

## Low-energy $J/\psi$ -Hadron Interactions from Quenched Lattice QCD

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The  $J/\psi$ -hadron interaction is a key ingredient in analyzing the  $J/\psi$  suppression in hot hadronic matter as well as the propagation of  $J/\psi$  in nuclei. As a first step to clarify the  $J/\psi$ -hadron interactions at low energies, we have calculated  $J/\psi$ - $\pi$ ,  $J/\psi$ - $\rho$  and  $J/\psi$ -nucleon scattering lengths by the quenched lattice QCD simulations with Wilson fermions for  $\beta = 6.2$  on  $24^3 \times 48$  and  $32^3 \times 48$  lattices. Using the Lüscher's method to extract the scattering length from the simulations in a finite box, we find an attractive interaction in the S-wave channel for all three systems: Among others, the  $J/\psi$ -nucleon interaction is most attractive. Possibility of the  $J/\psi$ -nucleon bound state is also discussed.

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## 1. Introduction

A possible signature of the formation of the quark-gluon plasma (QGP) in relativistic heavy ion collisions is the suppression of  $J/\psi$  due to color Debye screening [1]. In the  $Pb$ - $Pb$  collisions at CERN-SPS, the  $J/\psi$  suppression beyond the normal nuclear absorption has been discovered [2]. However, the data may be described either by the color Debye screening due to deconfined quarks and gluons or by absorption/dissociation due to comoving light hadrons. Recent BNL-RHIC data show a tendency that the  $J/\psi$  suppression is almost independent of collision energies between 62 GeV and 200 GeV. The magnitude of the suppression is less than those predicted from color Debye screening or from the absorption by comovers, which may be understood by the recombination of  $c$  and  $\bar{c}$  [3].

To understand the mechanism of  $J/\psi$  suppression in those experiments, we need more precise knowledge on the  $J/\psi$ -hadron interactions. In this report, we show our recent results of  $J/\psi$ -hadron scattering lengths calculated in quenched lattice QCD simulations. (See [4, 5, 6, 7, 8] and references therein for other approaches.)

## 2. Formulation

Suppose we have two hadrons in a finite box. The effect of their interaction appears as an energy shift  $\Delta E$  relative to the non-interacting case.  $\Delta E$  may be extracted from the correlator ratio  $R(t)$  for large  $t$ ;

$$R(t) = \frac{G_{J/\psi-H}(t)}{G_{J/\psi}(t)G_H(t)} \sim e^{-(E-m_{J/\psi}-m_H)t} = e^{-\Delta E t}, \quad (2.1)$$

where  $G_{J/\psi-H}(t)$  and  $G_H(t)$  are the  $J/\psi$ -hadron four-point function and the hadron two-point function, respectively.

In our calculation, we consider three scattering processes,  $J/\psi$ - $\pi$ ,  $J/\psi$ - $\rho$  and  $J/\psi$ - $N$  (nucleon), which are most important for  $J/\psi$  absorption by comoving hadrons in relativistic heavy ion collisions. Since each hadron has spins, we need to make spin projection to good total spin states of  $J/\psi$ -hadron two body system. For example, the  $J/\psi$ - $N$  case reads

$$G_{J/\psi-N}(t) = G_{1/2}(t)\hat{P}^{1/2} + G_{3/2}(t)\hat{P}^{3/2}, \quad (2.2)$$

where  $G_{1/2(3/2)}$  denotes the four-point function with a good spin quantum number ( $J = 1/2$  or  $3/2$ ) and  $\hat{P}^{1/2(3/2)}$  is the spin projection operator [9].

The Lüscher's formula tells us a relation between  $\Delta E$  and the scattering observables such as the scattering length and the scattering phase shift. For the S-wave scattering phase shift, it reads [10]

$$\tan \delta_0(q) = \frac{\pi^{3/2} \sqrt{q}}{Z_{00}(1, q)}, \quad (2.3)$$

with sign convention as which negative phase shift corresponds to repulsion. Here  $Z_{00}$  is a generalized zeta function defined by

$$Z_{00}(s, q) = \frac{1}{\sqrt{4\pi}} \sum_{\mathbf{n}} \frac{1}{(n^2 - q)^s}, \quad q = \left(\frac{pL}{2\pi}\right)^2, \quad (2.4)$$

$p$  and  $L$  are the relative momentum of the two hadrons and the spatial size of the box, respectively. Here it is assumed that the interaction range is finite. Outside the interaction range, the total energy of the system is related to the relative momentum  $p$  as

$$\sqrt{m_{J/\psi}^2 + p^2} + \sqrt{m_H^2 + p^2} = E. \quad (2.5)$$

Here positive (negative)  $p^2$  corresponds to the repulsion (attraction). If there are no interactions between the hadrons,  $p$  takes discrete momentum in the finite box as  $p^2 = (2\pi/L)^2 \cdot n$  ( $n = 0, 1, 2, \dots$ ). If there are interactions,  $p$  receives an extra contribution  $p_{\text{int}}$  and then a momentum squared divided by  $(2\pi/L)^2$  is no longer an integer.

The S-wave scattering length is defined as  $a_0 \equiv \lim_{p \rightarrow 0} \tan(p)/p$  and is related to the zeta function through Eq.(2.3) as

$$a_0 = \frac{L\sqrt{\pi}}{2Z_{00}(1, q)} \Big|_{n=0}, \quad (2.6)$$

where the scattering length is assigned to be negative (positive) for repulsion (weak attraction). The hadron scattering lengths based on the formulas Eqs.(2.3) and (2.6) has been extensively studied for  $\pi$ - $\pi$  and  $N$ - $N$  systems in Refs. [11, 12].

It is important here to discuss the asymptotic behavior of Eq.(2.6) for large  $L$  for the purpose of analyzing the system with attractive interactions. The large  $L$  expansion of the right hand side of Eq.(2.6) at  $q \sim 0$  leads to [10]

$$\Delta E = -\frac{2\pi a_0}{M_{\text{res}} L^3} \left( 1 + c_1 \left( \frac{a_0}{L} \right) + c_2 \left( \frac{a_0}{L} \right)^2 \right) + O(L^{-6}), \quad (2.7)$$

with  $c_1 = -2.837297$  and  $c_2 = 6.375183$ . Let us try to solve Eq.(2.7) in terms of  $a_0$  for given  $\Delta E$ . In the case that  $\Delta E > 0$ , both the expansion up to  $O(L^{-4})$  and that up to  $O(L^{-5})$  always have real and negative solutions. On the other hand, in the case that  $\Delta E < 0$ , the expansion up to  $O(L^{-4})$  gives no real solution for

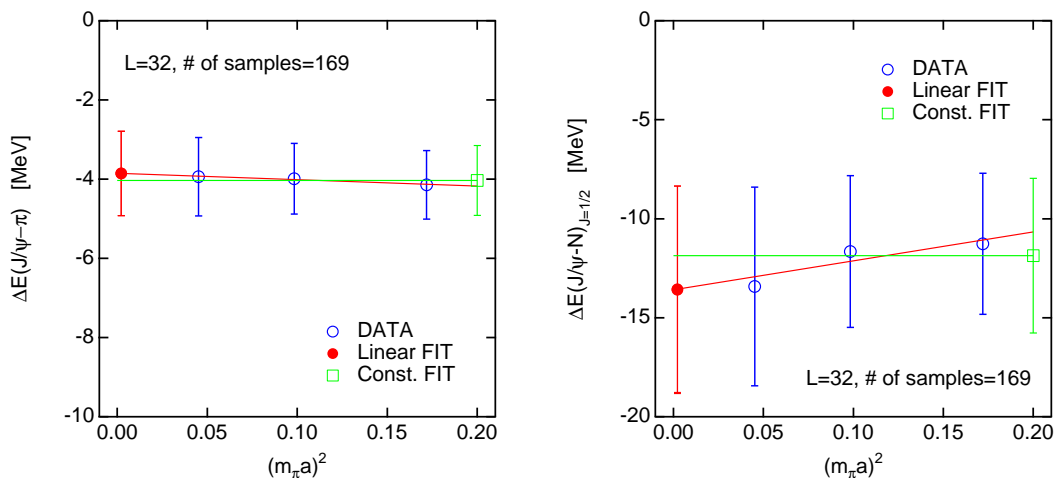
$$\Delta E < -\frac{\pi}{2|c_1|M_{\text{res}}L^2}, \quad (2.8)$$

although the expansion up to  $O(L^{-5})$  always has a real solution.

This observation implies that some care must be taken to use the expansion especially for relatively strong attraction. Indeed, our lattice data show that the  $J/\psi$  interaction with  $\pi$ ,  $\rho$  and  $N$  are all attractive and the condition Eq.(2.8) is met for  $J/\psi$ - $\rho$  and  $J/\psi$ - $N$  cases. Therefore, in our study, we use Eq.(2.6) directly without the large  $L$  expansion to extract  $a_0$ .

### 3. Results

In our simulation, we employed unimproved Wilson gauge action and Wilson fermion. We have  $\beta = 6.2$  on  $L^3 \times T = 24^3 \times 48$  and  $32^3 \times 48$  lattices with  $\kappa(\text{charm}) = 0.1360$  and  $\kappa(\text{light}) = 0.1520, 0.1506, 0.1489$ . In the physical unit, the lattice sizes are  $L \sim 1.6, 2.1$  fm, the lattice spacing is  $a \sim 0.067$  fm, and  $m_{J/\psi} \sim 3.0$  GeV and  $m_\pi \sim 0.6$ - $1.2$  GeV. The number of quenched



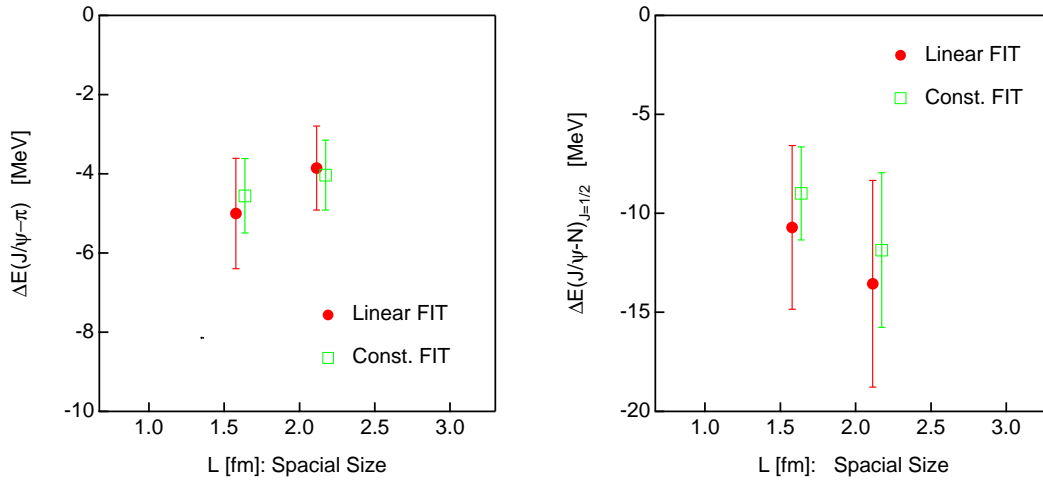
**Figure 1:** The left (right) panel shows the quark mass dependence of the energy shift for the  $J/\psi$ - $\pi$  ( $J/\psi$ - $N$ ) interaction in  $L = 32$  lattice. The horizontal axis is the pion mass squared  $(m_\pi a)^2$  and the vertical axis is the energy shift  $\Delta E$ . The open circles are the energy shift extracted from different  $\kappa$ . The circles are the linear extrapolated points at the physical point. The open squares are the points which are evaluated with an assumption that energy shift is independent of quark mass.

gauge configurations for smaller lattice is 161 and that for larger lattice is 169. Our error estimates are all based on the Jackknife method.

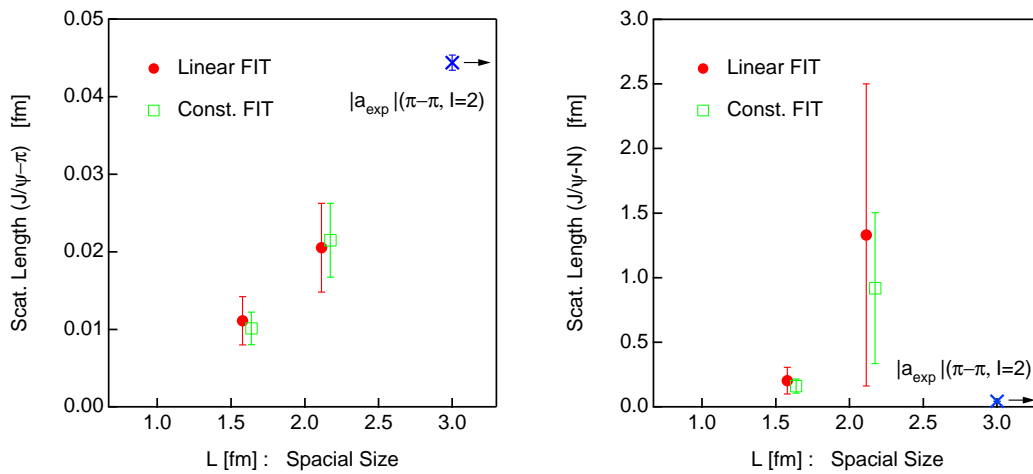
In Figure 1, we show the quark mass dependence of the energy shift  $\Delta E$  in the  $J/\psi$ - $\pi$  channel (the left panel) and the  $J/\psi$ - $N$  channel (the right panel). The open circles are the energy shift extracted from the correlator ratio for different quark masses. The circles and the open squares are the results of a linear fit in quark mass and of a simple average over the data with different quark masses, respectively. The open squares are the estimate of  $\Delta E$  without quark mass dependence as a reference. In both channels, we found that the interactions are attractive.

In Figure 2, we show the volume dependence of the energy shift in the  $J/\psi$ - $\pi$  channel (the left panel) and the  $J/\psi$ - $N$  channel (the right panel). The circles are the energy shifts from linear quark mass extrapolation to the physical point and the open squares are the results of using the constant quark mass dependence. Although the error bars are large in both channels, one can see the following tendency: In the  $J/\psi$ - $\pi$  channel, the absolute value of  $\Delta E$  decreases as  $L$  increases. On the other hand,  $\Delta E$  for  $J/\psi$ - $N$  has opposite tendency. However, to make firm conclusions on this, we need to increase statistics and also collect the data for larger  $L$ . If it turns out to be true in high statistics data, one may conclude that  $J/\psi$ - $\pi$  channel is attractive without a bound state, while  $J/\psi$ - $N$  may have a bound state [13].

Finally, in Figure 3, we show the volume dependence of the scattering lengths in the  $J/\psi$ - $\pi$  (left panel) and the  $J/\psi$ - $N$  (right panel) channels. The circles and the open squares are the results of linear quark mass extrapolation to the physical point and of fitting assumed constant quark mass dependence. To compare these absolute magnitudes of the  $J/\psi$ -hadron scattering length with that in the  $I = 2$   $\pi$ - $\pi$  channel, we put the empirical value of the  $\pi$ - $\pi$  scattering length by crosses. Note that  $I = 2$   $\pi$ - $\pi$  scattering is repulsive and we show its absolute value in the figure for comparison. The  $J/\psi$ - $\pi$  scattering length is negative and small compared to  $\pi$ - $\pi$ . This is partly because the size



**Figure 2:** The left (right) panel shows the volume dependence of the energy shift from the  $J/\psi-\pi$  ( $J/\psi-N$ ) interaction. The horizontal axis is the spatial size  $L$  and the vertical axis is the energy shift  $\Delta E$ . The circles indicate the energy shifts which are assumed to have linear quark mass dependence extrapolated to the physical point. The open squares show energy shifts estimated as if there is no quark mass dependence.



**Figure 3:** The left (right) panel shows the volume dependence of the scattering length in the  $J/\psi-\pi$  ( $J/\psi-N$ ) channel. The horizontal axis is the spatial size  $L$  and the vertical axis is the S-wave scattering length  $a_0$ . The circles (open squares) indicate the scattering lengths which are assumed to have linear (constant) quark mass dependence. The crosses show the sign-flipped empirical value for the  $\pi-\pi$  scattering lengths in the  $I = 2$  channel as reference points.

of  $J/\psi$  is small than the pion, and partly because only the gluonic exchange is allowed in the  $J/\psi$ - $\pi$  case. On the other hand, the scattering length for  $J/\psi$ - $N$  could be order of magnitude larger than  $J/\psi$ - $\pi$ , although the error bar is still quite large.

#### 4. Summary

In summary, we study the  $J/\psi$ -hadron scattering lengths by the quenched lattice QCD simulations. We found attractive interactions in all  $J/\psi$ - $\pi$ ,  $J/\psi$ - $\rho$  and  $J/\psi$ - $N$  channels. Furthermore, the  $J/\psi$ - $N$  scattering length is considerably larger than the  $J/\psi$ -meson scattering length. Also, we found a sizable volume dependence of scattering lengths in all three channels. There is an opposite tendency of the volume dependence between  $J/\psi$ - $\pi$  and  $J/\psi$ - $N$ . There are several future problems to be examined further: To study whether the attractive  $J/\psi$ - $N$  interaction could form a bound state, we need to have better statistics and simulations with larger lattice volumes, which is now under way. To confirm the validity of using the Lüscher's formula, we need to check whether the potential range is small enough in comparison to the lattice size. Moreover, we need more careful analysis of inelastic contribution in our correlator ratio such as the  $D$ - $\bar{D}$  contribution in the  $J/\psi$ - $\rho$  channel.

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