Supplementary Material to "First Constraints on Light Axions from the Binary Neutron Star Gravitational Wave Event GW170817"

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In the presence of the axions, the leading corrections to the GR binding energy and radiation power are given by

$$V_a = -\frac{Q_1 Q_2}{4\pi}\frac{e^{-m_a r}}{r}\left(1 - 16p\,GM\frac{e^{-m_a r}}{r}\right) \tag{1}$$

and

$$P_a = \frac{\left(\bar{Q}_1 M_2 - \bar{Q}_2 M_1\right)^2}{12\pi} r^2 \Omega^4 \left(1 - \frac{m_a^2}{\Omega^2}\right)^{3/2} \tag{2}$$

with

$$p \equiv \frac{1}{M}\left(\frac{Q_1}{Q_2}p_2 + \frac{Q_2}{Q_1}p_1\right) \tag{3}$$

and

$$\bar{Q}_{1,2} \equiv Q_{1,2}\left(1 - 8Gp_{2,1}\frac{e^{-m_a r}}{r}\right). \tag{4}$$

Here Ω is the orbital frequency, r denotes the separation between the two NSs of masses M_1 and M_2, and $M \equiv M_1 + M_2$ is the total mass. Comparing to Eqs. (4) and (6), we also include terms proportional to $p_{1,2}$ that could in principle arise due to the present of a generic scalar field. The value of $p_{1,2}$ is model dependent. For axions, these terms characterize the induced charge effect, and $p = (R_1 + R_2)/16GM$ when $m_a = 0$. Thus, we expect that the induced charge effect could become important at the late stage of inspirals for axions with small masses. However, taking into account this effect requires further studies on how $p_{1,2}$ relates to the parameters of the neutron stars and the axion field. Therefore, we neglected the induced charge effect in our analysis.

In TaylorF2 waveform temple, the phase $\Psi(f)$ in Eq. (7) can be calculated by using the stationary phase approximation,

$$\Psi(f) = 2\pi f t - \phi - \frac{\pi}{4} \tag{5}$$

with

$$t(f) = t_c - \int_{f_c}^{f} \frac{1}{P}\left(\frac{dE}{df'}\right) df' \tag{6}$$

and

$$\phi(f) = \phi_c - \int_{f_c}^{f} \frac{2\pi f'}{P}\left(\frac{dE}{df'}\right) df', \tag{7}$$

where E and P are the binding energy and radiation power of the binary system respectively. Note that E and P are functions of r and Ω, which are related by the modified Kepler's law $r(\Omega)$, and the GW frequency relates to the orbital frequency through $\Omega = \pi f$. Given the fact that the axion charge $Q_{1,2}$ must be small, we neglect terms of $\mathcal{O}(Q_{1,2}^4)$, $\mathcal{O}(Q_{1,2}^2 v^2)$ and higher when we evaluate Eqs. (6) and (7). In this case, the phase $\Psi(f)$ is given by Eq. (8) with

$$\Psi_a = \Psi_a^E + \begin{cases} \Psi_a^{P_>} & x > \alpha \\ \Psi_a^{P_<} & x \le \alpha \end{cases}, \tag{8}$$

where we have defined

$$\Psi_a^E = \frac{5}{64}\frac{\gamma_a e^{-\frac{\alpha}{x^{2/3}}}}{\eta x^{5/3}}\left[-4 - \frac{32x^{2/3}}{\alpha} - \frac{138x^{4/3}}{\alpha^2} - \frac{360x^2}{\alpha^3} + \frac{360x^{8/3}\left(e^{\frac{\alpha}{x^{2/3}}} - 1\right)}{\alpha^4} - \frac{21\sqrt{\pi}x^{5/3}e^{\frac{\alpha}{x^{2/3}}}\mathrm{Erf}\left(\frac{\alpha^{1/2}}{x^{1/3}}\right)}{\alpha^{5/2}}\right], \tag{9}$$

$$\Psi_a^{P>} = \frac{5}{254951424}\frac{\delta q^2}{\eta x^{16/3}}\Bigg[-\sqrt{x^2-\alpha^2}\left(-822640\alpha^2+\frac{227089x^6}{\alpha^4}+\frac{261342x^4}{\alpha^2}+671304x^2\right) + \frac{140049x^7}{\alpha^4}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\frac{\alpha^2}{x^2}\right)+320x\left(1183\alpha^2+\frac{512x^6}{\alpha^4}-\frac{684x^4}{\alpha^2}-741x^2\right)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\frac{\alpha^2}{x^2}\right) + 960x\left(-1183\alpha^2-\frac{80x^6}{\alpha^4}+\frac{684x^4}{\alpha^2}+741x^2\right)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\frac{\alpha^2}{x^2}\right)\Bigg]\,, \tag{10}$$

$$\Psi_a^{P<} = \frac{25\sqrt{\pi}}{1536}\frac{\delta q^2}{\alpha^{10/3}\eta}\frac{\Gamma\left(\frac{5}{3}\right)\Gamma\left(\frac{11}{3}\right)x-\Gamma\left(\frac{7}{6}\right)\Gamma\left(\frac{25}{6}\right)\alpha}{\Gamma\left(\frac{11}{3}\right)\Gamma\left(\frac{25}{6}\right)}\,. \tag{11}$$

Here Erf is the Gauss error function, ${}_2F_1$ is the hypergeometric function, and

$$\alpha \equiv GMm_a, \quad \eta \equiv \frac{M_1M_2}{M^2}, \quad x \equiv \pi GMf\,,$$
$$\delta q \equiv \frac{1}{4\sqrt{2\pi G}}\left(\frac{Q_1}{M_1}-\frac{Q_2}{M_2}\right). \tag{12}$$

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