

Metastable quark–antiquark droplets within the Nambu–Jona-Lasinio model

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Chemically non-equilibrated quark–antiquark matter is studied within the Nambu–Jona-Lasinio model. The equations of state of non-strange ($q = u, d$) and strange ($q = s$) $q\bar{q}$ systems are calculated in the mean-field approximation. The existence of metastable bound states with zero pressure is predicted at finite densities and temperatures $T \lesssim 50$ MeV. It is shown that the minimum energy per particle occurs for symmetric systems, with equal densities of quarks and antiquarks. At $T = 0$ these metastable states have quark number densities of about 0.5 fm^{-3} for $q = u, d$ and of 1 fm^{-3} for $q = s$. A first order chiral phase transition is found at finite densities and temperatures. The critical temperature for this phase transition is approximately 75 MeV (90 MeV) for the non-strange (strange) baryon-free quark–antiquark matter. For realistic choices of parameters, the model does not predict a phase transition in chemically equilibrated systems. Possible decay channels of the metastable $q\bar{q}$ droplets and their signatures in relativistic heavy-ion collisions are discussed.

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I. INTRODUCTION

Studying the equation of state (EOS) of strongly interacting matter is necessary for understanding the evolution of the early universe, properties of neutron stars and dynamics of heavy-ion collisions. First of all, one should know the degrees of freedom which are most relevant at a given energy density. When studying the EOS it is also important to identify the regions of possible phase transitions and to find all stable and metastable states which correspond to extrema of a thermodynamic potential. Presumably, lattice QCD calculations can be used to study these questions from “first principles”. At present, however, this approach gives reliable results only for a baryon-free matter in thermodynamic equilibrium.

Therefore, one should use effective theories when dealing with a wider class of multiparticle systems. The Nambu—Jona-Lasinio (NJL) model [1,2] is one of such effective theories which proved to be rather successful in describing the ground states of light hadrons in vacuum. The chiral symmetry of the strong interactions, most important for the low energy hadron physics, is explicitly implemented in this model. Although the model is non-renormalizable and does not contain gluons¹, it seems to be a reasonable approximation to QCD at scales intermediate between asymptotic freedom and confinement.

In this paper the NJL model is applied to investigate the properties of chemically non-equilibrated quark-antiquark ($q\bar{q}$) matter. The study of possible bound states and phase transitions is one of the main goals of the present work. The search for new states of strongly interacting systems, essentially different from normal nuclear matter, has already quite a long history. Using the linear σ -model, Lee and Wick predicted [3] a metastable superdense state of nucleonic matter at $T = 0$. This problem was further studied in Ref. [4] by including the repulsive vector interaction. The liquid-gas as well as the chiral phase transitions of hadronic

¹ Lattice calculations show that at temperatures $T \lesssim 150$ MeV the gluons acquire a large effective mass due to the color screening effects. In the present work, dealing mostly with such moderate temperatures, we assume that gluonic degrees of freedom are suppressed.

matter have been investigated within the framework of the relativistic mean-field model in Refs. [5–7]. Since the chiral symmetry is not respected by this model, its predictions become questionable at high baryon densities and temperatures. Attempts to generalize the hadronic models by implementing the main symmetries of QCD have been made in Refs. [8–10]. A chiral phase transition was found in nuclear matter only at very high baryon densities and temperatures when hadronic degrees of freedom are inappropriate.

The existence of some exotic multiquark and multihadron states has been discussed by various authors. For example, “stranglets” (bound states of strange and light quarks) have been proposed in Ref. [11]. The possibility of their formation in relativistic heavy-ion collisions has been considered in Ref. [12]. Arguments that multipion systems may be bound have been given in Ref. [13]. The phase transition of quark matter into a color superconducting state has been predicted recently [14,15] within the QCD-motivated models. The random matrix model has been used to study the phase diagram of quark matter in Ref. [16]. Possible signatures of QCD phase transitions have been discussed in Ref. [17].

The possibility of multiquark bound states and chiral phase transitions was studied by many authors (see e.g. [18–21]) within the NJL model. Attempts to apply the same model to nucleonic matter were made in Refs. [22–24]. Most of these studies have been done within the mean-field approximation. An attempt to go beyond this approximation, by including the mesonic degrees of freedom, has been made in Ref. [25]. The results of these works, obtained with different sets of model parameters, often contradict to each other. A common shortcoming appearing in many papers is the omission [20,21] or underestimation [18] of the repulsive vector interaction². It was shown in Ref. [19] that this interaction becomes rather important at large baryon densities. According to Ref. [19], at realistic values of the vector coupling constant the NJL model does not predict a first order phase transitions in quark matter.

² The vector interaction was not considered in the original version of the model [1].

All these results have been obtained by assuming that the conditions of statistical equilibrium are perfectly fulfilled in the matter. This implies two kinds of equilibration. The first, kinetic or thermal equilibrium, means that the occupation numbers of quarks and antiquarks coincide with the equilibrium distribution functions characterized by some temperature and chemical potentials. The second, chemical equilibrium, requires that the certain relations between chemical potentials of different particles must be fulfilled. These relations determine the equilibrium abundances of various species which can be reached at large times due to inelastic multiparticle interactions. First attempts to study non-equilibrated $q\bar{q}$ systems were made in Refs. [26–28] by using transport equations derived from the NJL model.

Off-equilibrium effects should inevitably accompany the formation and evolution of quark matter in relativistic heavy-ion collisions. There are many theoretical as well as experimental arguments in favour of large deviations from the chemical equilibrium, even in central collision of heaviest nuclei at c.m. bombarding energies $\sqrt{s} \gtrsim 10$ GeV per nucleon (i.e. for SPS energies and higher). This is a consequence of a short time available for the interaction of primary nuclei and for subsequent equilibration of particles at such high energies. The degree of the chemical equilibration has been investigated [29,30] on the basis of the parton cascade model. Strong deviations from chemical equilibrium were predicted for light quarks at intermediate stages of a heavy-ion collision at RHIC and LHC energies. Large nonequilibrium effects at SPS energies were found within the UrQMD model [31]. As follows from the thermal model analysis of experimental data [32], the hadronic matter is out of chemical equilibrium at late stages of relativistic nuclear collisions. The conclusion about a significant “overpopulation” of light quarks in 160 AGeV Pb+Pb collisions has been made in Ref. [33].

Motivated by these findings, below we study the properties of chemically non-equilibrated $q\bar{q}$ matter which is however in the thermal equilibrium. Formally, we introduce two chemical potentials for quarks and antiquarks with a given flavour and assume that their values do not satisfy in general the conditions of chemical equilibrium. It will be shown below that the EOS of chemically non-equilibrated $q\bar{q}$ matter is non-trivial and its phase structure is much richer as compared with the equilibrated matter.

In the next section we briefly formulate the SU(3) version of the NJL model used in the present paper. In Sec. 3 this model is applied to calculate the EOS of the non-strange and strange $q\bar{q}$ matter at arbitrary densities of quark and antiquarks. The numerical results are presented in Sec. 4. Sec. 5 is reserved for discussion and summary.

II. FORMULATION OF THE MODEL

We proceed from the SU(3) version of the NJL model suggested in Ref. [34]. Its interaction part is given by the local coupling between the quark color currents. After the Fierz transformation the color singlet part of the Lagrangian may be written as:

$$\begin{aligned} \mathcal{L} = & \bar{\psi} (i \hat{\partial} - \hat{m}_0) \psi + G_S \sum_{j=0}^8 \left[\left(\bar{\psi} \frac{\lambda_j}{2} \psi \right)^2 + \left(\bar{\psi} \frac{i\gamma_5 \lambda_j}{2} \psi \right)^2 \right] \\ & - G_V \sum_{j=0}^8 \left[\left(\bar{\psi} \gamma_\mu \frac{\lambda_j}{2} \psi \right)^2 + \left(\bar{\psi} \gamma_\mu \frac{\gamma_5 \lambda_j}{2} \psi \right)^2 \right], \end{aligned} \quad (1)$$

where $\lambda_1, \dots, \lambda_8$ are the SU(3) Gell-Mann matrices in flavour space, $\lambda_0 \equiv \sqrt{2/3} \mathbf{I}$ and $\hat{m}_0 = \text{diag}(m_{0u}, m_{0d}, m_{0s})$ is the matrix of bare (current) quark masses. At $\hat{m}_0 = 0$ this Lagrangian is invariant with respect to the $\text{SU}(3)_L \otimes \text{SU}(3)_R$ chiral transformation. The term with \hat{m}_0 leads to explicit breaking of chiral symmetry which is supposed to be small. The relation

$$G_S = 2 G_V \quad (2)$$

between the scalar (G_S) and vector (G_V) coupling constants follows from the QCD motivated initial Lagrangian [34].

In fact, the above model can be formulated in such a way that G_V/G_S is considered as an adjustable parameter, which may be chosen by fitting the observed masses of the vector mesons. Different calculations use values of G_V/G_S in the range 0.5–1 [34,35]. We would like to stress here that the choice $G_V = 0$, frequently made in the literature, is unrealistic for baryon-rich $q\bar{q}$ systems. In this case, the omission of the repulsive vector interaction results in an underestimation of pressure leading to incorrect results concerning the possibility of

phase transitions and the existence of (meta)stable states (see also Ref. [23]). On the other hand, as will be seen below, thermodynamic functions of baryon-free systems are insensitive to G_V .

In the mean-field (Hartree) approximation, used in the present paper, only the scalar and vector terms of Eq. (1) survive and the Lagrangian is diagonal in the flavour space:

$$\mathcal{L}_{\text{mfa}} = \sum_{f=u,d,s} \mathcal{L}_f, \quad (3)$$

where

$$\mathcal{L}_f = \bar{\psi}_f(i\hat{D} - m_f)\psi_f - \frac{G_S}{2} \langle \bar{\psi}_f\psi_f \rangle^2 + \frac{G_V}{2} \langle \bar{\psi}_f\gamma_\mu\psi_f \rangle^2. \quad (4)$$

Here ψ_f denote spinors for u, d, s quarks, $D_\mu = \partial_\mu + iG_V \langle \bar{\psi}_f\gamma_\mu\psi_f \rangle$, angular brackets correspond to quantum-statistical averaging and the constituent quark mass m_f is determined by the gap equations

$$m_f = m_{0f} - G_S \langle \bar{\psi}_f\psi_f \rangle. \quad (5)$$

Due to the absence of a flavour-mixing interaction, all extensive thermodynamical functions (energy density, pressure etc.) are additive in quark flavour. Therefore, in this approximation light ($f = u, d$) and strange ($f = s$) quark systems can be studied separately. In the following we consider isospin-symmetric $q\bar{q}$ systems, assuming equal numbers of u and d quarks (antiquarks), and disregard the difference between m_{0u} and m_{0d} .

The model parameters m_{0f}, G_S, Λ (the ultraviolet cutoff in 3-dimensional momentum space) can be fixed by reproducing the empirical values [36] of quark condensates $\langle \bar{u}u \rangle$, $\langle \bar{s}s \rangle$ and the observed values of π and K decay constants f_π, f_K ³ in the vacuum. We choose the following values of the model parameters [38]

$$m_{0u} = m_{0d} = 7 \text{ MeV}, m_{0s} = 132 \text{ MeV}, G_S = 24.5 \text{ GeV}^{-2}, \Lambda = 0.59 \text{ GeV}, \quad (6)$$

³These decay constants are related to the bare quark masses and the condensate densities by the GOR relations [37].

The calculation shows (see Sec. 3) that these parameters correspond to the values

$$f_\pi = 93 \text{ MeV}, \quad \langle \bar{u}u \rangle = (-230 \text{ MeV})^3, \quad m_u^{\text{vac}} = 300 \text{ MeV}, \quad (7)$$

$$f_K = 90 \text{ MeV}, \quad \langle \bar{s}s \rangle = (-250 \text{ MeV})^3, \quad m_s^{\text{vac}} = 520 \text{ MeV}, \quad (8)$$

where m_f^{vac} is the constituent quark mass in the vacuum.

III. EQUATION OF STATE OF QUARK-ANTIQUARK MATTER

In the mean field approximation the Lagrangian is quadratic in the quark fields and the calculation of thermodynamic functions is straightforward. Below the method of Ref. [18] is generalized for chemically non-equilibrated systems. The Coulomb and surface effects are disregarded and the characteristics of $q\bar{q}$ matter are assumed to be spatially homogeneous and time-independent. In this article we consider separately the systems with light ($f = u, d$) and strange ($f = s$) quarks. Unless stated otherwise, the flavour index f will be omitted. The main feature of a chemically non-equilibrated system is the existence of two chemical potentials, i.e. for quarks (μ) and antiquarks ($\bar{\mu}$) which are in general not strictly related to each other. If the equilibrium with respect to creation and annihilation of $q\bar{q}$ pairs were perfectly fulfilled, the above chemical potentials would satisfy the condition of chemical equilibrium,

$$\bar{\mu} = -\mu. \quad (9)$$

At given temperature T and quark density ρ Eq. (9) fixes the density of antiquarks $\bar{\rho}$ (see Fig. 8). Below we study a general case considering quark and antiquark densities and accordingly, their chemical potentials, as independent quantities.

The generalized expression for the partition function Z of the thermally equilibrated $q\bar{q}$ system may be written as

$$Z = \text{Sp} e^{-\tilde{H}/T} = e^{-\Omega/T}, \quad (10)$$

where

$$\widetilde{H} = H - \mu N - \bar{\mu} \bar{N}. \quad (11)$$

Here H is the Hamiltonian and N (\bar{N}) is the quark (antiquark) number operator. Up to an arbitrary additive constant the thermodynamic potential Ω is equal to $-PV$ where P and V are respectively the pressure and the volume of the $q\bar{q}$ matter. Below the quantum–statistical averaging of any operator A is made in accordance with the expression:

$$\langle A \rangle \equiv \text{Sp} (A e^{-\widetilde{H}/T}) / Z. \quad (12)$$

From the Lagrangian, Eq. (4), one obtains the equation of motion for the quark field ψ

$$(i \gamma^\mu \partial_\mu - m - G_V \rho_V \gamma^0) \psi = 0, \quad (13)$$

where m is the constituent quark mass and $\rho_V = \langle \bar{\psi} \gamma^0 \psi \rangle$ is the vector density. It can be expressed through the densities $\rho = \langle N \rangle / V$ and $\bar{\rho} = \langle \bar{N} \rangle / V$ as ⁴:

$$\rho_V = \rho - \bar{\rho}. \quad (14)$$

After the plane wave decomposition [39] of $\psi, \bar{\psi}$, the quantum states of quarks and antiquarks can be specified by momenta \mathbf{p} and discrete quantum numbers λ (helicity and color). Let $a_{\mathbf{p},\lambda}$ ($b_{\mathbf{p},\lambda}$) and $a_{\mathbf{p},\lambda}^+$ ($b_{\mathbf{p},\lambda}^+$) be the destruction and creation operators of a quark (an antiquark) in the state \mathbf{p}, λ . Then the operators H, N and \bar{N} can be expressed through the linear combinations of $a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda}$ and $b_{\mathbf{p},\lambda}^+ b_{\mathbf{p},\lambda}$. For example

$$\frac{N}{V} = \sum_{\mathbf{p},\lambda} a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda}, \quad \frac{\bar{N}}{V} = \sum_{\mathbf{p},\lambda} b_{\mathbf{p},\lambda}^+ b_{\mathbf{p},\lambda} \quad (15)$$

In the quasiclassical approximation the sum over momenta may be replaced by the 3–dimensional integral over momentum space:

$$\sum_{\mathbf{p}} = \frac{1}{(2\pi)^3} \int d^3 p. \quad (16)$$

⁴The baryonic density ρ_B of the $q\bar{q}$ system equals $1/3\rho_V$.

By using Eqs. (4)–(5) one can find the Hamiltonian density of the $q\bar{q}$ matter in the mean–field approximation. The resulting expression for the operator \tilde{H} reads as

$$\begin{aligned} \frac{\tilde{H}}{V} = & \sum_{\mathbf{p},\lambda} (E_{\mathbf{p}} - \mu_R) a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda} + \sum_{\mathbf{p},\lambda} (E_{\mathbf{p}} - \bar{\mu}_R) b_{\mathbf{p},\lambda}^+ b_{\mathbf{p},\lambda} \\ & - \sum_{\mathbf{p},\lambda} E_{\mathbf{p}} + \frac{(m - m_0)^2}{2G_S} - \frac{G_V \rho_V^2}{2}, \end{aligned} \quad (17)$$

where $E_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$ and $\mu_R, \bar{\mu}_R$ denote the “reduced” chemical potentials:

$$\mu_R = \mu - G_V \rho_V, \quad (18)$$

$$\bar{\mu}_R = \bar{\mu} + G_V \rho_V. \quad (19)$$

In Eq. (17) the gap equation

$$m = m_0 - G_S \rho_S, \quad (20)$$

has been used. Here $\rho_S \equiv \langle \bar{\psi} \psi \rangle$ is the scalar density of the $q\bar{q}$ system. It can be expressed as

$$\rho_S = \langle \sum_{\mathbf{p},\lambda} \frac{m}{E_{\mathbf{p}}} (a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda} + b_{\mathbf{p},\lambda}^+ b_{\mathbf{p},\lambda} - 1) \rangle. \quad (21)$$

Eqs. (17) and (21) contain divergent terms. They originate from the negative energy levels of the Dirac sea [34,24]. In the present paper these terms are regularized by introducing the three–dimensional momentum cutoff $\theta(\Lambda - |\mathbf{p}|)$ where $\theta(x) \equiv \frac{1}{2}(1 + \text{sgn } x)$. The cutoff momentum Λ is considered as an adjustable model parameter.

The quantum–statistical averaging of the operator $a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda}$ gives the occupation number of quarks in the state \mathbf{p}, λ . Direct calculation shows that it coincides with the Fermi–Dirac distribution for an ideal gas with the chemical potential μ_R :

$$\langle a_{\mathbf{p},\lambda}^+ a_{\mathbf{p},\lambda} \rangle \equiv n_{\mathbf{p}} = \left[\exp\left(\frac{E_{\mathbf{p}} - \mu_R}{T}\right) + 1 \right]^{-1}. \quad (22)$$

The analogous expression for the antiquark occupation number, $\bar{n}_{\mathbf{p}} = \langle b_{\mathbf{p},\lambda}^+ b_{\mathbf{p},\lambda} \rangle$, is obtained by the replacement $\mu_R \rightarrow \bar{\mu}_R$. At given densities of quarks and antiquarks, the chemical potentials μ_R and $\bar{\mu}_R$ are determined from the normalization conditions

$$\rho = \nu \sum_{\mathbf{p}} n_{\mathbf{p}}, \quad \bar{\rho} = \nu \sum_{\mathbf{p}} \bar{n}_{\mathbf{p}}, \quad (23)$$

where ν is the spin–color degeneracy factor. For a $q\bar{q}$ system with single flavour

$$\nu = 2N_c = 6. \quad (24)$$

After introducing the ultraviolet cutoff, Eq. (21) takes the form

$$\rho_S = \nu \sum_{\mathbf{p}} \frac{m}{E_{\mathbf{p}}} \left[n_{\mathbf{p}} + \bar{n}_{\mathbf{p}} - \theta(\Lambda - p) \right]. \quad (25)$$

The explicit form of the gap equation is given by Eqs. (20), (22), (25). The physical vacuum ($\rho = \bar{\rho} = \rho_V = 0$) corresponds to the limit $n_{\mathbf{p}} = \bar{n}_{\mathbf{p}} = 0$. As discussed above, the pressure can be represented as

$$P = \frac{T}{V} \ln Z + P_0, \quad (26)$$

where the additive constant P_0 is introduced to ensure that the vacuum pressure is zero, $P_{\text{vac}} = 0$. Direct calculation gives the result

$$P = P_K + \frac{G_V \rho_V^2}{2} - B(m). \quad (27)$$

Here the first (“kinetic”) term in the r.h.s. coincides with the total pressure of ideal gases of quarks and antiquarks having constituent mass m and chemical potentials μ_R and $\bar{\mu}_R$:

$$\begin{aligned} P_K &= \nu T \sum_{\mathbf{p}} \left\{ \ln \left[1 + \exp \left(\frac{\mu_R - E_{\mathbf{p}}}{T} \right) \right] + \ln \left[1 + \exp \left(\frac{\bar{\mu}_R - E_{\mathbf{p}}}{T} \right) \right] \right\} \\ &= \nu \sum_{\mathbf{p}} \frac{\mathbf{p}^2}{3E_{\mathbf{p}}} (n_{\mathbf{p}} + \bar{n}_{\mathbf{p}}). \end{aligned} \quad (28)$$

The third term in Eq. (27) gives the “bag” part of the pressure. The explicit expression for $B(m)$ can be written as

$$B(m) = \Phi(m_{\text{vac}}) - \Phi(m). \quad (29)$$

Here m_{vac} is the constituent quark mass in the vacuum and

$$\Phi(m) = \nu \sum_{\mathbf{p}} E_{\mathbf{p}} \theta(\Lambda - p) - \frac{(m - m_0)^2}{2G_S}. \quad (30)$$

From Eqs. (27)–(29) one may see that $P_{\text{vac}} = -B(m_{\text{vac}}) = 0$, i.e. the condition of vanishing vacuum pressure is automatically satisfied. The expression for $\Phi(m)$ can be calculated analytically,

$$\Phi(m) = \frac{\nu \Lambda^4}{8 \pi^2} \Psi\left(\frac{m}{\Lambda}\right) - \frac{(m - m_0)^2}{2 G_S}, \quad (31)$$

where

$$\Psi(x) \equiv 4 \int_0^1 dt t^2 \sqrt{t^2 + x^2} = \left(1 + \frac{x^2}{2}\right) \sqrt{1 + x^2} - \frac{x^4}{2} \ln \frac{1 + \sqrt{1 + x^2}}{x}. \quad (32)$$

As required by the thermodynamic consistency, the gap equation (20) should follow from the minimization of the thermodynamic potential Ω with respect to m ,

$$(\partial_m P)_{T, \mu, \bar{\mu}} = \frac{m_0 - m}{G_S} - \rho_S = 0. \quad (33)$$

Therefore, the quark mass is not an independent variable and the pressure can be considered as a function of T , μ and $\bar{\mu}$ only. The gap equation in vacuum is ⁵

$$\Phi'(m_{\text{vac}}) = 0. \quad (34)$$

This equation was solved numerically with the values of model parameters from Eq. (6). The calculations were performed separately for light ($f = u, d$) and strange ($f = s$) quark flavours. In this way one obtains the vacuum masses and scalar densities given in Eqs. (7)–(8). It is convenient to define the bag constants B_{0f} for each flavour separately, as the difference between the pressures of physical ($m_f = m_f^{\text{vac}}$) and perturbative ($m_f = m_{f0}$) vacua, i.e. $B_{0f} = B(m_{0f})$. These quantities are analogous to the bag constants introduced in the MIT bag model. For the present choice of parameters, the bag constants are

$$B_{0u} = B_{0d} \simeq 34.5 \text{ MeV/fm}^3, \quad B_{0s} \simeq 208 \text{ MeV/fm}^3. \quad (35)$$

Note, that for the isosymmetric matter, with equal numbers of u and d quarks, the resulting bag constant is $B_{0u} + B_{0d} \simeq 69 \text{ MeV/fm}^3 \simeq (152 \text{ MeV})^4$.

⁵The vacuum will be stable with respect to mass fluctuations if $\Phi''(m_{\text{vac}}) < 0$.

It is easy to verify that the following thermodynamic identities hold in the model

$$\rho = \partial_\mu P, \quad \bar{\rho} = \partial_{\bar{\mu}} P. \quad (36)$$

In the mean-field approximation the entropy density

$$\begin{aligned} s &= \partial_T P = \partial_T P_K = \\ &= -\nu \sum_{\mathbf{p}} \left[n_{\mathbf{p}} \ln n_{\mathbf{p}} + (1 - n_{\mathbf{p}}) \ln (1 - n_{\mathbf{p}}) + n_{\mathbf{p}} \rightarrow \bar{n}_{\mathbf{p}} \right] \end{aligned} \quad (37)$$

is the same as for the mixture of ideal gases of quarks and antiquarks. Using the thermodynamic relations for the free energy density

$$\mathcal{F} \equiv \frac{F}{V} = \mu \rho + \bar{\mu} \bar{\rho} - P = e - Ts, \quad (38)$$

one can calculate the energy density of the $q\bar{q}$ matter, e . From Eqs. (27), (37)–(38) one has

$$e \equiv \frac{E}{V} = e_K + \frac{G_V \rho_V^2}{2} + B(m). \quad (39)$$

Here the kinetic part, e_K , again corresponds to the mixture of quark and antiquark ideal gases:

$$e_K = \nu \sum_{\mathbf{p}} E_{\mathbf{p}} (n_{\mathbf{p}} + \bar{n}_{\mathbf{p}}). \quad (40)$$

The same expression for the energy density can be obtained by averaging the energy-momentum tensor $\langle T^{00} \rangle = \langle H \rangle / V$. According to Eqs. (29), (39)–(40), the vacuum energy density is equal to zero: $e_{\text{vac}} = B(m_{\text{vac}}) = 0$.

From the above formulae it is easy to check that the following differential relations

$$dP = \rho d\mu + \bar{\rho} d\bar{\mu} + s dT, \quad (41)$$

$$d\mathcal{F} = \mu d\rho + \bar{\mu} d\bar{\rho} - s dT, \quad (42)$$

hold for any process in a thermally equilibrated system. Using Eqs. (38), (42) one can derive the relations

$$\begin{aligned}
P &= (\rho \partial_\rho + \bar{\rho} \partial_{\bar{\rho}} - 1) \mathcal{F} \\
&= (\rho_+ \partial_{\rho_+} - 1) \mathcal{F} + \rho_V \partial_{\rho_V} \mathcal{F},
\end{aligned} \tag{43}$$

where $\rho_+ \equiv (\rho + \bar{\rho})/2$. From this expression it is evident that the pressure vanishes when the free energy per particle, $F/\langle N + \bar{N} \rangle = \mathcal{F}/(\rho + \bar{\rho})$, has a minimum in the $\rho - \bar{\rho}$ plane. As shall be seen below, at low enough temperatures, a whole line of zero pressure states exists in this plane. But only one point on this line, namely the one corresponding to the minimum of the free energy per particle should be regarded as a (meta)stable state. At $T = 0$ this state corresponds to the minimum of the energy per particle,

$$\epsilon \equiv \frac{E}{\langle N + \bar{N} \rangle} = \frac{e}{\rho + \bar{\rho}}. \tag{44}$$

Now we discuss the stability of the $q\bar{q}$ matter. Thermodynamically stable matter may exist provided the 2×2 matrix $||\partial_{\rho_i} \partial_{\rho_j} \mathcal{F}||$ (with $\rho_1 = \rho$, $\rho_2 = \bar{\rho}$) is positive [7]. If this condition is not fulfilled, the system will be unstable due to a spontaneous growth of isothermal density fluctuations (spinodal instability). From Eq. (42) it is evident that the above stability condition holds when the inequalities

$$\partial_\rho \mu > 0, \tag{45}$$

$$\partial_\rho \mu \partial_{\bar{\rho}} \bar{\mu} > \partial_{\bar{\rho}} \mu \partial_\rho \bar{\mu} \tag{46}$$

hold simultaneously. As will be shown below, outside the spinodal region, there exists a wider region of metastable states where the coexistence of two phases ($i = 1, 2$) with different densities and masses is thermodynamically favourable. This is possible when the Gibbs conditions

$$\mu_1 = \mu_2, \tag{47}$$

$$\bar{\mu}_1 = \bar{\mu}_2, \tag{48}$$

$$P_1 = P_2, \tag{49}$$

$$T_1 = T_2 \tag{50}$$

are satisfied.

In the mixed phase, the condition of the baryon charge conservation can be written as

$$\lambda(\rho_1 - \bar{\rho}_1) + (1 - \lambda)(\rho_2 - \bar{\rho}_2) = 3N_B/V, \quad (51)$$

where $\lambda \leq 1$ is the volume fraction of the phase 1 and N_B is the total baryon number of the $q\bar{q}$ system. From Eqs. (47)–(51) one can see that for $N_B \neq 0$ the pressure of the mixed state will in general be λ -dependent, i.e. the Maxwellian construction ($P(\lambda) = \text{const}$ at $T = \text{const}$) will be violated. The situation is similar to the liquid–gas phase transition in nuclear matter with fixed baryonic and electric charges [40,7]. We would like to stress here that by imposing the condition of chemical equilibrium, Eq. (9), one normally puts the system outside the region of the phase transition.

In the baryon-free $q\bar{q}$ matter ($\langle N - \bar{N} \rangle = 0$) we have $\rho = \bar{\rho}$ and $\mu = \bar{\mu}$ (see Eq. (23)). In this case Eqs. (45)–(51) become much simpler. In particular, the stability conditions, Eqs. (45)–(46), are equivalent to the requirement that the isothermal compressibility is positive:

$$(\partial_\rho P)_T = 2\rho(\partial_\rho \mu)_T > 0. \quad (52)$$

Here we have used Eqs. (41), (45).

It is interesting to note that one more instability may occur in a dense baryon-rich system due to the presence of strong vector potential. At sufficiently high baryonic density a finite $q\bar{q}$ droplet becomes unstable with respect to the spontaneous creation of $q\bar{q}$ pairs⁶. This process becomes possible when the vector field exceeds a certain critical value [42]

$$G_V|\rho - \bar{\rho}| > m + m_{\text{vac}}. \quad (53)$$

When this condition is satisfied, the upper energy levels of the Dirac sea reach the bottom of the positive energy continuum. A rough estimate of minimal baryon density necessary to

⁶ This phenomenon is analogous to the spontaneous electron–positron pair production in strong electromagnetic fields [41].

produce a $q\bar{q}$ pair can be obtained by omitting the first term in the r.h.s. of Eq. (53). In the case $f = u, d$ this gives the following estimate for the critical baryon density ρ_B^c :

$$\rho_B^c = \frac{1}{3}(\rho - \bar{\rho}) \gtrsim \frac{m_{\text{vac}}}{3G_V} \simeq 6.3\rho_0, \quad (54)$$

where $\rho_0 = 0.17 \text{ fm}^{-3}$ is the equilibrium density of normal nuclear matter. In the above estimate we have used our standard choice of the model parameters. At larger G_V the critical densities will be lower.

Before going to the results, let us consider the special case of zero temperature. Taking the limit $T \rightarrow 0$ in Eq. (22) and using Eq. (23) we have

$$n_{\mathbf{p}} \rightarrow \theta(\mu_R - E_{\mathbf{p}}) = \theta(p_F - p)\theta(\mu_R - m), \quad (55)$$

where

$$p_F = \sqrt{\mu_R^2 - m^2} = \left(\frac{6\pi^2\rho}{\nu} \right)^{1/3} \quad (56)$$

is the Fermi momentum of quarks. From Eq. (56) and the analogous expression for anti-quarks one gets the explicit formulae for the chemical potentials at $T = 0$:

$$\mu = \sqrt{m^2 + p_F^2} + G_V(\rho - \bar{\rho}), \quad (57)$$

$$\bar{\mu} = \sqrt{m^2 + \bar{p}_F^2} - G_V(\rho - \bar{\rho}). \quad (58)$$

From Eqs. (20), (25) and (55) we arrive at the following gap equation at zero temperature:

$$\frac{m_0 - m}{G_S} = \frac{\nu}{8\pi^2} \left[p_F^3 \Psi' \left(\frac{m}{p_F} \right) + \bar{p}_F^3 \Psi' \left(\frac{m}{\bar{p}_F} \right) - \Lambda^3 \Psi' \left(\frac{m}{\Lambda} \right) \right]. \quad (59)$$

Two comments are in order here. First, from Eqs. (57)–(58) one can see that the condition of chemical equilibrium, (9), can not be satisfied at $T = 0$. At zero temperature only pure baryon ($\bar{\rho} = 0$) or antibaryon ($\rho = 0$) matter should be considered as chemically equilibrated. Second, according to Eq. (59), the quark mass m becomes smaller than m_0 at sufficiently high p_F or \bar{p}_F . However, the applicability of the NJL model is questionable at very high densities. Indeed, as discussed in Ref. [43], the cutoff momentum Λ has to exceed all characteristic particle momenta. In particular, the inequality $\Lambda > \max(p_F, \bar{p}_F)$ must not be violated.

The explicit formulae for the energy density and the pressure at $T = 0$ are given by the relations:

$$\begin{aligned}
e &= \mu \rho + \bar{\mu} \bar{\rho} - P \\
&= \frac{\nu}{8\pi^2} \left[p_F^4 \Psi \left(\frac{m}{p_F} \right) + \bar{p}_F^4 \Psi \left(\frac{m}{\bar{p}_F} \right) \right] + \frac{G_V \rho_V^2}{2} + B(m).
\end{aligned} \tag{60}$$

The gap equation, Eq. (59), corresponds to the minimum of e at fixed ρ and $\bar{\rho}$.

IV. RESULTS

In this section we present the results of numerical calculations. They were performed by using the parameter set from Eq. (6). One should bear in mind that in some cases the results are rather sensitive to the particular choice of model parameters. These cases will be discussed in more detail. Special attention will be paid to qualitative effects of chemical non-equilibrium which, as we shall see later, strongly influence the thermodynamic properties of $q\bar{q}$ matter.

First, we study the non-strange matter consisting of light ($f = u, d$) quarks and antiquarks. The effects of isotopic asymmetry are disregarded i.e. we assume that $\rho_u = \rho_d$ and $\bar{\rho}_u = \bar{\rho}_d$. Second, we consider also the $s\bar{s}$ matter with arbitrary densities of quarks and antiquarks. In accordance with the SU(3) flavour symmetry of the interaction Lagrangian, the same coupling constants are used for strange and non-strange systems.

Let us discuss first the properties of the non-strange $q\bar{q}$ matter at zero temperature. As noted above, chemical equilibrium at $T = 0$ can be realized only at $\bar{\rho} = 0$ (in systems with positive baryon charge). In particular, the baryon-free matter, with equal densities of quarks and antiquarks, can not be chemically-equilibrated at $T = 0$. By using the formulae of Sec. 3 we calculated the quark mass m , chemical potentials $\mu, \bar{\mu}$, the pressure P and energy per particle ϵ as functions of densities ρ and $\bar{\rho}$. The results are given in Figs. 1–7⁷.

⁷Unless stated otherwise, for systems with $f = u, d$ we denote by P the total pressure of u and

Figure 1 shows the density dependence of ϵ , μ and m for baryon-free $q\bar{q}$ matter composed of light quarks. The rapid drop of the quark mass reflects the restoration of chiral symmetry at high densities. The calculation shows that $\epsilon(\rho)$ has a maximum at a relatively small quark density $\rho = \rho_B \ll \rho_0$. At a higher density, $\rho = \rho_A$, the energy per particle reaches a minimum $\epsilon = \epsilon_A < \epsilon(0) = m_{\text{vac}}$. Therefore, the metastable state of baryon-free $q\bar{q}$ matter is predicted at zero temperature. This state has zero pressure (see below Fig. 3) and its existence is possible only in a chemically non-equilibrium system.

In the considered case the vector density $\rho_V = 0$ and therefore the density dependence of $\epsilon = e_f/2\rho_f$ is determined by the balance between the first kinetic term in Eq. (60) and the bag part $B(m)$. At $\rho \rightarrow 0$ one has $m \simeq m_{\text{vac}}$ and $B(m) \simeq \frac{1}{2}B''(m_{\text{vac}})(m - m_{\text{vac}})^2$. The explicit expression for $B''(m)$ follows from Eqs. (29), (31). The condition of vacuum stability (see the footnote to Eq. (34)), $B''(m_{\text{vac}}) > 0$, can be easily verified. Substituting $p_F = \bar{p}_F = (\pi^2\rho_f)^{1/3}$ into Eq. (60), in the limit $p_F \ll m$ one has

$$\epsilon \simeq m + \frac{3}{10} \frac{p_F^2}{m} + \frac{B(m)}{2\rho_f}. \quad (61)$$

Minimizing this expression with respect to m , one finds that at low ρ_f the quark mass decreases linearly with density:

$$m \simeq m_{\text{vac}} - \frac{2\rho_f}{B''(m_{\text{vac}})}. \quad (62)$$

From Eqs. (61)–(62) one can get the approximate formulae

$$\epsilon \simeq m_{\text{vac}} + \frac{3}{10} \frac{p_F^2}{m_{\text{vac}}} - \frac{\rho_f}{B''(m_{\text{vac}})}. \quad (63)$$

One can see that at low densities Eqs. (62)–(63) qualitatively reproduce the results shown in Fig. 1. Indeed, the second term on the r.h.s. of Eq. (63) is proportional to $\rho^{2/3}$. At small ρ

d quarks: $P \equiv P_u + P_d = 2P_u$. In the same case ρ ($\bar{\rho}$) denotes the total density of light quarks (antiquarks), i.e. $\rho = \rho_u + \rho_d = 2\rho_u$. When necessary, the single-flavour values are introduced by the subscript f .

this term dominates and ϵ increases with density reaching a maximum at the point B. From Eqs. (38), (43) at $\rho = \bar{\rho}$, $T = 0$, one can obtain the relations

$$P/2\rho = \rho \partial_\rho \epsilon = \mu - \epsilon, \quad (64)$$

which show that $P = 0$ and $\mu = \epsilon$ at the extrema of ϵ .

At $\rho > \rho^{(B)}$, the energy and pressure start to decrease due to the chiral symmetry restoration effects. At sufficiently high densities, the quark mass approaches the bare mass. In lowest order one can take $m \simeq m_0 \ll p_F$ and substitute $m = m_0$ into the expressions for P and e given by Eq. (60). This gives the following approximate formulae

$$\mu \simeq p_F, \quad \epsilon \simeq \frac{3}{4} p_F + \frac{B_0}{2\rho_f}, \quad (65)$$

where $B_0 \equiv B(m_0)$ is the bag constant. From Eqs. (64)–(65) one can see that the pressure of a symmetric $q\bar{q}$ system changes its sign at the point A, where the kinetic (Fermi–motion) term and the bag part balance each other⁸:

$$P_f^{(A)} \simeq \frac{1}{2} \rho_f p_F - B_0 = 0. \quad (66)$$

At much higher densities the bag contribution may be neglected and we obtain the limit of a massless ideal Fermi–gas: $P \simeq \frac{1}{3} e \simeq \frac{1}{2} \rho p_F \propto \rho^{4/3}$.

Substituting $B_0 = B_{0u}$ from Eq. (35) we get the estimate $\rho_A = 2\rho_u^{(A)} \simeq 3.0 \rho_0$. This is close to the density of the metastable state obtained by the numerical calculation, which gives the values

$$\rho_A \simeq 2.93 \rho_0, \quad \epsilon_A = \mu_A \simeq 0.268 \text{ GeV}. \quad (67)$$

As will be seen below, the value ϵ_A is the absolute minimum of $\epsilon(\rho, \bar{\rho})$ in the whole $\rho - \bar{\rho}$ plane. It is interesting to note that the minimum energy value ϵ_A is below the vacuum

⁸According to these estimates, the origin of the above energy minimum is similar to that suggested in the MIT bag model to explain hadron properties in vacuum [44].

mass, m_{vac} . However, this state is in fact metastable due to the possibility of the annihilation, $q\bar{q} \rightarrow n\pi, n = 2, 3, \dots$

The "chemically-equilibrated" case of baryon-rich matter with $\bar{\rho} = 0$ is considered in Fig. 2. A comparison with the preceding figure shows that the quark mass drops with the quark density slower than in the baryon-free matter. Although the ρ -dependence of ϵ is rather flat, for the present parameter set the energy per particle has no extrema in the considered case. Formally one can use the same procedure to estimate the behaviour of ϵ at $\rho \rightarrow 0$ as was suggested above for the baryon-free matter. The resulting expression may be obtained by replacing the last term in Eq. (63) by $[G_V - 1/B''(m_{\text{vac}})]\rho_f/2$. Using this expression one can find a fictitious maximum of ϵ at $\rho \sim 6\rho_0$ which is not found in the numerical calculation. At such a high density, however, the assumptions $m \simeq m_{\text{vac}}, p_F \ll m$ used in the above low density expansion are not valid. Therefore, in the baryon-rich case the model predicts a qualitatively different equation of state than that for symmetric $q\bar{q}$ matter.

Figure 3 shows the density dependence of the pressure in the u, d matter at $T = 0$. Again, in the baryon-rich case, $P(\rho)$ is monotonically increasing function. On the other hand, the behaviour of the pressure in the symmetric system is non-trivial: there is a region of negative pressures at $\rho_B < \rho < \rho_A$. The interval with the negative compressibility ($\partial_\rho P < 0$) exists between points B and C. One can see from Eqs. (43), (45), that the system is mechanically unstable at $\rho_B < \rho < \rho_C$. The appearance of regions with negative P and $\partial_\rho P$ are necessary conditions for the existence of bound states and for the first order phase transitions, respectively.

One should bear in mind that the presence or absence of states with negative P and $\partial_\rho P$ in the quark matter with $\bar{\rho} = 0$ is quite sensitive to the choice of the model parameters. The similar conclusion was made in Refs. [19,23]. By taking small values of the vector coupling constant G_V and/or quark bare mass m_0 , one can find the bound states and phase transitions even in baryon-rich systems. This is clear from Fig. 4 which shows the sensitivity of the equation of state to the choice of G_V and m_0 in u, d matter with $\bar{\rho} = 0$. In the case $m_0 = 0$, bound states of the baryon-rich matter are possible only if $G_V \lesssim 0.35 G_S$. On

the other hand, in absence of the vector interaction, i.e. at $G_V = 0$, the model predicts bound states even for bare masses $m_0 \sim 5 - 8$ MeV, corresponding to correct values of the pion decay constant f_π . However, at the realistic choice of the model parameters we do not find any phase transitions or bound states in the chemically equilibrated matter. The stable quark droplets have been predicted in Ref. [14] within the generalized NJL model. As discussed in Ref. [23], the prediction is questionable since it was implicitly assumed that $G_V = 0$, $m_0 = 0$. With the same caution one should regard the results of Refs. [18,20,21], where the possibility of phase transitions in baryon-rich $q\bar{q}$ matter was studied within the NJL model, either without vector interaction or by assuming unrealistically small values of G_V .

Figures 5(a), (b) show the model predictions for arbitrary $q\bar{q}$ systems with light quarks at $T = 0$. The contours of equal energy per particle and pressure in the $\rho - \bar{\rho}$ plane are given in Fig. 5(a) and 5(b), respectively. Only the vertical and horizontal axes of this plane correspond to chemically equilibrated states at zero temperature. One can see that symmetric systems with $\rho = \bar{\rho} \simeq 3\rho_0$ indeed have the non-trivial minimum of the energy per particle and zero pressure. Shaded areas indicate the spinodal regions where the conditions (45)–(46) are violated. From Fig. 5(b) one can see that at $T = 0$ practically all unstable states are located inside the contour $P = 0$. The presence of a spinodal instability shows to the possibility of the phase coexistence in a wider density region. We shall discuss this issue in more detail at the end of this section.

The results for $s\bar{s}$ systems are presented in Figs. 6, 7. According to Fig. 6, the density dependence of m , ϵ and μ in the symmetric $s\bar{s}$ matter is qualitatively the same as for non-strange systems. The model predicts a metastable state (point A) with the parameters

$$\rho = \bar{\rho} \simeq 6.26 \rho_0, \quad \epsilon \simeq 0.451 \text{ GeV}. \quad (68)$$

Although the assumption $m \ll p_F$ is not so well-justified as for light quarks, Eq. (66) gives the estimate $\rho_A \simeq 5.8 \rho_0$ which is close to the numerical result. The noticeably higher density of the metastable state in the $s\bar{s}$ case is due to the higher bag constant of strange

quarks. Indeed, from Eq. (35) one can see that $B_{0s}/B_{0u} \sim 6$. Eq. (66) leads to the estimate $0.5 (B_{0s}/B_{0u})^{3/4} \simeq 1.9$ for the ratio of densities of the metastable states in strange and non-strange systems, rather close to the value obtained from numerical calculations.

As can be seen from Fig. 7(a), the symmetric $s\bar{s}$ system with the parameters given by Eq. (68), has the minimum energy as compared to other points in the $\rho - \bar{\rho}$ plane. According to Fig. 7(b), the model does not predict states negative P and $\partial_\rho P$ in baryon-rich matter with $\bar{\rho} = 0$. A special study with different vector coupling constants shows that, in the latter case, the states with $P < 0$ exist at $G_V/G_S < 0.175$. The analogous calculation shows that negative values of $\partial_\rho P$ are possible if $G_V/G_S < 0.215$. The shaded area again shows the region of spinodal instability. For strange $q\bar{q}$ matter the instability region extends to higher densities as compared to the non-strange matter.

Let us now discuss nonzero temperatures. Eq. (23) yields the relations

$$\frac{\rho}{T^3} = I_0 \left(\frac{m}{T}, \frac{\mu_R}{T} \right), \quad (69)$$

$$\frac{\bar{\rho}}{T^3} = I_0 \left(\frac{m}{T}, \frac{\bar{\mu}_R}{T} \right), \quad (70)$$

connecting the quark (μ_R) and antiquark ($\bar{\mu}_R$) reduced chemical potentials with the densities ρ and $\bar{\rho}$. The dimensionless function

$$I_0(x, y) = \frac{\nu}{2\pi^2} \int_0^\infty dt \frac{t^2}{\exp(\sqrt{x^2 + t^2} - y) + 1} \quad (71)$$

appears from the integration of the particle occupation numbers in momentum space.

Using Eqs. (27), (33), one may write the gap equation in the form

$$(\partial_m P_K)_{T, \mu_R, \bar{\mu}_R} = B'(m). \quad (72)$$

Substituting the expression for P_K from Eq. (28), we have

$$-\frac{B'(m)}{m T^2} = I_1 \left(\frac{m}{T}, \frac{\mu_R}{T} \right) + I_1 \left(\frac{m}{T}, \frac{\bar{\mu}_R}{T} \right). \quad (73)$$

Here $I_1(x, y)$ differs from $I_0(x, y)$ by the replacement $dt \rightarrow dt/\sqrt{t^2 + x^2}$ in the r.h.s. of Eq. (71). The above equation is the analogue of Eq. (59) for the case $T \neq 0$.

According to Eqs. (9), (18)–(19), chemical equilibrium in $q\bar{q}$ matter is achieved if $\mu_R = -\bar{\mu}_R$ ⁹. Using this condition and Eqs. (69)–(70), (73) it is easy to see that the densities ρ and $\bar{\rho}$ are not independent in chemically equilibrated systems: the density of antiquarks at a given T is fixed by the density of quarks, and vice versa. Figs. 8(a), (b) show the dependence $\bar{\rho}(\rho)$ at various temperatures for the non-strange and strange matter in chemical equilibrium. We see that up to rather high temperatures, $T \sim 100$ MeV for $f = u, d$ and $T \sim 150$ MeV for $f = s$, the equilibrium values of $\bar{\rho}$ or ρ are very small. This is especially evident for the regions far from the diagonal $\bar{\rho} = \rho$.

Figures 9 and 10 represent the mass and pressure isotherms in symmetric $q\bar{q}$ matter with light and strange quarks. These results are obtained by the simultaneously solving Eqs. (69)–(70), (73) with respect to m , μ_R and $\bar{\mu}_R$. Figs. 9(a), (b) show that the quark mass increases with T at a fixed ρ . Such a behaviour is a specific feature of chemically non-equilibrated systems. It can be qualitatively understood, at least in the limit $m \ll T$. Indeed, from the definition of $B(m)$ it is easy to show that the l.h.s of Eq. (73) decreases with m . On the other hand, by using Eq. (69) one can see that the r.h.s. of the same equation increases with ρ/T^3 . This follows from the fact that $I_i(0, y)$, $i = 1, 2$ are increasing functions of y . This explains the above mentioned behaviour of $m(T)$. Figs. 9–10 show also the density dependence of m and P in the limit of chemical equilibrium, i.e. at $\mu = \bar{\mu} = 0$. According to Eqs. (69), (73), (27)–(28), in this limit m , ρ and P are fully determined by the temperature only.

As can be seen from Figs. 10(a), (b), at a fixed density the pressure increases with T . Similar to the zero temperature case, one can find the spinodal instability regions with negative isothermal compressibilities $(\partial_\rho P)_T$. The calculations show that the stability condition (52) in symmetric $q\bar{q}$ matter is violated at temperatures $T < T_c$ where

⁹ Therefore, the symmetric matter, where $\mu_R = \bar{\mu}_R$ (see Eqs. (69)–(70)), may be chemically equilibrated only if $\mu_R = \bar{\mu}_R = 0$, which in this case is equivalent to $\mu = \bar{\mu} = 0$.

$$T_c \simeq \begin{cases} 76 \text{ MeV}, & f = u, d, \\ 92 \text{ MeV}, & f = s. \end{cases} \quad (74)$$

At lower temperatures the first order phase transition is possible. This means that the formation of the mixed phase is thermodynamically favourable.

The coexisting phases are characterized (for symmetric $q\bar{q}$ matter) by different densities $\rho_1 < \rho_2$ and quark masses $m_1 > m_2$, but they have equal chemical potentials ($\mu_1 = \mu_2 \equiv \mu$) and pressures. Formally such a situation is possible in the regions of temperature and chemical potential where the gap equation has several solutions for the quark mass. Using the formulae of Sec. 3, one arrives at the following set of equations for m_1, m_2 and μ at fixed T :

$$\begin{cases} P_K(m_1, \mu, T) - B(m_1) = P_K(m_2, \mu, T) - B(m_2), \\ \partial_{m_1} P_K(m_1, \mu, T) = B'(m_1), \\ \partial_{m_2} P_K(m_2, \mu, T) = B'(m_2). \end{cases} \quad (75)$$

The values of ρ_i ($i = 1, 2$) are obtained by substituting $m = m_i$ and $\mu_R = \mu$ into Eq. (69). The density points ρ_1 and ρ_2 are represented by dotted lines in Figs. 10(a), (b). These lines constrain the two-phase “binodal” regions shaded at the same plots. At each temperature the states with a negative compressibility are situated inside the interval $\rho_1 < \rho < \rho_2$. Similar to the well-known liquid–gas phase transition, the states between the spinodal and binodal lines are metastable with respect to the heterophase decomposition of $q\bar{q}$ matter.

As can be seen from Fig. 10, states with zero pressure exist at sufficiently low temperatures. The maximal temperature when the existence of such states is still possible equals $T_m \simeq 49$ (67) MeV for non-strange (strange) symmetric matter. At $T < T_m$ the states with $P = 0$ are in the metastable density region. Thin lines in Figs. 10(a), (b) represent the density dependence of the pressure in chemically equilibrated baryon-free matter. The points where this line intersects the pressure isotherms give the corresponding values of P and ρ for the case of chemical equilibrium. These points are located outside of the binodal region.

More detailed information concerning the phase transition in the baryon-free matter is given in Fig. 11. The phase coexistence lines in the $\mu - T$ plane are shown both for non-strange and strange baryon-free matter. One can clearly see that states with $\mu = \bar{\mu} = 0$ do not belong to the phase transition lines. Therefore, the discussed phase transition has essentially non-equilibrium nature. It does not occur in chemically equilibrium systems.

Figures 12(a), (b) represent the contour lines of ϵ and P for the non-strange $q\bar{q}$ mixture at $T = 50$ MeV. This temperature is close to the maximum temperature for the existence of zero pressure states. The comparison with Fig. 5 shows that the spinodal region occupies a smaller part of the $\rho - \bar{\rho}$ plane than in the case $T = 0$. From Fig. 8(a) we conclude that at such a low temperature the equilibrium quark-antiquark densities lie outside (below) the spinodal boundary. An analysis of the binodal boundaries in the $\rho - \bar{\rho}$ plane will be given in a future publication.

V. SUMMARY AND DISCUSSION

We have demonstrated that the NJL model predicts the existence of metastable states of $q\bar{q}$ matter out of chemical equilibrium. These novel states are most strongly bound for symmetric (net baryon-free) systems with equal numbers of quarks and antiquarks. For realistic model parameters there exists a region of quark chemical potentials and temperatures where a first order chiral phase transition is possible. It is shown that for the same set of parameters the first order phase transition does not occur in a chemically equilibrated system. This conclusion may also be valid for other models of strongly interacting matter. Up to now both the chiral and the deconfinement transition in hadronic and quark-gluon matter have been studied under the assumption that chemical equilibrium is fulfilled. As discussed above, this is most likely not the case in relativistic heavy-ion collisions, at least at some stages of matter evolution.

Let us briefly discuss possible signatures of the phase transitions and of the bound states predicted in this paper. Measurements of excitation functions of particle spectra in heavy-

ion collisions have been suggested [17] to detect a possible critical point in the $q\bar{q}$ matter phase diagram. In principle, the same idea can be applied to the search for these non-equilibrium phase transitions. However, an easier task might be to look experimentally for the decay of metastable $q\bar{q}$ droplets in nuclear collisions. It is clear that the formation of such droplets would be possible if dense and relatively cold baryon-free matter is created at an intermediate stage of the reaction. Such conditions may be realized at high bombarding energies, presumably, at RHIC and LHC. Selecting events with unusually high pion multiplicity and low $\langle p_T \rangle$ may increase the probability to observe such multi-quark droplets. Considerations similar to those suggested in Ref. [45] show that the subsequent hadronization of droplets may give rise to narrow bumps in hadron rapidity distributions. These bumps may be observable in an event-by-event analysis of π , K and ϕ spectra. It is also obvious, that the yields of $B\bar{B}$ pairs ($B = N, \Lambda, \Sigma, \Xi, \Omega \dots$) will be enhanced if they are produced through the decay of multi-quark-antiquark systems.

The understanding of the possible decay channels of the predicted metastable states is of great importance. In non-strange systems, the strongest decay channel should be the annihilation of $q\bar{q}$ pairs into pions. This channel is energetically open at least at the surface of the $q\bar{q}$ droplet. Indeed, according to Eq. (67), the average energy released in the annihilation of a $q\bar{q}$ pair is $\mu + \bar{\mu} = 2\epsilon_A \simeq 3.8 m_\pi$. This is sufficient to create up to three pions. The largest rate of annihilation is apparently given by the $q\bar{q} \rightarrow 2\pi$ channel. Indeed, the one-pion annihilation is suppressed in homogeneous matter due to the energy-momentum conservation. On the other hand, the relative probability of the $q\bar{q} \rightarrow 3\pi$ channel should be small due to the limited phase space volume available. However, the pion properties in the non-equilibrium $q\bar{q}$ medium have not yet been investigated. It may happen that pion-like excitations disappear due to the Mott transition [43]. It is also possible that the in-medium pion mass is increased as compared to the vacuum value. In both cases the annihilation inside the $q\bar{q}$ droplet will be suppressed.

The situation in a metastable $s\bar{s}$ droplet may be even more complicated. It is interesting that the direct annihilation of the $s\bar{s}$ pairs into hadrons is suppressed. Indeed, ac-

According to Eq. (68), the energy available in annihilation of a $s\bar{s}$ pair is equal on average to $\mu + \bar{\mu} = 2\epsilon \simeq 0.9$ GeV. This is lower than the threshold energies of the final hadronic states: ϕ (1020 MeV), $K\bar{K}$ (990 MeV) and $\rho\pi$ (910 MeV). In this case, annihilation is possible only for energetic $s\bar{s}$ pairs from the tails of the thermal distribution. The only open channel is $s\bar{s} \rightarrow 3\pi$. From the analogy to the $\phi \rightarrow 3\pi$ decay, which has the partial width $\Gamma_{\phi \rightarrow 3\pi} < 80$ keV, one can conclude that this channel should be quite weak.

However, in the deconfined $q\bar{q}$ matter, the strong decay channels $s\bar{s} \rightarrow u\bar{u}$, $d\bar{d}$ are possible, which may reduce significantly the life times of $s\bar{s}$ droplets. These processes will lead to the conversion of $s\bar{s}$ matter into a system where all three flavours are present. If sufficiently long times were available, chemical equilibrium between the flavours would be established. It will be interesting to check whether an admixture of $q\bar{q}$ pairs in strange baryon-rich systems is energetically favourable as compared with ordinary stranglets [11,12]. Detailed calculations of the formation probability and life times of metastable $q\bar{q}$ droplets are possible only within a dynamical approach.

One should investigate also the role of the mesonic degrees of freedom which have been disregarded in the present work. It is well known that mesons appear in the NJL model as bound states of a quark and an antiquark. The influence of mesonic degrees of freedom in chemically equilibrated $q\bar{q}$ matter has been studied in Ref. [25]. It was shown by using RPA that the contribution of mesons may be important at low quark densities.

In the present calculation the flavour mixing six-fermion interaction was neglected. This interaction has been introduced earlier [46] to reproduce the mass splitting of η and η' mesons. It has been shown in Ref. [23] that the flavour-mixing interaction only slightly changes the EOS of non-strange $q\bar{q}$ matter at $T = 0$. It will be interesting to investigate the role of this interaction in the case when chemical equilibrium is violated.

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- [1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
- [2] V.G. Vaks and A.I. Larkin, Sov. J. JETP **13**, 192 (1961).
- [3] T.D. Lee and G.C. Wick, in *Mesons in nuclei*, eds. M. Rho and D. Wilkinson, vol. 3, 1979, p. 35.
- [4] J. Boguta, Phys. Lett. **120B**, 34 (1983).
- [5] B.D. Serot and J.D. Walecka, Adv. in Nucl. Phys. **16**, 1 (1985);
Rep. Prog. Phys. **55**, 1855 (1997).
- [6] J. Theis, G. Graebner, G. Buchwald, J. Maruhn, W. Greiner, H. Stöcker and J. Polonyi, Phys. Rev. D **28**, 2286 (1983).
- [7] H. Müller and B.D. Serot, Phys. Rev. C **52**, 2072 (1995).
- [8] P.J. Ellis, E.K. Heide and S. Rudaz, Phys. Lett. **B282**, 271 (1992);
G. Carter, P.J. Ellis and S. Rudaz, Nucl.Phys. **A603**, 367 (1996).
- [9] I.N. Mishustin, J. Bondorf and M. Rho, Nucl. Phys. **A555**, 215 (1993).
- [10] P. Papazoglou, J. Schaffner, S. Schramm, D. Zschiesche, H. Stöcker and W. Greiner, Phys. Rev. C **55**, 1499 (1997).

- [11] E. Witten, Phys. Rev. D **30**, 292 (1984);
E. Farhi and R.L. Jaffe, *ibid.* **30**, 2379 (1984).
- [12] C. Greiner, P. Koch and H. Stöcker, Phys. Rev. Lett. **58**, 1825 (1987);
C. Greiner and H. Stöcker, Phys. Rev. D **44**, 3517 (1991).
- [13] I.N. Mishustin and W. Greiner, J. Phys. **G19**, L101 (1993).
- [14] M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. **B422**, 247 (1998).
- [15] R. Rapp, T. Schäfer, E.V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).
- [16] M.A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov and J.J. Verbarshot,
hep-ph/9804290; Phys. Rev D **58** [accepted for publication].
- [17] M. Stephanov, K. Rajagopal and E. Shuryak, hep-ph/9806219.
- [18] M. Asakawa and Y. Yazaki, Nucl. Phys. **A504**, 668 (1989).
- [19] S. Klimt, M. Lutz and W. Weise, Phys. Lett. **B249**, 386 (1990).
- [20] J. Cugnon, M. Jaminon and B. Van den Bossche, Nucl. Phys. **A598**, 515 (1996).
- [21] K.G. Klimenko and A.S. Vshivtsev, Zh. Eksp. Teor. Fiz. **84**, 1047 (1997)
[Sov. J. JETP **84**, 1057 (1997)]; K.G. Klimenko, hep-ph/9809218.
- [22] V. Koch, T.S. Biro, J. Kunz and U. Mosel, Phys. Lett. **B185**, 1 (1987);
V. Koch, B. Blättel and U. Mosel, *ibid.* **B194**, 331 (1987).
- [23] M. Buballa, Nucl. Phys. **A611**, 393 (1996);
M. Buballa and M. Oertel, *ibid.* **A642**, 39 (1998).
- [24] I.N. Mishustin, in Proc. Int. Conf. *Nuclear Physics at the Turn of the Millenium*
(Wilderness, South Africa, 1996), eds. H. Stöcker, A. Gallman, J.H. Hamilton,
World Scientific, 1997, p. 522.
- [25] P. Zhuang, J. Hüfner and S.P. Klevansky, Nucl. Phys. **A576**, 525 (1994).

- [26] W.M. Zhang and L. Wilets, Phys. Rev. C **45**, 1900 (1992).
- [27] A. Abada and J. Aichelin, Phys. Rev. Lett., **74**, 3130 (1995).
- [28] I.N. Mishustin and O. Scavenius, Phys. Lett. **B396**, 33 (1997).
- [29] K. Geiger and J.I. Kapusta, Phys. Rev. D **47**, 4905 (1993).
- [30] D.K. Srivastava, M.G. Mustafa and B. Müller, Phys. Rev. C **56**, 1064 (1997).
- [31] L.V. Bravina et al., nucl-th/9810036.
- [32] P. Braun-Munzinger and J. Stachel, Nucl. Phys. **A638**, 3c (1998).
- [33] J. Letessier and J. Rafelski, hep-ph/9810332; J. Phys. G [accepted for publication].
- [34] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991).
- [35] M. Lutz, S. Klimt and W. Weise, Nucl. Phys. **A542**, 521 (1992).
- [36] L. Reinders, J.H. Rubinstein and S. Yazaki, Phys. Rep. **205**, 2 (1985).
- [37] M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. **175**, 2195 (1968).
- [38] W. Ehehalt and W. Cassing, Nucl. Phys. **A602**, 449 (1996).
- [39] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
- [40] N.K. Glendenning, Phys. Rev. D **46**, 1274 (1992).
- [41] J. Schwinger, Phys. Rev. **82**, 664 (1951).
- [42] I.N. Mishustin, L.M. Satarov, J. Schaffner, H. Stöcker and W. Greiner, J. Phys. **G19**, 1303 (1993).
- [43] S.P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).
- [44] J.I. Kapusta, Phys. Rev. D **23**, 2444 (1981).

[45] I.N. Mishustin, hep-ph/9811307.

[46] S. Klimt, M. Lutz, U. Vogt and W. Weise, Nucl. Phys. **A516**, 429 (1990).

FIGURE CAPTIONS

FIG. 1. Energy per particle, chemical potential and constituent quark mass in symmetric matter with equal numbers of light quarks and antiquarks at zero temperature. ρ is the total density of u and d quarks. Point A shows the position of the metastable state.

FIG. 2. The same as in Fig. 1, but for asymmetric u, d matter with zero density of antiquarks.

FIG. 3. Pressure of $q\bar{q}$ matter with light quarks at zero temperature. The solid (dashed) line corresponds to the baryon-free (rich) system.

FIG. 4. Regions in the m_0-G_V plane where states with negative pressure (dark shading) and negative compressibility (light shading) exist in baryon-rich ($\bar{\rho} = 0$) quark matter at $T = 0$. Cross corresponds to parameters used in the present model.

FIG. 5. Contours of equal energy per particle (in GeV, upper part) and equal pressure (in GeV/fm³, lower part) in $q\bar{q}$ matter with light quarks at zero temperature. $\rho(\bar{\rho})$ is total density of u and d quarks (antiquarks). Shading indicates the regions of spinodal instability.

FIG. 6. The same as in Fig. 1, but for symmetric $s\bar{s}$ matter.

FIG. 7. The same as in Fig. 5, but for $s\bar{s}$ matter.

FIG. 8. Quark and antiquark densities, ρ and $\bar{\rho}$, in chemically equilibrated $q\bar{q}$ matter ($\bar{\mu} = -\mu$) at different temperatures (shown in MeV). Upper and lower part correspond to the non-strange and strange matter, respectively.

FIG. 9. Mass isotherms for non-strange (upper part) and strange (lower part) symmetric $q\bar{q}$ systems. Temperatures are shown in MeV near the corresponding curves. Thin solid lines show the density dependence of quark mass in chemically equilibrated matter ($\mu = \bar{\mu} = 0$).

FIG. 10. Pressure isotherms for baryon-free $q\bar{q}$ matter. Upper (lower) part represents model predictions for non-strange (strange) systems. Thin solid lines show the pressure of chemically equilibrated matter. The binodal regions are shaded.

FIG. 11. Phase coexistence lines in the $\mu - T$ plane for symmetric non-strange (solid line) and strange (dashed line) $q\bar{q}$ matter.

FIG. 12. The same as in Fig. 5, but for the temperature $T = 50$ MeV.

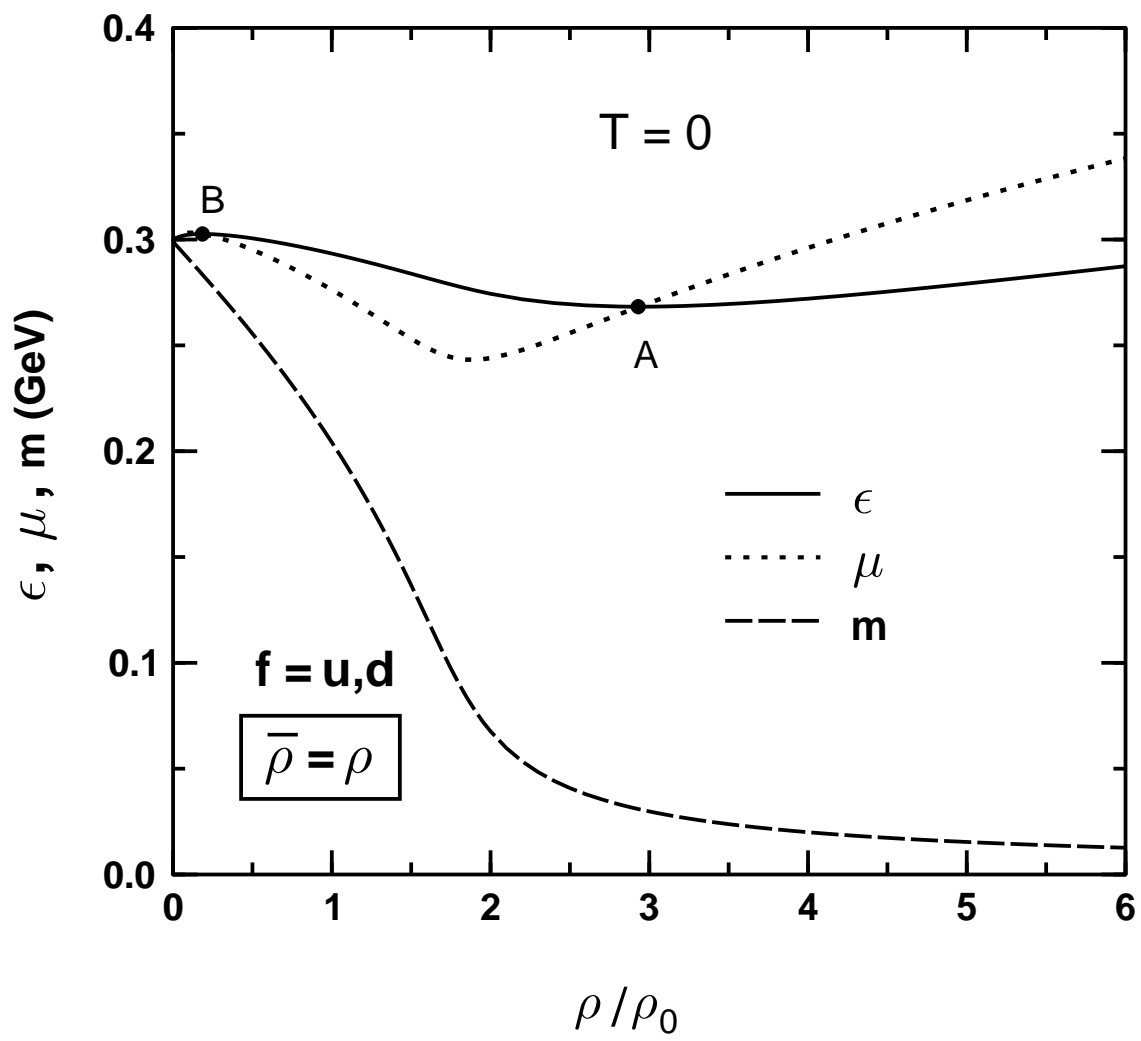


Fig. 1

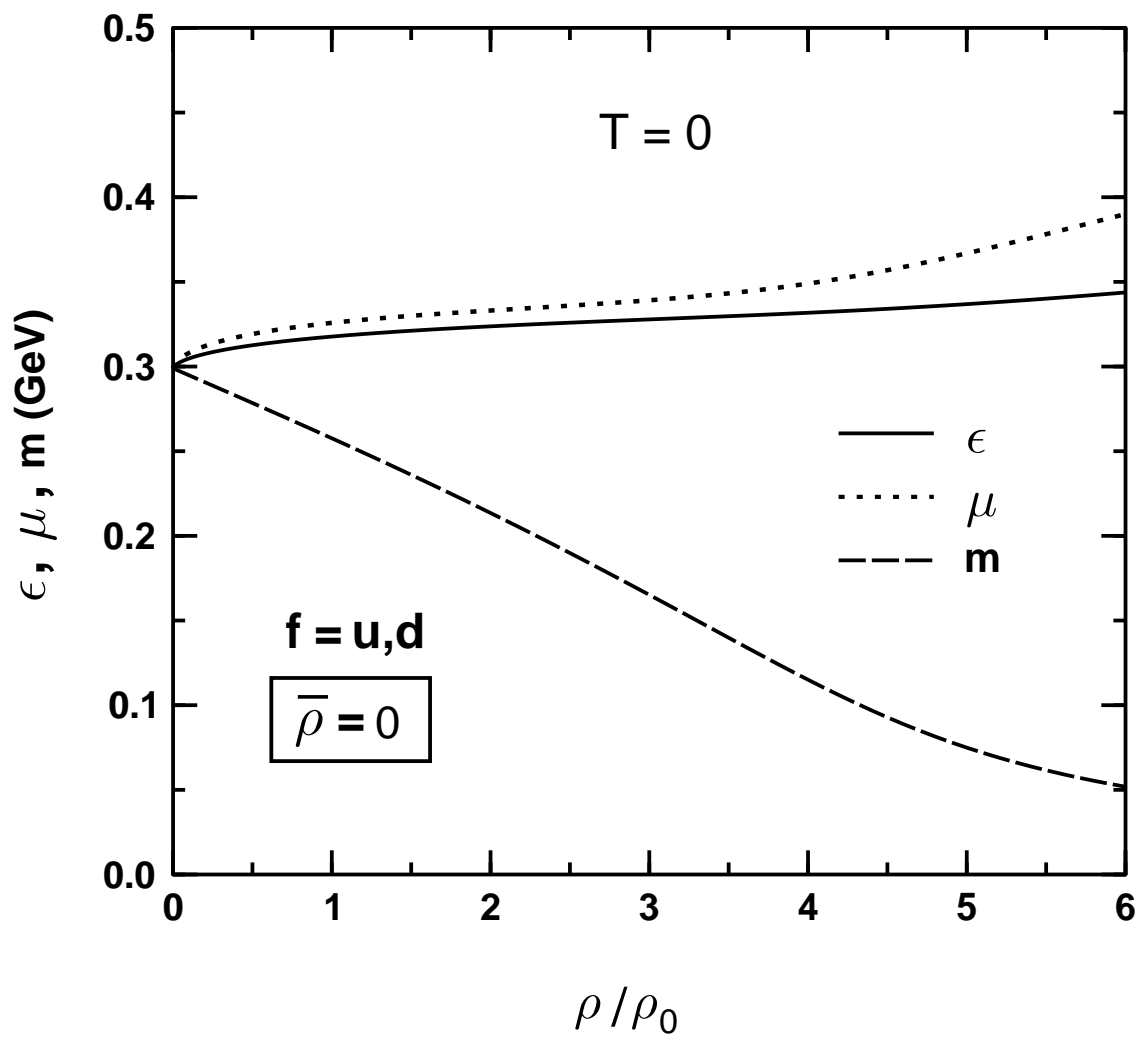


Fig. 2

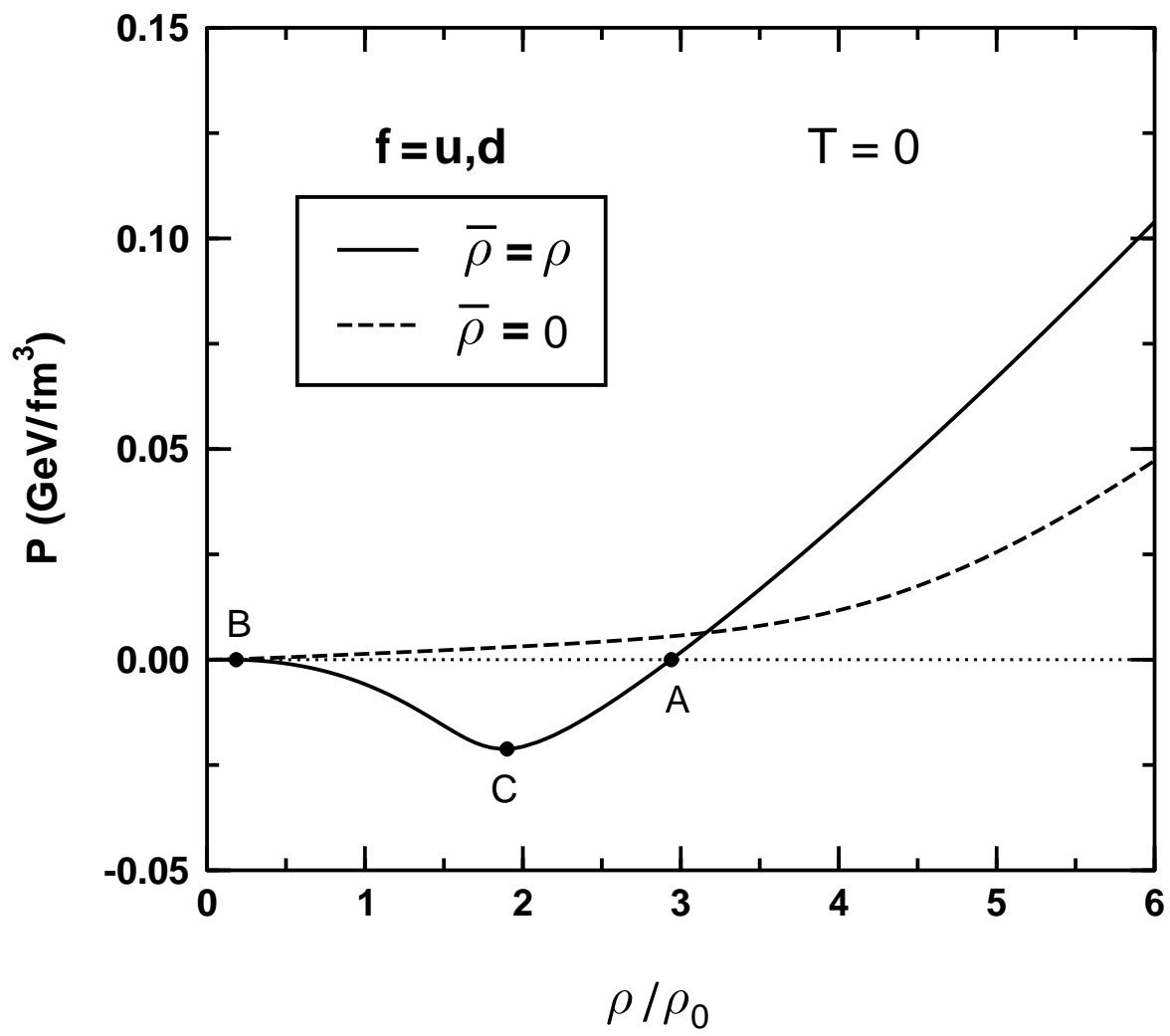


Fig. 3

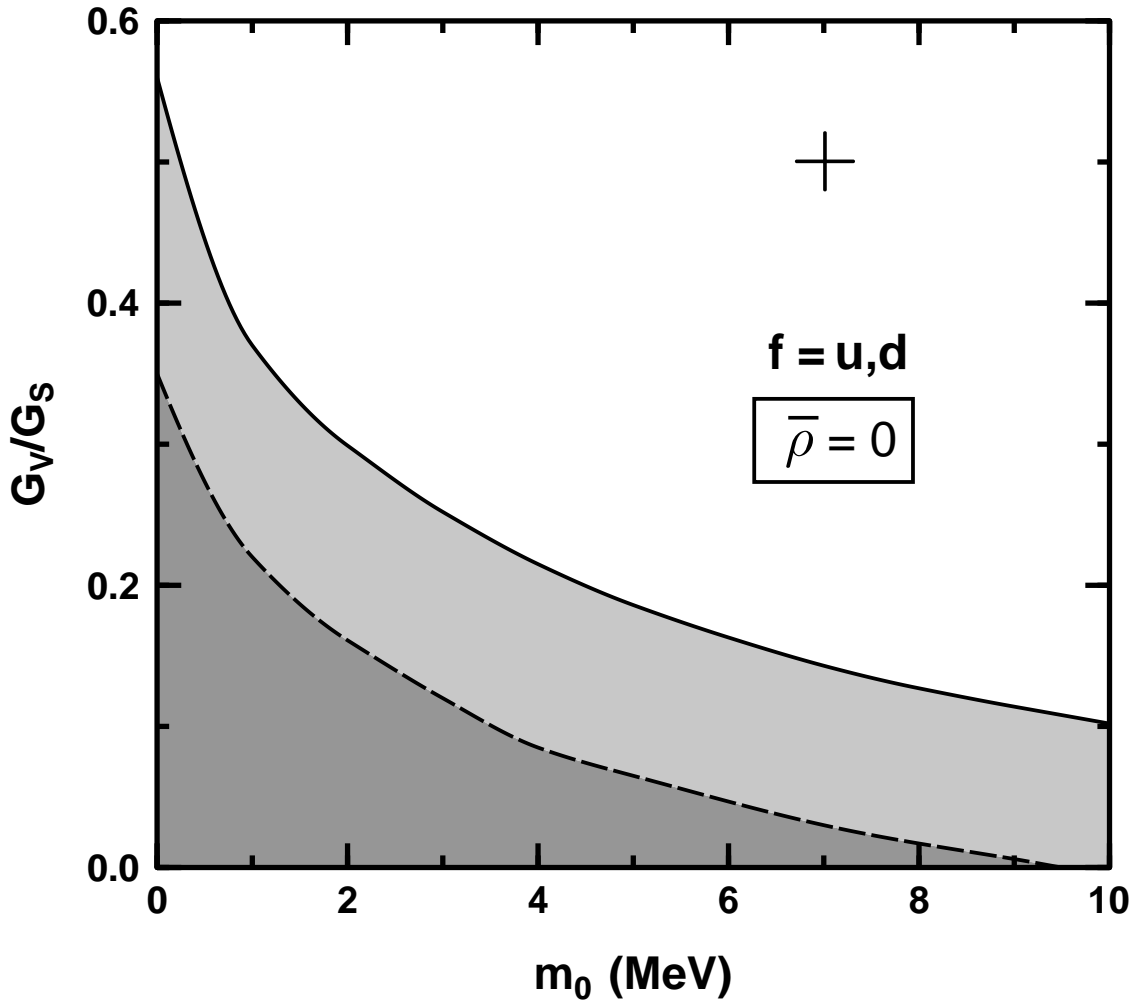


Fig. 4

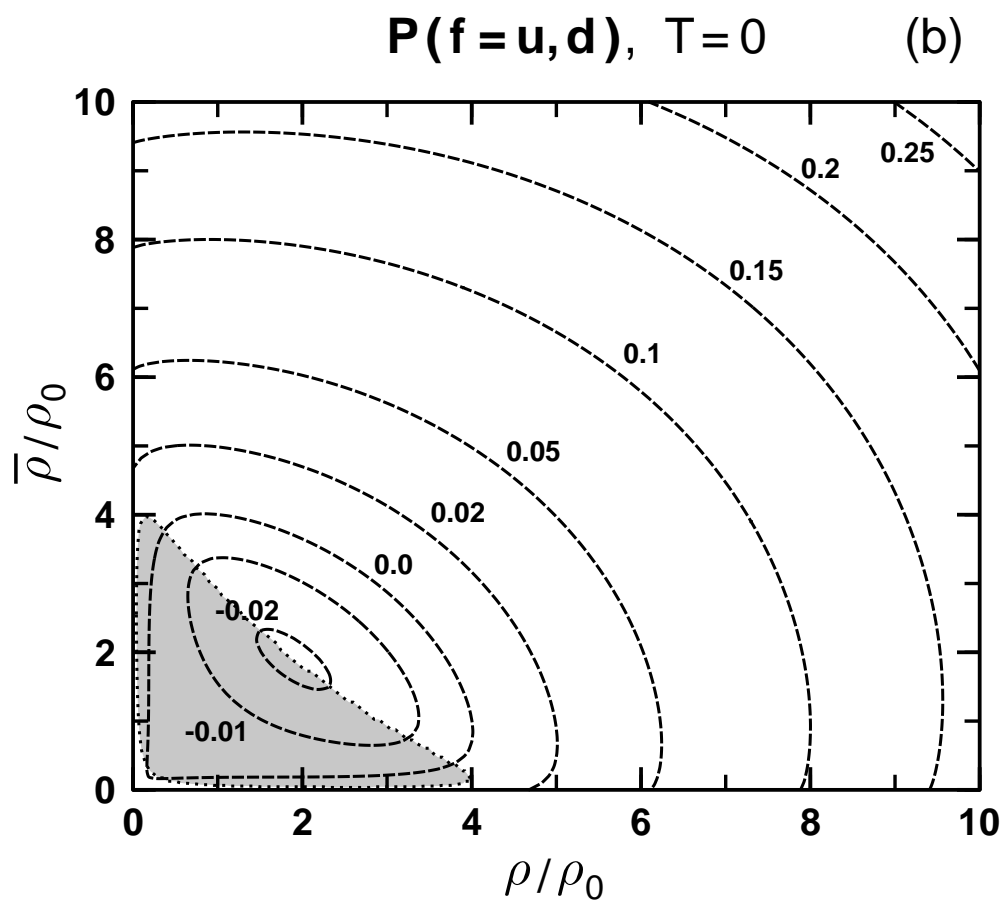
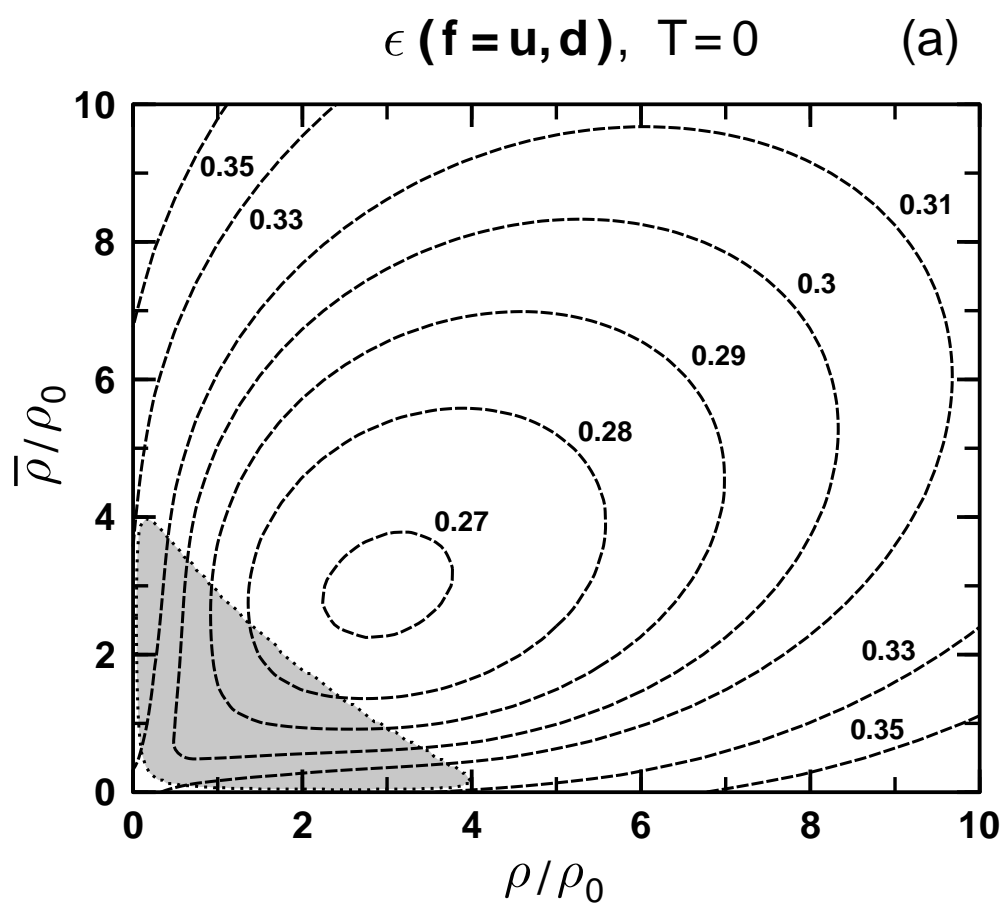


Fig. 5

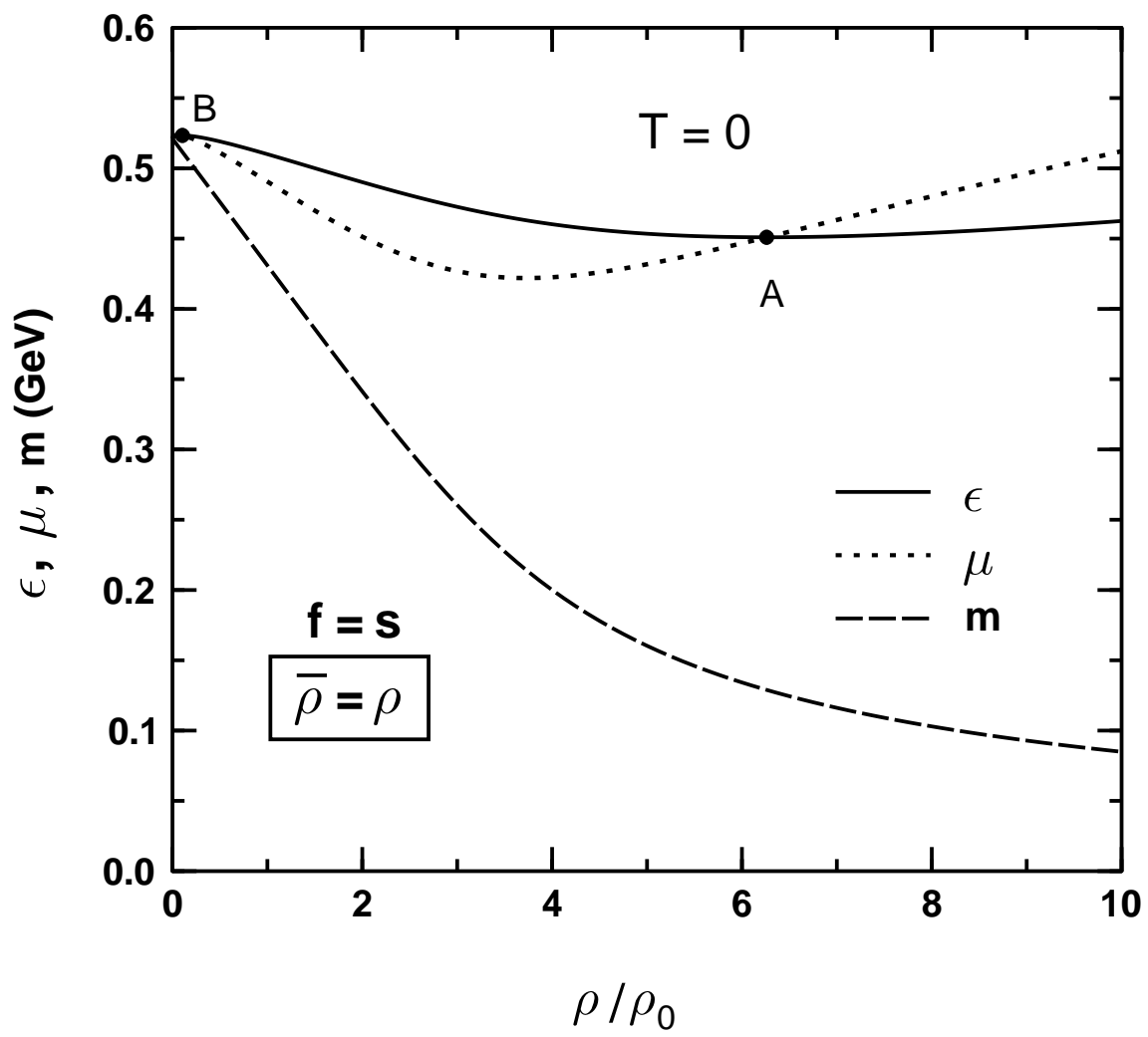


Fig. 6

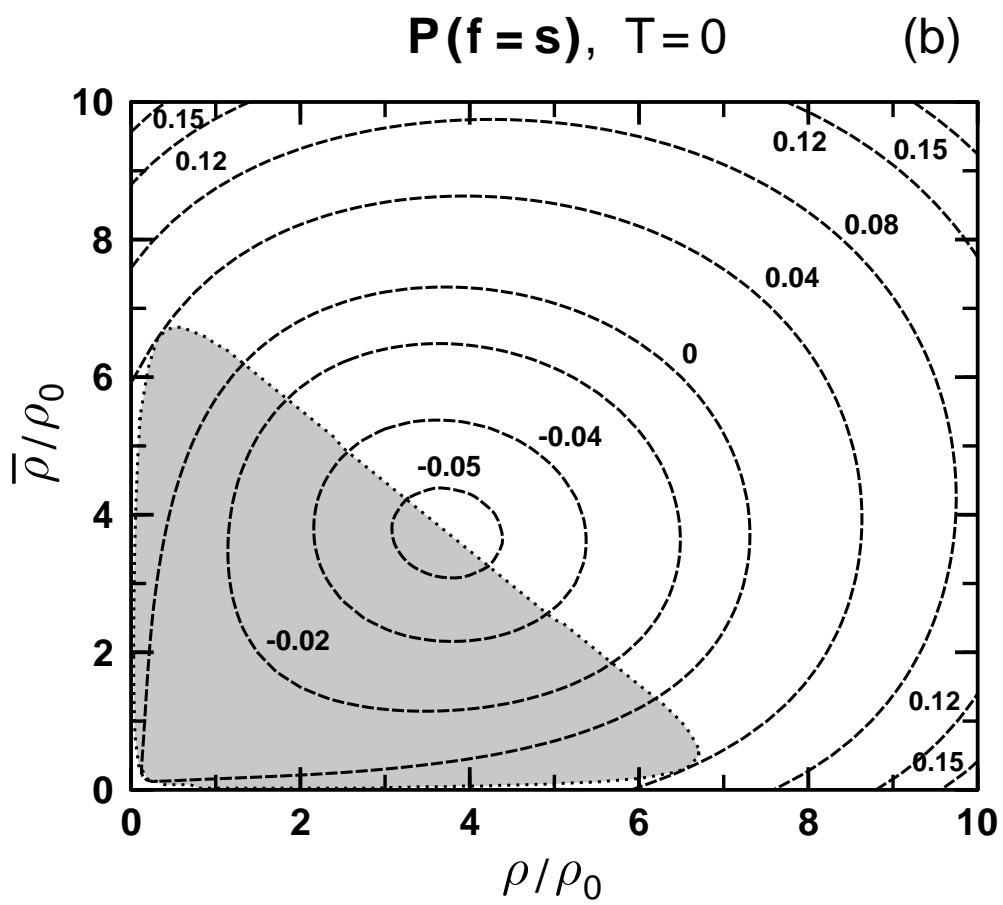
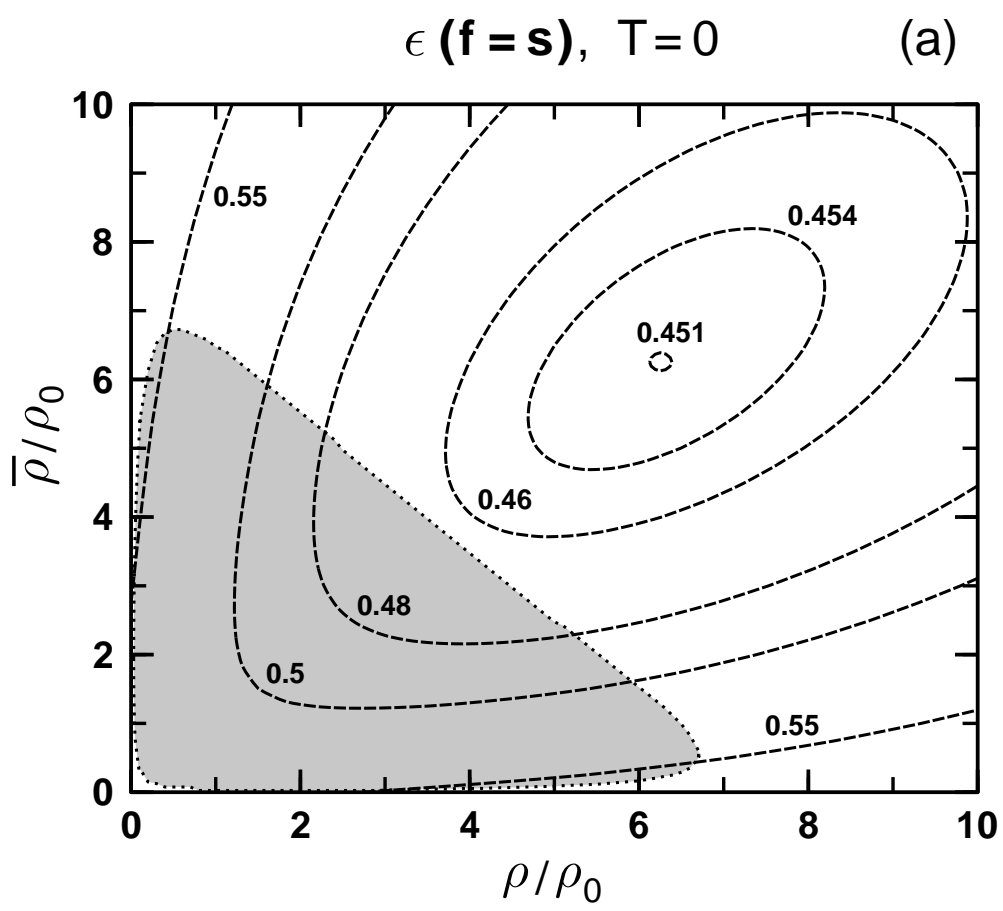
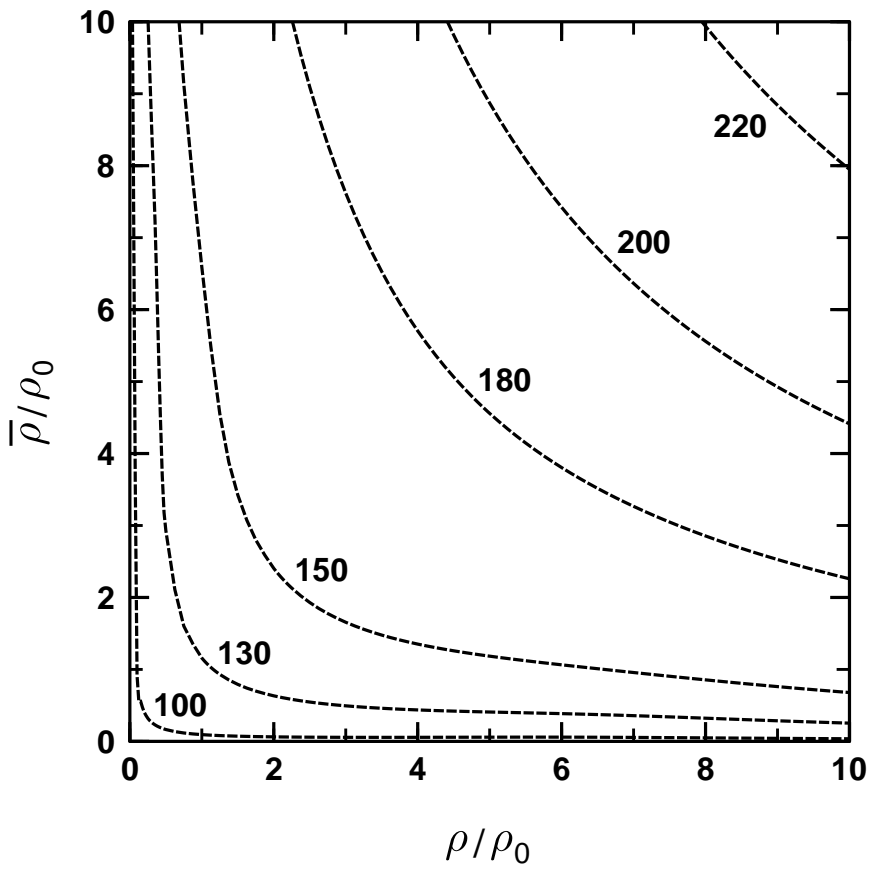


Fig. 7



f = s (b)

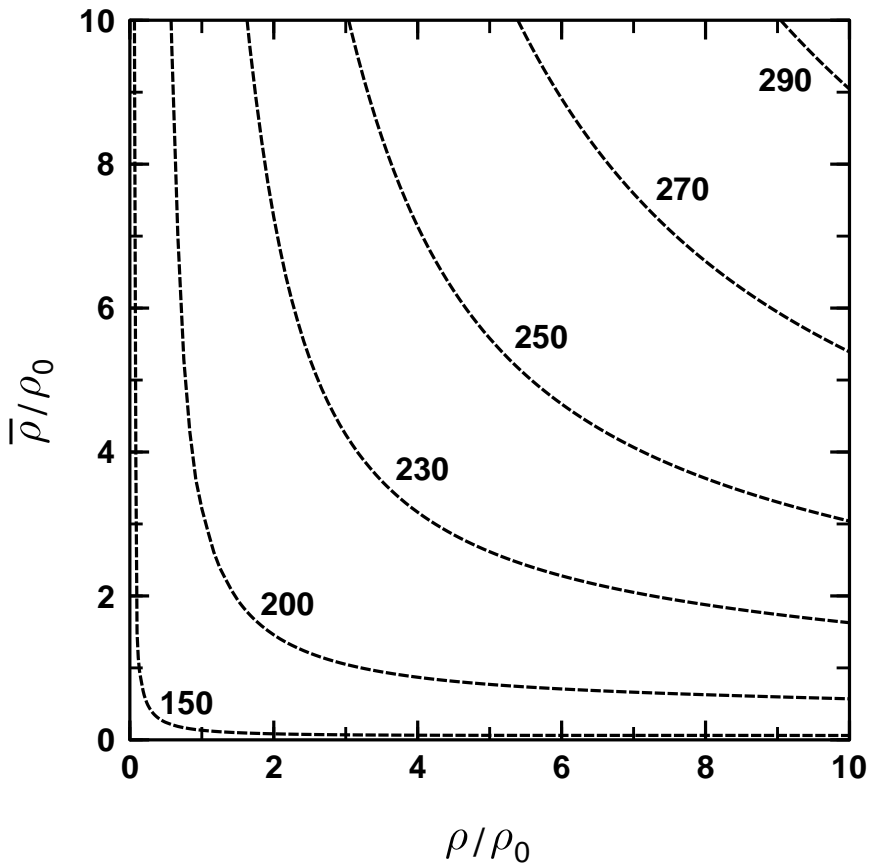
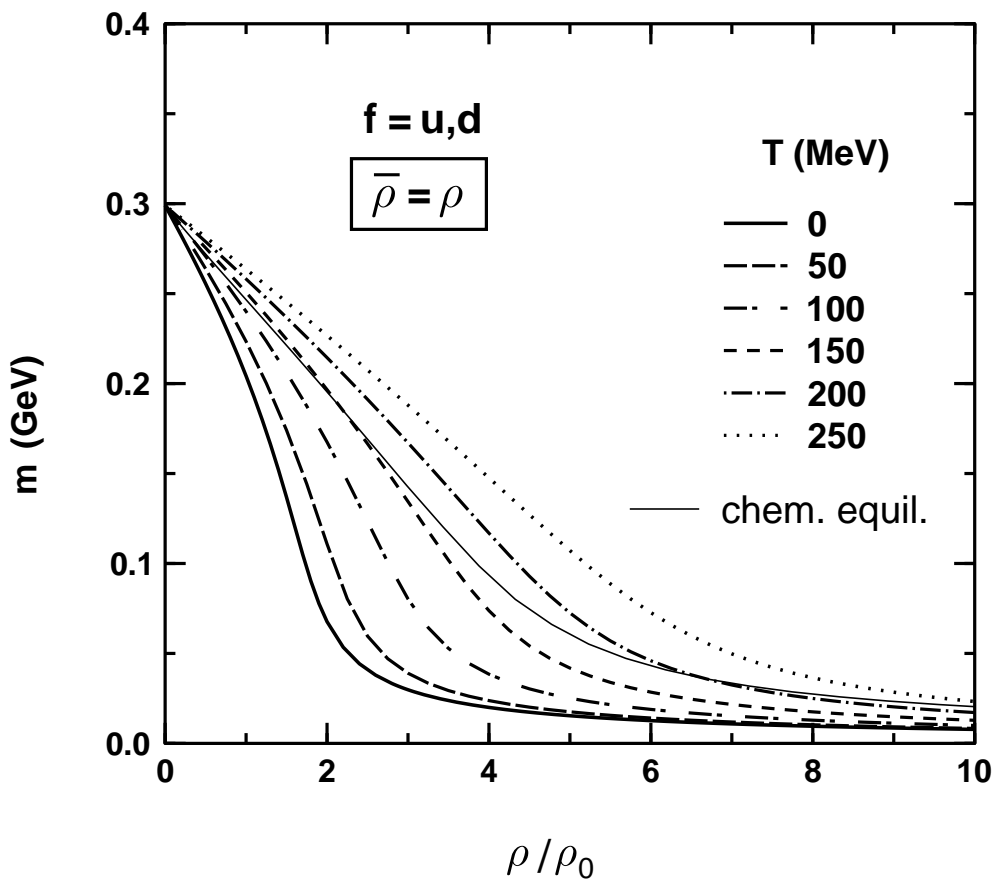


Fig. 8



(b)

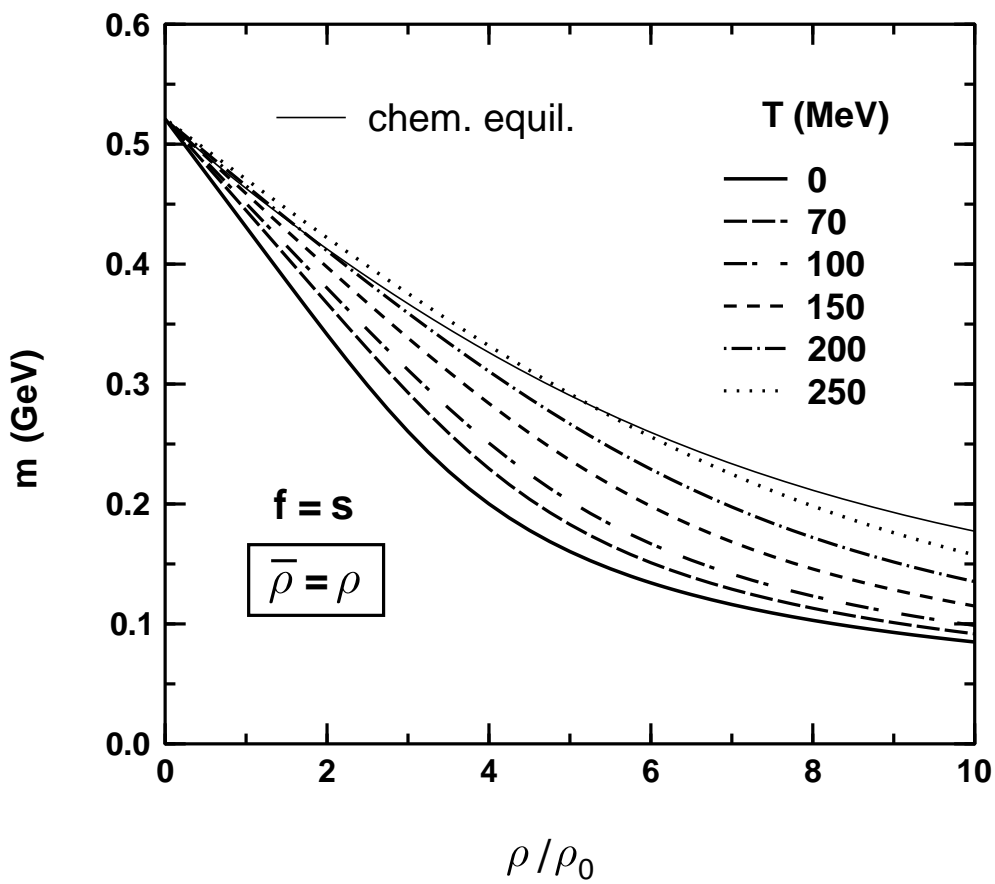
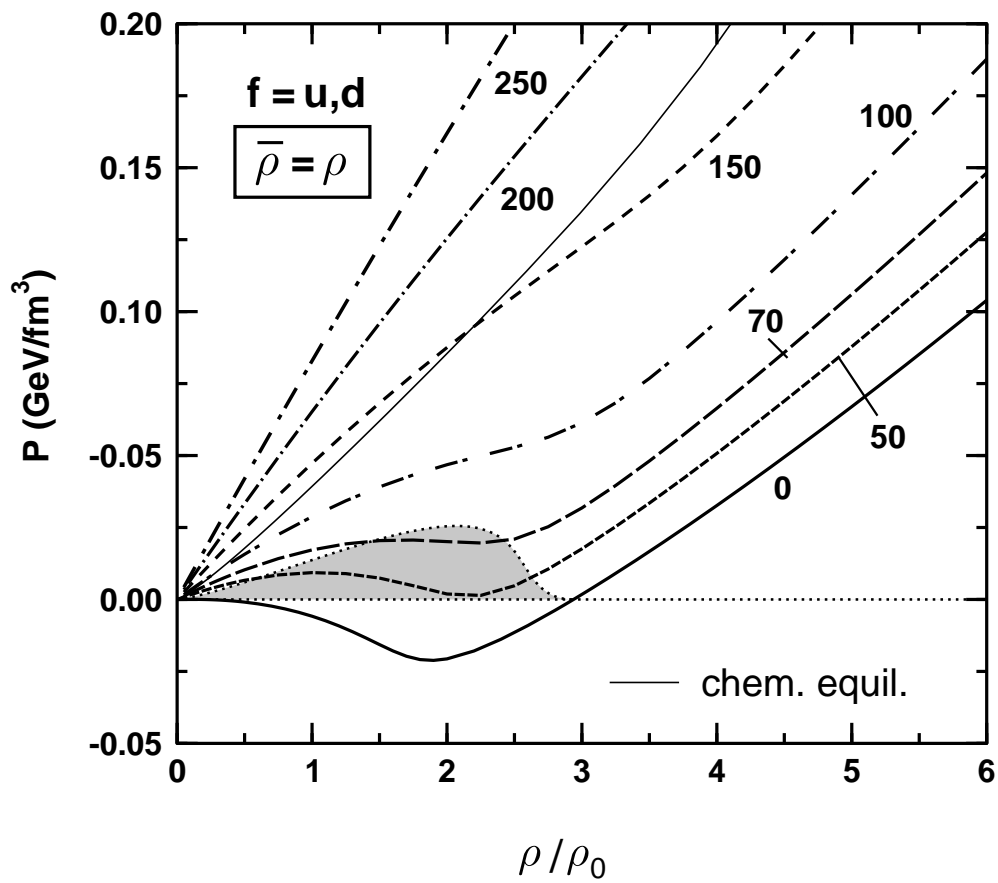


Fig. 9



(b)

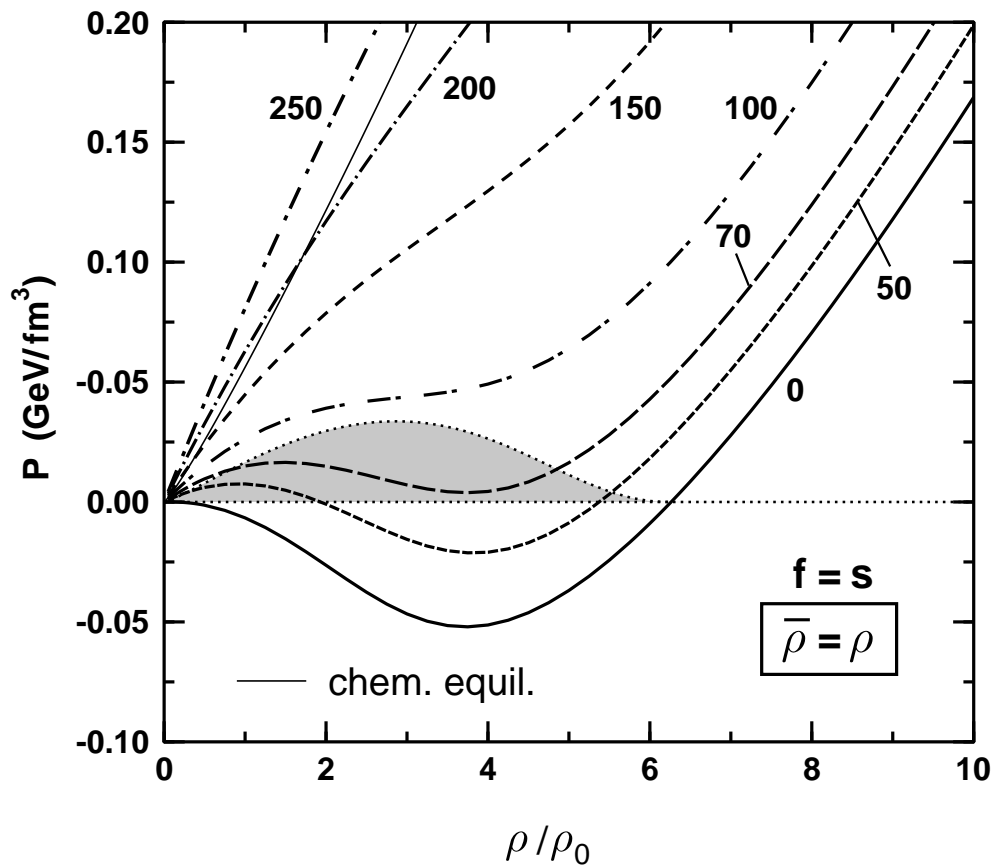


Fig. 10

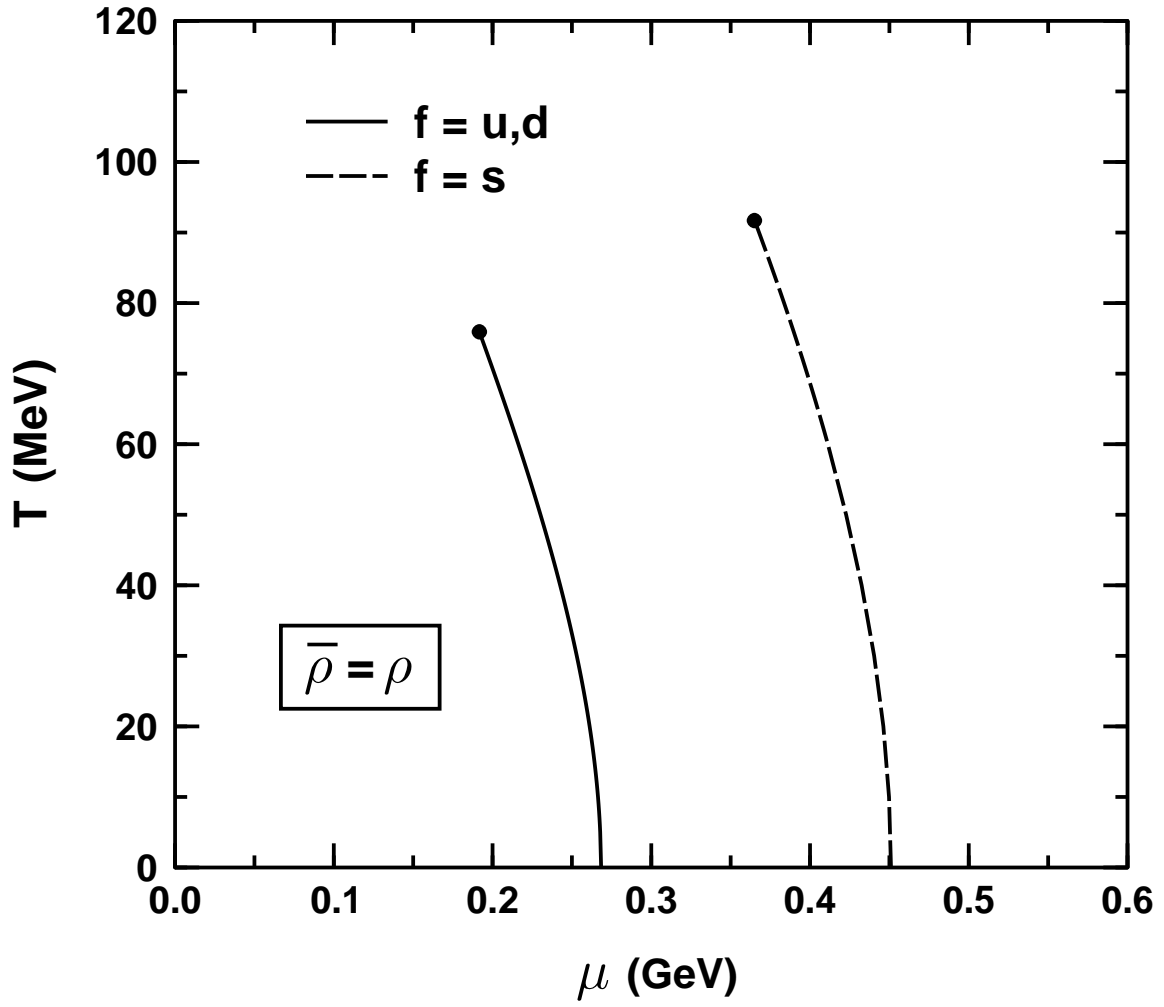
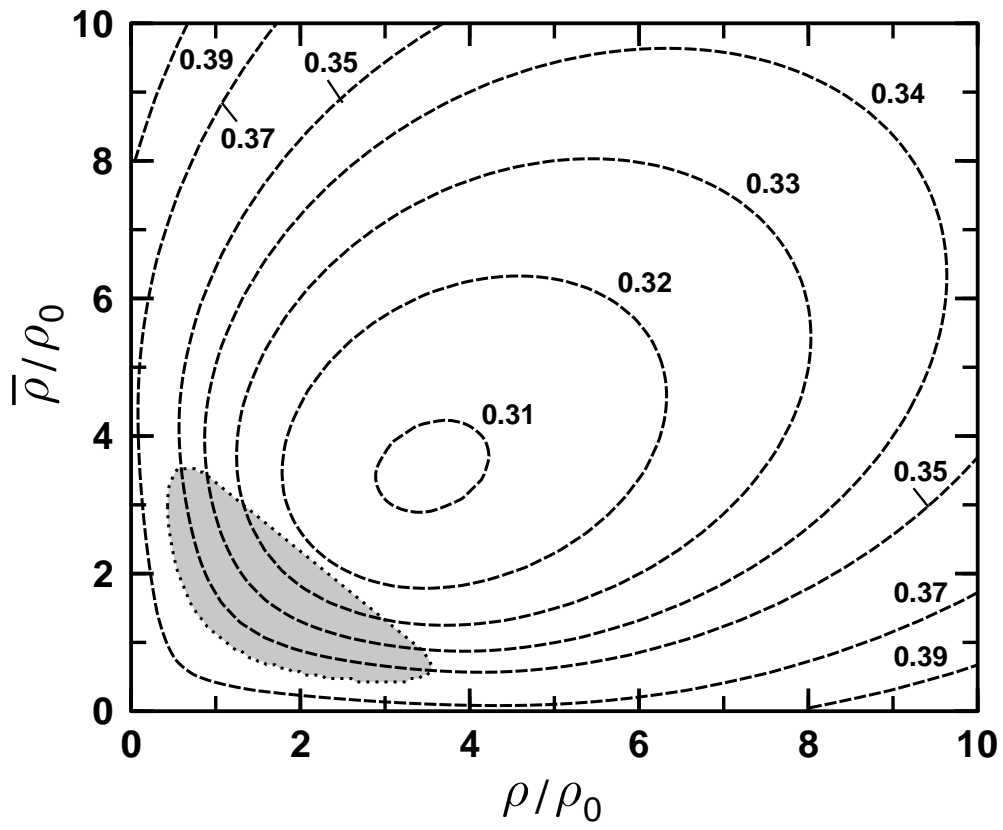


Fig. 11

$\epsilon (f = u, d), T = 50 \text{ MeV}$ (a)



$P (f = u, d), T = 50 \text{ MeV}$ (b)

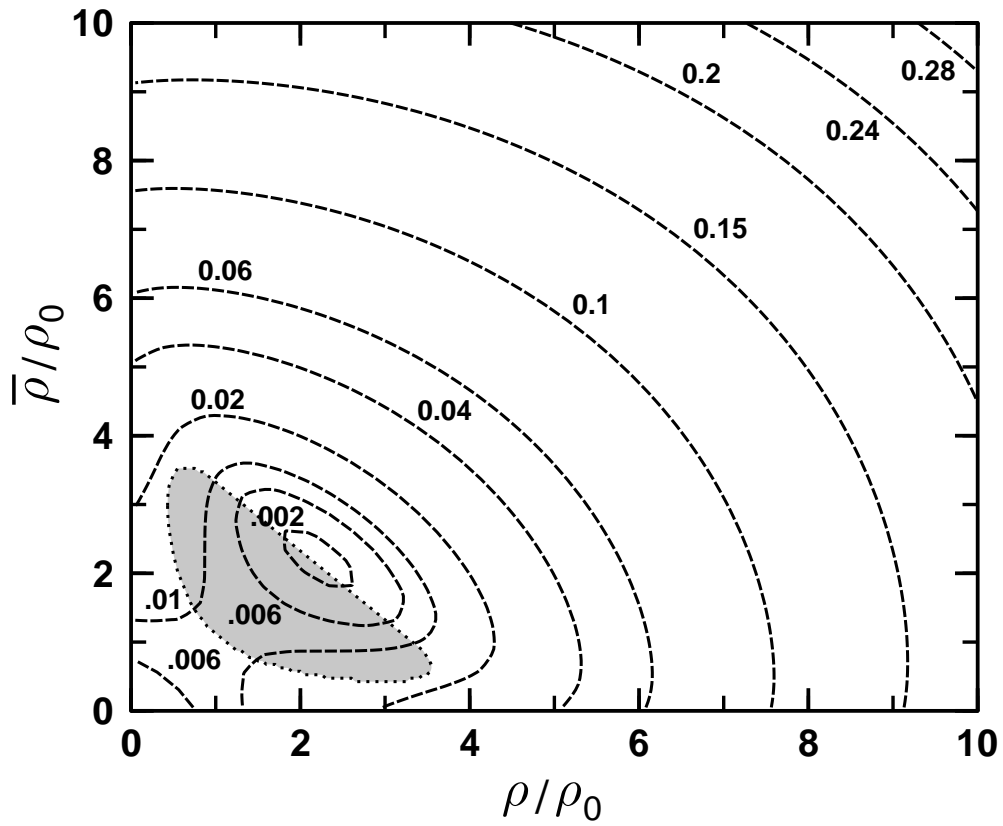


Fig. 12