to p(\lambda_3)K_S^0$	(θ_1,ϕ_1)	B_{λ_3}

The joint angular distribution is defined as

$$\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\phi_1} \propto \sum_{M,\lambda_1,\lambda_2,\lambda_3} D^{1*}_{M,\lambda_1-\lambda_2}(0,\theta_0,0) D^{1}_{M,\lambda_1'-\lambda_2}(0,\theta_0,0) A^{*}_{\lambda_1,\lambda_2} A_{\lambda_1',\lambda_2}$$
$$\times D^{1/2*}_{\lambda_1,\lambda_3}(\phi_1,\theta_1,0) D^{1/2}_{\lambda_1',\lambda_3}(\phi_1,\theta_1,0) |B_{\lambda_3}|^2, \qquad (4)$$

where the virtual photon spins $M = \pm 1.$ and the Wigner-D function[2] is $D^J_{m,n}(\phi,\theta,\gamma) = e^{-im\phi} d^J_{m,n}(\theta) e^{-in\gamma}$. Helicity amplitude A_{λ_1,λ_2} is related to the angular distribution parameters $\alpha_0 = \frac{|A_{\frac{1}{2},-\frac{1}{2}}|^2 - 2|A_{\frac{1}{2},\frac{1}{2}}|^2}{|A_{\frac{1}{2},-\frac{1}{2}}|^2 + 2|A_{\frac{1}{2},\frac{1}{2}}|^2}$, and helicity amplitude B_{λ_3} is related to the decay asymmetry parameter $\alpha^+_{pK^0_S} = |B_{\frac{1}{2}}|^2 - |B_{-\frac{1}{2}}|^2$ with normalization $|B_{\frac{1}{2}}|^2 + |B_{-\frac{1}{2}}|^2 = 1$. Then the joint angular distribution is simplified as

$$\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\phi_1} \propto 1 + \alpha_0 \cos^2\theta_0 + \mathcal{P}_T \alpha^+_{pK^0_S} \sin\theta_1 \sin\phi_1,$$
$$\mathcal{P}_T = \sqrt{1-\alpha_0^2} \cos\theta_0 \sin\theta_0 \sin\Delta_0, \qquad (5)$$

where $\Delta_0 = \delta_{\frac{1}{2},\frac{1}{2}} - \delta_{\frac{1}{2},-\frac{1}{2}}$ is the phase angle difference between the helicity amplitudes $A_{\frac{1}{2},\frac{1}{2}}$ and $A_{\frac{1}{2},-\frac{1}{2}}$, and $\mathcal{P}_T$ corresponds to a transverse polarization observable of the produced Λ_c^+. For the charge conjugation mode $\bar{\Lambda}_c^- \to \bar{p}K^0_S$, the formula of angular distribution is same, but with the parameter relations of $\bar{\Delta}_0 = \Delta_0$ and $\bar{\alpha}^-_{\bar{p}K^0_S} = -\alpha^+_{pK^0_S}$, when neglecting CP violation.

III. JOINT ANGULAR DISTRIBUTION FOR THE DECAYS $\Lambda_c^+ \to \Lambda\pi^+$ AND $\Sigma^+\pi^0$

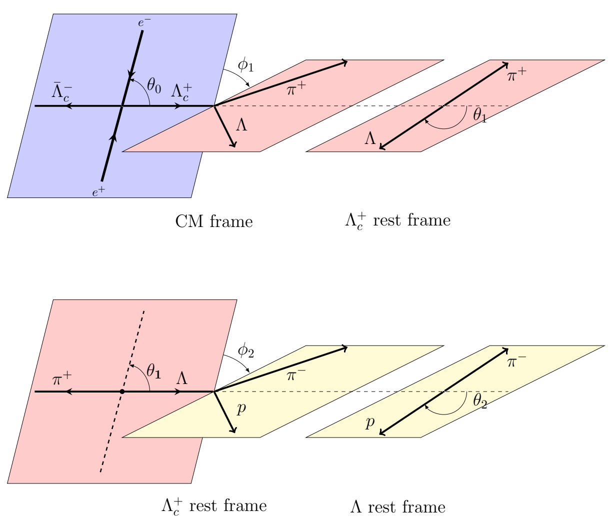

FIG. 2. Definition of the helicity frame for $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda \to p\pi^-$
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Figure 2 illustrates the definitions of the helicity angles for a two-step cascade decay $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda \to p\pi^-$. In the helicity system describing the $\Lambda_c^+ \to \Lambda\pi^+$ decay, the angle ϕ_1 is the angle between the $e^+\Lambda_c^+$ plane and $\Lambda\pi^+$ plane, and θ_1 is the polar angle of the Λ momentum in the rest frame of the Λ_c^+ with respect to the Λ_c^+ momentum in the CM frame. In the the helicity system describing the $\Lambda \to p\pi^-$ decay, the angle ϕ_2 is the angle between the $\Lambda\pi^+$ plane and $p\pi^-$ plane, and θ_2 is the polar angle of the proton momentum with respect to the opposite direction of π^+ momentum in the rest frame of Λ.

As listed in Table II, B_{λ_3} and C_{λ_4} are the helicity amplitudes of the $\Lambda_c^+ \to \Lambda\pi^+$ and $\Lambda \to p\pi^-$ decays, respectively.

TABLE II. Definition of decays, helicity angles and amplitudes in $\Lambda_c^+ \to \Lambda\pi^+$, where λ_i indicates the helicity values for the corresponding hadron.

Level	Decay	Angle	Amplitude
0	$e^+e^- \to \gamma^* \to \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$	θ_0	A_{λ_1,λ_2}
1	$\Lambda_c^+ \to \Lambda(\lambda_3)\pi^+$	(θ_1, ϕ_1)	B_{λ_3}
2	$\Lambda \to p(\lambda_4)\pi^-$	(θ_2, ϕ_2)	C_{λ_4}

The joint angular distribution is written as

$$\begin{aligned}
&\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\cos\theta_2 d\phi_1 d\phi_2} \\
&\propto 2 + 2\alpha_0\cos^2\theta_0 \\
&+ \sqrt{1-\alpha_0^2}\alpha_\Lambda \sin\Delta_0 \sin(2\theta_0)\sin\theta_1\cos\theta_2\sin\phi_1 \\
&+ \sqrt{1-\alpha_0^2}\alpha_\Lambda \sin\Delta_0 \sin(2\theta_0)\cos\theta_1\sin\theta_2\sin\phi_1 \\
&\times \sqrt{1-(\alpha^+_{\Lambda\pi^+})^2}\cos(\Delta_1^{\Lambda\pi^+}+\phi_2) \\
&+ \sqrt{1-\alpha_0^2}\alpha_\Lambda \sin\Delta_0 \sin(2\theta_0)\sin\theta_2\cos\phi_1 \\
&\times \sqrt{1-(\alpha^+_{\Lambda\pi^+})^2}\sin(\Delta_1^{\Lambda\pi^+}+\phi_2) \\
&+ \sqrt{1-\alpha_0^2}\sin\Delta_0\sin(2\theta_0)\sin\theta_1\sin\phi_1\alpha^+_{\Lambda\pi^+} \\
&+ 2\alpha_0\alpha_\Lambda\cos^2\theta_0\cos\theta_2\alpha^+_{\Lambda\pi^+} + 2\alpha_\Lambda\cos\theta_2\alpha^+_{\Lambda\pi^+},
\end{aligned} \tag{6}$$

where α_Λ denotes the decay asymmetry parameter in the weak hadronic decay $\Lambda \to p\pi^-$, $\Delta_0 = \delta_{\frac{1}{2},-\frac{1}{2}} - \delta_{\frac{1}{2},\frac{1}{2}}$ is the difference of phase angle for the helicity amplitudes A_{λ_1,λ_2} and $\Delta_1^{\Lambda\pi^+}$ is the difference of the phase angle between the helicity amplitudes $B_{-\frac{1}{2}}$ and $B_{\frac{1}{2}}$. For the case of the charge conjugation mode $\bar{\Lambda}_c^- \to \bar{\Lambda}\pi^-$, the formula is the same, but with the parameter relations of $\bar{\alpha}_{\bar{\Lambda}} = -\alpha_\Lambda$, $\bar{\alpha}^-_{\bar{\Lambda}\pi^-} = -\alpha^+_{\Lambda\pi^+}$, $\bar{\Delta}_0 = \Delta_0$, $\bar{\Delta}_1^{\bar{\Lambda}\pi^-} = -\Delta_1^{\Lambda\pi^+}$ on the assumption of CP conservation.

If the solid angles (θ_2, ϕ_2) for Λ decays are integrated out, one has

$$\begin{aligned}
\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\phi_1} &\propto 1 + \alpha_0\cos^2\theta_0 + \mathcal{P}_T\alpha^+_{\Lambda\pi^+}\sin\theta_1\sin\phi_1, \\
\mathcal{P}_T &= \sqrt{1-\alpha_0^2}\cos\theta_0\sin\theta_0\sin\Delta_0.
\end{aligned} \tag{7}$$

Equation (7) then returns to the Λ_c^+ one-step decay, which take the same form as given by Eq. (5). If the proton helicity angular θ_2 is only measured, one has

$$\frac{dN}{d\cos\theta_2} \propto 1 + \alpha^+_{\Lambda\pi^+}\alpha_\Lambda\cos\theta_2. \tag{8}$$

This equation indicates that even without information of $\mathcal{P}_T$, the decay asymmetry parameter $\alpha^+_{\Lambda\pi^+}$ can be accessed from the distribution of $\cos\theta_2$.

For the two-step decay $\Lambda_c^+ \to \Sigma^+\pi^0$, $\Sigma^+ \to p\pi^0$, helicity angles and amplitudes are defined as given in Tab. III, the formalism is analogous to that of $\Lambda_c^+ \to \Lambda\pi^+$, but replacing symbols of α_Λ with α_Σ, and $\alpha^+_{\Lambda\pi}$ with $\alpha^+_{\Sigma^+\pi^0}$.

IV. JOINT ANGULAR DISTRIBUTION FOR $\Lambda_c^+ \to \Sigma^0\pi^+$

Figure 3 illustrates definition of helicity angles for three-step decay $\Lambda_c^+ \to \Sigma^0\pi^+$, $\Sigma^0 \to \gamma\Lambda$ and $\Lambda \to p\pi^-$. In the helicity system describing the $\Lambda_c^+ \to \Sigma^0\pi^+$ decay, the angle ϕ_1 is the angle between the $e^+\Lambda_c^+$ plane and $\Sigma^0\pi^+$ plane, and θ_1 is the polar angle of the Σ^0 momentum in the rest frame of the Λ_c^+ with respect to the Λ_c^+ momentum in the CM frame. In the helicity system describing the $\Sigma^0 \to \gamma\Lambda$ decay, the angle ϕ_2 is the angle between the $\Sigma^0\pi^+$ plane

TABLE III. Definition of decays, helicity angles and amplitudes in $\Lambda_c^+ \to \Sigma^+\pi^0$, where λ_i indicates the helicity values for the corresponding hadron.

level	Decay	Angle	Amplitude
0	$e^+e^- \to \gamma^* \to \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$	θ_0	A_{λ_1,λ_2}
1	$\Lambda_c^+ \to \Sigma^+(\lambda_3)\pi^0$	(θ_1, ϕ_1)	B_{λ_3}
2	$\Sigma^+ \to p(\lambda_4)\pi^0$	(θ_2, ϕ_2)	C_{λ_4}

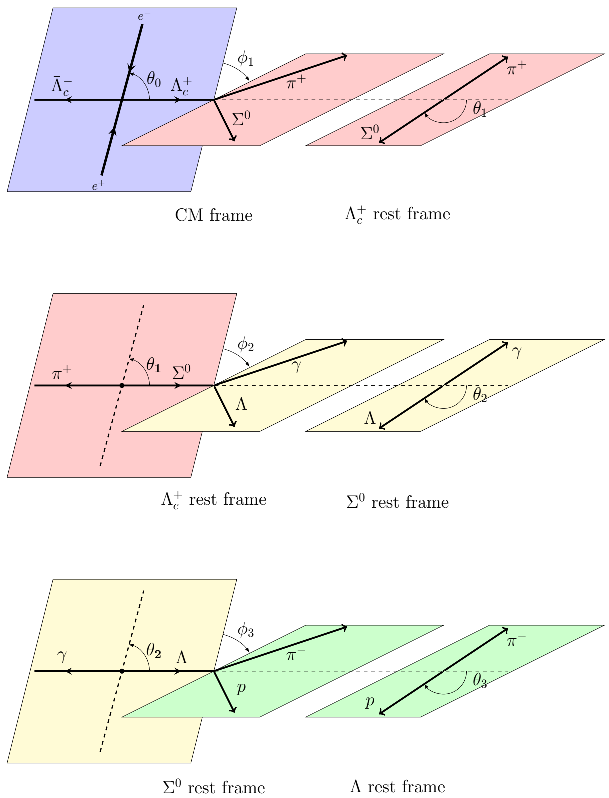

FIG. 3. Definition of the helicity frame for $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to \Sigma^0\pi^+$, $\Sigma^0 \to \gamma\Lambda$, $\Lambda \to p\pi^-$.

TABLE IV. Definition of decays, helicity angles and amplitudes in $\Lambda_c^+ \to \Sigma^0\pi^+$, where λ_i indicates the helicity values for the corresponding hadron.

Level	Decay	Angle	Amplitude
0	$e^+e^- \to \Lambda_c^+(\lambda_1)\bar{\Lambda}_c^-(\lambda_2)$	θ_0	A_{λ_1,λ_2}
1	$\Lambda_c^+ \to \Sigma^0(\lambda_3)\pi^+$	(θ_1, ϕ_1)	B_{λ_3}
2	$\Sigma^0 \to \Lambda(\lambda_4)\gamma(\lambda_5)$	(θ_2, ϕ_2)	C_{λ_4,λ_5}
3	$\Lambda \to p(\lambda_6)\pi^+$	(θ_3, ϕ_3)	F_{λ_4}

and $\gamma\Lambda$ plane, and θ_2 is the polar angle of the Λ momentum with respect to the opposite direction of π^+ momentum in the rest frame of Σ^0. In the helicity system describing the $\Lambda \to p\pi^-$ decay, ϕ_3 is the angle between the $\Lambda\gamma$ and $p\pi^-$ planes, while θ_3 is the polar angle of the proton with respect to the opposite direction of the photon momentum (from $\Sigma^0 \to \Lambda\gamma$) in the rest frame of Λ.

The helicity angles and amplitudes are defined in Table IV. The joint angular distribution is expressed as

$$
\begin{aligned}
&\frac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\cos\theta_2 d\cos\theta_3 d\phi_1 d\phi_2} \\
&\propto 2 + 2\alpha_0\cos^2\theta_0 \\
&-\sqrt{1-\alpha_0^2}\alpha_\Lambda \sin(2\theta_0)\sin\theta_1\cos\theta_2\cos\theta_3\sin\phi_1\sin\Delta_0 \\
&-\sqrt{1-\alpha_0^2}\alpha_\Lambda \sin(2\theta_0)\cos\theta_1\sin\theta_2\cos\theta_3\sin\Delta_0 \\
&\times\sqrt{1-(\alpha^+_{\Sigma^0\pi^+})^2}\sin(\Delta_1^{\Sigma^0\pi^+}+\phi_2) \\
&-\sqrt{1-\alpha_0^2}\alpha_\Lambda \sin(2\theta_0)\cos\phi_1\sin\theta_2\cos\theta_3\sin\Delta_0 \\
&\times\sqrt{1-(\alpha^+_{\Sigma^0\pi^+})^2}\sin(\Delta_1^{\Sigma^0\pi^+}-\phi_2) \\
&+\sqrt{1-\alpha_0^2}\sin(2\theta_0)\sin\theta_1\sin\phi_1\sin\Delta_0\alpha^+_{\Sigma^0\pi^+} \\
&-2\alpha_0\alpha_\Lambda\cos^2\theta_0\cos\theta_2\cos\theta_3\alpha^+_{\Sigma^0\pi^+} \\
&-2\alpha_\Lambda\cos\theta_2\cos\theta_3\alpha^+_{\Sigma^0\pi^+}.
\end{aligned} \tag{9}
$$

where $\Delta_1^{\Sigma^0\pi^+}$ is the phase angle difference for the helicity amplitudes $B_{\frac{1}{2}}$ and $B_{-\frac{1}{2}}$. For the corresponding charge-conjugate $\bar{\Lambda}_c^-$ decays, one has a similar formula, but with replacements $\bar{\alpha}_{\bar{\Lambda}} = -\alpha_\Lambda$, $\bar{\alpha}^-_{\bar{\Sigma}^0\pi^-} = -\alpha^+_{\Sigma^0\pi^+}$, $\bar{\Delta}_0 = \Delta_0$, $\bar{\Delta}_1^{\bar{\Sigma}^0\pi^-} = -\Delta_1^{\Sigma^0\pi^+}$

If the helicity angles of Λ and Σ^0 decays Ω_2=(θ_2, ϕ_2) and Ω_3=(θ_3, ϕ_3) are integrated out, one get the angular distribution

$$
\begin{aligned}
\tfrac{d\Gamma}{d\cos\theta_0 d\cos\theta_1 d\phi_1} &\propto 1+\alpha_0\cos^2\theta_0 + \mathcal{P}_T\alpha^+_{\Sigma^0\pi^+}\sin\theta_1\sin\phi_1, \\
\mathcal{P}_T &= \sqrt{1-\alpha_0^2}\cos\theta_0\sin\theta_0\sin\Delta_0.
\end{aligned} \tag{10}
$$

If the θ_2 and θ_3 angles are only measured, one has

$$
\frac{dN}{d\cos\theta_2 d\cos\theta_3} \propto 1 - \alpha^+_{\Sigma^0\pi^+}\alpha_\Lambda\cos\theta_2\cos\theta_3. \tag{11}
$$

This formula provides a way to measure the decay asymmetry parameter $\alpha^+_{\Sigma^0\pi^+}$ with no information of $\mathcal{P}_T$.

[1] T. D. Lee and C. N. Yang, Phys. Rev., **108**, 1645 (1957).
[2] Group Theory, Academic Press, New York, 1959.