A dimensional view on numeral systems

Hansjakob Seiler

August 1989
## CONTENTS

1. Introduction ................................................. 2  
2. Underlying cognitive-conceptual structure ................ 3  
3. Interface cognition-language: The dimension .............. 4  
4. Generalizations and their functional significance ....... 5  
4.1. "Atoms" .............................................. 5  
4.2. "Bases" .............................................. 9  
4.3. "Calculatory operations" ................................ 14  
4.3.1. Directionality ................................... 15  
4.3.2. Transitions and rule changes ....................... 18  
5. The diachronic perspective .................................. 23  

NOTES .................................................................. 28  
REFERENCES ......................................................... 29
1. Introduction

The Stanford Project on Language Universals began its activities in October 1967 and brought them to an end in August 1976. Its directors were Joseph H. Greenberg and Charles A. Ferguson. The Cologne Project on Language Universals and Typology [with particular reference to functional aspects], abbreviated UNITYP, had its early beginnings in 1972, but deployed its full activities from 1976 onwards and is still operating. This writer, who is the principal investigator, had the privilege of collaborating with the Stanford Project during spring of 1976.

The following lines are intended as a small tribute of gratitude to Joseph H. Greenberg, and this in more than one sense. Not only did this writer enjoy the stimulating atmosphere at the Stanford Project. Ever since, he sees in Greenberg's work, in its methodology and its results, a constant source of inspiration and of meditation for himself. One of the leading Greenbergian ideas, that of implicational generalizations, has been integrated as a fundamental principle in the construction of continua and of universal dimensions as proposed by UNITYP.

It is hoped that the following considerations on numeral systems will be apt to bear witness to this situation. They would be unthinkable without Greenberg's pioneering work on "Generalizations about numeral systems" (Greenberg 1978:249 ff., henceforth referred to as Greenberg, NS). Further work on this domain and on other comparable domains almost inevitably leads one to the view that generalizations of the Greenberg type have a functional significance and that a dimensional framework is apt to bring this to the fore. This is the view on
linguistic behaviour as being purposeful, and on language as a problem-solving device. The problem consists in the linguistic representation of cognitive-conceptual ideas. The solution is represented by the corresponding linguistic structures in their diversity; and the task of the linguist consists in reconstructing the program and subprograms underlying the process of problem-solving. It is claimed that the construct of continua and of universal dimensions makes these programs intelligible.

2. Underlying cognitive-conceptual structure

The idea that numeration would represent a universal linguistic dimension may derive some plausibility from the fact that the underlying cognitive-conceptual operation, viz. the act of counting, is of a dimensional nature. It involves a starting point, a recursive operation of adding 1, and a limit number (Greenberg, NS:253). These arithmetical operations presuppose an understanding of something conceptually deeper, and it is the merit of the genetic epistemologist J. Piaget to have brought this to the fore (Piaget 1961:277 ff.). According to him the concept of number results from a synthesis of two different operations, one classificatory, the other serializing in a spatio-temporal order. In their Principia Mathematica Russell and Whitehead had defined number as the class of equivalent classes. This means that number is based on a term-by-term correspondence. Their example: What the twelve months of the year, the twelve apostles, the twelve symbols of the Zodiac, and the twelve Marshals of Napoléon have in common is precisely and only their twelve-hood, and not any other of their individual properties. Each element counts as an arithmetic unit, its particular properties remain outside of consideration. These classes form a hierarchy where the class of 12 includes the class of 11 and is included by the class of 13, etc. However, if elements as the ones mentioned in the example are viewed as stripped of their properties (except that of number), they become
indistinguishable, so that we would get the tautology \( A+A=A \), instead of the intended \( A+A=B \) (i.e. \( 1+1=2 \)). In order to distinguish these otherwise indistinguishable elements it is necessary to introduce ordering relations, i.e. to order the elements in space or to count them, one after the other.

We shall see that the combination of these two fundamental principles that are cognitively relevant even outside of language proper have their importance for linguistic representation as well. We shall distinguish between atoms, corresponding to the arithmetical units; bases that are instrumental in the packing strategy, i.e. in marking class inclusion; and calculatory operations as the explicit means for marking ordering relations.

3. Interface cognition-language: The dimension

The more recent stages in the development of the dimensional model of UNITYP have been described in Seiler (1986:20 ff.) and Seiler (1987:250 ff.). A universal dimension is a topological ordering in a continuum of different linguistic procedures, called techniques, all fulfilling the purpose of representing a common universal concept. In our case this universal concept is number, or rather: the operation of counting. The ordering is linear, and is determined by the formal and semantic similarities vs. dissimilarities of the linguistic structures representing the output of the techniques. Each technique is, in turn, constituted by a number of interacting continua or parameters, each comprising a certain range of variation: They are subdimensions within the overall dimension. In our case the techniques are called "atoms", "bases", and "calculatory operations". Their respective parameters are presented and discussed in the corresponding sections.
Dimensions are structured by two complementary and conversely correlated functional principles called indicativity and predicativity, and a third functional principle called iconicity. Indicativity means that the concept is linguistically represented by deictic means, by pointing it out. The technique of "atoms" is predominantly indicative, and their deictic character is particularly salient in the accompanying gestures. Predcativity means that the concept is predicated on, i.e. defined, and that is what "calculatory operations" do. "Bases", finally, is a technique that is neither predominantly deictic nor predominantly predicative; instead, it is a direct representation, based on criteria of relational similarity, similarity, that is, between properties of the concept and properties of the linguistic representation. Note that these three modes of representation are related to the three Peircian semiotic modes of representation, viz. index, symbol, and icon. However, in contradistinction to Peirce, these are not conceived of as disjunct static categories, but rather as dynamic principles copresent in every technique and in every linguistic structure, although with varying degrees of dominance. Dominant indicativity is correlated with non-dominant predicativity, and vice-versa. As for iconicity, it has its preferred peak at certain turning points where neither indicativity nor predicativity seem to be dominant and where often syntactic or semantic rules change. In numeration the area of transition around a base as a turning point is precisely the place where most irregularities are encountered. It is hoped that by integrating them into the dimensional framework they will lose some of their baffling and irrational appearance.

4. Generalizations and their functional significance

4.1. "Atoms"

Greenberg (NS:256) uses this term for the set of numerals "which receive simple lexical representations". These are normally the lowest numerals beginning with
expressions for 1 and 2. Simple lexical representations may, however, be a result rather than a primary criterion. In what follows we shall apply the term for that particular set of numerals that has the highest potential of being recursively used in cycles or with bases. This would exclude bases from atoms, while they are included along with "simple atoms" in Greenberg (l.c.). There are examples of bases which, at least originally, received composite lexical representation: English hundred, Old-Norse hundrad < *hunda-rap, a compound with hund '100' as first, and -rap (Gothic raþjan 'to count') 'number' as second member.

Atoms are of a highly indicative-indexical character. This means that their representation is basically by pointing. For atoms more than for any other set of numerals it is true that they "are never used without accompanying gestures" (Greenberg NS:256; see also the detailed account by Majewicz 1981:194 ff.). The obvious frame of reference here is constituted by the set of fingers of each hand, plus, eventually, the set of toes. It is by virtue of these fixed frames of reference that a direct assignment of a numerical value to the numeral becomes possible. There is no need for, and only sporadic use of, calculatory operations, and serialization is irregular: Compare Greenberg's examples of Montagnais (Athapaskan) with 7 expressed as either (10-3) or (8-1); 8 as (4x2), and 9 as (10-1) (Greenberg NS:260). In Yurok (Northern California) as described by R.H. Robins (1958:86 ff.; 1985:723 ff.) the numerals for 7, 8, and 9 are obviously related to the words for 'index or pointing finger' (7), 'long (third) finger' (8), and 'little finger' (9), respectively. However, the 'ring finger' and the 'thumb' have no numerical value and are thus missing out in the serialization.

In a situation where gestures pointing to fingers and other parts of the body are essential in counting, the formal distinctiveness of the corresponding numerals is of minor importance. The extreme case, viz. complete identity
of the numerals, is cited by Greenberg (NS:257, following Koch-Grünberg 1928:316; see also Majewicz 1981:197) for the language of the Kaliana Indians in South America, where counting goes meyakan 1, meyakan 2, meyakan 3, meyakan 4 with accompanying gestures involving fingers and toes. A less extreme case is presented by languages where in finger counting the same numerical expressions are used for such pairs as 1 and 6, 2 and 7, 3 and 8, 4 and 9, 5 and 10. These overlaps conditioned by the symmetrical shape of the hands seem to be quite wide-spread (Majewicz 1981:198 ff.). Another manifestation of this same tendency of weak formal distinctiveness of low numerals, i.e. atoms, is shown by partial mutual assimilation of adjacent numerals: In Russian, the successions of 7 sem', 8 vōsem', and of 9 dévjat', 10 désjat' would be examples.

A direct consequence of the indicative-deictic character of atoms is their close relationship to referential functions. In many languages, as, e.g., in French, the numeral for 1 un functions also as an article. In Ancient Indo-European languages the ordinal series began with two determiners, as in the Latin unus '1st', alter, or alius, 'other, 2nd', and only with tertius '3rd' are we in the numeral series (Winter 1953:3 ff.). We have here an area of convergence between the dimensions of numeration and determination, respectively (Seiler 1978:301 ff.). Another association of low numerals with the referential system of a language is shown by their sensitivity to gender distinction. This notoriously occurs with numerals 1,2,3, with an eventual tapering-off of the number of distinctions. Agreement in gender and number is a technique within the dimension of apprehension, a technique with strong predominance of the functional principle of indicativity. As I have shown (Seiler 1986:113 ff.), its function is to indicate constancy of reference. Thus, we find here another convergence, viz. between numeration and apprehension, respectively.
We do not think that the set of atoms should be delimited from the set of non-atoms on the basis of a strict yes/no decision. The essential criterion is functional: highest potential of recursive utilization. Surely, by virtue of the human counting capacity these must be the lower numerals. The extension of this set, however, may vary from one language to another, and also in diachronic perspective. In a decimal system the numerals for 11-19 have a potential of recursive use which is not as high as that for numerals 1-9, yet higher than that of any further numerals. In fact, we find some of the "irregularities" just mentioned - "irregularities" they are from a one-sidedly serializing point of view - in numerals from 11 to 19: Thus the succession in Welsh of 15 pymtheg (5+10), 16 un ar bymtheg (1 on (5+10)), 17 dau ar bymtheg (2 on (5+10)), 18 deunaw (2x9), 19 pedwar ar bymtheg (4 on (5+10) (Williams 1980:40). A series of additive operations is interrupted by a multiplication in 18=(2x9). In a comparable way, Breton interrupts an additive series 16=(6+10), 17=(7+10), 19=(9+10) by a multiplicative 18=(3x6). It is certainly not by accident that such phenomena within the second decade occur in languages showing a vigesimal system.

These are cases of counting by unsystematic operating, i.e. by unsystematic calculating, comparable to the cases formed within the series of ones. The set of non-atoms begins with systematic calculating. Transparency of the numeral expression may be an indicator for this. The cut-off point (on this notion see Greenberg, NS:272) is language-specific. Thus, French shows 11 onze, 12 douze, 13 treize, 14 quatorze, 15 quinze, 16 seize with diminished transparency, but from 17 dix-sept onwards we find perfect transparency (10+7) and a word order higher summand preceding lower summand, which suits calculatory purposes better than the reverse order (Greenberg, NS:274). In Spanish, as against French and Italian, the cut-off is between 15 quince and 16 dieciséis.
To conclude this section we might say that the set of atoms is functionally founded. It is linguistically characterized not by a single feature but by a number of interacting parameters, each being constituted by a certain range of variation. They are:

- connection with gestures
- direct assignment of number value
- referentiality
- lack of transparency
- lack of systematic operation

4.2. "Bases"

In Greenberg (NS:270) a base is defined as a serialized multiplicand. In English 10, 100, 1000, 1000000 are bases. We should like to retain the term, but apply it to a wider range of phenomena, where the decisive definitory criteria would again be functional.

Bases are marks of hierarchical packing. Packs are classes of numerals, defined both extensionally, viz. by their correspondence to numerical value, and intensionally, viz. by the predominance of certain rule-types. Thus, in English we have packs of tens, of hundreds, etc., in French we have packs of tens, hundreds, etc., and certain packs of twenties; in Efik and other Kwa languages we have packs of fives, twenties, and hundreds (Welmers 1973:298).

What are the procedures for marking a base? They are not normally of a pointing or deictic or indicative character. Nor are they predicative in the sense of forming part of a systematic calculus. The strategy is based on the third semiotic option, the iconic. As I have shown (Seiler 1988:2 ff.), iconic representation is determined by relational similarity between properties of the representandum and properties of the linguistic expression. The most natural simile for a pack of denumerable objects is constituted by the human body. The
human body is in many instances the source for designating bases.

Api (New Hebrides) shows the following system (Dantzig 1940:25):

1  
   tai  6  otai  'new one'
2  
   lua  7  olua  'new two'
3  
   tolu  8  otolu  'new three'
4  
   vari  9  ovari  'new four'
5  
   luna 'hand'  10  lua luna 'two hands'

In many languages all over the world 'hand' is the obvious mark for a pack of 5. In some languages 20 is expressed by a phrase referring to something like 'the whole person' e.g. Jukun (Nigeria-Cameroon borderland area), which has a vigesimal system (Weimers 1973:295).

Other similia may also be used for base marking. Greenberg (NS:272) reports the words for 'road' with value 100, 'road large' with value 1000, and 'road large old' with value 1000000 in Yuchi (Macro-Siouan). The similarity, here, seems to consist in spatial extension.

These and similar cases fit into the more general principle that bases behave like substantives (Greenberg NS:287). The corresponding base may then simply assume the meaning of 'the pack of...', e.g. 'the pack of ten', French la dixaine, German der Zehner, Greek dékas, etc. The syntactic behavior is accordingly. Thus, in Russian we find, beginning with 50 pjat' desjat and through 80, constructions where desjat represents an old genitive contrasting with the form of the simple numeral 10 désjat'. And likewise, beginning with 500 pjat'sot and through 900, constructions including a genitive, contrasting with the form 100 sto.²

A direct consequence of the iconic origin of base markings is their often approximative numerical value assignment. As I have shown (Seiler 1988:4 ff.), iconic
representation is bound to be approximative and often polyvalent as the criteria for similarity exhibit these very characteristics. For base markings this means, among other things, that they can be reinterpreted with different numerical value assignments. One of the better known cases is the Germanic use of 'hundred' with the value of 120 (and of 'thousand' with the value of 1200). It is the so-called Grosshundert 'duodecimal or long hundred' (and Grosstausend). In Old Icelandic tírótt hundrad 'ten-reckoned hundred' (-róedr = Goth. ga-réjan 'to reckon') is distinguished from tolróett hundrad 'twelve-reckoned hundred'. P. Sommer (1951:65 ff.) is certainly right in rejecting the idea of an interfering duodecimal system. If 12 were really a base of a rudimentary duodecimal system, then the next base would be 144, and not 120. The source for the reinterpretation of the Indo-European decimal 'hundred' as 120 seems to be geographically located around the North Atlantic and the Baltic Sea, and the primary use seems to have to do with trading fish and other goods that come 'by the dozen' or by the 'Grosshundert', where the remaining 3 or 20, respectively, represent a margin for discount (cf. "cheaper by the dozen", i.e. "12 for the price of 10") (see also Kluge-Sötze EW 26 s.v. "Grosshundert").

Another, even more intriguing case is the value of Danish tyve. Danish exhibits a partially vigesimal system, its base being tyve with the value 20. But in fyrretyve = 40 tyve has the value of 10 (4x10), and likewise in the less transparent tredive = 30 (3x10). Only with 50 = halv-tred-sinds-tyve, literally 'half the third-times-twenty, i.e.'two and a half times twenty' (on these formations see below, 4.1), tyve assumes again its etymologically justified value of 20; and this value persists from 50 to 90. Much of the baffling character of this system disappears when we realize that base marking, by virtue of its predominantly iconic origin, is "entitled" to poly- or plurivalence.
If base marking is essentially of an iconic origin, it nevertheless happens quite frequently that this origin becomes blurred or obliterated by phonological and morphological changes. After all, bases in numeration are used to work on, mostly in their capacity of serialized multiplicands (Greenberg, NS:270). This means that they are recursively used thereby undergoing truncation and other modifications. An instructive case is presented by Cahuilla, a Southern Californian Uto-Aztecan language (Seiler 1977:330; 1988:22). The first thirteen numerals are as follows:

1 súpłe 6 kwansúpłe
2 wih 7 kwänwih
3 pāh 8 kwänpāh
4.wičiw 9 kwänwičiw
5..namakwánaną 10 nánčúmi

The system is quinary. Numerals from 6 to 9 are formed on the basis of 5 by additive juxtaposition of the digits: (5+1) = 6, (5+2) = 7, etc.; likewise for 15 = (10+(5+1)), etc. Namakwánaną = 5 contains the lexeme kwánaną 'half' and, as first element, the possessed form for 'hand', thus: 'my hand or my hands - half' (i.e. 'half of the fingers of my hands'). This would correspond to the iconic uses for 'hand' shown in the examples presented earlier in this section. But the Cahuilla expression has undergone changes: 'My hand' is né-ma, but the first element in 5 is definitely nama-. Furthermore, there has been truncation of namakwánaną 5 to kwän- in the expressions from 6 to 9. Nánčúmi = 10 is a base. It contains the lexeme čúmi- 'finish' plus, apparently, 'my hand' as first element, thus: '(the number of fingers of) my hands-finished'. Again, né-ma 'my hand' is changed, this time to name-. After 10 numeration follows a different rule, using the
superessive link (on this term see Greenberg NS:265) \( p \)-eta 'on top of it'.

A last property of bases deriving from their iconic nature has to do with the relationship of iconicity within a dimension. Here, the principle of iconicity must be compared with the two other universal functional principles, that of indicativity, and that of predicativity. Our inquiry into the dimensions described by \textsc{unityp} thus far has shown us that the principle of iconicity may freely combine with either of the two other principles, but that there is a preferred place or peak of iconicity where the other two principles neutralize each other, being about equal in force. This is usually the place where morpho-syntactic rules change drastically, e.g. from nominal syntagm to verbal syntagm in the dimension of \textsc{possession} (Seiler 1983:55 ff.), or from government to modification in \textsc{participation} (Seiler 1984:108 ff.). It is furthermore the place where we find, for one and the same function, a multiplicity of options in the expressive means. We termed this critical or "catastrophic" point as the turning point in the dimension. It became apparent that around the turning point there is an area of transition where some of the characteristics of dominant iconicity also obtain, viz. multiple choice, and plurivalence (Seiler 1988:12 ff.).

These considerations seem to favor the view that bases are dimensional turning points and that numeration is a dimension with normally more than one turning point. This, however, must be further substantiated by highlighting the operational aspects of numeration, i.e. those aspects which have to do with rules, and, in particular, with serialization.

We conclude by saying that the notion of base in functional terms is linguistically characterized not by a single feature, but by a number of interacting parameters,
each being constituted by a certain range of variation. Here they are:

- packing
- iconic marking
- fluctuation in numerical value assignment; reinterpretation
- obliteration of iconicity
- marking of turning point.

4.3. "Calculatory operations"

Calculatory operations are, besides atoms and bases, the third constitutive factor in the construction of numeral systems. They are linguistically implemented by syntactic rules and by rules of semantic interpretation. Our interest, here, is focused on the workings of these rules with regard to the three aforementioned functional principles that are constitutive for a dimension, i.e. indicativitiy, iconicity, and predicativity. We found that atoms are characterized by predominant indicativity, i.e. by their direct assignment of number values, by their referentiality, and by their lack of systematic operations. Both semantic and syntactic rules seem to be of minor import here, which, of course, does not mean that atoms are not amenable to empirical generalizations. As for bases, we found iconicity to be their predominant characteristic, which, in turn, is based on criteria of relational similarity. As with atoms, number assignment in bases is direct and immediate, but in contradistinction to atoms it is not effectuated by pointing but relies on criteria of similarity. As these are subject to multiple interpretation and fluctuation, syntactic and semantic rules seem to intervene here in a rather unsystematic and idiosyncratic way.

The true domain of syntactic and semantic rules seems to be the domain of predominant predicativity where numerals are being operationally construed. The prototypical manifestation of this is serialization.
The steps in the syntactic and semantic analysis of numerical systems have been completely outlined in Greenberg (NS:263 ff.), and there is little that we wish to add in the limits of this study. What we are primarily interested in is the workings of serialization and semantosyntactic rules in their interaction with bases and with atoms.

4.3.1. Directionality

At first sight it seems natural that serialization follows the course of the arithmetical operation of adding one, yielding continuously increasing numerical values, both before and after bases. Thus, in German we find 98 achthundertneunzig, 99 neunundneunzig, 100 hundert (=base), 101 hundert(und)eins, 102 hundert(und)zwei, etc. After the base there is a rule change in word order of the constituents from lower-higher (LH) to higher-lower (HL). Moreover, the conjunctive particle und is obligatory before the base, but optional thereafter. But serialization continues progressively.

Now, bases, we said, are markers for packing or hierarchical classification, and serialized ordering is only possible within a given frame of hierarchical classification. A base like 100 is more than just a position one step above 99 and one step below 101 (as, e.g., 98 is (97+1) and (99-1)). It is a "base" in the literal sense, i.e. a base to work on and to work from. A priori and logically there is no reason why such working should be unidirectional, i.e. only progressive. It might as well be regressive. However, there is a strong natural bias for progressive serialization which is grounded in the fact that 1, when considered as the starting point of any numeral system, entails only progressive serialization, and that in the recursive use of lowest numerals this same procedure is perpetuated.
Yet, regressive construction of numerals is more frequent and more wide-spread than is commonly assumed. Several types may be distinguished, but in all of them regressivity is structurally and systematically related to a base.

The simplest type is represented by subtraction, as, e.g. in Latin, where 18 is duodeviginti, i.e. 'two from twenty' and 19 is undeviginti 'one from twenty'. The minuend, viginti 20, is a multiple of base 10 (cf. Greenberg, NS:260), and the subtractive procedure is continued with all the multiples up to 90; it stops with base 100, where 98 is nonaginta octo 'ninety-eight' and not *duodecentum, and analogously with 99. Note that subtraction works only with digits in the immediate neighborhood of a base, in the Latin case with the last two.

Subtraction is found in numerous languages of Africa, e.g. Efik, Yoruba (Welmers 1973:289 ff.), the Americas, e.g. Tunica (Haas 1941:84), in Finnish, in Biblical Welsh (Hurford 1975:136 ff.), in Ainu (Hurford, op.cit. 239). An intricate alternation of addition and subtraction is exhibited in the numeral system of Yoruba (Welmers 1973:301 ff.): The digits of 1 through 4 are added to 10 or any of its multiples. The digits from 5 down to 1 are subtracted from the next higher multiple of ten. There are new units for 20 and 30, 40 and higher multiples of 20, through nine twenties, combine with an alternant form of 20 with the stem of the appropriate digit. Odd multiples of 10, beginning with 50, represent subtractions from the next higher multiple of 20. An irregularity is introduced in the combinations for 185 through 189. These are additions to 180. There is a new unit for 200, which functions as a base. Combinations for 190 through 199 are subtractions of 10 down to 1 from this unit (Welmers 1973:303). Both kinds of irregularities last mentioned may lose some of their irrationality when viewed in the light of a principled fluctuation between several options in the neighborhood of
bases (see 4.3.2.). Subtraction here occurs, again, in the immediate neighborhood of base 200.

Another type of regressivity might be termed "anticipatory counting" (term used by Stampe 1976:602 ff.)⁵. If a hierarchical class or pack of numerals is delimited by two successive bases, one representing the lower-bound limit and the other the upper-bound limit, serialization may take either one or the other as its point of reference. In anticipatory counting the reference point is the upper-limit base, as with subtraction, but, in contradistinction to the latter, serialization is progressive. The procedure is found in the Mayan group, in some Finno-Ugric languages, in Danish, and presumably in quite a few other languages.

Finnish combines subtraction and anticipatory counting: It subtracts for eigths and nines; for the rest it counts upwards toward the next 10: 11 = yksi-toista 'one-of-the-second', 12 = kaksi-toista 'two-of-the-second', and likewise 21 = yksi-kolmatta 'one-of-the-third', etc.

Danish, as we have seen (4.2.) , shows somewhat unsystematic operating regarding the tens up to and including 40. Beginning with 50 we find a consistent vigesimal procedure taking the next-higher 20 as a reference point, where twenties are counted by ordinals: 'third, fourth, fifth twenty' and the uneven tens indicated by fractioning. The expressions are morphologically complex, and there are fuller forms, now somewhat archaic, and truncated forms, which are more current nowadays. But an understanding of the workings of this system must come from the fuller forms, which also underlie the formation of the corresponding ordinals (Hurford 1975:117 ff.).
50 = halv-tred -sinds-tyve  
1/2 -third-times-twenty = 2 1/2 x 20  
60 = tre -sinds-tyve  
three-times-twenty = 3 x 20  
70 = halv-fjerd -sinds-tyve  
1/2 -fourth-times-twenty = 3 1/2 x 20  
80 = fir -sinds-tyve  
four-times-twenty = 4 x 20  
90 = halv-fem -sinds-tyve  
1/2 -fifth-times-twenty = 4 1/2 x 20  

Once more we find that this kind of regressivity in anticipation is linked up with the notion of base.

4.3.2. Transitions and rule changes

We hypothesized (above, 4.2.) that bases are dimensional turning points and that numeration is a dimension with normally more than one turning point.

In order to substantiate the hypothesis, let us inquire somewhat further into the workings of rules with regard to bases and their immediate neighborhood.

Quite frequently, a base marks the change of a syntactic and/or semantic rule, as in the German examples 99 neunundneunzig, 100 hundert, 101 hundert(und)eins introduced above (4.3.1.). Although such rule changes are frequent around bases in many languages, they are by no means necessary. Modern Greek shows a transition around base 100 without any intervening changes: 99 enēnda enēa (90+9), 100 ekatón, 101 ekatón éna (100+1).

Regardless of whether a base marks a change of rules or not, we find in the neighborhood of bases a number of closely connected phenomena that, albeit not lending themselves to absolute generalizations, would nevertheless seem to occur with more than chance frequency. The phenomena are: 1) General irregularities in the constituency of the numerals immediately preceding and
following the base. 2) Numerals around the base follow a rule which differs from the earlier as well as from the later serialization rules. 3) Coexistence of more than one competing options of choice among different rules (multiple choice situation). It is as if we had, around the base, a transition zone of turbulences between two series of consistent operations. We might also compare it to the intermediate stage in the gear-shift between two speeds of a car.

The reason for the turbulences as mentioned under 1) - 3) seems to lie in the particular semiotic status of the base as the turning point between serializations: It is, as we stated, predominantly iconic, i.e. originating from criteria of similarity, and not deriving from compositional rules of constant serialization. It interrupts serialization, thereby causing disturbances before serialization resumes and either continues the earlier rule or follows a different rule. Interruption as a factor causing turbulences in a transition zone becomes even more plausible if we remember that a base potentially allows for either progressive or recursive operation.

With base 10 many languages show a special rule for 11 and 12, before, with 13, serialization goes on along a more persistent rule:

In the Germanic languages the respective numerals are: Gothic ain-lif 11, twa-lif 12, German elf, zwölf, and their etymologies point to compounds with the digits for 1 and 2 and a root *likʷ. 'to be left over', thus literally 'one - left over', 'two - left over', with the elliptic base 10 to be supplied. After that, serialization continues as in German dreizehn 13, vierzehn 14, etc. However, the corresponding forms of Lithuanian show that the 'left - over' rule for 11 and 12 was an option that could eventually be kept further on: 10 dvideimt, 11 vėžuliškia, 12 dvyliskia, 13 tryliškia, 14 keturiolika and so forth till 20. The respective etymologies show, as in German, compounds
with the digits as first element, and the root *likʷ- as second.

Basque (Araujo 1975:141) exhibits 10 hamař as a base, and from 11 to 19 compounds with 10 plus digits. From 13 onwards the comparison with the digits is straightforward, for 11 and 12 it is irregular:

1 bat      11 hameka
2 biga     12 hamabi
3 hiru     13 hamahiru
4 lauř     14 hamalauř

and so forth through 18. But compare 9 bederaci with 19 hemereci:

19 is irregular in the vicinity of 20 which constitutes another base; the system is vigesimal. In this latter case the numeral immediately preceding the base is affected. Comparable irregularities for numerals 11 and 12 and an onset of regular serialization with 13 are reported for Yukatec Maya (Guitel 1975:396 ff.).

There may be counter-examples, viz. where rule changes and/or irregularities occur without the intervention of a discernible base. Russian 40 sórok may be such a case. The preceding decades are 20 dvádcat‘, 30 trídcat‘, while the decades following sórok are 50 pjat‘desjat, 60 šest‘desjat. Whereas the earlier decades represent compositional contractions of the corresponding digits with the form 10 désjat‘, the decades after 40 show an old endingless genitive -desjat. Furthermore, numerals from 50 to 80 decline both parts of the composition. The origin of 40 sórok is a problem of long standing for Slavists. Sórok seems to have replaced an older Četyre desqte ('four tens') and occurs only in Russian. Among the numerous etymologies the least improbable seems to be related to an old term in the trade of furs: In documents of the 14th and 15th century sórok meant a 'bundle of fourty sable-skins'. Thus,
sórok could be related to soróčka 'shirt'. This would be paralleled by Old Norse serkr 1) 'shirt', 2) '200 furs' (Vasmer 1955:698).

This possibly iconic character of the numeral 40 might make one wonder whether sórok at some time and in some circumscribed geographical and sociological area constituted a base of a rudimentary, and not very successful vigesimal system. Another enigmatic point, in the Russian numeral system is 90 devjanósto which seems to have replaced an earlier devjatdesját, preserved in Ukrainian. It is a "turbulence" which, in accordance with our generalization, occurs in the last decade preceding base 100.

Another possible exception may be presented by Welsh (and, in a parallel way, by Breton) where, as shown above (4.1.), a serialization rule 16 = (1 on (5+10)), 17 = (2 on (5+10)), 19 = (4 on (5+10)) is interrupted by 18 = (2x9). However, the situation may appear less exceptional when seen in the light of a comparison with Breton. Here we have a serialization from 11 onwards with 16 = (6+10), 17 = (7+10), 19 = (9+10), and an interruption with 18 triwec'h 'three six(es)' = (3x6). Not only is this an option which differs from the Welsh (2x9), but we have also in Breton a dialectal variant for 18 eitek (8+10) which is regular (Press 1986:86). All in all we find in these Celtic languages a multiple choice situation in the neighboring 18s and 19s of base 20. This would be in accordance with our generalization at the beginning of 4.3.2.

A clear example of a multiple choice situation in the neighborhood of a base is the case of the last three decades approaching base 100 in French and its following dialectal variants:
Allegedly the "façon de compter par vingtaines" is due to Celtic influence. I do not see any cogent reason for this. Vigesimal systems can surely arise spontaneously - compare the Danish case, or the case of English score. It is a fact, however, that in the older stages of the French language the extension of the vigesimal system was far greater, encompassing both lower decades, e.g. 60 trois-vingts and higher ones: 120 six-vingts, 140 sept-vingts, apparently up to 360 dix-huit-vingts (Damourette et Pichon 1911-1940, Vol.VI:493). Some fossilized relics of these stages have survived to this day: The name of a Paris hospital is Hôpital des quinze-vingts; it was founded in the 13th century by Louis IX to accommodate 300 blind veterans (Ifrah 1981/1986:64). Apart from that, open manifestations of vigesimality have survived only in the last two decades preceding 100. Note that 70 soixante-dix is a mixture between decimal and vigesimal. In pure vigesimal terms we would expect trois-vingt-dix. Furthermore, the behavior of the intervening digits agrees with the behavior in the earlier decades and is markedly different from the behavior in the eighth and ninth decade. Compare

20 vingt 60 soixante 70 soixante-dix
21 vingt et un 61 soixante et un 71 soixante et onze
22 vingt-deux 62 soixante-deux 72 soixante-douze

with
Incidentally, the sequence 20 vingt, 21 vingt et un, 22 vingt-deux, 23 vingt-trois is one more example of an irregularity in the immediate neighborhood of 20, a base of a cycle, where regular serialization resumes with 22.

5. The diachronic perspective

It is one of our major tenets that the UNITYP dimensions represent a primary locus of language change and of typological differentiation within the respective domains. The dimension of numeration as outlined above may now be added to the ones we studied before. It seems to offer a particularly favorable testing ground because of the strong conceptual-cognitive support that comes from its underlying mathematical structure. The major task, here, would consist in showing that actual historical changes can be best understood in the light of the dimensional framework, and, specifically, in relation to the three dynamic functional principles as pointed out in the foregoing: indicativity as the dominant principle in atoms, iconicity dominant in bases, and predicativity dominant in calculatory operations. If this can be shown it would mean that the dimension and its constitutive functional principles correspond to something that speakers actually do or rather: that goes on in their heads when they construct a dimension like that of numeration.

Only a few hints can be given here. The task is considerable and would require detailed research that should preferably start with different stages of one and the same language and remain within the limits of one particular language family. This shall be done in a separate study.
The following pairs of opposite dynamic forces seem to be at work in the historical changes that are actually observed: differentiation vs. assimilation and "transparentization" vs. obliteration. The pairs are only partly interdependent, and their relation to atoms, bases, and calculatory operations is not one-to-one.

It seems natural that atoms as labels are weakly transparent and that their differentiation is of minor importance since, at least originally, they are used with accompanying gestures. The similarity between the Russian numerals for 7 and 8, and those for 9 and 10 has already been mentioned (4.1.); it is doubtlessly due to an assimilatory process. Another example would be Latin 4 quattuor and 5 quinque with initial qu- in both cases whereas the reconstructed IE forms are *kʷetw₁ for 4 but *penkʷe- with initial p- for 5. Yet, the opposing force of differentiation has been at work in the low numerals of Modern German, where, in the special situation of telecommunication, the old feminine form 2 zwö has been reactivated in order to remedy the disturbing similarity between 2 zwei and 3 drei.

It seems natural, furthermore, that numerals representing calculatory operations will favor forms with transparent compositionality, and that therefore less transparent forms are made more transparent in the course of history. An example, presented in more detail below, would be the Germanic decades from 20 to 60 as compared to the corresponding Indo-European decades reconstructed on the converging evidence of Greek, Latin, Celtic, Tocharian, and Indo-Iranian (20-50). Yet, calculatory operations normally result in serialization, and numerals in a series have some formal elements in common and some elements in which they differ. Occasionally, the common basis carries more emphasis than the difference, and then we get assimilations. An example would be the decades from 70 to 90 in some Germanic languages (see below).
As for bases, we have pointed out their iconic origins (4.2.). It is this aspect that will favor transparent formations. Yet, we also said that bases are used to work on, specifically in the recursive construction of higher numerals. From this point of view, it seems plausible that their compositionality is of minor importance and becomes obliterated. It was furthermore said that bases, in accordance with their iconic nature, are subject to reinterpretation. An example of an obliteration, and an example of reinterpretation, were presented above (4.2.).

Among the numerous attempts to account for the history of Indo-European decad formations and their continuation in the Germanic languages there is one highly respected by specialists (e.g. Szemerényi 1960:32 f.) which we shall briefly summarize, because it provides pertinent illustrations to what has just been outlined: the account by F. Sommer (1951:48 ff.).

In Germanic the formation of the last three decades approaching base 100 differs markedly from the formation of the decades 20 to 60. Thus, in Gothic, we find 20 twai tigjus, 30 greis tigjus, 40 fidwör tigjus, 50 fimf tigjus, 60 saihs tigjus. They contain the plural tigjus (from singular *tigus 'decad') preceded by the corresponding digits. The formations are perfectly transparent, and, in all likelihood, this transparent character is due to an innovation. The motivation for this, as hypothesized by Sommer (op.cit.:56), consisted in the completely irregular shape of the digits 2-6, furthermore in the dissimilarity between these and the corresponding first members of the decadic compounds, and finally in the lack of any recursive stretches in the decades 20-60.

The decades from 70 onwards and including 100 all end in -tēhund: Gothic 70 sibuntēhund, 80 ahtautēhund, 90 niuntēhund, 100 taīhuntēhund. These contain the easily recognizable digits as first members, but the remainder of these formations does not agree with the converging
evidence of the other major IE daughter languages and is therefore to be considered an innovation. Sommer (op.cit.50) sees the starting point in the base 100: An equivocal situation had arisen there because of the Germanic interpretation of the inherited word for 'hundred' as 'long hundred' = 120, which we mentioned earlier (4.2.). The situation called for a remedy in the sense of more precision. It was found in the coinage of Proto-Germanic *texuntōn xundan, a genitive syntagm, literally 'of the decades hundred' in contradistinction to the 'hundred of the dodecads'. As compared with this Proto-Germanic form the Gothic continuation 100 taihuntēhund presupposes the otherwise attested changes -on > -o > -ē. None of the Indo-European sister languages shows comparable formations for 100.

An innovated *texuntōn xundan 'hundred of the decades' could as well be understood as meaning 'ten decades'. There was a strong formal assonance between this xundan 'hundred' and the inherited *-xundā as reconstructed for the decades 70-90 and meaning 'decad' which must have favored this interpretation. Thus, for 90, an inherited Proto-Germanic *niun-ē(?)-xundā was changed to *niuntōn xundan, in analogy to 100 *texuntōn xundan, and *xundan in this remodelled form for 90 referred to the decad. Finally, the numerals for 70 and 80 followed suit.

To sum up, the analogical changes started from base 100 and were motivated by the need for more precision. They spread in regression, working backwards till 70. They stopped there because the earlier decades had already been remodelled in the sense of increased transparency.

If Sommer's account of these historical changes can be accepted, this would neatly illustrate the dynamism connected with a base in the construction of a numeral system. The dynamism as manifested in diachrony turns out to parallel the synchronic dynamism which was pointed out under the label of directionality (4.3.1.). There it was
stated that a base like 100 was an operational base to work on and to work from, and that logically there was no reason why such working should be unidirectional. Diachronic considerations seem to support the view that both progressive and regressive constructivism has psychological reality.

The bases were also viewed as turning points in the dimension of numeration. It remains to be seen how the behavior of bases in numeration as characterized in the foregoing may cast new light on the nature of turning points within the dimensions studied earlier by UNITYP.
Notes

1. On the notion of "packing", see Hurford (1975:67 ff.).

2. Five and multiples of five appear to assume base-like functions in Russian in that they represent turning points for rule change. On this notion see below 4.3.2.

3. Other cases of reinterpretation of bases are cited in Greenberg (NS:289 ff.).

4. Compare Greenberg's pertinent remark on the "psychological reality of the notion of a base" (NS:290).

5. In Greenberg (NS:258) this is termed the "going-on" operation; Hurford (1975:235) uses the term "overcounting".
References


In der Reihe akup erscheinen die Arbeiten des Kölner Universalienprojekts (DFG-Projekt, Leitung Prof. Dr. Hansjakob Seiler).

* = vergriffen

17. LEHMANN, Ch. 1975. "Determination, Bezugsnomen und Pronomen im Relativsatz".
18. SEILER, H. 1975. "Language Universals and Interlinguistic Variation".
22. VAN DEN BOOM, H. 1976. "Bedeutungsexplikation und materiale Implikation".


32. HOLENSTEIN, E. 1978. "Von der Hintergebarkeit der Sprache (und der Erlanger Schule)".

33. RAMAT, P. 1978. "Y-a-t-il une typologie profonde? (Quelques considérations théoriques (et pratiques))".


40. STACHOWIAK, F.-J. 1981. "Zum funktionaI-operationalen Ansatz in der sprachlichen Universalienforschung aus psycholinguistischer Sicht". 

LEHMANN, Ch. 1981. "On some current views of the language universal".
SERZISKO, F. 1981. "Gender, noun class and numeral classification: a scale of classificatory techniques".

41. CLASEN, B. 1981. "Inhärenz und Etablierung".


43. SEILER, H. 1982. "Possessivity, Subject and Object".

44. MOSEL, U. 1982. "Possessive constructions in Tolai".

45. LEHMANN, Ch. 1982. "Rektion und syntaktische Relationen".

46. LEHMANN, Ch. 1982. "Twenty-four questions on linguistic typology and a collection of answers".

47. HEINE, B. & REH, M. 1982. "Patterns of grammaticalization in African languages".


53. UNITYP-FORSCHERGRUPPE. 1983. "Beiträge zum Problembereich Skalen und Kontinua".


56. KÖLVER, U. 1984. "Local Prepositions and serial verb constructions in Thai".

57. SERZISKO, F. 1984. "ORIENTIERUNG".


59. BRETTSCHEIDER, G. 1984. "PARTIZIPATION verknüpft mit NEKTION".

60. DROSSARD, W. 1984. "KAUSATIVIERUNG und TRANSITIVIERUNG im Tagalog".

MATSUBARA, T. 1984. "Das Problem der KAUSATIVIERUNG am Beispiel ja-
panischer Kausationsausdrücke".


63. DROSSARD, W. 1986. "KASUSMARKIERUNG und die Zentralität von Partizipanten".

64. DROSSARD, W. 1986. "Verbklassen".
LEHMANN, Ch. 1986. "Relationality and the grammatical operation".

65. SEILER, H. 1987. "Language Typology in the UNITYP model".


68. DROSSARD, W. 1987. "Transitivität (vs. TRANSITIVIERUNG) und Intransitivität (vs. INTRANSITIVIERUNG) unter typologischem Aspekt".

69. QUADRANTI, P. 1988. "Kant, Piaget et UNITYP".
"Entwurf einer operationalen Morphologie".

70. MÜLLER-BARDEY, Th. 1988. "Typologie der Subjektverkettung ("Switch reference")".

71. LEHMANN, Ch. 1988. "Studies in general comparative linguistics".

PREMPER, W. 1988. "Zum Problem der lexikalischen Kausation (mit Daten aus dem Arabischen)".

73. SEILER, H. 1988. "L'Iconicité en perspective fonctionnelle".


