Measurement of collective flow in heavy-ion collisions using particle-pair correlations

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We present a new type of flow analysis, based on a particle-pair correlation function, in which there is no need for an event-by-event determination of the reaction plane. Consequently, the need to correct for dispersion in an estimated reaction plane does not arise. Our method also offers the option to avoid any influence from particle misidentification. Using this method, streamer chamber data for collisions of Ar+KCl and Ar+BaI_2 at 1.2 GeV/nucleon are compared with predictions of a nuclear transport model.

Many intermediate-energy heavy ion experiments have been directed toward the goal of inferring properties of the nuclear equation of state (EOS) [1]. In parallel with this effort, theoretical work in the area of nuclear transport models has focused on the task of identifying the most appropriate experimental observables for probing the EOS and on the related task of establishing a quantitative connection between such observables and the EOS [2]. Many factors, both theoretical and experimental, have contributed to the current lack of a consensus on even a relatively coarse characterization of the compressional potential energy at maximum density (in other words, a characterization of the EOS as relatively "hard" or "soft"). One such factor, for example, arises from the fact that detector inefficiencies and distortions can be difficult to simulate and quantify (particularly in the case of a 4π detector), and this leads to systematic uncertainties in measurements of collective flow. This paper presents a new form of collective flow analysis for two data sets from the Bevalac streamer chamber. The most noteworthy feature of this new method is that it is designed to minimize the type of systematic uncertainty mentioned above; more specifically, the influences of particle misidentification and dispersion of the reaction plane can be removed.

For a nonzero impact parameter, the beam direction ($\mathbf{z}$) and the line joining the centers of the nuclei determine the reaction plane, i.e., the x-z plane. The azimuthal angle of a fragment in this coordinate system is

$$\phi=\tan^{-1}(p_y/p_x) . \quad (1)$$

We assume that the distribution function of ϕ in an interval of rapidity centered on y_1 can be described by an expression of the form

$$\frac{d\sigma}{d\phi}=A(y_1)[1+\lambda(y_1)\cos\phi] . \quad (2)$$

Data [3,4] from the Diogène and Plastic Ball detectors support this assumption for rapidities other than the midrapidity region where the "squeeze-out" [5] effect can result in a more complex distribution. In the present study, we restrict our analysis to forward rapidities (see below). The maximum azimuthal anisotropy, as defined by Welke *et al.* [6], is

$$R=\frac{1+\lambda}{1-\lambda} . \quad (3)$$

The method proposed by Welke *et al.* [6] for determining R in an experiment involves estimating ϕ in Eqs. (1) and (2) using the relation $\phi=\phi_{obs}-\phi_R$, where ϕ_{obs} is the observed azimuth of a fragment, and ϕ_R is the estimated azimuth of the reaction plane as determined from the observed fragments in the final state. This method requires that the resulting R be corrected upward, to allow for the fact that ϕ_R is distributed about $\phi=0$ with a finite dispersion. Each step in this procedure is a possible source of systematic uncertainty. In particular, it is normally necessary to include the full acceptance of the detector to obtain the minimum possible dispersion; as a consequence, inefficiencies anywhere in the acceptance will influence the final result. We propose an azimuthal correlation function analysis which yields a value of R while circumventing both the need for event-by-event estimation of the reaction plane and the need for a correction for dispersion. An additional benefit of the correlation function method is that it becomes practical to confine our analysis to an acceptance region where the detector efficiency is high.

We assume that collective flow, as parametrized by Eq.

(2), is the only correlation that influences the azimuthal distributions. The main factors that can potentially affect this assumption are the Coulomb interaction and the effect of quantum statistics for identical particles. These two factors only affect particle pairs with low relative momentum $|\mathbf{p}_1-\mathbf{p}_2|<50$ MeV/c. Both effects can be neglected in the present analysis, because particle pairs with relative momentum in this range make up only 3% of the total pair population. From Eq. (2) the probability of observing two fragments with azimuthal angles ϕ_1 and ϕ_2 is

$$\frac{d^2\sigma}{d\phi_1 d\phi_2}=A^2(1+\lambda\cos\phi_1)(1+\lambda\cos\phi_2)\ , \quad (4)$$

and the distribution probability of ψ, the angle between the transverse momenta of two correlated particles, has the form

$$P(\psi)=A^2(1+0.5\lambda^2\cos\psi)\ . \quad (5)$$

Adapting the approach of interferometry analysis [7], we define the azimuthal correlation function as

$$C(\psi)=\frac{P_{\rm cor}(\psi)}{P_{\rm uncor}(\psi)}\ , \quad (6)$$

where $P_{\rm cor}(\psi)$ is the observed ψ distribution for pairs in which both fragments are selected from the same event and $P_{\rm uncor}(\psi)$ is the ψ distribution for uncorrelated pairs generated by "event mixing", i.e., by randomly selecting each member of a pair from a different event with the same multiplicity. Collective flow shows up as $C(\psi)>1$ at small ψ and as $C(\psi)<1$ at large ψ, and the magnitude of an observed flow can be characterized by the value of λ in Eq. (5) that best fits the data for $C(\psi)$.

The experimental samples used in this paper contain a total 1357 1.2A-GeV Ar beam events with observed charged multiplicity $M\geq 30$. Of these, 571 were collisions on a KCl target and the remaining 786 on a BaI_2 target. The condition $M\geq 30$ selects just over 20% of the inelastic cross section in the case of the KCl target and just under 40% in the case of the BaI_2 target. Flow analyses of these data in terms of in-plane transverse momentum have been reported previously [8,9], and further experimental details can be found elsewhere [8,10].

Although a streamer chamber can provide only limited statistics, the visual scanning method leads to a high efficiency for finding all tracks emerging from an interaction point, for correctly measuring rigidities and angles over all possible event and track configurations, and for rejecting tracks unrelated to the primary interaction vertex. Particular attention was paid to these matters in processing the data used in the present study; all reconstructed events were checked at least once by an observer other than the original measurer and were remeasured where necessary. On the other hand, we have only a limited ability to distinguish between the various positively charged fragment species at middle to backward rapidities in the streamer chamber. The analysis method described above does not require any knowledge of the identity of each fragment, except when deciding whether the fragment passes the cut to select forward rapidities. Simulations indicate that our fragment identification is relatively good in this rapidity region. In contrast, a commonly used flow analysis — the mean in-plane transverse momentum per nucleon as a function of rapidity — uses fragment identification information on both axes and is more sensitive to possible particle misidentification.

The model [11] used in this study is a microscopic Monte Carlo simulation which can be considered a solution of the Vlasov-Uehling-Uhlenbeck (VUU) equation. This model incorporates the effect of the EOS through a momentum-independent mean-field potential $U(\rho)=a\rho+b\rho^\gamma$, where ρ is the nuclear density, and a, b, and γ are constants. $\gamma=2$ corresponds to an incompressibility $K=380$ MeV and lies in the range of what is normally considered to be a "stiff" EOS while $\gamma=\frac{7}{6}$ corresponds to $K=200$ MeV, usually described as a "soft" EOS. In general, model predictions must be filtered to simulate the detector acceptance and inefficiencies before being compared with the experimental data; however, the azimuthal correlation function analysis is designed so that no filtering is required beyond applying the appropriate cuts described below.

Figure 1 shows the distributions of polar angle in the laboratory frame, $dN/d\theta$, for Ar+KCl and Ar+BaI_2, normalized according to positively charged fragments per bin per event. No kinematic cuts have been applied. VUU events have been generated over the full range of possible impact parameters, and the predictions shown in Fig. 1 are based on a subset of these events, selected using the same minimum multiplicity requirement as experiment (see below). The VUU simulation neglects clusters, and as expected, its lack of fragments with $Z\geq 2$ leads to a prediction that is too high in the smallest θ bin. The

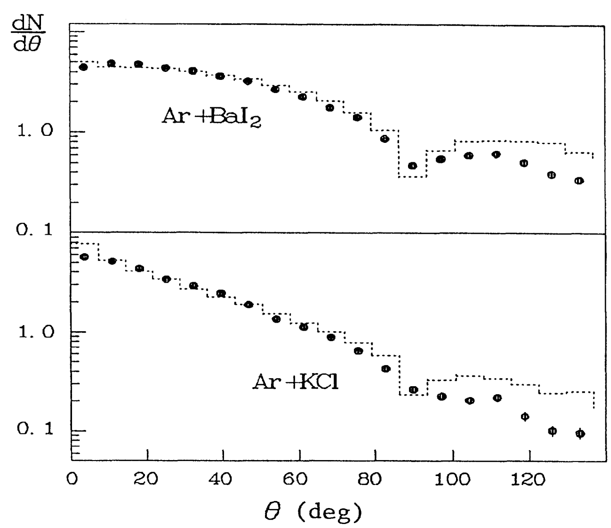

FIG. 1. Distributions of laboratory polar angle for fragments from 1.2A-GeV Ar+KCl events with $M^*\geq 16$ and from 1.2A-GeV Ar+BaI_2 events with $M^*\geq 17$. The solid circles show the experimental data, and the dotted line denotes the VUU calculation for the soft EOS.

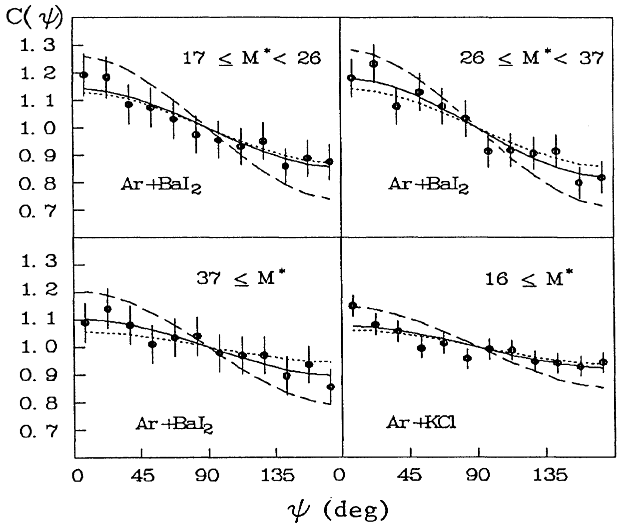

FIG. 2. Azimuthal correlation function for fragments with rapidity $y_{\rm lab} \geq 0.75 y_{\rm beam}$. The solid circles show the experimental data, and the solid line indicates the fit to these data using Eq. (5) with $A=1$. The dotted and dashed lines indicate the fits to the VUU calculations for a soft and hard EOS, respectively.

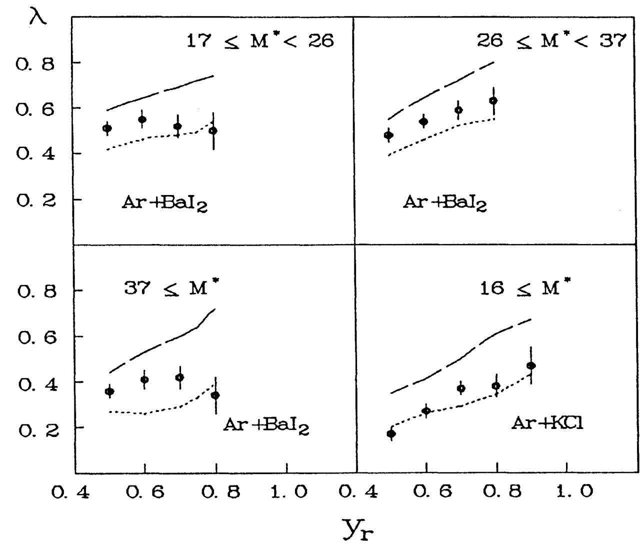

FIG. 3. Fitted λ values as a function of the rapidity cut used to select fragments emitted forward in the center-of-mass frame, where $y_r = y/y_{\rm beam}$ evaluated in the laboratory frame. As before, the dotted and dashed lines denote the VUU calculation for soft and hard equations of state.

effect of energy loss and absorption in the target are incorporated in the VUU calculation in Fig. 1; nevertheless, discrepancies are evident near $\theta = 90^\circ$ and above, where target spectators dominate. These discrepancies again can be attributed at least in part to the absence of clustering in the model. Between the two spectator-dominated regions, the detector filter does not have an important influence on $dN/d\theta$ predictions and the agreement between VUU and experiment is good; hence we define a reduced multiplicity M^*, counting only fragments with $8^\circ < \theta < 85^\circ$. Fragments outside this range are not included in any subsequent analysis.

Figure 2 shows the azimuthal correlation function for fragments with rapidity greater than $0.75 y_{\rm beam}$ for Ar+BaI_2 in three M^* intervals, each containing about 260 events, and for Ar+KCl in a single M^* interval. The solid, dotted, and dashed curves are χ^2-minimized fits of Eq. (5) to the experiment and to the VUU predictions with soft and hard EOS, respectively. The fitted value λ and its error Δλ for each of these curves, and the maximum azimuthal anisotropy R and its error ΔR for each case, are listed in Table I. As the reaction plane is known *a priori* in the case of the model, the maximum azimuthal anisotropy can also be calculated from Eqs. (1) and (2). The results of this calculation are tabulated as R_d. The R and R_d values listed in Table I agree within statistical errors. This finding is consistent with the azimuthal correlation function analysis being unaffected by dispersion effects. Another consequence of this property of the azimuthal correlation function is that random inefficiencies for finding tracks only reduce statistics, whereas under the same circumstances, an analysis based on a determination of the event reaction plane would suffer increased dispersion and generally larger systematic uncertainties. For example, if we randomly discard 40% of the particles in each Ar+KCl event, the maximum azimuthal anisotropy R calculated by the azimuthal correlation function analysis is 2.3±0.4 (experiment),

TABLE I. Best-fit parameters for the azimuthal distributions shown in Fig. 2.

	Ar+BaI_2, $y_{\rm lab} \geq 0.75 y_{\rm beam}$									Ar+KCl, $y_{\rm lab} \geq 0.75 y_{\rm beam}$		
	$17 \leq M^* < 26$			$26 \leq M^* < 37$			$37 \leq M^*$			$M^* \geq 16$		
	expt	hard	soft	expt	hard	soft	expt	hard	soft	expt	hard	soft
λ	0.54	0.73	0.51	0.61	0.76	0.54	0.46	0.64	0.33	0.39	0.54	0.35
Δλ	0.05	0.03	0.03	0.04	0.02	0.02	0.06	0.03	0.03	0.04	0.02	0.02
R	3.3	6.4	3.1	4.1	7.3	3.3	2.7	4.6	2.0	2.3	3.3	2.1
ΔR	0.5	0.8	0.2	0.5	0.7	0.2	0.4	0.5	0.1	0.2	0.2	0.1
R_d		6.3	3.4		6.9	3.8		4.2	2.1		3.5	2.2
ΔR_d		0.6	0.4		0.8	0.4		0.5	0.2		0.3	0.2

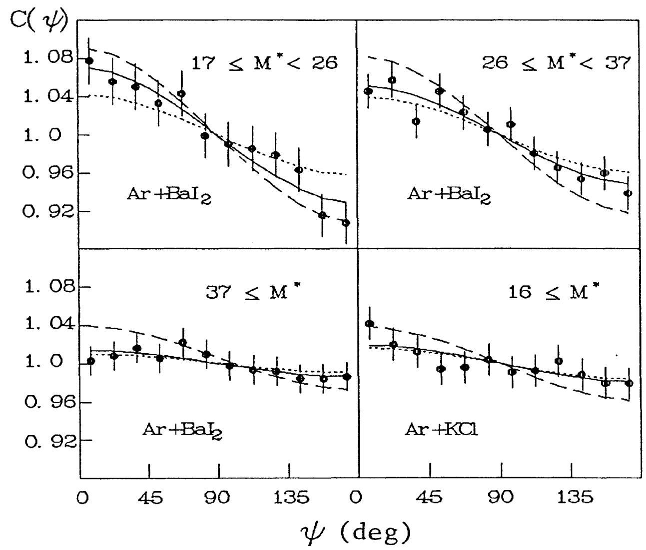

FIG. 4. As in Fig. 2, except that fragments up to a maximum polar angle ($\theta_{up}=29°$ in the case of Ar+KCl and $\theta_{up}=34°$ in the case of $Ar+BaI_2$) have been included in the analysis, rather than fragments selected by a rapidity cut.

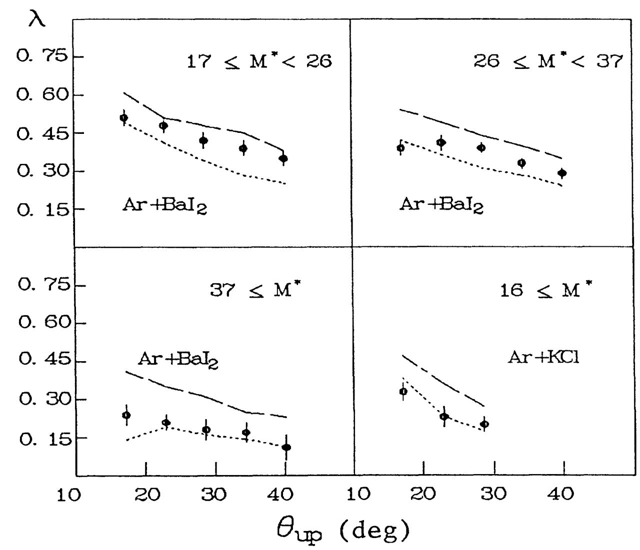

FIG. 5. As in Fig. 3, except that fragments up to a maximum polar angle θ_{up} have been included in the analysis, rather than fragments selected by a rapidity cut.

2.1±0.3 (soft EOS), and 3.5±0.4 (hard EOS). These results are consistent within statistical errors with those obtained when all tracks were used in the same analysis.

Figure 3 shows best-fit values of λ, the parameter in the azimuthal correlation function, for different forward rapidity intervals. Inferred values of the EOS stiffness are generally intermediate between hard and soft. Some nuclear transport models include a prescription for incorporating momentum-dependent interactions (MDI) [12]; the effect of adding MDI is to consistently enhance the flow signature for a given EOS. On the other hand, possible modification [9,13–15] of nucleon-nucleon collision cross sections in the nuclear medium beyond the final-state Pauli blocking already incorporated in current transport models could either increase or decrease flow signature for a given EOS. Hence it is probably premature to reach any definitive conclusion about the stiffness of the EOS. At the present time, two main inferences can be drawn from the VUU model comparisons: first, in common with an earlier analysis of the same data [9], our results do not consistently favor the same EOS for all combinations of target mass, multiplicity, and rapidity interval; second, the azimuthal correlation function method provides a sensitivity to the EOS that is comparable to the conventional transverse flow analysis [9,16].

To illustrate that a useful flow analysis is possible even if fragment identification information is completely disregarded, the above analysis has been repeated using a polar angle (θ) cut in place of the rapidity cut. In Fig. 4 the upper limit of the analyzed polar angle range is 34° for $Ar+BaI_2$ and 29° for Ar+KCl. The parameters for the curves in Fig. 4 and the corresponding maximum azimuthal anisotropies are listed in Table II. Figure 5 presents the same results as shown in Fig. 3, except that the horizontal axis gives the upper limit of the polar angle range instead of the lower limit of the rapidity range. As expected, the azimuthal correlation function analysis with a θ cut results in lower values of λ and somewhat poorer sensitivity to the EOS, although the qualitative features of the comparison are largely unchanged. This form of

TABLE II. Best-fit parameters for the azimuthal distributions shown in Fig. 4.

	$Ar+BaI_2$, $\theta_{lab}\le 34°$, $17\le M^*<26$			$26\le M^*<37$			$37\le M^*$			Ar+KCl, $\theta_{lab}\le 29°$, $M^*\ge 16$		
	expt	hard	soft	expt	hard	soft	expt	hard	soft	expt	hard	soft
λ	0.38	0.43	0.29	0.33	0.40	0.29	0.17	0.25	0.14	0.20	0.28	0.18
$\Delta\lambda$	0.03	0.02	0.02	0.03	0.02	0.02	0.04	0.02	0.02	0.03	0.02	0.02
R	2.2	2.5	1.8	2.0	2.3	1.8	1.4	1.7	1.3	1.5	1.8	1.4
ΔR	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
R_d		2.6	1.9		2.2	1.9		1.6	1.4		1.8	1.5
ΔR_d		0.1	0.1		0.1	0.1		0.1	0.1		0.1	0.1

flow analysis can readily be applied to emulsion measurements and raises the prospect of following the energy dependence of flow from Bevalac to Synchrophasotron and on to Alternating Gradient Synchrotron energies using existing emulsion data.

In summary, the azimuthal distribution of particles in the final state for collisions of Ar+KCl and $Ar+BaI_2$ at $1.2A$ GeV are studied using an azimuthal correlation function analysis. This method allows us to study collective flow with similar sensitivity compared with previous analyses, and because it involves only the angle between the transverse momenta of particle pairs, the complications associated with reaction plane dispersion in conventional flow analyses do not arise. Two alternative prescriptions for the azimuthal correlation function analysis are presented — one in which minimal use is made of fragment identification information and a second version in which particle identification is completely disregarded, but sensitivity to the nuclear equation of state is somewhat reduced. In either case, our experimental findings can readily be compared with models that do not incorporate final-state clustering, and there is no need for filtering of predictions beyond what is needed to simulate the experimental multiplicity selection.

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