Determination of the Σ^+ Timelike Electromagnetic Form Factors

Determination of the \(\Sigma^{2}\) Hindings of \$\frac{1}{2}\), X. C. Al²³. R. Alberti²⁶, A. Amoroso^{74,4,74C}, M. R. An⁶, Q. An^{71,25}, Y. Bu⁷, Q. Balima⁷, I. Balossino⁷⁴, Y. Pau⁷⁸, Y. Butoschaya^{1,4}, A. Amoroso^{74,4,74C}, M. R. An⁶, Q. An^{71,25}, Y. Butoschaya^{1,4}, D. Baloschaya^{1,4}, A. Butoschaya^{1,4}, D. Baloschaya^{1,4}, A. Butoschaya^{1,4}, A. Butoschaya^{1,4},

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Based on data samples collected with the BESIII detector at the BEPCII collider, the process $e^+e^- \to \Sigma^+\bar{\Sigma}^-$ is studied at center-of-mass energies $\sqrt{s}=2.3960,\ 2.6454,\$ and 2.9000 GeV. Using a fully differential angular description of the final state particles, both the relative magnitude and phase information of the Σ^+ electromagnetic form factors in the timelike region are extracted. The relative phase between the electric and magnetic form factors is determined to be $\sin\Delta\Phi=-0.67\pm0.29$ (stat) ±0.18 (syst) at $\sqrt{s}=2.3960$ GeV, $\Delta\Phi=55^\circ\pm19^\circ$ (stat) $\pm14^\circ$ (syst) at $\sqrt{s}=2.6454$ GeV, and $78^\circ\pm22^\circ$ (stat) $\pm9^\circ$ (syst) at $\sqrt{s}=2.9000$ GeV. For the first time, the phase of the hyperon electromagnetic form factors is explored in a wide range of four-momentum transfer. The evolution of the phase along with four-momentum transfer is an important input for

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Hyperons have a very similar quark composition to that of nucleons, except that one or more of the up or down quarks is replaced by strange quarks. Together with the nucleons, they form a spin-1/2 baryon octet under SU(3) symmetry [1, 2]. As one of the fundamental physics observables of the baryons, electromagnetic form factors (EMFFs) provide a valuable perspective for understanding baryon structure [3–5] by probing internal charge and current distributions [6–9]. The EMFFs are analytic functions of the four-momentum transfer squared (q^2) , and they can be divided into spacelike $(q^2 < 0)$ and timelike $(q^2 > 0)$ regions [10, 11]. The former are often measured using electron-baryon elastic scattering experiments, while the latter use electronpositron annihilation into baryon antibaryon pairs or the reverse reaction. However, owing to the difficulties in producing stable and high-quality hyperon beams, it is challenging to study the EMFFs of hyperons in the spacelike region. Currently, only a few experiments have measured the EMFFs of hyperon in the spacelike region by elastic scattering of hyperon beam off atomic electrons, and the range of $|q^2|$ for exploring EMFFs is limited due to kinematic constraints [12]. On the other hand, hyperons can be readily produced in electron-positron annihilation above their pair production thresholds. Therefore, the hyperon EMFFs are usually studied in the timelike region via $e^+e^- \rightarrow \gamma^* \rightarrow Y\bar{Y}$, where Y represents a hyperon with spin 1/2, and these can be related to the spacelike region via dispersion relations [13].

A large number of measurements are available in the literature for the effective form factors (G_{eff}) of SU(3) baryons, which are extracted from production cross sections for $e^+e^- \rightarrow \gamma^* \rightarrow B\bar{B}$ under the assumption of the electric form factor ($|G_E|$) equal to magnetic form factor ($|G_M|$) [14–29]. Previous measurements also exist for the modulus of EMFF ratios $|G_E/G_M|$, which are obtained by analyzing one-dimensional angular distributions [19, 20, 24, 26]. However, according to the optical theorem, the form factors at the lowest order for the spacelike region are real due to the Hermiticity of the electromagnetic Hamiltonian, while in the timelike region they are complex [30, 31]. Thus, a complete knowledge of EMFFs includes the relative phase $\Delta\Phi$ between electric and magnetic form factors, G_E and G_M . Since a nonzero $\Delta\Phi$ ensures a transverse polarization for the produced baryons [32], $\Delta\Phi$ can be extracted from the polarization. The transverse hyperon polarization is selfanalyzed in their weak decays, while the polarization of nucleons needs additional dedicated devices to be measured.

The only previous determination of the $|G_E/G_M|$ and $\Delta\Phi$ for a baryon was performed at BESIII using the exclusive process $e^+e^-\to\Lambda\bar{\Lambda}$ at $\sqrt{s}=2.396$ GeV. The

relative phase of the Λ EMFFs was extracted by fitting the angular distributions [22]. Many theoretical activities [33–38] arose after this measurement. In Ref. [34], the EMFF ratio and their relative phase are also predicted for Σ hyperons, with a different dependence on the center-of-mass (c.m.) energy from the Λ case, reflecting complex dynamics. Though the G_{eff} and $|G_E/G_M|$ of the Σ hyperons have been measured by various experiments [26, 29, 39, 40], the extraction of $\Delta\Phi$ for Σ is still unavailable. Thus, measurements of Σ EMFFs can provide deeper insight into $\bar{Y}Y$ dynamics. Moreover, analyticity implies that the EMFFs tend to be real at large fourmomentum transfer squared in the timelike region [35]. Since $\sin \Delta \Phi_{\Lambda}$ has previously been found to be significantly different from zero [22], this indicates that the asymptotic threshold has not yet been reached for the q^2 so far studied. The phase measurement in a broader four-momentum transfer squared range is thus important to ascertain the asymptotic behavior of the hyperons and to investigate its dynamical mechanisms [35].

In this letter, we present a study of $e^+e^- \rightarrow \Sigma^+\bar{\Sigma}^-$ at three energy points, $\sqrt{s}=2.3960$, 2.6454, and 2.9000 GeV, with a total integrated luminosity of 239.84 pb⁻¹ collected with the Beijing Spectrometer (BESIII) at the Beijing Electron Positron Collider (BEPCII). The first energy point, 2.3960 GeV, is in close proximity to the production threshold for Σ^+ hyperon pairs $(2M_{\Sigma^+}=2.3788~{\rm GeV})$, where M_{Σ^+} represents the nominal mass of the Σ^+ [41]. Here 2.6454 GeV is a combined dataset of 2.6444 GeV and 2.6464 GeV. The $|G_E/G_M|$ ratio and the relative phase $\Delta\Phi$ are determined using a fully differential angular expression. The formalism is described in Ref. [42].

The description of the design and performance of the BESIII detector can be found in Ref. [43]. The Monte Carlo (MC) samples used to optimize event selection criteria are generated using a GEANT4-based [44] simulation software package. The CONEXC [45] generator is used to generate signal MC samples and includes higher order processes with one radiative photon in the final state. The input cross section of line shape for $e^+e^- \to \Sigma^+\bar{\Sigma}^-$ is obtained from Ref. [26]. The phase space (PHSP) model in EVTGEN [46, 47] is used to generate six million MC events to calculate the normalization factors in the multidimensional fits. The inclusive MC sample is generated with a HYBRID generator [48] for background analysis at each energy point.

Two different reconstruction methods are used to select $\Sigma^+\bar{\Sigma}^-$ pairs, according to the c.m. energy. At $\sqrt{s}=2.3960$ GeV, due to the low tracking efficiency for low-momentum tracks, a single-tag method is used to select the process $e^+e^- \to \Sigma^+\bar{\Sigma}^- \to \bar{p}\pi^0 + X$, where X denotes

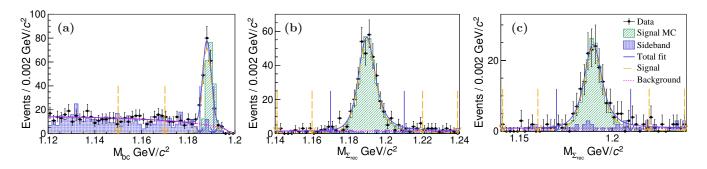


FIG. 1. The distributions of (a) M_{bc} at 2.3960 GeV, (b) $M_{\Sigma_{\text{rec}}}$ at 2.6454 GeV, and (c) $M_{\Sigma_{\text{rec}}}$ at 2.9000 GeV. The black dots with error bars are data. The histograms filled with green diagonal lines represent the signal MC samples, and the histograms filled with purple shading represent the backgrounds estimated by the sidebands. The purple solid lines are the total fit result. The yellow dash-dotted and magenta dotted lines are the signal and background shapes, respectively. The signal and background regions used for further angular analysis are indicated by purple solid-line arrows and yellow dashed-line arrows, respectively.

inclusive decays of the Σ^+ . At higher c.m. energies, both proton and antiproton are selected in the process $e^+e^- \to \Sigma^+\bar{\Sigma}^-$. To improve the detection efficiency, only one π^0 is reconstructed by two photons.

Charged tracks are reconstructed in the main drift chamber (MDC) as in Ref. [49]. Combined information of the specific ionization energy loss ($\mathrm{d}E/\mathrm{d}x$) in the MDC and the time of flight (TOF) is used to calculate particle identification (PID) probabilities for the pion, kaon, and proton hypotheses. The particle type with the highest probability is assigned for the track. At \sqrt{s} = 2.3960 GeV, only the $\mathrm{d}E/\mathrm{d}x$ is used for PID since the charged tracks cannot reach the TOF detector due to low momenta. Photon candidates are reconstructed from clusters of energy deposited in the electromagnetic calorimeter (EMC) as in Ref. [49]. To reject showers from charged tracks, the angle between the shower direction and the track extrapolated to the EMC must be greater than 20 degrees in the single-tag reconstruction.

In the single-tag reconstruction at $\sqrt{s} = 2.3960$ GeV, at least one good charged track, identified as an antiproton, is required. At least two good photons are required in each event. The $\bar{\Sigma}^-$ candidates are selected by looping over all possible $\bar{p}\gamma\gamma$ combinations. Two variables, ΔE and $M_{\rm bc}$, which reflect energy and momentum conservation, are used to select $\bar{\Sigma}^-$ candidates. Here $\Delta E \equiv E - E_{\rm beam}$ is the energy difference, where E is the total measured energy of the $\bar{\Sigma}^-$ and E_{beam} is the beam energy, and $M_{\rm bc} \equiv \sqrt{E_{\rm beam}^2/c^4 - P_{\Sigma^-}^2/c^2}$ is the beamconstrained mass and P is the magnitude of measured total momentum of the $\bar{\Sigma}^-$ candidate. Further selection criteria on the $\gamma\gamma$ invariant mass $(M_{\gamma\gamma})$ and ΔE , 0.126 $< M_{\gamma\gamma} < 0.139 \text{ GeV}/c^2 \text{ and } -0.013 < \Delta E < 0.005 \text{ GeV},$ are applied. After the above selections, the distribution of $M_{\rm bc}$ at $\sqrt{s}=2.3960$ GeV is shown in Fig. 1(a).

In the reconstruction with one missing π^0 at \sqrt{s}

2.6454 and 2.9000 GeV, a good event must have at least two good charged tracks identified to be one proton and one antiproton. At least two good photons are selected, and π^0 candidates are reconstructed from pairs of photons as in Ref. [49]. At least one good π^0 candidate is required. To further remove potential background and improve the mass resolution, a two-constraint (2C) kinematic fit under the $e^+e^- \to p\bar{p}\pi^0\pi^0$ hypothesis is performed. The fit requires total energy-momentum conservation, and the $\gamma\gamma$ invariant mass is constrained to the nominal π^0 mass, while the other π^0_{miss} is treated as a missing particle with free three-momentum. For events with more than one $\pi^0_{\gamma\gamma}$ candidate, by looping over the $\pi^0_{\gamma\gamma}$ candidates in the kinematic fit, the best $\pi^0_{\gamma\gamma}$ is selected with the minimum χ^2_{2C} which is further required to be less than 15. The $\pi^0_{\gamma\gamma}$ is then paired with either the proton or antiproton depending on which combination gives the minimum $|M_{(p\pi^0_{\gamma\gamma}/\bar{p}\pi^0_{\gamma\gamma})} - M_{\Sigma^+}|$ and the best combination is denoted as $\Sigma_{\rm tag}$. The signal region in the invariant mass of $\Sigma_{\rm tag}$ is chosen as $1.175 < M_{\Sigma_{\rm tag}} <$ 1.200 GeV/ c^2 . The recoiling mass spectrum against Σ_{tag} , $M_{\Sigma_{\rm rec}}$, after the previously described selections, is shown in Figs. 1(b)(c).

Both the inclusive MC sample and the data sideband are used to study the potential background events. The main background, found in the inclusive MC sample, includes processes from e^+e^- annihilation events with the same final states as the signal, with an additional photon, and with intermediate states like $\Lambda,~\Sigma$ and Δ baryons. The background in the inclusive MC sample is smooth. The sideband regions are defined as -0.040 $<\Delta E<$ -0.031 GeV and 0.028 $<\Delta E<$ 0.037 GeV for $\sqrt{s}=2.3960$ GeV, and 1.135 $< M_{\Sigma_{\rm tag}}<$ 1.150 GeV/ c^2 and 1.225 $< M_{\Sigma_{\rm tag}}<$ 1.240 GeV/ c^2 for other energy points. As shown in Fig. 1, the backgrounds in the sideband regions in both $M_{\rm bc}$ and $M_{\Sigma_{\rm rec}}$ are smooth, so no further selection is applied.

To extract the signal yield, a simultaneus fit of $M_{\rm bc}$ and $M_{\Sigma_{\rm rec}}$ is applied. In the fit, the probability density functions (PDF) of signal events are described by MC-simulated shapes, extracted from the signal MC sample, convolved with a Gaussian function. The PDFs of background events are described by an Argus function [50] at $\sqrt{s}=2.3960$ GeV and a linear function at $\sqrt{s}=2.6454$ and 2.9000 GeV. The best fit results are shown in Fig. 1. The numbers of signal events are 207 ± 17 , 364 ± 21 , and 168 ± 15 at 2.3960, 2.6454, and 2.9000 GeV, respectively, and the corresponding MC selection efficiencies are 11.33%, 34.39%, and 33.58%, respectively. Furthermore, a cross-check of the Born cross section with the previous

BESIII results [26] is performed to ensure the reliability of the selection method. To ensure a pure sample for the further angular distribution analysis, tighter selections are applied on both $M_{\rm bc}$ and $M_{\Sigma_{\rm rec}}$, requiring $1.185 < M_{\rm bc} < 1.191~{\rm GeV}/c^2$ and $1.170 < M_{\Sigma_{\rm rec}} < 1.210~{\rm GeV}/c^2$ as indicated with arrows in Fig. 1. The background fractions are 12.7%, 7.7%, and 10.2% at 2.3960, 2.6454, and $2.9000~{\rm GeV}$, respectively.

Following Refs. [42, 51], the joint angular distribution $W(\xi)$ of $e^+e^- \to \Sigma^+ (\to p\pi^0)\bar{\Sigma}^- (\to \bar{p}\pi^0)$ can be expressed as

$$\mathcal{W}(\xi) \propto \mathcal{F}_0(\xi) + \alpha \mathcal{F}_5(\xi) + \alpha_1 \alpha_2 [\mathcal{F}_1(\xi) + \sqrt{1 - \alpha^2} \cos(\Delta \Phi) \mathcal{F}_2(\xi) + \alpha \mathcal{F}_6(\xi)] + \sqrt{1 - \alpha^2} \sin(\Delta \Phi) [-\alpha_1 \mathcal{F}_3(\xi) + \alpha_2 \mathcal{F}_4(\xi)],$$
(1)

where ξ is a five-dimensional vector, $\xi = (\theta_{\Sigma^+}, \theta_1, \theta_2, \phi_1, \phi_2)$; θ_{Σ^+} is the angle between the Σ^+ hyperon and positron beam; θ_1 (θ_2) and ϕ_1 (ϕ_2) are the polar and azimuthal angles of the proton (antiproton) with respect to the Σ^+ and $\bar{\Sigma}^-$ helicity frame, respectively; and α_1 and α_2 are the decay asymmetry parameters of the Σ^+ and $\bar{\Sigma}^-$. The set of angular distribution functions $\mathcal{F}_i(\xi)$ (i=0,1,...,6) are obtained in Ref. [42]. Owing to limited statistics, we assume CP to be conserved and $\alpha_1 = -\alpha_2 = -0.980$ [41]. The α is the angular distribution parameter describing the ratio of the two helicity amplitudes in $e^+e^- \to \Sigma^+\bar{\Sigma}^-$ and $\Delta\Phi$ is their relative phase. The α relates to $|G_E/G_M|$ via [52]

$$|G_E/G_M| = \sqrt{\frac{s(1-\alpha)}{4M_{\Sigma^+}^2(1+\alpha)}}.$$
 (2)

Since only one hyperon is reconstructed at $\sqrt{s} = 2.3960 \,\text{GeV}$, θ_1 and ϕ_1 are integrated at this energy point, and the angular distribution becomes

$$W(\xi) \propto \mathcal{F}_0(\xi) + \alpha \mathcal{F}_5(\xi) + \sqrt{1 - \alpha^2} \sin(\Delta \Phi) \alpha_2 \mathcal{F}_4(\xi).$$
(3)

The parameters α and $\Delta\Phi$ can be extracted by a multidimensional maximum likelihood fit to data. The joint likelihood function for observing N events in the data sample is

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(\xi_i; \alpha, \Delta\Phi) = \prod_{i=1}^{N} \mathcal{CW}(\xi_i; \alpha, \Delta\Phi) \epsilon(\xi_i), \quad (4)$$

where $\mathcal{P}(\xi_i; \alpha, \Delta\Phi)$ is the probability density function of ξ_i , i is the corresponding event index, and $\epsilon(\xi_i)$ is the

efficiency of each event. The normalization factor \mathcal{C} is given by $\mathcal{C}^{-1} = \int \mathcal{W}(\xi; \alpha, \Delta \Phi) \epsilon(\xi) d\xi$ and evaluated by the PHSP signal MC sample. The parameters α and $\Delta \Phi$ are extracted by minimizing the likelihood function

$$S = -\ln \mathcal{L}_{\text{Data}} + \ln \mathcal{L}_{\text{Bkg}}, \tag{5}$$

where $\mathcal{L}_{\mathrm{Data}}$ is the corresponding likelihood value of data and $\mathcal{L}_{\mathrm{Bkg}}$ represents the background, estimated with data events in the background region indicated in Fig. 1 and normalized to the signal region. The best fit results for α , $\Delta\Phi$, and (or) $\sin(\Delta\Phi)$ are summarized in Table I, where only $\sin(\Delta\Phi)$ can be extracted at 2.3960 GeV due to the application of a single-tag method and the lack of sufficient angular distribution information.

Furthermore, the nonzero $\Delta\Phi$ will lead to a dependence of the polarization on the scattering angle of the Σ^+ [32, 51]:

$$P_y = -\frac{\sqrt{1 - \alpha^2 \sin \theta_{\Sigma^+} \cos \theta_{\Sigma^+}}}{1 + \alpha \cos^2 \theta_{\Sigma^+}} \sin(\Delta \Phi). \tag{6}$$

Experimentally, the P_y is determined by

$$P_y = \frac{m}{N} \sum_{i=1}^{N_k} \frac{(3+\alpha)(n_{1,y}^i + n_{2,y}^i)}{(\alpha_1 - \alpha_2)(1 + \alpha \cos^2 \theta_{\Sigma^+}^i)},\tag{7}$$

where N is the total number of events in the dataset and m=8 is the number of bins in $\cos\theta_{\Sigma^+}$; N_k denotes the number of events in the k-th $\cos\theta_{\Sigma^+}$ bin; and $n_{1,y}$ $(n_{2,y})$ is the projection of a proton (antiproton) perpendicular to the scattering plane in the rest frame of Σ^+ $(\bar{\Sigma}^-)$. To test the goodness of the fit results, the signal MC sample is generated using Eqs. (1) and (3) and inputting the measured parameters from the data.

\sqrt{s} (GeV)	2.3960	2.6454	2.9000		
α	$-0.47 \pm 0.18 \pm 0.09$	$0.41 \pm 0.12 \pm 0.06$	$0.35 \pm 0.17 \pm 0.15$		
$\Delta\Phi$ (°)	$-42 \pm 22 \pm 14 \ (-138 \pm 22 \pm 14)$	$55 \pm 19 \pm 14$	$78 \pm 22 \pm 9$		
$\sin \Delta \Phi$	$-0.67 \pm 0.29 \pm 0.18$				
$ G_E/G_M $	$1.69 \pm 0.38 \pm 0.20$	$0.72 \pm 0.11 \pm 0.06$	$0.85 \pm 0.16 \pm 0.15$		

TABLE I. Fit results for α , $\Delta\Phi$ (°), $\sin(\Delta\Phi)$, and $|G_E/G_M|$ at each energy point.

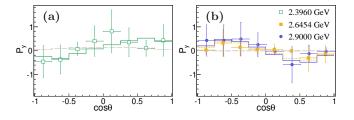


FIG. 2. The polarization P_y as a function of the scattering angle at 2.3960 GeV (a) and 2.6454 and 2.9000 GeV (b). The open squares, solid squares, and dots are data. The histograms with solid lines (dotted line at 2.6454 GeV) are signal MC samples based on the fit results, and the histograms with the gray dashed lines are the PHSP signal MC samples at each energy point.

The angular-dependent transverse polarization of Σ is obtained as shown in Fig. 2.

The sources of systematic uncertainties are summarized in Table II. For the first four sources in Table II. uncertainties are caused by the event selection and are evaluated by varying the selection criteria. For the fifth to eighth sources in Table II, the uncertainties from the fit procedure are estimated with alternative fits by varying the signal region, changing the sideband selections, and varying the fixed decay parameters (α_1, α_2) by $\pm 1\sigma$, individually. The maximum difference with the nominal value is taken as the uncertainty. To estimate the systematic uncertainty of the fit method, 500 sets of signal MC samples with the parameters from Table I are generated and fitted to obtain the distribution of the output parameters, and the difference between the input and averaged output values is assigned as the systematic uncertainty. Some inconsistencies between data and MC simulation are observed in the $M_{\rm bc}$ distribution, as shown in Fig. 1(a). To estimate their effect on the final results, the measurement of beam energy and the calibration of the $\bar{\Sigma}^-$ momentum are investigated. For the E_{beam} calibration, we generate three MC samples with different c.m. energies, defined around 2.3960 GeV in steps of 1 MeV, that is, 2.3950, 2.3970, and 2.3980 GeV, and choose the one that gives the best description of the data in the fit procedure. For the $\bar{\Sigma}^-$ momentum calibration, ten MC samples are generated, with different scale factors for the three-momentum of antiproton in

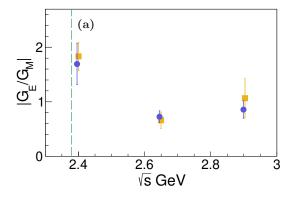
each sample. The scale factors are defined in steps of 0.001 from 1.040 to 1.049, and we choose the one giving the best description of the data in the fit procedure. The differences between the updated and nominal results are taken as the systematic uncertainties. In Table II, the individual uncertainties are assumed to be uncorrelated and are added in quadrature.

TABLE II. The systematic uncertainties for α , $\Delta\Phi$ (°), and $\sin(\Delta\Phi)$ at each energy point (in GeV).

Source	2.3960		2.6454		2.9000	
Source	α	$\sin(\Delta\Phi)$	α	$\Delta\Phi$	α	$\Delta\Phi$
ΔE cut	0.03	0.02				
$\gamma\gamma$ mass window	0.04	0.06				
χ^2_{2C} cut			0.04	5	0.08	5
$\Sigma_{\rm tag}$ mass window			0.00	3	0.06	2
Signal region	0.05	0.16	0.04	9	0.05	4
Sideband region	0.02	0.06	0.02	9	0.09	5
$lpha_1$			0.01	0	0.00	1
$lpha_2$	0.00	0.01	0.01	0	0.00	1
Fit method	0.00	0.01	0.02	2	0.03	2
E_{beam} calibration	0.03	0.00				
Momentum calibration	0.04	0.01				
Total	0.09	0.18	0.06	14	0.15	9

In summary, the process $e^+e^- \to \Sigma^+\bar{\Sigma}^-$ is studied at 2.3960, 2.6454, and 2.9000 GeV. Using a joint angular distribution analysis, the final results for $|G_E/G_M|$, the relative phase $\Delta\Phi$, and $\sin\Delta\Phi$ are summarized in Table I and plotted in Fig. 3, where the relative phase of the Σ^+ hyperon is measured for the first time in a wide four-momentum transfer range.

The precision of $|G_E/G_M|$ is improved compared with the previous measurement [26] at 2.6454 and 2.9000 GeV. Since only the sine value of $\Delta\Phi$ can be extracted at 2.3960 GeV, the two solutions are plotted as shown in Fig. 3(b), and there is a significant discrepancy between our experimental result for $\Delta\Phi$ and the theoretical predictions from the $\bar{Y}Y$ potential model [34]. On the other hand, in Fig. 3(b), $\Delta\Phi$ is less than zero at 2.3960 GeV and greater than zero at 2.6454 GeV, which implies that there may be at least one $\Delta\Phi=0^\circ$ between these two energy points. Such an evolution will be an important input for understanding its asymptotic behavior [35] and the dynamics of baryons. Moreover, the fact that the



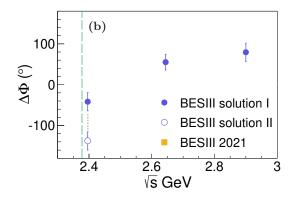


FIG. 3. Results for $|G_E/G_M|$ (a) and the relative phase $\Delta\Phi$ (b) from this work (purple dots). The yellow squares in (a) denote the previous results from BESIII [26]. The open circle in (b) represents the second solution of $\Delta\Phi$ at 2.3960 GeV. The vertical dashed lines indicate the production threshold for $e^+e^- \to \Sigma^+\bar{\Sigma}^-$, where $|G_E/G_M| = 1$ and $\Delta\Phi = 0^\circ$ by definition.

relative phase is still increasing at 2.9000 GeV indicates that the asymptotic threshold has not yet been reached.

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