

# Measurement of the $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ cross sections from $\sqrt{s} = 2.000$ to $3.080$ GeV

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ABSTRACT: Based on  $e^+e^-$  collision data collected at center-of-mass energies from 2.000 to 3.080 GeV by the BESIII detector at the BEPCII collider, a partial wave analysis is performed for the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ . The results allow the Born cross sections of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ , as well as its subprocesses  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$  to be measured. The Born cross sections for  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  are consistent with previous measurements by BaBar and SND, but with substantially improved precision. The Born cross section lineshape of the process  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  is consistent with a vector meson state around 2.2 GeV with a statistical significance of  $3.2\sigma$ . A Breit-Wigner fit determines its mass as  $M_Y = (2164.1 \pm 9.6 \pm 3.1)$  MeV/ $c^2$  and its width as  $\Gamma_Y = (32.4 \pm 21.1 \pm 1.5)$  MeV, where the first uncertainties are statistical and the second ones are systematic, respectively.

KEYWORDS:  $e^+e^-$  Experiments, Particle and Resonance Production, Vector Meson Production

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## 1 Introduction

The vector meson state  $Y(2175)$ , denoted as  $\phi(2170)$  by the Particle Data Group [1], is one of the more interesting particles in the field of light-flavored hadron spectroscopy. It was first observed by the BaBar collaboration [2] and subsequently investigated by the Belle, BES and BESIII collaborations [3–13]. Several interpretations have been proposed for the  $\phi(2170)$  state, such as a conventional  $3^3S_1$  or  $2^3D_1$   $s\bar{s}$  state [14–17], a  $s\bar{s}g$  hybrid [18, 19], a tetraquark state [20–23], a  $\Lambda\bar{\Lambda}(^3S_1)$  bound state [24–26], and a  $\phi K\bar{K}$  resonance state [27].

Studies of the  $\phi(2170)$  have been carried out using various final states such as  $\phi\eta$  [7, 28],  $\phi\eta'$  [8],  $\phi f_0(980)$  [2–6, 9, 29],  $K^+K^-$  [10, 30, 31],  $K_S^0K_L^0$  [11, 32],  $K^*(892)^+K^-$  [12],  $K_2^*(1430)^+K^-$  [12],  $K^*(892)^+K^*(892)^-$  [13] and other charged excited  $K\bar{K}$  states [13]. None of these final states is dominant, and the product of the  $e^+e^-$  partial width and the branching fraction (BF) of each final state is consistently below 100 eV. The partial decay width  $\Gamma(\phi\eta)$  [7, 28] is less than  $\Gamma(\phi\eta')$  [8], which disfavors the hybrid interpretation [18, 19]. This result can be explained by the hadronic transition of a strangeonium-like meson along with  $\eta - \eta'$  mixing [33]. The partial width of  $K^*(892)^+K^*(892)^-$  is significantly greater than that of  $K_1(1400)^+K^-$ , as predicted for the  $2^3D_1$  or  $3^3S_1$  state [14–17]. However, the BESIII collaboration has observed a clear structure in the cross section line shape of  $K_1(1400)^+K^-$  around 2.2 GeV [13], but no enhancement in the cross section line shape for  $K^*(892)^+K^*(892)^-$  [13], which disfavors the  $2^3D_1$  or  $3^3S_1$  prediction. The BESIII collaboration has also measured a larger partial width of  $K_2^*(1430)^+K^-$  compared to  $K^*(892)^+K^-$

at center-of-mass (c.m.) energies ( $\sqrt{s}$ ) from 2.000 to 3.080 GeV [12], which contradicts the prediction that the  $\phi(2170)$  is the  $2^3D_1$  strangeonium state [16]. More precise measurements of the decay properties of the  $\phi(2170)$  are desirable to better understand the nature of the  $\phi(2170)$ .

The  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  [34] and  $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp$  [28] processes have been investigated by the BaBar collaboration using the initial state radiation (ISR) technique. A Dalitz amplitude analysis was performed for  $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp$ , leading to the determination of the isoscalar and isovector cross sections for  $K^*(892)\bar{K}$ . A distinct asymmetry between neutral and charged channels is observed in the Dalitz plot for  $K_S^0 \pi^\mp$  and  $K^\pm \pi^\mp$  within  $\sqrt{s} = 2 - 3$  GeV. It may be related to a similar effect observed in the radiative decay rates of the neutral and charged  $K_2^*(1430)$  [28]. The SND collaboration has studied  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  at  $\sqrt{s} = 1.3 - 2.0$  GeV, and the cross sections have been measured at a statistical uncertainty level of 10%-30% [35].

In this paper, we present a partial wave analysis (PWA) of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  based on 19 data samples collected by the BESIII experiment, ranging from  $\sqrt{s} = 2.000$  to 3.080 GeV and corresponding to an integrated luminosity of  $647 \text{ pb}^{-1}$  [36, 37]. The Born cross section of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  and its sub-processes  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$  are measured. Throughout the paper charge conjugated processes are also included by default.

## 2 BESIII detector and Monte Carlo simulation

The BESIII detector [38] records symmetric  $e^+e^-$  collisions provided by the BEPCII storage ring [39], which operates with a peak luminosity of  $1 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$  in the range of  $\sqrt{s}$  from 2.0 to 4.95 GeV. BESIII has collected large data samples in this energy region [40]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T (0.9 T in 2012) magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the  $dE/dx$  resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end cap region is 110 ps.

Simulated samples produced with GEANT4 based [41] Monte Carlo (MC) software, which includes the geometric description [42] of the BESIII detector and the detector response, are used to optimize the event selection criteria, estimate backgrounds, and determine the detection efficiency. The signal MC samples for the processes  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ ,  $K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$  are generated by CONEXC [43] using an amplitude model with parameters fixed to the PWA results. For background studies, inclusive hadronic events are generated with a hybrid generator that includes CONEXC, LUARLW [44] and PHOKHARA [45].

### 3 Event selection and background analysis

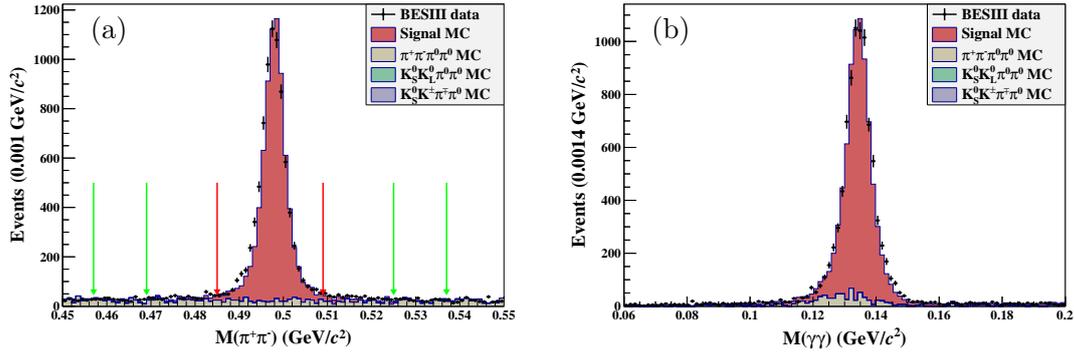
The signal process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  is reconstructed with  $K_S^0 \rightarrow \pi^+\pi^-$ ,  $\pi^0 \rightarrow \gamma\gamma$ , and  $K_L^0$  treated as a missing particle. Signal candidates are required to have two charged pions with zero net charge and at least two photons.

Charged tracks detected in the MDC are required to be within a polar angle ( $\theta$ ) range of  $|\cos\theta| < 0.93$ . Here,  $\theta$  is defined with respect to the  $z$ -axis, which is the symmetry axis of the MDC. Each  $K_S^0$  candidate is reconstructed from two oppositely charged tracks satisfying that the distance of closest approach to the interaction point (IP) must be less than 20 cm along the  $z$ -axis. The two charged tracks are assigned as  $\pi^+\pi^-$  without imposing further particle identification criteria. They are constrained to originate from a common vertex and are required to have an invariant mass within  $|M(\pi^+\pi^-) - m_{K_S^0}| < 12 \text{ MeV}/c^2$ , where  $M(\pi^+\pi^-)$  is the invariant mass of  $\pi^+\pi^-$  pair with kinematics updated by the vertex fit and  $m_{K_S^0}$  is the  $K_S^0$  nominal mass [1]. The decay length of the  $K_S^0$  candidate is required to be greater than twice the vertex resolution away from the IP.

Photon candidates are identified using showers in the EMC. The deposited energy of each shower must be more than 25 MeV in the barrel region ( $|\cos\theta| < 0.80$ ) and more than 50 MeV in the end cap region ( $0.86 < |\cos\theta| < 0.92$ ). To exclude showers that originate from charged tracks, the angle subtended by the EMC shower and the position of the closest charged track at the EMC must be greater than  $10^\circ$  as measured from the IP. To suppress electronic noise and showers unrelated to the event, the difference between the event start time and the EMC time of the photon candidate is required to be within  $[0, 700]$  ns.

To suppress background and improve the kinematic resolution, a one-constraint (1C) kinematic fit imposing energy-momentum conservation is carried out under the  $K_S^0 K_L^0 \gamma\gamma$  hypothesis with  $K_L^0$  treated as a missing particle. If there are more than two photons in an event, the combination with the minimum  $\chi_{1C}^2$  is retained for further analysis, and candidate events are required to satisfy  $\chi_{1C}^2 < 30$ . To suppress the contamination from the process  $e^+e^- \rightarrow \gamma_{\text{ISR}} K_S^0 K_L^0$ , an additional 1C kinematic fit is performed under the hypothesis of  $\gamma K_S^0 K_L^0$ , and only events which satisfy  $\chi_{1C}^2 < \chi_{1C}^2(\gamma K_S^0 K_L^0)$  are retained. To remove  $K_L^0$  showers in the EMC that could be mistaken as photons, the angles between the candidate EMC shower and the  $K_L^0$  momentum after the kinematic fit are required to be greater than  $20^\circ$ . Each signal candidate is required to have the invariant mass of the two photons within the  $\pi^0$  mass region ( $|M(\gamma\gamma) - m_{\pi^0}| < 0.015 \text{ GeV}/c^2$ ).

Potential background sources are studied by analyzing inclusive  $e^+e^- \rightarrow \text{hadrons}$  and exclusive  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ ,  $K_S^0 K^\pm \pi^\mp \pi^0$  and  $K_S^0 K_L^0 \pi^0 \pi^0$  MC samples after applying the same event selection criteria. The dominant background process is  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ . Exclusive  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$  events are generated by PHOKHARA [45] based on the results of the BaBar collaboration [46]. The  $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp \pi^0$  and  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0 \pi^0$  events are generated by CONEXC based on the dressed cross sections for  $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp \pi^0$  and  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0 \pi^0$  from the BaBar experiment [34, 47] with a phase space model and re-weighted to improve the agreement with BESIII data using a multidimensional gradient-boosting algorithm (HEPML) [48], respectively. The exclusive  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ ,  $K_S^0 K^\pm \pi^\mp \pi^0$  and  $K_S^0 K_L^0 \pi^0 \pi^0$  samples, which have been normalized to

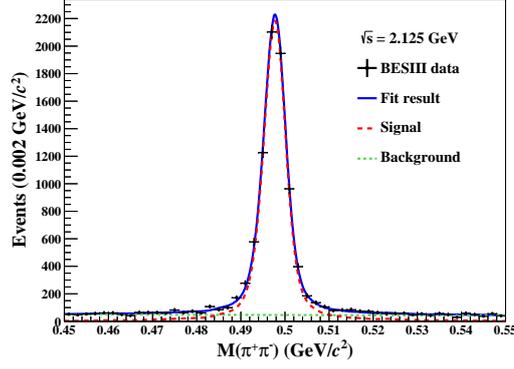


**Figure 1.** Distributions of (a)  $M(\pi^+\pi^-)$  and (b)  $M(\gamma\gamma)$  at  $\sqrt{s} = 2.125$  GeV, where the (black) dots with error bars are data, and the shaded histogram are stacked MC samples of signal process,  $\pi^+\pi^-\pi^0\pi^0$ ,  $K_S^0 K_L^0 \pi^0 \pi^0$  and  $K_S^0 K^\pm \pi^\mp \pi^0$ . The region between red arrows is the signal region, and the regions between the green arrows are the sideband regions.

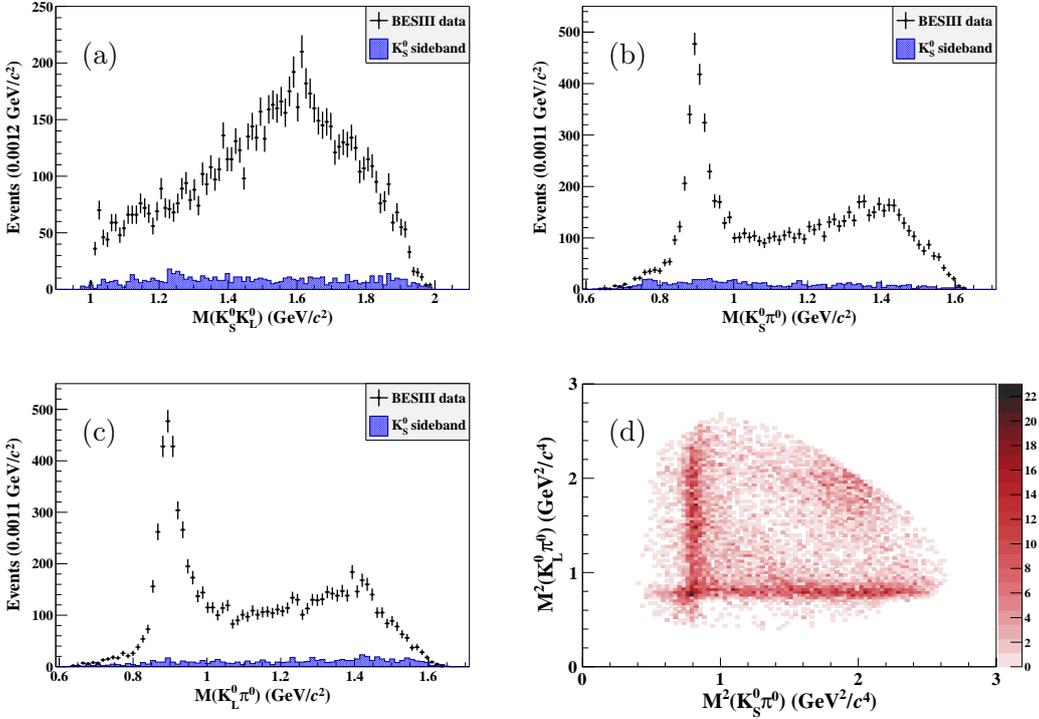
the experimental integrated luminosity, are used to evaluate the numbers of background events. The contribution of  $K_S^0$  peaking background events from  $e^+e^- \rightarrow K_S^0 K^\pm \pi^\mp \pi^0$  and  $K_S^0 K_L^0 \pi^0 \pi^0$  is at a level of 0.10% – 0.40% for different energy points, which is negligible in following fit. Figure 1 shows distributions of the invariant masses of  $\pi^+\pi^-$ ,  $M(\pi^+\pi^-)$  and  $\gamma\gamma$ ,  $M(\gamma\gamma)$  without the  $K_S^0$  and  $\pi^0$  mass window requirements, respectively. Non- $K_S^0$  events are characterized by a flat shape in  $M(\pi^+\pi^-)$  and are estimated with the events in the  $K_S^0$  sideband, which is defined by  $0.022 < |M(\pi^+\pi^-) - m_{K_S^0}| < 0.035$  GeV/ $c^2$ .

The signal yields of the  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  process are obtained by performing an unbinned maximum likelihood fit to the  $M(\pi^+\pi^-)$  spectrum. The signal component is described by the signal MC-simulated shape convolved with a Gaussian function which describes the difference between data and MC simulation. The mean value and width of the Gaussian function are separately float parameters at different energy points. The background function is parameterized by a first-order polynomial function. The corresponding fit result for data taken at  $\sqrt{s} = 2.125$  GeV is shown in figure 2. The same event selection criteria and fit procedure are applied for all data samples at the nineteen c.m. energies.

In order to improve the resolution of kinematic variables, the remaining  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  events are subjected to a three-constraint (3C) kinematic fit, which, in addition to imposing energy and momentum conservation, further constrains the  $\pi^0$  and  $K_S^0$  masses to their PDG values [1]. After all above criteria, the invariant mass spectra of  $K_S^0 K_L^0$ ,  $K_S^0 \pi^0$ ,  $K_L^0 \pi^0$  and the invariant masses squared of  $K_S^0 \pi^0$  versus  $K_L^0 \pi^0$  are shown in figure 3, where the  $K^*(892)^0$  structure is clear. For the invariant mass spectra of  $K_S^0 K_L^0$ ,  $K_S^0 \pi^0$  and  $K_L^0 \pi^0$ , the contributions of background events which are obtained by the  $K_S^0$  sideband are smooth and confirm that there is no peaking structure. Those non- $K_S^0$  events are used to estimate the background contributions and those  $K_S^0$  peaking backgrounds are negligible in the following amplitude analysis.



**Figure 2.** Fit to the  $M(\pi^+\pi^-)$  distribution at  $\sqrt{s} = 2.125$  GeV, where the black dots with error bars are data, the blue solid curve is the total fit result, the green dashed curve indicates the fitted background shape, and the red dashed curve is the fitted signal shape.



**Figure 3.** Distributions of (a)  $M(K_S^0 K_L^0)$ , (b)  $M(K_S^0 \pi^0)$  and (c)  $M(K_L^0 \pi^0)$ , where the (black) dots with error bars are data, and the shaded histograms are non- $K_S^0$  events estimated by the  $K_S^0$  sideband. (b) Distribution of  $M^2(K_S^0 \pi^0)$  versus  $M^2(K_L^0 \pi^0)$ . All plots are based on data at  $\sqrt{s} = 2.125$  GeV.

#### 4 Amplitude analysis

Based on the GPUPWA framework [49], a PWA is performed on the surviving candidate events to identify the intermediate processes present in  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ . The

quasi-two-body decay amplitudes in the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  with two sequential decays  $e^+e^- \rightarrow R_1 \pi^0 \rightarrow K_S^0 K_L^0 \pi^0$  and  $e^+e^- \rightarrow K_S^0(K_L^0)R_2 \rightarrow K_S^0 K_L^0 \pi^0$  are considered and constructed using the covariant tensor amplitude formalism [50], where  $R_1$  and  $R_2$  are the intermediate states that can decay to  $K_S^0 K_L^0$  and  $K_S^0(K_L^0)\pi^0$ , respectively.

According to Ref. [50], the general form for the decay amplitude of a mother particle (vector meson  $Y$ ) is

$$A(m) = Y_\mu(m)A^\mu = Y_\mu(m) \sum_i \Lambda_i U_i^\mu, \quad (4.1)$$

where  $Y_\mu(m)$  is the polarization vector of  $Y$ ,  $m$  is the spin projection of  $Y$ , and  $U_i^\mu$  is the  $i$ -th partial-wave amplitude with coupling strength determined by a complex parameter  $\Lambda_i$ . The partial wave amplitudes  $U_i$  used in the analysis are constructed with the four momenta of daughter particles according to the expressions given in Ref. [50]. The differential cross-section can be written as

$$\frac{d\sigma}{d\Phi_n} = \frac{1}{2} \sum_m A^\mu A^{*\mu} = \frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_m U_i^\mu U_j^{*\mu}. \quad (4.2)$$

The propagator of intermediate resonance is parameterized by a relativistic Breit-Wigner (BW) function with an invariant mass dependent width [51]

$$\text{BW}(s) = \frac{1}{m^2 - s - i\sqrt{s}\Gamma(s)}, \quad (4.3)$$

$$\Gamma(s) = \Gamma_0(m^2) \left(\frac{m^2}{s}\right) \left(\frac{p(s)}{p(m^2)}\right)^{2l+1}, \quad (4.4)$$

where  $s$  is the invariant mass squared of the daughter particle,  $m$  and  $\Gamma_0$  are the mass and width of the intermediate resonance, respectively,  $l$  is the orbital angular momentum for a daughter particle, and  $p(s)$  or  $p(m^2)$  is the momentum of a daughter particle in the rest frame of the resonance with mass  $\sqrt{s}$  or  $m$ . To include the resolution effect for the narrow  $\phi$  resonance, the BW function is convolved with a Gaussian function.

The relative magnitudes and phases of the individual intermediate processes are determined by performing an unbinned maximum likelihood fit using MINUIT [52], where the magnitude and phase of the reference amplitude  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  are fixed to 1 and 0, respectively, while those of other amplitudes are free parameters of the fit.

Conservation of  $J^{PC}$  for intermediate states, in the process  $e^+e^- \rightarrow R_1 \pi^0 \rightarrow K_S^0 K_L^0 \pi^0$ , allows both  $\mathcal{P}$  and  $\mathcal{F}$  wave contributions both in  $e^+e^- \rightarrow R_1 \pi^0$  and  $R_1 \rightarrow K_S^0 K_L^0$ . In the case of  $e^+e^- \rightarrow K_S^0(K_L^0)R_2 \rightarrow K_S^0 K_L^0 \pi^0$ , the contributions of  $\mathcal{P}$ ,  $\mathcal{D}$  and  $\mathcal{F}$  waves are all allowed both in the primary and secondary processes. The PWA fit procedure starts by including the  $K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$  as the initial baseline solutions, and then adds one at a time other possible intermediate states which can decay to  $K_S^0(K_L^0)\pi^0$  or  $K_S^0 K_L^0$ . The masses and widths of possible intermediate resonances are fixed to their PDG values [1]. Intermediate states are included in the solution if the statistical significance is greater than  $5\sigma$ , where the statistical significance is evaluated from the changes in likelihood and degrees of freedom with and without the corresponding amplitude included in the PWA fit. The

Process	Significance		
	2.125 GeV	2.396 GeV	2.900 GeV
$\phi\pi^0$	$13.1\sigma$	$8.6\sigma$	$9.7\sigma$
$\phi(1680)\pi^0$	$11.1\sigma$	$12.2\sigma$	$8.3\sigma$
$K^*(892)^0\bar{K}^0$	$>30\sigma$	$>30\sigma$	$>30\sigma$
$K_2^*(1430)^0\bar{K}^0$	$29.2\sigma$	$5.7\sigma$	$5.1\sigma$
$K(1680)^0\bar{K}^0$	$9.8\sigma$	$8.4\sigma$	$7.6\sigma$

**Table 1.** Statistical significances of the intermediate states for data at  $\sqrt{s} = 2.125, 2.396$  and  $2.900$  GeV.

Process	Fraction (%)		
	2.125 GeV	2.396 GeV	2.900 GeV
$\phi\pi^0$	$0.78\pm 0.56$	$0.87\pm 0.61$	$1.82\pm 1.11$
$\phi(1680)\pi^0$	$2.39\pm 1.23$	$5.96\pm 2.10$	$5.22\pm 1.50$
$K^*(892)^0\bar{K}^0$	$79.89\pm 1.12$	$86.01\pm 1.38$	$72.65\pm 2.11$
$K_2^*(1430)^0\bar{K}^0$	$7.42\pm 0.83$	$1.93\pm 0.57$	$1.85\pm 0.82$
$K(1680)^0\bar{K}^0$	$3.00\pm 1.11$	$6.73\pm 1.91$	$5.82\pm 1.96$

**Table 2.** Fit fractions of the intermediate states for data at  $\sqrt{s} = 2.125, 2.396$  and  $2.900$  GeV.

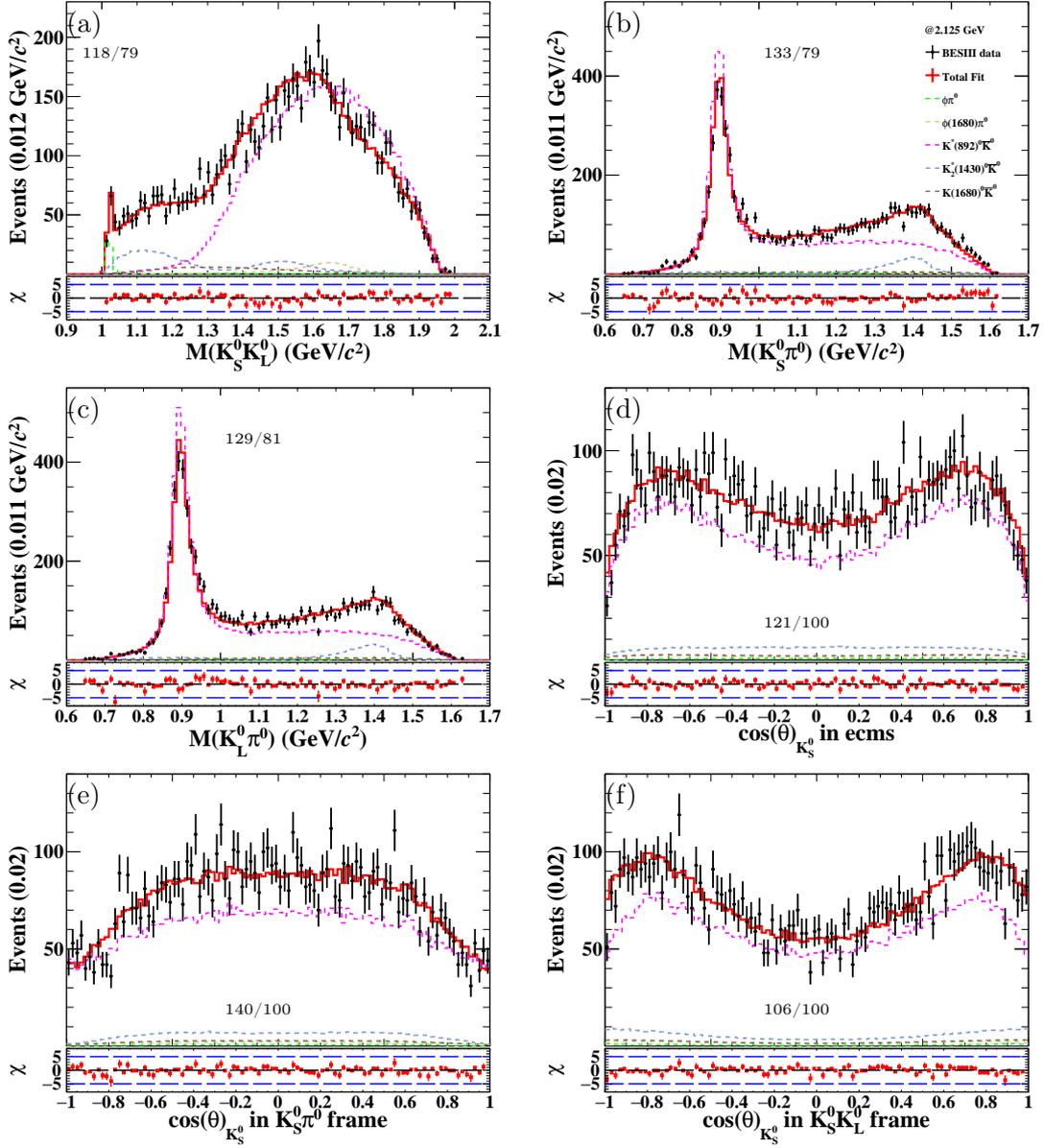
direct decay process without an intermediate resonance is treated as a consecutive quasi-two-body decay of a very broad resonance decaying into  $K_S^0 K_L^0$  or  $K_S^0(K_L^0)\pi^0$  and modeled as a  $1^-$  phase space distribution [12]. The procedure is repeated until a best solution is obtained.

The above strategy is implemented individually on the experimental data sets collected at  $\sqrt{s} = 2.125, 2.396$  and  $2.900$  GeV, which have the largest luminosities and yields among the nineteen data sets. The best solution for data at  $\sqrt{s} = 2.125$  GeV includes the processes  $e^+e^- \rightarrow \phi\pi^0, \phi(1680)\pi^0, K^*(892)^0\bar{K}^0, K_2^*(1430)^0\bar{K}^0$  and  $K(1680)^0\bar{K}^0$ . The statistical significances of the intermediate states and fit fractions for  $\sqrt{s} = 2.125, 2.396$  and  $2.900$  GeV are listed in table 1 and table 2, respectively. For other sixteen data samples with lower luminosities and limited statistics, the same intermediate components are used in the nearby c.m. energies with higher statistics. The intermediate component candidates of  $\sqrt{s} = 2.000, 2.050, 2.100, 2.150, 2.175, 2.200,$  and  $2.232$  GeV are same as  $\sqrt{s} = 2.125$  GeV. The intermediate component candidates of  $\sqrt{s} = 2.309, 2.386, 2.644$  and  $2.646$  GeV are same as  $\sqrt{s} = 2.396$  GeV. The remaining datasets use the same intermediate components as  $\sqrt{s} = 2.900$  GeV. The data for each energy point is fitted individually.

The invariant mass spectra, angular distributions and fit results for  $\sqrt{s} = 2.125$  GeV are shown in figure 4. The  $\chi^2/\text{nbin}$  value is displayed on each figure, where nbin is the number of bins of each figure and  $\chi^2$  is defined as:

$$\chi^2 = \sum_{i=1}^{\text{nbin}} \frac{(n_i - \nu_i)^2}{n_i}, \quad (4.5)$$

where  $n_i$  and  $\nu_i$  are the number of events for the data and the fit projections in the  $i$ -th



**Figure 4.** Superposition of data and the PWA fit projections for invariant mass distributions of (a)  $K_S^0 K_L^0$ , (b)  $K_S^0 \pi^0$  and (c)  $K_L^0 \pi^0$ , and the  $\cos \theta$  distributions of (d)  $K_S^0$  in  $e^+e^-$  c.m. frame, (e)  $K_S^0$  in  $K_S^0 \pi^0$  rest frame and (f)  $K_S^0$  in  $K_S^0 K_L^0$  rest frame at  $\sqrt{s} = 2.125$  GeV. The  $\chi^2/\text{nbin}$  value is displayed on each figure. The pull projection of the residuals is shown beneath each distribution correspondingly. Different styles of the curves denote different components.

bin of each figure, respectively.

## 5 Born cross sections measurement

The Born cross section for  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  is obtained at each c.m. energy using

$$\sigma(\sqrt{s}) = \frac{N_{\text{sig}}}{\mathcal{L} \cdot \epsilon \cdot (1 + \delta) \cdot \frac{1}{|1-\Pi|^2} \cdot \mathcal{B}}, \quad (5.1)$$

where  $N_{\text{sig}}$  is the number of signal events,  $\mathcal{L}$  is the integrated luminosity,  $\epsilon$  is the efficiency obtained by weighting MC simulation according to the PWA results,  $\mathcal{B}$  is the product of BFs in the full decay chain  $\mathcal{B} = \mathcal{B}(K_S^0 \rightarrow \pi^+\pi^-) \cdot \mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = 68.39\%$ , which is taken from the PDG [1],  $\frac{1}{|1-\Pi|^2}$  is the vacuum polarization (VP) factor [53], and  $1 + \delta$  is the ISR correction factor, which is obtained by a QED calculation [54]. Both  $\epsilon$  and  $1 + \delta$  depend on the line shape of cross sections and are determined by an iterative procedure [11, 55]. The Born cross section for an intermediate process,  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  or  $K_2^*(1430)^0 \bar{K}^0$ , at each energy is obtained with the same approach, where  $N_{\text{sig}}$  is replaced with the product of the total number of surviving events and the corresponding fraction relative to the total obtained according to the PWA results, and  $\mathcal{B}$  is replaced with the product of the BFs of the decays  $K_S^0 \rightarrow \pi^+\pi^-$ ,  $\pi^0 \rightarrow \gamma\gamma$  and that of the intermediate state from the PDG [1]. The Born cross sections are listed in tables 3, 4 and 5, separately for the processes  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ ,  $K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$ , respectively.

## 6 Systematic uncertainties

Two categories of systematic uncertainties are considered in the measurement of the Born cross sections. The first category includes the integrated luminosity and those associated with event selection and reconstruction, *i.e.*,  $K_S^0$  reconstruction, requirement on the number of charged tracks ( $N_{\text{charge}}$ ), photon reconstruction, kinematic fit,  $\pi^0$  invariant mass requirement, ISR and VP correction factors (Rad), BF, and  $M(\pi^+\pi^-)$  fit (Fit). These contributions are evaluated as follows:

1. The uncertainty associated with the integrated luminosity is 1% and estimated by using large angle Bhabha events [36].
2. The uncertainty concerning  $K_S^0$  reconstruction is studied with control samples of  $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$  and  $J/\psi \rightarrow \phi K_S^0 K^\pm \pi^\mp$ . The result shows that the difference in efficiency between data and MC simulation is 1% per  $K_S^0$  [56].
3. The uncertainty of the requirement on the number of charged tracks is estimated with a control sample of  $J/\psi \rightarrow K_S^0 K_L^0 \pi^0$ . The difference in efficiency between data and MC simulation with and without this requirement is taken as the uncertainty.
4. The uncertainty concerning photon detection efficiency is studied with a control sample of  $e^+e^- \rightarrow K^+ K^- \pi^+ \pi^- \pi^0$  [57]. The result shows that the difference in detection efficiency between data and MC simulation is 1% per photon.
5. The uncertainty related to the kinematic fit is studied with a control sample of  $J/\psi \rightarrow K_S^0 K_L^0 \pi^0$ . The difference in efficiency between data and MC simulation with and without the kinematic fit is taken as the uncertainty.

$\sqrt{s}$ (GeV)	$N_{\text{sig}}$	$\mathcal{L}$ (pb $^{-1}$ )	$\epsilon$	$1+\delta$	$\frac{1}{ 1-\Pi ^2}$	$\mathcal{B}$	$\sigma$ (pb)
2.0000	880.6 $\pm$ 32.3	10.1 $\pm$ 0.1	15.3%	1.22	1.037	0.6839	662.7 $\pm$ 24.3 $\pm$ 25.7
2.0500	308.4 $\pm$ 19.2	3.34 $\pm$ 0.03	15.8%	1.20	1.038	0.6839	682.1 $\pm$ 42.5 $\pm$ 35.2
2.1000	941.7 $\pm$ 33.8	12.2 $\pm$ 0.1	16.4%	1.18	1.039	0.6839	557.8 $\pm$ 20.0 $\pm$ 21.7
2.1250	8175.0 $\pm$ 98.9	108 $\pm$ 1	16.6%	1.17	1.039	0.6839	545.4 $\pm$ 6.6 $\pm$ 17.0
2.1500	228.1 $\pm$ 16.8	2.84 $\pm$ 0.02	17.1%	1.13	1.040	0.6839	582.2 $\pm$ 42.9 $\pm$ 30.8
2.1750	678.7 $\pm$ 28.4	10.6 $\pm$ 0.1	16.5%	1.23	1.040	0.6839	444.5 $\pm$ 18.6 $\pm$ 20.6
2.2000	772.8 $\pm$ 30.5	13.7 $\pm$ 0.1	16.2%	1.25	1.040	0.6839	391.2 $\pm$ 15.4 $\pm$ 16.4
2.2324	594.1 $\pm$ 26.7	11.9 $\pm$ 0.1	16.0%	1.28	1.041	0.6839	342.9 $\pm$ 15.4 $\pm$ 14.6
2.3094	846.1 $\pm$ 32.8	21.1 $\pm$ 0.1	15.5%	1.31	1.041	0.6839	277.3 $\pm$ 10.8 $\pm$ 13.9
2.3864	741.3 $\pm$ 30.1	22.5 $\pm$ 0.2	15.5%	1.34	1.041	0.6839	224.2 $\pm$ 9.1 $\pm$ 10.1
2.3960	2146.1 $\pm$ 51.2	66.9 $\pm$ 0.5	15.2%	1.34	1.041	0.6839	220.4 $\pm$ 5.3 $\pm$ 7.7
2.6444	595.2 $\pm$ 27.7	33.7 $\pm$ 0.2	15.0%	1.45	1.039	0.6839	114.4 $\pm$ 5.3 $\pm$ 4.6
2.6464	615.4 $\pm$ 27.8	34.0 $\pm$ 0.3	15.0%	1.45	1.039	0.6839	117.2 $\pm$ 5.3 $\pm$ 4.7
2.9000	1100.7 $\pm$ 37.1	105 $\pm$ 1	14.0%	1.65	1.033	0.6839	64.8 $\pm$ 2.2 $\pm$ 2.3
2.9500	124.0 $\pm$ 13.0	15.9 $\pm$ 0.1	13.3%	1.68	1.029	0.6839	49.6 $\pm$ 5.2 $\pm$ 3.0
2.9810	146.0 $\pm$ 13.8	16.1 $\pm$ 0.1	13.9%	1.66	1.025	0.6839	56.1 $\pm$ 5.3 $\pm$ 3.2
3.0000	144.5 $\pm$ 13.9	15.9 $\pm$ 0.1	13.7%	1.67	1.021	0.6839	56.6 $\pm$ 5.4 $\pm$ 3.6
3.0200	143.8 $\pm$ 13.6	17.3 $\pm$ 0.1	13.7%	1.71	1.014	0.6839	51.1 $\pm$ 4.8 $\pm$ 2.5
3.0800	963.8 $\pm$ 35.3	126 $\pm$ 1	12.8%	1.83	0.915	0.6839	52.3 $\pm$ 1.9 $\pm$ 1.9

**Table 3.** The measured Born cross sections for  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ , where the first uncertainties are statistical and the second ones are systematic.

6. The uncertainty from the  $\pi^0$  invariant mass window requirement is studied with a control sample of  $J/\psi \rightarrow K_S^0 K_L^0 \pi^0$ . The efficiency difference between data and MC simulation with and without the  $\pi^0$  mass window requirement is taken as the uncertainty.
7. The uncertainty of the VP and ISR correction factors is obtained with the accuracy of the radiation function, which is about 0.5% [53], and has an additional contribution from the cross section line shape, which is estimated by varying the model parameters of the fit to the cross section. All parameters are randomly varied within their uncertainties, and the resulting parametrization of the line shape is used to recalculate  $(1 + \delta)\epsilon$  and the corresponding cross section. This procedure is repeated one thousand times, and the standard deviation of the resulting cross sections is taken as the systematic uncertainty. The systematic uncertainty associated with the VP and ISR correction factor is evaluated as the quadratic sum of contributions from the QED theory and line shape parametrization [10].
8. The uncertainty associated with the BFs from the PDG [1] is 0.08%, including both  $\mathcal{B}(K_S^0 \rightarrow \pi^+ \pi^-) = (69.20 \pm 0.05)\%$  and  $\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) = (98.823 \pm 0.034)\%$ .

$\sqrt{s}$ (GeV)	$N_{\text{sig}}$	$\mathcal{L}$ (pb $^{-1}$ )	$\epsilon$	$1+\delta$	$\frac{1}{ 1-\Pi ^2}$	$\mathcal{B}$	$\sigma$ (pb)
2.0000	845.4 ± 45.3	10.1 ± 0.1	17.0%	1.07	1.037	0.2279	1942.5 ± 104.0 ± 100.0
2.0500	225.4 ± 22.8	3.34 ± 0.03	17.2%	1.08	1.038	0.2279	1533.4 ± 155.4 ± 100.7
2.1000	772.9 ± 52.7	12.2 ± 0.1	17.6%	1.07	1.039	0.2279	1420.4 ± 96.9 ± 68.9
2.1250	6530.7 ± 120.8	108 ± 1	17.7%	1.05	1.039	0.2279	1376.2 ± 25.5 ± 46.0
2.1500	200.6 ± 24.4	2.84 ± 0.02	17.1%	1.10	1.040	0.2279	1585.8 ± 193.1 ± 92.4
2.1750	516.1 ± 31.7	10.6 ± 0.1	17.3%	1.13	1.040	0.2279	1055.1 ± 64.8 ± 52.1
2.2000	575.5 ± 33.6	13.7 ± 0.1	16.9%	1.14	1.040	0.2279	918.5 ± 53.6 ± 46.4
2.2324	472.0 ± 26.9	11.9 ± 0.1	16.9%	1.16	1.041	0.2279	851.5 ± 48.5 ± 38.6
2.3094	637.6 ± 32.5	21.1 ± 0.1	16.4%	1.20	1.041	0.2279	648.1 ± 33.0 ± 35.5
2.3864	595.6 ± 31.5	22.5 ± 0.2	16.2%	1.23	1.041	0.2279	559.5 ± 29.6 ± 31.2
2.3960	1845.8 ± 53.0	66.9 ± 0.5	16.4%	1.23	1.041	0.2279	578.4 ± 16.6 ± 23.9
2.6444	438.0 ± 25.5	33.7 ± 0.2	15.5%	1.35	1.039	0.2279	262.0 ± 15.3 ± 12.2
2.6464	452.9 ± 25.9	34.0 ± 0.3	15.5%	1.35	1.039	0.2279	268.5 ± 15.3 ± 12.4
2.9000	799.7 ± 35.6	105 ± 1	14.9%	1.50	1.033	0.2279	145.8 ± 6.5 ± 6.2
2.9500	94.8 ± 11.0	15.9 ± 0.1	14.3%	1.54	1.029	0.2279	115.5 ± 13.4 ± 9.0
2.9810	111.6 ± 11.7	16.1 ± 0.1	14.4%	1.56	1.025	0.2279	132.0 ± 13.8 ± 9.9
3.0000	108.0 ± 11.5	15.9 ± 0.1	14.2%	1.58	1.021	0.2279	130.5 ± 13.9 ± 9.5
3.0200	104.3 ± 10.8	17.3 ± 0.1	14.2%	1.59	1.014	0.2279	115.7 ± 12.0 ± 8.1
3.0800	695.9 ± 33.4	126 ± 1	13.3%	1.73	0.915	0.2279	115.1 ± 5.5 ± 4.4

**Table 4.** The measured Born cross sections for  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$ , where the first uncertainties are statistical and the second ones are systematic.

9. The uncertainty caused by the  $M(\pi^+\pi^-)$  fit includes the descriptions of signal shape, background shape and fit range. The nominal MC-simulated shape convolved with a Gaussian function is replaced by a pure MC-simulated shape, and the nominal background shape is replaced by a second-order polynomial function, and the differences with the nominal results are taken as the uncertainties. The fit range is varied by  $\pm 10$  MeV/ $c^2$  at both boundaries, and the largest difference is taken as the uncertainty. The uncertainties from these three sources are added in quadrature and taken as the total uncertainty from the  $M(\pi^+\pi^-)$  fit.

The second category of uncertainties includes those associated with the PWA fit, *i.e.*, fit parameters (FPar), BW parametrization (BW), resonance parameters (Par), extra additional resonances (Extra), background estimation in PWA fit (Bkg), Blatt-Weisskopf barrier factor (BWf) and efficiency difference of data and MC simulation for  $K_S^0$  and  $\pi^0$  (EC). Fits with alternative scenarios are performed, and the changes of signal yields are taken as systematic uncertainties associated with the PWA fit. These contributions are evaluated as follows:

1. The uncertainty from the fit parameters is estimated by the standard deviation of re-calculated efficiencies derived from one thousand groups of randomly generated fit

$\sqrt{s}$ (GeV)	$N_{\text{sig}}$	$\mathcal{L}$ (pb $^{-1}$ )	$\epsilon$	$1+\delta$	$\frac{1}{ 1-\Pi ^2}$	$\mathcal{B}$	$\sigma$ (pb)
2.0000	$80.1 \pm 28.2$	$10.1 \pm 0.1$	20.0%	1.06	1.037	0.1140	$315.2 \pm 111.2 \pm 17.9$
2.0500	$11.1 \pm 5.0$	$3.34 \pm 0.03$	20.4%	1.07	1.038	0.1140	$127.5 \pm 58.2 \pm 9.0$
2.1000	$27.3 \pm 13.7$	$12.2 \pm 0.1$	20.7%	1.07	1.039	0.1140	$84.6 \pm 42.4 \pm 5.2$
2.1250	$606.2 \pm 68.3$	$108 \pm 1$	20.5%	1.08	1.039	0.1140	$214.2 \pm 24.1 \pm 9.0$
2.1500	$9.7 \pm 5.1$	$2.84 \pm 0.02$	23.4%	0.95	1.040	0.1140	$129.6 \pm 68.1 \pm 9.1$
2.1750	$59.1 \pm 28.6$	$10.6 \pm 0.1$	26.1%	0.88	1.040	0.1140	$204.1 \pm 98.7 \pm 12.2$
2.2000	$113.1 \pm 54.7$	$13.7 \pm 0.1$	25.8%	0.96	1.040	0.1140	$304.0 \pm 147.0 \pm 17.0$
2.2324	$55.8 \pm 25.2$	$11.9 \pm 0.1$	23.8%	1.03	1.041	0.1140	$171.9 \pm 77.8 \pm 10.0$
2.3094	$54.5 \pm 21.6$	$21.1 \pm 0.1$	22.3%	1.04	1.041	0.1140	$94.3 \pm 37.4 \pm 5.8$
2.3864	$28.0 \pm 11.5$	$22.5 \pm 0.2$	22.3%	1.07	1.041	0.1140	$45.3 \pm 18.7 \pm 2.5$
2.3960	$41.5 \pm 12.3$	$66.9 \pm 0.5$	21.7%	1.07	1.041	0.1140	$22.3 \pm 6.6 \pm 1.1$
2.6444	$21.5 \pm 8.2$	$33.7 \pm 0.2$	22.0%	1.07	1.039	0.1140	$22.9 \pm 8.8 \pm 1.3$
2.6464	$22.3 \pm 8.4$	$34.0 \pm 0.3$	22.0%	1.07	1.039	0.1140	$23.5 \pm 8.9 \pm 1.3$
2.9000	$20.4 \pm 9.1$	$105 \pm 1$	22.7%	1.08	1.033	0.1140	$6.7 \pm 3.0 \pm 0.3$
2.9500	$2.2 \pm 1.1$	$15.9 \pm 0.1$	22.5%	1.08	1.029	0.1140	$4.8 \pm 2.4 \pm 0.4$
2.9810	$2.5 \pm 1.1$	$16.1 \pm 0.1$	22.9%	1.08	1.025	0.1140	$5.3 \pm 2.4 \pm 0.4$
3.0000	$4.7 \pm 2.2$	$15.9 \pm 0.1$	22.8%	1.08	1.021	0.1140	$10.3 \pm 4.7 \pm 0.8$
3.0200	$2.2 \pm 1.1$	$17.3 \pm 0.1$	23.0%	1.08	1.014	0.1140	$4.5 \pm 2.1 \pm 0.3$
3.0800	$8.7 \pm 3.7$	$126 \pm 1$	23.6%	1.08	0.915	0.1140	$2.6 \pm 1.1 \pm 0.1$

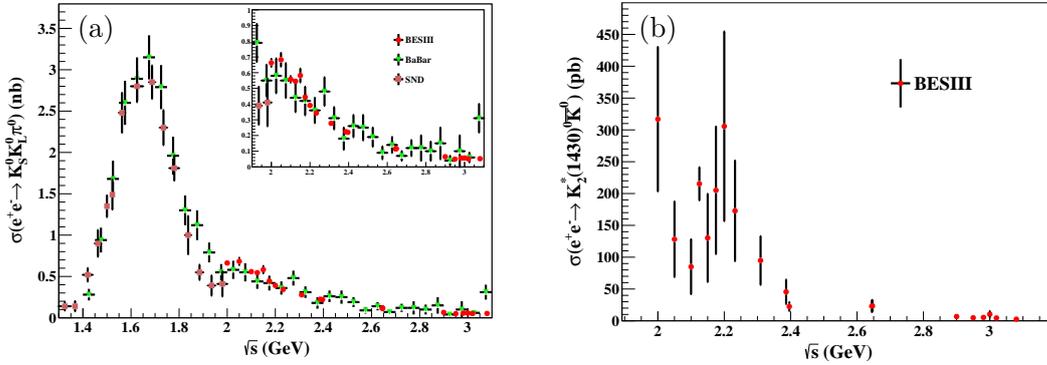
**Table 5.** The measured Born cross sections for  $e^+e^- \rightarrow K_2^*(1430)^0 \bar{K}^0$ , where the first uncertainties are statistical and the second ones are systematic.

parameters using a correlated multi-variable Gaussian function.

2. The uncertainty associated with the BW parametrization is estimated by replacing the nominal one with a BW parametrization with a fixed width.
3. The uncertainty related to the resonance parameters is estimated by performing alternative fits shifting the world-average parameter value by its error from the PDG [1].
4. The uncertainty concerning the extra additional resonances is estimated by performing alternative fits with all components whose significances are greater than  $3\sigma$ .
5. The uncertainty of the background estimation in the PWA fit is estimated by increasing or reducing the sideband events by 50%.
6. The partial wave amplitudes [50] include the Blatt-Weisskopf barrier factor. The associated uncertainty is estimated by varying the radius of the centrifugal barrier from 0.7 to 1.0 fm.
7. The uncertainty from the efficiency difference on data and MC simulation is estimated by performing alternative fits with the efficiency correction factor obtained

Source	Uncertainty(%)
Luminosity	1.00
$K_S^0$ reconstruction	1.00
Requirement on $N_{\text{charge}}$	0.70
Photon reconstruction	2.00
Kinematic fit	0.70
$\pi^0$ mass window	0.19
BF	0.08
Total	2.65

**Table 6.** The 100% correlated systematic uncertainties for the Born cross section of  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ .



**Figure 5.** The Born cross sections for (a) the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  and (b) the process  $e^+e^- \rightarrow K_2^*(1430)^0 \bar{K}^0$ . The red dots are the measured results from BESIII, where errors include both statistical and systematic uncertainties. The green triangles and brown squares are the results from BaBar and SND, respectively.

from control samples of  $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$  and  $J/\psi \rightarrow \pi^+ \pi^- \pi^0$  for  $K_S^0$  and  $\pi^0$  [56, 57], respectively.

Assuming all the sources of systematic uncertainties as independent, the total systematic uncertainty is obtained by adding them in quadrature. The 100% correlated uncertainties for the Born cross sections of  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ ,  $K^*(892)^0 \bar{K}^0$  and  $K_2^*(1430)^0 \bar{K}^0$  are listed in table 6. The total systematic uncertainties are listed in tables 7-9.

## 7 Fit to the lineshape

The Born cross sections for the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  are shown in figure 5(a). The results are consistent with the previous results from BaBar and SND. The Born cross sections for the intermediate process  $e^+e^- \rightarrow K_2^*(1430)^0 \bar{K}^0$  and  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  are shown in figures 5(b) and 6, respectively.

A  $\chi^2$  fit, incorporating the correlated and uncorrelated uncertainties among different energy points, is performed to determine the resonance parameters for the Born cross

$\sqrt{s}$ (GeV)	Rad	Fit	FPar	BW	Par	Extra	Bkg	BWf	EC	Total
2.0000	0.50	1.09	2.40	0.30	0.70	0.41	0.20	0.21	0.10	2.84
2.0500	0.50	2.72	2.50	1.01	2.11	0.40	0.20	0.21	0.10	4.43
2.1000	0.50	0.79	2.40	0.22	1.02	0.40	0.40	0.17	0.10	2.84
2.1250	0.50	1.24	0.80	0.10	0.22	0.38	0.10	0.13	0.10	1.63
2.1500	0.50	3.41	2.60	0.82	1.10	0.43	0.30	0.31	0.10	4.57
2.1750	0.50	1.52	3.00	0.51	1.53	0.40	0.20	0.40	0.10	3.81
2.2000	0.50	1.56	2.50	0.34	1.11	0.41	0.20	0.22	0.10	3.25
2.2324	0.50	2.13	2.20	0.60	1.02	0.32	0.20	0.17	0.10	3.35
2.3094	0.50	3.21	2.40	0.50	1.10	0.31	0.20	0.22	0.10	4.24
2.3864	0.50	1.60	3.00	0.50	1.10	0.30	0.20	0.21	0.10	3.67
2.3960	0.50	1.53	1.40	0.20	0.60	0.30	0.20	0.21	0.10	2.27
2.6444	0.50	2.39	1.41	0.22	1.02	0.30	0.20	0.40	0.10	3.06
2.6464	0.50	2.33	1.40	0.22	1.02	0.30	0.20	0.40	0.10	3.00
2.9000	0.50	1.55	1.30	0.50	0.68	0.29	0.20	0.13	0.10	2.28
2.9500	0.51	4.12	2.57	1.21	2.10	0.33	0.40	0.31	0.10	5.49
2.9810	0.50	3.83	2.20	1.30	1.69	0.31	0.30	0.30	0.10	4.96
3.0000	0.50	4.73	2.20	1.20	2.06	0.30	0.20	0.25	0.10	5.77
3.0200	0.50	2.85	1.70	1.23	1.91	0.30	0.20	0.30	0.10	4.08
3.0800	0.50	1.33	1.70	0.60	1.02	0.31	0.20	0.20	0.10	2.55

**Table 7.** Uncorrelated systematic uncertainties (%) for the Born cross section of  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  at each c.m. energy associated with the ISR and VP correction factors (Rad), the  $M(\pi^+\pi^-)$  fit (Fit), the fit parameters in PWA (FPar), the BW parametrization (BW), the resonance parameters (Par), the extra additional resonances (Extra), the Blatt-Weisskopf barrier factor (BWf) and Efficiency difference (EC).

sections of  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$ . The fit probability density function is a coherent sum of a continuum component and a resonant component. The cross section is modeled as:

$$\sigma = \left| \frac{\sqrt{12\pi\Gamma_Y\Gamma_Y^{e^+e^-}\mathcal{B}}}{M_Y^2 - s - iM_Y\Gamma(\sqrt{s})} \sqrt{\frac{P(\sqrt{s})}{P(M_Y)}} e^{i\phi_Y} + c_1 \frac{\sqrt{P(\sqrt{s})}}{s^{c_2}} \right|^2, \quad (7.1)$$

where  $M_Y$  and  $\Gamma_Y$  are the mass and width of the resonance;  $\phi_Y$  is the relative phase between the continuum component and the resonance;  $\Gamma_Y^{e^+e^-}$  is its partial width to  $e^+e^-$ ;  $\mathcal{B}$  is the BF of  $Y \rightarrow K^*(892)^0 \bar{K}^0$ ; and  $c_1$  and  $c_2$  are additional parameters of the fit.  $\Gamma(\sqrt{s})$  is defined as  $\Gamma_Y(\frac{P(\sqrt{s})}{P(M_Y)})$ , where  $P(\sqrt{s}) = \int |A_{K^*(892)^0 \bar{K}^0}|^2 d\Phi_3$  is the phase-space factor for the relative orbital angular momentum  $L = 1$  of the process  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0 \rightarrow K_S^0 K_L^0 \pi^0$  and  $\Phi_3$  is three-body phase space [1]. The amplitude  $A$  is the partial wave amplitude in the covariant Rarita-Schwinger tensor formalism [50] and is described as:

$$A_{K^*(892)^0 \bar{K}^0} = -\epsilon_{\mu\nu\lambda\sigma} p_{(K_S^0 K_L^0 \pi^0)}^\sigma \tilde{T}_{K^*(892)^0 \bar{K}^0}^{(1)\nu} \cdot f_{K^0 \pi^0}^{K^*(892)^0} \cdot \tilde{T}_{K^0 \pi^0}^{(1)\lambda}, \quad (7.2)$$

where  $\tilde{T}$  is the covariant tensor,  $f$  is the Breit-Wigner propagator,  $\epsilon_{\mu\nu\lambda\sigma}$  is the Levi-Civita symbol, and the other operators can be found in Ref. [50].

$\sqrt{s}$ (GeV)	Rad	Fit	BW	Par	Extra	Bkg	BWf	EC	Total
2.0000	0.50	1.09	2.90	1.90	0.71	1.31	1.91	0.30	4.41
2.0500	0.50	2.72	3.30	1.22	0.70	2.42	3.10	0.30	6.01
2.1000	0.50	0.79	2.22	1.71	0.70	1.77	2.01	0.30	4.06
2.1250	0.50	1.24	1.10	0.21	0.68	0.20	0.73	0.30	2.04
2.1500	0.50	3.41	2.72	1.23	0.73	1.22	2.01	0.30	5.20
2.1750	0.50	1.52	2.51	1.01	0.70	1.05	2.40	0.30	4.16
2.2000	0.50	1.56	2.43	1.32	0.71	1.62	2.22	0.30	4.30
2.2324	0.50	2.13	1.79	1.10	0.52	1.11	1.67	0.30	3.69
2.3094	0.50	3.21	2.80	1.05	0.51	1.03	1.42	0.30	4.79
2.3864	0.50	1.60	3.72	1.04	0.51	1.04	2.20	0.30	4.90
2.3960	0.50	1.53	2.30	0.53	0.50	0.58	1.13	0.30	3.18
2.6444	0.50	2.39	1.82	0.72	0.51	1.32	1.70	0.30	3.84
2.6464	0.50	2.33	1.82	0.72	0.51	1.29	1.70	0.30	3.80
2.9000	0.50	1.55	1.60	0.38	0.69	1.77	1.32	0.30	3.29
2.9500	0.51	4.12	2.81	3.01	0.73	3.00	3.18	0.30	7.34
2.9810	0.50	3.83	2.90	3.01	0.71	2.42	3.17	0.30	6.99
3.0000	0.50	4.73	2.80	1.53	0.70	2.41	2.68	0.30	6.81
3.0200	0.50	2.85	3.23	1.22	0.70	1.82	4.20	0.30	6.47
3.0800	0.50	1.33	1.50	0.50	0.71	1.00	1.27	0.30	2.78

**Table 8.** Uncorrelated systematic uncertainties (%) for the Born cross section of  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$  at each c.m. energy associated with the ISR and VP correction factors (Rad), the  $M(\pi^+\pi^-)$  fit (Fit), the BW parametrization (BW), the resonance parameters (Par), the extra additional resonances (Extra), the Blatt-Weisskopf barrier factor (BWf) and Efficiency difference (EC).

In total, there are six free parameters in the fit:  $M_Y, \Gamma_Y, \phi_Y, c_1, c_2$  and the product of  $\Gamma_Y^{e^+e^-} \mathcal{B}$ . The fit results are shown in figure 6, and the resonance parameters are listed in table 10. The fit has  $\chi^2/\text{ndf} = 18.95/13$  with two solutions for the resonance with identical mass and width, but different relative phase  $\phi_Y$  and  $\Gamma_Y^{e^+e^-} \mathcal{B}$ . The mass and width of the resonance are  $M_Y = (2164.1 \pm 9.6)$  MeV/ $c^2$  and  $\Gamma_Y = (32.4 \pm 21.1)$  MeV, respectively, where the uncertainties are statistical only. The significance of the resonance is determined to be  $3.2\sigma$  by comparing the change of  $\chi^2$  ( $\Delta\chi^2 = 18.27$ ) and the change of ndf ( $\Delta\text{ndf} = 4$ ) between the nominal fit and the fit without the resonance. Figure 6(b) shows a large interference between resonance and continuum components. The uncertainties (statistical and systematic) of the measured Born cross sections have been included when fitting the line shape, determining the resonance parameters and estimating the significance of the resonance.

Besides the uncertainties of individual cross-section measurements, the fit to the line-shape is also affected by the uncertainty of the BEPCII c.m. energy and the description of the continuum. The uncertainty of the c.m. energy calibration is estimated as 0.1% and is ignored in the determination of resonance parameters [36]. To evaluate the systematic uncertainty associated with the lineshape model, the continuum term  $c_1 \frac{\sqrt{P(\sqrt{s})}}{s^{c_2}}$  is

$\sqrt{s}$ (GeV)	Rad	Fit	BW	Par	Extra	Bkg	BWf	EC	Total
2.0000	0.50	1.09	3.41	2.02	1.00	1.73	2.07	0.10	5.03
2.0500	0.50	2.72	4.10	2.10	1.00	2.77	2.21	0.10	6.52
2.1000	0.50	0.79	3.30	2.27	1.00	2.72	2.21	0.10	5.49
2.1250	0.50	1.24	2.02	1.01	1.00	1.10	1.29	0.10	3.27
2.1500	0.50	3.41	4.01	1.82	1.00	2.32	2.08	0.10	6.47
2.1750	0.50	1.52	3.61	2.10	1.00	1.80	2.10	0.10	5.35
2.2000	0.50	1.56	2.60	2.10	1.00	2.09	2.21	0.10	4.91
2.2324	0.50	2.13	2.78	2.12	1.31	2.02	2.01	0.10	5.17
2.3094	0.50	3.21	2.50	2.10	1.30	1.70	2.10	0.10	5.50
2.3864	0.50	1.60	2.49	2.10	1.30	1.82	2.10	0.10	4.78
2.3960	0.50	1.53	2.00	1.49	1.31	1.47	1.70	0.10	3.96
2.6444	0.50	2.39	2.09	1.70	1.31	2.10	2.00	0.10	4.83
2.6464	0.50	2.33	2.10	1.70	1.31	2.10	2.00	0.10	4.81
2.9000	0.50	1.55	2.01	1.38	1.50	2.00	1.92	0.10	4.30
2.9500	0.51	4.12	3.20	2.50	1.53	3.00	2.20	0.10	7.06
2.9810	0.50	3.83	3.14	2.72	1.52	3.07	2.11	0.10	6.94
3.0000	0.50	4.73	3.13	2.51	1.52	3.20	2.20	0.10	7.47
3.0200	0.50	2.85	3.19	2.51	1.52	3.10	2.00	0.10	6.38
3.0800	0.50	1.33	3.01	1.70	1.51	2.20	1.92	0.10	4.96

**Table 9.** Uncorrelated systematic uncertainties (%) for the Born cross section of  $e^+e^- \rightarrow K_2^*(1430)^0 \bar{K}^0$  at each c.m. energy associated with the ISR and VP correction factors (Rad), the  $M(\pi^+\pi^-)$  fit (Fit), the BW parametrization (BW), the resonance parameters (Par), the extra additional resonances (Extra), the Blatt-Weisskopf barrier factor (BWf) and Efficiency difference (EC).

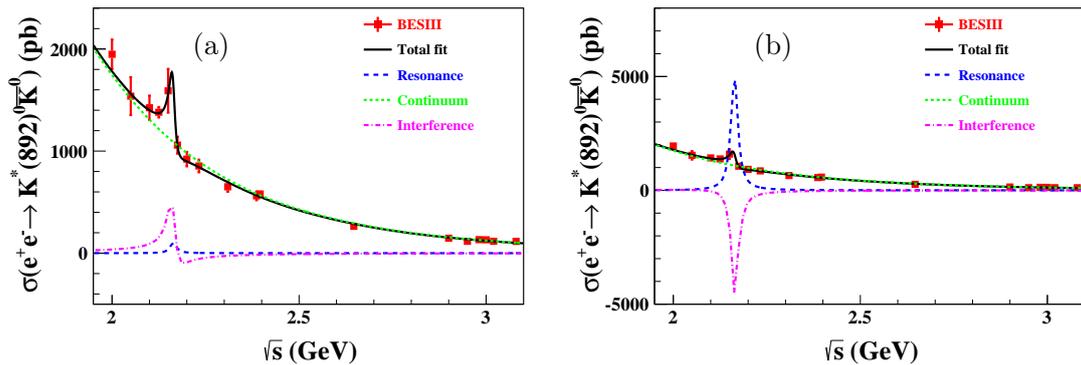
Parameter	Solution 1	Solution 2
$M_Y$ (MeV/ $c^2$ )	$2164.1 \pm 9.6(\text{stat.}) \pm 3.1(\text{syst.})$	
$\Gamma_Y$ (MeV)	$32.4 \pm 21.1(\text{stat.}) \pm 1.5(\text{syst.})$	
$\Gamma_Y^{e^+e^-} \mathcal{B}$ (eV)	$1.0 \pm 0.2(\text{stat.}) \pm 0.1(\text{syst.})$	$73.6 \pm 4.5(\text{stat.}) \pm 2.0(\text{syst.})$
$\phi_Y$ (rad)	$2.5 \pm 0.5(\text{stat.}) \pm 0.1(\text{syst.})$	$-1.7 \pm 0.1(\text{stat.}) \pm 0.1(\text{syst.})$
Significance	$3.2\sigma$	

**Table 10.** The fit results obtained from  $e^+e^- \rightarrow K^*(892)^0 \bar{K}^0$ .

replaced with an exponential function of the form  $c_0 \cdot e^{-p_0(\sqrt{s}-M_{th})}$ , where  $c_0$  and  $p_0$  are free parameters and  $M_{th} = m_{K^*(892)^0} + m_{\bar{K}^0}$  is the mass threshold for  $K^*(892)^0 \bar{K}^0$  production [1, 12]. The difference of the parameters from the nominal results are taken as the systematic uncertainties.

## 8 Summary

In summary, a partial wave analysis of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  is performed for nineteen data samples collected in the BESIII experiment with center-of-mass energies ranging



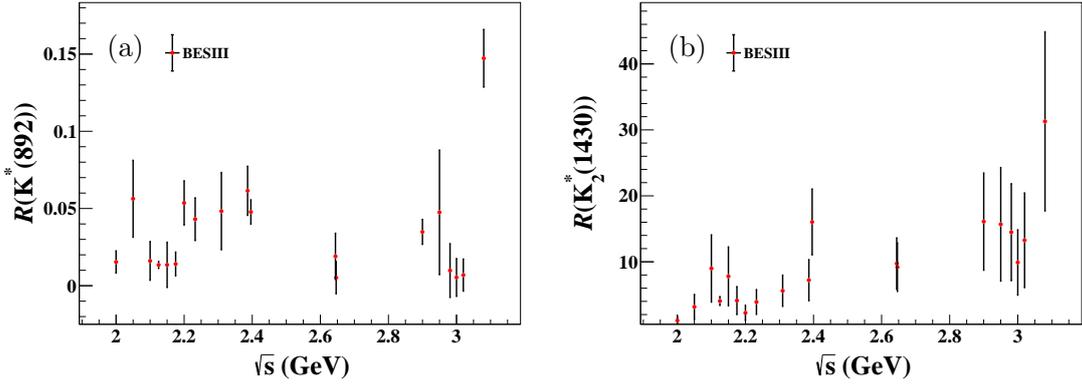
**Figure 6.** The Born cross section and fit curves for  $e^+e^- \rightarrow K^*(892)^0\bar{K}^0$ , (a) and (b), corresponding to the two solutions in table 10. Rectangles with error bars are BESIII data, where errors include both statistical and systematic uncertainties. The solid black curves represent the total fit result, the dashed blue curves for the resonance and the dashed green curves for the continuum component, and the dash-dotted pink curves for the interference between the resonance and continuum components.

from 2.000 to 3.080 GeV corresponding to a total integrated luminosity of  $647 \text{ pb}^{-1}$ . The Born cross sections of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ , as well as those for the intermediate processes  $e^+e^- \rightarrow K^*(892)^0\bar{K}^0$  and  $K_2^*(1430)^0\bar{K}^0$  are measured by performing the partial wave analysis on each data sample individually, where the charge conjugated processes are also included. The measured Born cross sections of the process  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$  are consistent with earlier results by BaBar [34] and SND [35], while the precision is significantly improved. The Born cross section lineshape of the process  $e^+e^- \rightarrow K^*(892)^0\bar{K}^0$  is consistent with a resonant structure around 2.2 GeV with a statistical significance of  $3.2\sigma$ . A Breit-Wigner fit yields its mass  $M_Y = (2164.1 \pm 9.6 \pm 3.1) \text{ MeV}/c^2$  and width  $\Gamma_Y = (32.4 \pm 21.1 \pm 1.5) \text{ MeV}$ . The resonance parameters, especially the very narrow width, are very close to the BESIII results measured through the  $\phi\eta$  channel [7] of the  $\phi(2170)$  meson [1].

The previous BESIII measurement [12] with the charged channel  $e^+e^- \rightarrow K^+K^-\pi^0$  showed that  $K_2^*(1430)^+K^-$  is the dominant component, with the fraction of  $K^*(892)^+K^-$  at the 2-10% level. However, in this study with the neutral channel  $e^+e^- \rightarrow K_S^0 K_L^0 \pi^0$ ,  $K^*(892)^0\bar{K}^0$  is dominant, while  $K_2^*(1430)^0\bar{K}^0$  is at the 5% level in the BESIII c.m. energy region. The asymmetry is also observed by BaBar [28] in the production of  $K^*(892)^0\bar{K}^0$ ,  $K^*(892)^+K^-$ ,  $K_2^*(1430)^0\bar{K}^0$  and  $K_2^*(1430)^+K^-$ . To quantify the effect, we define relative ratios of the Born cross sections:

$$\begin{aligned}
 R(K^*(892)) &= \frac{\sigma(e^+e^- \rightarrow K^*(892)^+K^-)}{\sigma(e^+e^- \rightarrow K^*(892)^0\bar{K}^0)}, \\
 R(K_2^*(1430)) &= \frac{\sigma(e^+e^- \rightarrow K_2^*(1430)^+K^-)}{\sigma(e^+e^- \rightarrow K_2^*(1430)^0\bar{K}^0)}.
 \end{aligned}
 \tag{8.1}$$

The corresponding results for the relative ratio are summarized in figure 7. The ratio



**Figure 7.** Relative ratio distributions for (a)  $K^*(892)$  and (b)  $K_2^*(1430)$ .

of Born cross section measurements of the process  $e^+e^- \rightarrow K^*(892)^+K^-$  to the process  $e^+e^- \rightarrow K^*(892)^0\bar{K}^0$  is less than 0.2 and that for  $K_2^*(1430)$  is in the region of 0-40, where the statistical and systematic uncertainties are included. If we apply the isospin decomposition for the decay from isospin vector ( $\rho^*$ ) or isospin scalar ( $\omega^*, \phi^*$ ) state to the final state  $K^*\bar{K}$ , the ratio of the yields in the neutral and charged  $K^*\bar{K}$  should be 1. On the other hand, the electromagnetic interaction also contributes to the production of  $e^+e^- \rightarrow K^*\bar{K}$ , and it does not require isospin conservation. Future experimental and theoretical studies are needed to understand the observed phenomenon.

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