

# The Necessity of Timekeeping in Adversarial Queueing

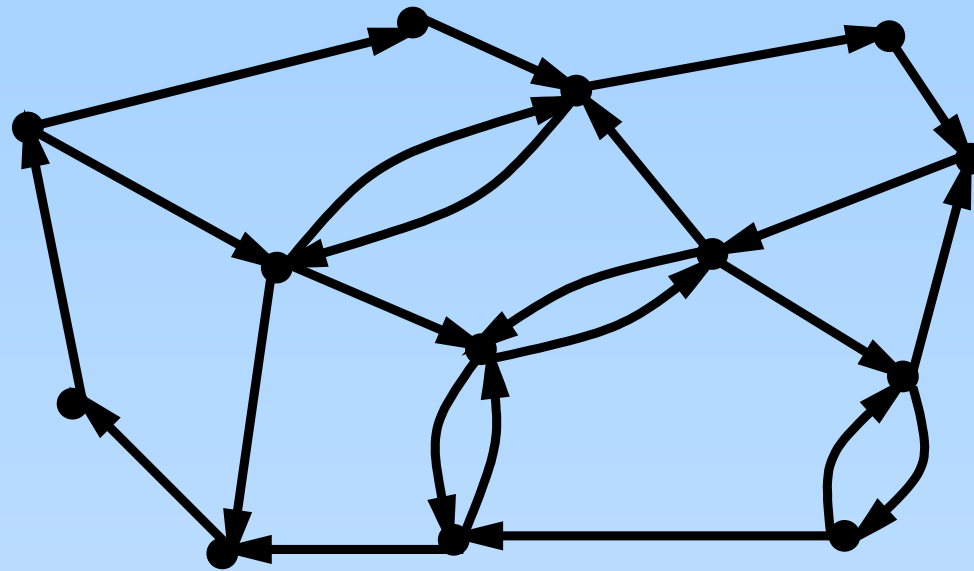
Maik Weinard  
Institute of Computer Science  
University of Frankfurt

13.5.05

# Routing

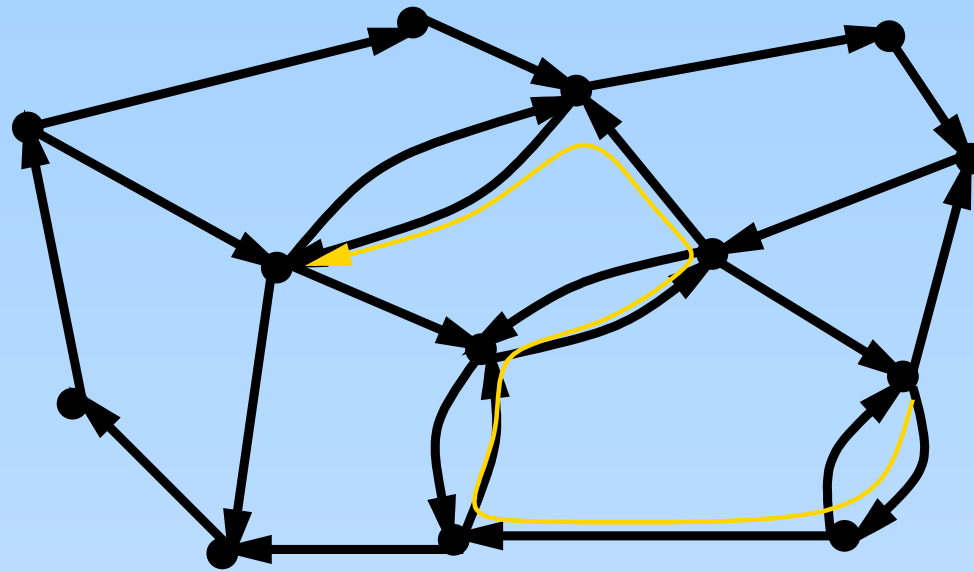
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Graph  $G$



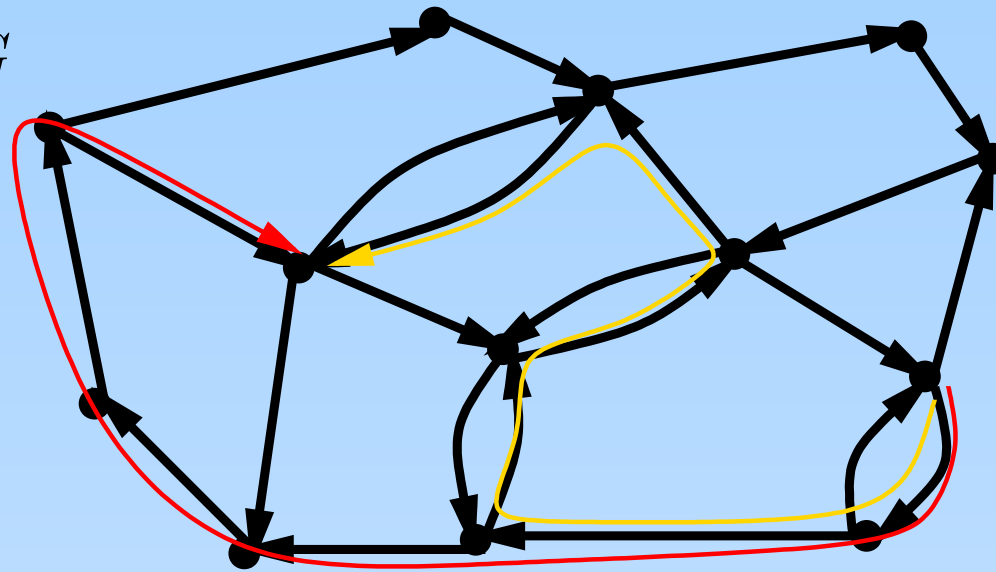
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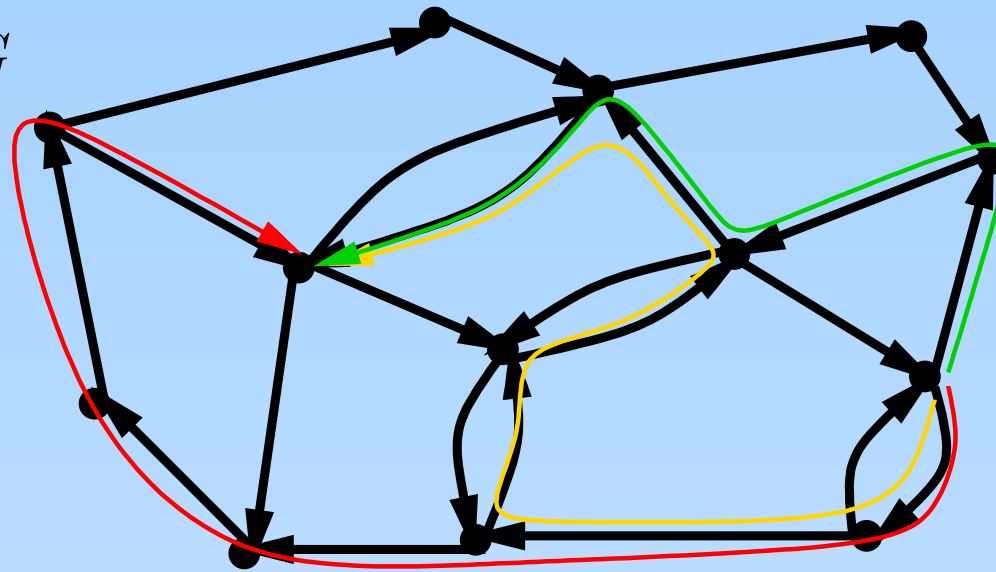
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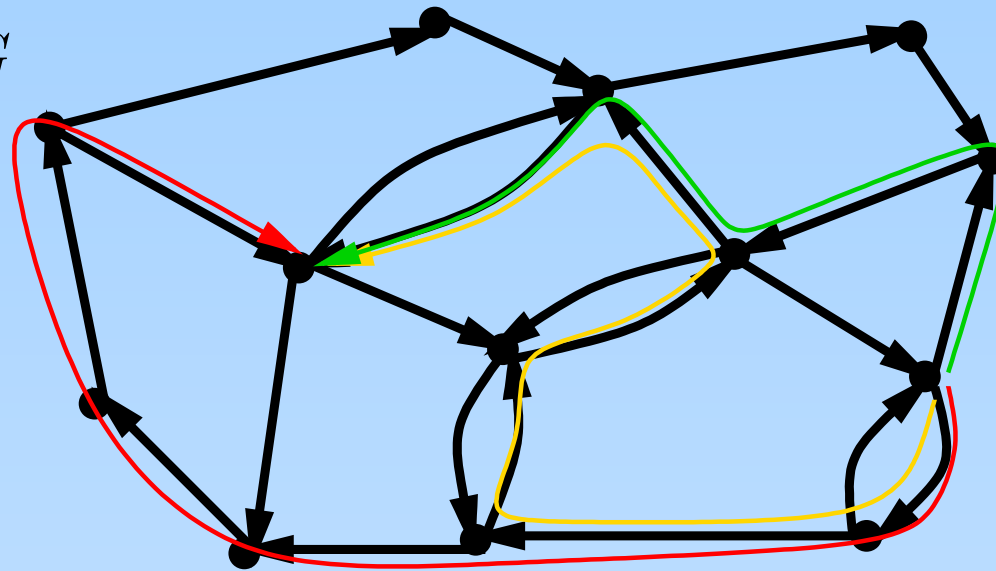
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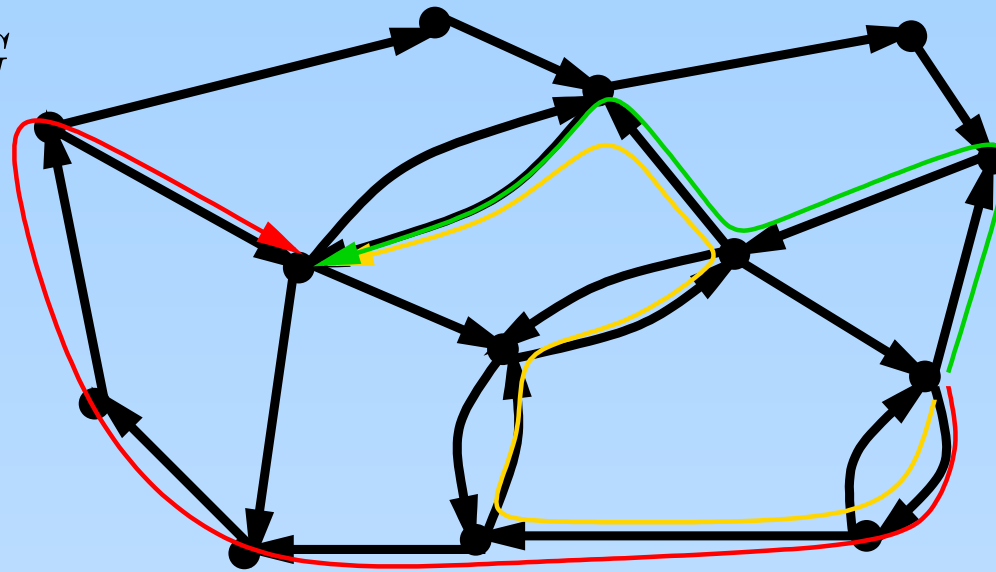
Graph  $G$



Routing policies assign simple paths.

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Routing policies assign simple paths.

We assume unit capacity and unit speed edges.



# Queueing

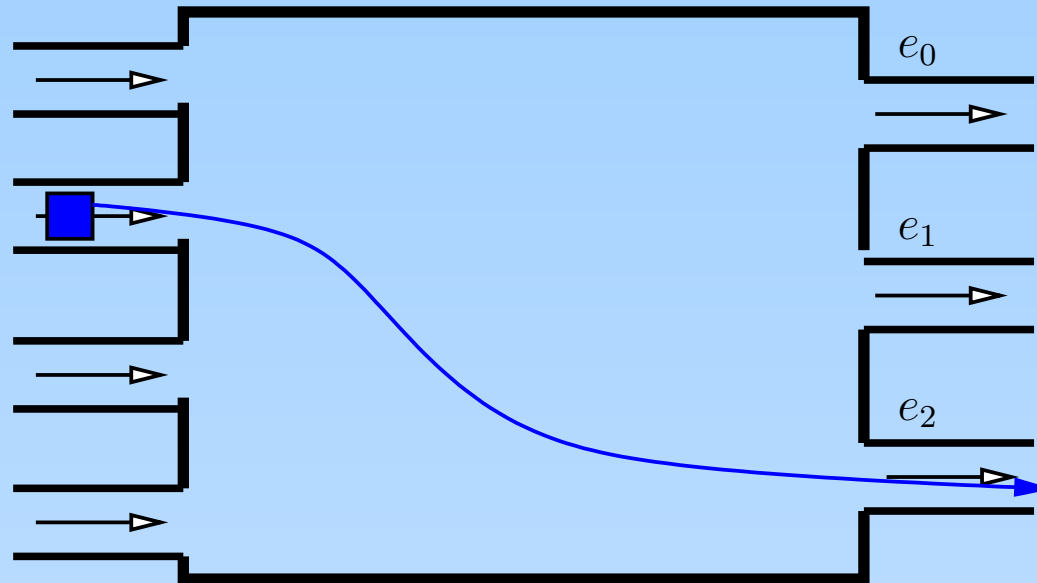
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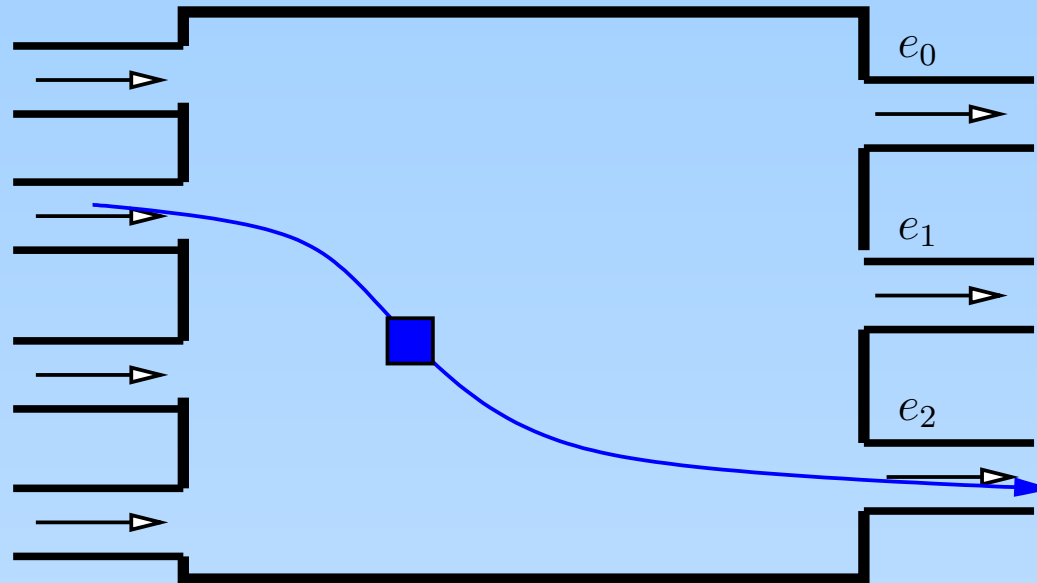
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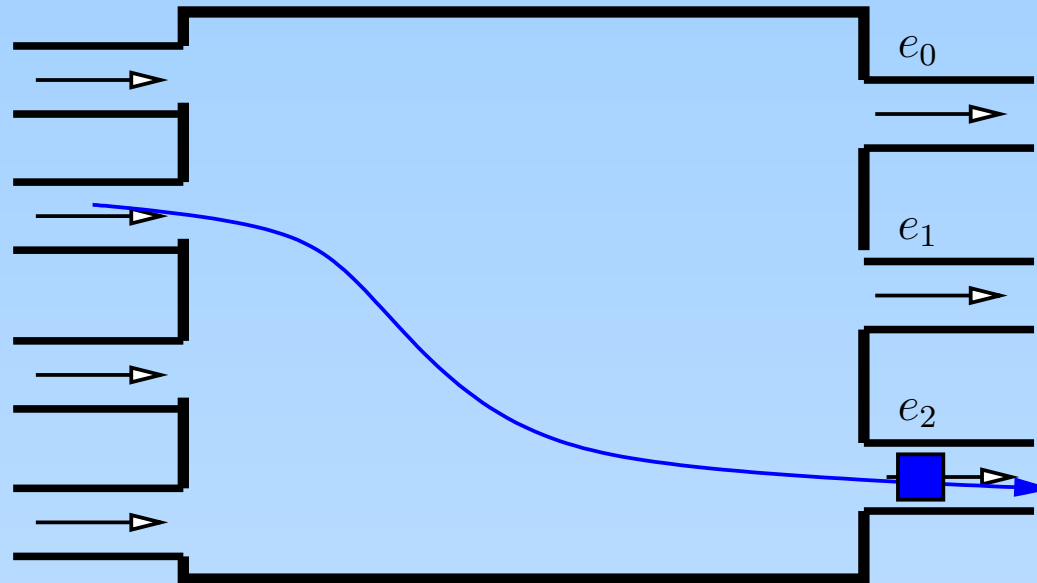
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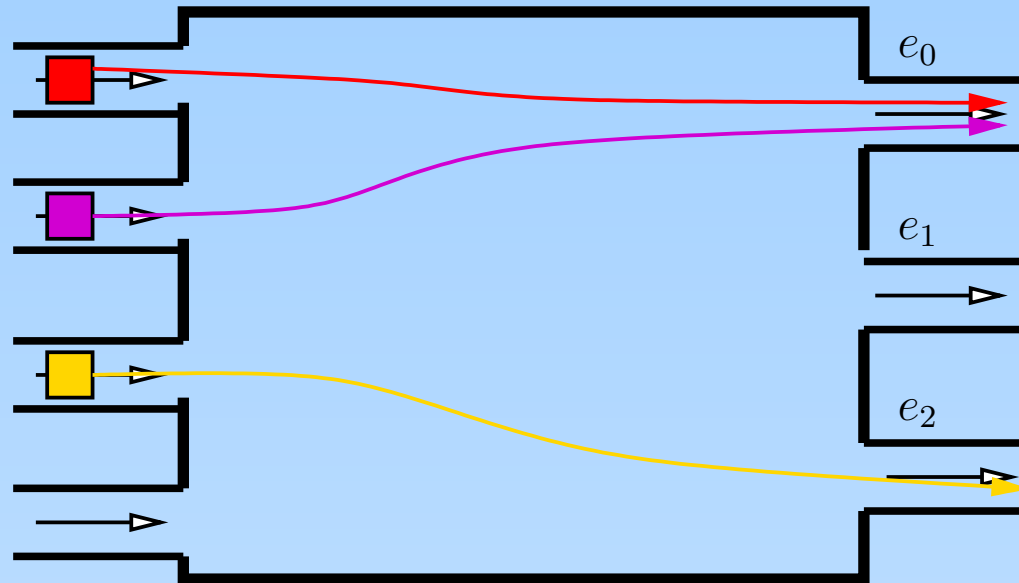
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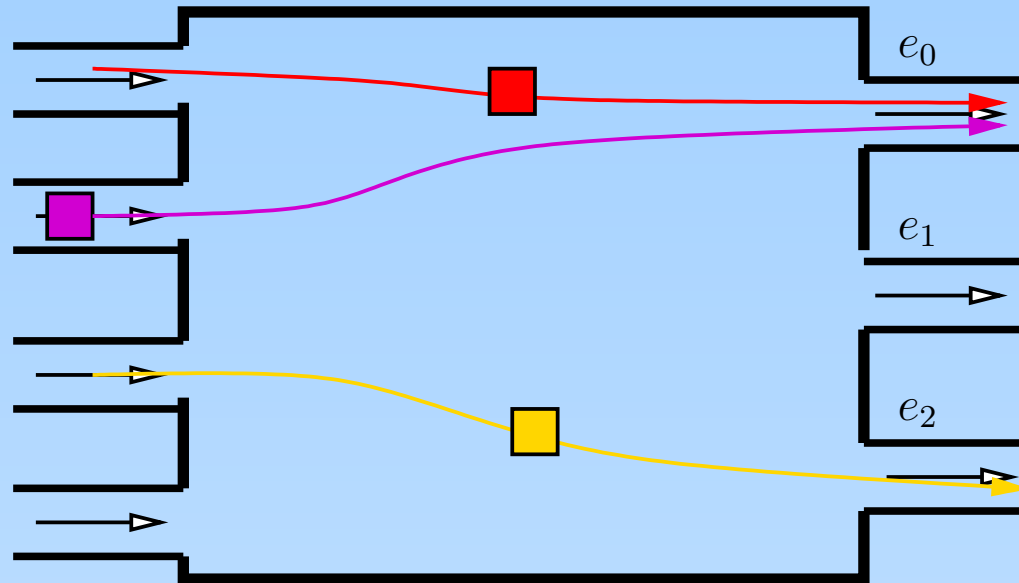


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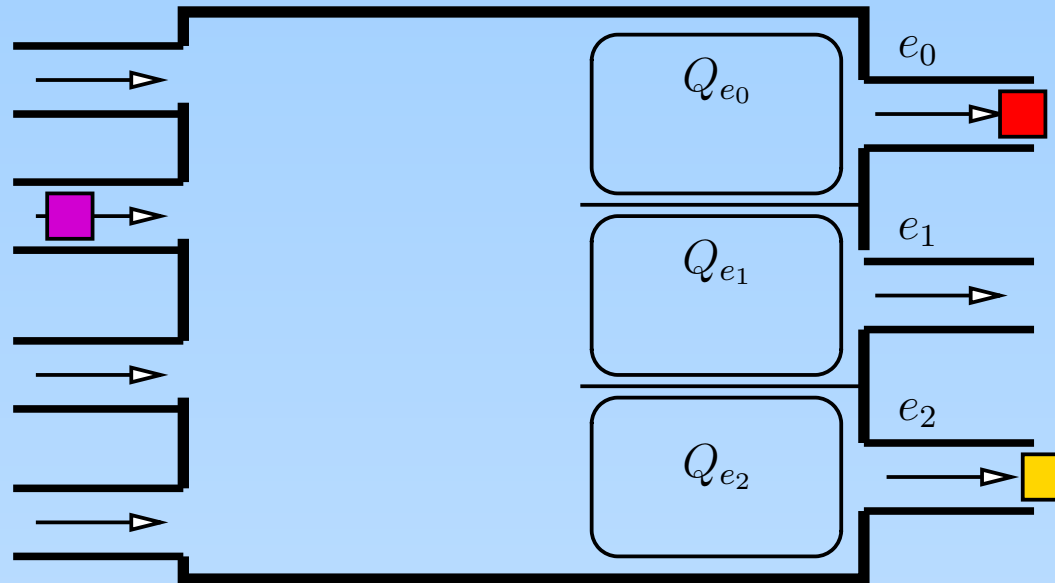
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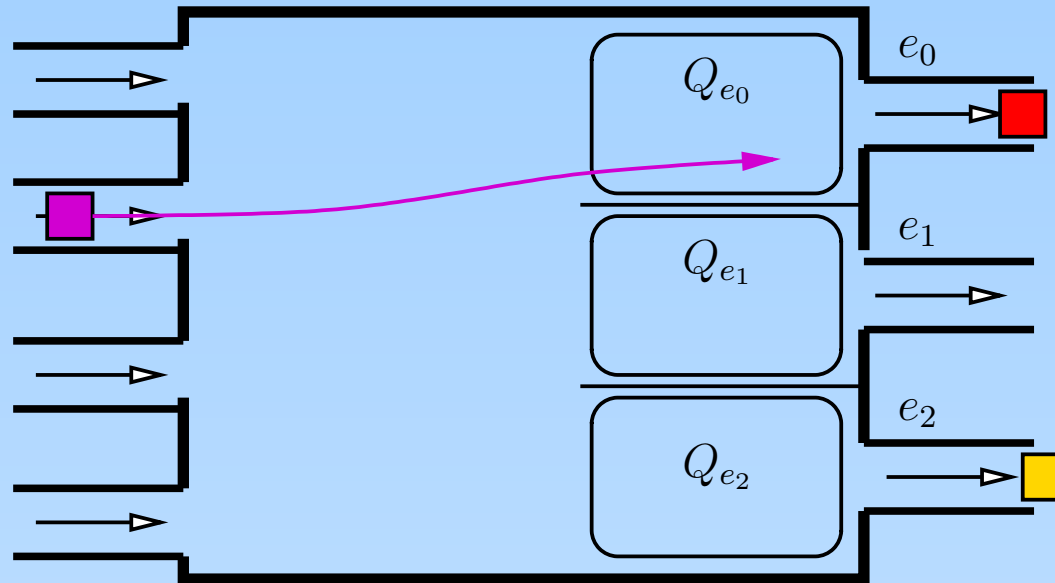
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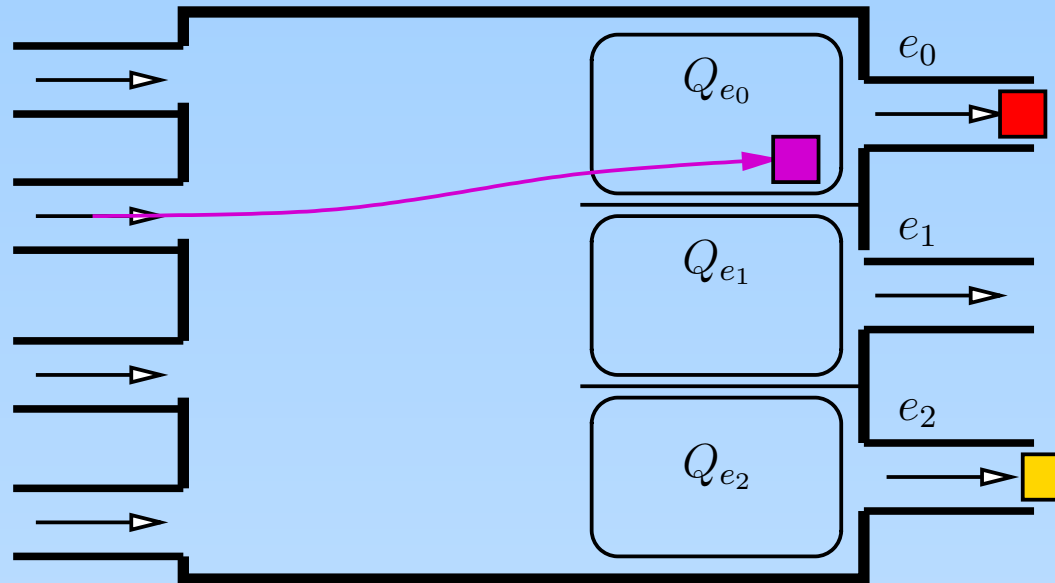
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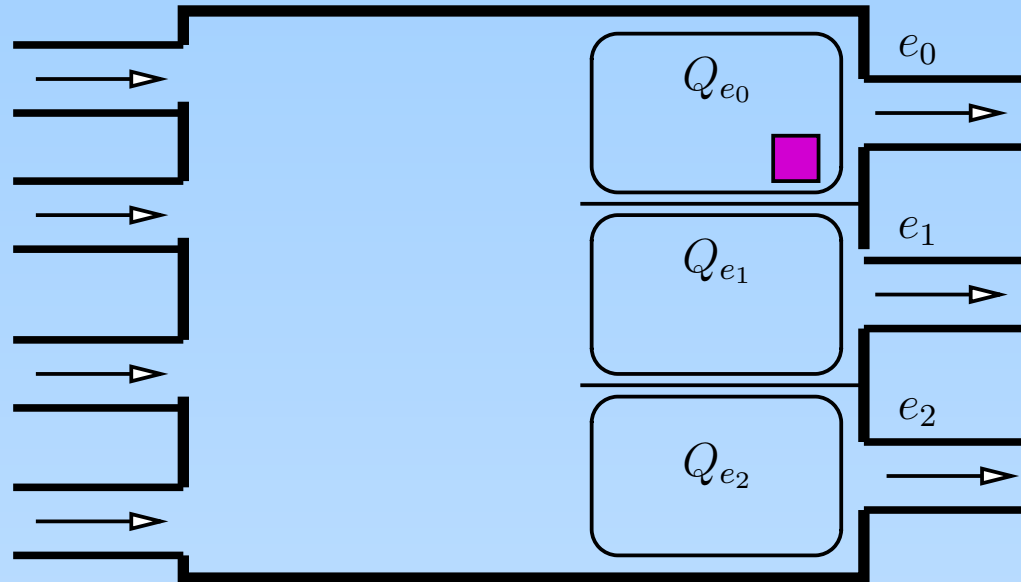
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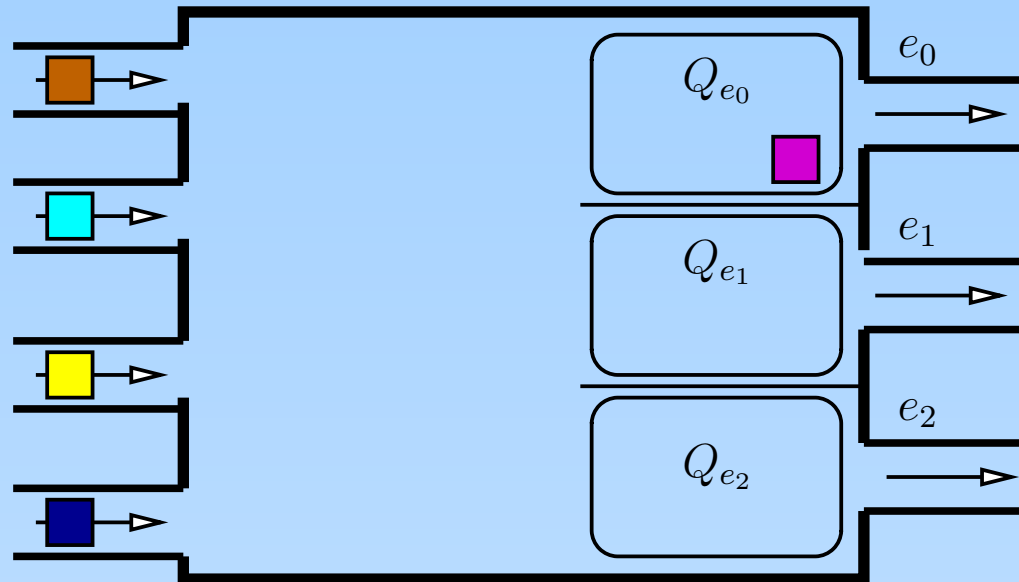
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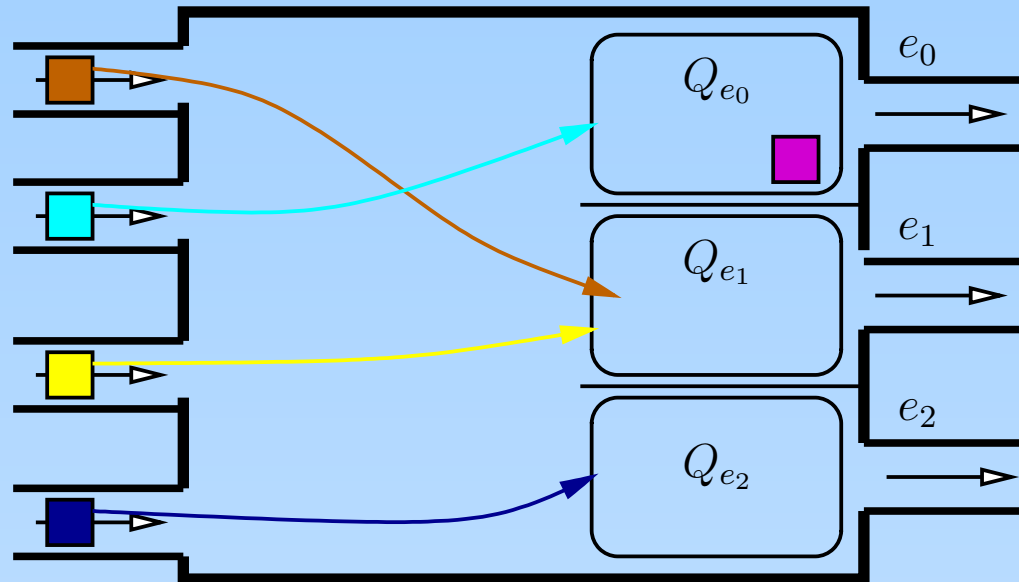
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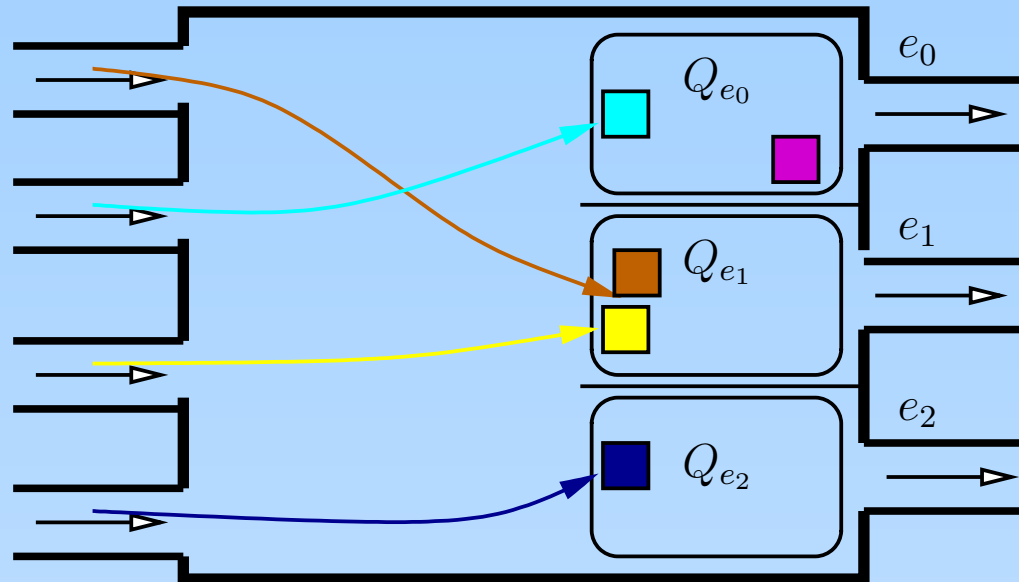


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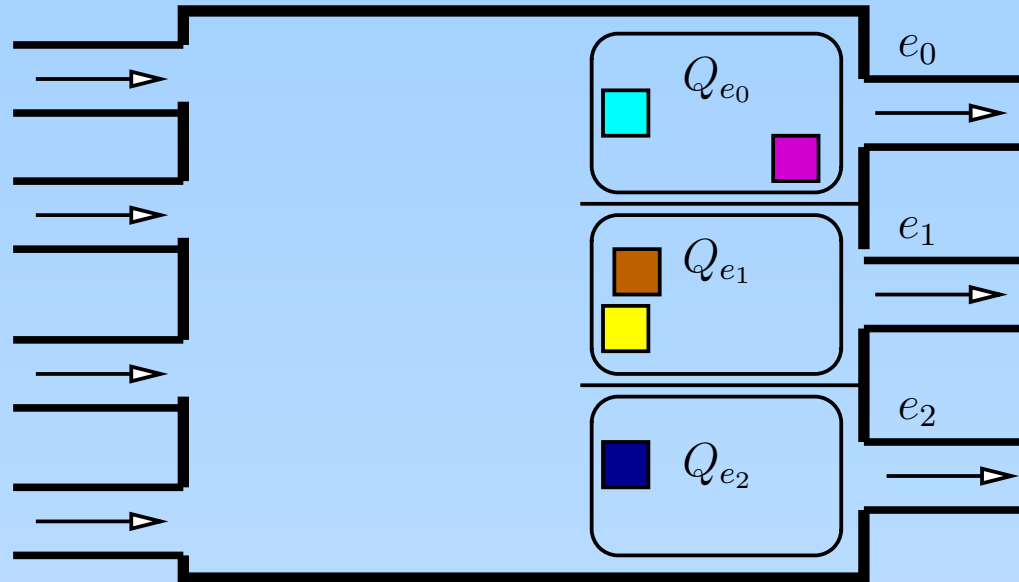




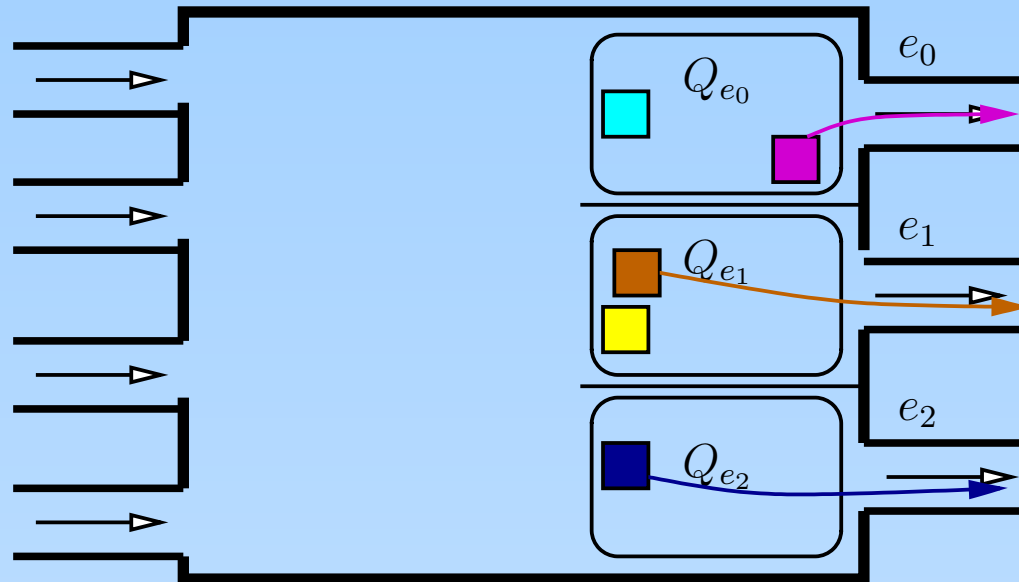
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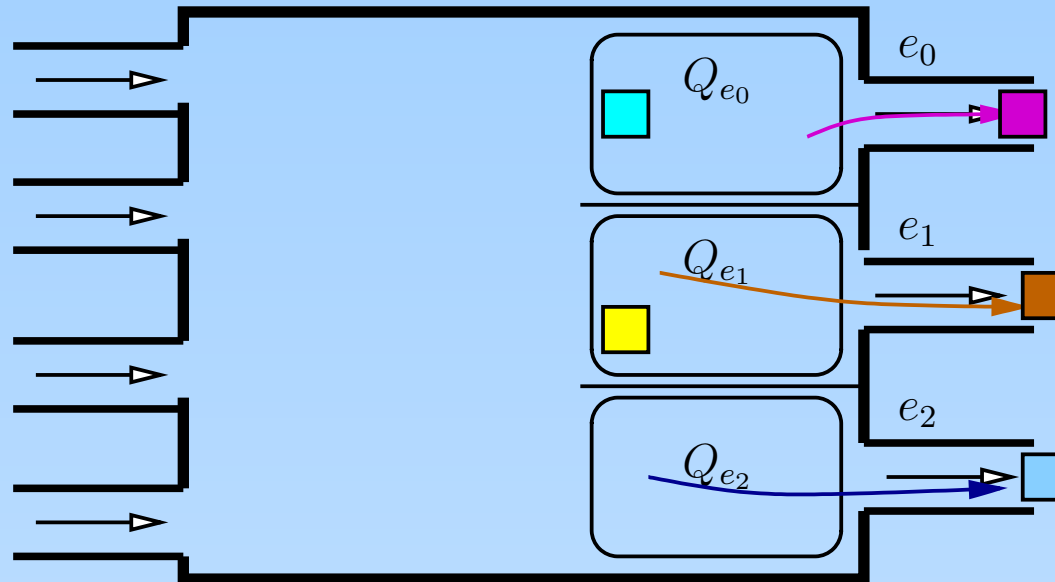
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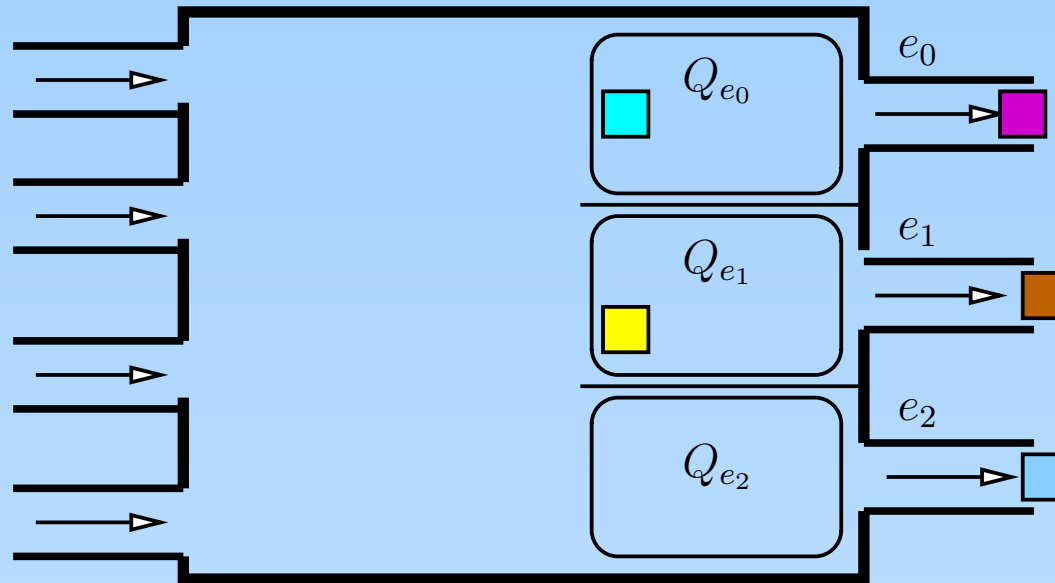
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Queueing strategies decide which packet may proceed.

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- **Only restriction:** Adversary may not straightforwardly overload edges.



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- $b$  is the burstiness.



# Stability



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A queueing policy is *stable* on a graph  $G$  against  $(r, b)$  adversaries, if for every sequence of insertions into  $G$  by an  $(r, b)$ -adversary the number of packets in  $G$  is upper bounded by  $c(r, b, G)$ .



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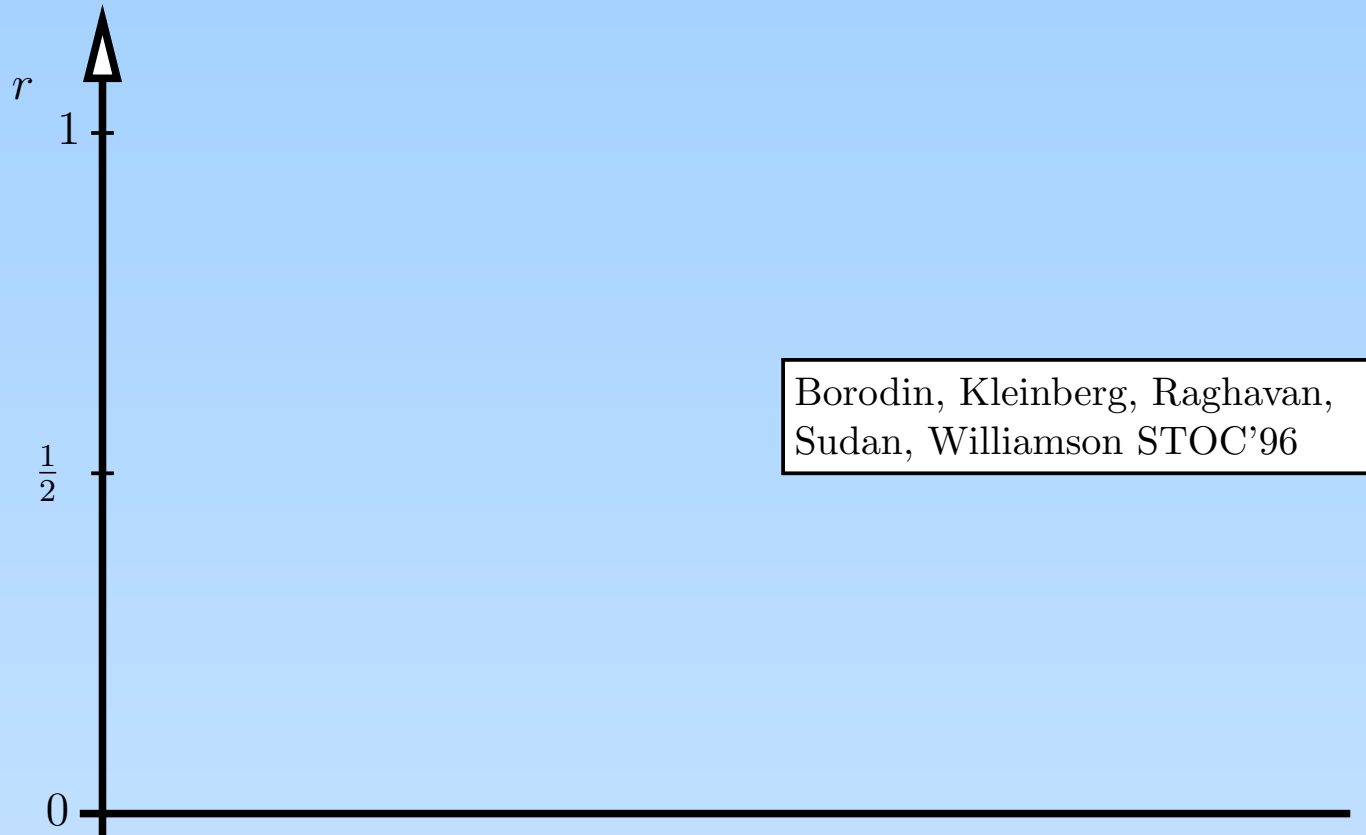
A queueing strategy is *universally stable* if it is stable on every graph against every  $(r, b)$ -adversary with  $r < 1$ .



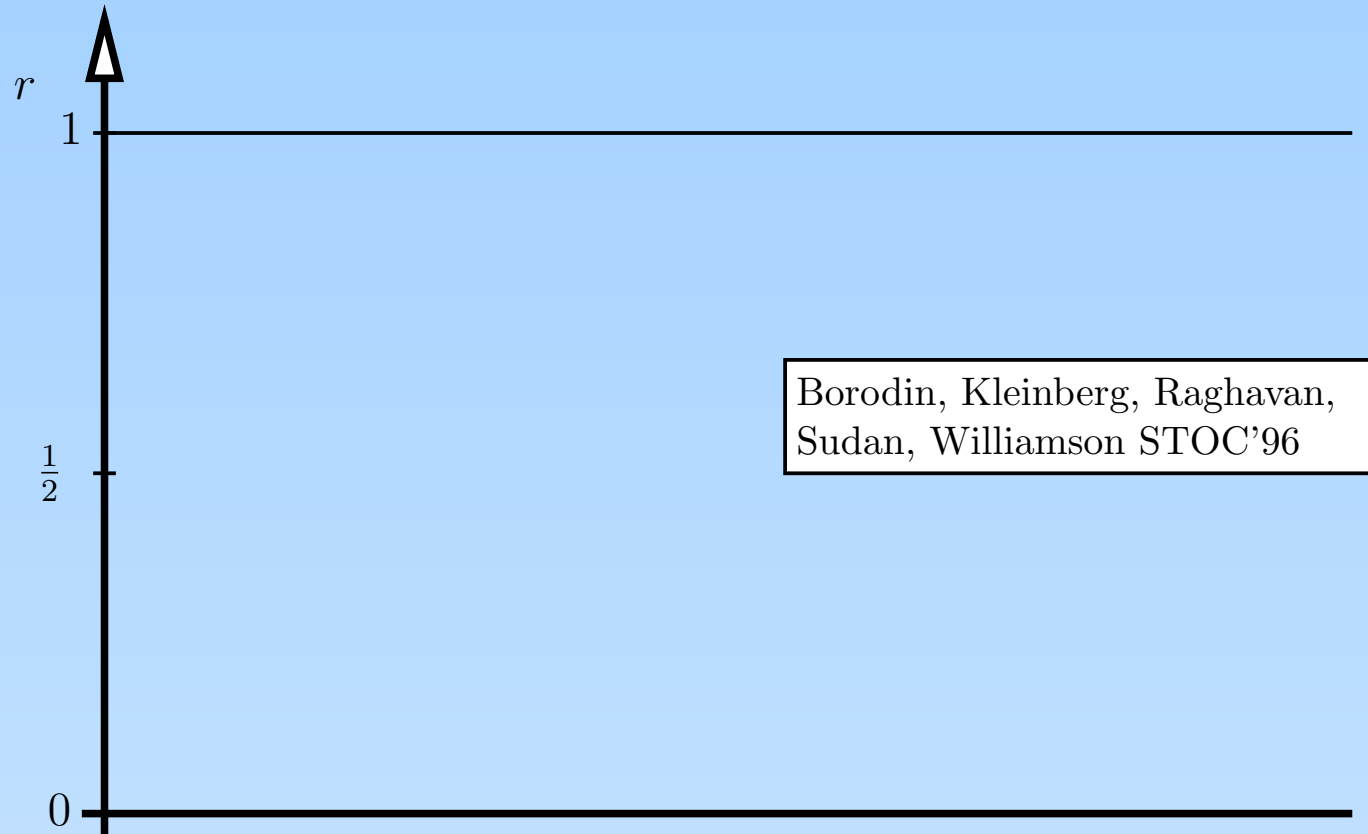


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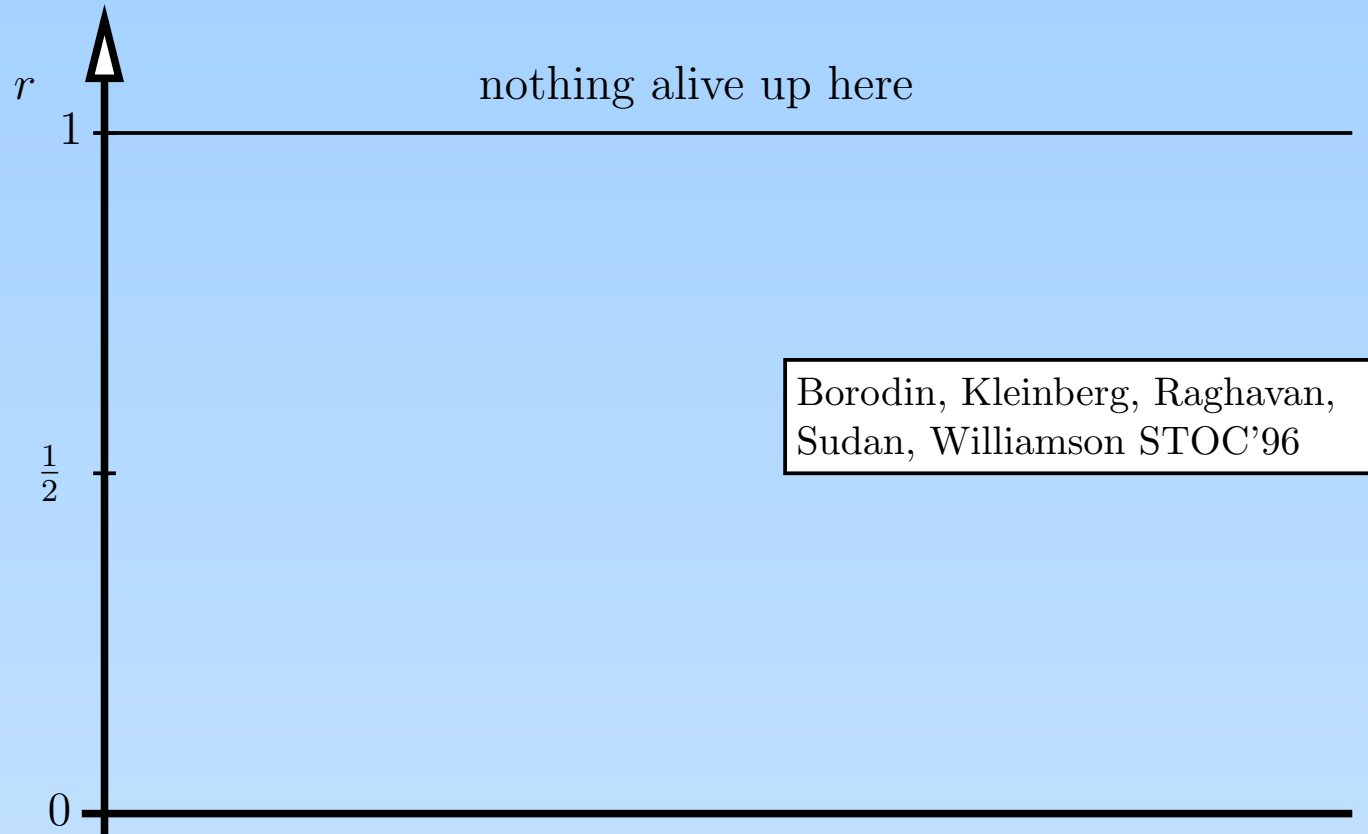
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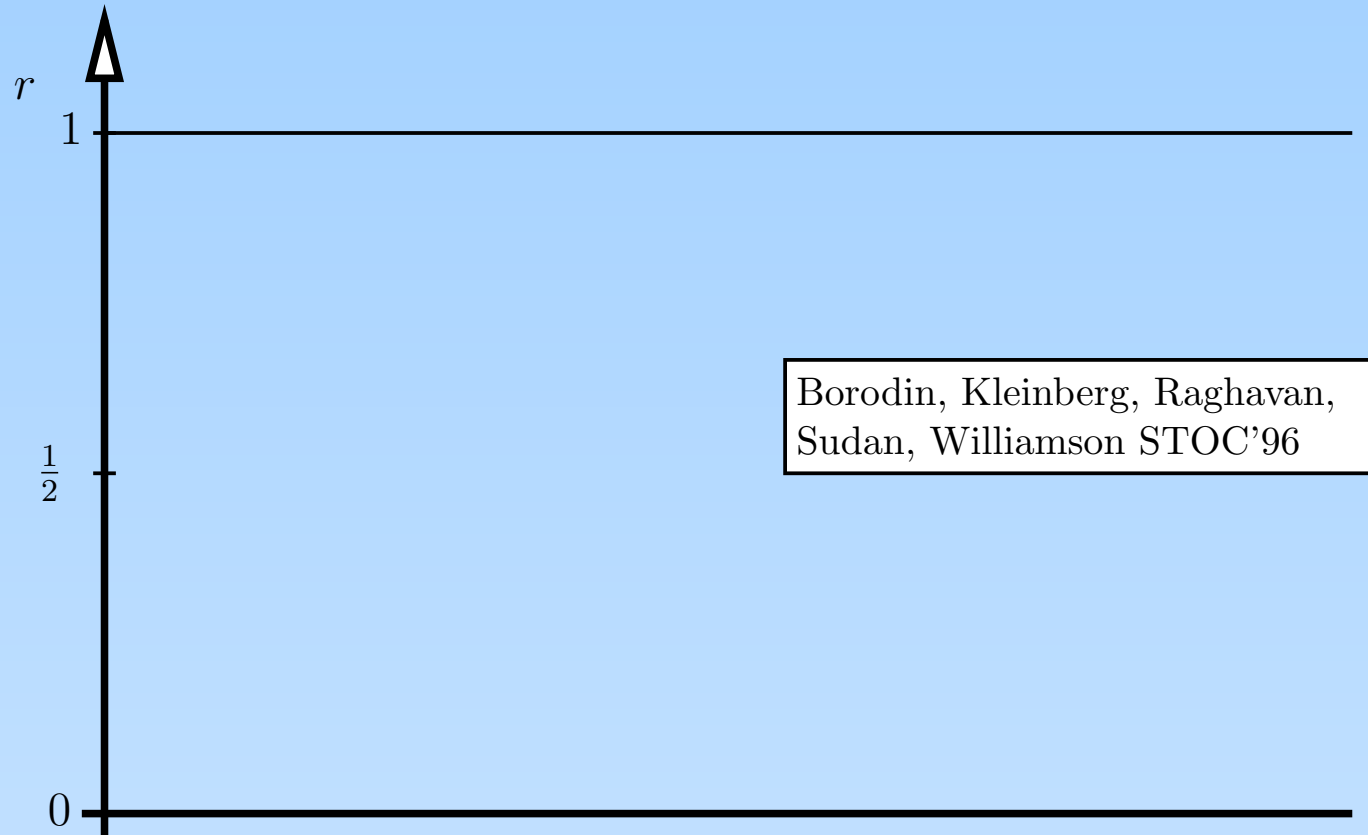
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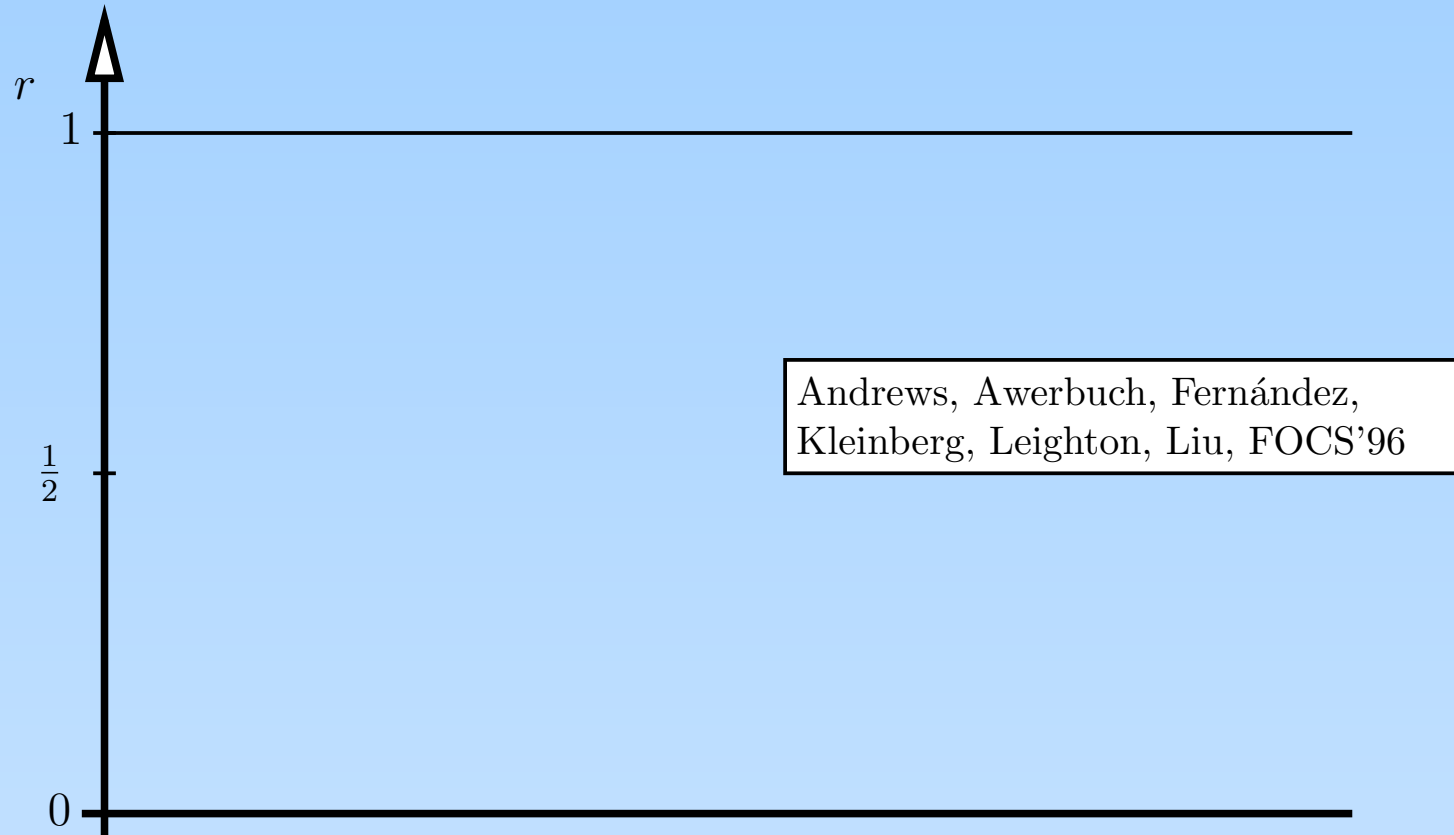
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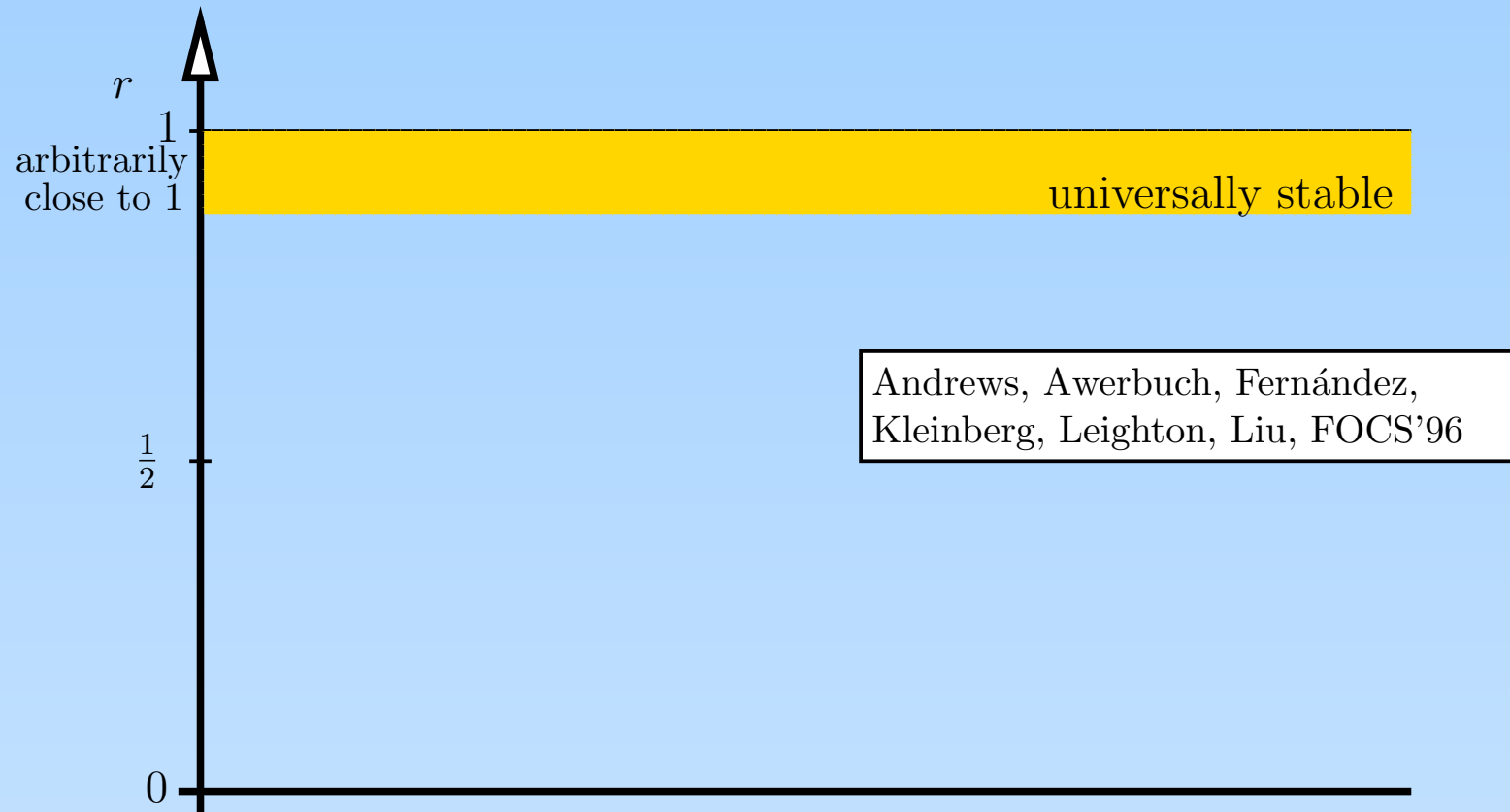
Borodin, Kleinberg, Raghavan,  
Sudan, Williamson STOC'96



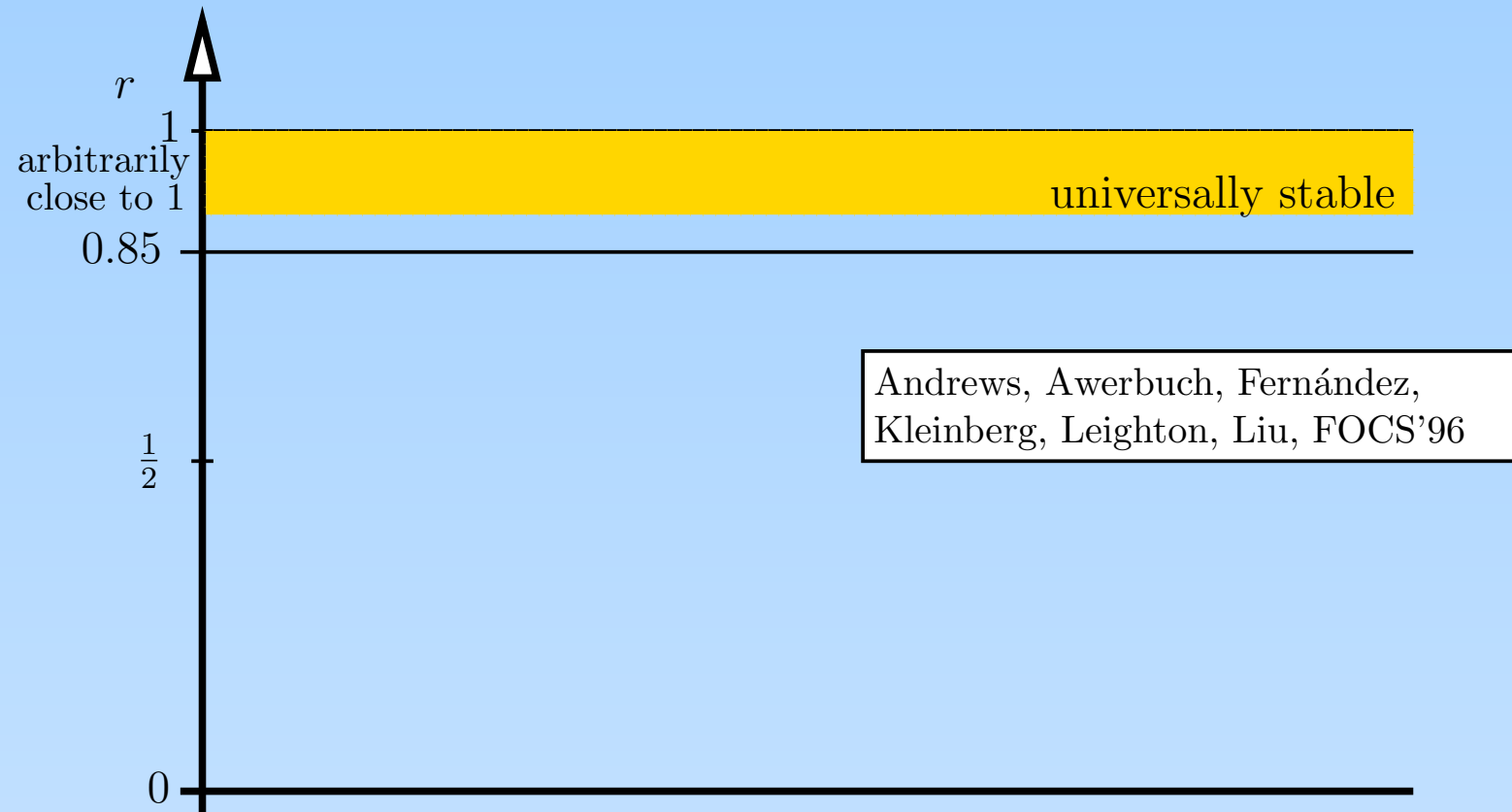
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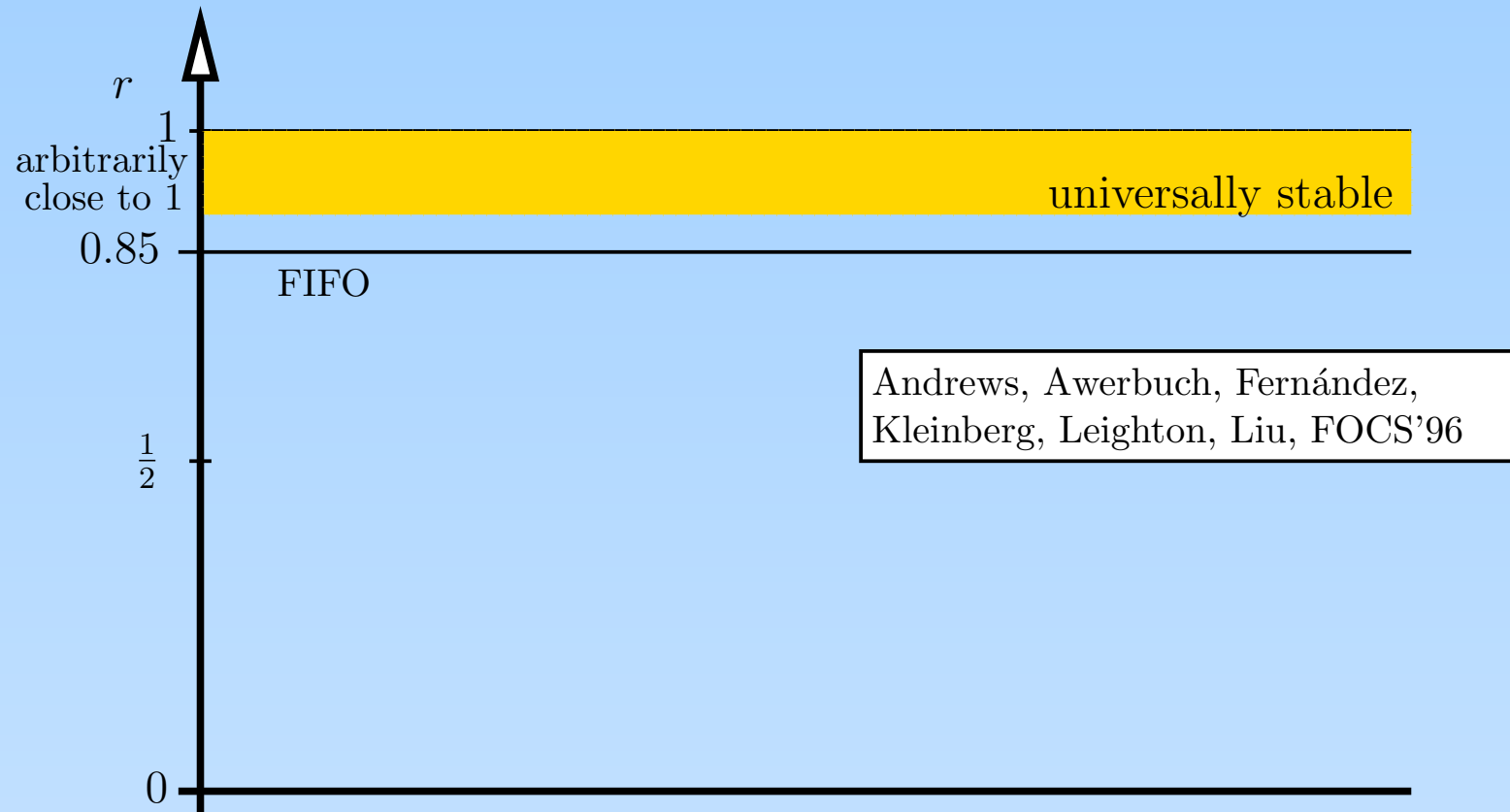


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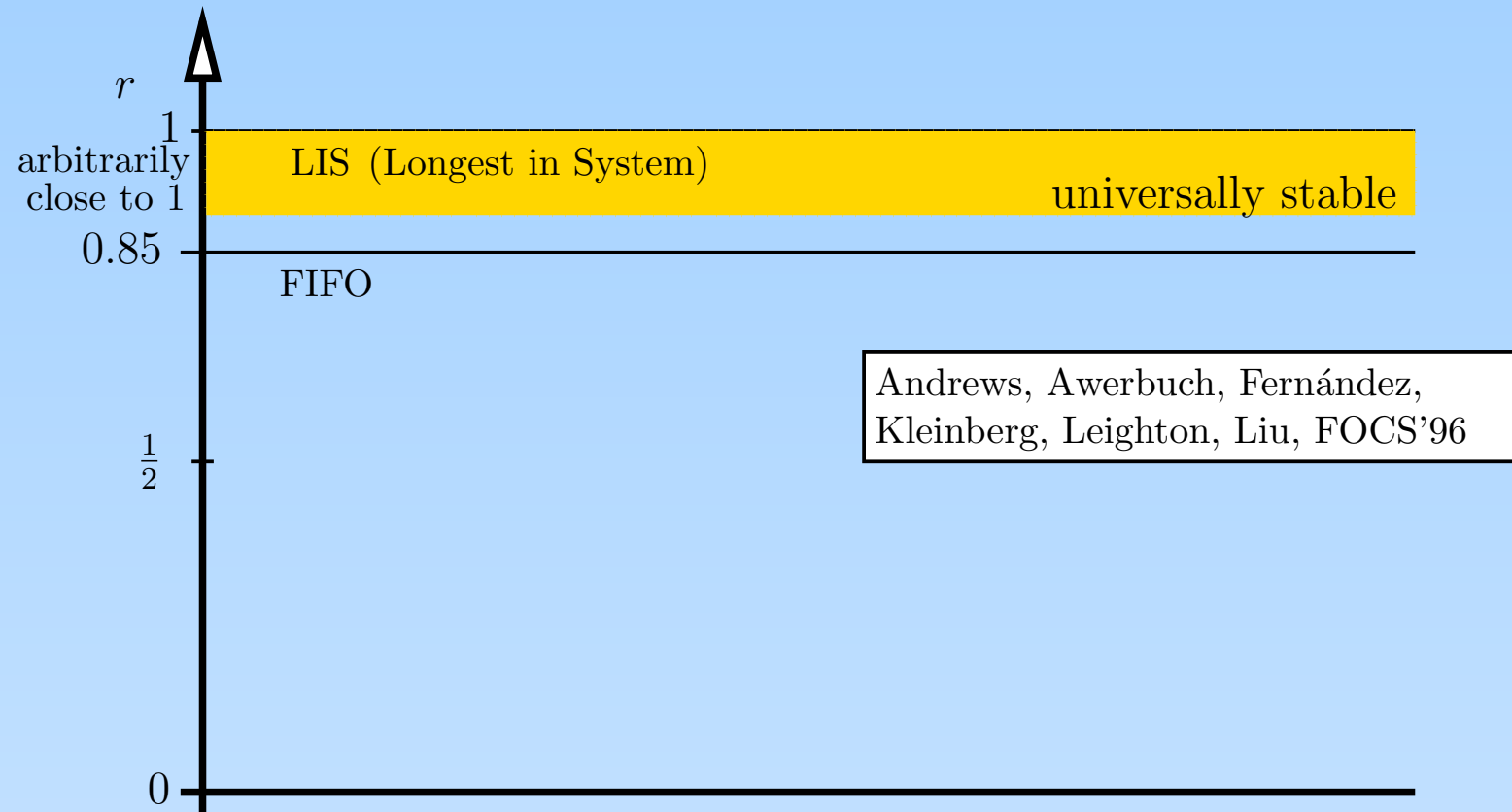




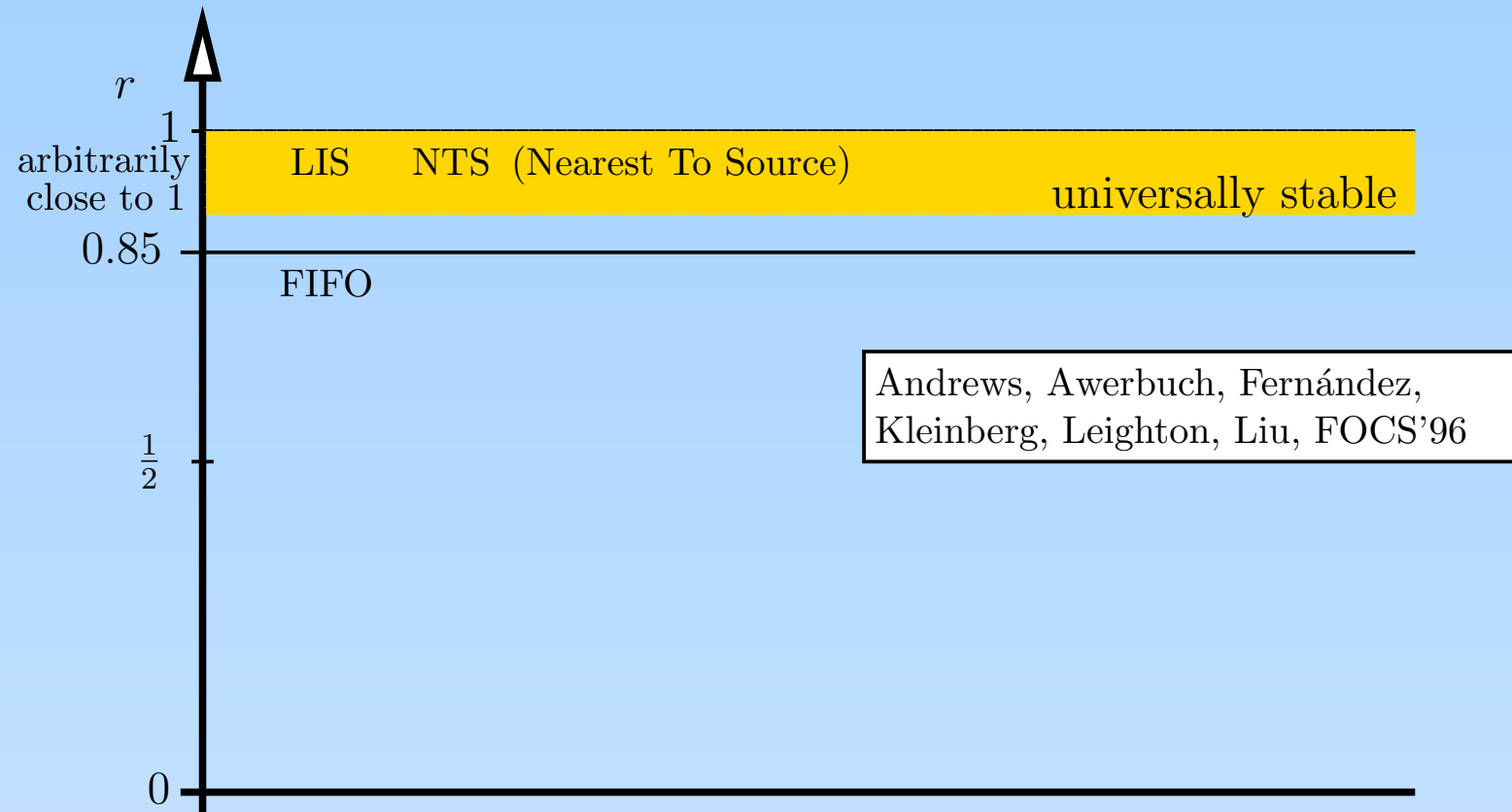
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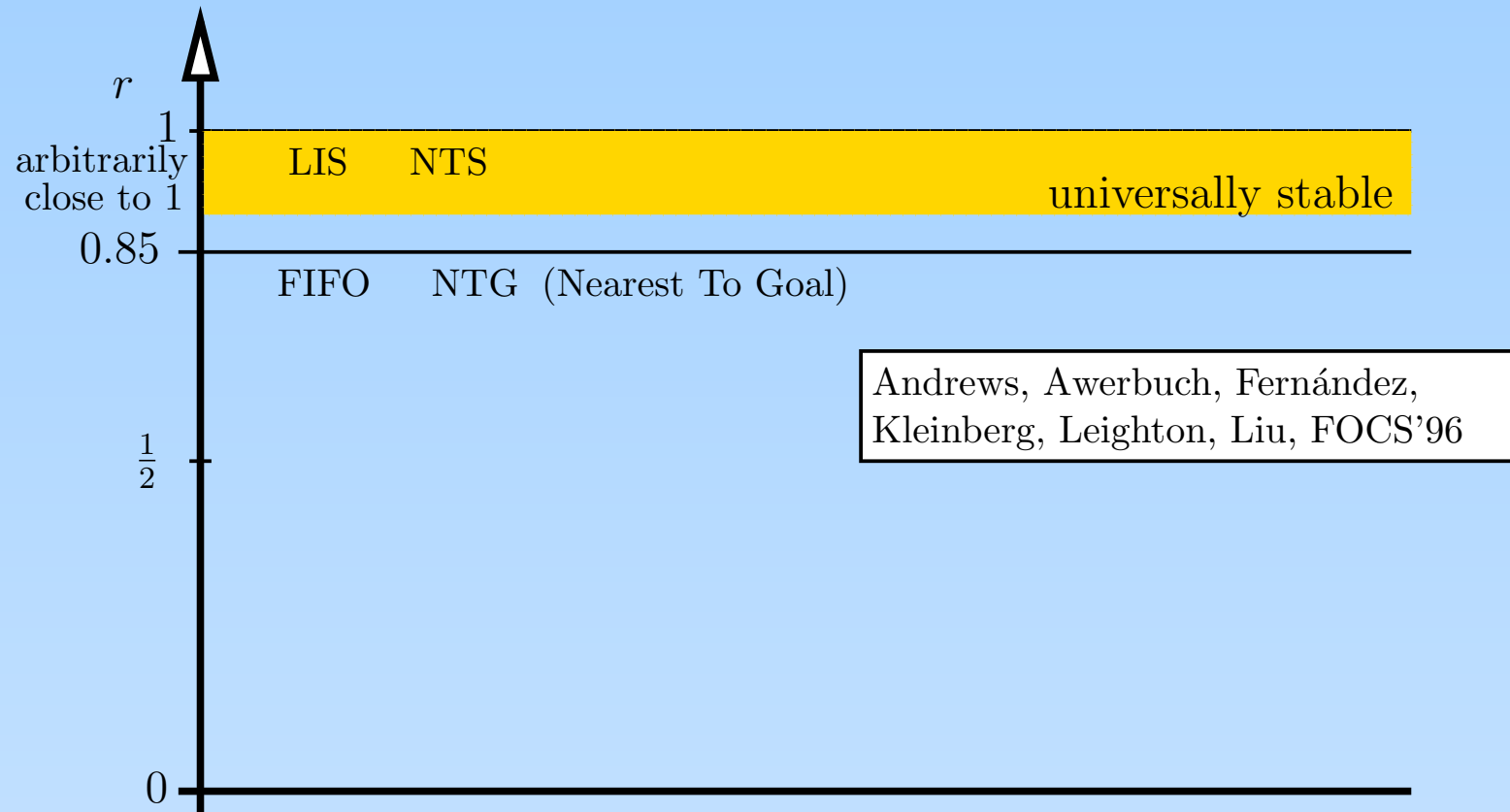
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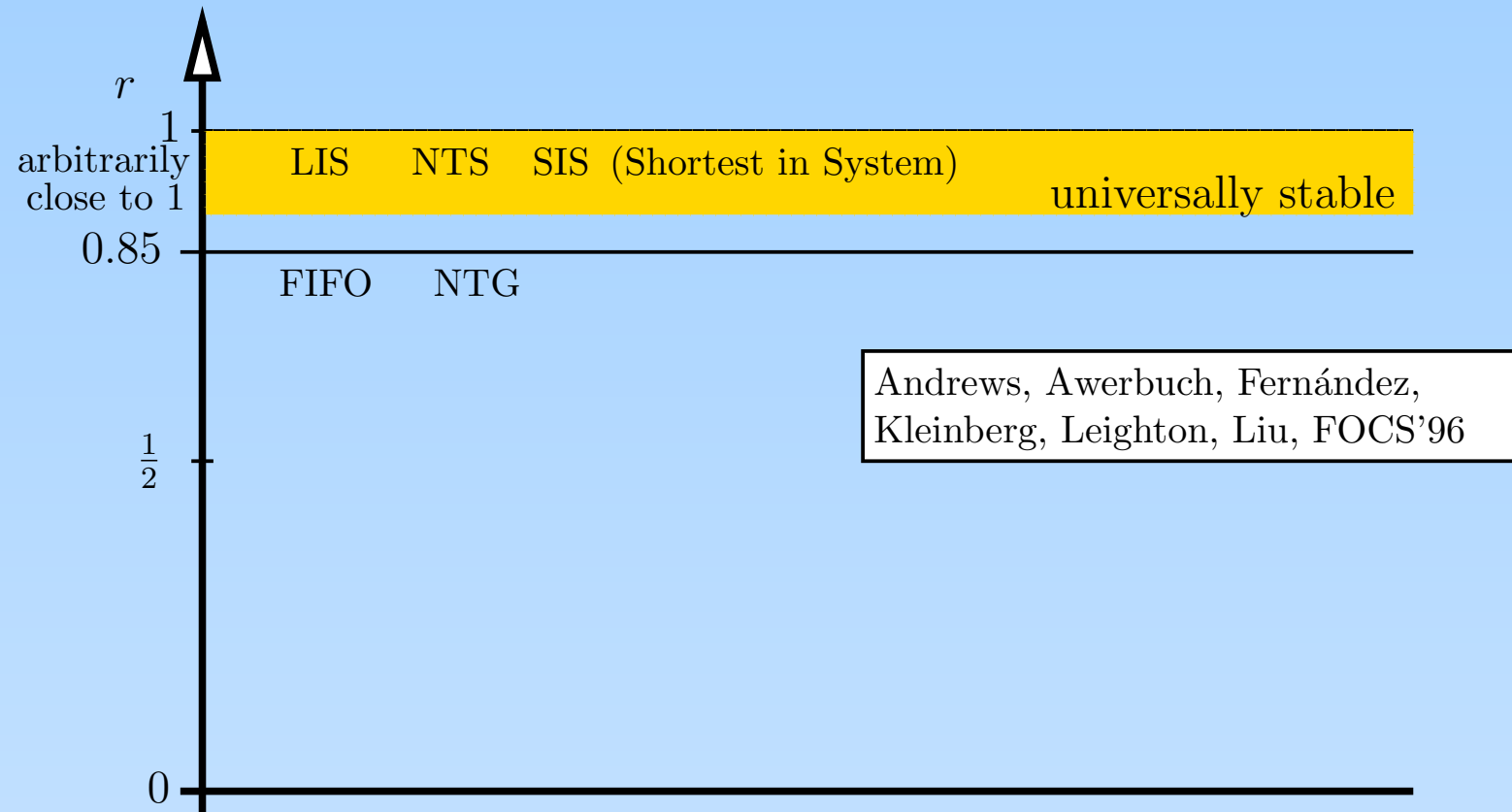
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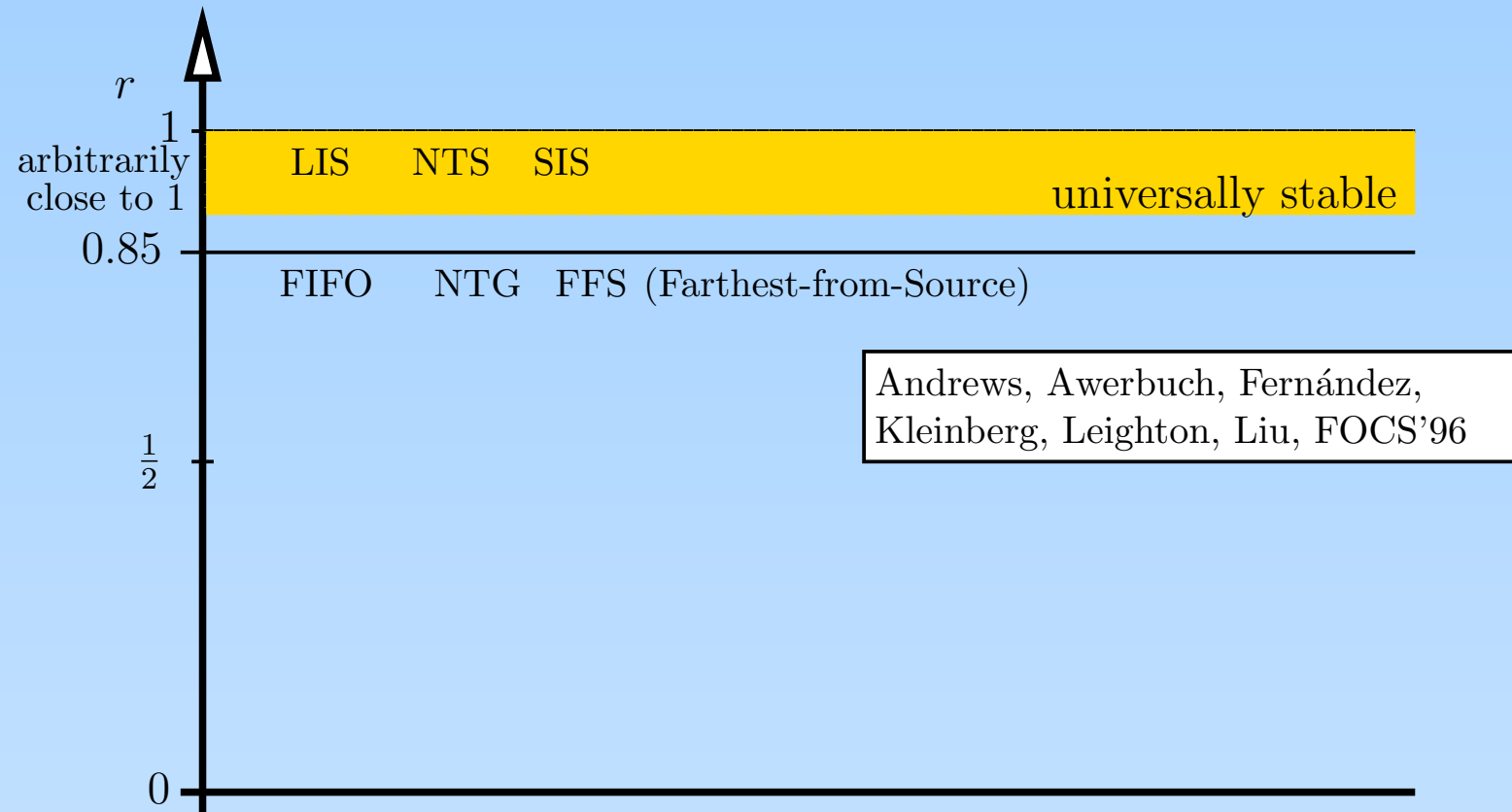
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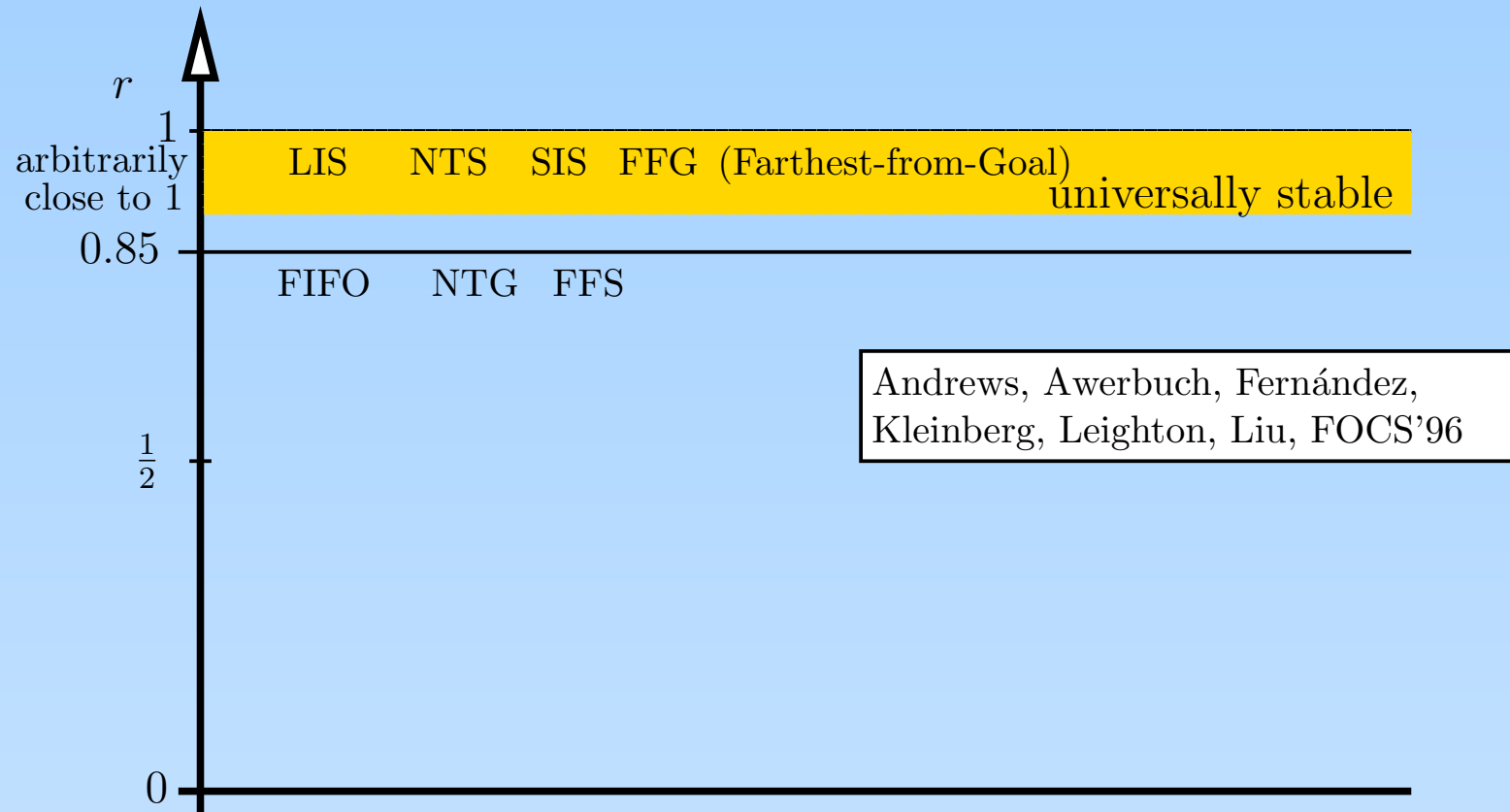
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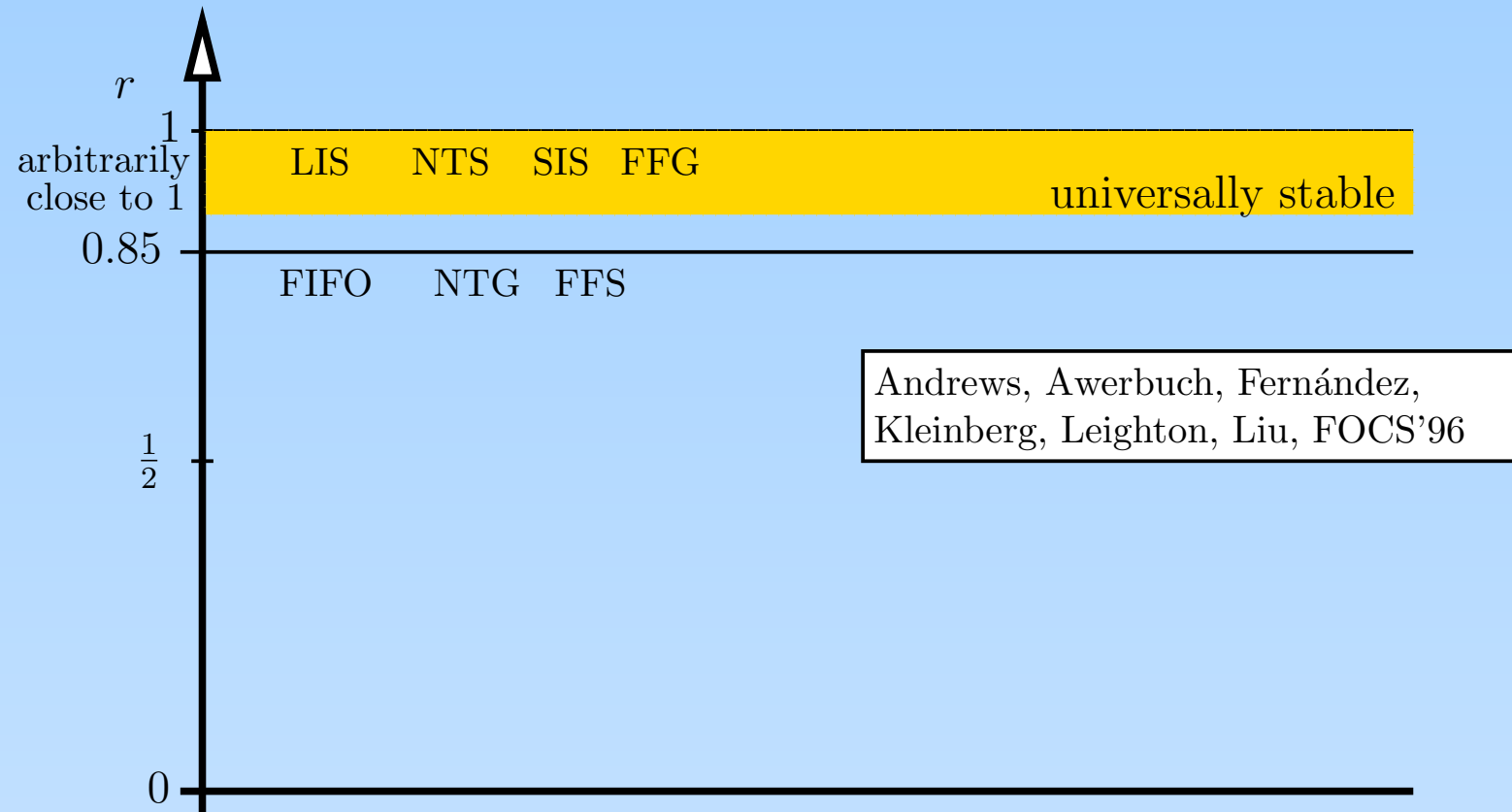
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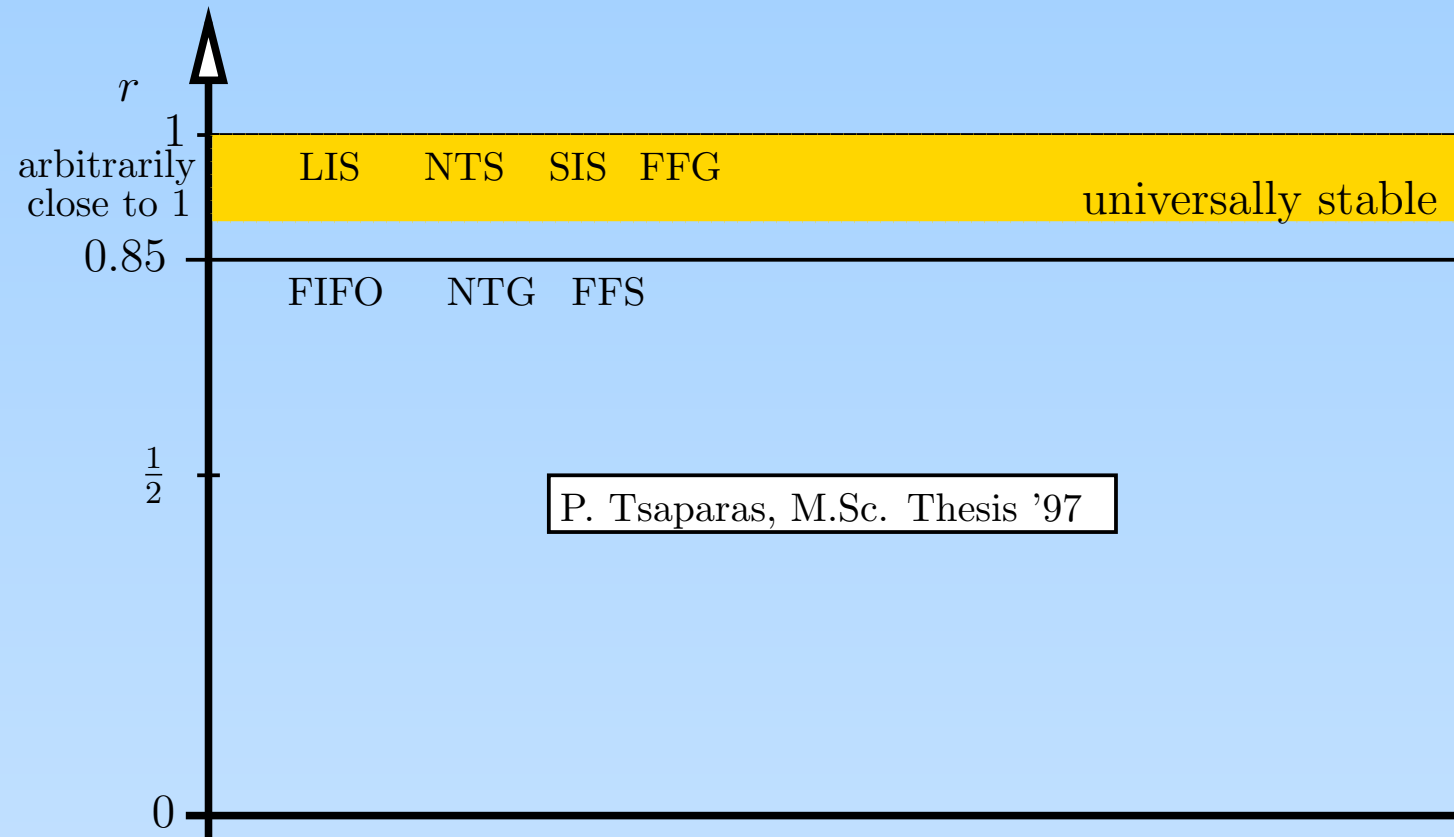


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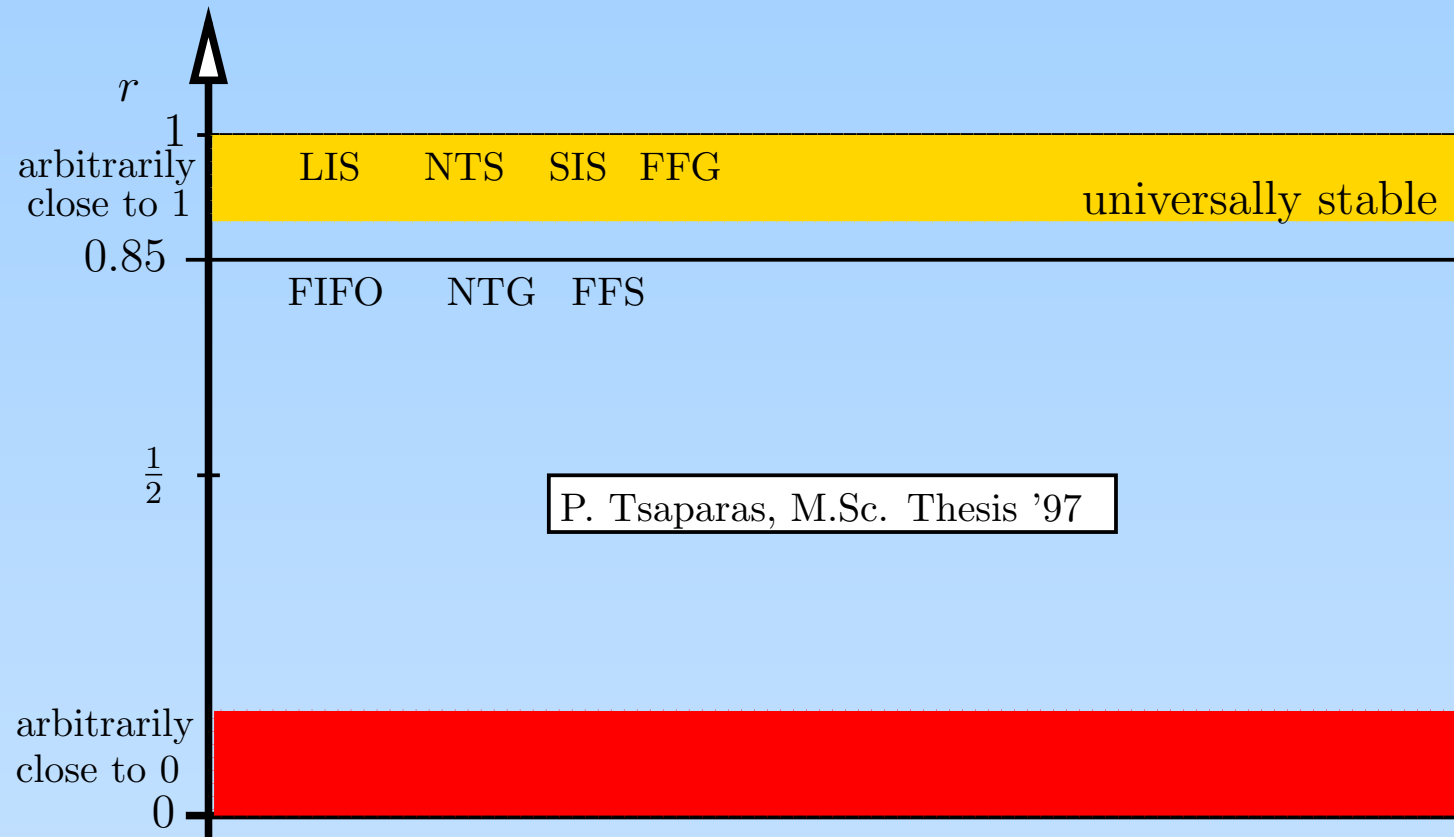




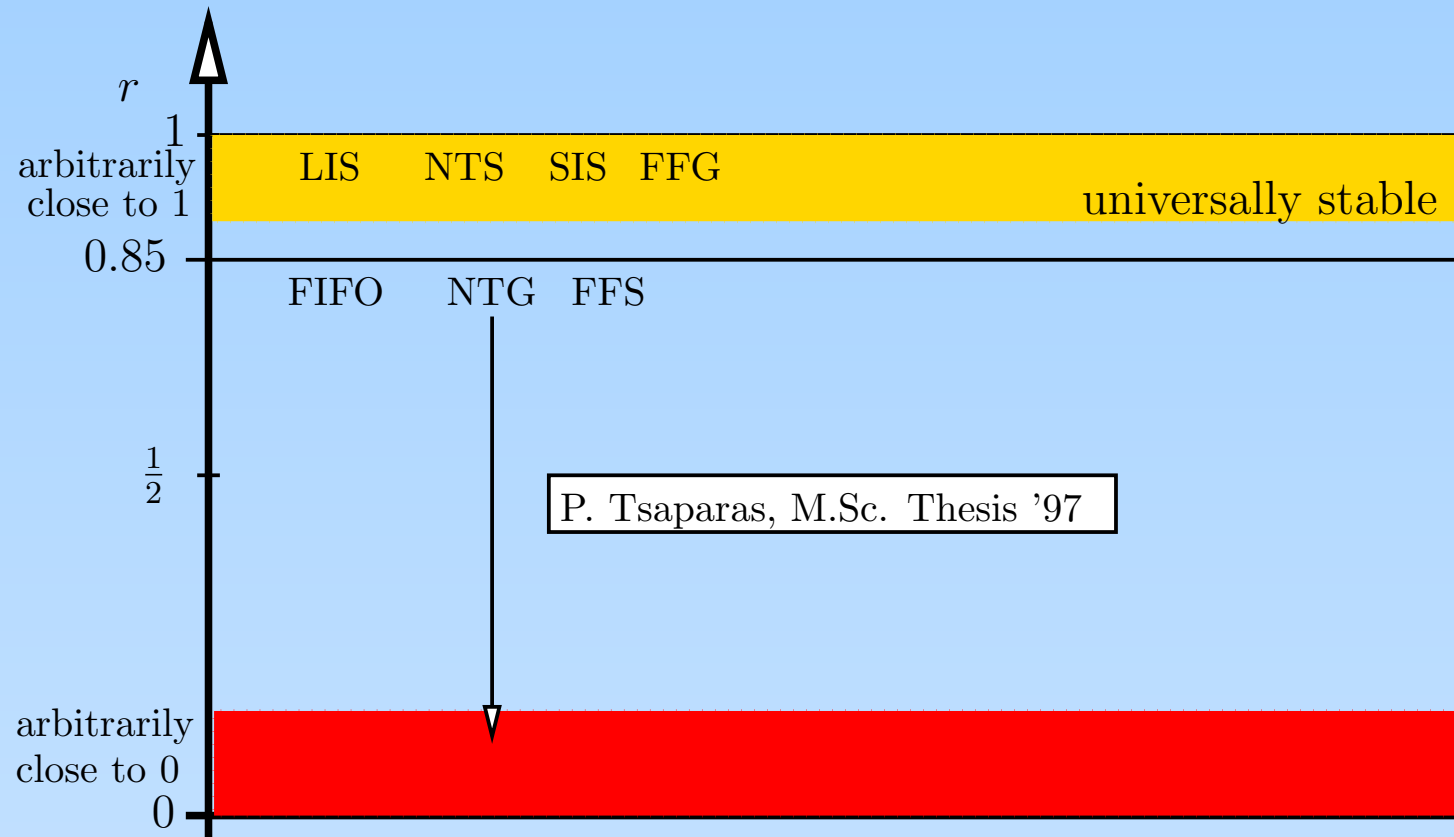
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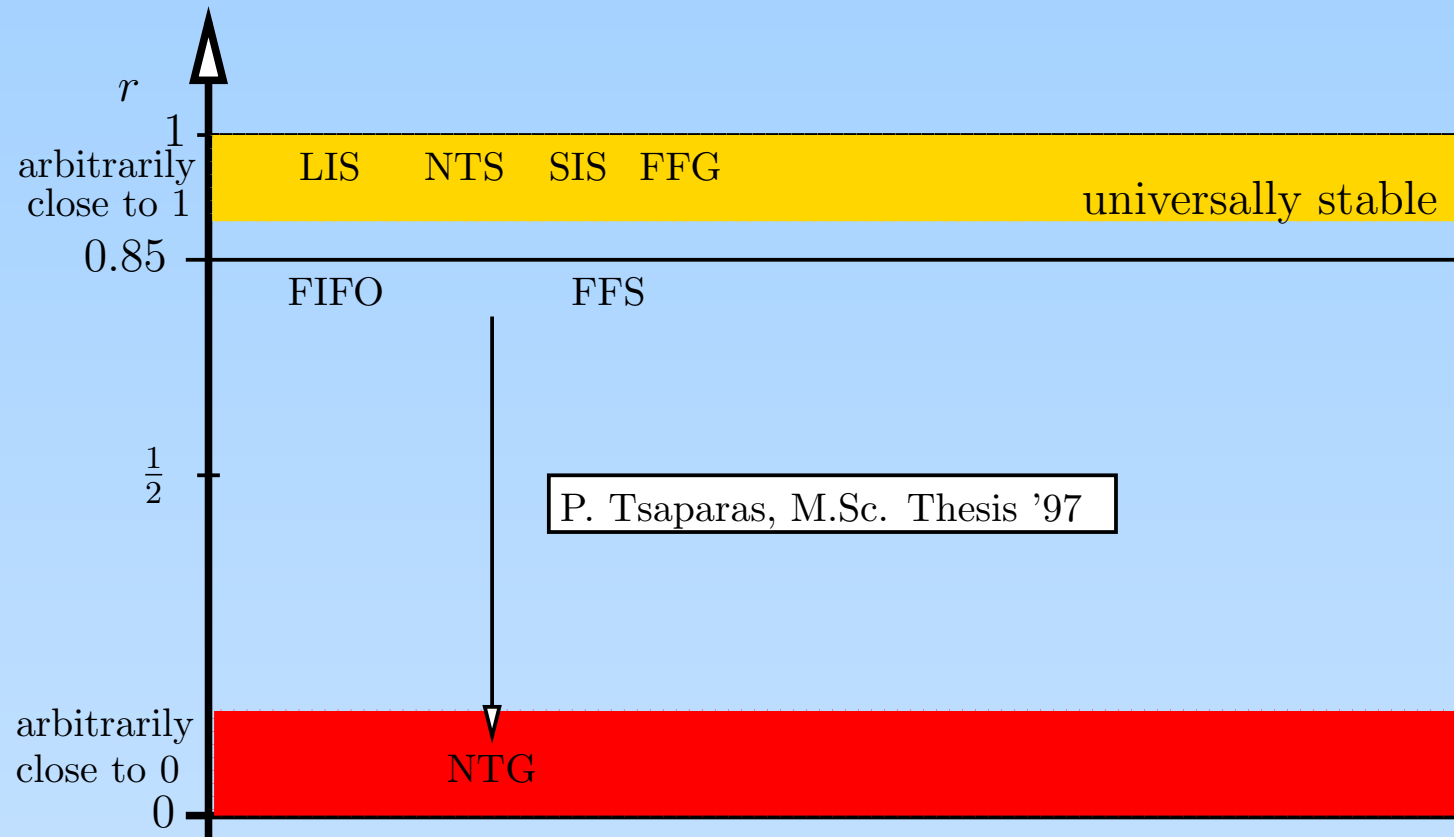
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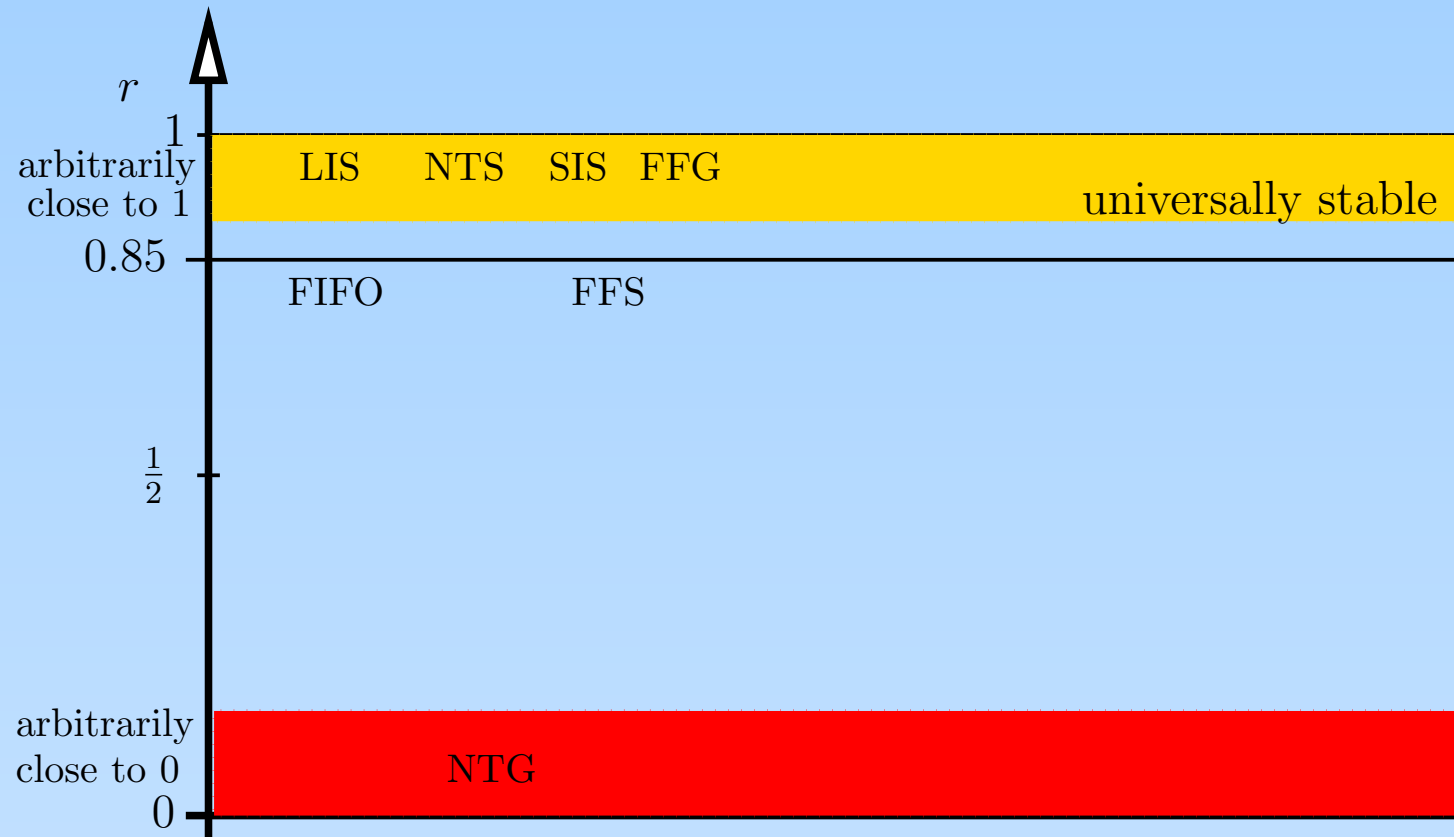
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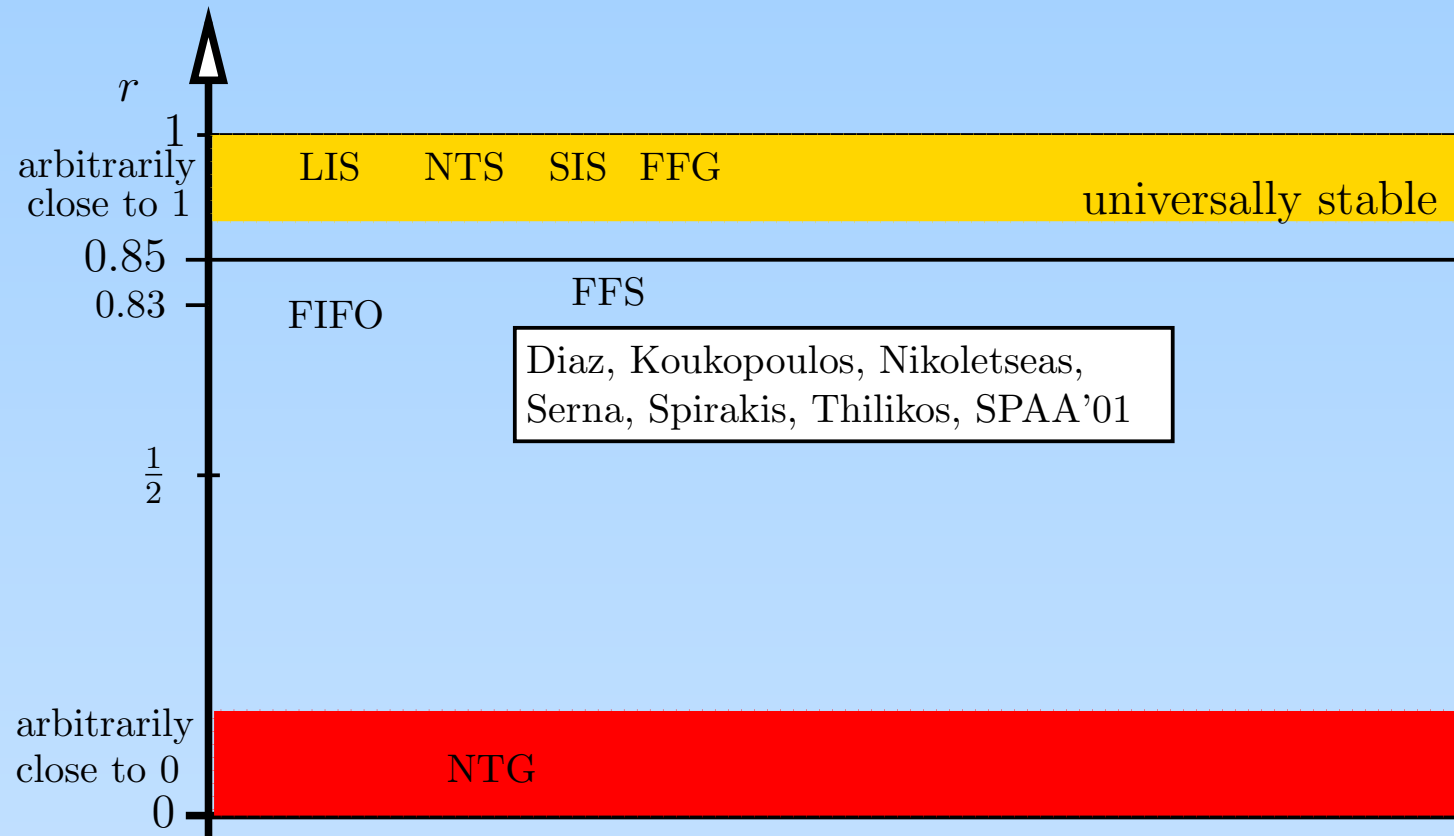
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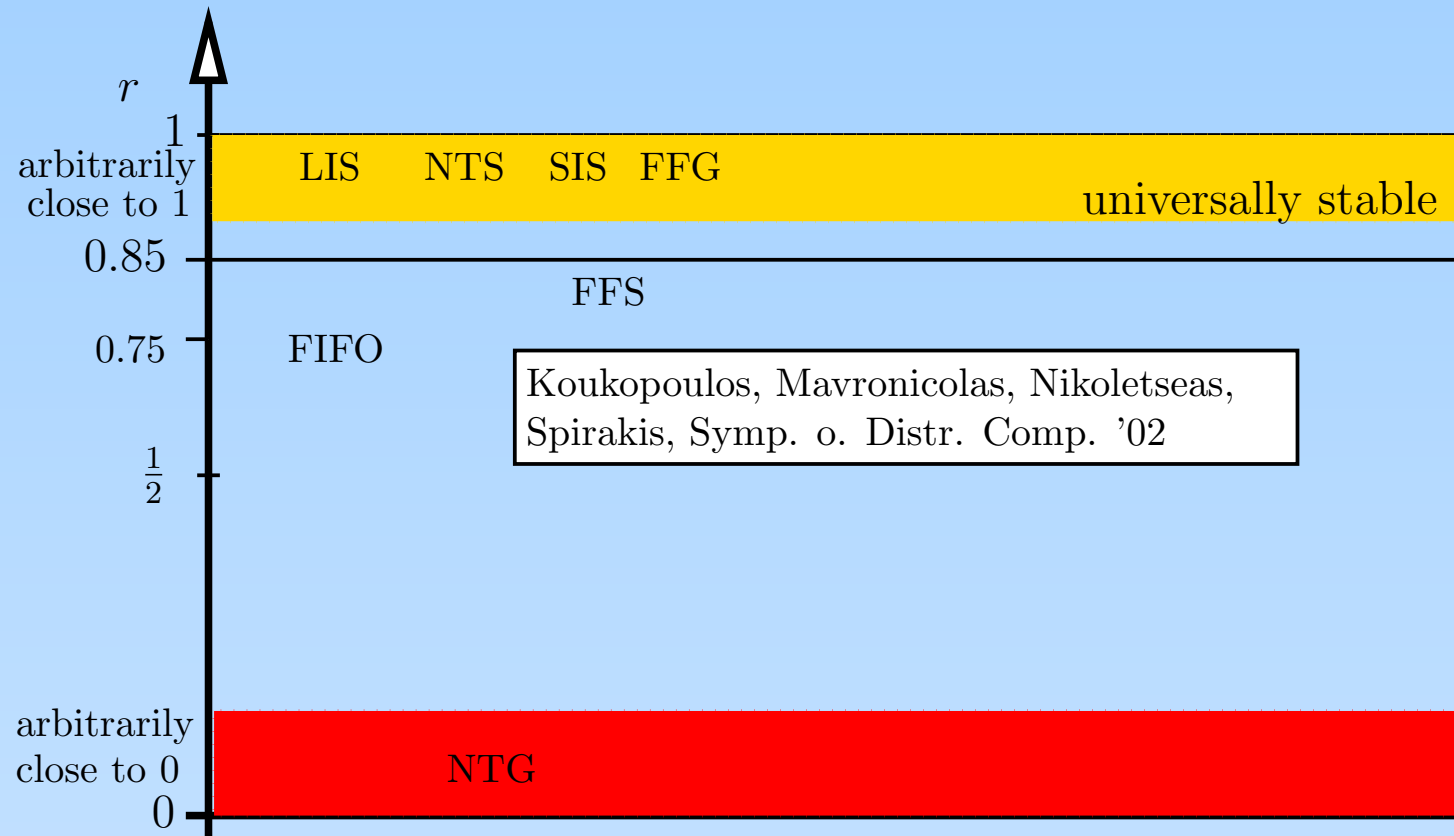
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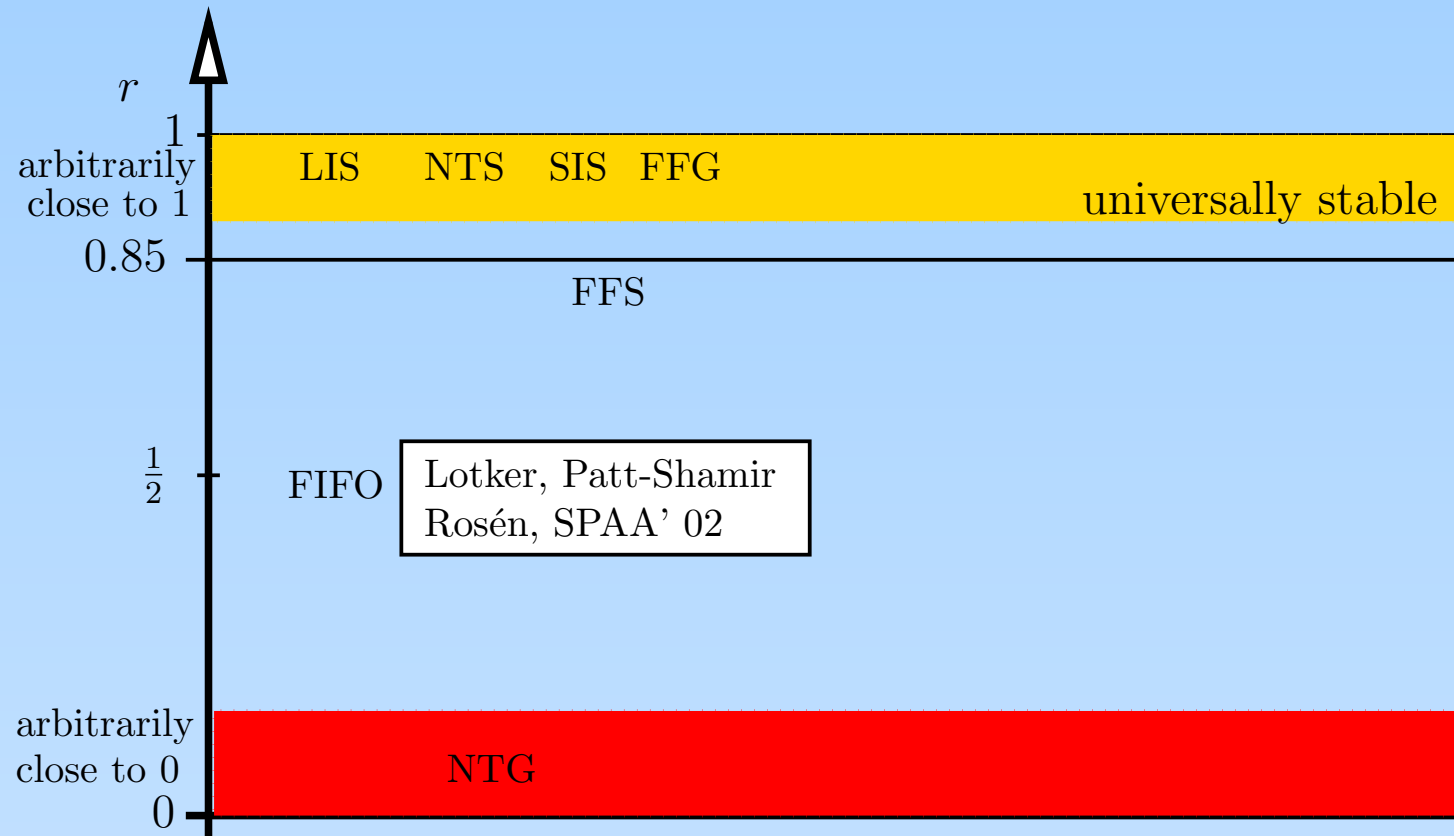
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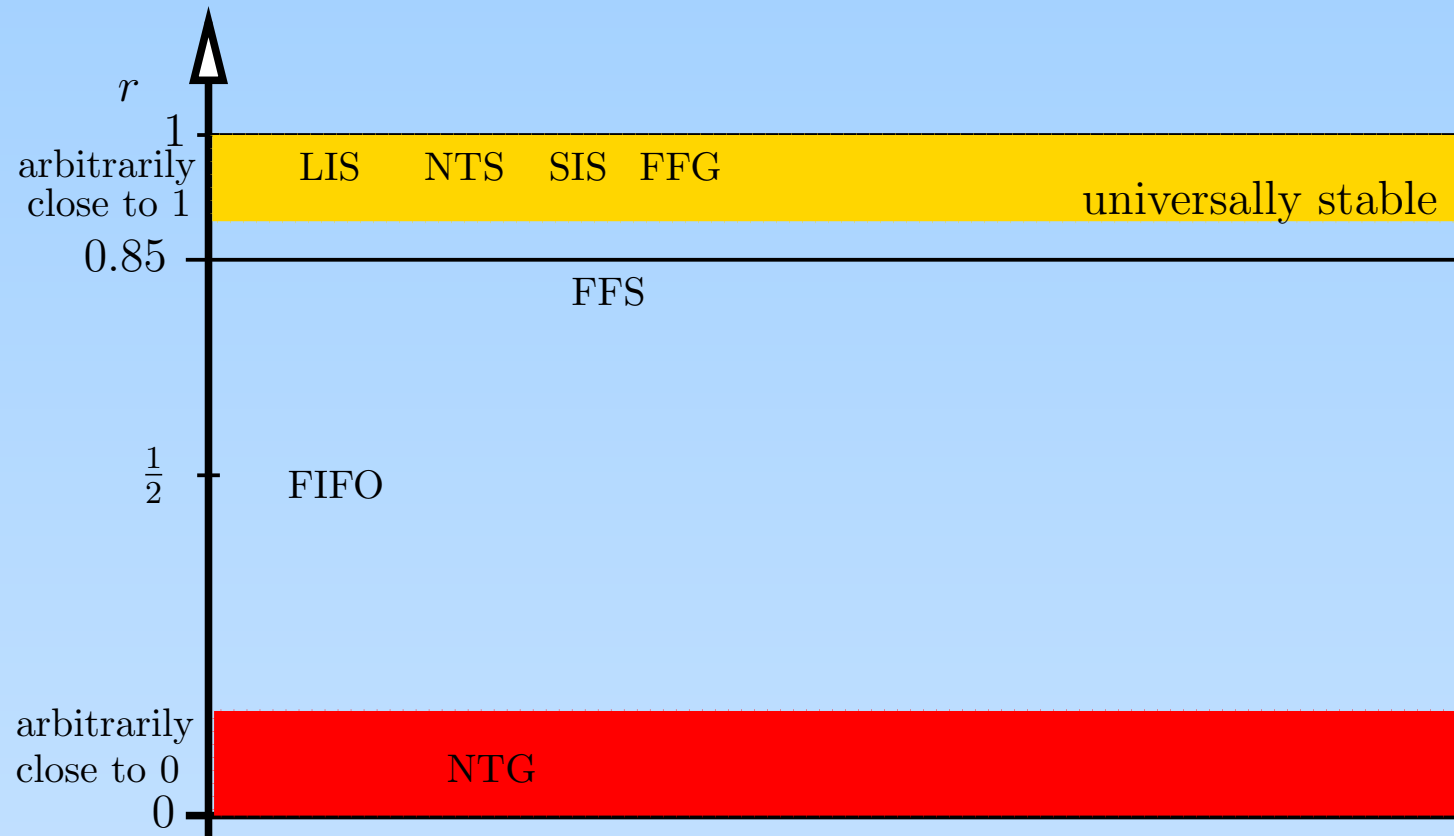


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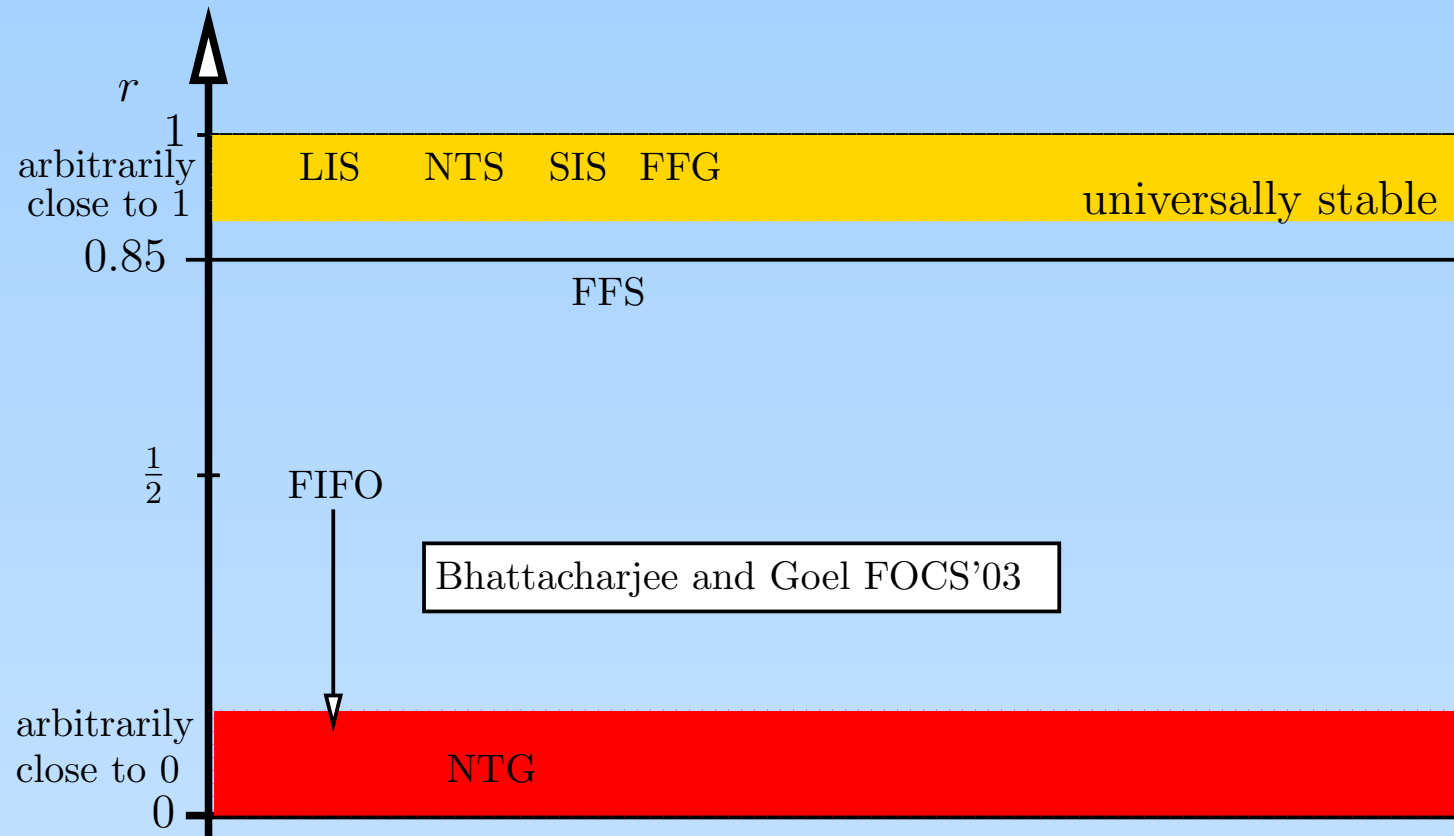




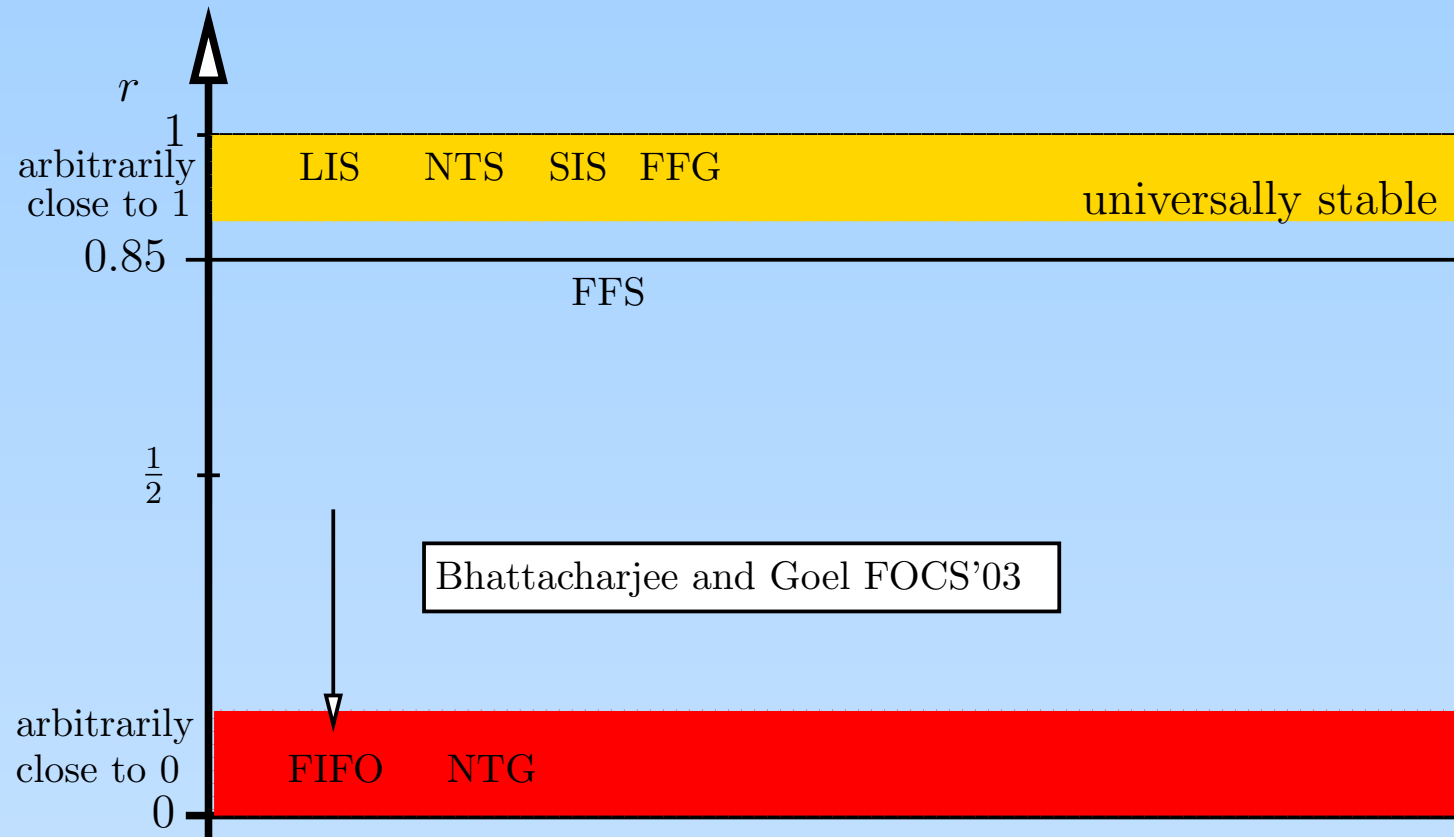
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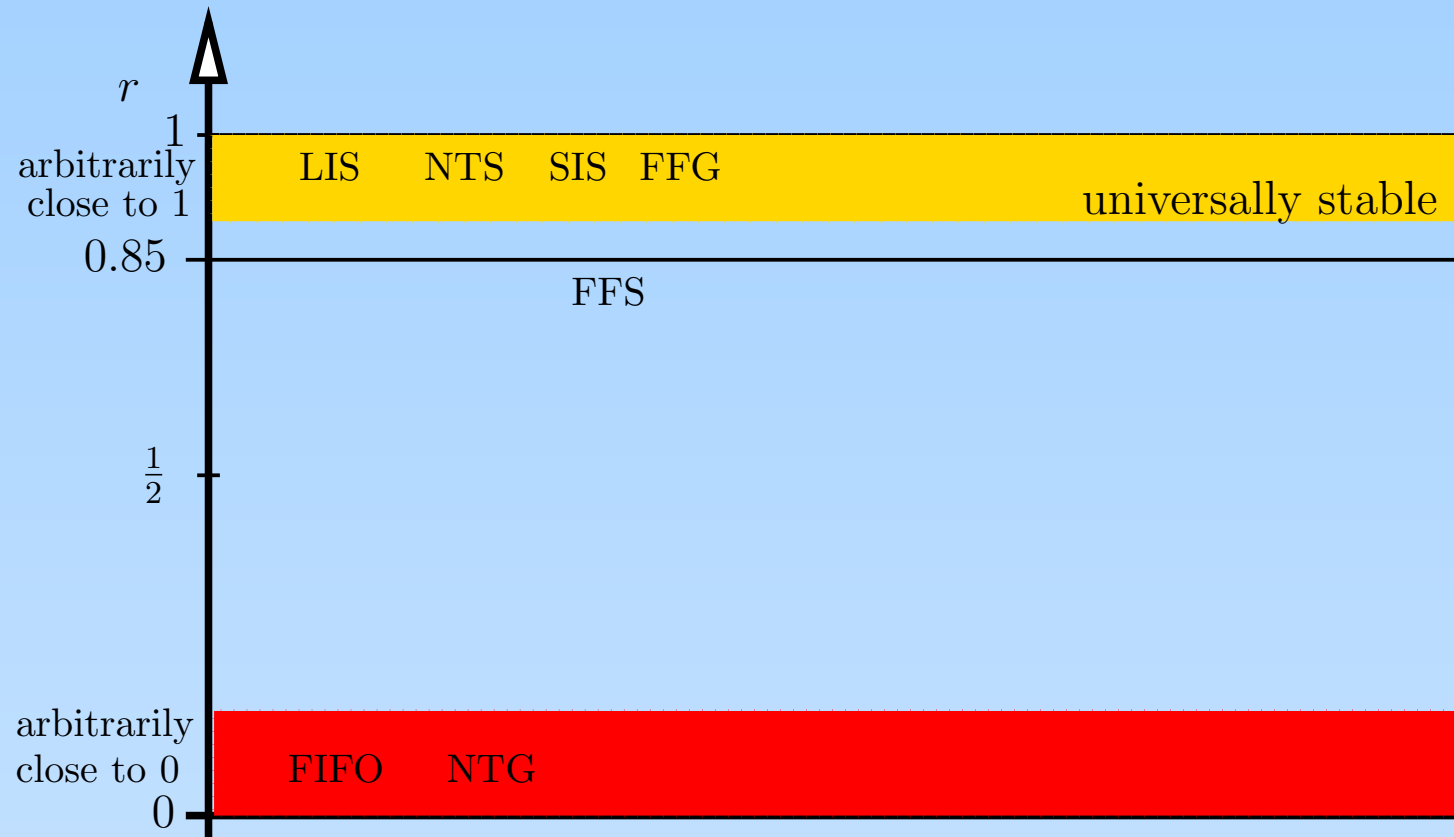
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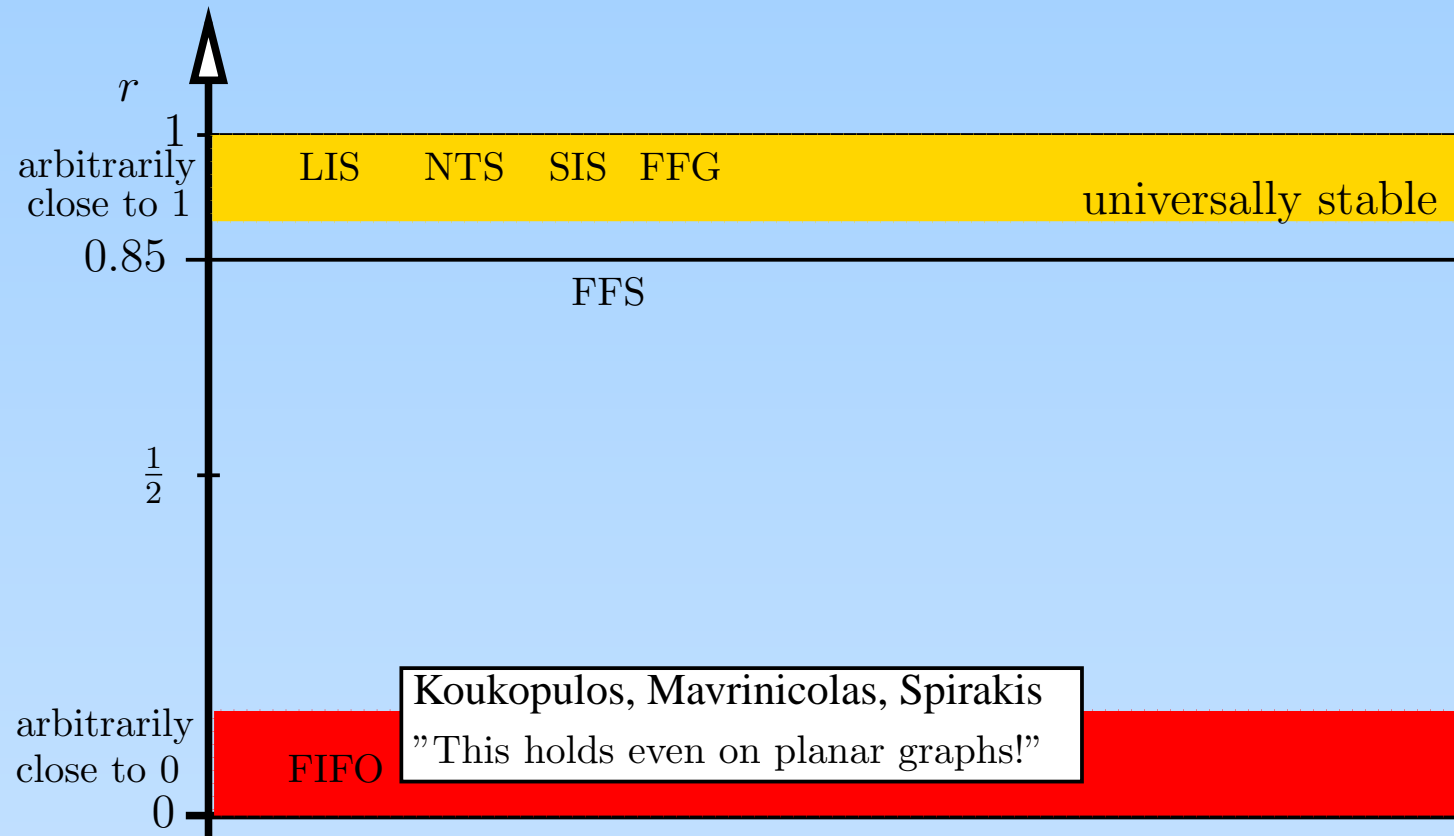
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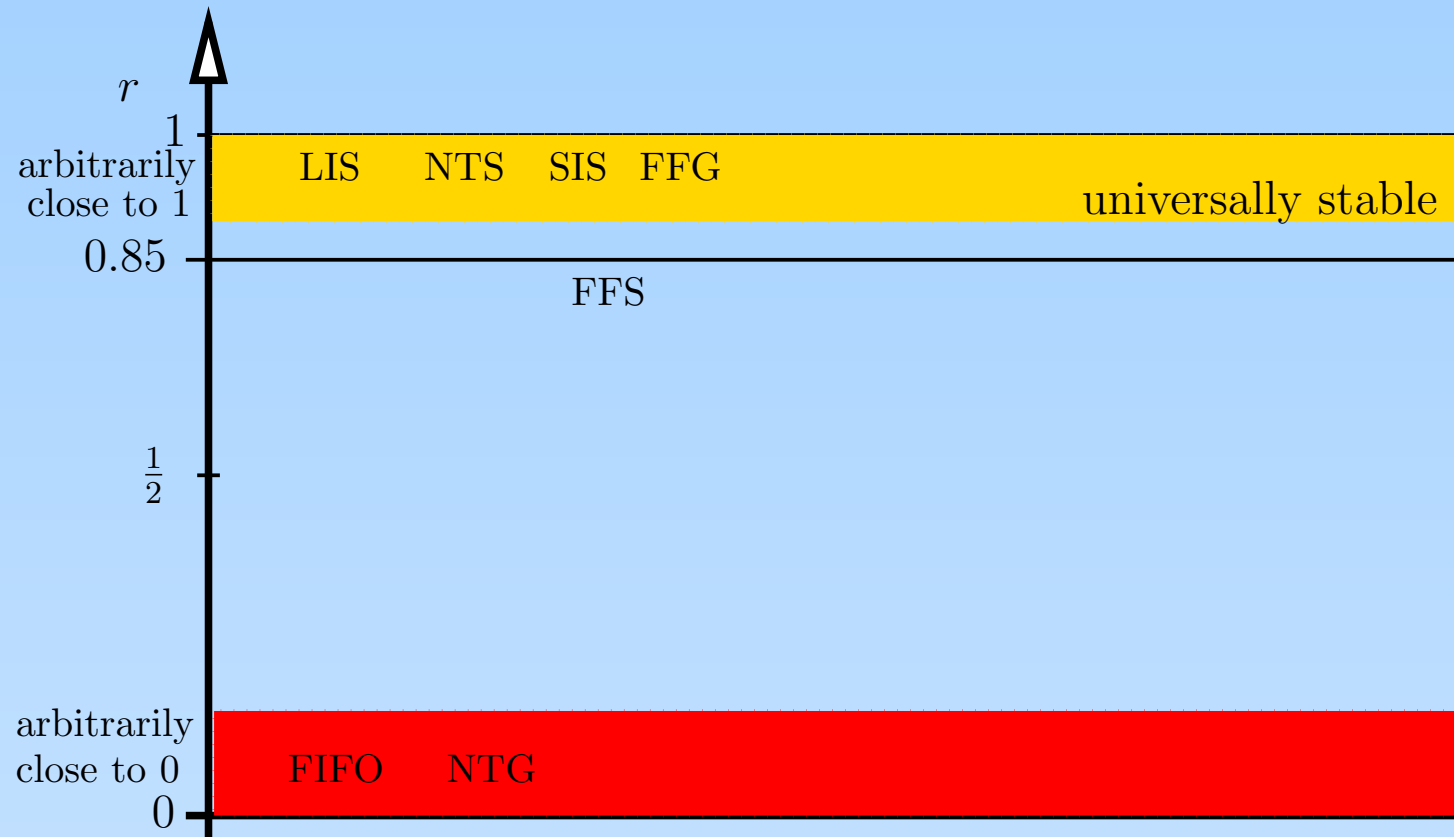
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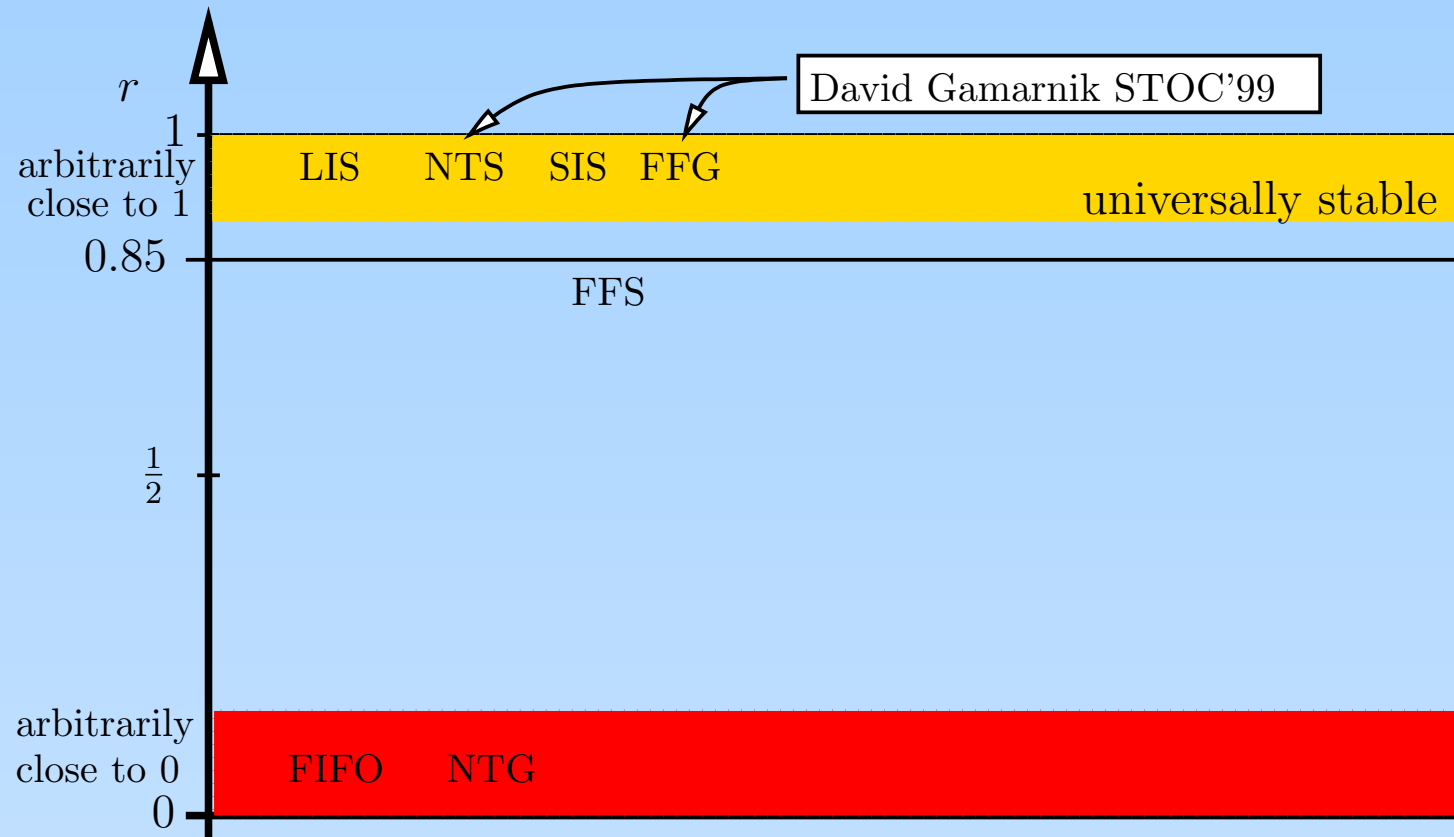
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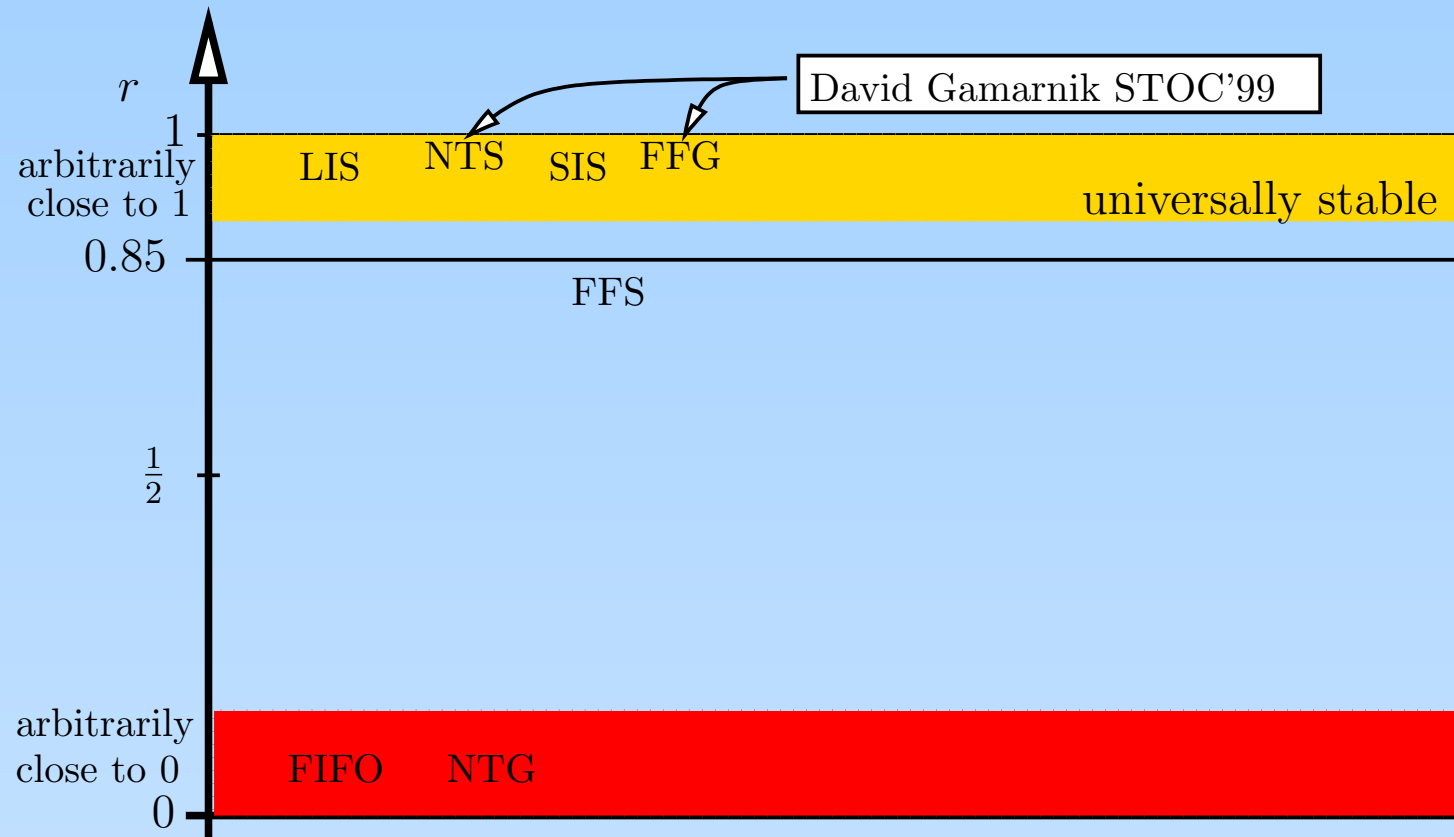
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- **Question:** How difficult do strategies with polynomial delay need to be?





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At every contested edge a packet of maximum priority is advanced.



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- Longest-In-System uses a packets age.



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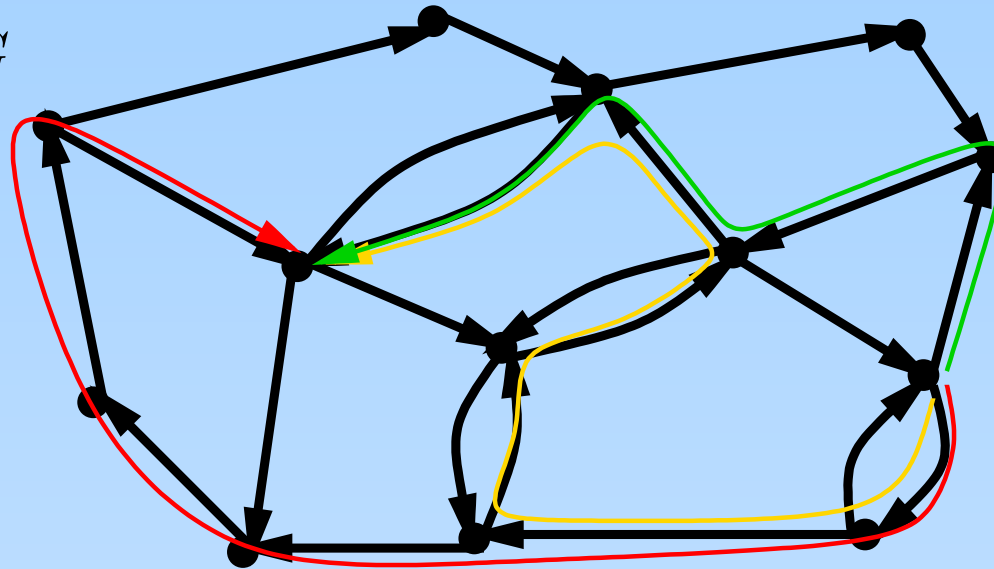
The only *reasonable* quantities not used are times:

- Longest-In-System uses a packets age.
- FIFO uses a packets current waiting time.



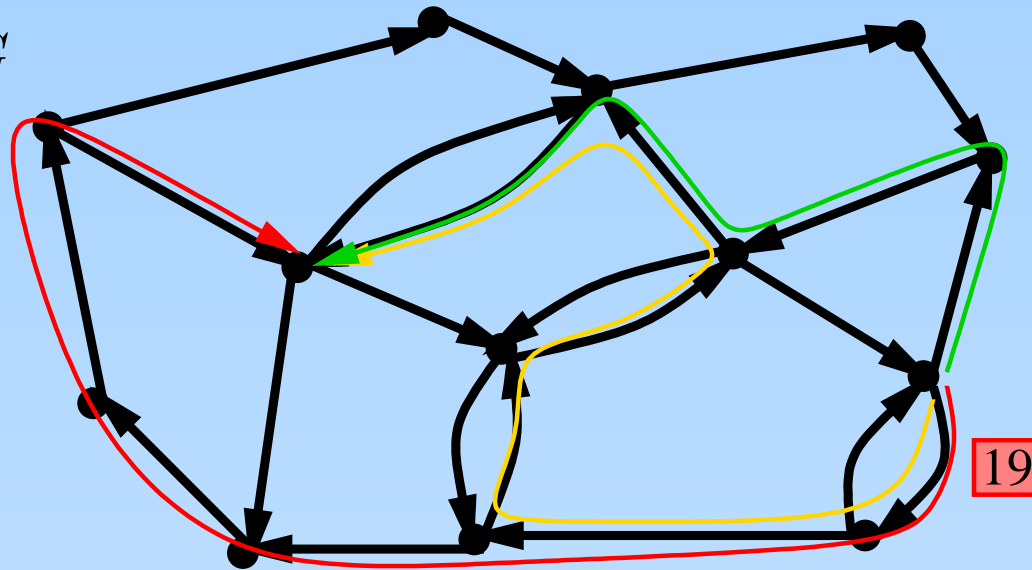
# WTS-Strategies

Graph  $G$



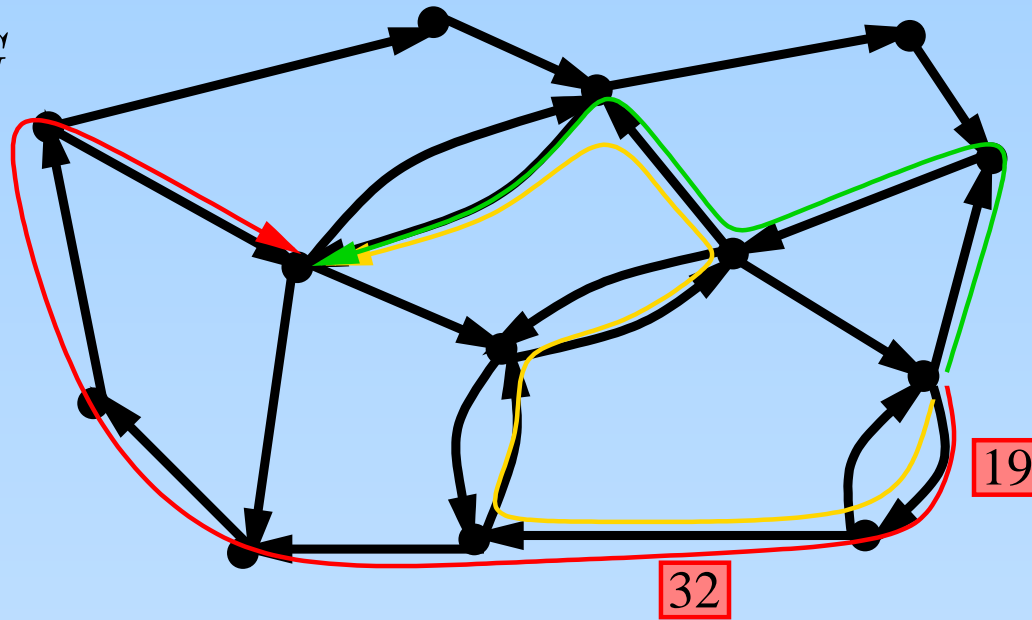
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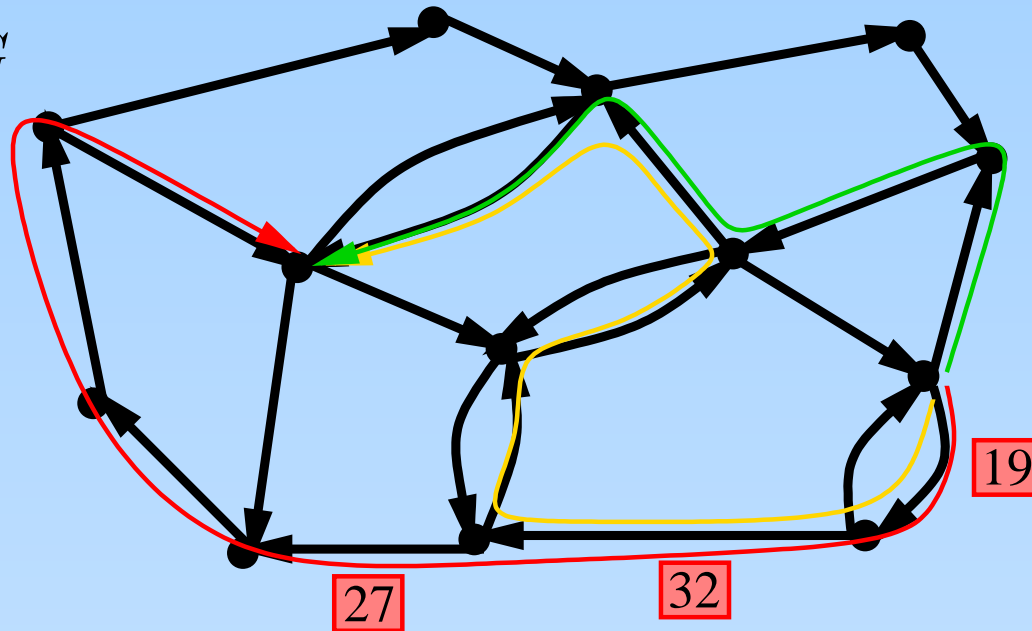
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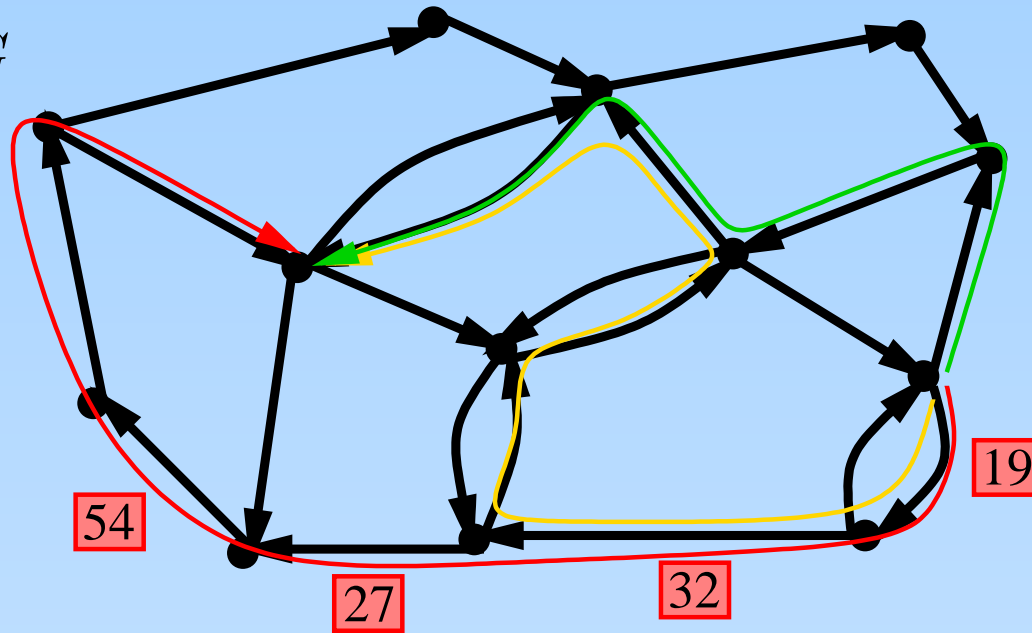
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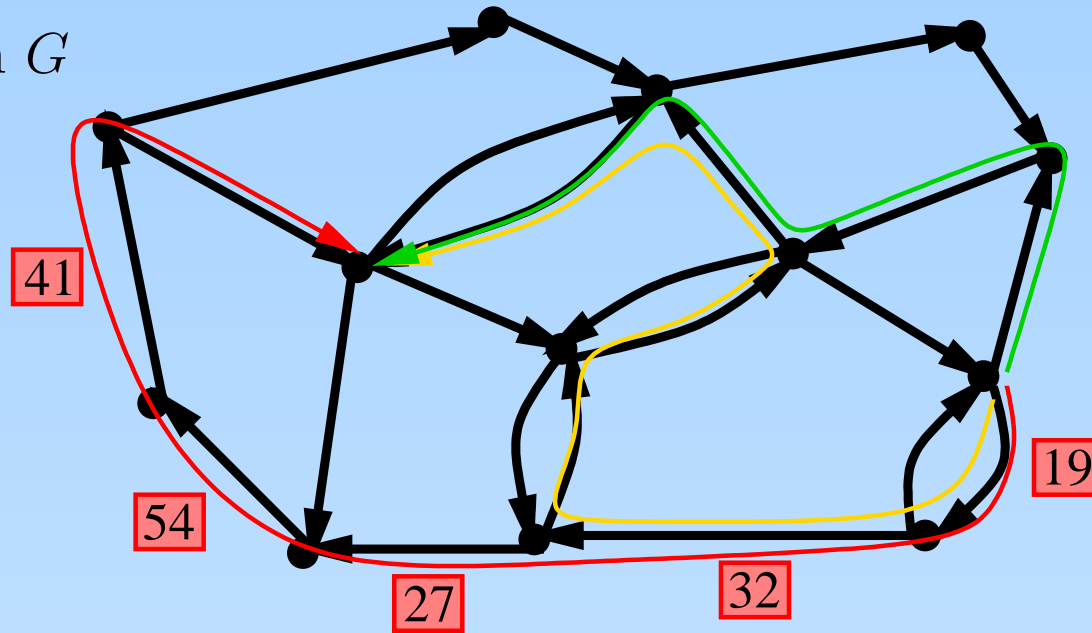
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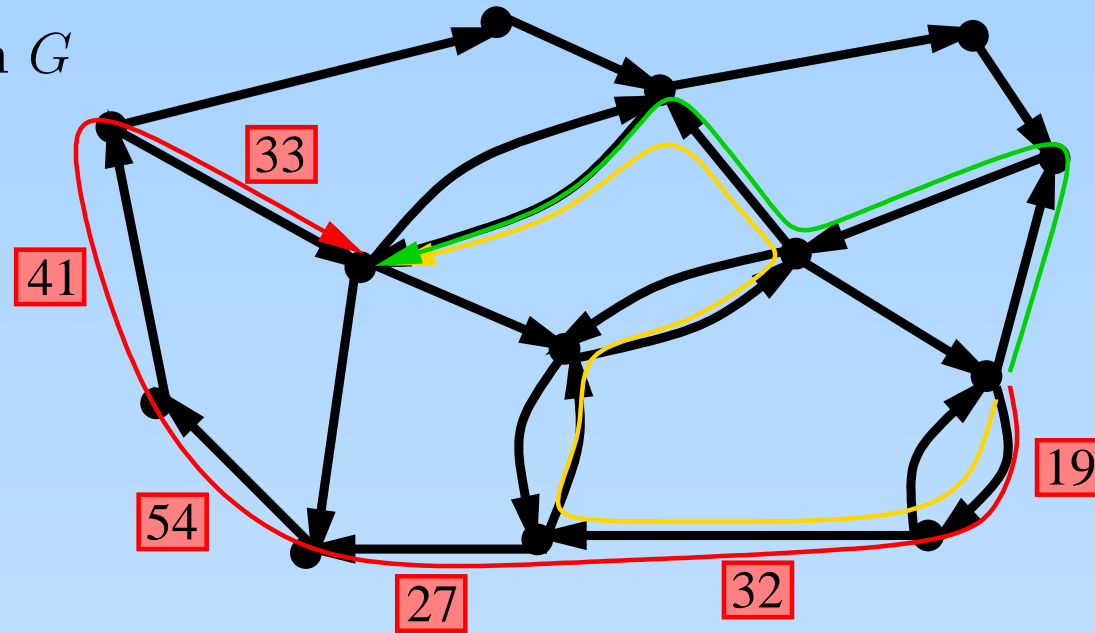
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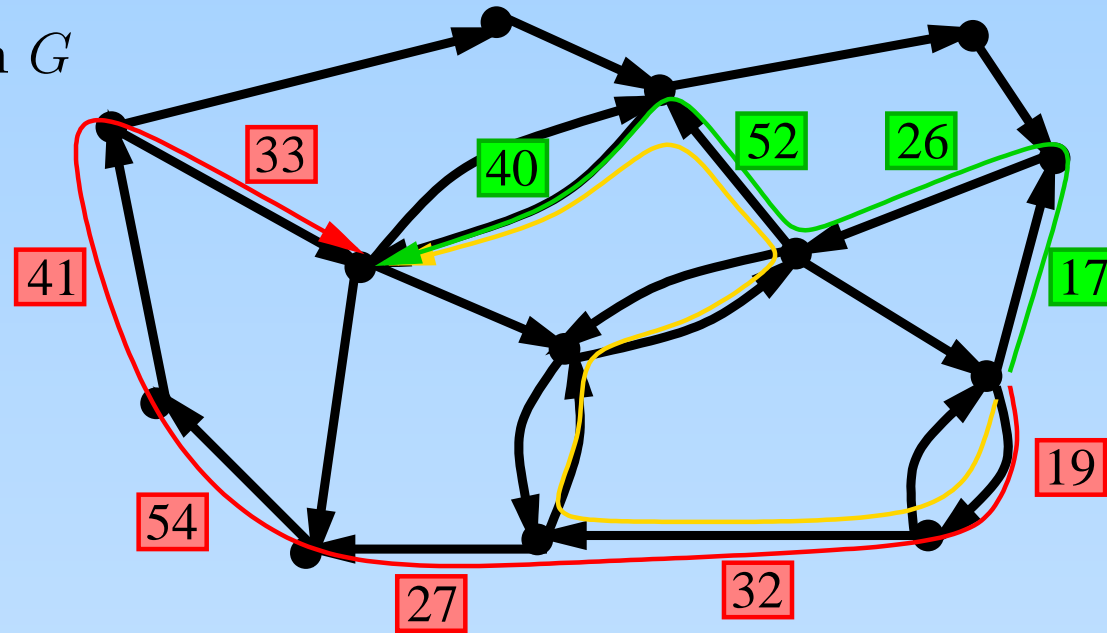
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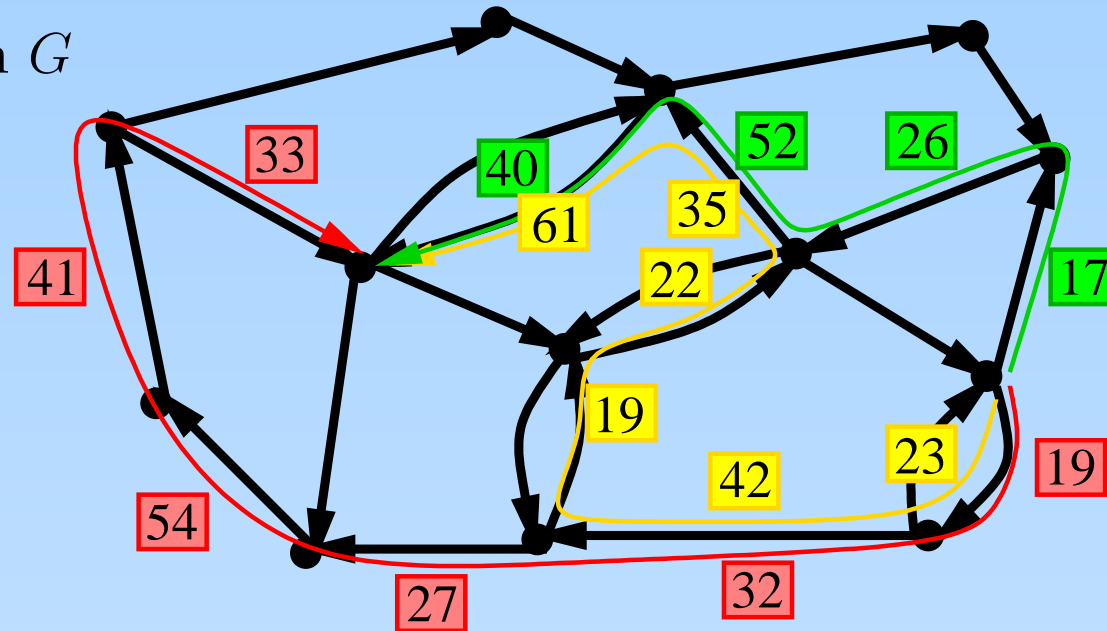
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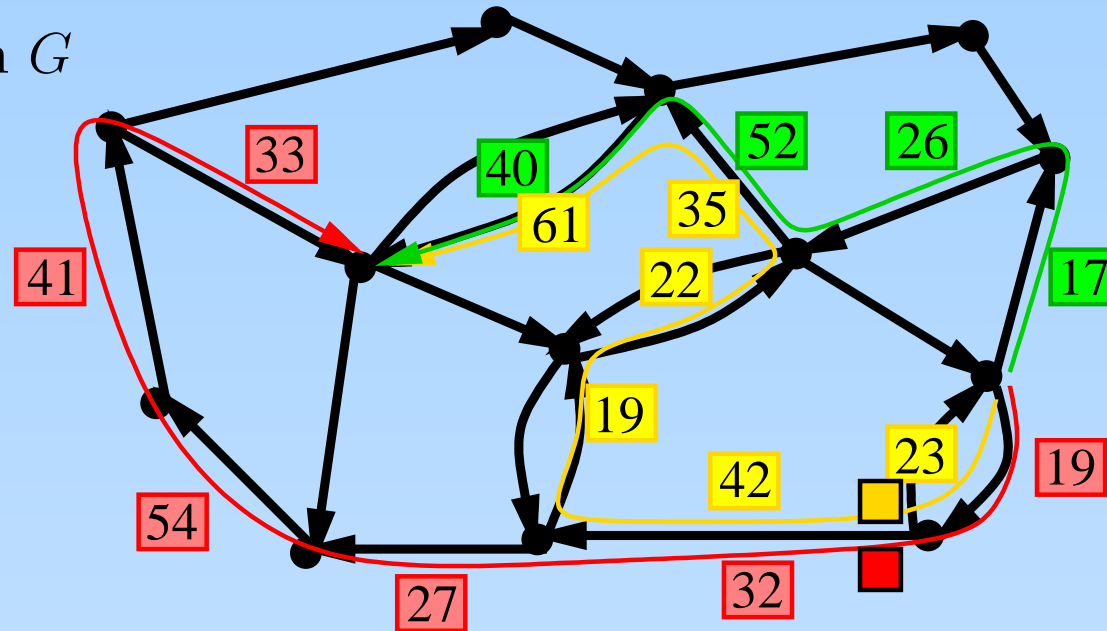
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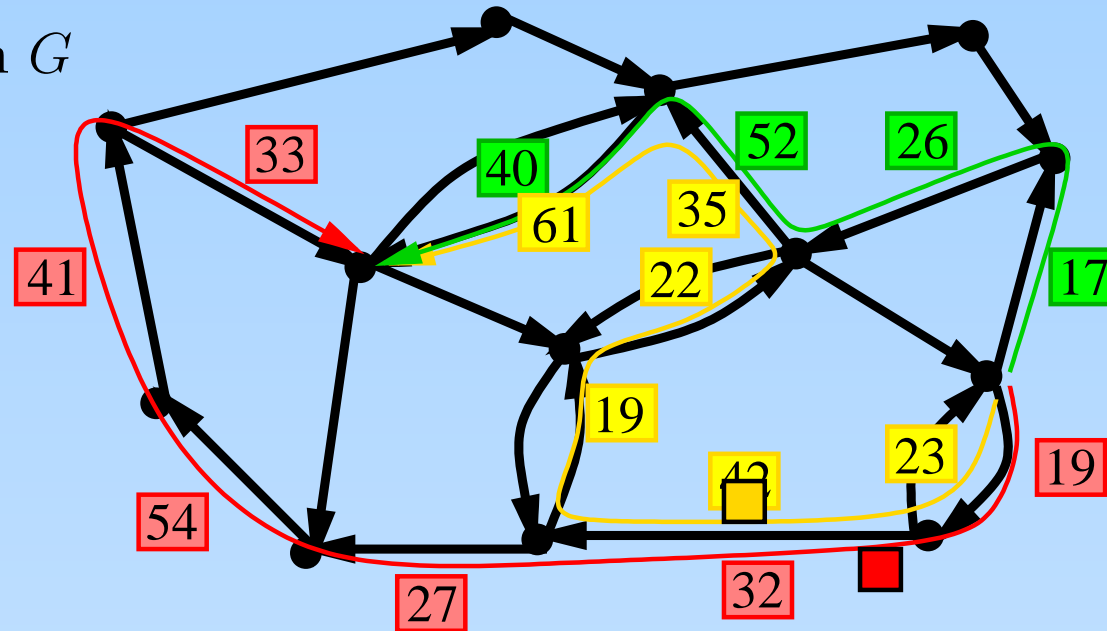
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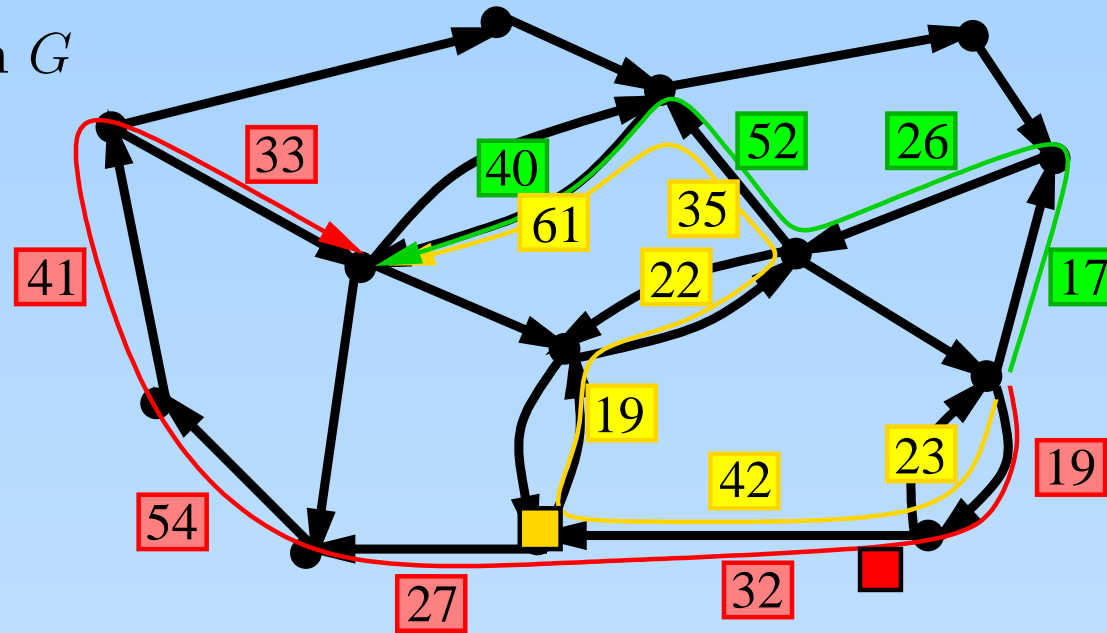
Graph  $G$





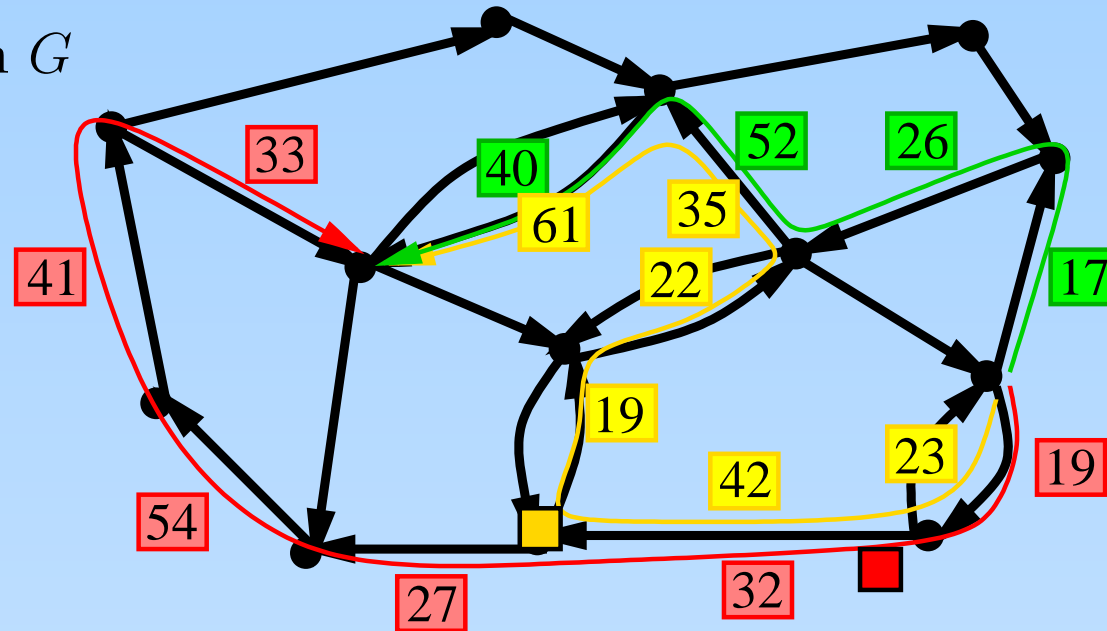
# WTS-Strategies

Graph  $G$



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Graph  $G$



We seek a negative result about all WTS-strategies.

# Results



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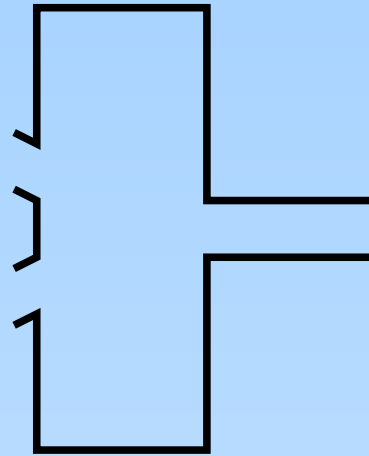
**Problem:** Provide a family of graphs, so that for every possible assignment of priorities, a jam of exponentially many packets can be created.



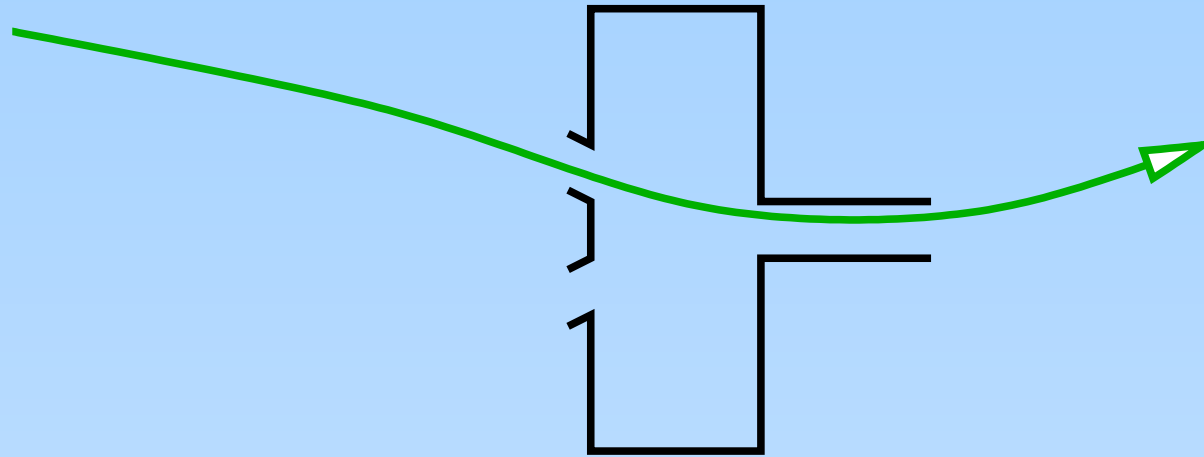
# How to create a traffic jam



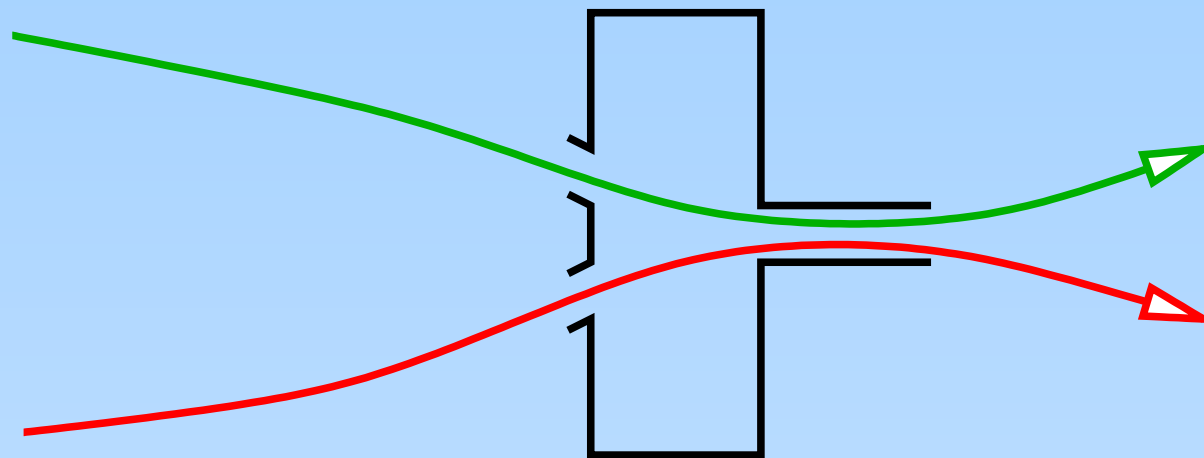
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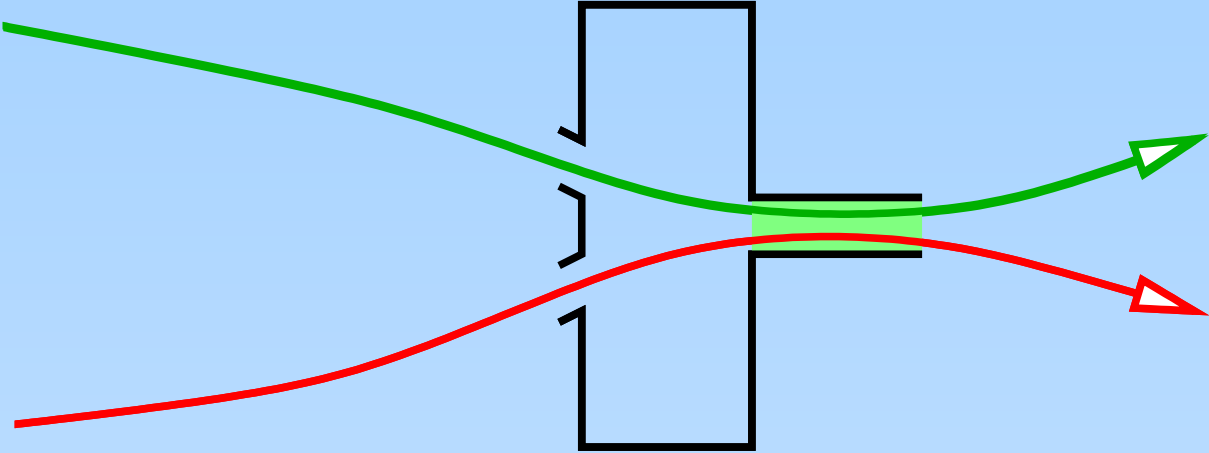
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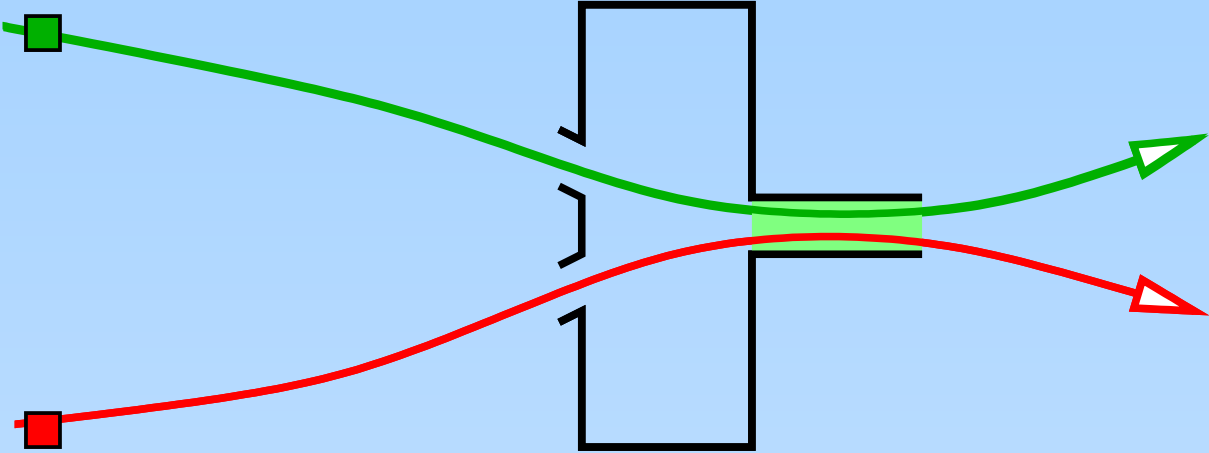


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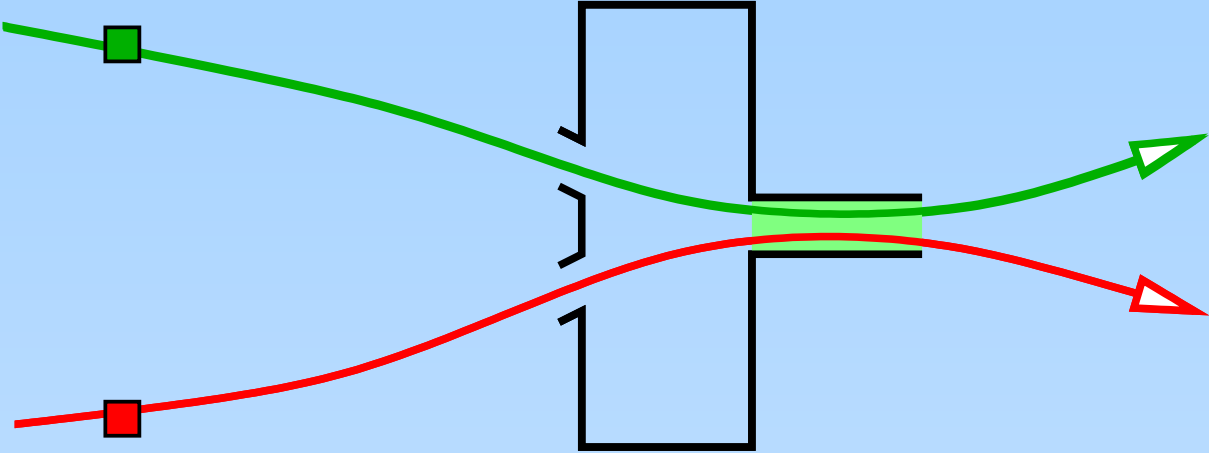




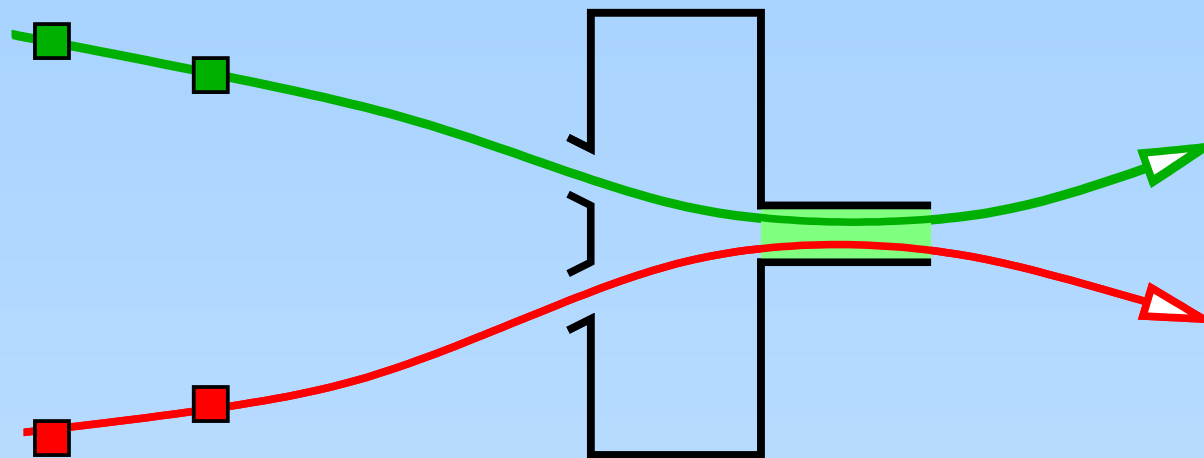
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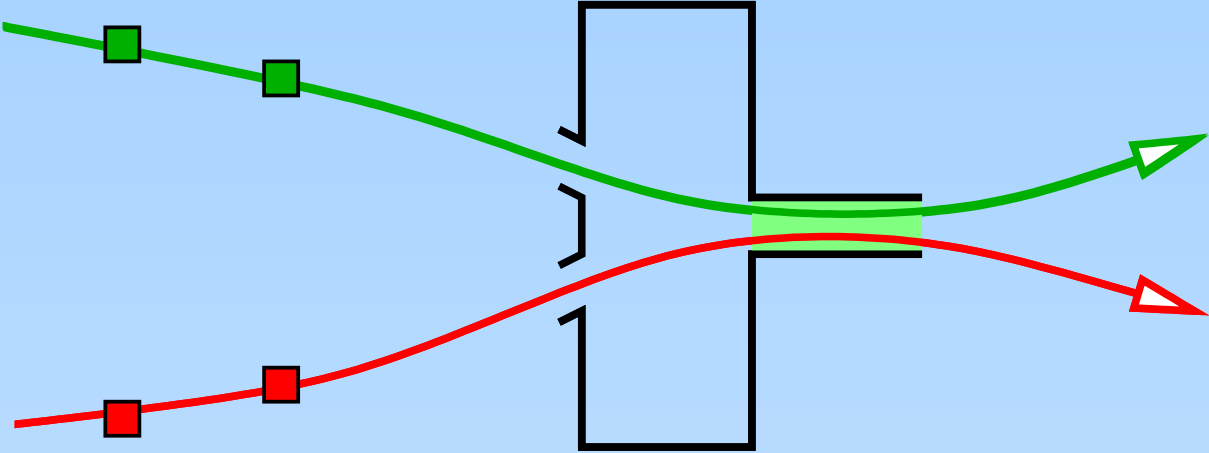
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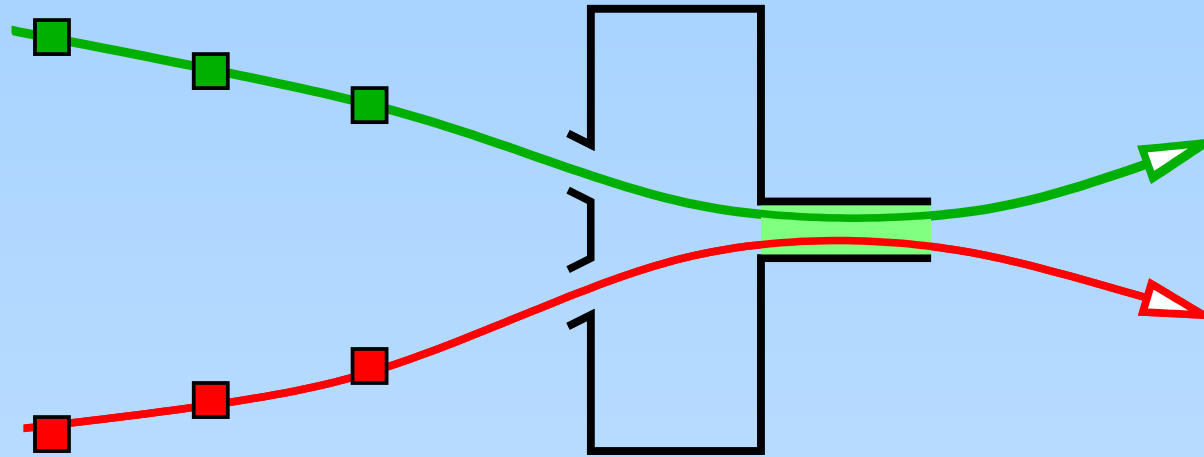
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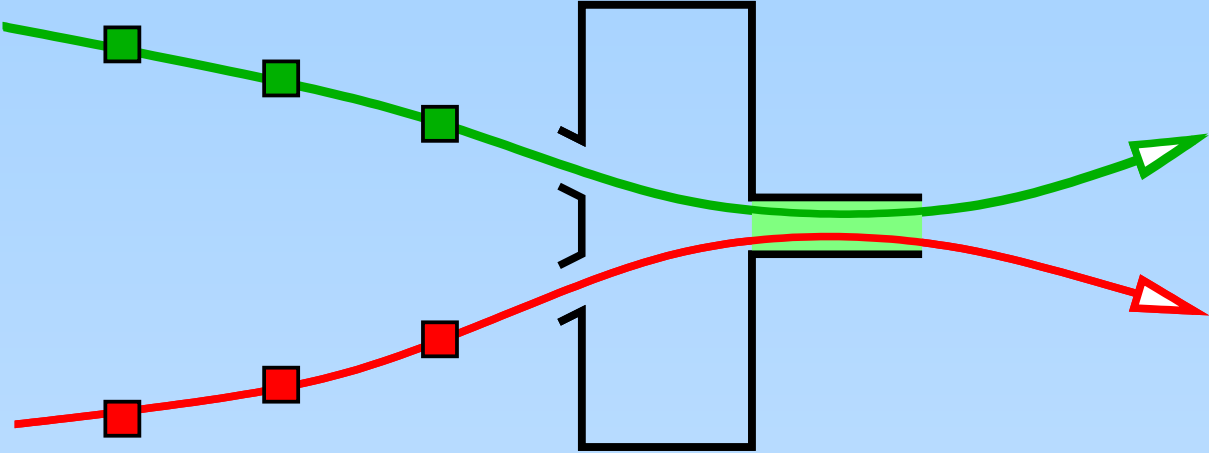
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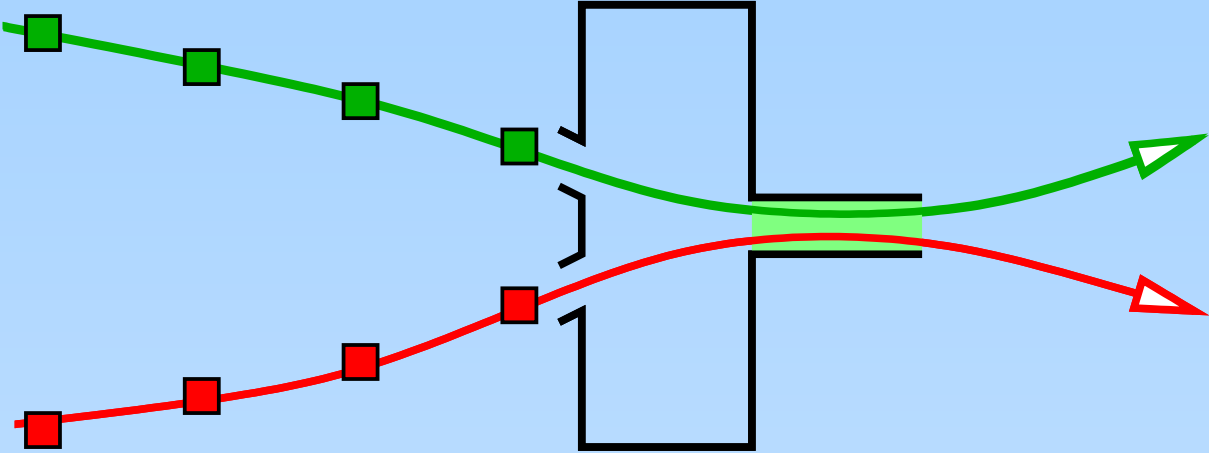
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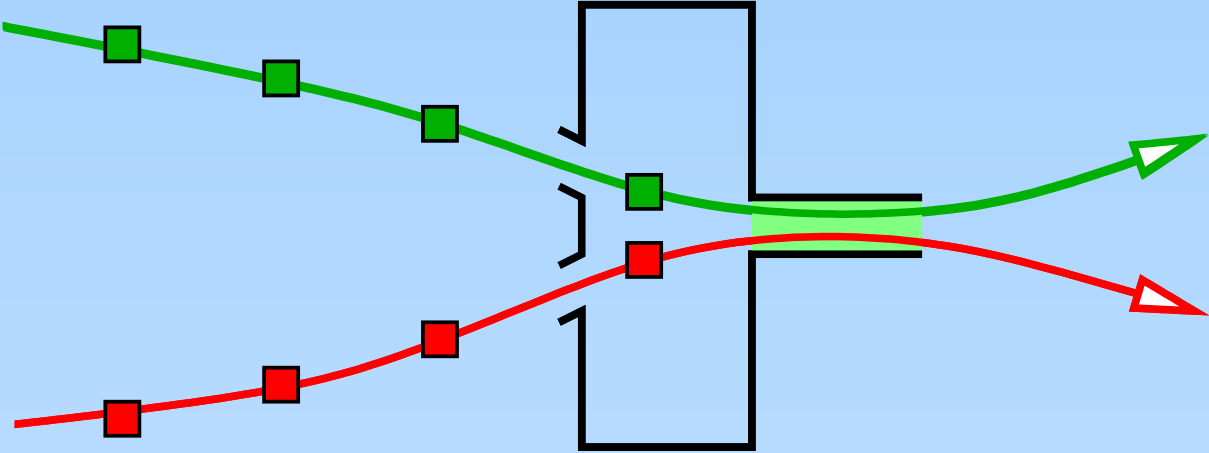
# How to create a traffic jam



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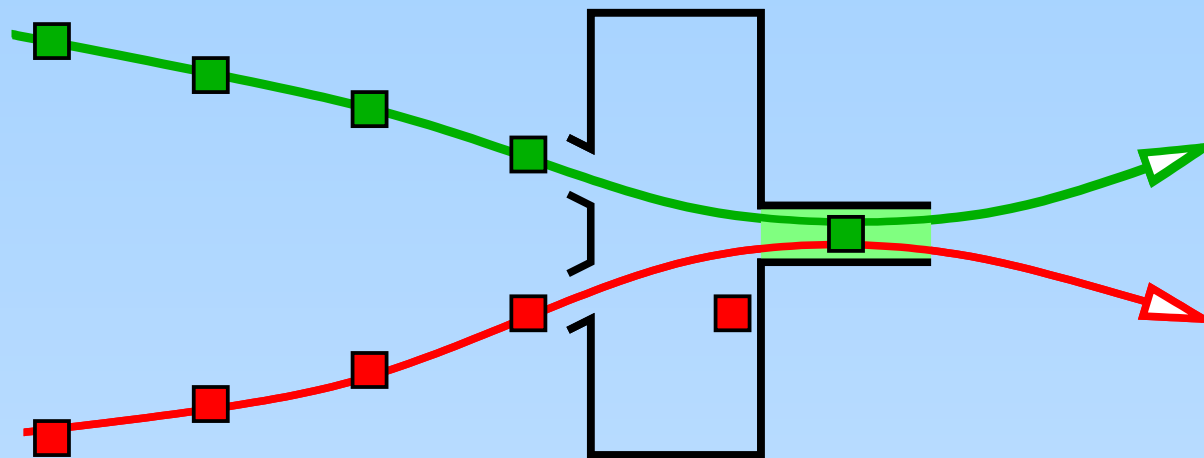


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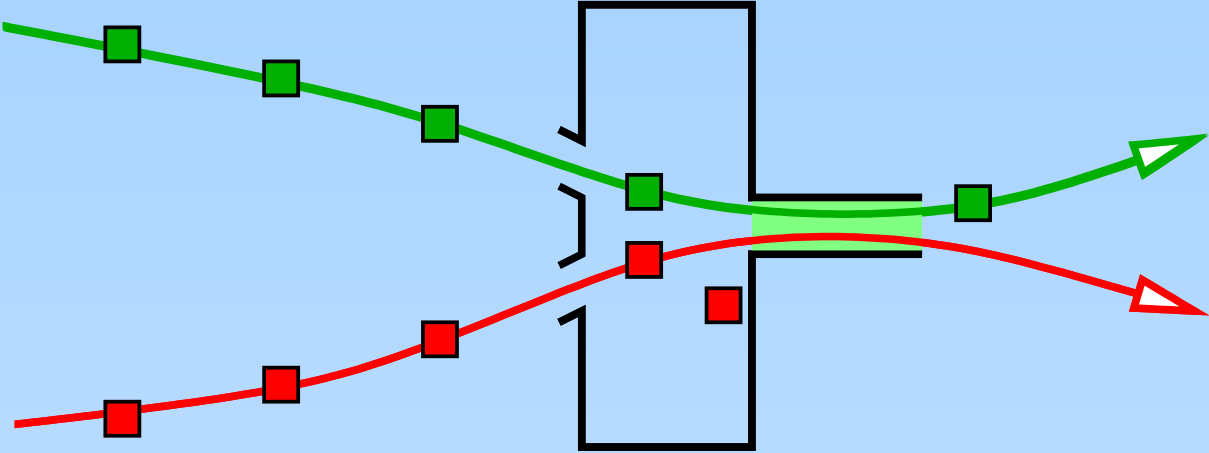




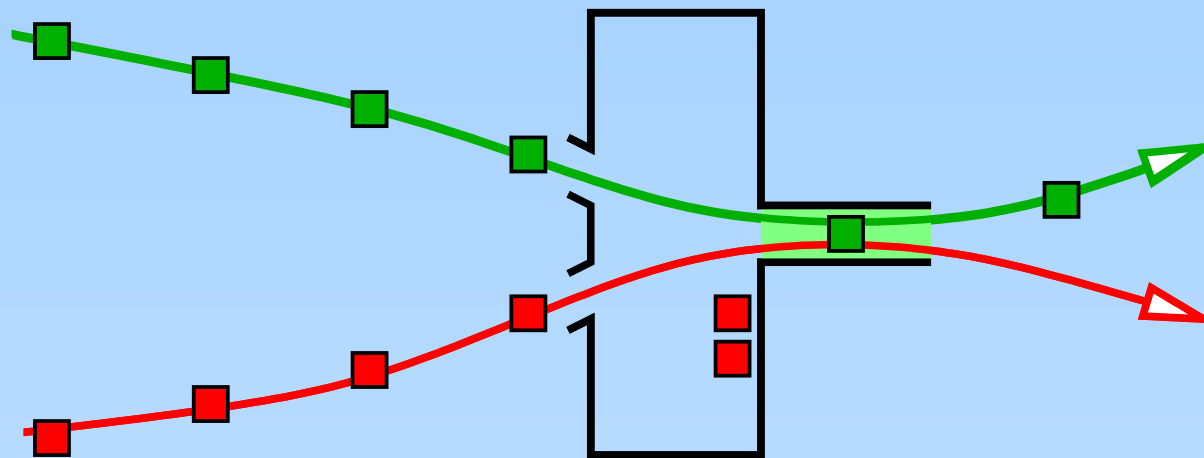
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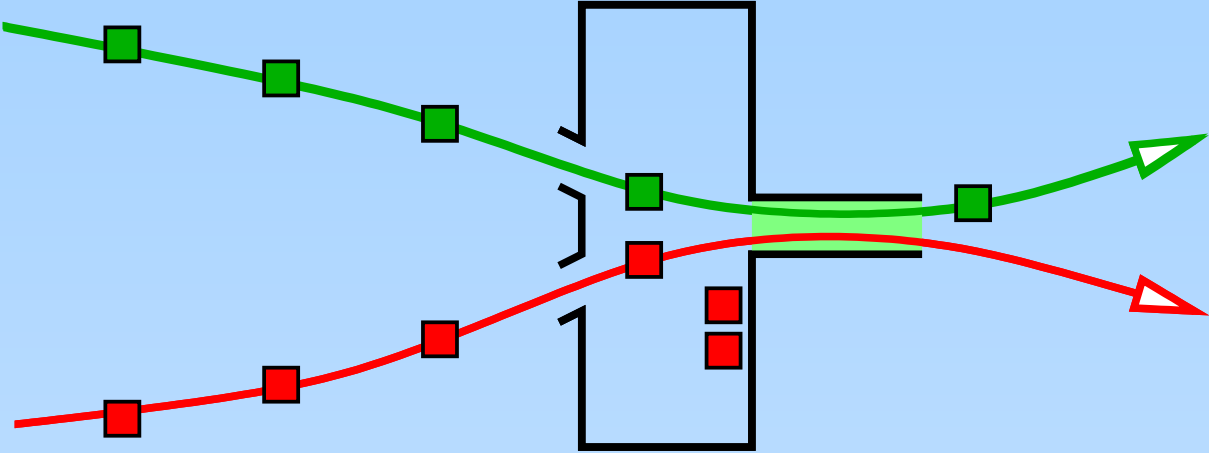
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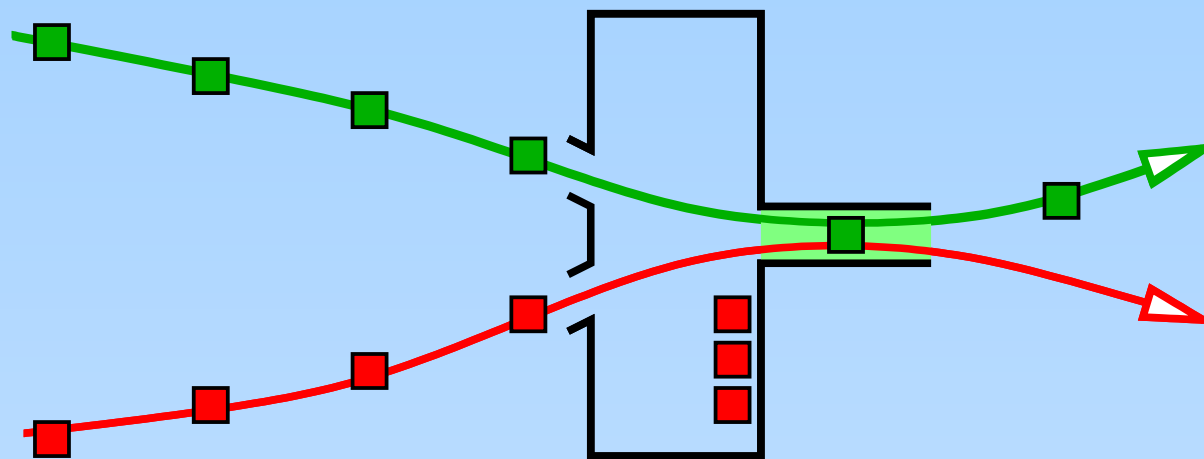
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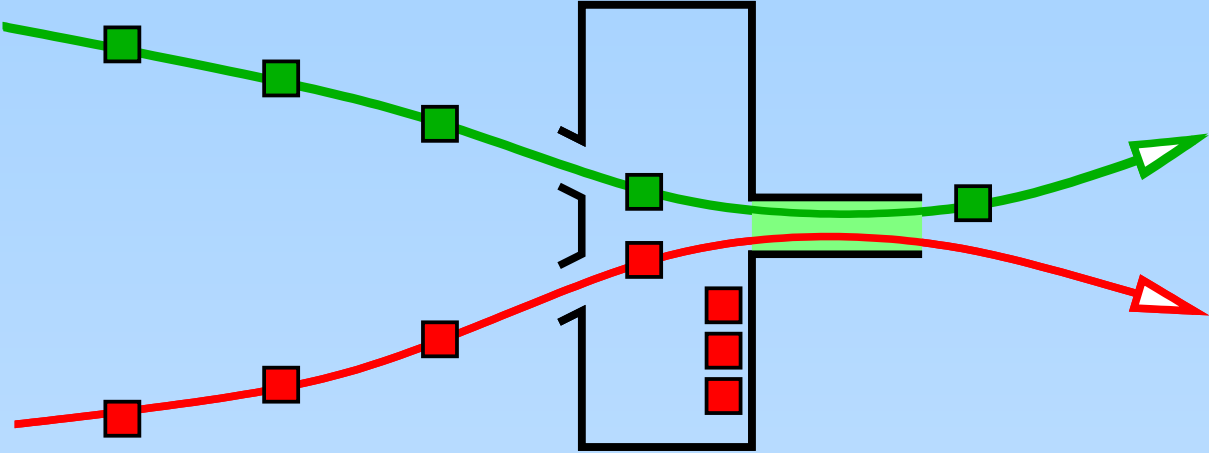
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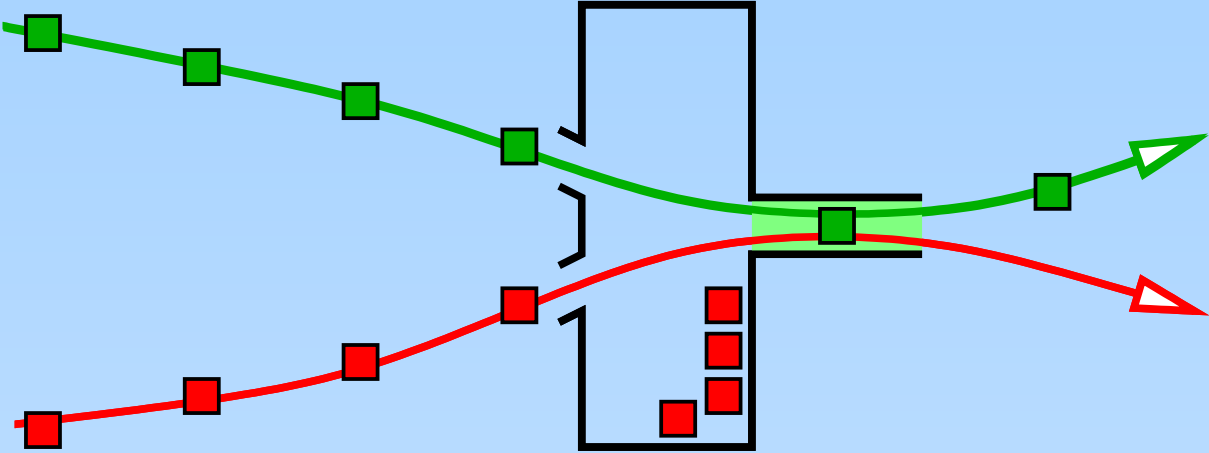
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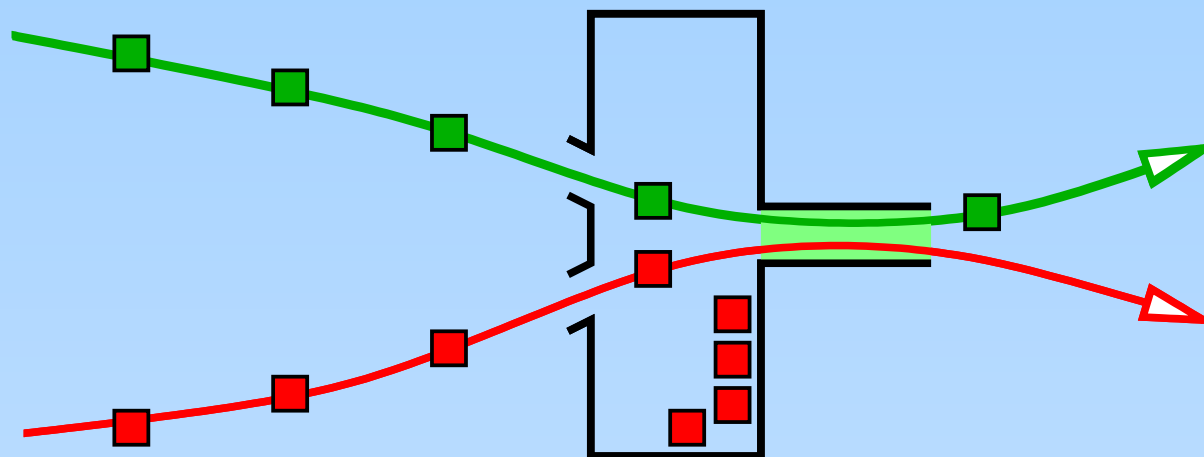
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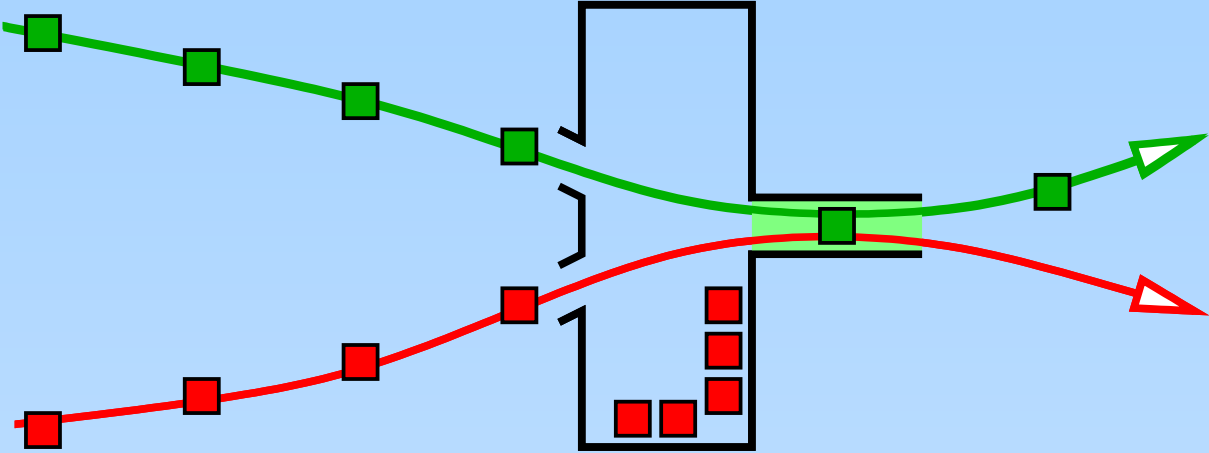


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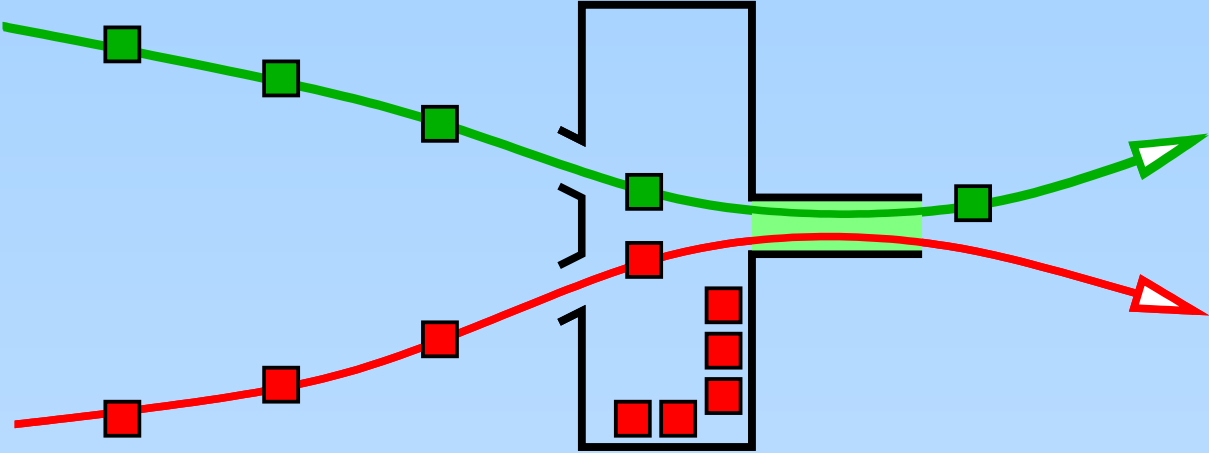




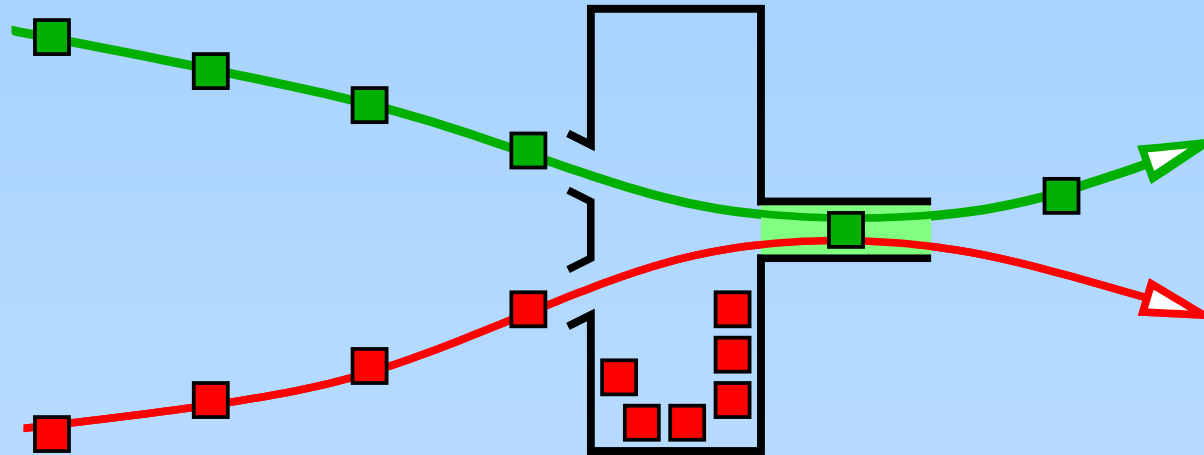
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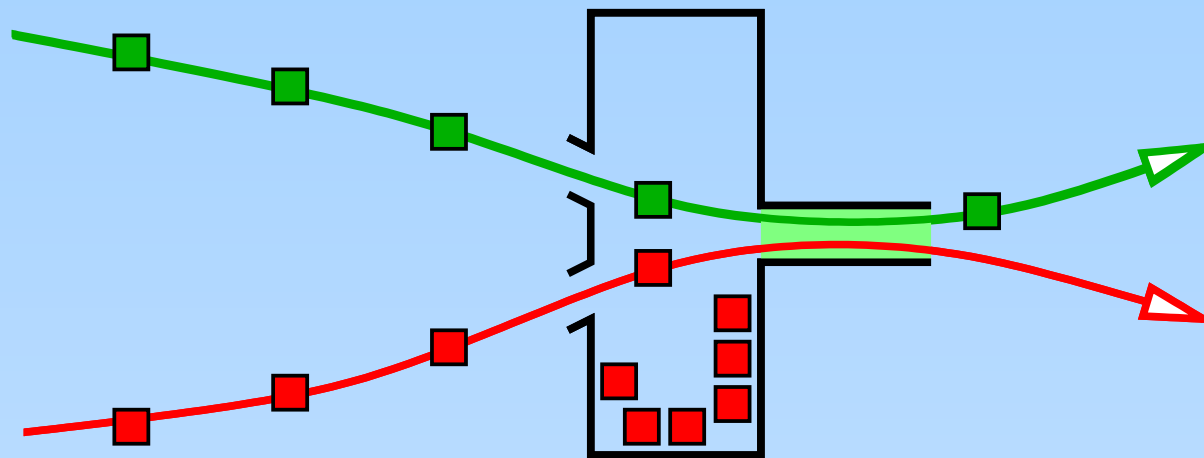
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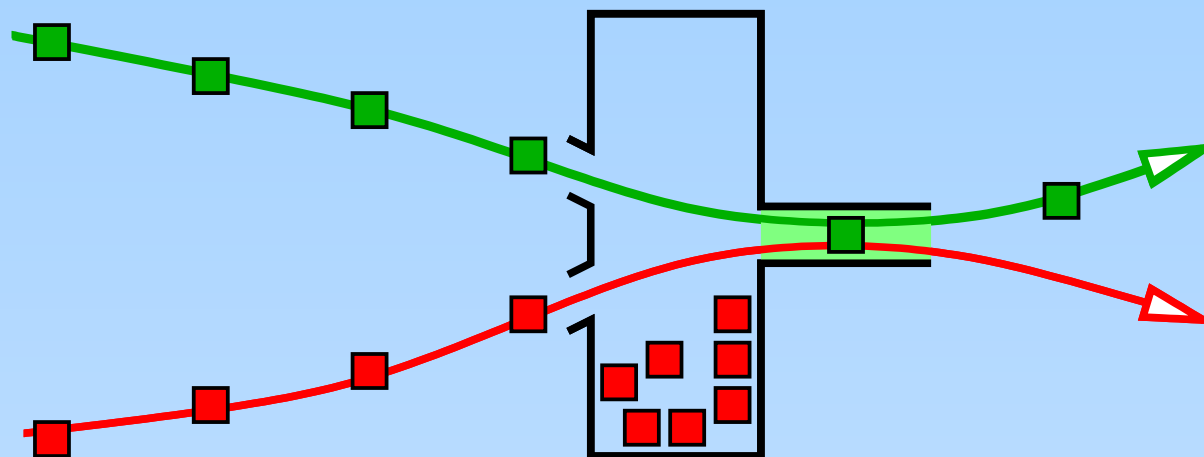
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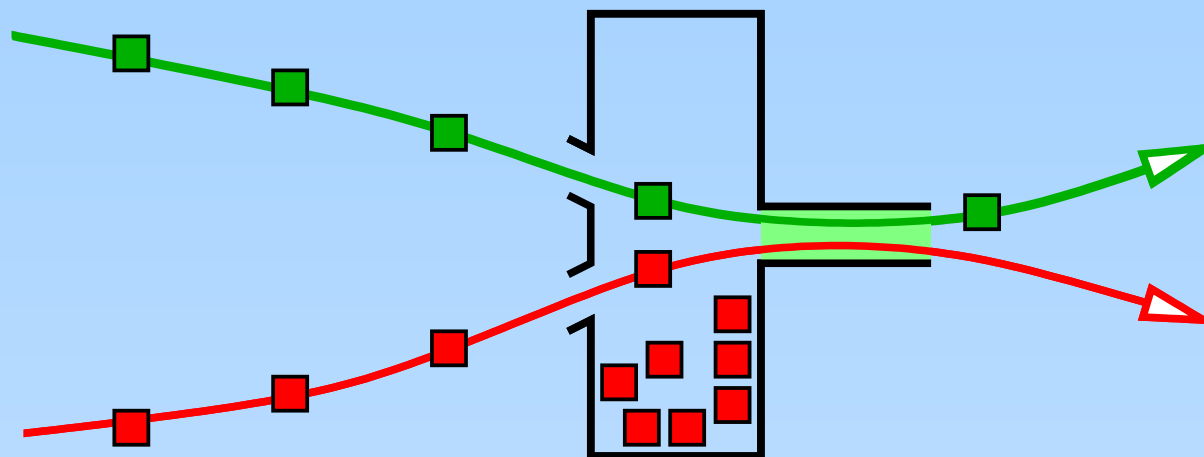
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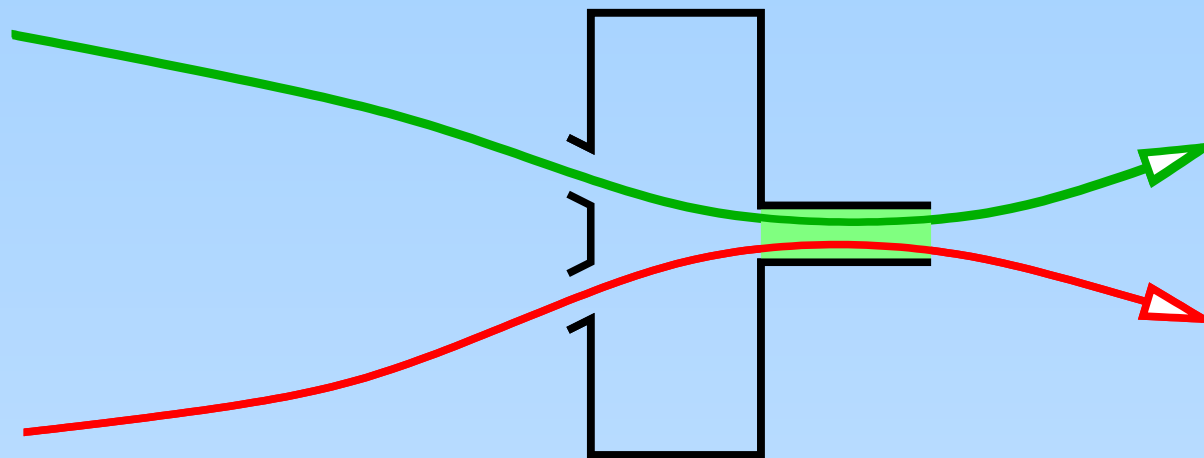


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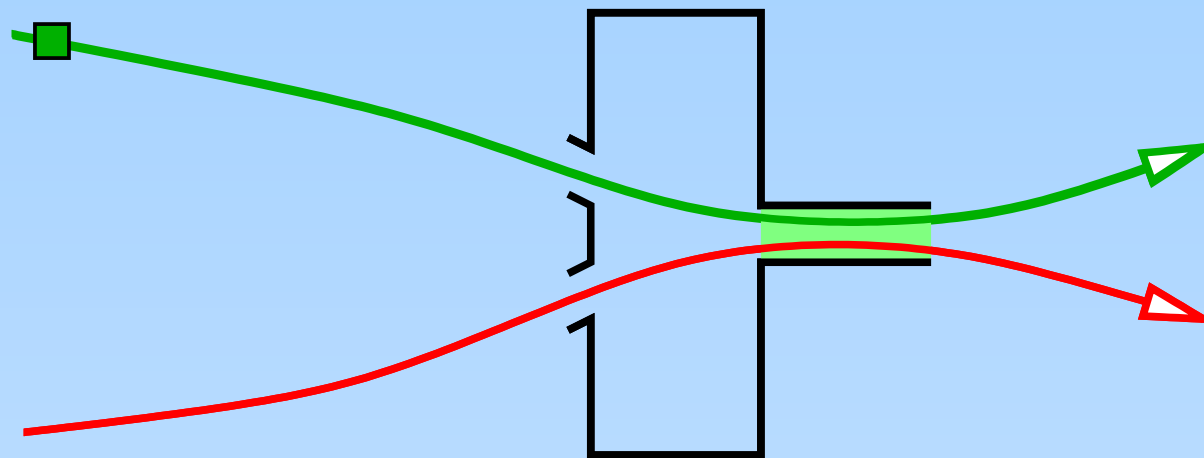
Joint edge is overloaded.

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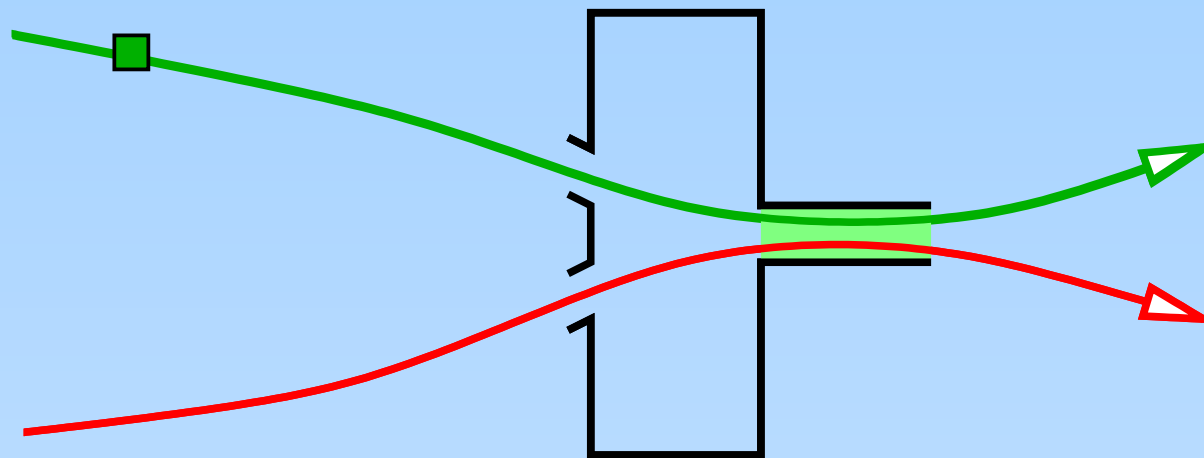
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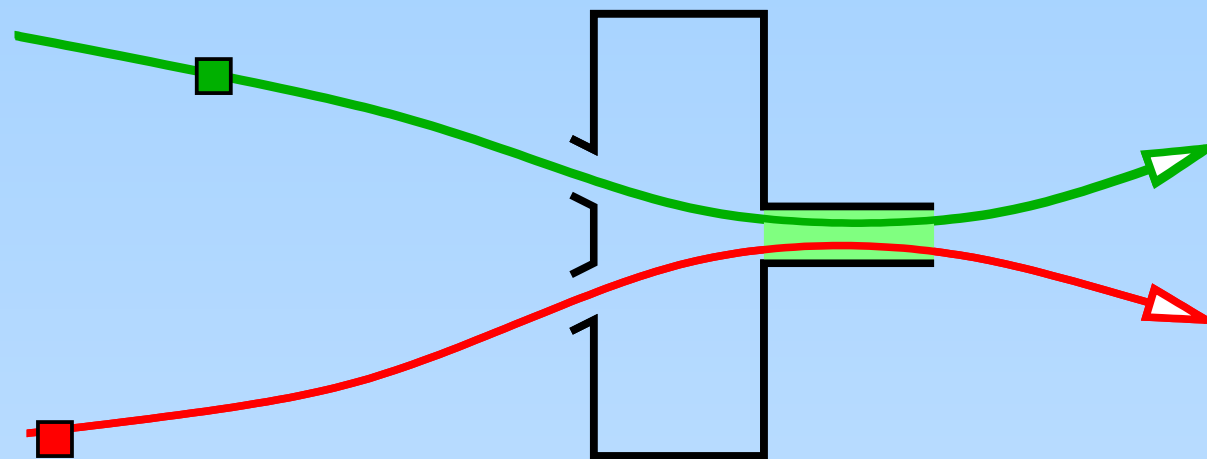


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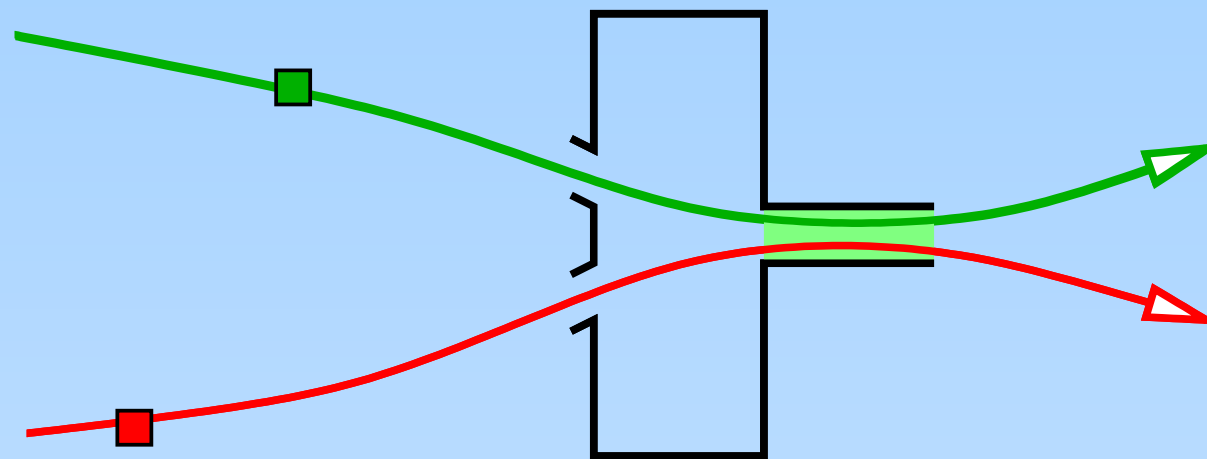
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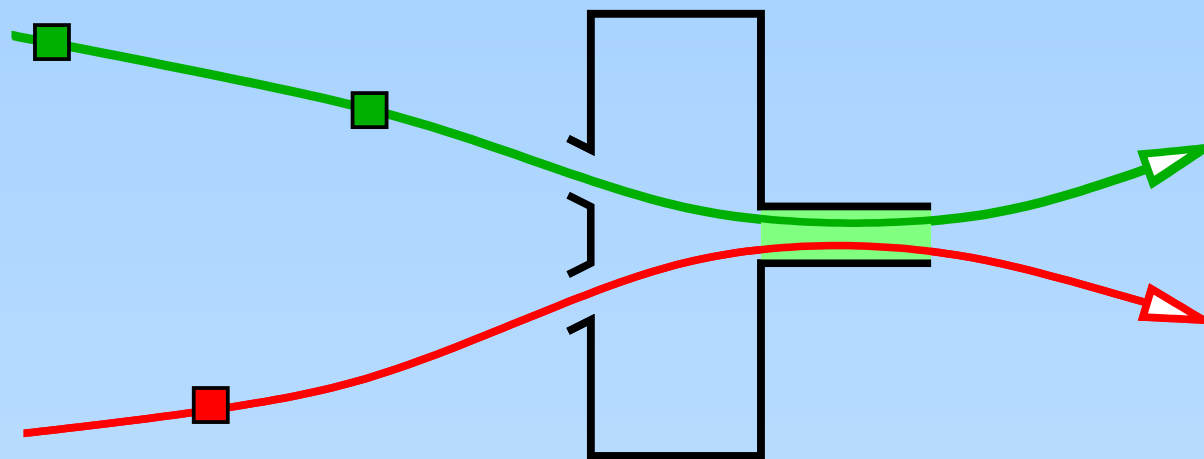
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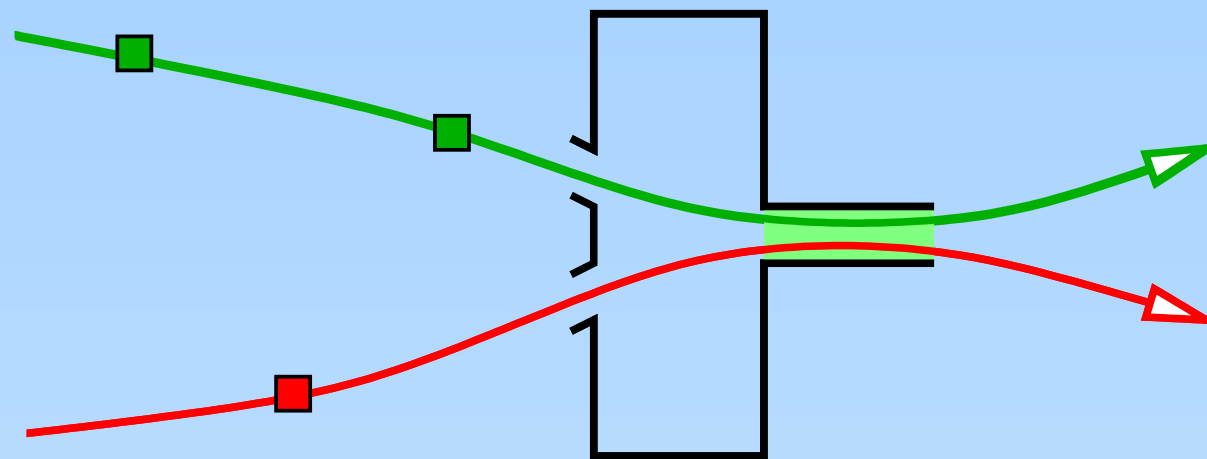
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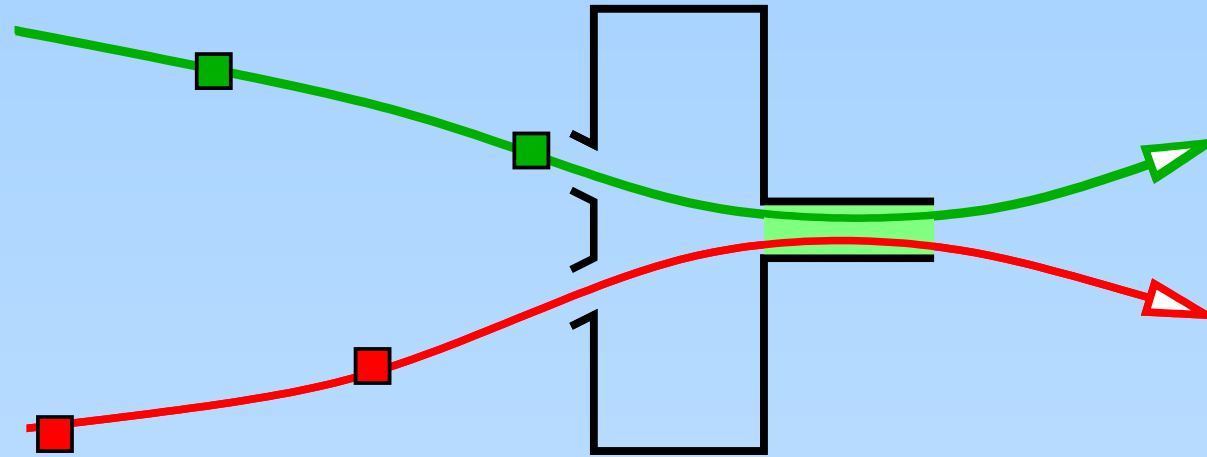
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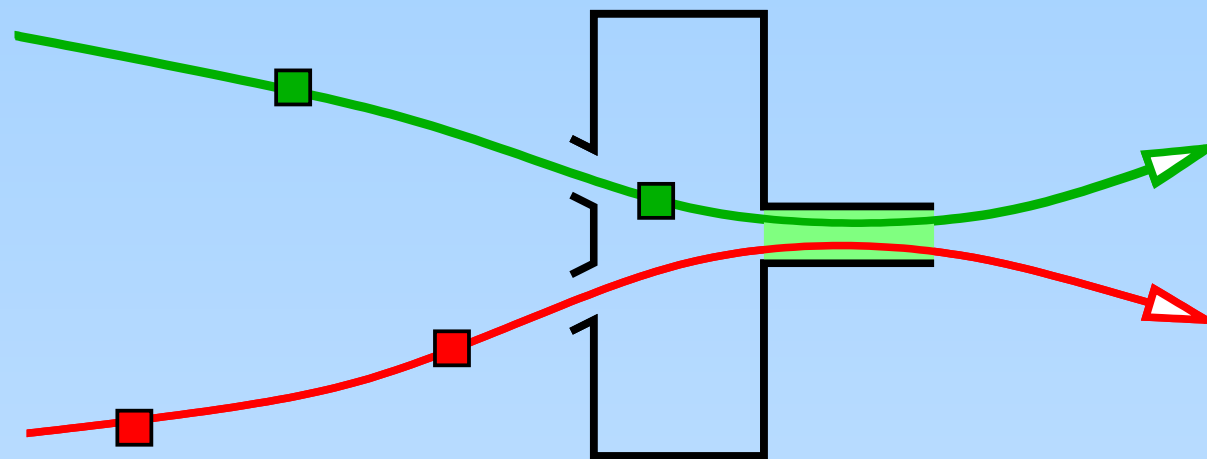
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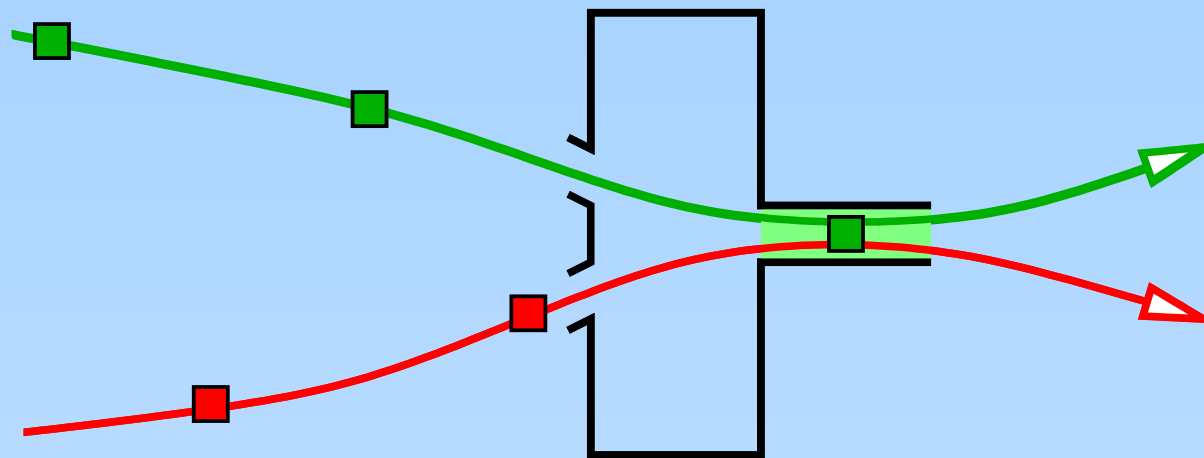
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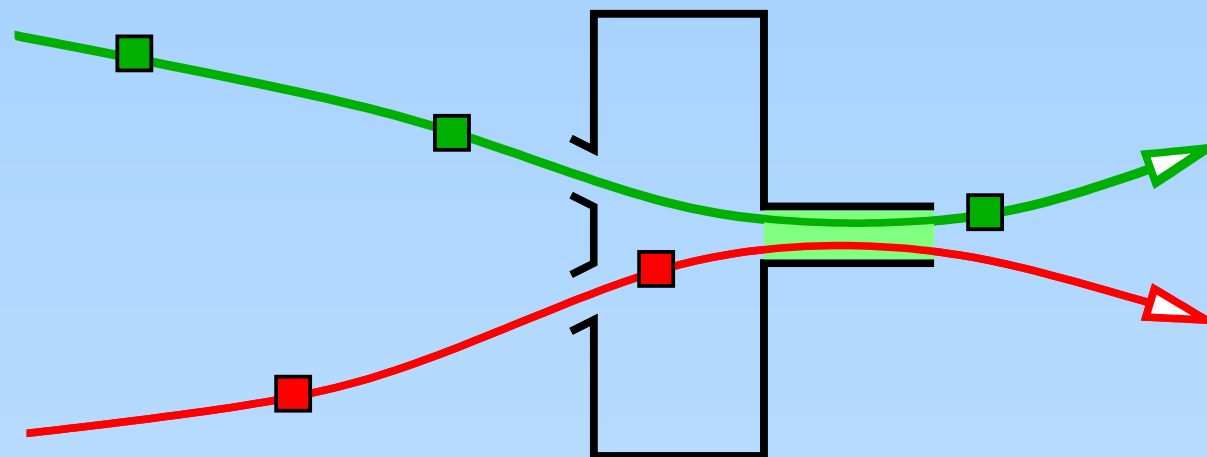
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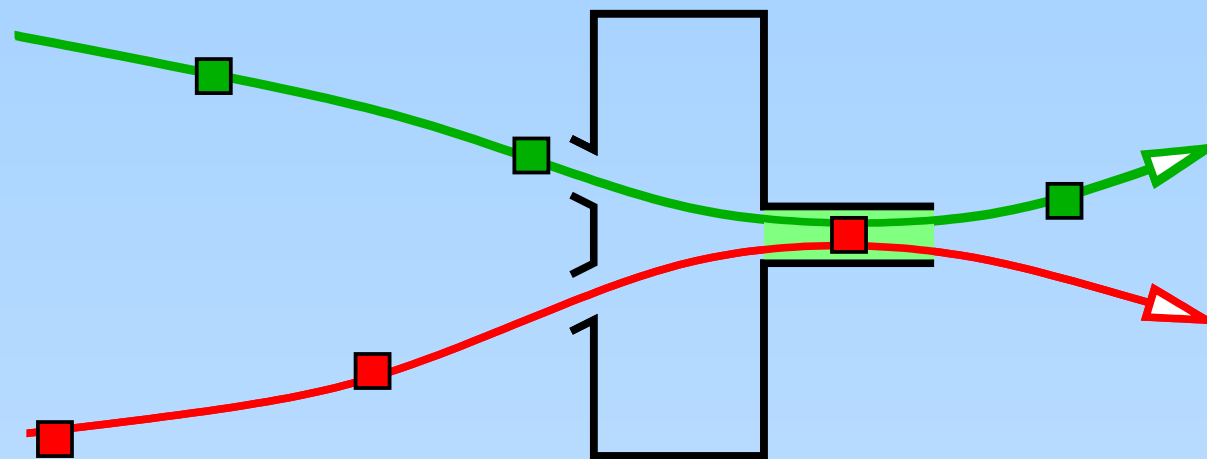


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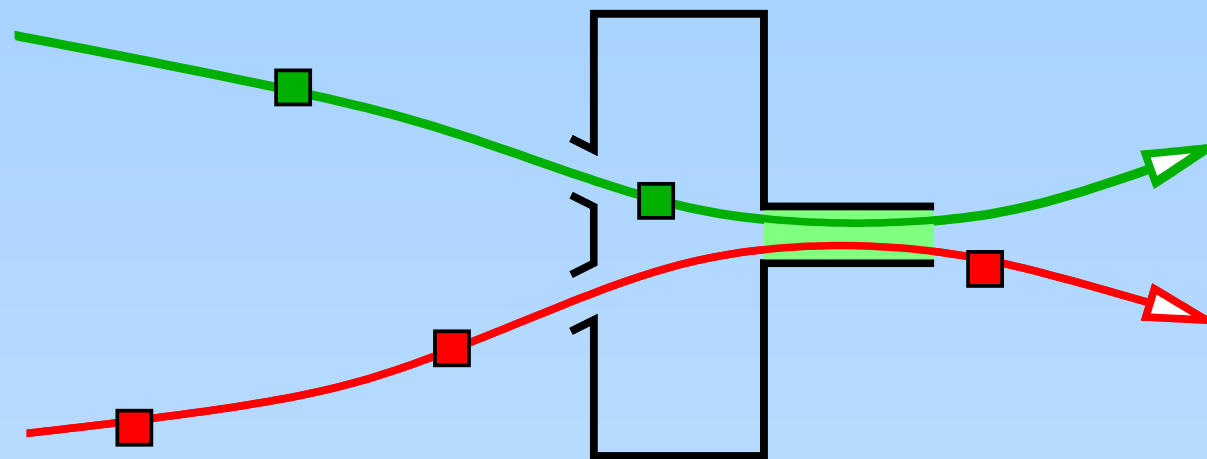
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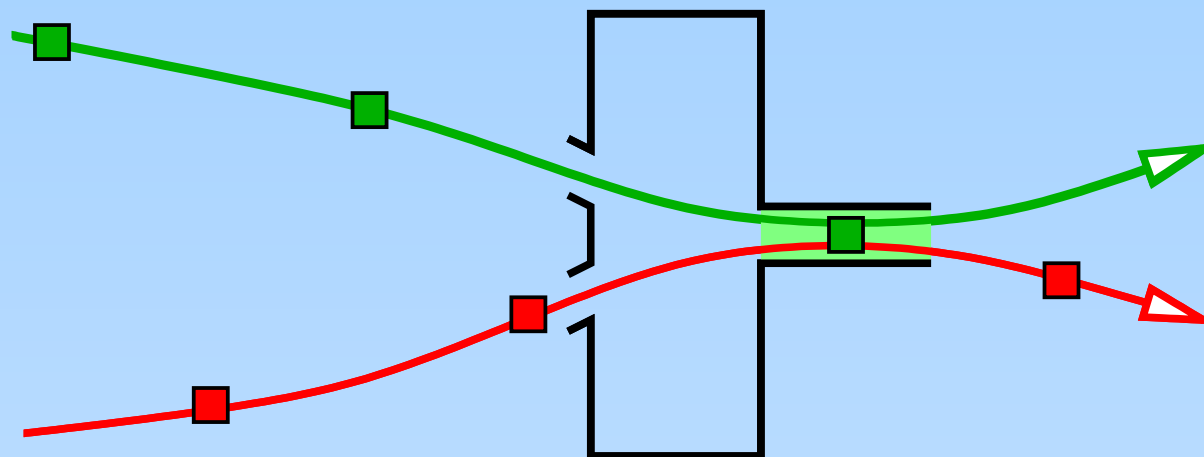
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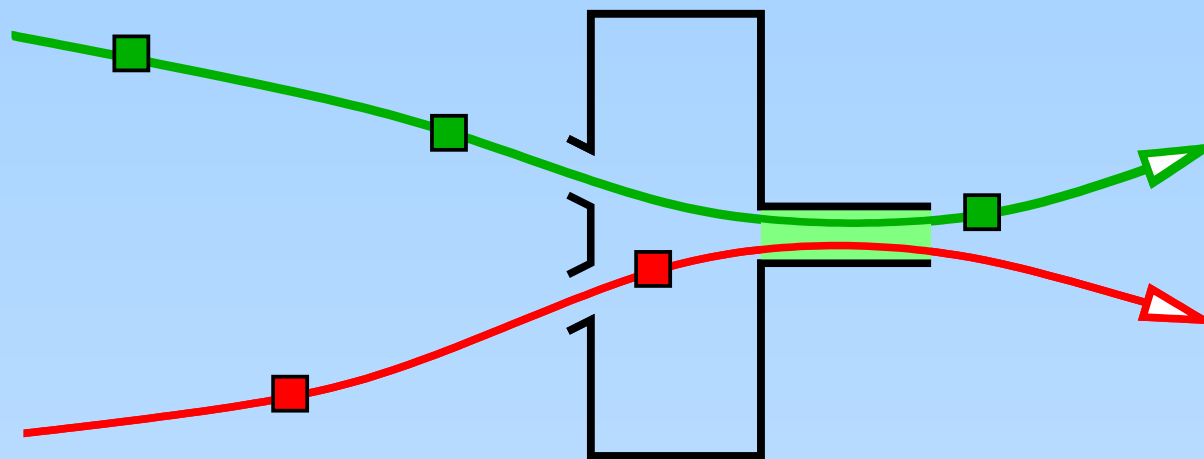
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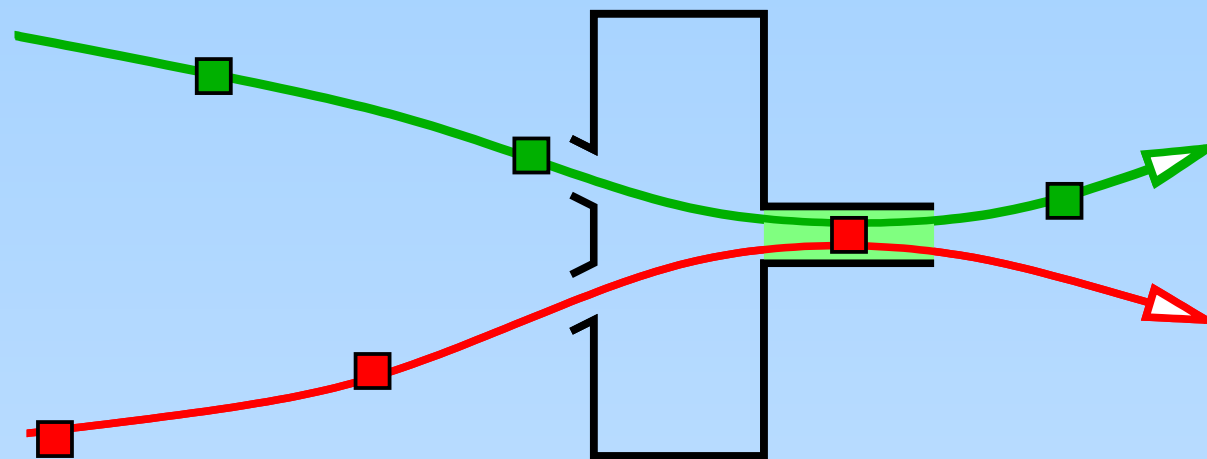
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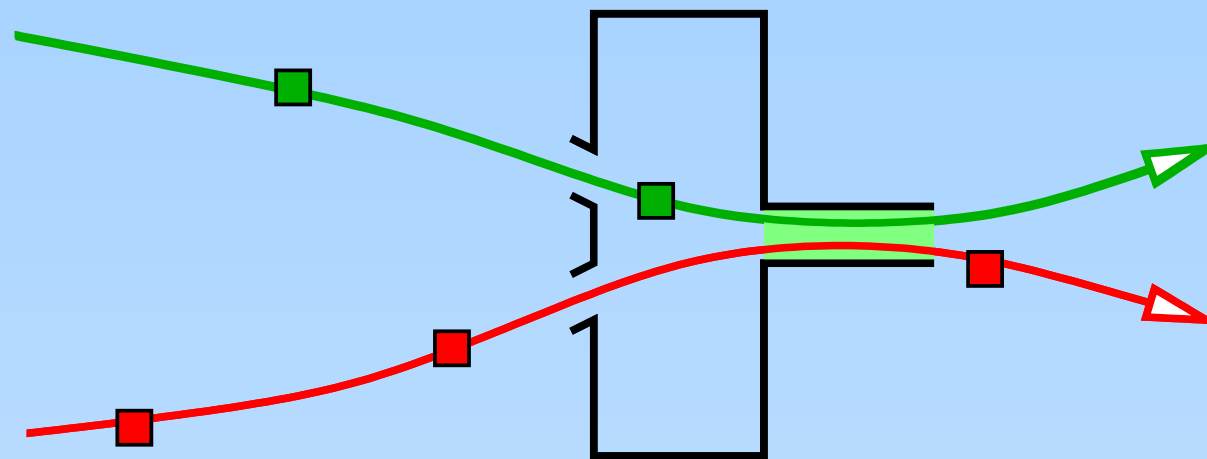
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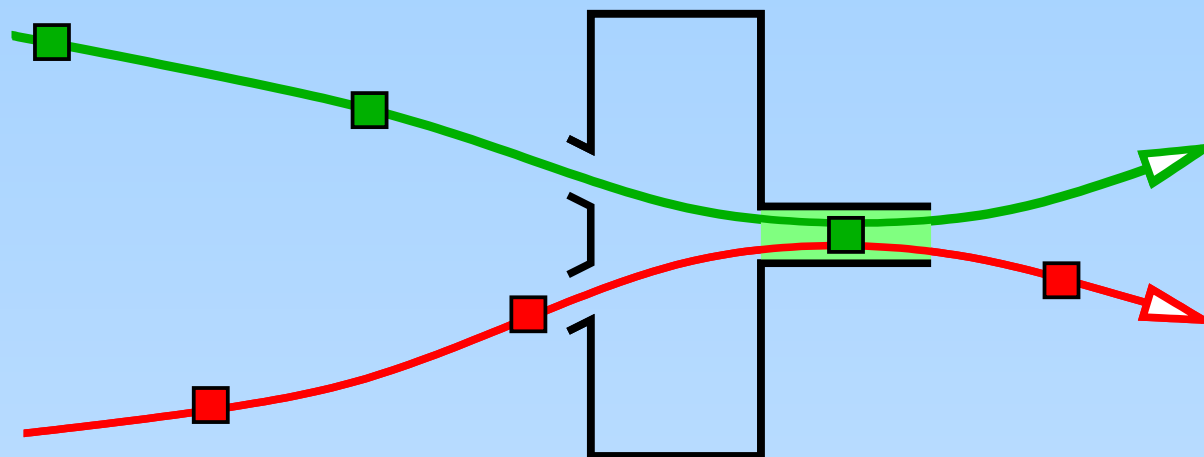
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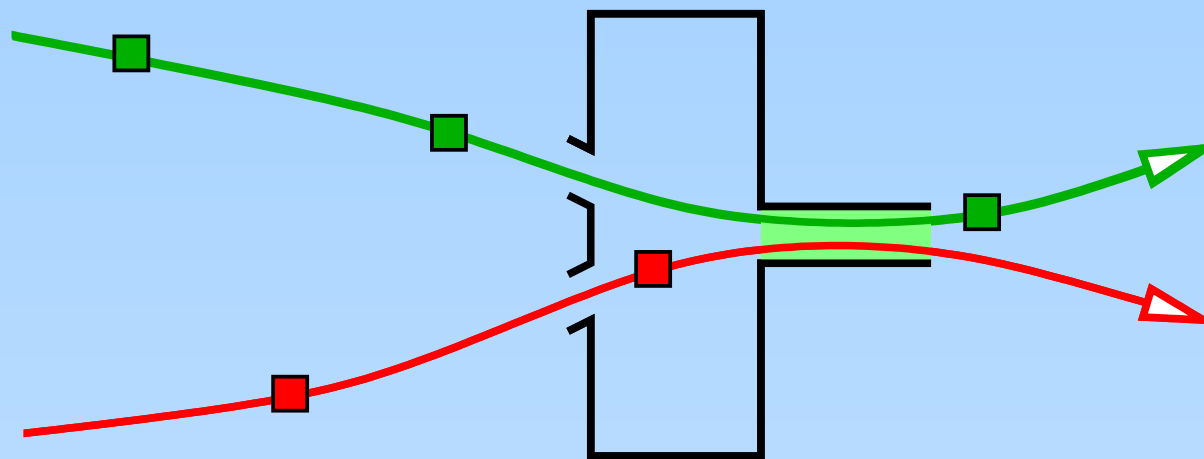
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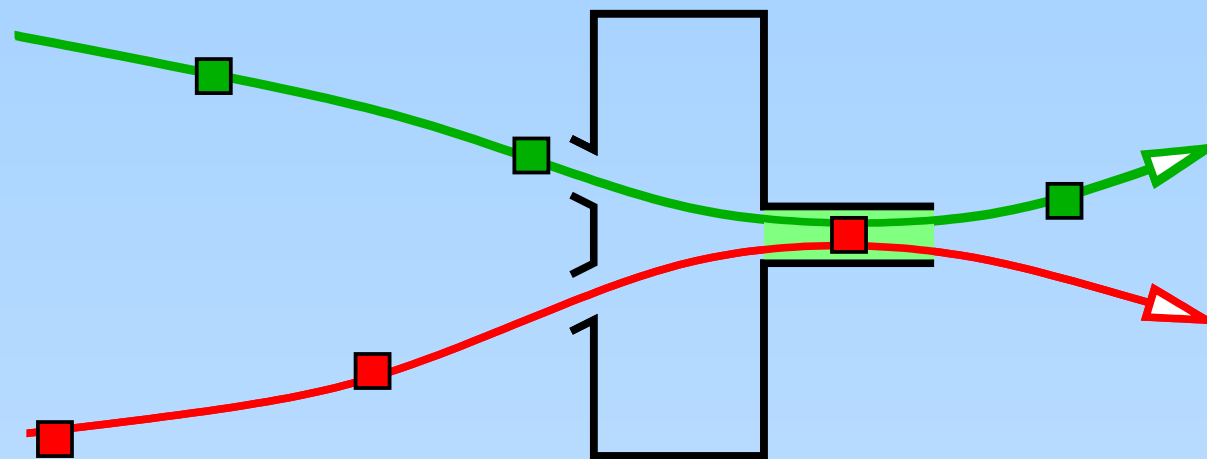


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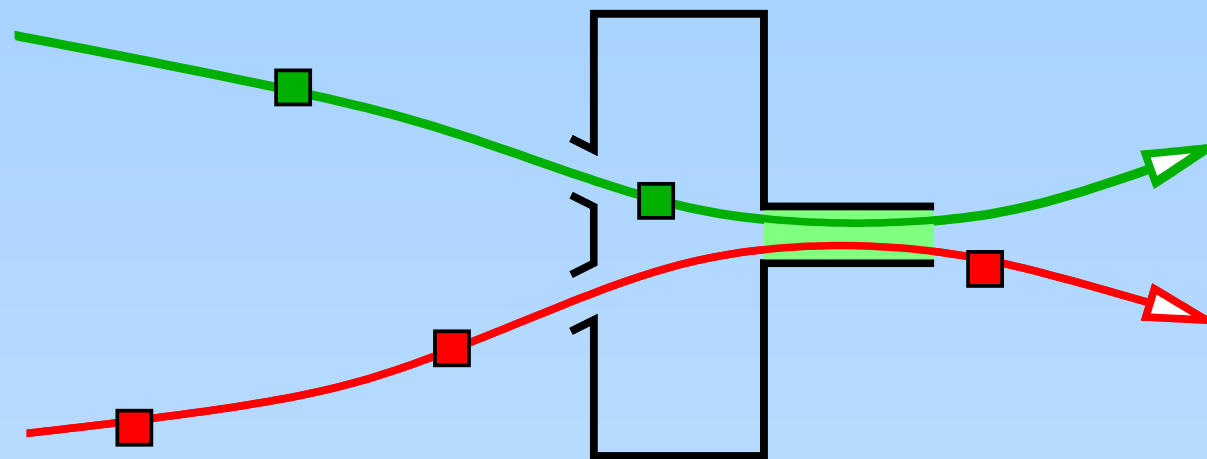
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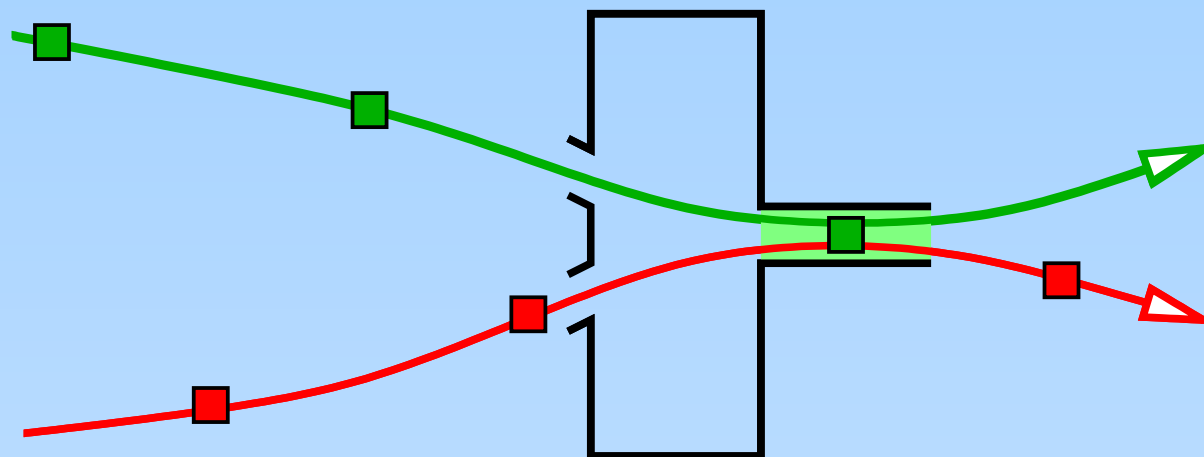
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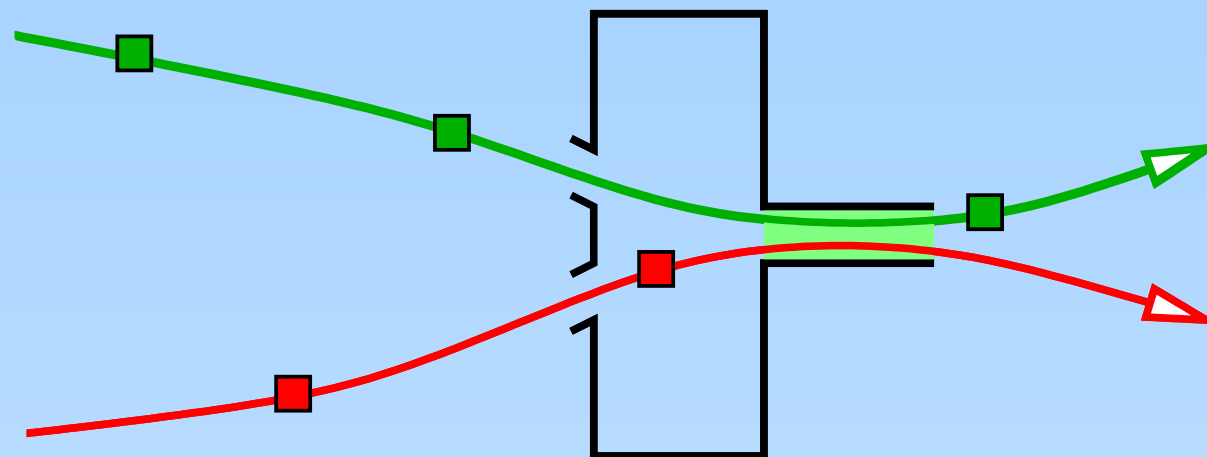
## How to create a traffic jam



Joint edge is overloaded.

For legal insertions no jam is created.

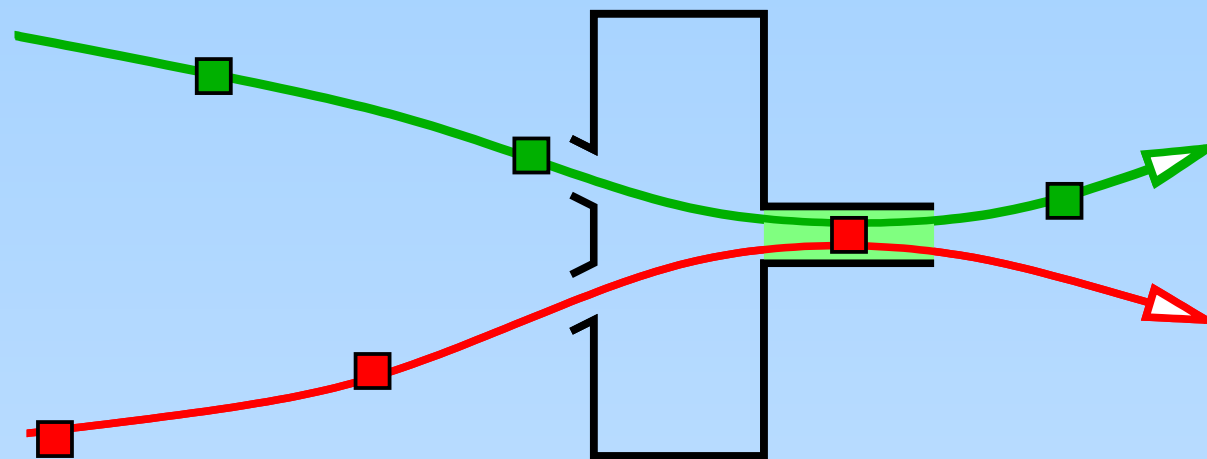
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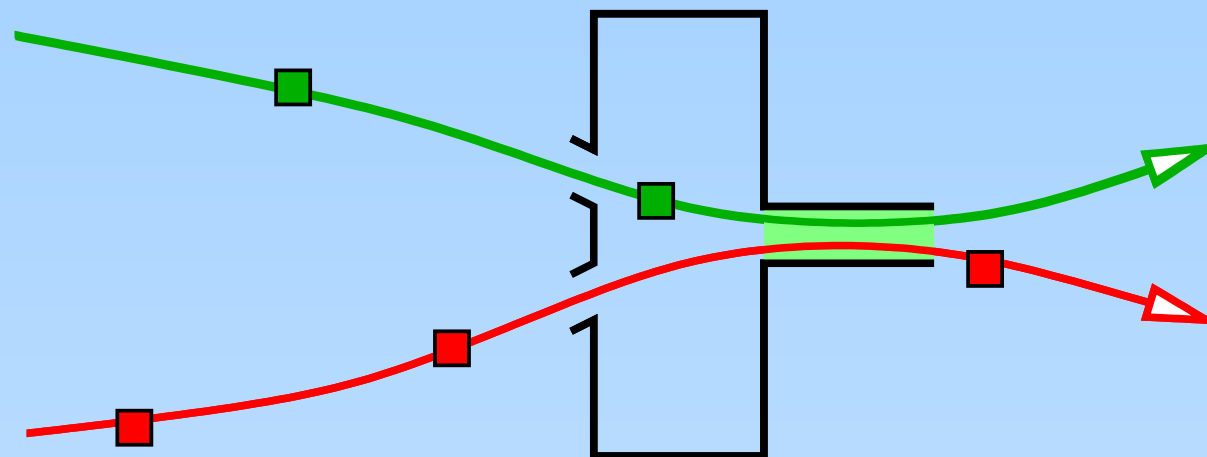
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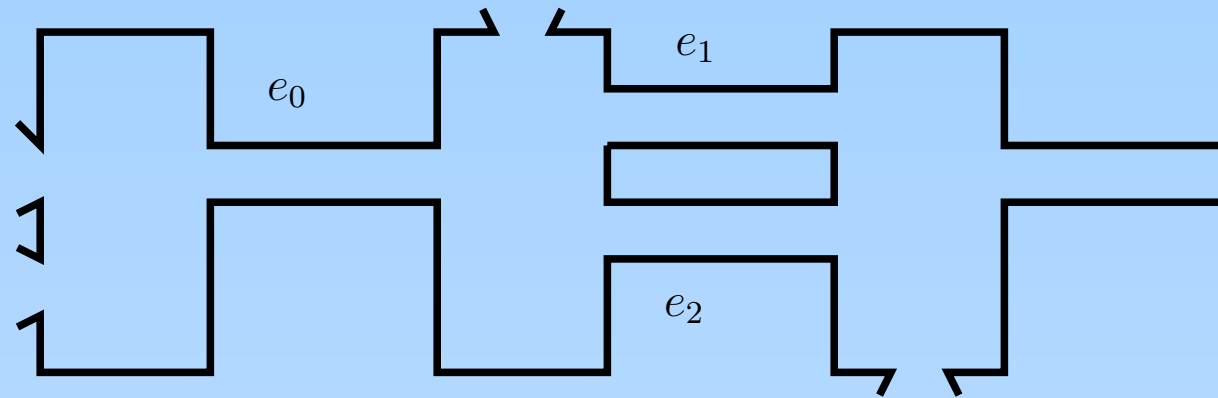
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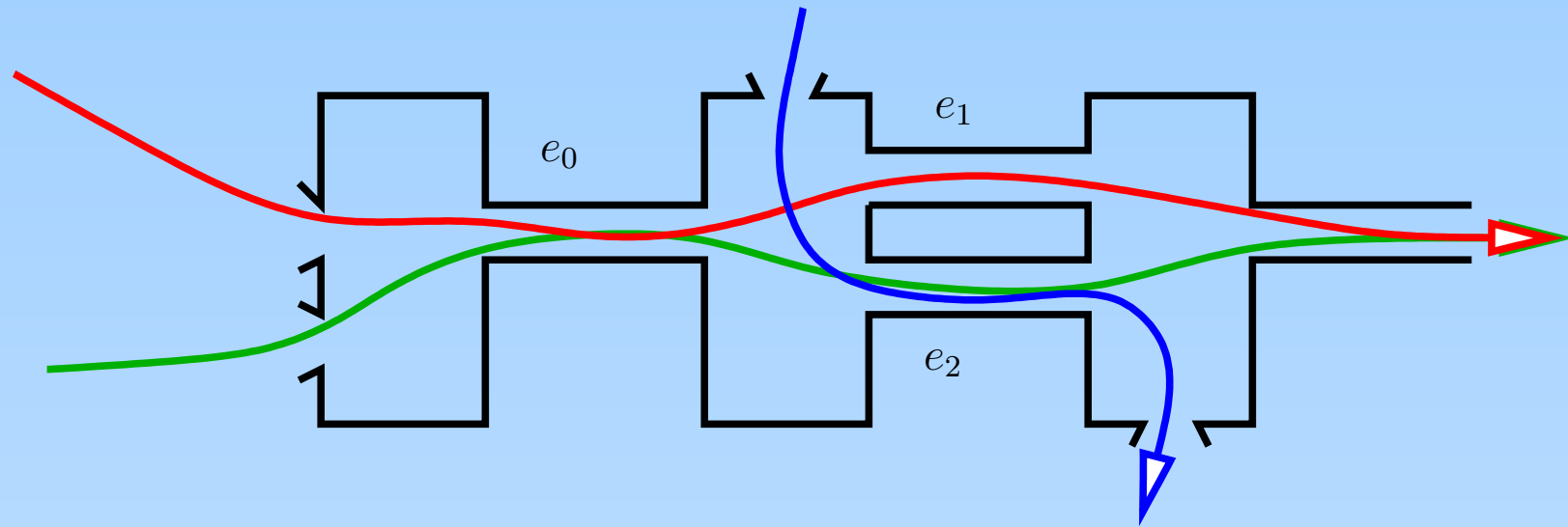
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# Piling up packets

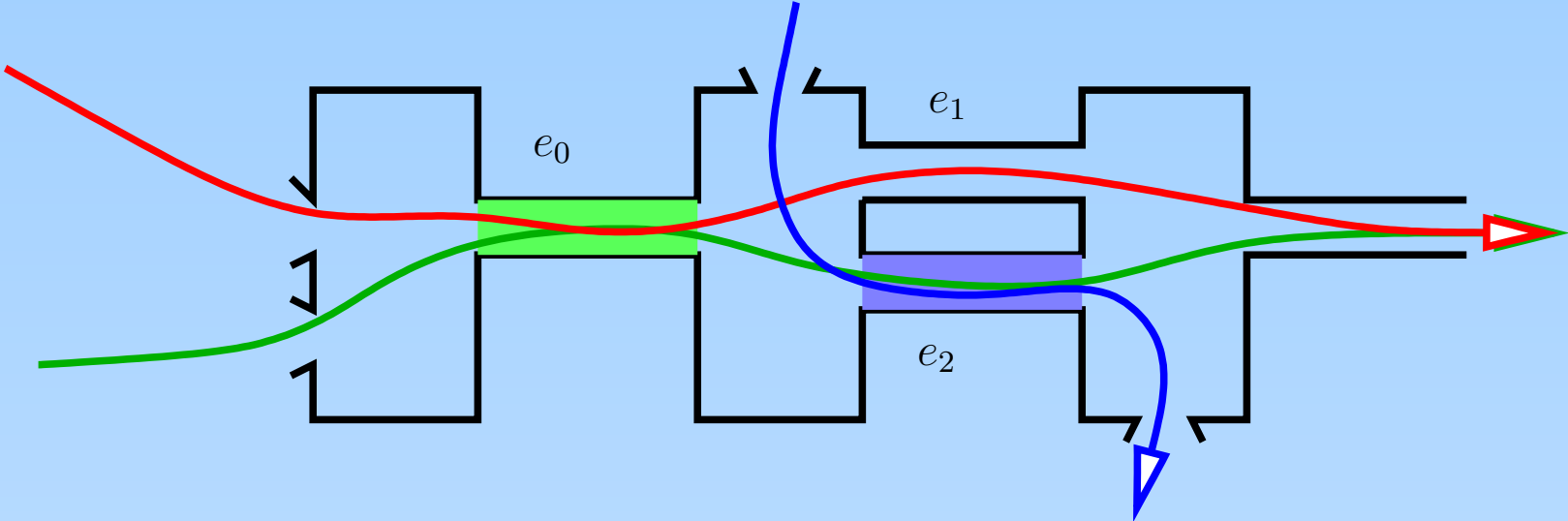




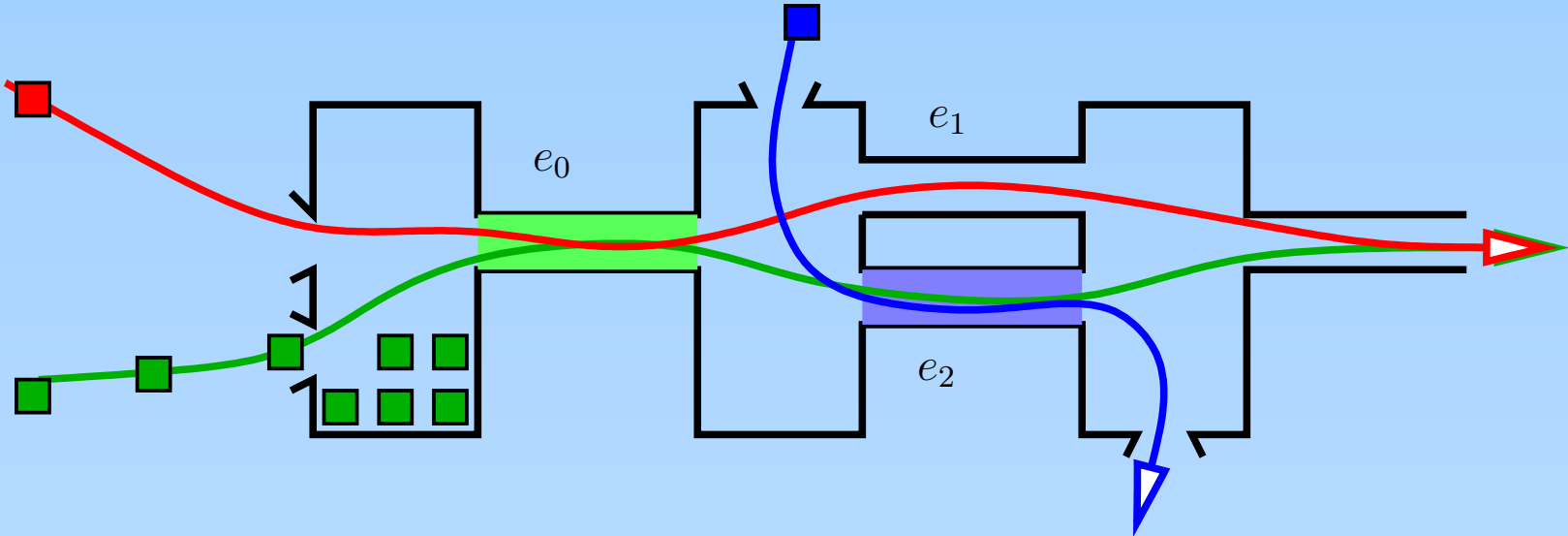
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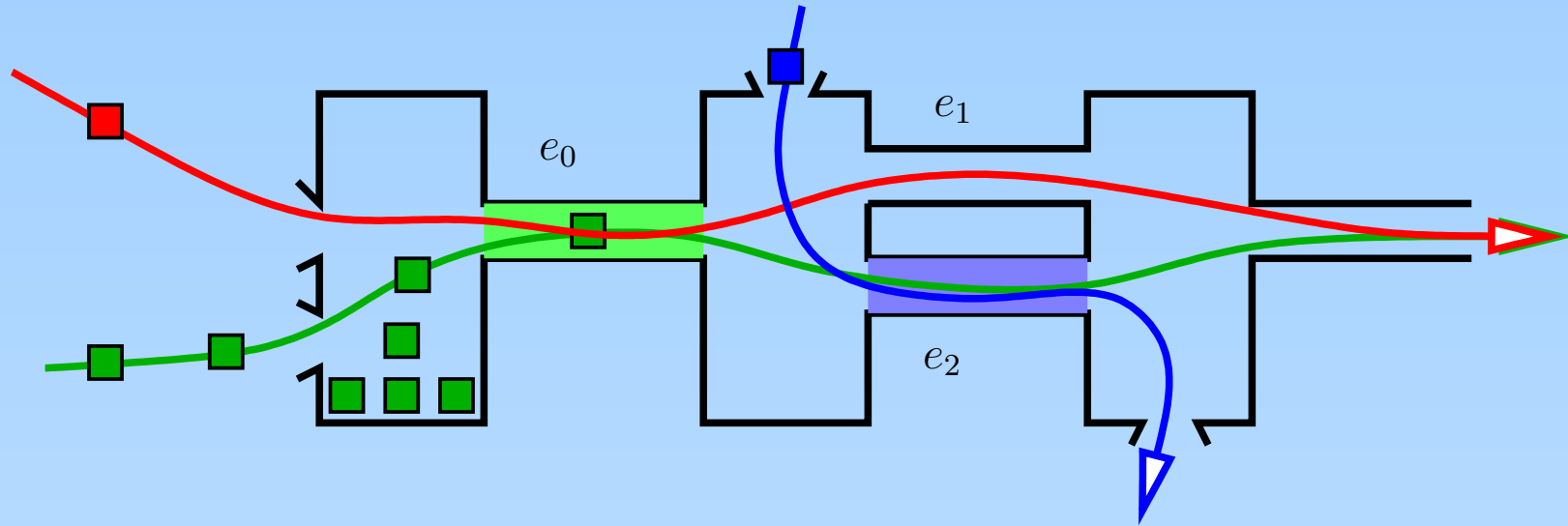
# Piling up packets



# Packets	
# green	$s$
# red	$rs$
# blue	$rs$



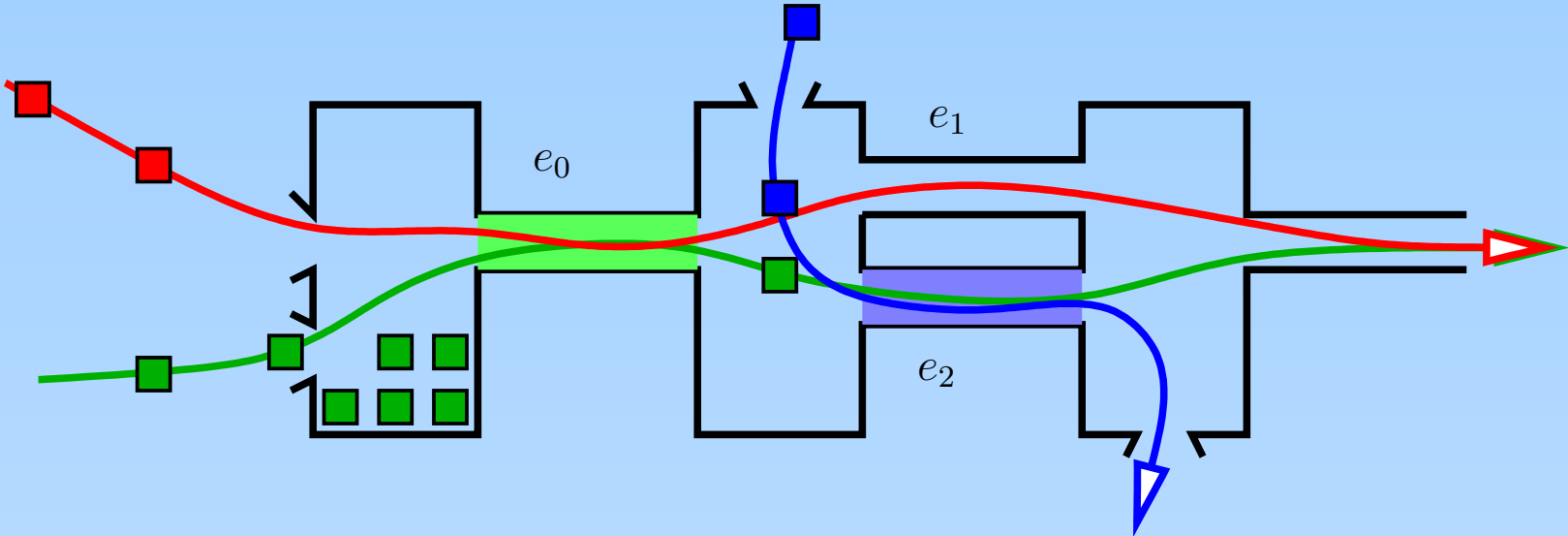
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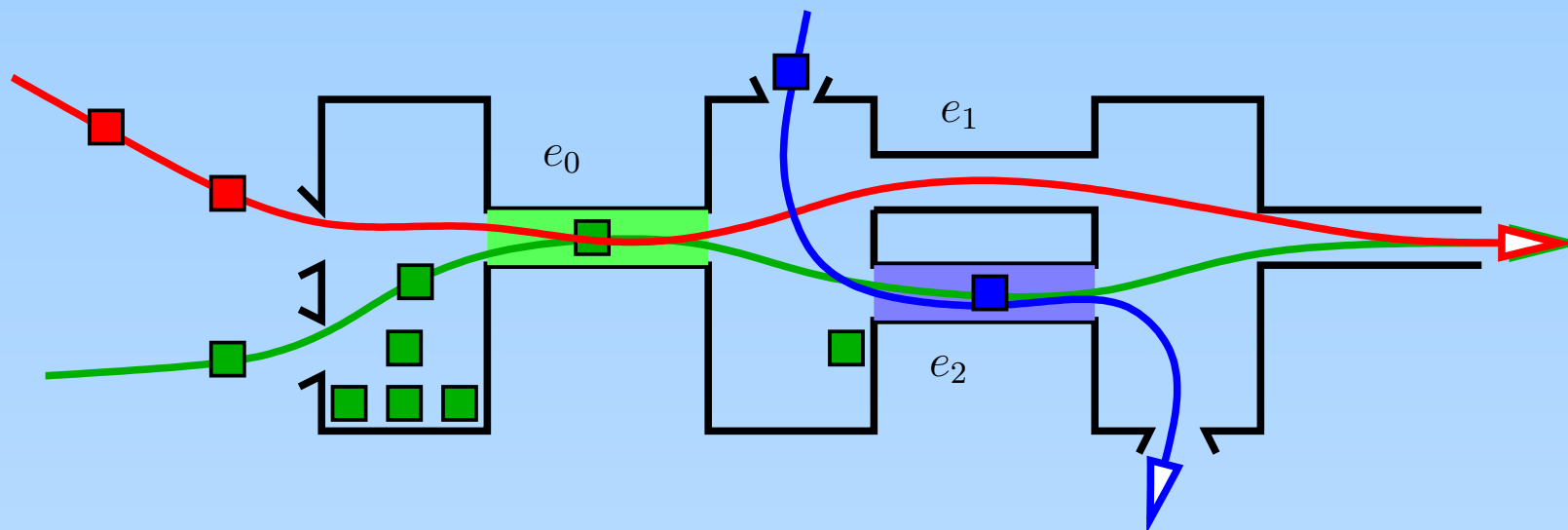
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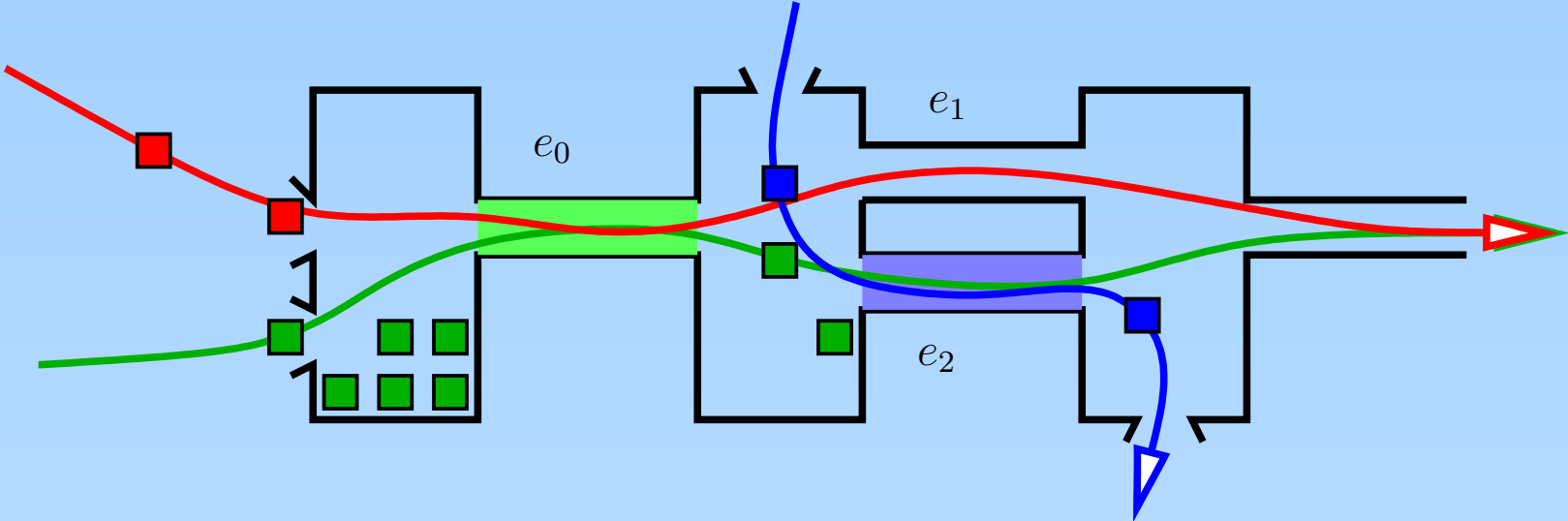
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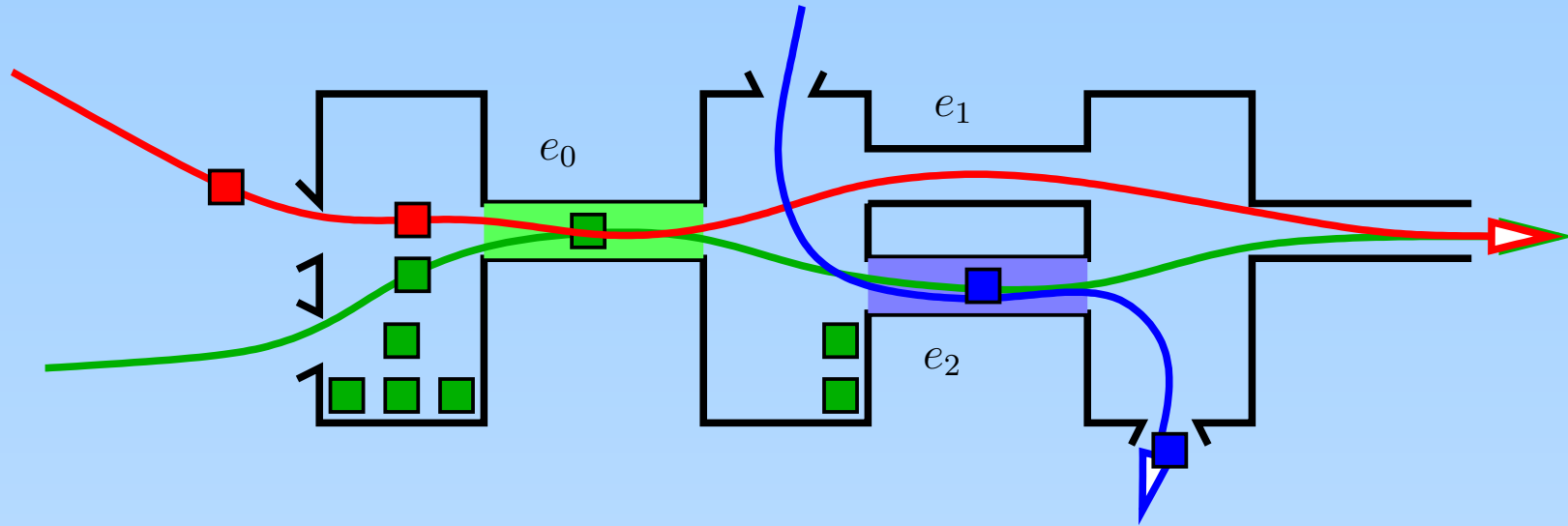
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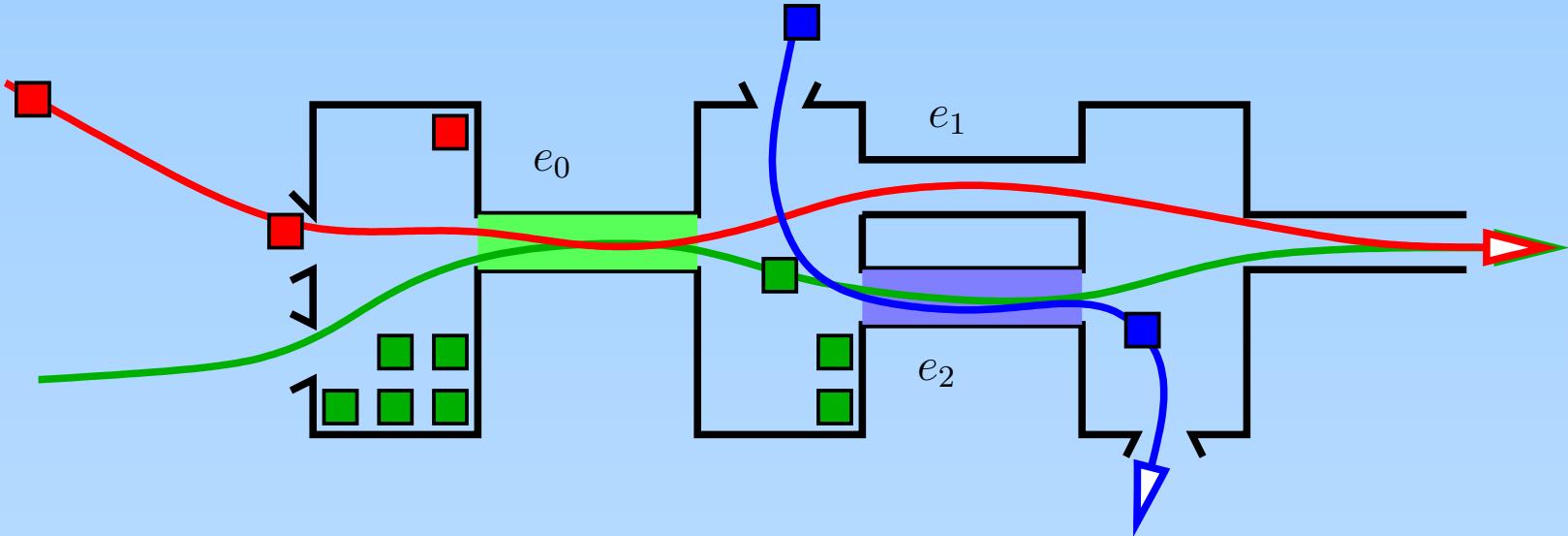


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# green	$s$
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# blue	$rs$





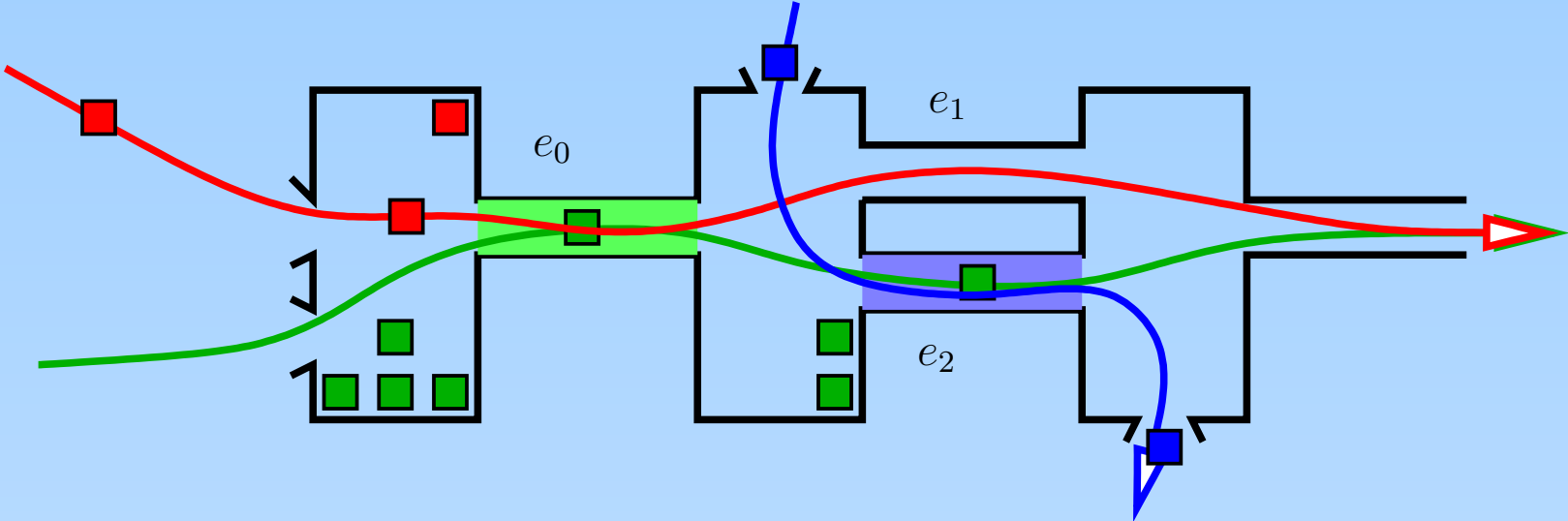
# Piling up packets



# Packets	
# green	$s$
# red	$rs$
# blue	$rs$



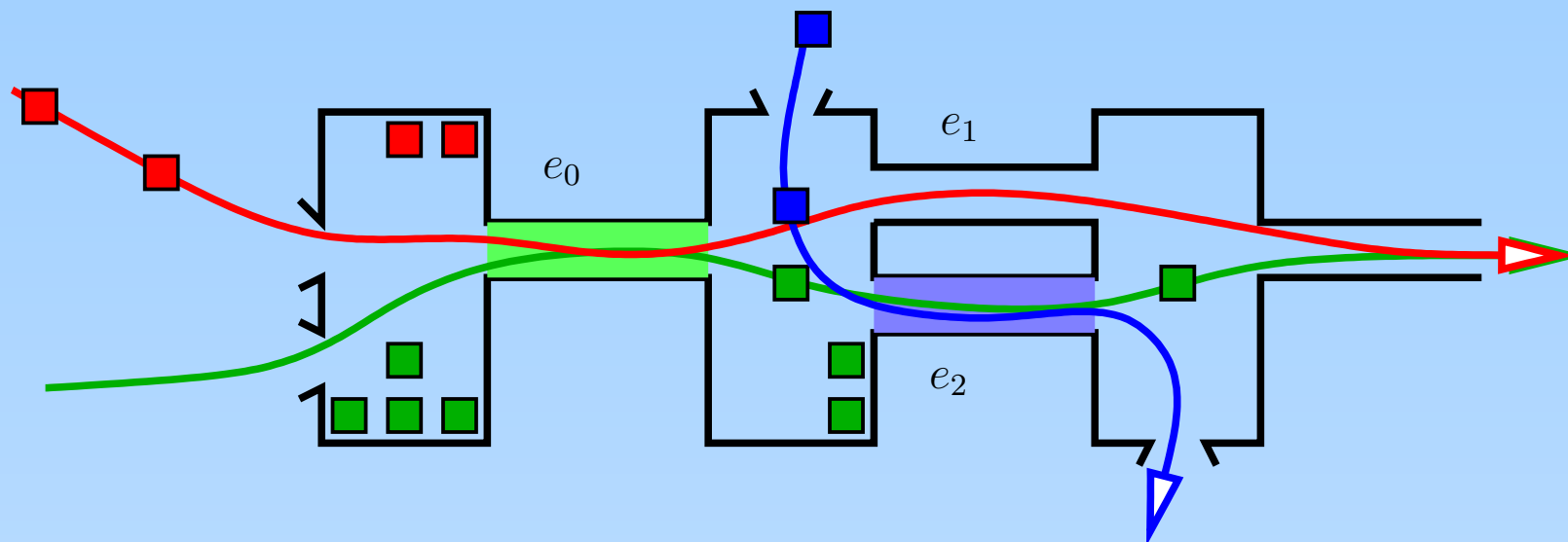
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# blue	$rs$



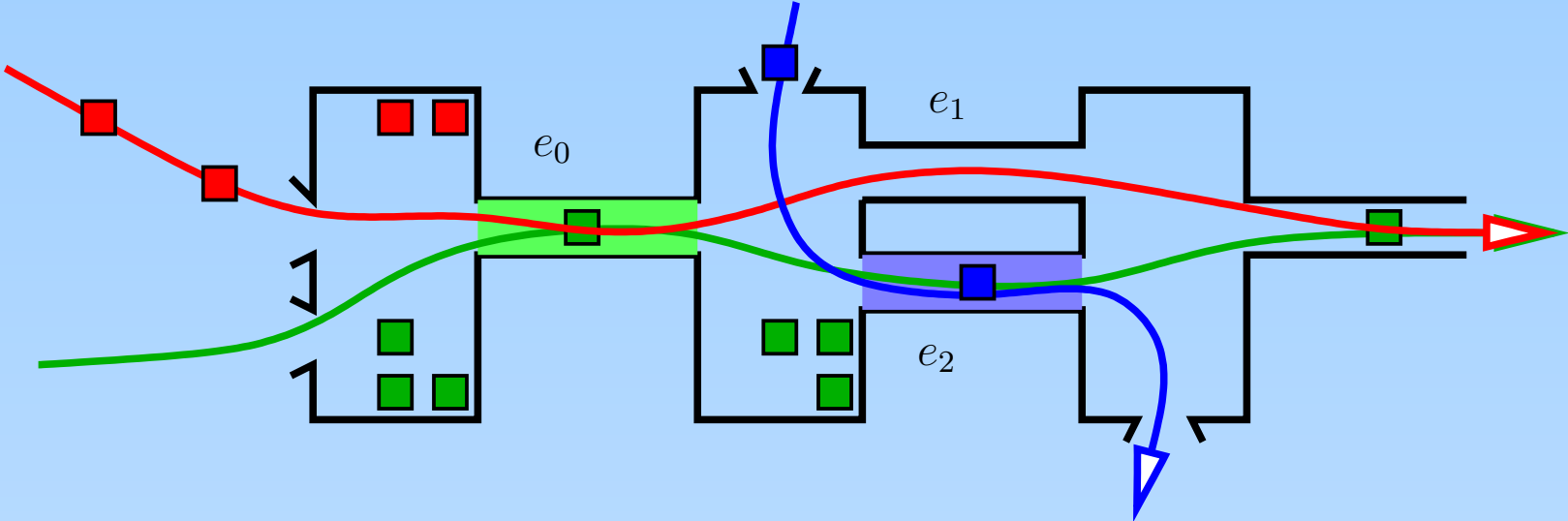
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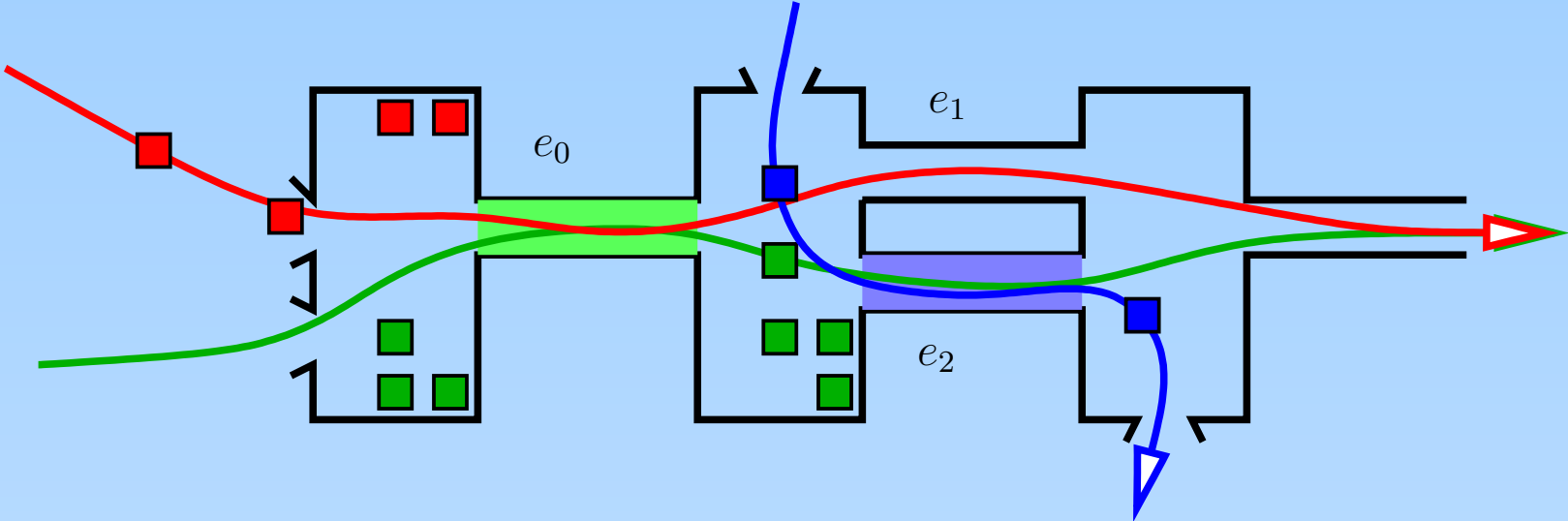
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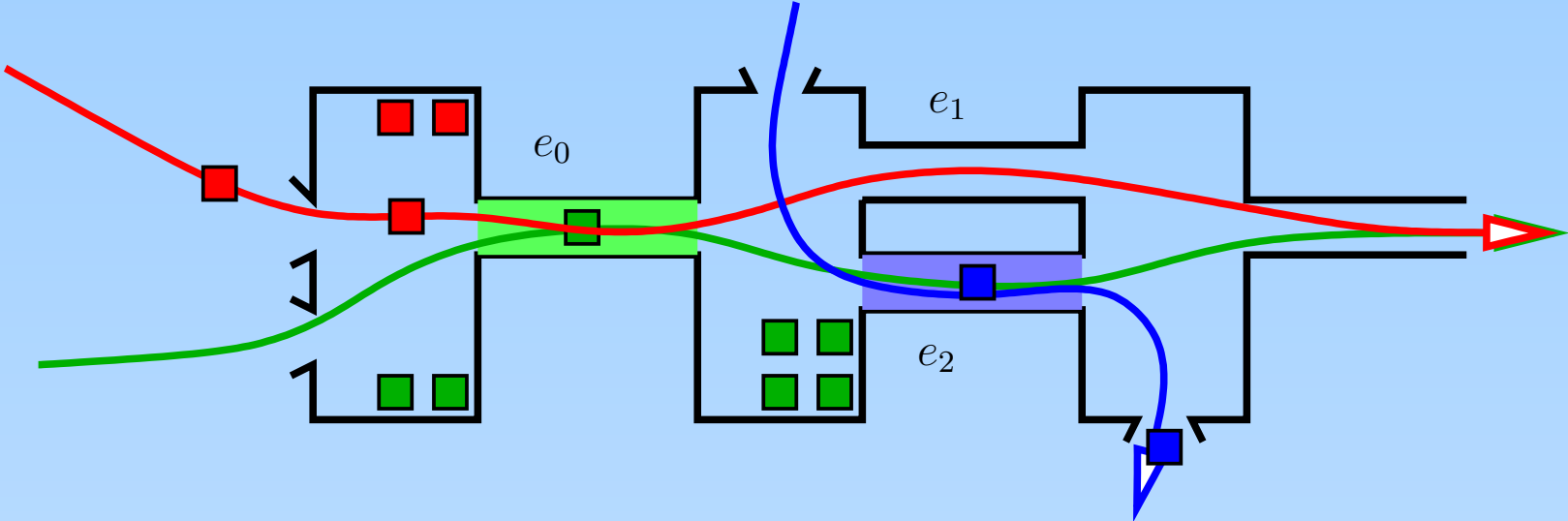
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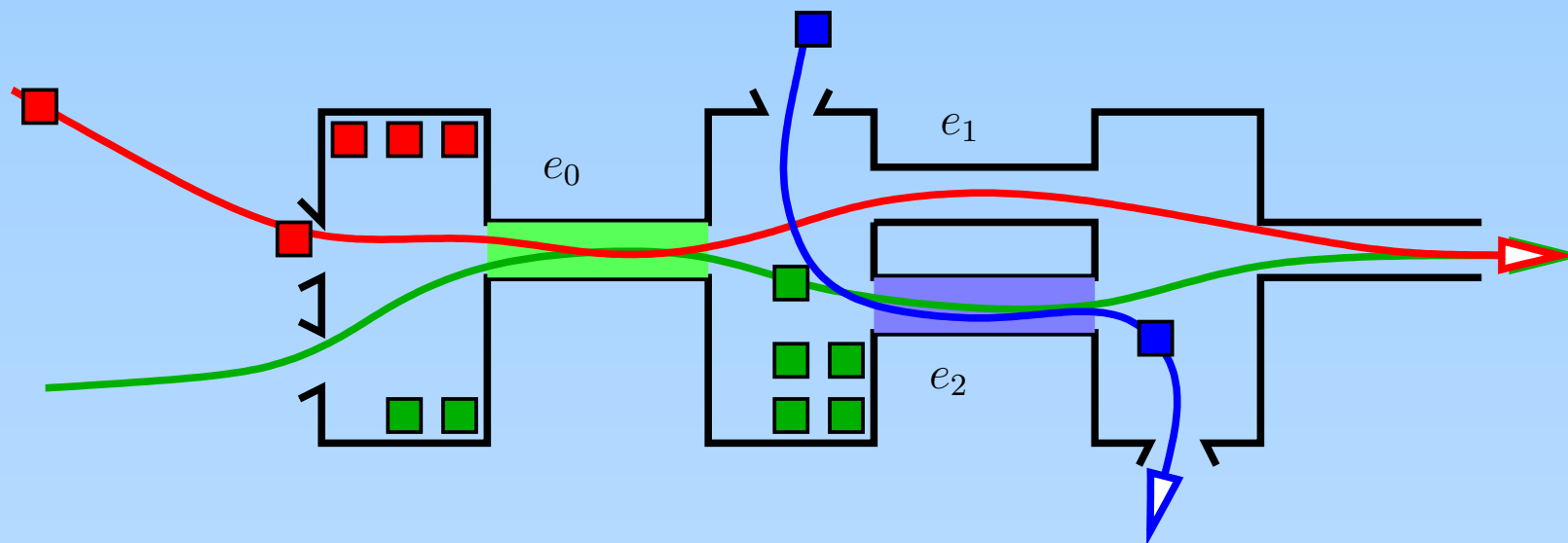
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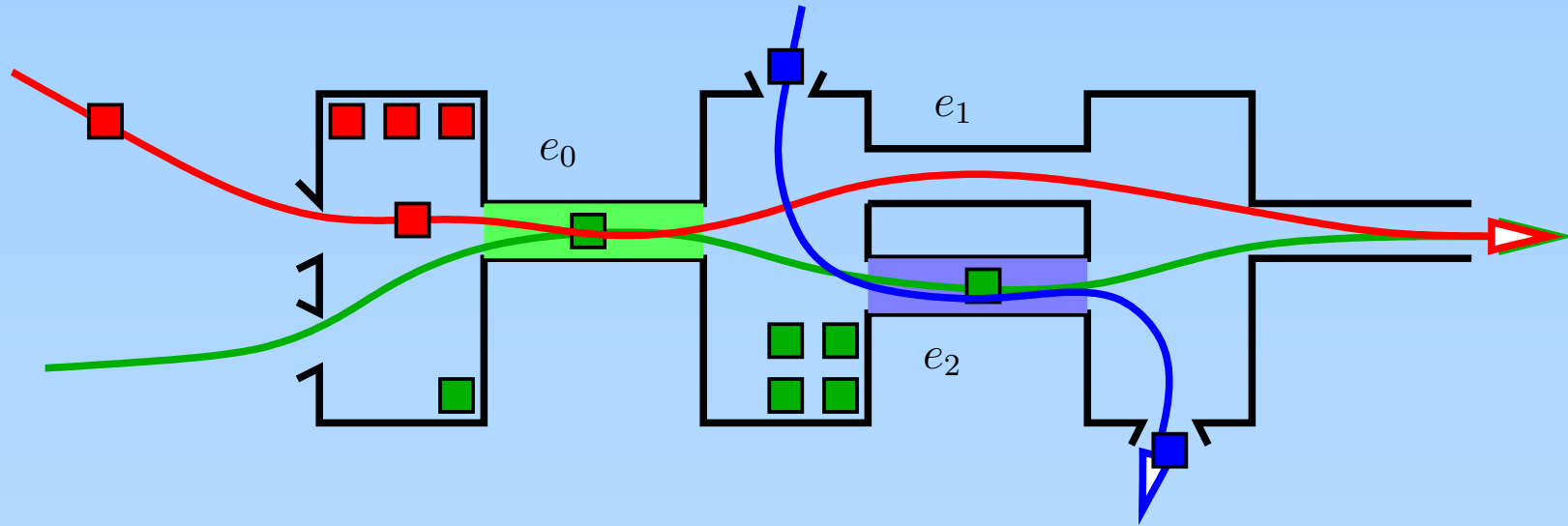
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# Packets	
# green	$s$
# red	$rs$
# blue	$rs$

No red packet traverses the system.

# Piling up packets



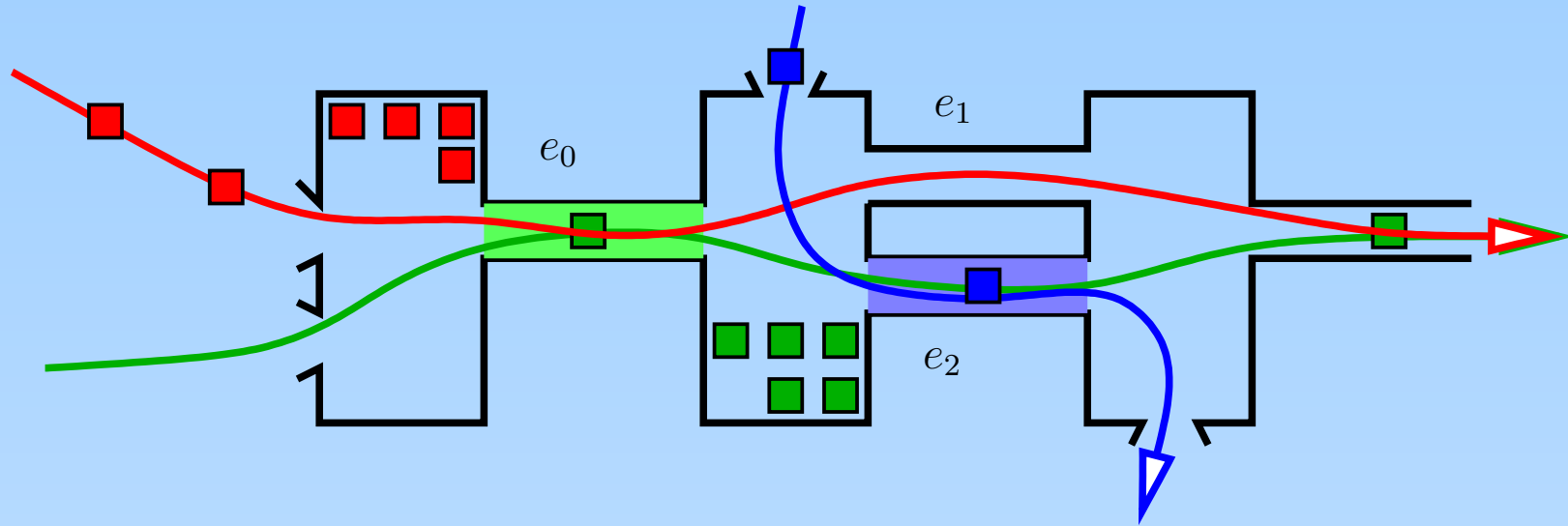
# Packets	
# green	$s$
# red	$rs$
# blue	$rs$

No red packet traverses the system.  
 Only  $(1 - r) \cdot s$  green packets slip through.





# Piling up packets

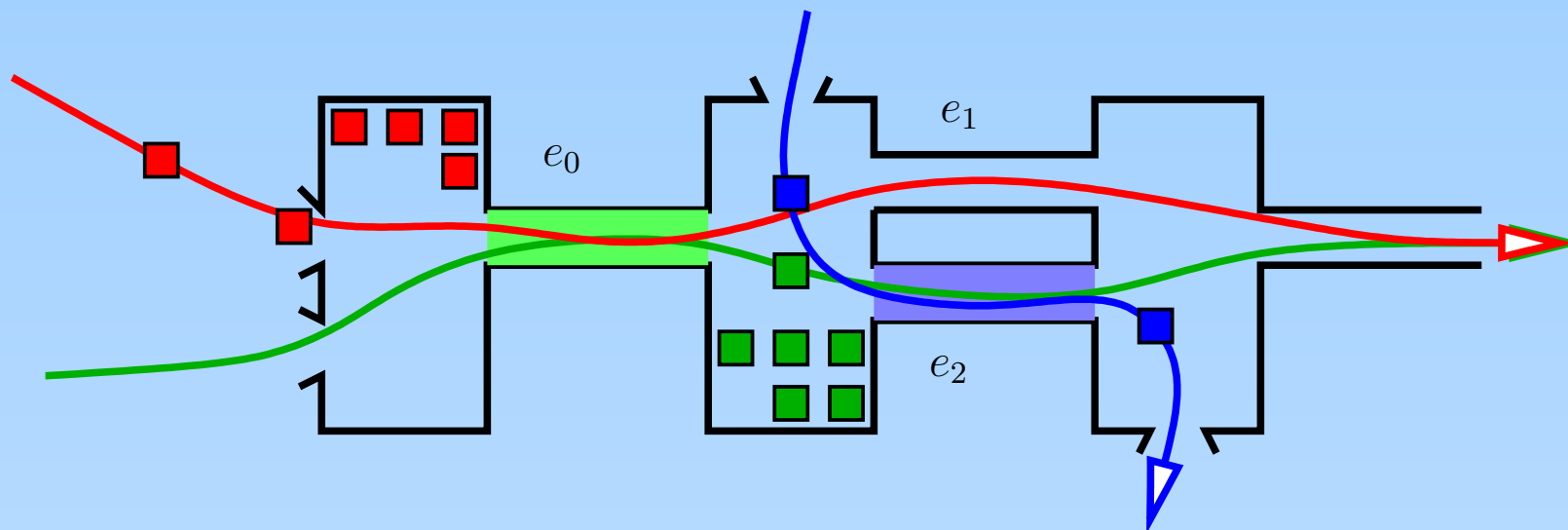


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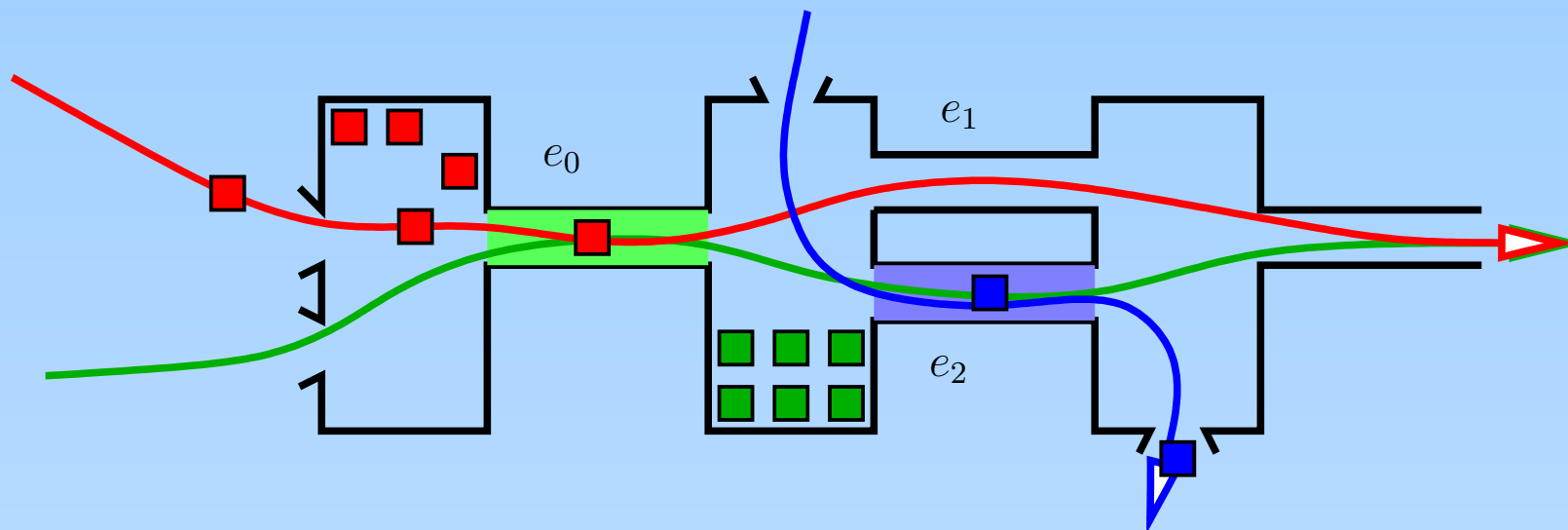
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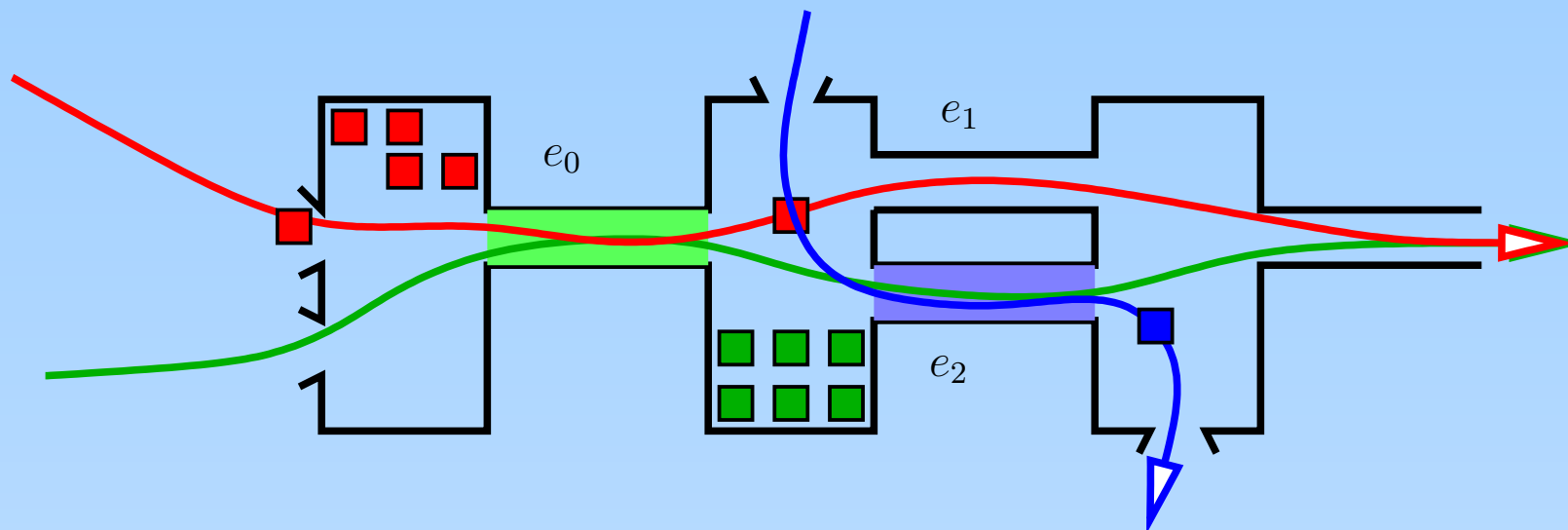
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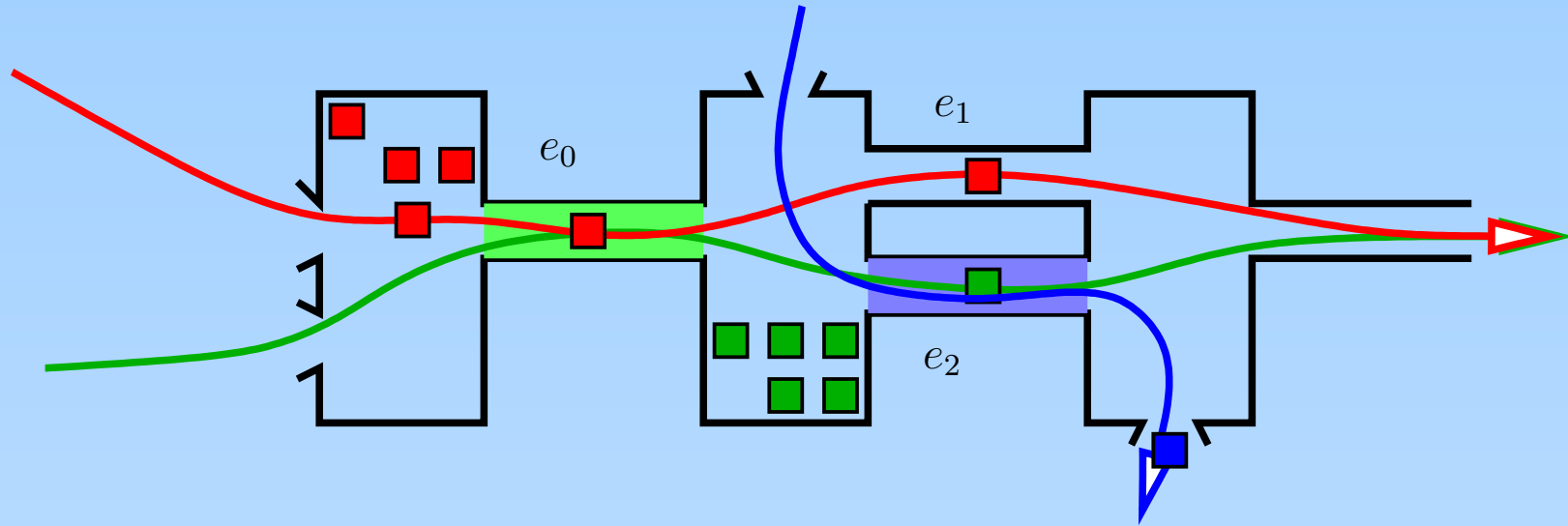
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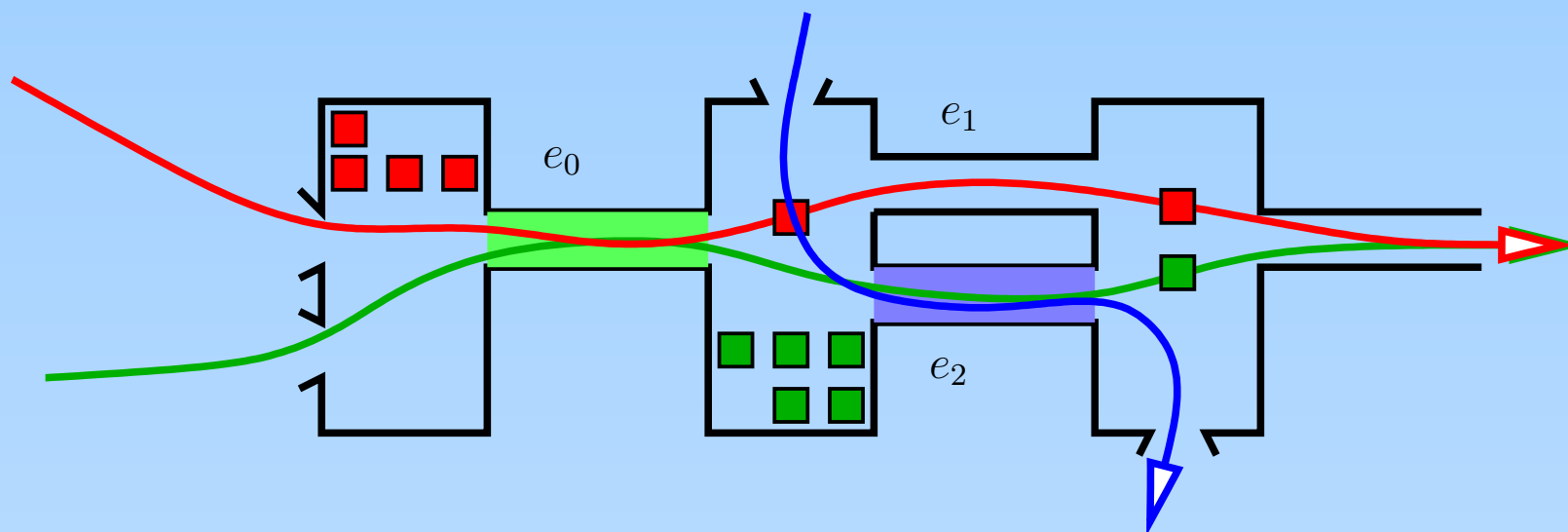


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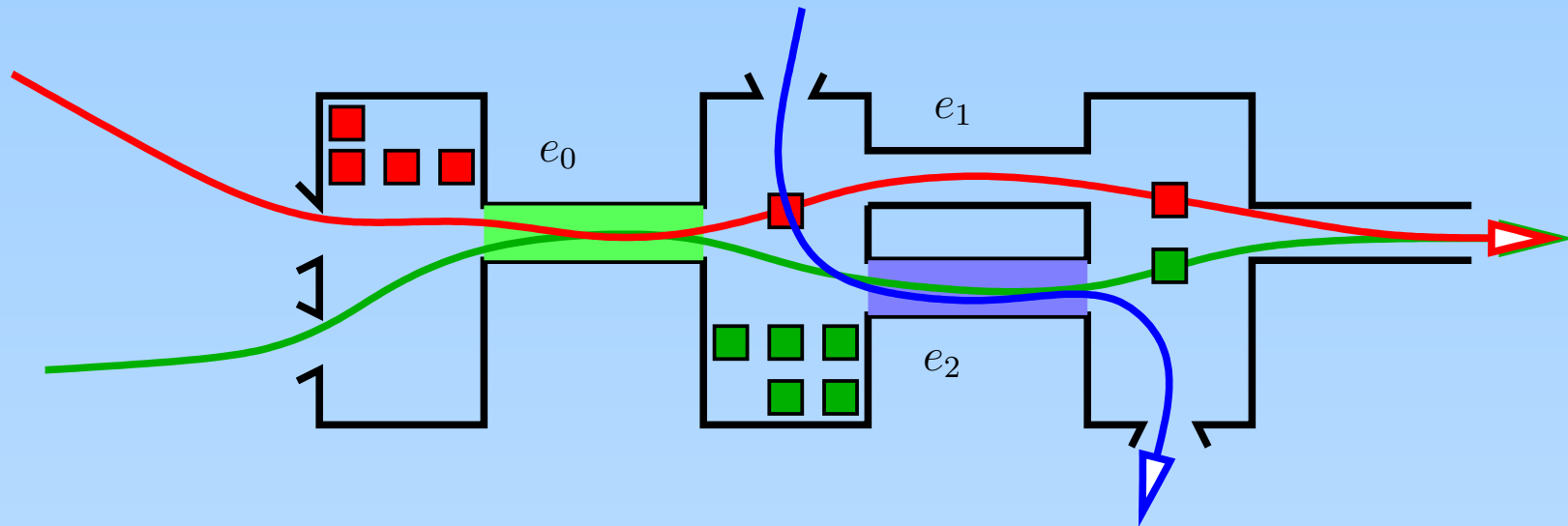
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 Remaining packets:  $2r \cdot s$ .

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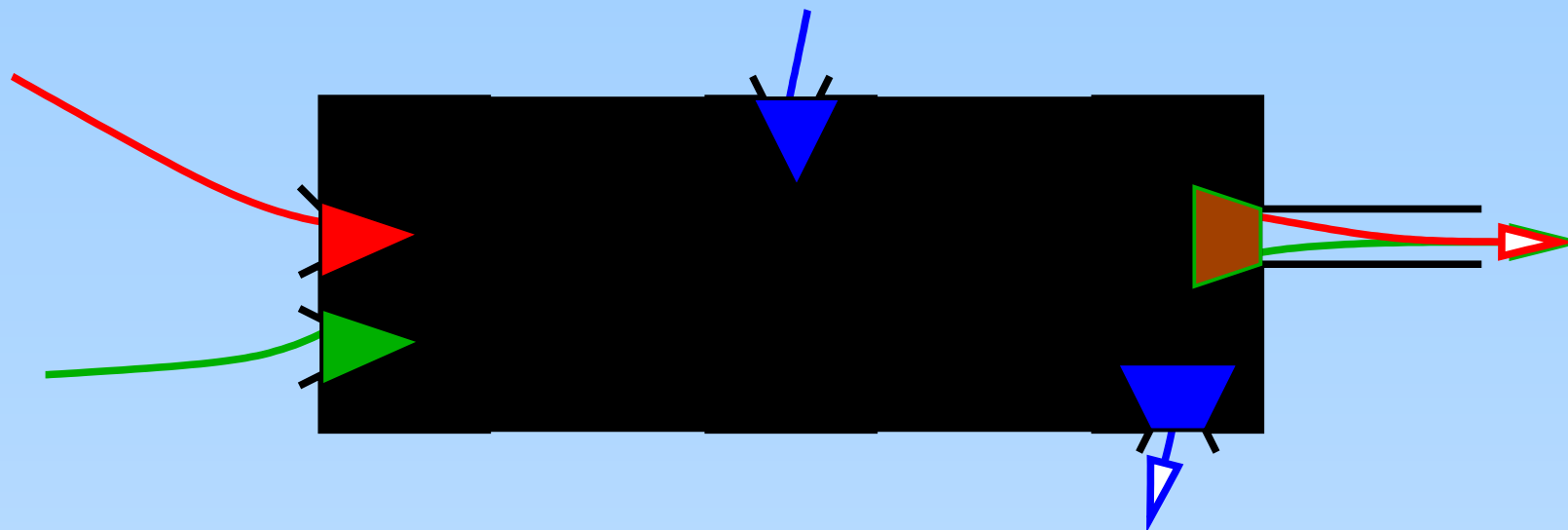
Only  $(1 - r) \cdot s$  green packets slip through.

Remaining packets:  $2r \cdot s$ .

For  $r > \frac{1}{2}$  the set of packets grows by a multiplicative factor.



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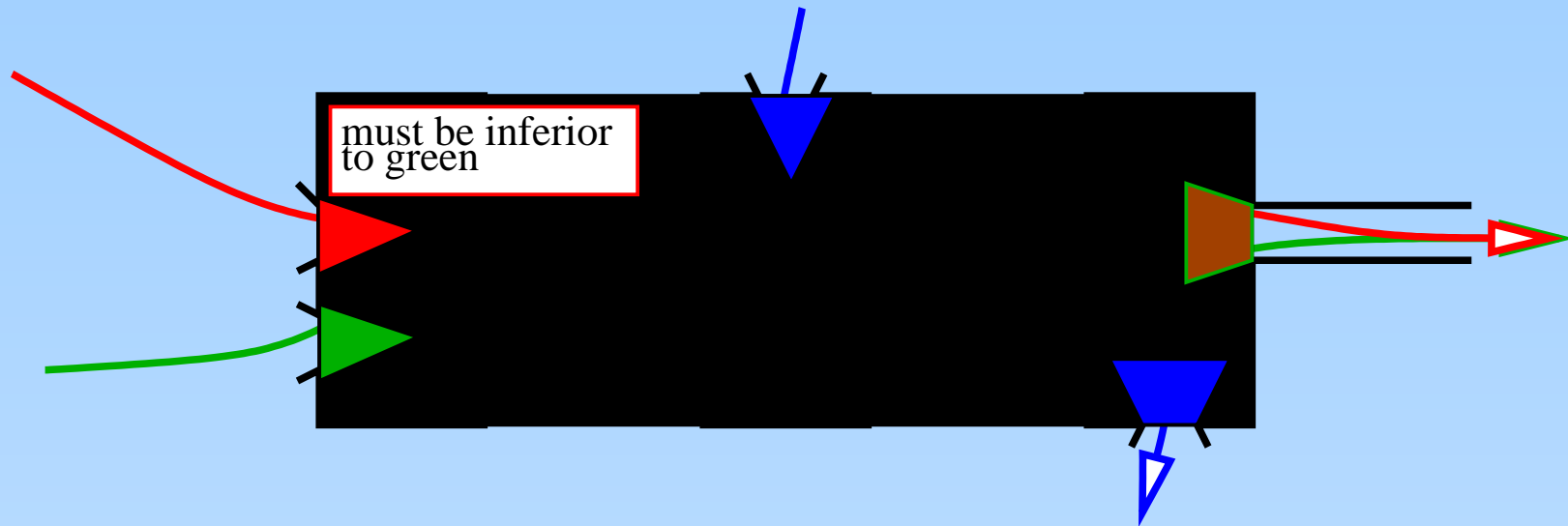
No red packet traverses the system.

Only  $(1 - r) \cdot s$  green packets slip through.

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# green	$s$
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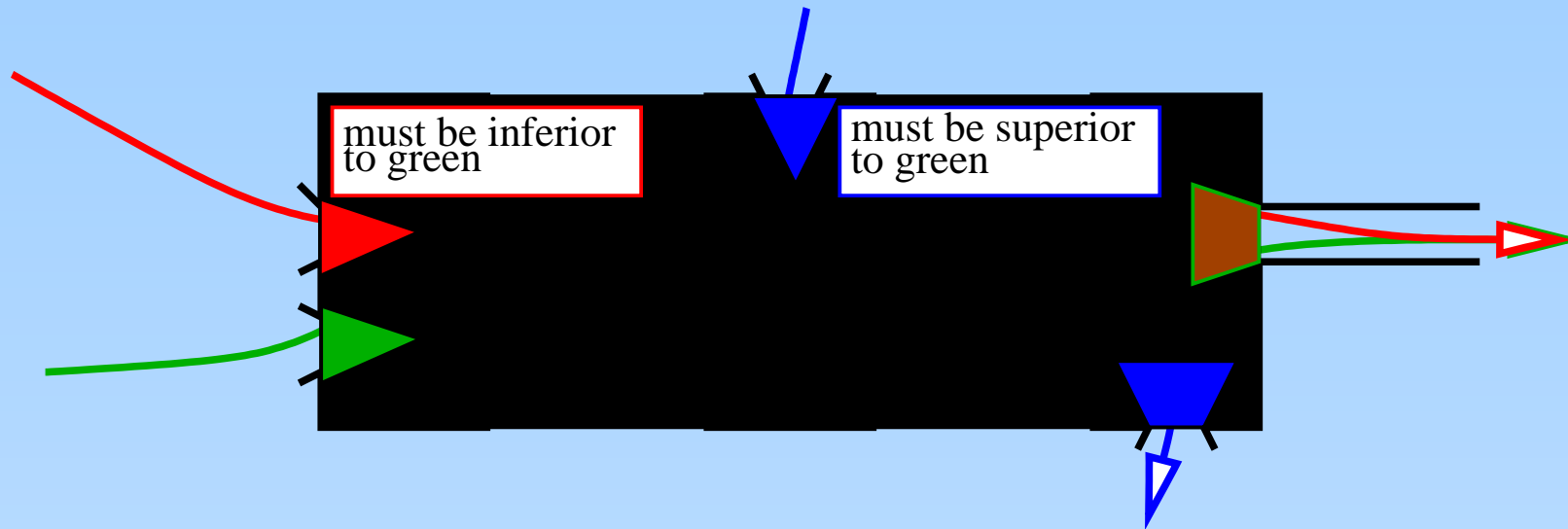
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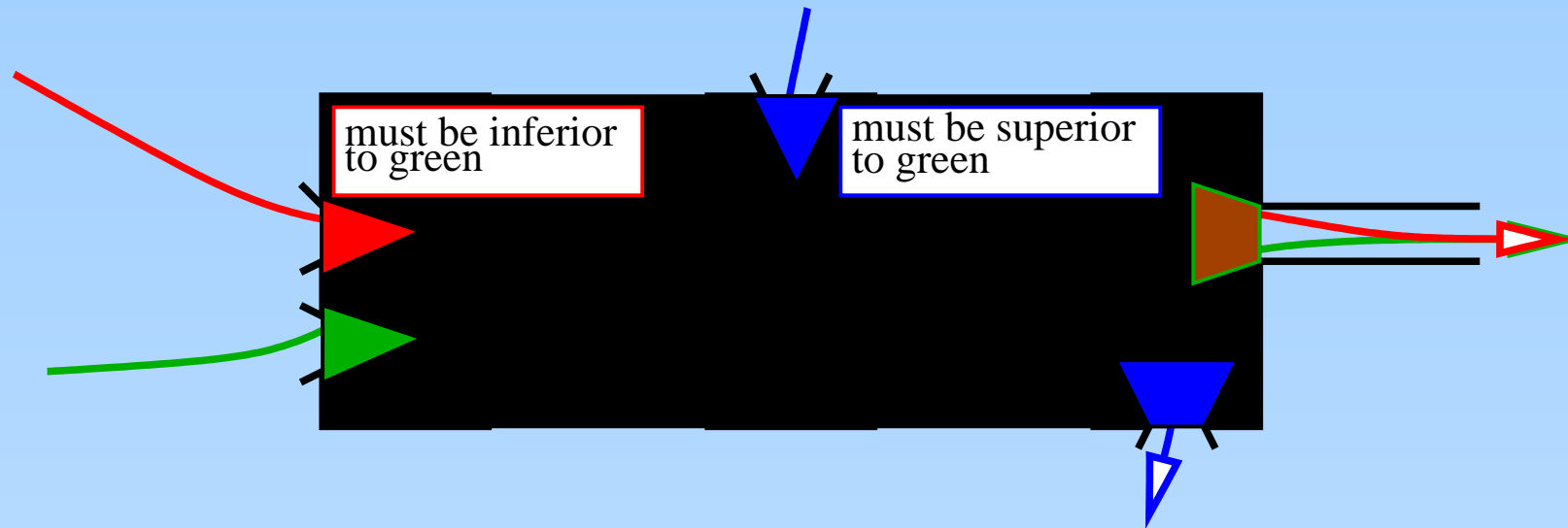
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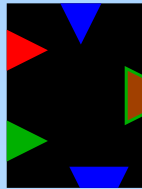
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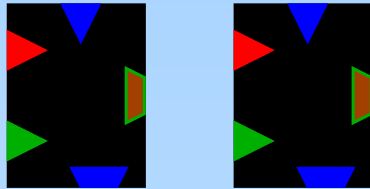
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# Concatenating Gadgets

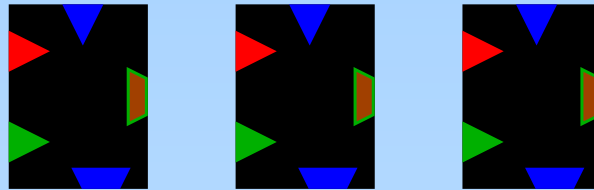
# Concatenating Gadgets



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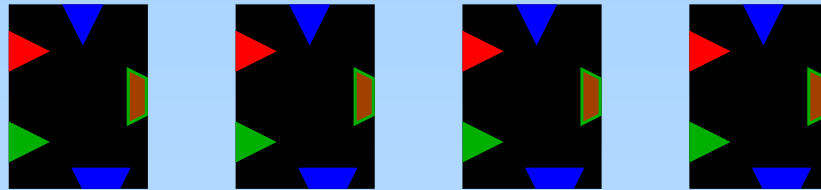


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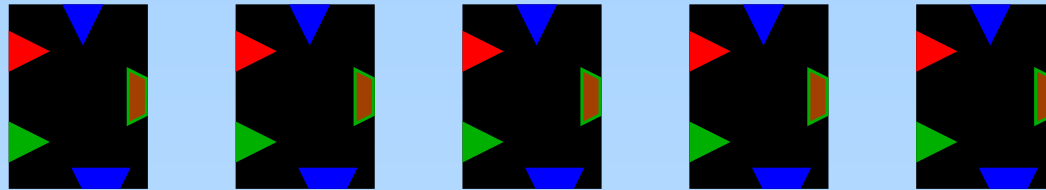




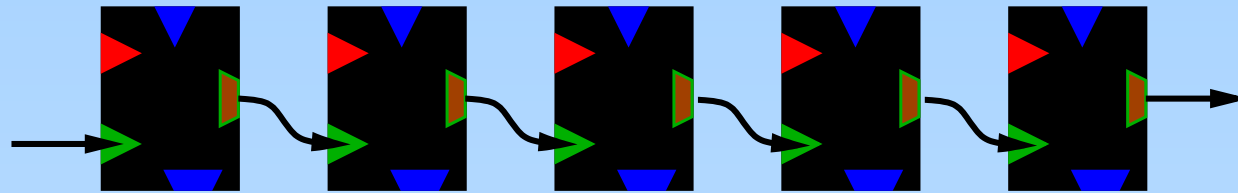
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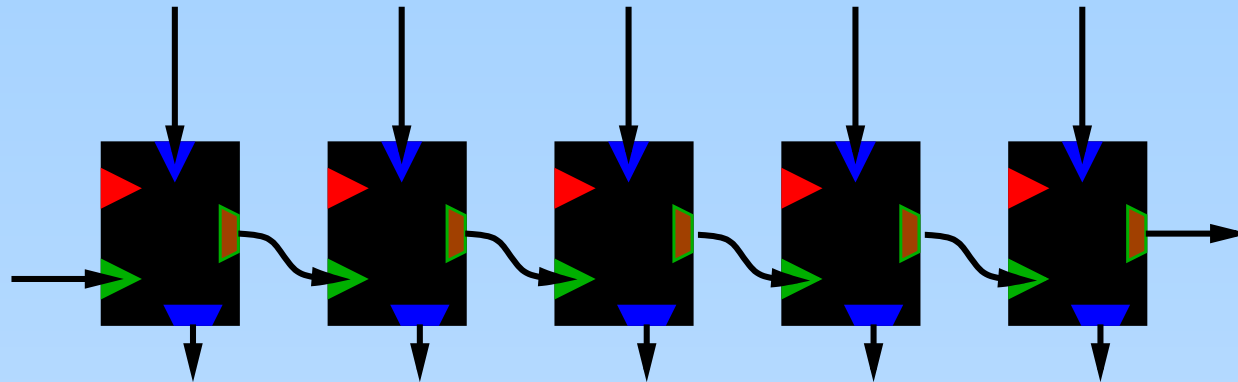
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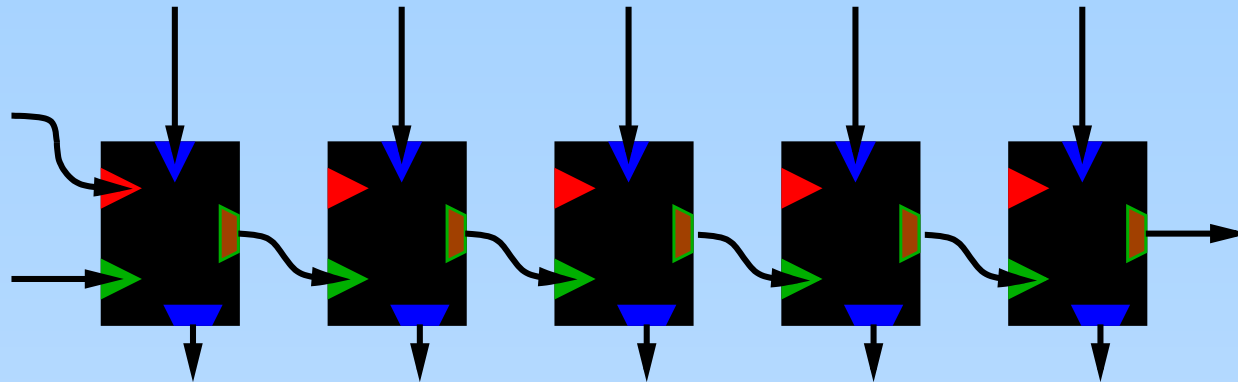
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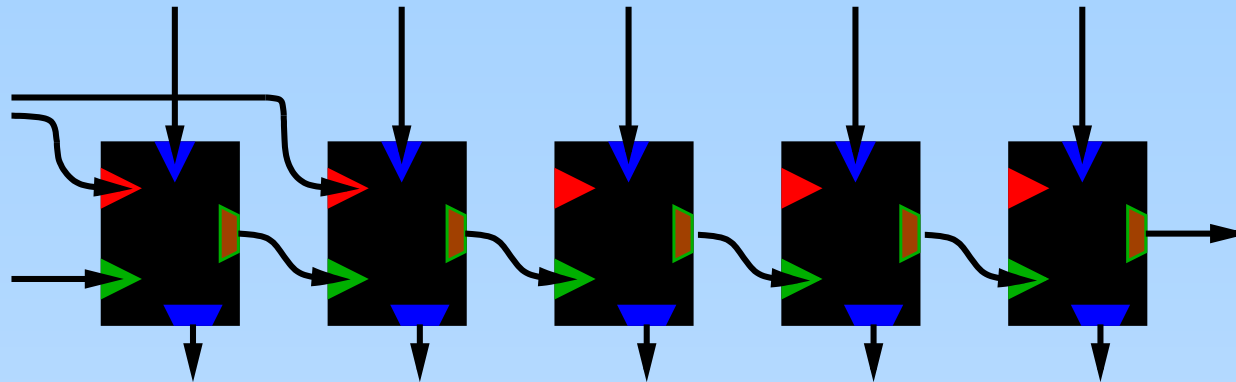
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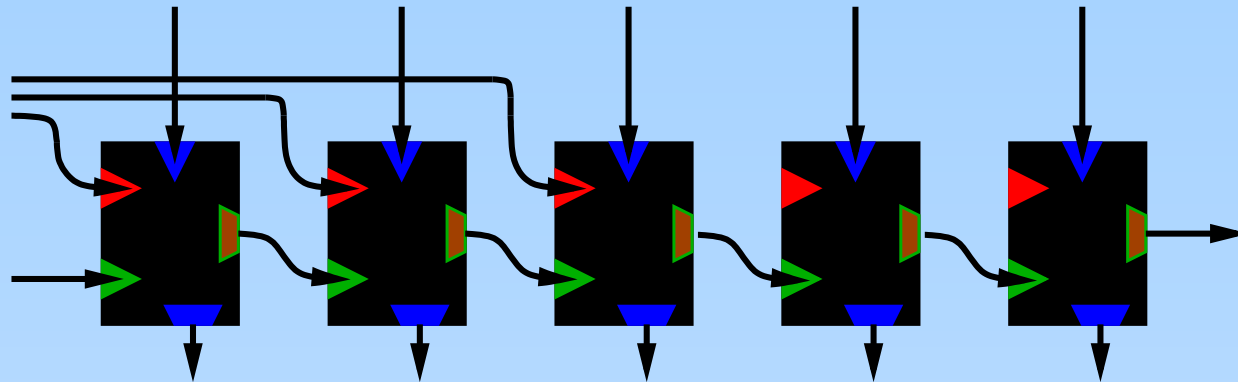
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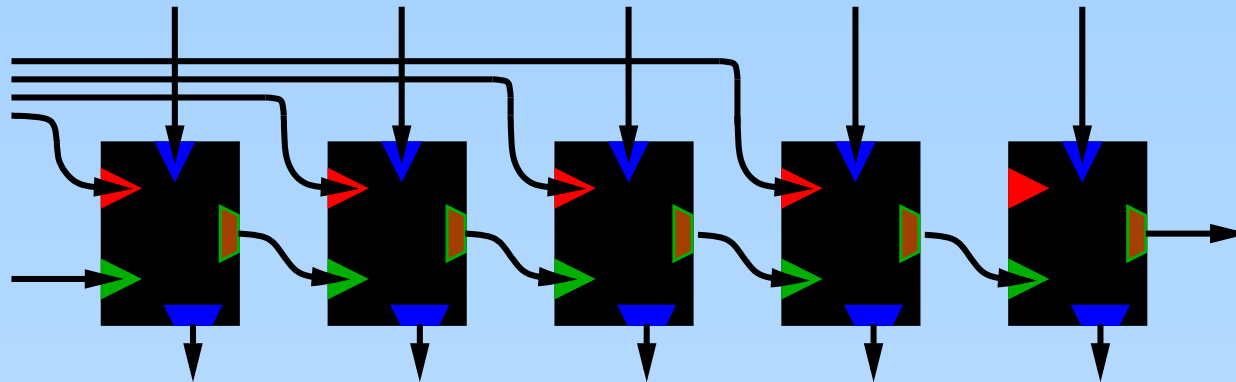
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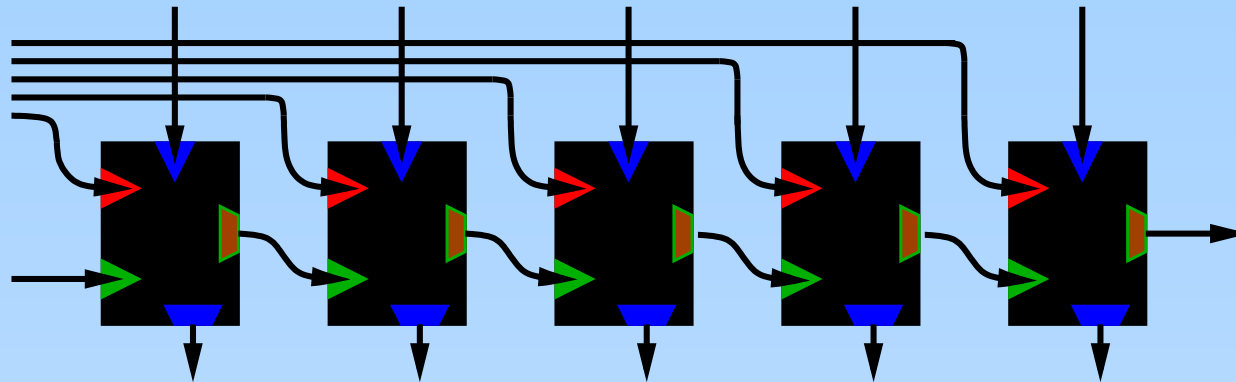


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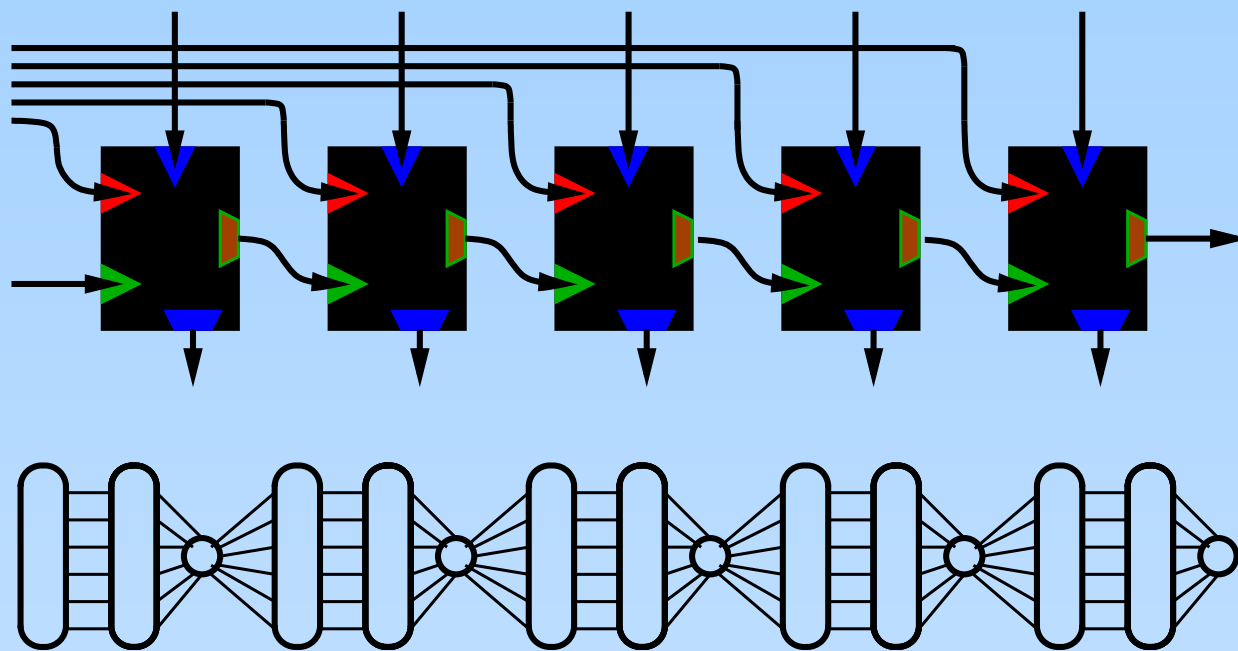




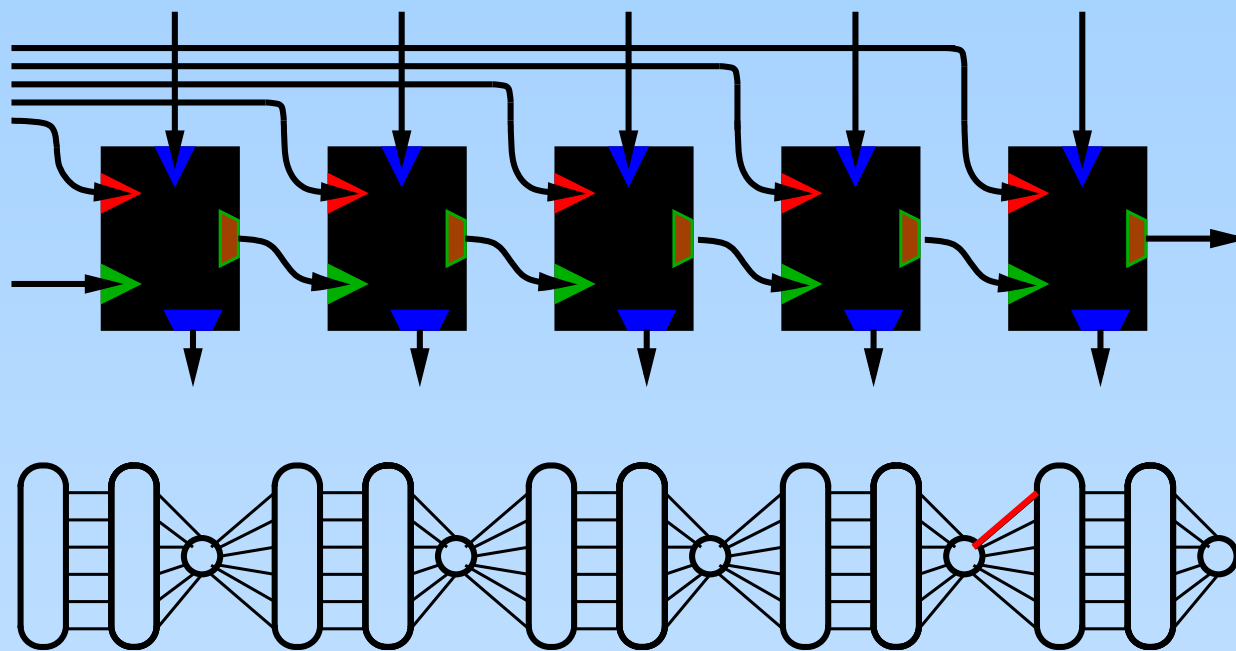
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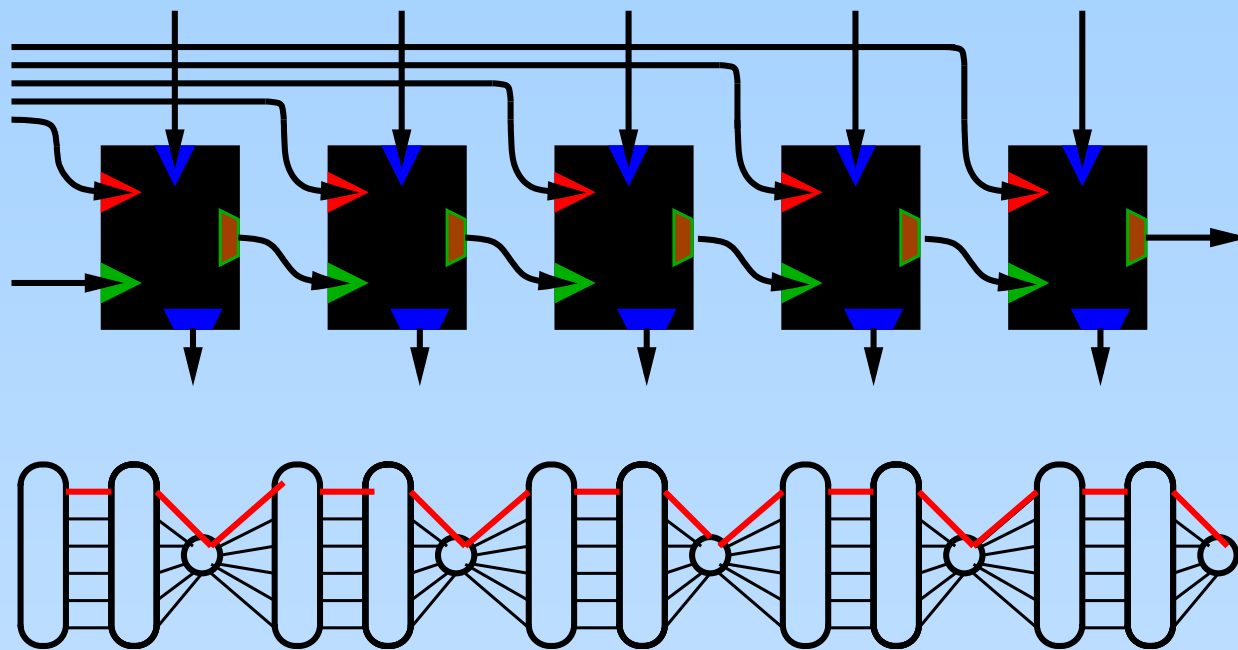
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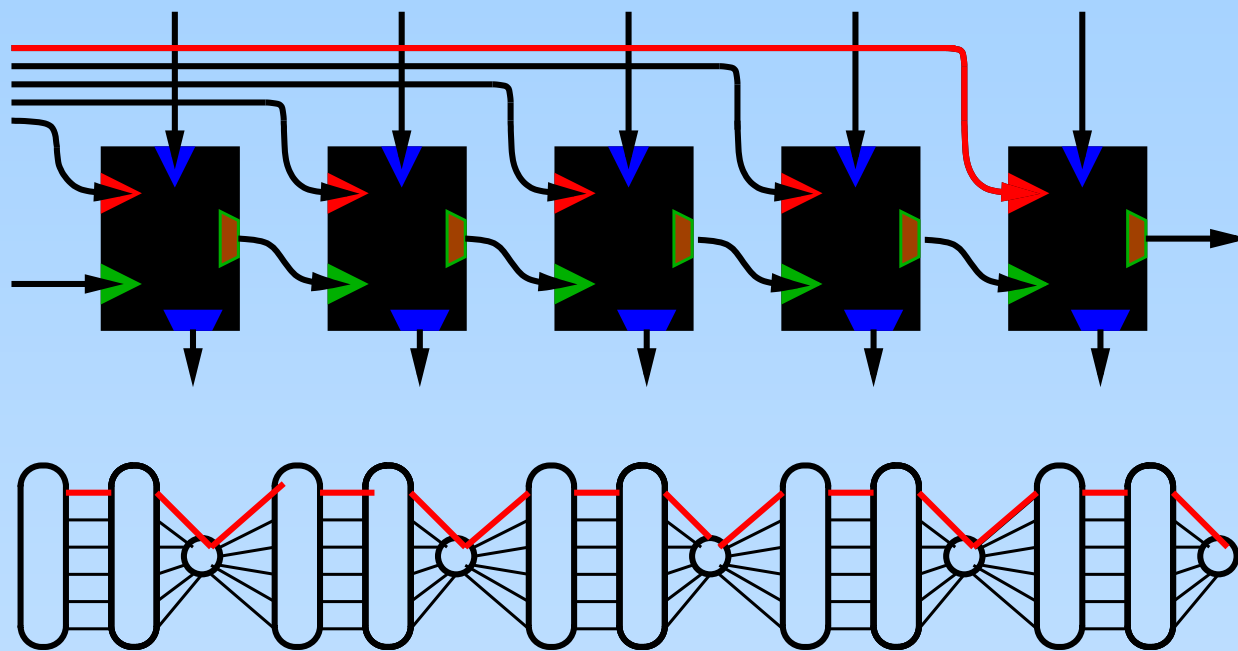
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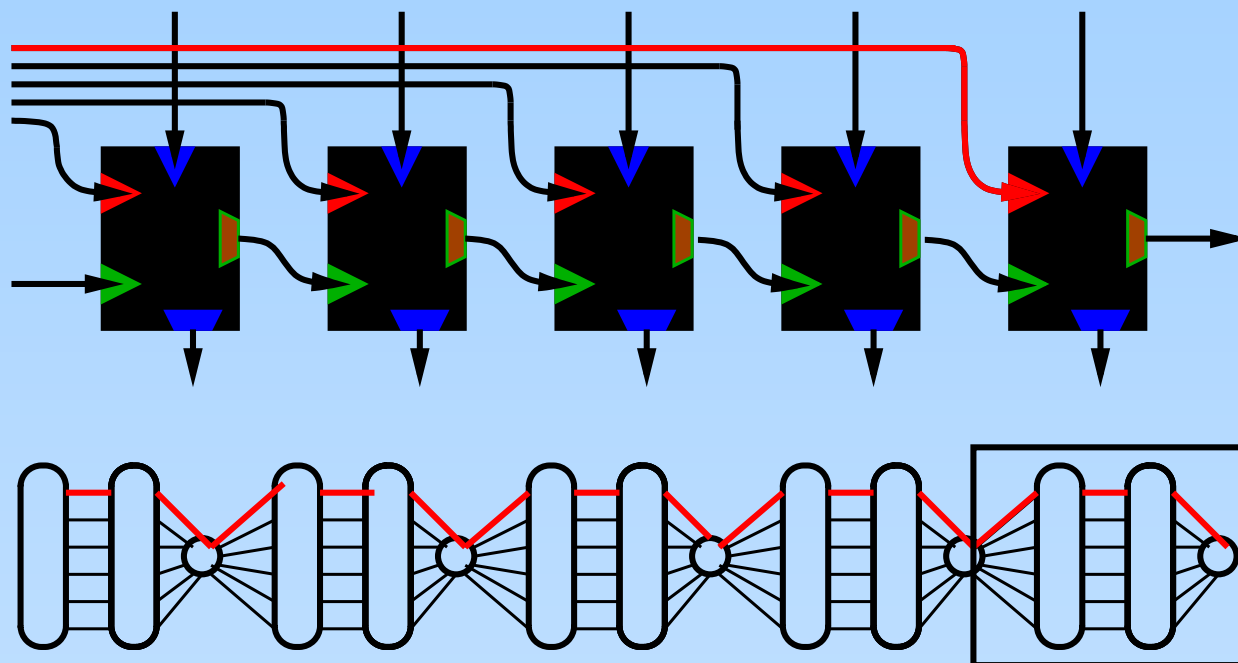
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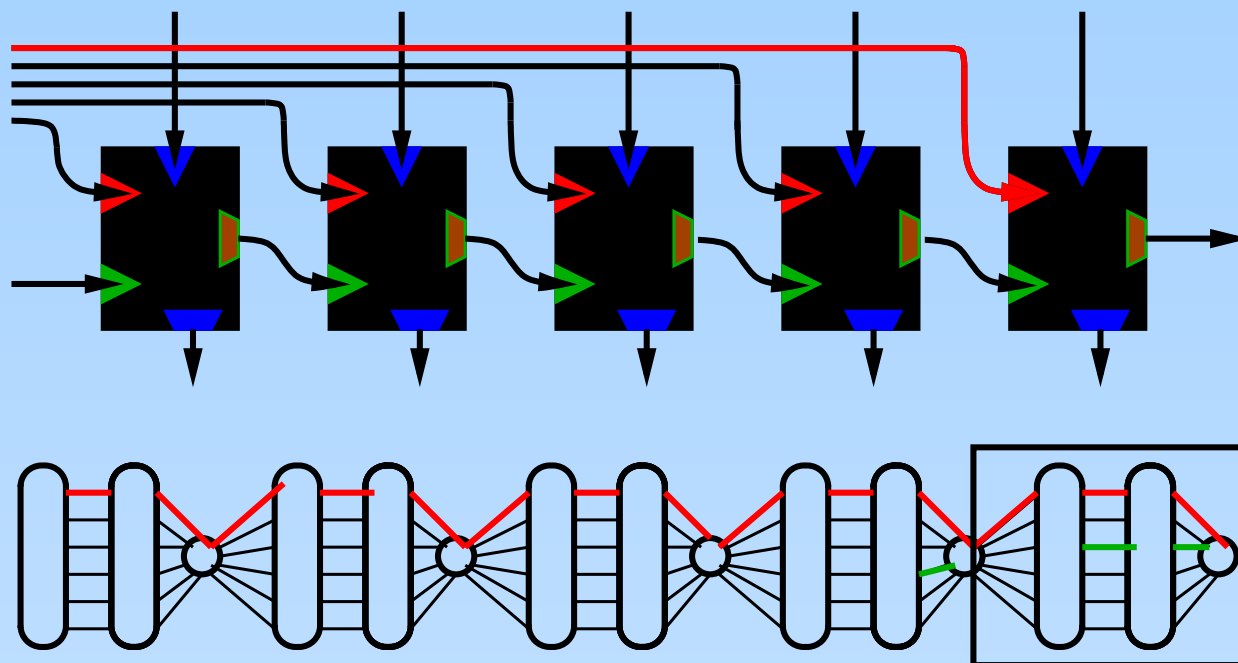
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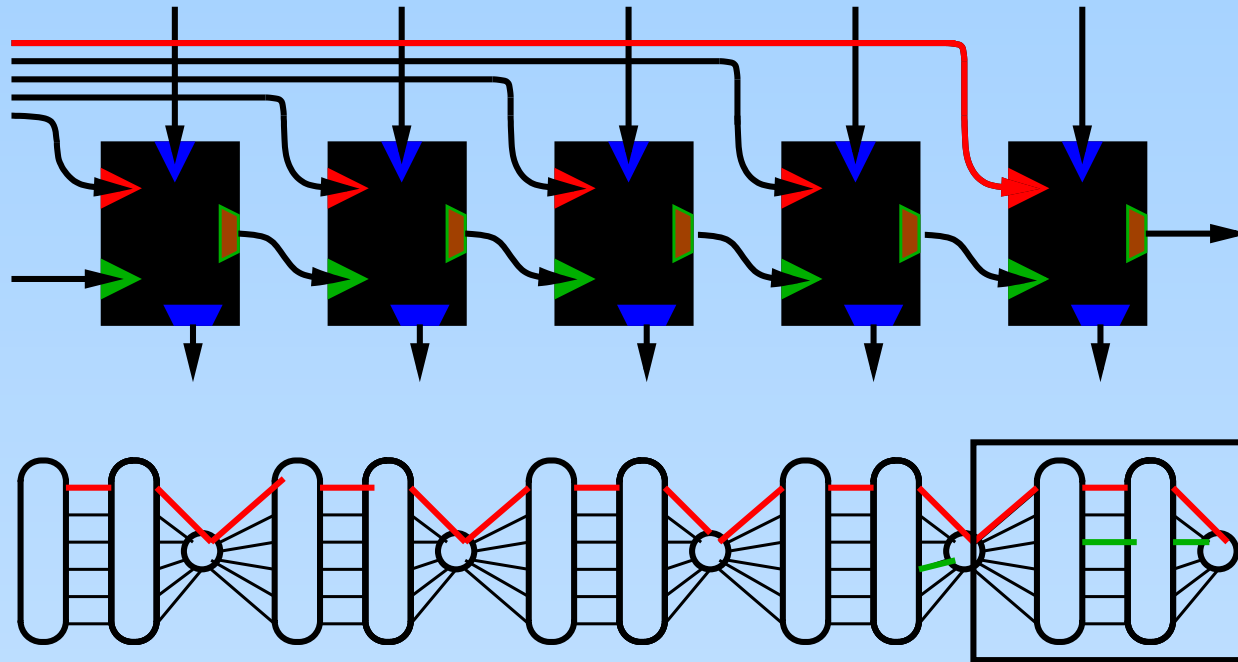
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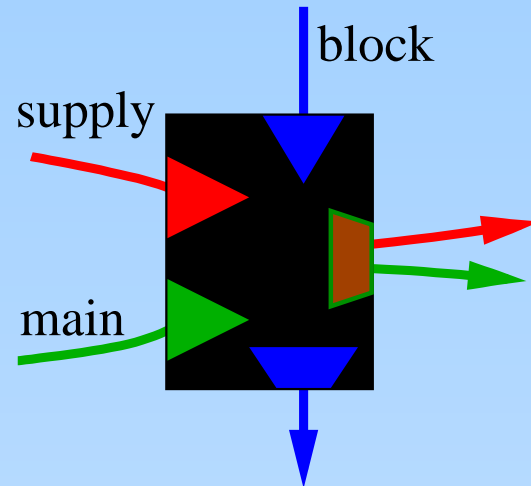
How can we get (blue) paths capable of blocking?



# Idea: Improve Gadget

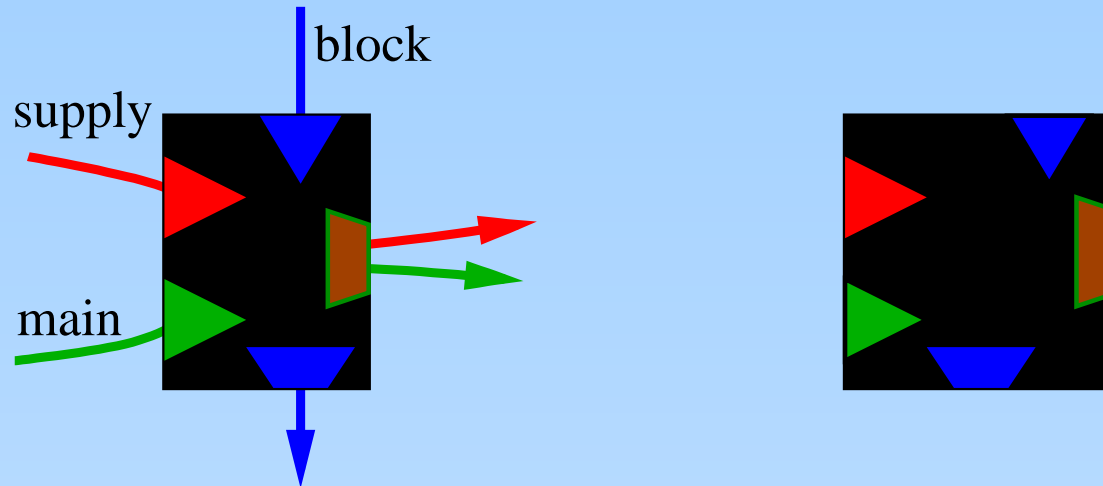


## Idea: Improve Gadget



If green packets cannot be blocked by blue packets...

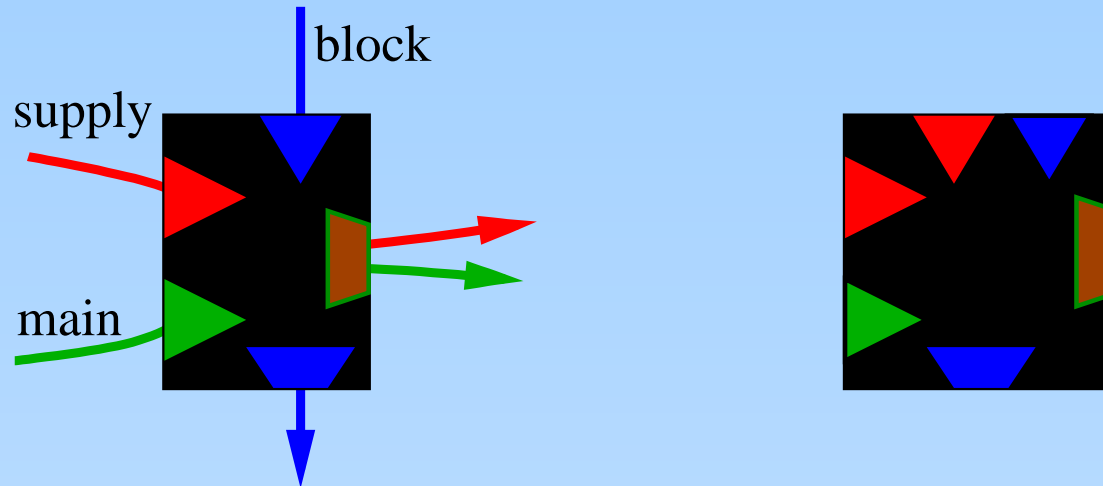
## Idea: Improve Gadget



If green packets cannot be blocked by blue packets...

... then blue packets can be blocked by green packets.

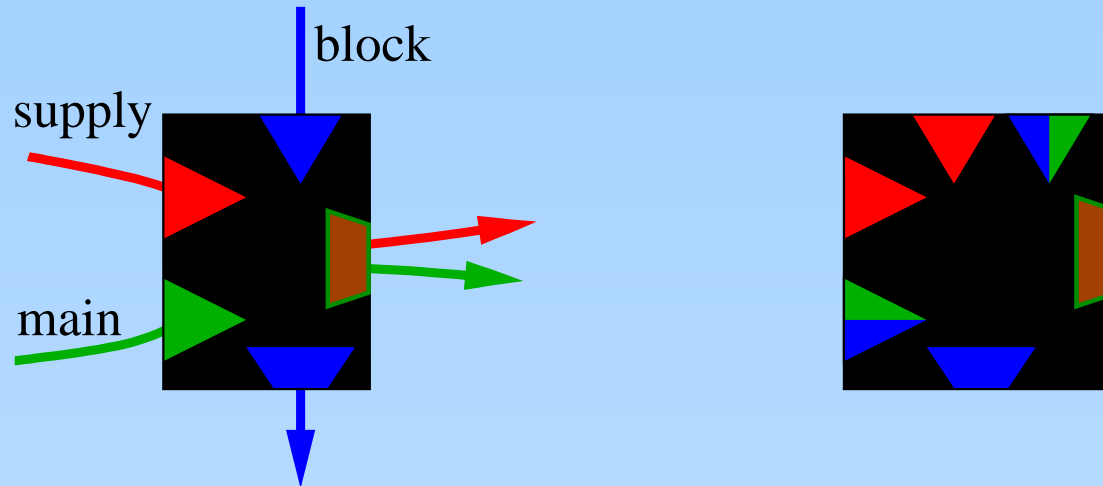
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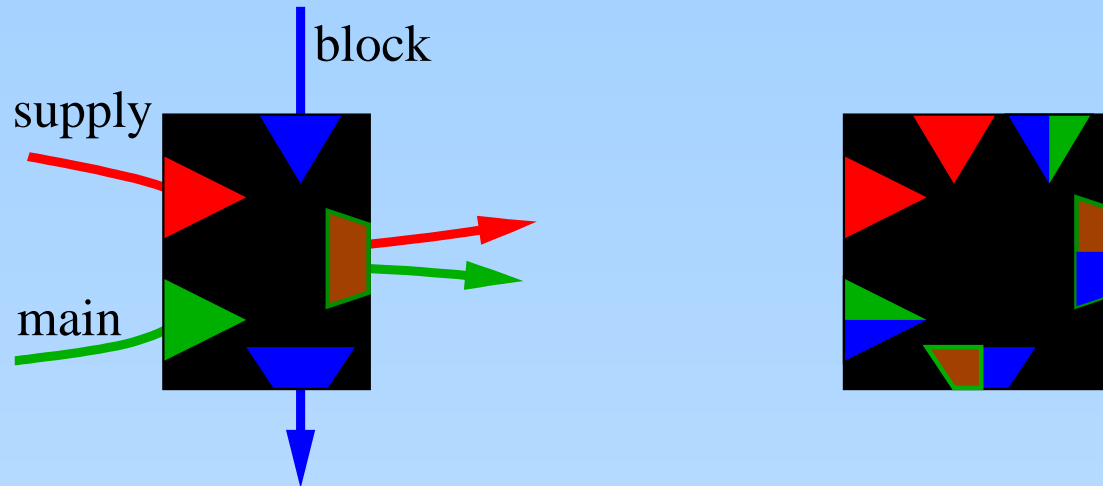
... then blue packets can be blocked by green packets.

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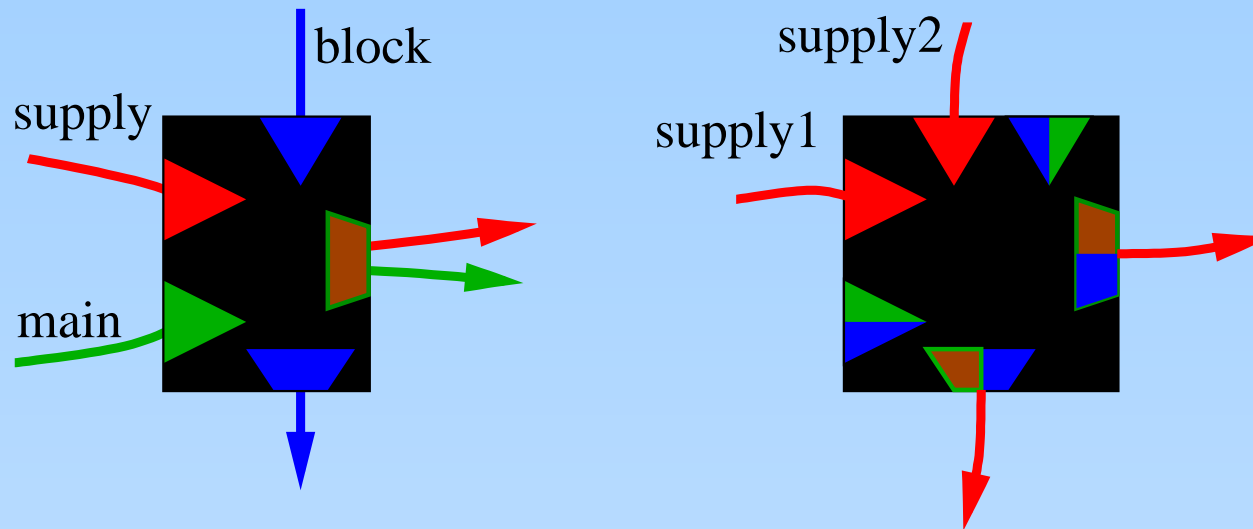
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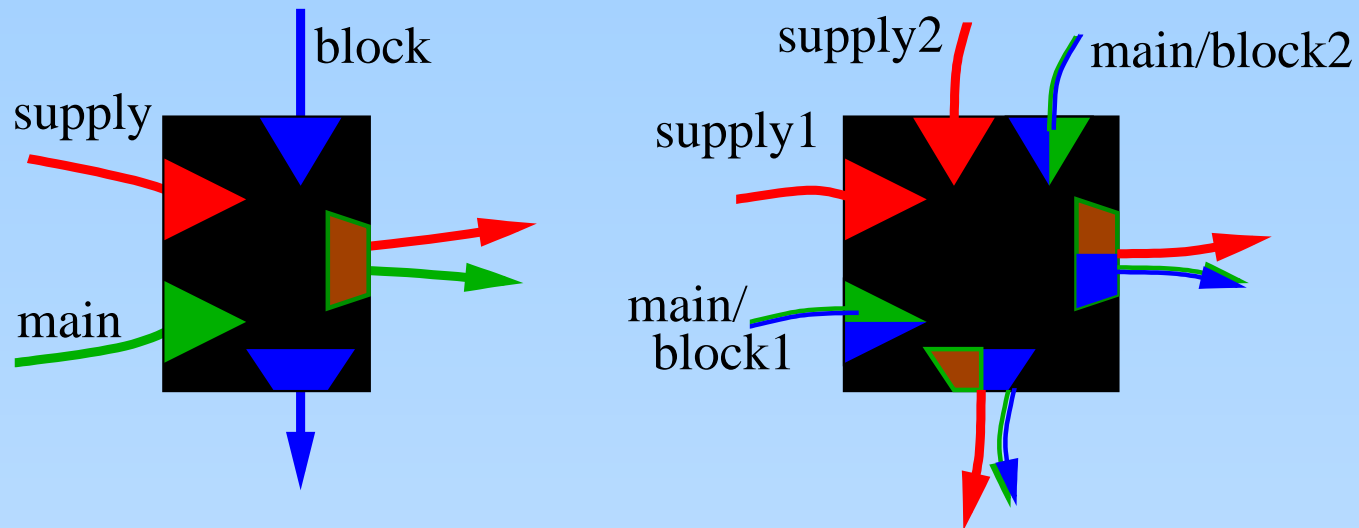
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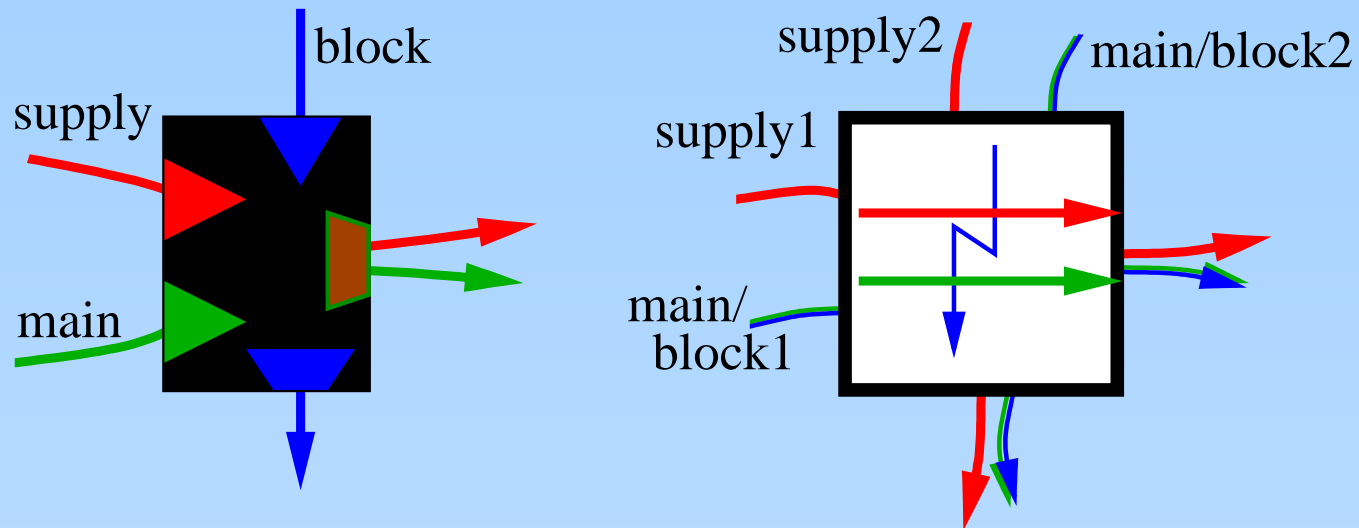


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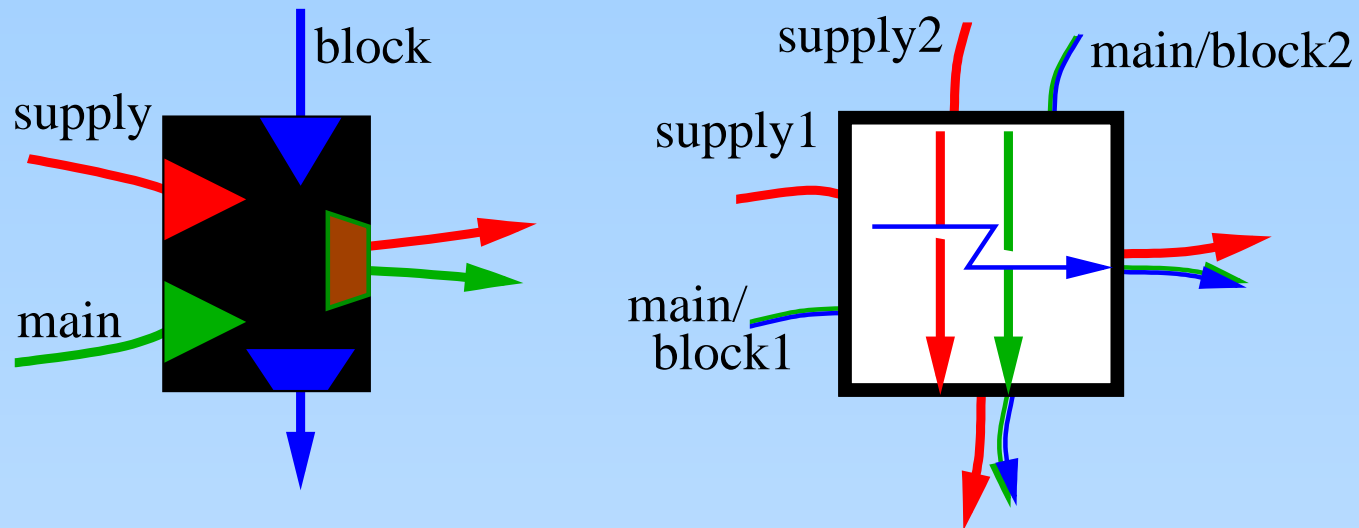


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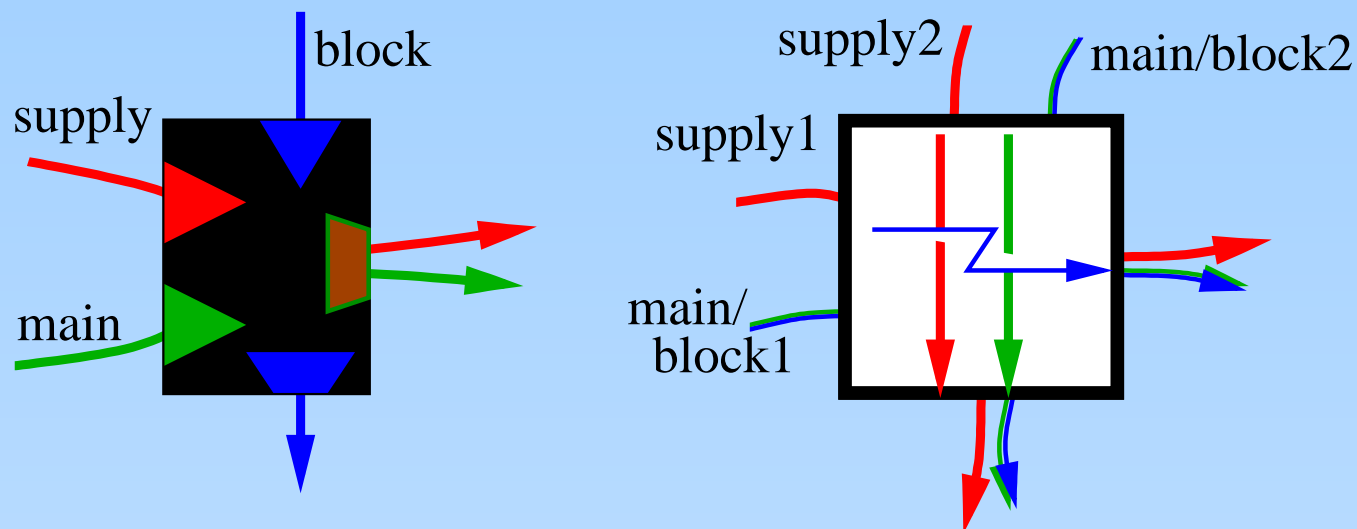
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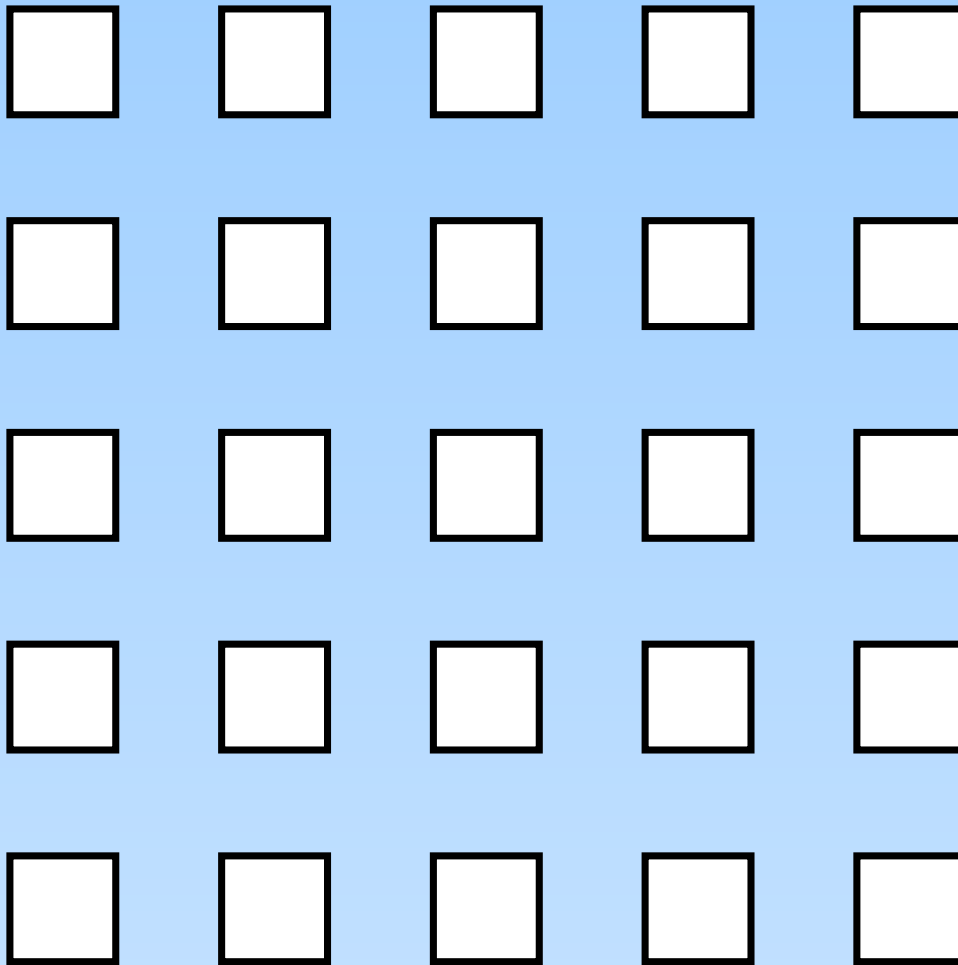
... then blue packets can be blocked by green packets.

Gadget works in at least one orientation.

# Arranging a Gadget Matrix



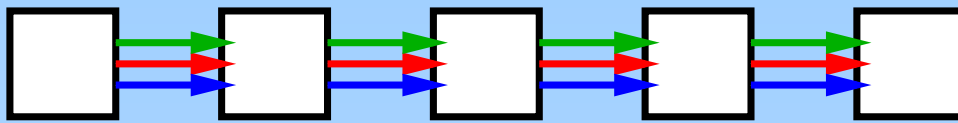
## Arranging a Gadget Matrix



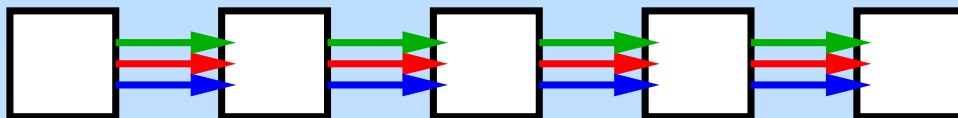
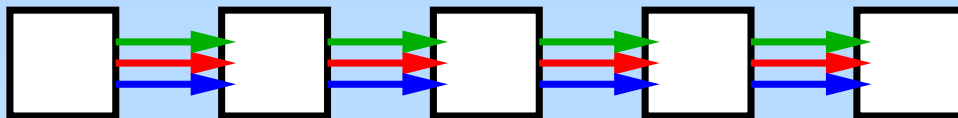
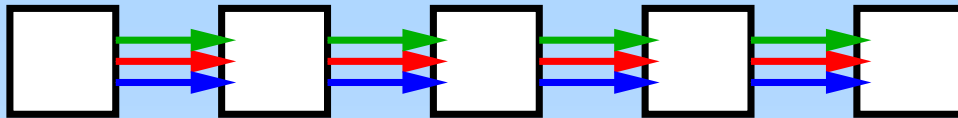
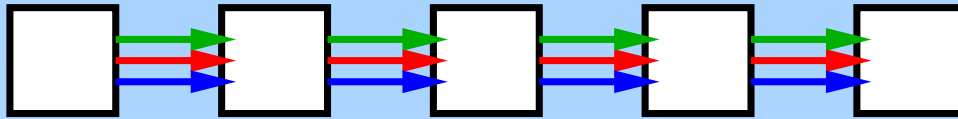
A  $k \times k$  matrix of gadgets.



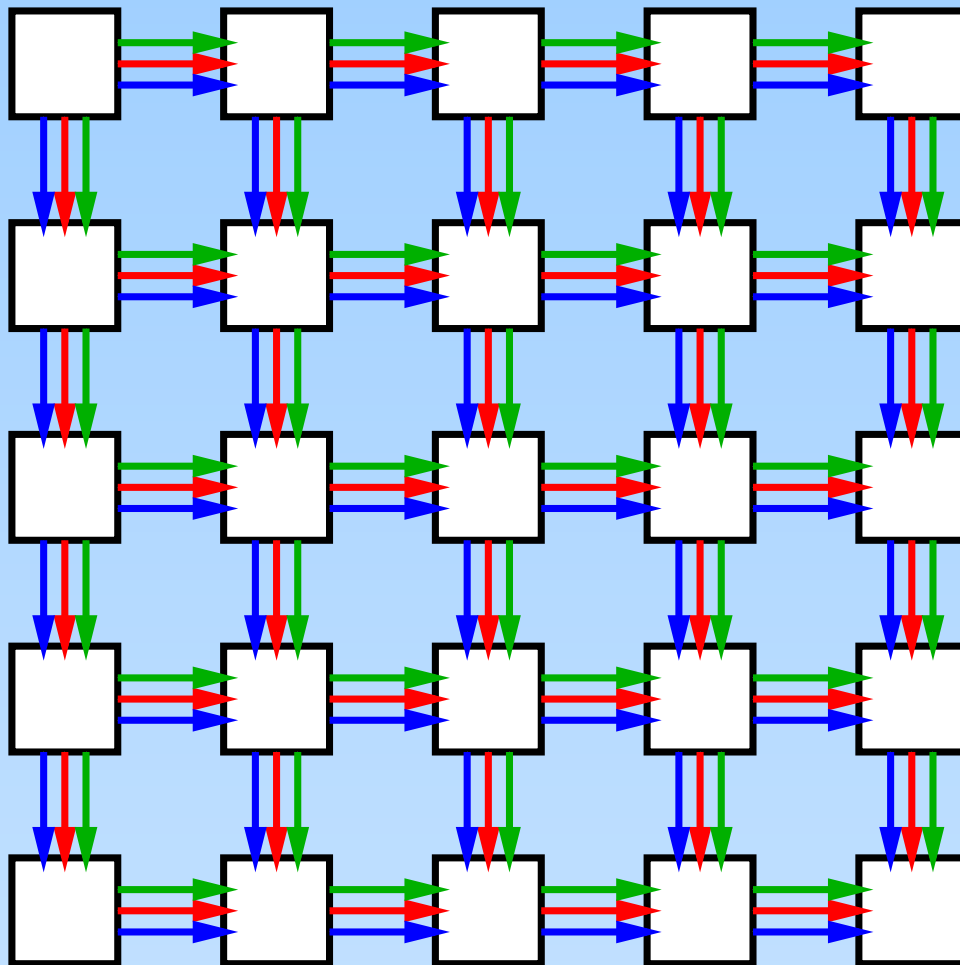
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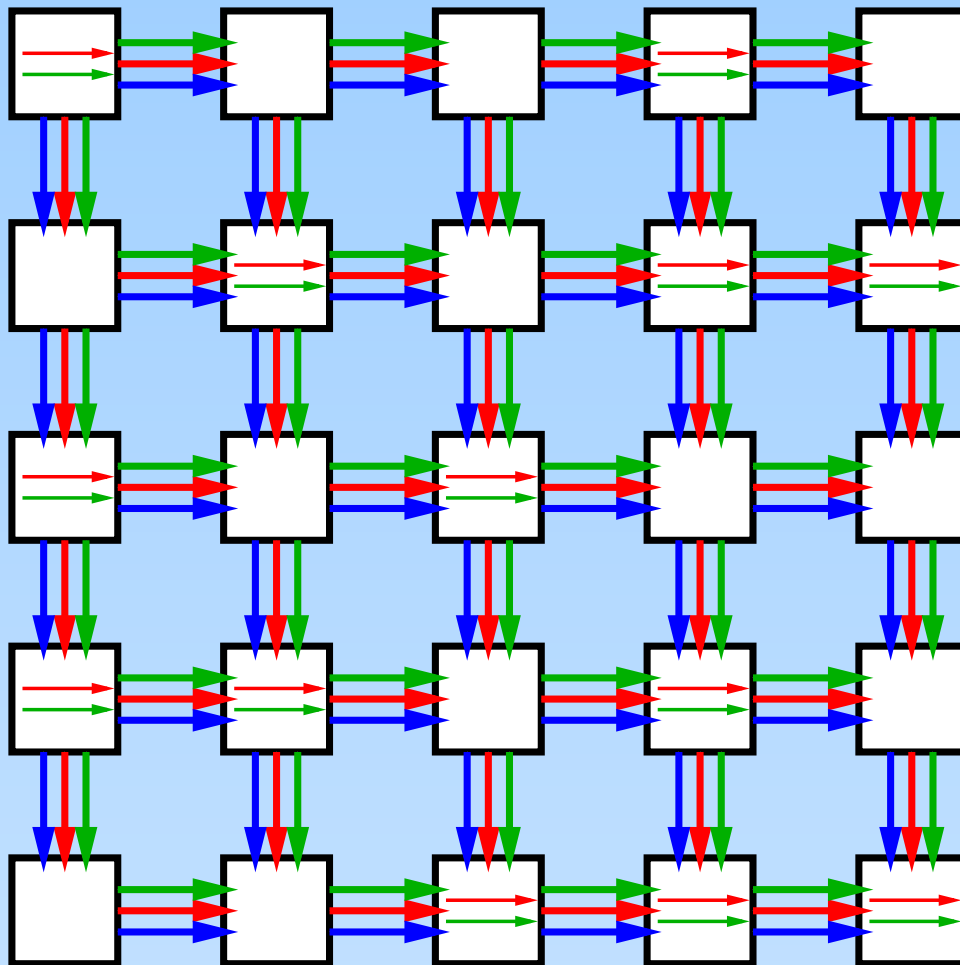
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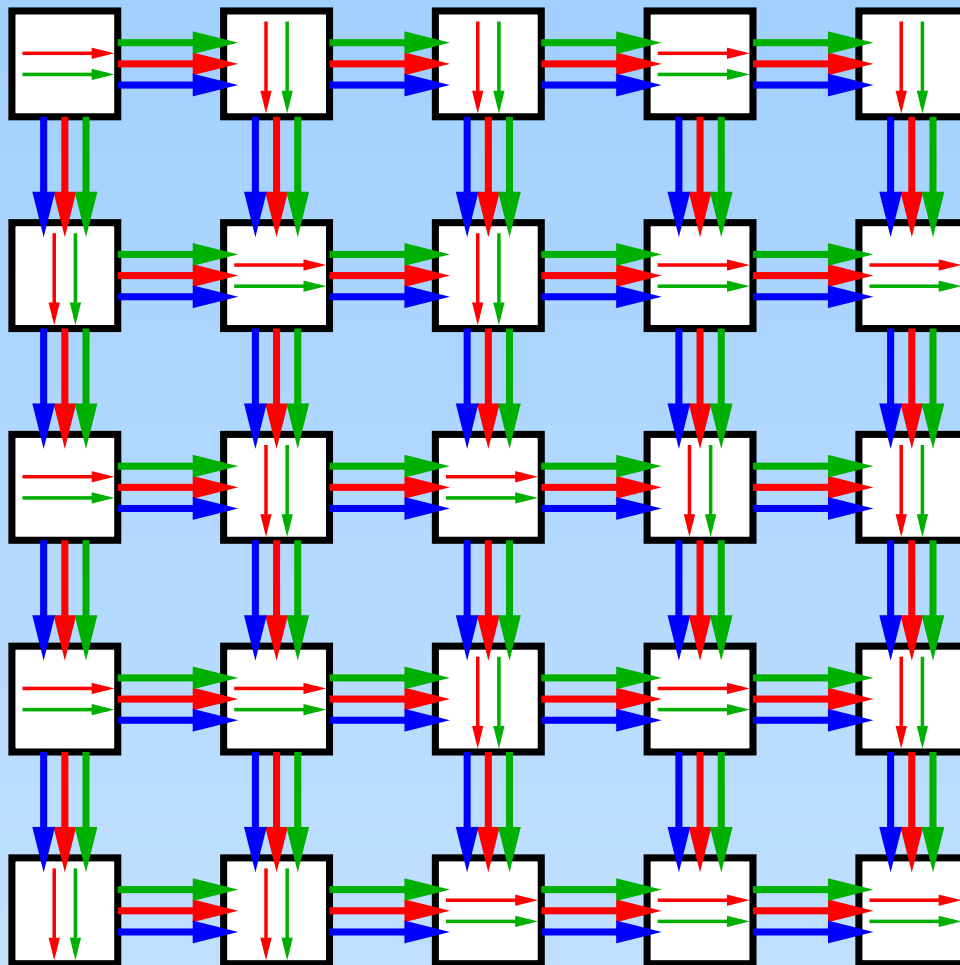
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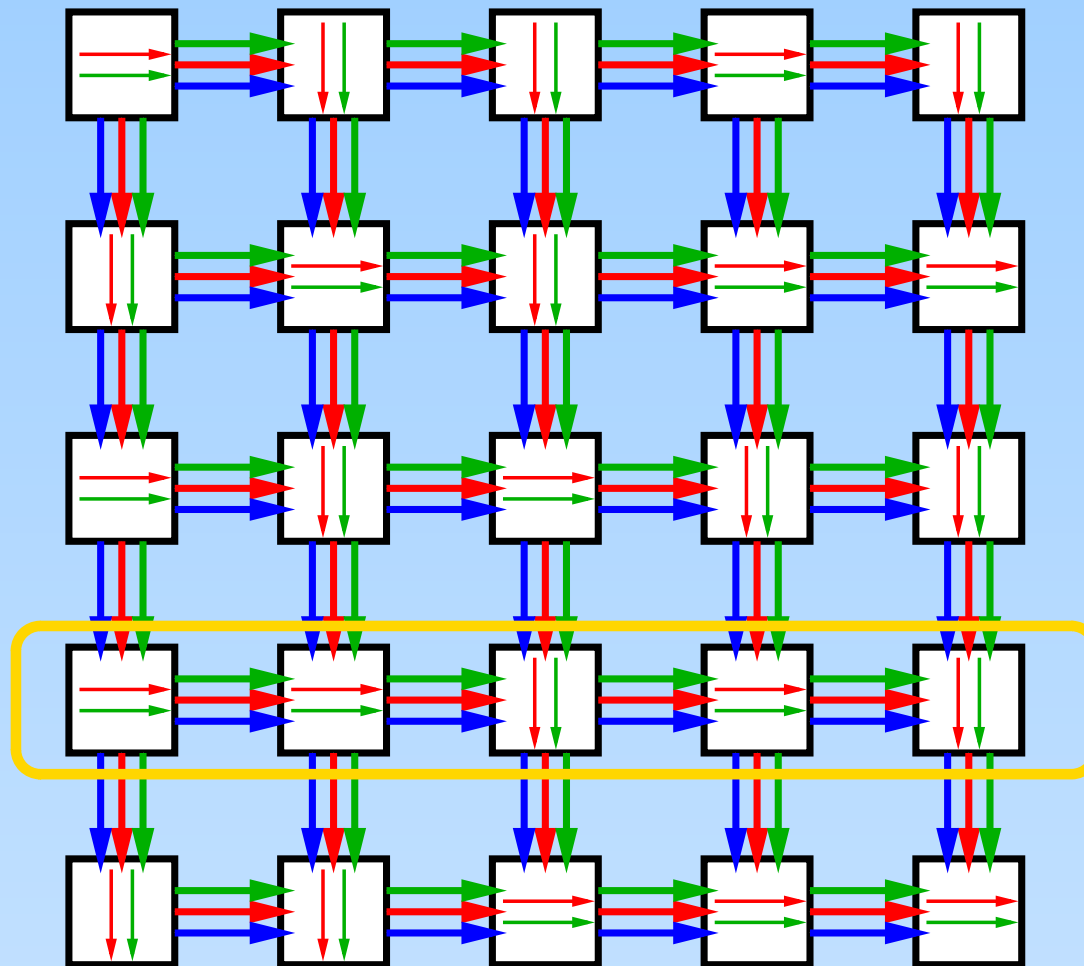
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Diameter:  $\Theta(k)$ .

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$2^{\Theta(k)}$  packets can be piled up.

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**THANK YOU**

