Supplementary materials of Investigation of the $\Delta I = 1/2$ rule and test of CP symmetry through the measurement of decay asymmetry parameters in Ξ^- decays

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PROJECTION OF THE ANGULAR AMPLITUDE: MOMENTS OF POLARIZATION AND SPIN-CORRELATION

The complete angular amplitude of the decay $J/\psi \to \Xi^-\bar{\Xi}^+ \to \Lambda(p\pi^-)\pi^-\bar{\Lambda}(\bar{n}\pi^0)\pi^+ + c.c.$ is written as [1]

$$\mathcal{W}(\xi;\omega) = \sum_{\mu,\nu=0}^{3} C_{\mu\nu} \sum_{\mu'\nu'=0}^{3} a^{\Xi}_{\mu\mu'} a^{\bar{\Xi}}_{\nu\nu'} a^{\Lambda}_{\mu'0} a^{\bar{\Lambda}}_{\nu'0}, \tag{1}$$

where ξ is one row matrix containing nine helicity angles and ω is one row matrix containing eight parameters defined as

$$\begin{aligned} \xi &= (\theta_{\Xi}, \theta_{\Lambda}, \phi_{\Lambda}, \theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}}, \theta_p, \phi_p, \theta_{\bar{n}}, \phi_{\bar{n}}), \\ \omega &= \left(\alpha_{J/\psi}, \Delta\Phi_{J/\psi}, \alpha_{\Xi}, \phi_{\Xi}, \bar{\alpha}_{\Xi}, \bar{\phi}_{\Xi}, \alpha_{\Lambda -}, \bar{\alpha}_{\Lambda 0}\right). \end{aligned} \tag{2}$$

Here $C_{\mu,nu}(\theta_{\Xi}, \alpha_{J/\psi}, \Delta\Phi_{J/\psi})$ is a 4×4 spin density matrix, written as

$$C_{\mu\nu} = \left(1 + \alpha_{J/\psi}\cos^2\theta_{\Xi}\right) \begin{pmatrix} 1 & 0 & P_y & 0 \\ 0 & C_{xx} & 0 & C_{xz} \\ -P_y & 0 & C_{yy} & 0 \\ 0 & -C_{xz} & 0 & C_{zz} \end{pmatrix}, \tag{3}$$

where P_y represents the contribution of the θ_{Ξ}-dependent Ξ^- and $\bar{\Xi}^+$ polarizations and C_{xx}, C_{yy}, C_{zz}, C_{xz} are the spin-correlation coefficients defined as follow,

$$\begin{aligned} P_y &= \sqrt{1-\alpha^2_{J/\psi}} \frac{\cos\theta_{\Xi}\sin\theta_{\Xi}}{1+\alpha_{J/\psi}\cos^2\theta_{\Xi}} \sin(\Delta\Phi_{J/\psi}), \\ C_{xx} &= \frac{\sin^2\theta_{\Xi}}{1+\alpha_{J/\psi}\cos^2\theta_{\Xi}}, \\ C_{yy} &= \frac{\alpha_{J/\psi}\sin^2\theta_{\Xi}}{1+\alpha_{J/\psi}\cos^2\theta_{\Xi}}, \\ C_{zz} &= -\frac{\alpha+\sin^2\theta_{\Xi}}{1+\alpha_{J/\psi}\cos^2\theta_{\Xi}}. \end{aligned} \tag{4}$$

The polarization and spin correlations of the process $e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+$ are shown in Fig. 1.

SYSTEMATIC UNCERTAINTIES

The main sources of systematic uncertainties include the reconstruction and selection of the signal candidates, the background estimation and the fit procedure, which are summarized in Table II.

The efficiency of π^0 reconstruction is investigated by using a control sample $J/\psi \to \Sigma^+(p\pi^0)\pi^-\bar{\Lambda}(\bar{p}\pi^+) + c.c.$, which is of the similar final state and the hyperon's decay length. The efficiency from $\pi^\pm$ reconstruction is investigated by using a control sample $J/\psi \to \Xi^-\bar{\Xi}^+ \to \Lambda(p\pi^-)\pi^-\bar{\Lambda}(\bar{p}\pi^+)\pi^+$. The efficiency differences between data and MC simulation are obtained. Based on these, we obtain the ratio of the efficiencies between data and MC simulation, $w_{ij} = \frac{\varepsilon^{\rm data}_{ij}}{\varepsilon^{\rm MC}_{ij}}$, where i represents the i-th bin of the polar angle $\cos\theta$ and j represents the j-th bin of the momentum of π^0 momentum or the transverse momentum of $\pi^\pm$. The ratios are used to re-weight the signal MC samples of the process $e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+$ in the log-likelihood fit. The systematic uncertainties are estimated by smearing the ratios with a Gaussian distribution and repeating the fit procedure by 100 trials. A Gaussian function is used to model the 100 trials for each parameter and its width is taken as the systematic uncertainty.

The systematic uncertainties related to the selection criteria, *e.g.* the decay points and the invariant masses of Λ and Ξ^-, the polar angle of Ξ^-, the invariant mass of (anti)neutron and the χ^2 of kinematic fit are studied by varying individual requirements around the nominal ones and repeating the fit. For each test, i, the selection criteria are varied in the range R by steps s. The changes of the fitted parameters compared to the nominal results, $\Delta_i = |\omega - \omega_i|$ and the uncorrelated uncertainties, $\sigma_{uc,i} = \sqrt{|\sigma^2_{\omega} - \sigma^2_{\omega,i}|}$ are calculated, where σ_{ω} and $\sigma_{\omega,i}$ are the uncertainties of

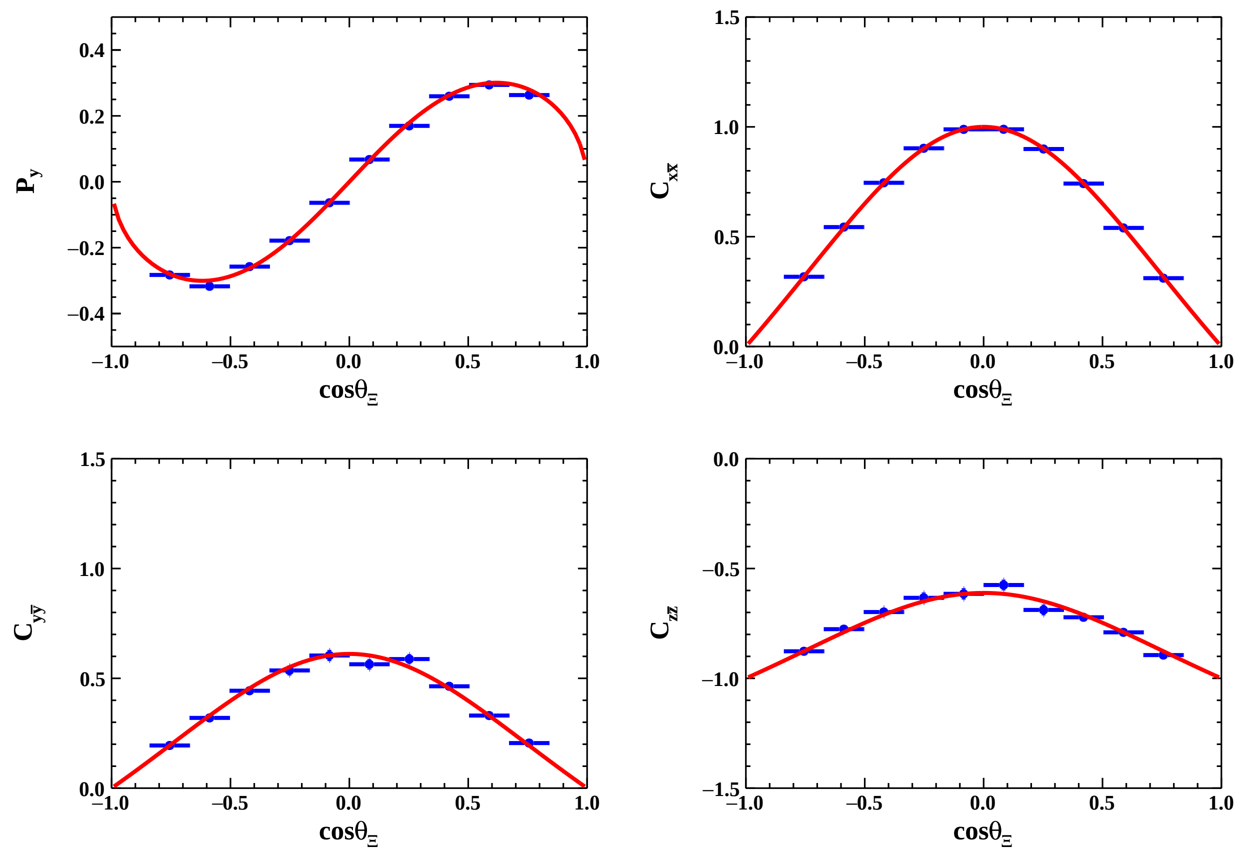

FIG. 1. The polarization P_y and spin correlations, $C_{x\bar{x}}, C_{y\bar{y}}$ and $C_{z\bar{z}}$ of the reaction $e^+e^- \to J/\psi \to \Xi^-\bar{\Xi}^+$. The data points with error bars are determined independently in each $\cos\theta_\Xi$ bin. The red solid curves are the expected angular dependence obtained with the parameters $\alpha_{J/\psi}, \Delta\Phi_{J/\psi}$ from the global maximum log-likelihood fit.

TABLE I. The nominal cut, varied range, step and standard deviation of the selection criteria in the studies of systematic uncertainties.

Source	Nominal cut	Range R	Step s	Standard deviation
Λ decay length (cm)	0	[−1, 3.5]	0.5	1.2
$\lvert M_{p\pi} - m_\Lambda\rvert$ (MeV/c^2)	11	[1, 19]	2	2.0
Ξ decay length (cm)	0	[−1, 3.5]	0.5	0.5
$\lvert M_{p\pi\pi} - m_\Xi\rvert$ (MeV/c^2)	11	[1, 19]	2	3.0
$\lvert\cos\theta_\Xi\rvert$	0.84	[0.74, 0.92]	0.02	0.1
$\lvert M_n - m_n\rvert$ (MeV/c^2)	15	[5, 23]	2	5.0
$\chi^2_{\rm KMFIT}$	200	[0, 200]	20	-

the nominal and varied results, respectively [2]. The nominal cut, varied range, and step for each selection criterion are listed in Table I.

No strong trending behavior is observed for these tests, and a linear function fit is applied to the obtained distribution for each case. For the decay lengths of Λ and Ξ, the values of the linear fit at one standard deviation, 1.17 cm and 0.53 cm, which are larger than the nominal position 0 cm, are taken as the systematic uncertainties. If most of the systematic tests ω_i (more than 7) are larger or smaller than the nominal ones ω, asymmetric systematic uncertainties are assigned, otherwise the symmetric systematic uncertainties are assigned. The difference between the nominal value and the value from the linear fit are taken as one side of the systematic uncertainty. The other side of the systematic uncertainty is 0. For the invariant masses of Ξ, Λ, and neutron and the polar angle of Ξ, the values of the linear fit at one standard deviation of the kinematic variables both larger and smaller than the nominal position are calculated.

For each parameter, if two fit values are both larger or smaller than the nominal value, the largest deviation is taken as one side of systematic uncertainty; the other side of the systematic uncertainty is 0. If two fit values have opposite signs, they are taken as the asymmetric systematic uncertainties. For the χ^2 of kinematic fit, the differences between the nominal values and the values of the linear fit at 100 are taken as the systematic uncertainties.

The uncertainties associated with the combinatorial backgrounds are estimated by smearing the parameters ω of model within one standard deviation and varying it number within $\pm 1\sigma$ in the log-likelihood fit. The uncertainty of the number of combinatorial background is obtained from the fit of the missing mass distribution. The uncertainties associated with the resonant backgrounds are estimated by varying the number of each background within $\pm 1\sigma$ in the fit, individually. The largest changes of the fitted parameters are assigned as the systematic uncertainties. The number of resonant background is determined as

$$N_{\rm bkg} = N_{J/\psi} \times \Pi_i \mathcal{B}_i \times \varepsilon, \tag{5}$$

where $N_{J/\psi}$ is the total number of J/ψ events, $\mathcal{B}_i$ is the branching fraction of the ith decay mode, and ε is the detection efficiency. The detection efficiency is determined by using the exclusive MC sample. The uncertainty of the number of resonant background is determined via the error propagation formula,

$$\sigma_{N_{\rm bkg}} = N_{\rm bkg} \times \sqrt{\left(\frac{\sigma_{N_{J/\psi}}}{N_{J/\psi}}\right)^2 + \sum_i \left(\frac{\sigma_{\mathcal{B}_i}}{\mathcal{B}_i}\right)^2 + \left(\frac{\sigma_\varepsilon}{\varepsilon}\right)^2}. \tag{6}$$

The definition of 2D sideband region is shown in Fig. 2. The sideband region is defined as $0.016 < |M_{p\pi\pi} - M_{\Xi}^{\rm PDG}| < 0.038$ GeV/c^2 and $0.016 < |M_{n\pi^0\pi} - M_{\Xi}^{\rm PDG}| < 0.038$ GeV/c^2. The background yield in the signal region is estimated by the normalized sideband events $N_{\rm bkg,sideband} = 0.25 N_A$, where N_A is the number of events in the sideband region. The factor 0.25 is the area of the signal region to the sideband region. The uncertainties associated with non-resonant background are estimated by comparing the changes of the fitted parameters with and without subtracting this background.

To validate the fit procedure, 1000 pseudo-data-sets, in which each of them has the same statistics as data, are generated with the production and decay asymmetry parameters derived from in the nominal measurement. For each pseudo-data-set, the fit procedure is performed. The pull of the fit result of parameters is defined as

$$P(\omega) = \frac{\omega^{\rm pseudo} - \omega^{\rm input}}{\sigma^{\rm pseudo}}, \tag{7}$$

where $\omega^{\rm input}$ stands for the input value of the parameters, $\omega^{\rm pseudo}$ and $\sigma^{\rm pseudo}$ are the fit result and its uncertainty of the parameters for each pseudo-data-set. The pull distributions are fitted with Gaussian functions. The sum of the center value of difference and its uncertainty is taken as the systematic uncertainties associated with the fit procedure,

$$\sigma_{\rm io} = (|\mu_{\rm pull} + \sigma_\mu|) \times \sigma_{\rm stat.}, \tag{8}$$

where $\mu_{\rm pull}$ and σ_μ are the mean value and its statistical uncertainty of the pull distribution, respectively. The $\sigma_{\rm stat.}$ is the statistical uncertainty.

[1] M. Ablikim *et al.* (BESIII collaboration), Nature **606**, 64 (2022), arXiv:2105.11155 [hep-ex].
[2] O. Behnke, K. Kröninger, T. Schörner-Sadenius, and G. Schott, eds., *Data analysis in high energy physics: A practical guide to statistical methods* (Wiley-VCH, Weinheim, Germany, 2013).

TABLE II. The systematic uncertainties on the production and decay asymmetry parameters.

		$\alpha_{J/\psi}$	$\Delta\Phi_{J/\psi}$	α_{Ξ^-}	ϕ_{Ξ^-}	$\alpha_{\bar{\Xi}}$	$\phi_{\bar{\Xi}^-}$	α_0	α_+	α_-	$\bar{\alpha}_0$
Ξ charged decay	Λ Decay Length	$^{+0.0087}_{-0.0000}$	$^{+0.0000}_{-0.0156}$	$^{+0.0020}_{-0.0000}$	$^{+0.0007}_{-0.0007}$	$^{+0.0000}_{-0.0012}$	$^{+0.0000}_{-0.0091}$	$^{+0.0064}_{-0.0000}$	$^{+0.0008}_{-0.0008}$	$^{+0.0019}_{-0.0019}$	$^{+0.0001}_{-0.0001}$
	$\lvert M_{p\pi} - m_\Lambda\rvert$	$^{+0.0009}_{-0.0000}$	$^{+0.0009}_{-0.0012}$	$^{+0.0000}_{-0.0003}$	$^{+0.0005}_{-0.0002}$	$^{+0.0002}_{-0.0001}$	$^{+0.0002}_{-0.0008}$	$^{+0.0000}_{-0.0004}$	$^{+0.0016}_{-0.0000}$	$^{+0.0000}_{-0.0012}$	$^{+0.0000}_{-0.0012}$
	Ξ Decay Length	$^{+0.0054}_{-0.0000}$	$^{+0.0075}_{-0.0000}$	$^{+0.0000}_{-0.0023}$	$^{+0.0000}_{-0.0065}$	$^{+0.0000}_{-0.0022}$	$^{+0.0000}_{-0.0088}$	$^{+0.0012}_{-0.0012}$	$^{+0.0026}_{-0.0000}$	$^{+0.0024}_{-0.0024}$	$^{+0.0000}_{-0.0039}$
	$\lvert M_{p\pi\pi} - m_\Xi\rvert$	$^{+0.0039}_{-0.0000}$	$^{+0.0070}_{-0.0000}$	$^{+0.0006}_{-0.0009}$	$^{+0.0000}_{-0.0027}$	$^{+0.0000}_{-0.0020}$	$^{+0.0009}_{-0.0012}$	$^{+0.0000}_{-0.0016}$	$^{+0.0000}_{-0.0010}$	$^{+0.0000}_{-0.0020}$	$^{+0.0000}_{-0.0014}$
Ξ neutral decay	$\chi^2_{\rm kmf}$	$^{+0.0004}_{-0.0000}$	$^{+0.0030}_{-0.0000}$	$^{+0.0000}_{-0.0012}$	$^{+0.0000}_{-0.0000}$	$^{+0.0000}_{-0.0002}$	$^{+0.0011}_{-0.0000}$	$^{+0.0000}_{-0.0029}$	$^{+0.0000}_{-0.0027}$	$^{+0.0001}_{-0.0001}$	$^{+0.0000}_{-0.0023}$
	$\lvert\cos\theta_\Xi\rvert$	$^{+0.0017}_{-0.0000}$	$^{+0.0019}_{-0.0000}$	$^{+0.0000}_{-0.0002}$	$^{+0.0002}_{-0.0001}$	$^{+0.0003}_{-0.0000}$	$^{+0.0001}_{-0.0002}$	$^{+0.0000}_{-0.0007}$	$^{+0.0006}_{-0.0000}$	$^{+0.0000}_{-0.0016}$	$^{+0.0000}_{-0.0017}$
	$\lvert M_n - m_n\rvert$	$^{+0.0002}_{-0.0001}$	$^{+0.0026}_{-0.0074}$	$^{+0.0000}_{-0.0011}$	$^{+0.0023}_{-0.0024}$	$^{+0.0014}_{-0.0013}$	$^{+0.0000}_{-0.0014}$	$^{+0.0002}_{-0.0028}$	$^{+0.0012}_{-0.0026}$	$^{+0.0019}_{-0.0006}$	$^{+0.0016}_{-0.0022}$
	$\varepsilon_{\pi^\pm}$	0.0046	0.0070	0.0004	0.0012	0.0004	0.0010	0.0014	0.0015	0.0016	0.0013
	ε_{π^0}	0.0045	0.0158	0.0016	0.0013	0.0015	0.0014	0.0028	0.0023	0.0027	0.0029
Fitting procedure	I/O	0.0004	0.0011	0.0001	0.0008	0.0002	0.0006	0.0009	0.0003	0.0004	0.0003
Background	Combinatorial	0.0010	0.0033	0.00035	0.0010	0.0004	0.0012	0.0021	0.0014	0.0007	0.0015
	Resonant	0.0001	0.0002	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
	Non-resonant	0.0016	0.0040	0.0012	0.0022	0.0011	0.0001	0.0047	0.0016	0.0015	0.0045
	Total	$^{+0.0129}_{-0.0066}$	$^{+0.0210}_{-0.0250}$	$^{+0.0029}_{-0.0036}$	$^{+0.0042}_{-0.0081}$	$^{+0.0025}_{-0.0040}$	$^{+0.0028}_{-0.0130}$	$^{+0.0089}_{-0.0077}$	$^{+0.0049}_{-0.0053}$	$^{+0.0050}_{-0.0055}$	$^{+0.0060}_{-0.0080}$

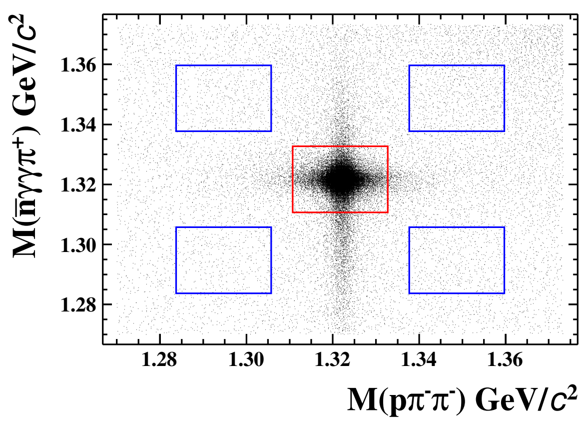

FIG. 2. The definition of the sideband in $M(p\pi^-\pi^-)$ versus $M(\bar{n}\gamma\gamma\pi^+)$ distribution. The red rectangle shows the signal region and the blue rectangles show the sideband regions.