Moments of event observable distributions and many-body correlations

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Abstract

We investigate event-by-event fluctuations for ensembles with non-fixed multiplicity. Moments of event observable distributions, like total energy distribution, total transverse momentum distribution, etc, are shown to be related to the multi-body correlations present in the system. For classical systems, these moments reduce in the absence of any correlations to the moments of particle inclusive momentum distribution. As a consequence, a zero value for the recently introduced Φ-variable is shown to indicate the vanishing of two-body correlations from one part, and of correlations between multiplicity and momentum distributions from the other part. It is often misunderstood as a measure of the degree of equilibration in the system.

Keywords: Event-by-event fluctuations; many-body correlations

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One of the main goals of relativistic heavy-ion collisions is the study of hadronic matter under extreme conditions of temperature and density. This offers the unique opportunity to investigate the possible phase transition of hadronic matter to quark-gluon plasma. As is well known from statistical mechanics [1,2], large fluctuations may occur at such a phase transition. These fluctuations are maximum at the critical point where large portions of the system become strongly correlated. Moreover, the investigation of these fluctuations is now possible with the advent of large acceptance detectors which allow for the first time an event-by-event analysis of the data. Already, event-by-event fluctuations of the transverse momentum distributions have been proposed to provide information about the heat capacity [3–6], or about a possible equilibration of the system [7–12]. On the experimental side, first preliminary result of the NA49 collaboration seem to indicate the absence of any non-statistical fluctuations in the mean transverse momentum distribution for Pb-Pb collisions at 160 AGeV [13].

In this letter, we propose a general method to investigate the presence of many-body correlations and of non-statistical fluctuations in momentum distributions of multiparticle events. The method applies to all p-p, p-A, or A-A collisions. The recently introduced Φ-variable [7] appears to be one of the moments proposed to give evidence for the presence or no of these correlations.

Consider the global observable defined for each event by:

$$Z = \sum_{i=1}^{N} y(\vec{p_i}), \tag{1}$$

where N indicates the multiplicity of the event considered and $y(\vec{p_i})$ is any function which depends of the momentum of particle i in the event. This quantity could be for instance the energy, the transverse momentum, etc. We are interested by the fluctuations of this global observable from event to event and in particular by the moments

$$\langle Z^k \rangle = \frac{1}{M} \sum_{j=1}^{M} \left[\sum_{i=1}^{N_j} y(\vec{p_i}) \right]^k , \tag{2}$$

where M is the total number of events and N_j indicates the multiplicity of event j.

Consider now the N-body distribution function $f_N(N, \vec{p_1}, \cdots, \vec{p_N})$ which gives the probability for a system of N particles where particle 1 has a momentum $\vec{p_1}$, particle 2 a momentum $\vec{p_2}$, and so on. Since we want to describe systems with different multiplicity, the distribution function $f_N(N, \vec{p_1}, \cdots, \vec{p_N})$ may as well depend on the multiplicity N. It is defined such that

$$\int d\vec{p_1} \cdots d\vec{p_N}\ f_N(N, \vec{p_1}, \cdots, \vec{p_N}) = P(N), \tag{3}$$

where $P(N)$, the probability of finding the system with exactly N particles regardless of their momenta, is normalized according to :

$$\sum_{N=0}^{\infty} P(N) = 1. \tag{4}$$

The reduced s-body distribution functions ($s < N$) for a system of indistinguishable particles is given by [1,2]:

$$f_s(N,\vec{p_1},\cdots,\vec{p_s}) = \frac{N!}{(N-s)!}\int d\vec{p_{s+1}}\cdots d\vec{p_N}\ f_N(N,\vec{p_1},\cdots,\vec{p_N}). \tag{5}$$

From the above definitions, we have:

$$\int d\vec{p_1}\cdots d\vec{p_s}\ f_s(N,\vec{p_1},\cdots,\vec{p_s}) = \frac{N!}{(N-s)!}P(N). \tag{6}$$

After these definitions, the moments of the event variable Z are defined as:

$$\langle Z^k\rangle = \sum_{N=0}^{\infty}\int d\vec{p_1}\cdots d\vec{p_N}\ \left[\sum_{i=1}^{N} y(\vec{p_i})\right]^k f_N(N,\vec{p_1},\cdots,\vec{p_N}), \tag{7}$$

and in particular,

$$\langle Z\rangle = \sum_{N=0}^{\infty}\int d\vec{p_1}\ y(\vec{p_1})\ f_1(N,\vec{p_1}); \tag{8}$$

$$\langle Z^2\rangle = \sum_{N=0}^{\infty}\left[\int d\vec{p_1}\ y^2(\vec{p_1})\ f_1(N,\vec{p_1}) + \int d\vec{p_1}d\vec{p_2}\ y(\vec{p_1})y(\vec{p_2})\ f_2(N,\vec{p_1},\vec{p_2})\right]; \tag{9}$$

$$\begin{aligned}\langle Z^3\rangle = \sum_{N=0}^{\infty}\Bigg[\int d\vec{p_1}\ y^3(\vec{p_1})\ f_1(N,\vec{p_1}) &+ \int d\vec{p_1}d\vec{p_2}\ y^2(\vec{p_1})y(\vec{p_2})\ f_2(N,\vec{p_1},\vec{p_2}) \\ &+ \int d\vec{p_1}d\vec{p_2}d\vec{p_3}\ y(\vec{p_1})y(\vec{p_2})y(\vec{p_3})\ f_3(N,\vec{p_1},\vec{p_2},\vec{p_3})\Bigg],\end{aligned} \tag{10}$$

and so on for the higher moments. One sees that the fluctuations of the event observable Z are related to the higher n-body correlations; the second moment is related to 2-body correlations, the third moment to 2- and 3-body correlations and so on. Note also that these moments are related to the possible correlation of the multiplicity of particles to the s-body momentum distributions $f_s(N,\vec{p_1},\cdots,\vec{p_s})$[1]. This result is similar to that obtained in [14].

Let us now answer the following question: how do the above defined moments of the event variable Z reduce in the absence of any correlation? If no correlations are present in the system, the s-body distribution functions, consistent with Eqs.(3-6), read:

$$f_s(N,\vec{p_1},\cdots,\vec{p_s}) = \frac{N!}{(N-s)!}P(N)\ \tilde{f}_1(\vec{p_1})\cdots\tilde{f}_1(\vec{p_s}), \tag{11}$$

[1]It seems that what makes the s-body momentum distributions possibly depend on the particle multiplicity are precisely the multi-body correlations. In the absence of correlations, the many-body distribution functions consist of a product of one-body distribution functions. In this case, every particle in the system does not feel the presence of the other particles. Its distribution function should not then depend on whether there is only one particle or many of them, hence it should not depend on the multiplicity of particles in the system.

with

$$\int d\vec{p}\ \widetilde{f}_1(\vec{p}) = 1. \tag{12}$$

Eqs.(8-10) reduce then to

$$\langle Z\rangle = \langle N\rangle \int d\vec{p}\ y(\vec{p})\ \widetilde{f}_1(\vec{p}); \tag{13}$$

$$\langle Z^2\rangle = \langle N\rangle \int d\vec{p}\ y^2(\vec{p})\ \widetilde{f}_1(\vec{p}) + \Big(\langle N^2\rangle - \langle N\rangle\Big)\left[\int d\vec{p}\ y(\vec{p})\ \widetilde{f}_1(\vec{p})\right]^2; \tag{14}$$

$$\begin{aligned}\langle Z^3\rangle = \langle N\rangle \int d\vec{p}\ y^3(\vec{p})\ \widetilde{f}_1(\vec{p}) + \Big(\langle N^2\rangle - \langle N\rangle\Big)\left[\int d\vec{p}\ y^2(\vec{p})\ \widetilde{f}_1(\vec{p})\right]\left[\int d\vec{p}\ y(\vec{p})\ \widetilde{f}_1(\vec{p})\right] \\ + \Big(\langle N^3\rangle - 3\langle N^2\rangle + 2\langle N\rangle\Big)\left[\int d\vec{p}\ y(\vec{p})\ \widetilde{f}_1(\vec{p})\right]^3,\end{aligned} \tag{15}$$

where

$$\langle N^k\rangle = \sum_{N=0}^{\infty} N^k\ P(N). \tag{16}$$

1. Note that in the absence of correlations, the distribution function $\widetilde{f}_1(\vec{p})$ coincides with the inclusive one-particle distribution function. Indeed, the inclusive one-particle distribution function is defined[2] as:

$$f^{incl}(\vec{p}) = \frac{1}{\langle N\rangle}\sum_{N=0}^{\infty} f_1(N,\vec{p}), \tag{17}$$

which reduces in the absence of correlations to

$$f^{incl}(\vec{p}) = \frac{1}{\langle N\rangle}\sum_{N=0}^{\infty} NP(N)\ \widetilde{f}_1(\vec{p}) \equiv \widetilde{f}_1(\vec{p}). \tag{18}$$

2. Note also that by a judicious choice of the particle variable $y(\vec{p})$ in such a way that in the absence of correlations, the mean value of $y(\vec{p})$ vanishes,

$$\int d\vec{p}\ y(\vec{p})\ \widetilde{f}_1(\vec{p}) = 0 \tag{19}$$

[2]The inclusive particle distribution function is defined such as the average value of a given particle observable O is given by: $\langle O\rangle = \frac{1}{N_{tot}}\sum_{i=1}^{N_{tot}} O(\vec{p_i}) = \frac{1}{N_{tot}}\sum_{i=1}^{M}\sum_{j=1}^{N_i} O(\vec{p_{i,j}})$ where M is the number of events and $\vec{p_{i,j}}$ is the momentum of particle j in event i. N_{tot} is the total number of particles in all events; it is given by $N_{tot} = \sum_{i=1}^{M} N_i = M\langle N\rangle$ with $\langle N\rangle = \frac{1}{M}\sum_{i=1}^{M} N_i$. One obtains then $\langle O\rangle = \frac{M}{N_{tot}}\frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{N_i} O(\vec{p_{i,j}}) = \frac{1}{\langle N\rangle}\sum_{N=0}^{\infty}\int d\vec{p}\ O(\vec{p})\ f_1(N,\vec{p_1})$ (see Eq.(8)).

the moments of the event observable Z will be exactly proportional to the moments of the particle observable $y(\vec{p})$. A good choice of the particle variable $y(\vec{p})$ is:

$$y(\vec{p}) = x(\vec{p}) - \bar{x} \tag{20}$$

where $x(\vec{p})$ is any function of momentum $\vec{p}$ (kinetic-, transverse-energy, transverse momentum, ... etc) and

$$\bar{x} = \int d\vec{p}\, x(\vec{p})\ f^{incl}(\vec{p}) \tag{21}$$

Note that with this choice, and according to Eqs.(8,17), the average value of Z is always zero, even in the presence of strong correlations.

It appears then that, in the absence of any correlations in the system and by a judicious choice of the particle variable $y(\vec{p})$, the moments of the event variable Z are exactly proportional to the inclusive moments of the particle variable $y(\vec{p})$:

$$\langle Z \rangle = \langle N \rangle \int d\vec{p}\ y(\vec{p})\ f^{incl}(\vec{p}) \equiv 0 \tag{22}$$

$$\langle Z^2 \rangle = \langle N \rangle \int d\vec{p}\ y^2(\vec{p})\ f^{incl}(\vec{p}) \tag{23}$$

$$\langle Z^3 \rangle = \langle N \rangle \int d\vec{p}\ y^3(\vec{p})\ f^{incl}(\vec{p}) \tag{24}$$

The proportionality factor is the average number of particles $\langle N \rangle$. If multi-body correlations are present in the system, Eqs.(23,24) do not hold any longer. A non-zero value for the quantities

$$\frac{\langle Z^k \rangle}{\langle N \rangle} - \bar{y^k} \qquad\qquad (k > 1) \tag{25}$$

with

$$\bar{y^k} = \int d\vec{p}\ y^k(\vec{p})\ f^{incl}(\vec{p}), \tag{26}$$

would then indicate the presence of strong many-body correlations and non-statistical fluctuations in the system. A non-vanishing value of, e.g., $\frac{\langle Z^2 \rangle}{\langle N \rangle} - \bar{y^2}$ indicates the presence of two-body correlations from one part, and of possible correlations between the particle multiplicity and the momentum distributions from the other part. The absence of correlations between the particle multiplicity and the momentum distributions alone does not necessarily imply a zero value for this quantity [7,12]. The generalization of this method to different particle distributions, as for instance hadronic type distributions [8], is straightforward. In this case, the N-particle momentum distributions $f_N(N, \vec{p_1}, \cdots, \vec{p_N})$ are replaced by the N-particle hadronic-type distribution functions $f_N(N, h_1, \cdots, h_N)$, where h_i indicates the hadronic-type of particle i, with the normalization

$$\sum_{h_1=1}^{N_h} \cdots \sum_{h_N=1}^{N_h} f_N(N, h_1, \cdots, h_N) = P(N). \tag{27}$$

Here N_h indicates the number of all possible hadron types. The event observable is defined in this case as

$$Z = \sum_{i=1}^{N} y(h_i), \tag{28}$$

where $y(h_i)$ is any function of the hadronic type of particle i. The vanishing of the quantities in Eq.(25) would indicate in this case an uncorrelated hadron-type production in the system.

A remark here is in order: There is a common misinterpretation of the quantity $\Phi = \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\bar{y^2}}$: it has been argued that a zero-value of this quantity would indicate an equilibration (thermal or chemical) of the system [8,11–13]. We have shown that such a zero value of Φ merely indicates the absence of 2-body correlations and of correlations of the particle multiplicity with momentum distributions. No specific form for the distribution function was assumed. It is true that a possible equilibration of the system implies a completely uncorrelated system and gives a zero-value for this quantity and all higher moments, but a zero value for Φ does not necessarily imply a complete equilibration of the system.

Note that the derivation of the quantities in Eq.(25) was done for a system of classical particles. For quantum systems, the s-body distribution functions (Wigner functions) can not be written as a product of uncorrelated one-body distribution functions as in Eq.(11). It is clear that the only source of correlations in a classical system is the existence of interactions between the particles. However, in a quantum system, there exists another source of correlations: the existence of quantum-statistical boson or fermion constraints. These are present even in an ideal gas of non-interacting particles. Due to the presence of these minimal correlations coming from the quantum nature (bosonic or fermionic) of the particles, the moments of the event observable Z do not reduce for a quantum system to the moments of inclusive particle momentum distributions as in the classical case (Eqs.(22-24)), and the quantities in Eq.(25) will not vanish, even for a gas of independent particles [10].

In conclusion, we have investigated the moments of event observable distributions for systems with non-fixed multiplicity. These moments are shown to be related to the higher many-body correlations. In the absence of any correlations, these moments reduce for classical systems to the moments of inclusive particle momentum distribution. Moreover, we have shown that a non-zero value for the quantities defined in Eq.(25) indicates the presence of many-body correlations and non-statistical fluctuations in the momentum distributions of multiparticle processes. We have also shown that the vanishing of the Φ-variable does not necessarily indicate an equilibration (thermal or chemical) of the system.

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