

## Improved measurement of the branching fractions of the inclusive decays

$$D^+ \rightarrow K_S^0 X \text{ and } D^0 \rightarrow K_S^0 X$$

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By analyzing  $2.93 \text{ fb}^{-1}$  of  $e^+e^-$  collision data taken at the center-of-mass energy of 3.773 GeV with the BESIII detector, the branching fractions of the inclusive decays  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  are measured to be  $(33.11 \pm 0.13 \pm 0.36)\%$  and  $(20.75 \pm 0.12 \pm 0.20)\%$ , respectively, where the first uncertainties are statistical and the second are systematic. These results are consistent with the world averages of previous measurements, but with much improved precision.

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## I. INTRODUCTION

Weak decays of charmed mesons offer an ideal laboratory for improving our understanding of both strong and weak interactions. The  $D^+$  and  $D^0$  mesons decay dominantly into final states involving  $K^-$  and  $\bar{K}^0$  via Cabibbo-favored processes. The branching fractions of the inclusive decays  $D^{+(0)} \rightarrow K^0/\bar{K}^0 X$ , where  $X$  denotes any possible particle combinations, were first measured by Mark-III [1] in 1991 and later by BES [2] in 2006. Assuming  $\mathcal{B}(K^0/\bar{K}^0 \rightarrow K_S^0) = 0.5$  and considering the world average branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , the branching fractions of  $D^{+(0)} \rightarrow K_S^0 X$  are expected to be  $(30.5 \pm 2.5)\%$  and  $(23.5 \pm 2.0)\%$ , respectively [3]. In recent years, the precision of the branching fractions of known decay modes has been significantly improved, and many new channels have been observed. Following this progress, the sums of the branching fractions of the exclusive  $D^+$  and  $D^0$  decays involving  $K_S^0$  are found to be  $(31.68 \pm 0.32)\%$  and  $(18.16 \pm 0.72)\%$ , as summarized in the Appendix. In contrast, however, the branching fractions of the inclusive decays  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  suffer from large uncertainties due to the limited sample sizes used to perform the measurements. Improved determinations of these inclusive decay branching fractions will help to quantify and guide the search for the missing decay modes involving a  $K_S^0$  meson.

In this paper, we report the improved measurements of the branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , from the analysis of  $2.93 \text{ fb}^{-1}$  of  $e^+e^-$  collision data [4]

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taken at the center-of-mass energy  $\sqrt{s}$  of 3.773 GeV with the BESIII detector. The charge-conjugated processes are implied throughout the discussion.

## II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector [5] records symmetric  $e^+e^-$  collisions provided by the BEPCII storage ring [6], which operates in the center-of-mass energy range from 2.0 to 4.95 GeV, with a peak luminosity of  $1 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  achieved at  $\sqrt{s} = 3.773 \text{ GeV}$ . BESIII has collected large data samples in this energy region [7]. The cylindrical core of the BESIII detector covers 93% of the full solid angle and consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field [8]. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identification modules interleaved with steel. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the specific ionization energy loss (dE/dx) resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution in the TOF barrel region is 68 ps, while that in the end-cap region is 110 ps.

Simulated samples produced with the Geant4-based [9] Monte Carlo (MC) package, which includes the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes the beam-energy spread and initial-state radiation in the  $e^+e^-$  annihilations modeled with the generator KKMC [10]. The inclusive MC samples consist of the production of  $D\bar{D}$  pairs with consideration of quantum coherence for all neutral  $D$  modes, the non- $D\bar{D}$  decays of the  $\psi(3770)$ , the initial-state radiation production of the  $J/\psi$  and  $\psi(3686)$  states, and the continuum processes. The known decay modes are modeled with EvtGen [11] using the branching fractions taken from Particle Data Group (PDG) [3], and the remaining unknown decays of the charmonium states are modeled by LUNDCHARM [12].

Final-state radiation from charged final-state particles is incorporated using the PHOTOS package [13].

The three-body decays  $D^+ \rightarrow K_S^0 \pi^+ \pi^0$  and  $D^+ \rightarrow K_S^0 \pi^+ \eta$  are modeled with a modified data-driven generator BODY3 [11], which was developed to simulate different intermediate states in data for a given three-body final state. The Dalitz plot from data, corrected for backgrounds and efficiencies, is taken as input for the BODY3 generator. The decay  $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$  is modeled with the partial-wave analysis model from Ref. [14]. Other decays reported in recent BESIII measurements [15–17] are modeled by constructing a cocktail of possible intermediate resonances.

### III. ANALYSIS METHOD

At  $\sqrt{s} = 3.773$  GeV,  $D^0 \bar{D}^0$  or  $D^+ D^-$  pairs are produced without accompanying hadron(s), thereby offering a clean environment to investigate hadronic  $D$  decays with the double-tag (DT) method [18]. To measure the branching fractions of the hadronic decays of  $D^+(D^0)$  mesons, as a first step the single-tag  $D^-(\bar{D}^0)$  mesons are selected by using the hadronic decay modes  $D^- \rightarrow K^+ \pi^- \pi^-$  and  $\bar{D}^0 \rightarrow K^+ \pi^-$ . The number of the expected single-tag  $D^-(\bar{D}^0)$  mesons is given by

$$N_{\text{tag}} = 2N_{D\bar{D}} \mathcal{B}_{\text{tag}} \epsilon_{\text{tag}}, \quad (1)$$

where  $N_{D\bar{D}}$  is the total number of  $D^+ D^-$  ( $D^0 \bar{D}^0$ ) pairs,  $\mathcal{B}_{\text{tag}}$  is the branching fraction of  $D^-(\bar{D}^0)$  decay into the tag mode, and  $\epsilon_{\text{tag}}$  is the efficiency of reconstructing the single-tag  $D^-(\bar{D}^0)$  mesons.

The double-tag sample is comprised of events in which the signal decay  $D^{+(0)} \rightarrow K_S^0 X$  can be selected in the presence of the single-tag  $D^-(\bar{D}^0)$  mesons. The number of the expected double-tag events is given by

$$N_{\text{DT}} = 2N_{D\bar{D}} \mathcal{B}_{\text{tag}} \mathcal{B}_{\text{sig}} \epsilon_{\text{DT}}, \quad (2)$$

where  $\mathcal{B}_{\text{sig}}$  is the branching fraction of  $D^{+(0)} \rightarrow K_S^0 X$ , and  $\epsilon_{\text{DT}}$  is the efficiency of detecting the double-tag events. The branching fraction of the signal decay is then

$$\mathcal{B}_{\text{sig}} = \frac{N_{\text{DT}}}{N_{\text{tag}} \epsilon_{\text{sig}}}, \quad (3)$$

where  $\epsilon_{\text{sig}} = \frac{\epsilon_{\text{DT}}}{\epsilon_{\text{tag}}}$  is the effective efficiency of reconstructing  $D^{+(0)} \rightarrow K_S^0 X$ .

Quantum correlations between the  $D^0 \bar{D}^0$  pair modify the double-tag rates. The measured branching fraction of  $D^0 \rightarrow K_S^0 X$  tagged by  $\bar{D}^0 \rightarrow K^+ \pi^-$  must be corrected to

$$\mathcal{B}_{\text{sig}}^{\text{cor}} = \frac{1}{1 - C_f(2f_{CP^+} - 1)} \mathcal{B}_{\text{sig}}, \quad (4)$$

where  $f_{CP^+}$  is the fractional  $CP$ -even content of the decay  $D^0 \rightarrow K_S^0 X$ , and  $C_f$  is the strong-phase factor

$$C_f = \frac{2r \cos \delta}{1 + r^2}. \quad (5)$$

Here  $r = 0.0587 \pm 0.0002$  [19] is the ratio between doubly Cabibbo-suppressed and Cabibbo-favored amplitudes for  $D^0/\bar{D}^0 \rightarrow K^\pm \pi^\mp$ , and  $\delta = (191.7_{-3.8}^{+3.6})^\circ$  [19] is the strong-phase difference between two amplitudes [20,21]. These inputs yield  $C_f = (-11.5 \pm 0.2)\%$  for  $D^0/\bar{D}^0 \rightarrow K^\pm \pi^\mp$ .

The  $CP$ -even fraction  $f_{CP^+}$  of  $D^0 \rightarrow K_S^0 X$  decays is not known, but can be measured from the  $D^0 \bar{D}^0$  dataset. Following Refs. [21,22], the  $CP$ -even fraction is calculated by

$$f_{CP^\pm} = \frac{N^\pm}{N^+ + N^-},$$

$$N^\pm = \frac{M_{\text{measured}}^\pm}{S^\pm},$$

and

$$S_\pm = \frac{S_{\text{measured}}^\pm}{1 - \eta_\pm y_D},$$

where  $N^\pm$  is the ratio between double-tag and single-tag yields with  $CP$ -odd(even) tags,  $M_{\text{measured}}^\pm$  is the number of double-tag candidates for a signal channel versus  $CP$ -odd(even) tags,  $S_{\text{measured}}^\pm$  is the number of single-tag candidates for  $CP$ -even(odd) decay modes, and  $S^\pm$  is the number of the corrected single-tag candidates for  $CP$ -even(odd) decay modes. Finally,  $\eta_\pm = \pm 1$  for  $CP$ -even(odd) mode and  $y_D = (6.47 \pm 0.24) \times 10^{-3}$  is the mixing parameter taken from the latest average from HFLAV [19].

### IV. YIELDS OF SINGLE-TAG $D^-(\bar{D}^0)$ MESONS

Charged kaons and charged pions are selected with the same selection criteria as those used in our previous works [23–25]. All charged tracks, apart from those from  $K_S^0$  decays, are required to have a polar angle  $\theta$  with respect to the MDC symmetry axis within the MDC acceptance  $|\cos \theta| < 0.93$ , a distance of closest approach with respect to the interaction point along the beam direction  $|V_z| < 10$  cm, and a distance of closest approach with respect to the interaction point in the plane transverse to the beam direction  $|V_{xy}| < 1$  cm. Particle identification for charged kaons and pions is performed by exploiting combined  $dE/dx$  and TOF information. The confidence levels for pion and kaon hypotheses ( $CL_\pi$  and  $CL_K$ ) are calculated. Charged tracks satisfying  $CL_K > CL_\pi$  and  $CL_\pi > CL_K$  are assigned as charged kaons and pions, respectively.

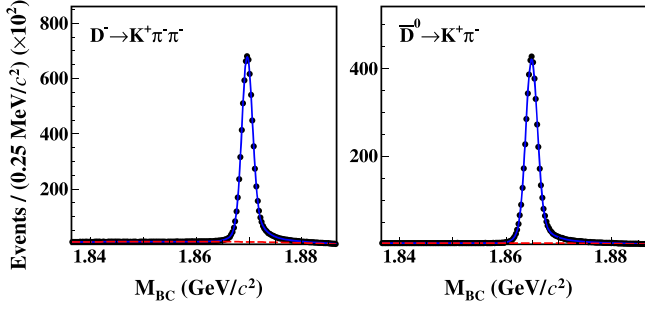


FIG. 1. Best fits to the  $M_{BC}$  distributions of the single-tag candidates for  $D^- \rightarrow K^+ \pi^- \pi^-$  and  $\bar{D}^0 \rightarrow K^+ \pi^-$ . The points with error bars are data. The blue solid curves are the fit results. The red dashed curves are the fitted combinatorial background shapes.

When selecting  $\bar{D}^0 \rightarrow K^+ \pi^-$  candidates, contamination from cosmic rays and Bhabha events is suppressed with the following requirements [26]. First, the two charged tracks must have a TOF time difference less than 5 ns and they must be inconsistent with being a muon-antimuon or an electron-positron pair. Second, in the event there must be at least one EMC shower with an energy greater than 50 MeV or at least one additional charged track detected in the MDC.

The tagged  $D^- (\bar{D}^0)$  mesons are identified using the energy difference  $\Delta E \equiv E_{\bar{D}} - E_{\text{beam}}$  and the beam-constrained mass  $M_{BC} \equiv \sqrt{E_{\text{beam}}^2 - |\vec{p}_{\bar{D}}|^2}$ . Here,  $E_{\text{beam}}$  is the beam energy, and  $\vec{p}_{\bar{D}}$  and  $E_{\bar{D}}$  are the momentum and energy of the  $D^- (\bar{D}^0)$  candidate in the rest frame of the  $e^+ e^-$  system, respectively. For each tag mode, if there are multiple candidates in an event, only the one with the smallest  $|\Delta E|$  is kept. The tagged  $\bar{D}$  candidates are required to be within  $\Delta E \in (-25, 25)$  MeV.

To determine the yields of single-tag  $D^- (\bar{D}^0)$  mesons for individual tag modes, binned maximum-likelihood fits are performed on the corresponding  $M_{BC}$  distributions of the accepted single-tag candidates [23–25]. In the fits, the  $D^- (\bar{D}^0)$  signal is modeled by an MC-simulated shape convolved with a double-Gaussian function describing the resolution difference between data and MC simulation. The combinatorial background is described by an ARGUS function [27]. The best fit results for the  $M_{BC}$  distributions are shown in Fig. 1. The yields of the single-tag  $D^-$  and  $\bar{D}^0$  mesons are  $798935 \pm 1011$  and  $529227 \pm 761$ , respectively, where the uncertainties are statistical only. The efficiencies of reconstructing the single-tag  $D^- (\bar{D}^0)$  mesons are estimated to be  $(51.90 \pm 0.08)\%$  and  $(65.60 \pm 0.09)\%$ , respectively, by analyzing the inclusive MC sample.

## V. YIELDS OF DOUBLE-TAG $D^{+ (0)}$ MESONS

The  $K_S^0$  mesons are reconstructed in the  $K_S^0 \rightarrow \pi^+ \pi^-$  decay mode. The charged pions making up the  $K_S^0$  candidates are required to satisfy  $|V_z| < 20$  cm and

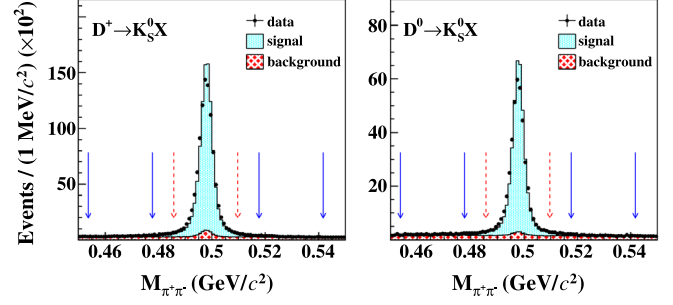


FIG. 2. Distributions of  $M_{\pi^+ \pi^-}$  of the candidates for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ . The red dashed and blue solid arrow pairs show the  $K_S^0$  signal and sideband regions, respectively. The points with error bars are data. The histograms are the inclusive MC sample.

$|\cos \theta| < 0.93$ . To improve the resolution of the  $K_S^0$  invariant mass, a primary vertex and a  $K_S^0$  decay vertex fit are performed on the  $\pi^+ \pi^-$  pairs. To suppress the combinatorial background, we require  $L/\sigma_L > 2$ , where  $\sigma_L$  is the uncertainty of the flight distance  $L$ . The invariant mass of the chosen charged pions,  $M_{\pi^+ \pi^-}$ , is required to be within  $|M_{\pi^+ \pi^-} - M_{K_S^0}| < 12$  MeV/ $c^2$ , which corresponds to about  $\pm 3\sigma$  around the known  $K_S^0$  mass ( $M_{K_S^0}$ ) [3]. To estimate the combinatorial  $\pi^+ \pi^-$  background, the  $K_S^0$  sideband region is defined by  $20 < |M_{\pi^+ \pi^-} - M_{K_S^0}| < 44$  MeV/ $c^2$ , which corresponds to about  $\pm 5\sigma$  to  $\pm 11\sigma$  away from the known  $K_S^0$  mass. The definitions of the  $K_S^0$  signal and sideband regions are indicated in Fig. 2. The background is mainly from the  $D\bar{D}$  and  $q\bar{q}$  ( $q = u, d, s$ ) continuum process. Especially, events with the wrong tag and one real  $K_S^0$  meson of the signal side will form a peak in the  $M_{\pi^+ \pi^-}$  distribution but do not form a peak in the  $M_{BC}$  distribution. There is some discrepancy in resolution between data and MC simulation, and this effect will be taken into account in the systematic uncertainty of  $K_S^0$  reconstruction.

The distributions of the recoil mass squared, momentum, and  $\cos \theta$  of the  $K_S^0$  candidates from the selected  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  events are shown in Fig. 3. The experimental and MC distributions are in good agreement.

The decay yields of the signals are determined from the fits to the  $M_{BC}$  distributions of the accepted double-tag events in data. However, combinatorial background events with  $M_{\pi^+ \pi^-}$  in the  $K_S^0$  signal region may form a peaking background in the  $M_{BC}$  distribution. So, the net signal yields are determined by

$$N_{\text{DT}}^{\text{net}} = N_{K_S^0 \text{sig}} - f_{\text{co}} \cdot N_{K_S^0 \text{sb}}, \quad (6)$$

where  $N_{K_S^0 \text{sig}}$  and  $N_{K_S^0 \text{sb}}$  are the fitted double-tag yields in the  $K_S^0$  signal and sideband regions, respectively; the sideband scaling factor  $f_{\text{co}}$  is taken as 0.5 under the

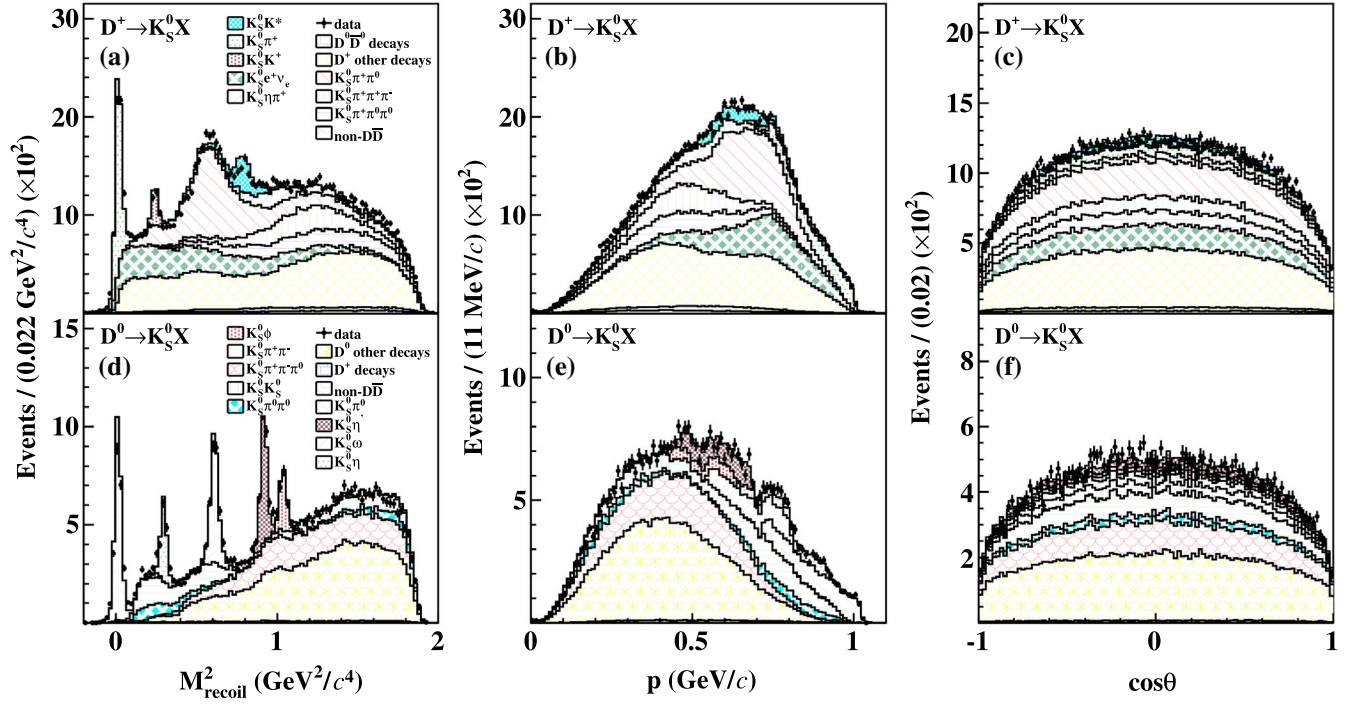


FIG. 3. Distributions of the [(a) and (d)] recoil mass squared, [(b) and (e)] momentum, and [(c) and (f)]  $\cos \theta$  of the  $K_S^0$  candidates from the accepted candidates for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ . The points with error bars are data. The histograms are the inclusive MC sample.

assumption that the background contribution in the  $M_{\pi^+\pi^-}$  spectrum is flat. Figure 4 shows the  $M_{BC}$  distributions of the accepted double-tag events in data with  $M_{\pi^+\pi^-}$  in the  $K_S^0$  signal and sideband regions. We perform similar fits to

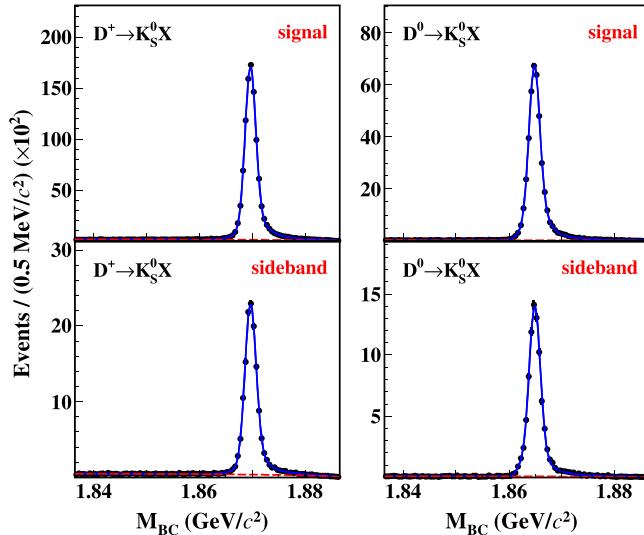


FIG. 4. Best fits to the  $M_{BC}$  distributions of the double-tag events in data. The upper and lower plots correspond to events with  $M_{\pi^+\pi^-}$  in the  $K_S^0$  signal and sideband regions, respectively. The points with error bars are data, the blue solid curves are the fit results, and the red dashed curves are the fitted combinatorial background shapes.

these spectra as for the fits to the single-tag candidates. In these fits, the parameters of the smeared double-Gaussian resolution functions are fixed at the values obtained from the fits to the single-tag  $D^-$  ( $\bar{D}^0$ ) candidates. The fit results are also shown in Fig. 4. From these fits, we obtain the yields of  $N_{K_S^0 \text{sig}}$  and  $N_{K_S^0 \text{sb}}$ , as summarized in Table II.

The efficiencies of reconstructing the inclusive decays  $D^{+(0)} \rightarrow K_S^0 X$  are determined by analyzing the inclusive MC sample ( $\epsilon_{\text{sig}}$ ). Inserting the numbers of  $N_{\text{DT}}^{\text{net}}$ ,  $N_{\text{tag}}$ , and  $\epsilon_{\text{sig}}$  into Eq. (3), we obtain the branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ .

In order to account for the effect of quantum correlations on the branching fraction of  $D^0 \rightarrow K_S^0 X$ , we apply the correction given in Eq. (4). This procedure requires knowledge of the  $CP$ -even fraction of these decays, which is measured in the data by using the  $CP$ -even tag

TABLE I. Measurements of  $S_{\text{measured}}^{\pm}$ ,  $M_{\text{measured}}^{\pm}$ ,  $f_{CP+}$  and the correction factor for  $D^0 \rightarrow K_S^0 X$  due to the quantum correlation effect.

| $CP$ tag mode               | $\bar{D}^0 \rightarrow K^+ K^-$ | $\bar{D}^0 \rightarrow K_S^0 \pi^0$ |
|-----------------------------|---------------------------------|-------------------------------------|
| $S_{\text{measured}}^{\pm}$ | $57779 \pm 287$                 | $70512 \pm 311$                     |
| $M_{\text{measured}}^{\pm}$ | $4760 \pm 81$                   | $4068 \pm 78$                       |
| $f_{CP+}$                   | $0.413 \pm 0.010$               |                                     |
| Correction factor           | $1.0204 \pm 0.0024$             |                                     |

TABLE II. Numbers of single-tag  $D^-(\bar{D}^0)$  mesons ( $N_{\text{tag}}$ ), fitted double-tag yields in the  $K_S^0$  signal and sideband regions ( $N_{K_S^0\text{sig}}$  and  $N_{K_S^0\text{sb}}$ ), net numbers of signal events ( $N_{\text{DT}}^{\text{net}}$ ), efficiencies of finding  $D^{+(0)} \rightarrow K_S^0 X$  in the presence of the single-tag  $D^-(\bar{D}^0)$  meson ( $\epsilon_{\text{sig}}$ ), and measured branching fractions of  $D^{+(0)} \rightarrow K_S^0 X$  ( $\mathcal{B}_{\text{sig}}$ ). The efficiency values include the branching fraction of  $K_S^0 \rightarrow \pi^+\pi^-$  and the correction factors described in the text. The uncertainties are statistical only.

| Signal mode               | $N_{\text{tag}}$  | $N_{K_S^0\text{sig}}$ | $N_{K_S^0\text{sb}}$ | $N_{\text{DT}}^{\text{net}}$ | $\epsilon_{\text{sig}}$ (%) | $\mathcal{B}_{\text{sig}}$ (%) |
|---------------------------|-------------------|-----------------------|----------------------|------------------------------|-----------------------------|--------------------------------|
| $D^+ \rightarrow K_S^0 X$ | $798935 \pm 1011$ | $102186 \pm 365$      | $13981 \pm 140$      | $95195 \pm 372$              | $35.99 \pm 0.08$            | $33.11 \pm 0.13$               |
| $D^0 \rightarrow K_S^0 X$ | $529227 \pm 761$  | $42166 \pm 215$       | $8688 \pm 97$        | $37822 \pm 220$              | $34.45 \pm 0.10$            | $20.75 \pm 0.12$               |

$\bar{D}^0 \rightarrow K^+K^-$  and the  $CP$ -odd tag  $\bar{D}^0 \rightarrow K_S^0\pi^0$ . The yields of these tags and the resulting value for  $f_{CP^+}$  are given in Table I. The correction factor is determined to be  $1.0204 \pm 0.0024$ .

The net efficiencies, including all factors discussed above, and the final values for the branching fractions are summarized in Table II.

## VI. SYSTEMATIC UNCERTAINTIES

With the double-tag method, the branching-fraction measurements have a greatly reduced sensitivity to systematic bias arising from the selection criteria on the tag side.

The systematic uncertainties in the yields of the single-tag  $D^-$  or  $\bar{D}^0$  mesons have been estimated in Ref. [23] to be 0.50%, by examining the relative change between data and MC simulation yields when varying the fit range, signal shape, and end point of the ARGUS function.

The  $K_S^0$  reconstruction efficiencies are examined in two aspects. The  $\pi^\pm$  tracking efficiencies are examined by using hadronic  $D\bar{D}$  events by missing  $\pi^\pm$  as a control sample. The efficiencies associated with  $K_S^0$  mass window, primary vertex and  $K_S^0$  decay vertex fit are examined by the hadronic  $D\bar{D}$  events, with  $D^0$  or  $D^+$  decaying into  $K_S^0\pi^+\pi^-$ ,  $K_S^0\pi^+\pi^-\pi^0$ ,  $K_S^0\pi^0$ ,  $K_S^0\pi^+$ ,  $K_S^0\pi^-\pi^0$ , and  $K_S^0\pi^+\pi^-\pi^-$ . The polar angle distribution of the control sample is well consistent with that in the signal decays, therefore its effect on the data-MC difference of  $K_S^0$  reconstruction efficiency is negligible. The momentum weighted data-MC differences of the  $K_S^0$  reconstruction efficiency are  $(0.19 \pm 0.70)\%$  and  $(0.01 \pm 0.68)\%$  for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , respectively. To be conservative, we assign 0.89% and 0.69% as the systematic uncertainty due to  $K_S^0$  reconstruction for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , respectively.

Two methods are used to estimate the systematic uncertainty associated with the  $K_S^0$  multiplicity. First, we vary the components of the decays containing one, two, and three  $K_S^0$  mesons by  $\pm 1\sigma$  according to their known branching fractions. The changes in the detection efficiencies for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , 0.05% and 0.07% respectively, are assigned as the uncertainties. Second, we measure the detection efficiencies for all known exclusive decays and determine the change of the reweighted

detection efficiency by varying each of the decays within  $\pm 1\sigma$  according to the known branching fractions. By adding in quadrature all the changes in the detection efficiencies, we obtain 0.20% for  $D^+ \rightarrow K_S^0 X$  and 0.28% for  $D^0 \rightarrow K_S^0 X$ , which are assigned as the uncertainties. We assign the larger effect coming from the two methods as the corresponding systematic uncertainty for each signal decay.

To estimate the systematic uncertainty due to the choice of the  $K_S^0$  sideband region, we examine the effects on the branching fractions by shifting the  $K_S^0$  sideband region by  $\pm 2$  or  $\pm 4$  MeV/ $c^2$ . If the difference of the remeasured branching fraction is greater than the statistical uncertainty after considering the signal correlations, it is assigned as the systematic uncertainty; otherwise, it is neglected. Following this procedure, we assign 0.15% for  $D^+ \rightarrow K_S^0 X$  and 0.10% for  $D^0 \rightarrow K_S^0 X$  as the corresponding systematic uncertainties.

The uncertainty on the correction factor for quantum correlations is propagated to the branching fraction of  $D^0 \rightarrow K_S^0 X$  decays. The uncertainties due to the limited signal double-tag MC samples for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  are estimated to be 0.25% and 0.28%, respectively. The uncertainty of the branching fraction of  $K_S^0 \rightarrow \pi^+\pi^-$  is 0.07%, taken from the PDG [3].

The uncertainty arising from the scale factor  $f_{\text{co}}$  used in Eq. (6) is assessed with the alternative scale factors derived from the inclusive MC sample, which are 0.549 and 0.534 for  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , respectively. The changes

TABLE III. Relative systematic uncertainties (in %) in the branching fraction measurements. A “...” denotes that there is no systematic uncertainty.

| Source                       | $D^+ \rightarrow K_S^0 X$ | $D^0 \rightarrow K_S^0 X$ |
|------------------------------|---------------------------|---------------------------|
| $N_{\text{tag}}$             | 0.50                      | 0.50                      |
| $K_S^0$ reconstruction       | 0.89                      | 0.69                      |
| $K_S^0$ multiplicity         | 0.20                      | 0.28                      |
| $K_S^0$ sideband             | 0.15                      | 0.10                      |
| Quantum-correlation effect   | ...                       | 0.24                      |
| MC statistics                | 0.25                      | 0.28                      |
| Quoted branching fraction    | 0.07                      | 0.07                      |
| Scale factor $f_{\text{co}}$ | 0.02                      | 0.05                      |
| Total                        | 1.08                      | 0.98                      |



TABLE IV. Comparisons of the branching fractions of  $D^{+(0)} \rightarrow K_S^0 X$  measured in this study and Mark-III, BES, and PDG. For the values from BES, Mark-III, and PDG, we take half of the branching fractions of  $D \rightarrow K^0/\bar{K}^0 X$ .  $\mathcal{B}_{\text{exclusive}}^{\text{sum}}$  denotes the total branching fractions summing over all known exclusive  $D^{+(0)}$  decays involving a  $K_S^0$ .

| Decay mode                | Mark-III (%) [1]          | BES (%) [2]               | PDG (%) [3]    | This study (%)            | $\mathcal{B}_{\text{exclusive}}^{\text{sum}}$ (%) |
|---------------------------|---------------------------|---------------------------|----------------|---------------------------|---|
| $D^+ \rightarrow K_S^0 X$ | $30.60 \pm 3.25 \pm 2.15$ | $30.25 \pm 2.75 \pm 1.65$ | $30.5 \pm 2.5$ | $33.11 \pm 0.13 \pm 0.36$ | $31.68 \pm 0.32$                                  |
| $D^0 \rightarrow K_S^0 X$ | $22.75 \pm 2.50 \pm 1.60$ | $23.80 \pm 2.40 \pm 1.50$ | $23.5 \pm 2.0$ | $20.75 \pm 0.12 \pm 0.20$ | $18.16 \pm 0.72$                                  |

of the remeasured branching fractions, 0.02% and 0.05%, are assigned as the corresponding systematic uncertainties.

The systematic uncertainties are summarized in Table III. Assuming that all of these components are independent, they are summed in quadrature to give the totals 1.08% and 0.98% for the branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$ , respectively.

## VII. SUMMARY

By analyzing  $2.93 \text{ fb}^{-1}$  of  $e^+e^-$  collision data taken at the center-of-mass energy of 3.773 GeV with the BESIII detector, the branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  are measured to be  $(33.11 \pm 0.13 \pm 0.36)\%$  and  $(20.75 \pm 0.12 \pm 0.20)\%$ , respectively, where the first uncertainties are statistical and the second are systematic. These results are consistent with previous measurements by Mark-III [1] and BES [2] within uncertainties, as summarized in Table IV. Compared with the average values from the PDG, the precision of the branching fractions of  $D^+ \rightarrow K_S^0 X$  and  $D^0 \rightarrow K_S^0 X$  is improved by factors of 7.1 and 7.6, respectively. Summing over the branching fractions of the known  $D^{+(0)}$  decay modes containing  $K_S^0$ , we obtain  $\mathcal{B}_{\text{exclusive}}^{\text{sum}}(D^+ \rightarrow K_S^0 X) = (31.68 \pm 0.32)\%$  and  $\mathcal{B}_{\text{exclusive}}^{\text{sum}}(D^0 \rightarrow K_S^0 X) = (18.16 \pm 0.72)\%$ . The differences between the inclusive and exclusive decay branching fractions are  $(1.43 \pm 0.44)\%$  and  $(2.59 \pm 0.76)\%$  for  $D^+$  and  $D^0$  decays, respectively. These results indicate that there may be some missing decay modes involving  $K_S^0$  for both  $D^+$  and  $D^0$  yet to be observed.

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## APPENDIX: BRANCHING FRACTIONS OF THE KNOWN EXCLUSIVE $D^{+(0)}$ DECAYS INVOLVING $K_S^0$

Tables V and VI summarize the branching fractions of the known exclusive  $D^+$  and  $D^0$  decays involving  $K_S^0$ , respectively.

TABLE V. Initial and final states contributing to the inclusive decay  $D^+ \rightarrow K_S^0 X$  and the corresponding branching fractions.

| Initial state                     | $\mathcal{B}$ (%)        | Final state                       | $\mathcal{B}$ (%)   |
|-----------------------------------|--------------------------|-----------------------------------|---------------------|
| $K_S^0 \pi^+$                     | $1.49 \pm 0.03$ [28]     | $K_S^0 \pi^+$                     | $1.49 \pm 0.03$     |
| $\bar{K}^0 e^+ \nu_e$             | $8.73 \pm 0.10$ [3]      | $K_S^0 e^+ \nu_e$                 | $4.37 \pm 0.05$     |
| $\bar{K}^{*0} e^+ \nu_e$          | $5.40 \pm 0.10$ [3]      | $K_S^0 \pi^0 e^+ \nu_e$           | $0.89 \pm 0.02$     |
| $\bar{K}^0 \mu^+ \nu_\mu$         | $8.76 \pm 0.19$ [3]      | $K_S^0 \mu^+ \nu_\mu$             | $4.380 \pm 0.095$   |
| $\bar{K}^{*0} \mu^+ \nu_\mu$      | $5.27 \pm 0.15$ [3]      | $K_S^0 \pi^0 \mu^+ \nu_\mu$       | $0.88 \pm 0.02$     |
| $K_S^0 K_S^0 K^+$                 | $0.254 \pm 0.013$ [3]    | $K_S^0 K_S^0 K^+$                 | $0.254 \pm 0.013$   |
| $K_S^0 K_L^0 K^+$                 | $0.508 \pm 0.026$ [3]    | $K_S^0 K_L^0 K^+$                 | $0.508 \pm 0.026$   |
| $K_S^0 K_S^0 \pi^+$               | $0.270 \pm 0.013$ [3]    | $K_S^0 K_S^0 \pi^+$               | $0.270 \pm 0.013$   |
| $\bar{K}^0 \pi^+ \pi^+ \pi^-$     | $6.20 \pm 0.18$ [3]      | $K_S^0 \pi^+ \pi^+ \pi^-$         | $3.10 \pm 0.09$     |
| $\bar{K}^0 K^+ K^- \pi^+$         | $0.048 \pm 0.010$ [3]    | $K_S^0 K^+ K^- \pi^+$             | $0.024 \pm 0.005$   |
| $K_S^0 \pi^+ \pi^0$               | $7.36 \pm 0.21$ [3]      | $K_S^0 \pi^+ \pi^0$               | $7.36 \pm 0.21$     |
| $\bar{K}^0 K^{*+}$                | $1.738 \pm 0.182$ [29]   | $K_S^0 K^{*+}$                    | $0.869 \pm 0.091$   |
| $\bar{K}_1^0(1270) e^+ \nu_e$     | $0.231 \pm 0.030$ [30]   | $K_S^0 \pi^0 \pi^0 e^+ \nu_e$     | $0.0055 \pm 0.0007$ |
| $\bar{K}_1^0(1270) e^+ \nu_e$     | $0.231 \pm 0.030$ [30]   | $K_S^0 \pi^+ \pi^- e^+ \nu_e$     | $0.039 \pm 0.005$   |
| $\bar{K}_1^0(1270) \mu^+ \nu_\mu$ | $0.231 \pm 0.030$ [30]   | $K_S^0 \pi^+ \pi^- \mu^+ \nu_\mu$ | $0.039 \pm 0.005$   |
| $\bar{K}_1^0(1270) \mu^+ \nu_\mu$ | $0.231 \pm 0.030$ [30]   | $K_S^0 \pi^0 \pi^0 \mu^+ \nu_\mu$ | $0.0055 \pm 0.0007$ |
| $K_S^0 \pi^+ \eta$                | $1.309 \pm 0.048$ [25]   | $K_S^0 \pi^+ \eta$                | $1.309 \pm 0.048$   |
| $K_S^0 K^+ \eta$                  | $0.0185 \pm 0.0050$ [25] | $K_S^0 K^+ \eta$                  | $0.0185 \pm 0.0050$ |
| $K_S^0 \pi^+ \pi^0 \eta$          | $0.12 \pm 0.03$ [25]     | $K_S^0 \pi^+ \pi^0 \eta$          | $0.12 \pm 0.03$     |
| $K_S^0 K^+ \pi^+ \pi^-$           | $0.189 \pm 0.013$ [15]   | $K_S^0 K^+ \pi^+ \pi^-$           | $0.189 \pm 0.013$   |
| $K_S^0 K^- \pi^+ \pi^+$           | $0.227 \pm 0.013$ [15]   | $K_S^0 K^- \pi^+ \pi^+$           | $0.227 \pm 0.013$   |
| $K_S^0 K^+ \pi^0 \pi^0$           | $0.058 \pm 0.013$ [15]   | $K_S^0 K^+ \pi^0 \pi^0$           | $0.058 \pm 0.013$   |
| $K_S^0 K_S^0 \pi^+ \pi^0$         | $0.134 \pm 0.021$ [15]   | $K_S^0 K_S^0 \pi^+ \pi^0$         | $0.134 \pm 0.021$   |
| $K_S^0 K_L^0 \pi^+ \pi^0$         | $0.268 \pm 0.042$ [15]   | $K_S^0 K_L^0 \pi^+ \pi^0$         | $0.268 \pm 0.042$   |
| $\bar{K}^0 K^+$                   | $0.604 \pm 0.024$ [31]   | $K_S^0 K^+$                       | $0.302 \pm 0.012$   |
| $K_S^0 K^+ \pi^0$                 | $0.507 \pm 0.003$ [31]   | $K_S^0 K^+ \pi^0$                 | $0.507 \pm 0.003$   |
| $\phi \pi^+$                      | $0.570 \pm 0.015$ [32]   | $K_S^0 K_L^0 \pi^+$               | $0.193 \pm 0.006$   |
| $\bar{K}^0 \pi^+ \eta'$           | $0.380 \pm 0.042$ [33]   | $K_S^0 \pi^+ \eta'$               | $0.190 \pm 0.021$   |
| $K_S^0 \omega \pi^+$              | $0.707 \pm 0.050$ [16]   | $K_S^0 \omega \pi^+$              | $0.707 \pm 0.050$   |
| $K_S^0 \pi^+ \pi^0 \pi^0$         | $2.2516 \pm 0.1068$ [34] | $K_S^0 \pi^+ \pi^0 \pi^0$         | $2.2516 \pm 0.1068$ |
| $K_S^0 \pi^+ \pi^+ \pi^- \pi^0$   | $0.4001 \pm 0.0550$ [34] | $K_S^0 \pi^+ \pi^+ \pi^- \pi^0$   | $0.4001 \pm 0.0550$ |
| $K_S^0 \pi^+ \pi^0 \pi^0 \pi^0$   | $0.126 \pm 0.050$ [34]   | $K_S^0 \pi^+ \pi^0 \pi^0 \pi^0$   | $0.126 \pm 0.050$   |
| $\phi \pi^+ \pi^0$                | $0.579 \pm 0.055$ [35]   | $K_S^0 K_L^0 \pi^+ \pi^0$         | $0.196 \pm 0.019$   |
| Sum                               | ...                      | ...                               | $31.68 \pm 0.32$    |

TABLE VI. Initial and final states contributing to the inclusive decay  $D^0 \rightarrow K_S^0 X$  and the corresponding branching fractions.

| Initial state                   | $\mathcal{B}$ (%)         | Final state                       | $\mathcal{B}$ (%)   |
|---------------------------------|---------------------------|-----------------------------------|---------------------|
| $K_S^0 \omega$                  | $1.11 \pm 0.12$ [3]       | $K_S^0 \omega$                    | $1.11 \pm 0.12$     |
| $K_S^0 \eta' \pi^0$             | $0.252 \pm 0.027$ [3]     | $K_S^0 \eta' \pi^0$               | $0.252 \pm 0.027$   |
| $K^- a_0^+$                     | $0.059 \pm 0.018$ [3]     | $K_S^0 K^+ K^-$                   | $0.003 \pm 0.001$   |
| $K_S^0 \phi$                    | $0.444 \pm 0.015$ [3]     | $K_S^0 \phi$                      | $0.444 \pm 0.015$   |
| $\phi K_L^0$                    | $0.4440 \pm 0.0304$ [3]   | $K_S^0 K_L^0 K_L^0$               | $0.151 \pm 0.010$   |
| $\bar{K}^{*0} \pi^0$            | $1.82 \pm 0.07$ [3]       | $K_S^0 \pi^0 \pi^0$               | $0.303 \pm 0.012$   |
| $\phi \gamma$                   | $0.00282 \pm 0.00019$ [3] | $K_S^0 K_L^0 \gamma$              | $0.0010 \pm 0.0001$ |
| $K_S^0 \pi^+ \pi^-$             | $2.80 \pm 0.18$ [3]       | $K_S^0 \pi^+ \pi^-$               | $2.80 \pm 0.18$     |
| $K_S^0 K^+ \pi^-$               | $0.217 \pm 0.034$ [3]     | $K_S^0 K^+ \pi^-$                 | $0.217 \pm 0.034$   |
| $K_S^0 K^- \pi^+$               | $0.33 \pm 0.05$ [3]       | $K_S^0 K^- \pi^+$                 | $0.33 \pm 0.05$     |
| $K_S^0 K_S^0 K^- \pi^+$         | $0.059 \pm 0.013$ [3]     | $K_S^0 K_S^0 K^- \pi^+$           | $0.059 \pm 0.013$   |
| $K_S^0 K_L^0 K^- \pi^+$         | $0.1180 \pm 0.0013$ [3]   | $K_S^0 K_L^0 K^- \pi^+$           | $0.1180 \pm 0.0013$ |
| $K_S^0 K_S^0 K^+ \pi^-$         | $0.059 \pm 0.013$ [3]     | $K_S^0 K_S^0 K^+ \pi^-$           | $0.059 \pm 0.013$   |
| $K_S^0 K_L^0 K^+ \pi^-$         | $0.1180 \pm 0.0013$ [3]   | $K_S^0 K_L^0 K^+ \pi^-$           | $0.1180 \pm 0.0013$ |
| $K_S^0 \rho^0 \pi^+ \pi^-$      | $0.11 \pm 0.07$ [3]       | $K_S^0 \pi^+ \pi^- \pi^+ \pi^-$   | $0.11 \pm 0.07$     |
| $K^{*-} \pi^+ \pi^+ \pi^-$      | $0.10 \pm 0.07$ [3]       | $K_S^0 \pi^+ \pi^- \pi^+ \pi^-$   | $0.033 \pm 0.023$   |
| $K_S^0 \pi^+ \pi^- \pi^0$       | $4.09 \pm 0.60$ [3]       | $K_S^0 \pi^+ \pi^- \pi^0$         | $4.09 \pm 0.60$     |
| $\phi \pi^+ \pi^-$              | $0.02 \pm 0.01$ [3]       | $K_S^0 K_L^0 \pi^+ \pi^-$         | $0.0068 \pm 0.0034$ |
| $\bar{K}^{*0} \gamma$           | $0.042 \pm 0.007$ [3]     | $K_S^0 \pi^0 \gamma$              | $0.007 \pm 0.001$   |
| $K_S^0 \pi^0 \eta$              | $1.006 \pm 0.046$ [25]    | $K_S^0 \pi^0 \eta$                | $1.006 \pm 0.046$   |
| $K_S^0 K_S^0 \eta$              | $0.0130 \pm 0.0062$ [25]  | $K_S^0 K_S^0 \eta$                | $0.0130 \pm 0.0062$ |
| $K_S^0 K_L^0 \eta$              | $0.0270 \pm 0.0124$ [25]  | $K_S^0 K_L^0 \eta$                | $0.0270 \pm 0.0124$ |
| $K_S^0 \pi^+ \pi^- \eta$        | $0.280 \pm 0.021$ [25]    | $K_S^0 \pi^+ \pi^- \eta$          | $0.280 \pm 0.021$   |
| $K_S^0 \pi^0 \pi^0 \eta$        | $0.176 \pm 0.026$ [25]    | $K_S^0 \pi^0 \pi^0 \eta$          | $0.176 \pm 0.026$   |
| $K_S^0 K^- \pi^+ \pi^0$         | $0.132 \pm 0.016$ [15]    | $K_S^0 K^- \pi^+ \pi^0$           | $0.132 \pm 0.016$   |
| $K_S^0 K_S^0 \pi^+ \pi^-$       | $0.053 \pm 0.009$ [15]    | $K_S^0 K_S^0 \pi^+ \pi^-$         | $0.053 \pm 0.009$   |
| $K_S^0 K_L^0 \pi^+ \pi^-$       | $0.106 \pm 0.018$ [15]    | $K_S^0 K_L^0 \pi^+ \pi^-$         | $0.106 \pm 0.018$   |
| $K_S^0 K^- \pi^+ \pi^0$         | $0.132 \pm 0.016$ [15]    | $K_S^0 K^- \pi^+ \pi^0$           | $0.132 \pm 0.016$   |
| $K_S^0 K^+ \pi^- \pi^0$         | $0.065 \pm 0.008$ [15]    | $K_S^0 K^+ \pi^- \pi^0$           | $0.065 \pm 0.008$   |
| $\phi \pi^0$                    | $0.117 \pm 0.004$ [32]    | $K_S^0 K_L^0 \pi^0$               | $0.040 \pm 0.001$   |
| $\phi \eta$                     | $0.018 \pm 0.005$ [32]    | $K_S^0 K_L^0 \eta$                | $0.006 \pm 0.001$   |
| $K_S^0 \pi^0 \omega$            | $0.848 \pm 0.055$ [16]    | $K_S^0 \pi^0 \omega$              | $0.848 \pm 0.055$   |
| $K_S^0 \pi^0 \pi^0 \pi^0$       | $0.591 \pm 0.042$ [34]    | $K_S^0 \pi^0 \pi^0 \pi^0$         | $0.591 \pm 0.042$   |
| $K_S^0 \pi^+ \pi^- \pi^0 \pi^0$ | $0.327 \pm 0.045$ [34]    | $K_S^0 \pi^+ \pi^- \pi^0 \pi^0$   | $0.327 \pm 0.045$   |
| $K_S^0 \eta'$                   | $0.949 \pm 0.016$ [36]    | $K_S^0 \eta'$                     | $0.949 \pm 0.016$   |
| $K_S^0 \pi^0$                   | $1.240 \pm 0.022$ [36]    | $K_S^0 \pi^0$                     | $1.240 \pm 0.022$   |
| $K^{*-} e^+ \nu_e$              | $2.03 \pm 0.66$ [37]      | $K_S^0 \pi^- e^+ \nu_e$           | $0.68 \pm 0.22$     |
| $\bar{K}^{*0} \pi^- e^+ \nu_e$  | $0.158 \pm 0.034$ [37]    | $K_S^0 \pi^- e^+ \nu_e$           | $0.079 \pm 0.017$   |
| $K^{*-} \mu^+ \nu_\mu$          | $1.91 \pm 0.62$ [37]      | $K_S^0 \pi^- \mu^+ \nu_\mu$       | $0.63 \pm 0.20$     |
| $K_1^-(1270) e^+ \nu_e$         | $0.109 \pm 0.018$ [38]    | $K_S^0 \pi^0 \pi^- e^+ \nu_e$     | $0.026 \pm 0.004$   |
| $K_1^-(1270) \mu^+ \nu_\mu$     | $0.109 \pm 0.018$ [38]    | $K_S^0 \pi^0 \pi^- \mu^+ \nu_\mu$ | $0.026 \pm 0.004$   |
| $K_S^0 K_S^0$                   | $0.0170 \pm 0.0015$ [39]  | $K_S^0 K_S^0$                     | $0.0170 \pm 0.0015$ |
| $K_S^0 K_S^0 K_S^0$             | $0.072 \pm 0.007$ [39]    | $K_S^0 K_S^0 K_S^0$               | $0.072 \pm 0.007$   |
| $K_S^0 K_L^0 K_L^0$             | $0.216 \pm 0.021$ [39]    | $K_S^0 K_L^0 K_L^0$               | $0.216 \pm 0.021$   |
| $K_S^0 K_S^0 K_L^0$             | $0.216 \pm 0.021$ [39]    | $K_S^0 K_S^0 K_L^0$               | $0.216 \pm 0.021$   |
| Sum                             | ...                       | ...                               | $18.16 \pm 0.72$    |

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