FORMATION OF SMALL-SCALE STRUCTURE IN SUSY CDM

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The lightest supersymmetric particle, most likely the lightest neutralino, is one of the most prominent particle candidates for cold dark matter (CDM). We show that the primordial spectrum of density fluctuations in neutralino CDM has a sharp cut-off, induced by two different damping mechanisms. During the kinetic decoupling of neutralinos, non-equilibrium processes constitute viscosity effects, which damp or even absorb density perturbations in CDM. After the last scattering of neutralinos, free streaming induces neutralino flows from overdense to underdense regions of space. Both damping mechanisms together define a minimal mass scale for perturbations in neutralino CDM, before the inhomogeneities enter the non-linear epoch of structure formation. We find that the very first gravitationally bound neutralino clouds ought to have masses above $10^{-6}M_\odot$, which is six orders of magnitude above the mass of possible axion miniclusters.

1. Introduction

Recent measurements support the idea that there is a significant amount of cold dark matter (CDM) in the Universe. An analysis of the temperature anisotropies in the cosmic microwave background (CMB) gives for the CDM mass density $\Omega_{\rm cdm}h^2 = 0.13^{+0.03}_{-0.02}$ (all data with weak h-prior)[1], while from the same analysis the mass density of the baryons is much smaller $\Omega_{\rm b}h^2 = 0.023^{+0.003}_{-0.003}$. The latter is consistent with the measurement of the primordial deuterium abundance in high-redshift hydrogen clouds, which gives the most precise determination of the baryon density[2], $\Omega_{\rm b}h^2 = 0.0205 \pm 0.0018$. Large galaxy redshift surveys support the CMB measurements: from the analysis of 160,000 galaxies Percival et al[3] find $(\Omega_{\rm b} + \Omega_{\rm cdm})h = 0.20 \pm 0.03$ and $\Omega_{\rm b}/(\Omega_{\rm b} + \Omega_{\rm cdm}) = 0.15 \pm 0.07$. Taking various observations together, Turner[4] estimates that the non-baryonic mass in the Universe, which is just the mass of CDM, is given by $\Omega_{\rm cdm} = 0.29 \pm 0.04$.

The characteristic feature of CDM is its non-relativistic equation of state at the time when the Universe contains enough matter within one Hubble volume to form one galaxy. The two leading particle candidates for CDM are the axion, a very light particle with a mass in the range $m_a \sim (10^{-6} - 10^{-5})$ eV, which is cold since it has been produced in a Bose condensate, and the neutralino, a heavy particle with a mass $M_{\tilde{\chi}} \sim (50-500)$ GeV, which is cold by virtue of its large mass. In the constrained minimal supersymmetric extension of the standard model (MSSM), the lightest neutralino is most likely the bino.

For almost 20 years there has been a strong working hypothesis: axion CDM and neutralino CDM cannot be distinguished by purely cosmological observations because both particle candidates interact only via gravity, as far as cosmology goes. This fact has made the study of structure formation on large scales > 1 Mpc simple—the microphysics of CDM is irrelevant. However, at the galactic scale and below, various particle candidates might be distinguishable. To learn more about the nature of CDM, we study the small-scale structure of neutralino CDM with emphasis on the very first, purely gravitationally bound, neutralino clouds (for details, see our recent work[5]).

2. Chemical and kinetic decoupling

There are two distinct temperature scales for CDM particles that are massive and weakly interacting and obey a thermal history. For temperatures $T > T_{\rm cd}$, binos are kept in chemical equilibrium with all fermions in the heat bath via the annihilation processes $\widetilde{\chi} + \widetilde{\chi} \leftrightarrow \overline{F} + F$, at the rate

$$\Gamma_{\rm ann} \approx 10^{-3}\, M_{\widetilde{\chi}}\, \frac{{M_{\widetilde{\chi}}}^4}{\left({M_{\widetilde{F}}}^2 + {M_{\widetilde{\chi}}}^2\right)^2}\, \frac{{M_{\widetilde{F}}}^4 + {M_{\widetilde{\chi}}}^4}{\left({M_{\widetilde{F}}}^2 + {M_{\widetilde{\chi}}}^2\right)^2}\, x^{-\frac{5}{2}}\, {\rm e}^{-x} \,. \tag{1}$$

Here, $x = M_{\widetilde{\chi}}/T$ and $M_{\widetilde{F}}$ denotes the universal sfermion mass. The numerical prefactor is the effective neutralino–fermion coupling, the second factor shows the dependence on the MSSM parameters and the x-dependent factor gives the temperature dependence in units of the neutralino mass and is proportional to the bino number density. Chemical decoupling of binos happens at the temperature $T_{\rm cd} = M_{\widetilde{\chi}}/x_{\rm cd}$, when the neutralino annihilation rate becomes comparable to the Hubble rate:

$$x_{\rm cd} \approx \ln \left[10^{-4}\, \frac{M_{\rm Pl} \left({M_{\widetilde{F}}}^4 + {M_{\widetilde{\chi}}}^4\right) {M_{\widetilde{\chi}}}^3}{\left({M_{\widetilde{F}}}^2 + {M_{\widetilde{\chi}}}^2\right)^4} \right] \,. \tag{2}$$

Scanning the MSSM parameter space, we typically find $x_{\rm cd} \approx 22$, see Figure 1. The chemical decoupling temperature $T_{\rm cd}$ increases with increasing bino mass, since the neutralino number density is suppressed with $\exp(-M_{\widetilde{\chi}}/T)$. As a consequence, the annihilation rate approaches the Hubble rate faster for larger bino masses. For a fixed bino mass, $T_{\rm cd}$ increases for an increasing sfermion mass, since the interaction range decreases. (Note that this discussion is not correct on a quantitative level, since we neglect here important effects like co-annihilation, in order to render an analytic discussion possible.)

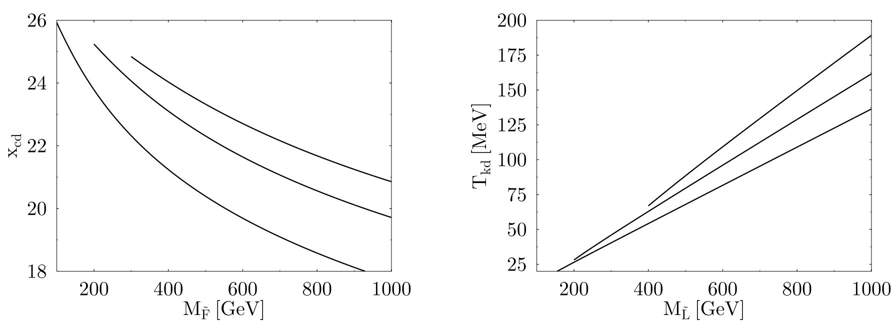

Figure 1. Chemical and kinetic decoupling as a function of the universal sfermion (slepton) mass for bino masses $M_{\widetilde{\chi}} \in \{50, 100, 200\}$ GeV (from bottom to top).

For temperatures $T_{\rm kd} < T < T_{\rm cd}$ neutralinos are in local thermal equilibrium (lte) with all fermions of the heat bath. Equilibrium is maintained due to elastic scattering processes $\widetilde{\chi} + \{\overline{F}, F\} \leftrightarrow \widetilde{\chi} + \{\overline{F}, F\}$ at the rate

$$\Gamma_{\rm el} \approx 10^{-2}\, M_{\widetilde{\chi}}\, \frac{{M_{\widetilde{\chi}}}^4}{\left({M_{\widetilde{\rm F}}}^2 - M_{\widetilde{\chi}}^2\right)^2}\, x^{-5}\, . \qquad (3)$$

Kinetic decoupling of binos takes place when the relaxation time $\tau_{\rm relax}$ in the bino system exceeds the Hubble time. The relaxation time $\tau_{\rm relax} = N\tau_{\rm coll}$ differs from the collision time $\tau_{\rm coll} = 1/\Gamma_{\rm el}$ by the number $N(T)$ of elastic scatterings needed to keep or establish lte in the bino system. This number is given by the relative momentum transfer per elastic scattering, $N(T) = (\Delta P_{\widetilde{\chi}}/P_{\widetilde{\chi}})^{-1} \approx M_{\widetilde{\chi}}/T$, with $\Delta P_{\widetilde{\chi}} = \sqrt{< t >} \approx 3T/\sqrt{2}$ denoting the rms of the Mandelstam variable t. Kinetic decoupling happens at

$$T_{\rm kd} \approx \left[10^2 \, \frac{M_{\widetilde{\chi}}{}^{\alpha}\left(M_{\widetilde{F}}{}^2 - M_{\widetilde{\chi}}{}^2\right)^2}{M_{\rm Pl}}\right]^{\frac{1}{3+\alpha}} , \qquad (4)$$

with $\alpha = 0$ if mistakenly $\tau_{\rm coll} \equiv \tau_{\rm relax}$ is assumed and $\alpha = 1$ when the number of scatterings N is taken into account. We typically find $T_{\rm kd} = (10-100)$ MeV, see Figure 1. The kinetic decoupling temperature is increasing with increasing bino mass because the momentum transfer to the heat bath is decreasing. As a consequence the number of elastic scattering processes needed to keep or establish lte is increasing.

We find $T_{\rm cd} \gg T_{\rm kd}$, because $\Gamma_{\rm ann}/\Gamma_{\rm el} \approx 10^{-1} x^{5/2} \exp{(-x)} \ll 1$ for $T < T_{\rm cd} < M_{\widetilde{\chi}}$; the number density of neutralinos is Boltzmann suppressed with respect to the number densities of the relativistic fermions. This is the reason for the temperature hierarchy $T_{\rm cd} \gg T_{\rm kd} > T_{\rm ls}$, with $T_{\rm ls}$ denoting the temperature at which binos scatter for the last time.

3. Local transport coefficients

During the process of kinetic decoupling, non-equilibrium processes constitute themselves as viscosity phenomena in the bino system. We therefore use hydrodynamics for the description. For $T \gg T_{\rm cd}$, CDM and the heat bath can be described by a single ideal fluid. For $T_{\rm cd} > T > T_{\rm kd}$ the CDM fluid is strongly coupled to the radiation fluid (rad), which keeps CDM in lte. Around $T_{\rm kd}$, the CDM fluid decouples from the radiation fluid, in which lte persists, since $\Omega_{\rm cdm} = (a/a_{\rm eq})\Omega_{\rm rad} \ll \Omega_{\rm rad}$ for $T \gg T_{\rm eq}$.

The resulting non-equilibrium processes in the CDM fluid can be taken into account by additional Lorentz tensors $\mathbf{J}^{(1)}$ and $\mathbf{T}^{(1)}$ in the current density $\mathbf{J}_{\rm cdm}$ and the energy momentum tensor $\mathbf{T}_{\rm cdm}$ of the CDM fluid. We fix the ambiguities in the relativistic description of imperfect fluids by demanding $\mathbf{J}^{(1)} \equiv \mathbf{0}$ and $\mathbf{T}^{(1)} = \zeta \, \mathbf{h}\nabla \cdot \mathbf{U} + \eta \, \mathbf{W}^{(\rm T)} + \chi \, {\rm Sym}\left(\mathbf{U} \otimes \mathbf{Q}^{(\rm T)}\right)$. $\mathbf{U}$ denotes the adiabatic velocity field and $\mathbf{h}$ projects on the hypersurface perpendicular to it. The first term is the bulk viscosity, describing the flow of $\mathbf{U}$ in the hypersurface defined by $\mathbf{h}$. The second term is the shear viscosity and describes the bending of $\mathbf{U}$ in the direction perpendicular to the adiabatic current. The last term is the heat conduction.

The strength of the dissipative processes is given by the local transport coefficients ζ, η and χ. An efficient method for calculating these coefficients was proposed by Silk[6]. It has been applied in great detail to the case of a relativistic fluid by Weinberg[7]. A generalisation to an arbitrary equation of state can be found in our recent work[5].

The main idea is to compare the Lorentz tensors in the hydrodynamical description with the Lorentz tensors in the kinetic description. In the kinetic description we make the ansatz $F^{(1)}(\omega, n, x) = \mathbf{A}(\omega, x) + \mathbf{B}(\omega, x) \cdot \mathbf{n} + \mathbf{C}(\omega, x) \cdot (\mathbf{n} \otimes \mathbf{n} + 1/3\, \mathbf{h})$ for the CDM phase-space distribution describing the non-equilibrium state of the bino system. Here, ω denotes the projection of the momentum on the velocity field and $\mathbf{n}$ denotes a vector perpendicular to the adiabatic current. We calculate[5] the coefficients in the expansion of $F^{(1)}$ into irreducible polynoms in $\mathbf{n}$ and $\mathbf{h}$. For the tensor structure we find $\mathbf{A} \propto \nabla \cdot \mathbf{V}$, $\mathbf{B} \propto \mathbf{Q}^{(\mathrm{T})}$ and $\mathbf{C} \propto \mathbf{W}^{(\mathrm{T})}$. The scalar $\mathbf{A}$ generates dissipative processes, which are not perpendicular to the adiabatic current. This contribution deserves special care.

Calculating the energy momentum tensor in the kinetic description and comparing it with $\mathbf{T}^{(1)}$, we find the local transport coefficients $\zeta = 5\rho_{\mathrm{cdm}}/3\Gamma_{\mathrm{el}}$, $\eta = \rho_{\mathrm{cdm}}/\Gamma_{\mathrm{el}}$ and $\chi \equiv 0$ in first order in $1/\Gamma_{\mathrm{el}}$ and $1/x$. All coefficients are decreasing with an increasing elastic scattering rate, since elastic scattering allows the transfer of derivations from lte to the heat bath. Heat conduction is a subdominant process for non-relativistic particles and vanishes in leading order.

4. Acoustic absorption

The dissipative processes presented in the last section transfer energy and momentum in the direction perpendicular to the adiabatic flow. This provides a damping mechanism for acoustic perturbations in CDM[7]. The damping of density inhomogeneities is given by $\mathrm{Im}(\omega)$ as calculated in linear perturbation theory for relativistic hydrodynamics. For an acoustic wave with wave number k we find

$$\frac{\delta\rho_{\widetilde{\chi}}}{\rho_{\widetilde{\chi}}} \propto \exp\left[-\frac{3}{2}\int_0^{t(T_{\mathrm{kd}})} \mathrm{d}t\, \frac{k^2}{\Gamma_{\mathrm{el}}}\right] = \exp\left[\left(-\frac{M_{\mathrm{d}}}{M}\right)^{2/3}\right], \qquad (5)$$

where $M_{\mathrm{d}} \approx 3 \cdot 10^{-8} (\mathrm{GeV}^2/M_{\widetilde{\chi}} T_{\mathrm{kd}})^{3/2} (\Omega_{\widetilde{\chi}} h^2) M_\odot$ is the characteristic damping scale. We find $M_{\mathrm{d}} \approx 10^{-9} M_\odot$, see Figure 2. Thus, only acoustic perturbations with masses $M > M_{\mathrm{d}}$, contained in the overdense volume, are not absorbed and enter the free streaming regime.

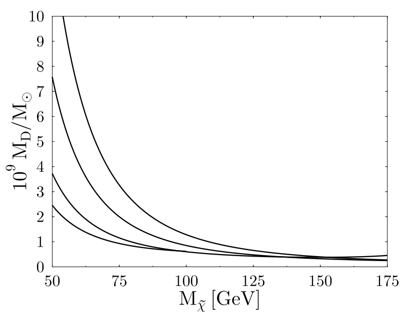

Figure 2. Acoustic damping scale as a function of the bino mass for sfermion masses $M_{\widetilde{F}} \in \{150, 200, 300, 400\}$ GeV (from bottom to top).

5. Free streaming

For temperatures $T < T_{\rm ls}$, CDM does freely streaming on geodesics. As a consequence, CDM propagates from overdense to underdense regions, thus smearing out local inhomogeneities. We find for the induced damping scale $M_{\rm fs}(a) = M_{\rm d} \ln^3(a/a_{\rm ls})$. Free streaming becomes the leading damping mechanism once the universe has doubled its size after last scattering. It is interesting to calculate the free streaming scale at the time of matter-radiation equality, when CDM density perturbations start to grow linearly with the expansion factor. We typically find $M_{\rm fs}(a_{\rm eq}) \approx 10^{-6} M_\odot$.

6. Conclusions

We have shown that collisional damping and free streaming smear out all power of primordial density inhomogeneities in bino CDM below $10^{-6} M_\odot$ by the time of matter-radiation equality. This is in striking contrast to claims in the literature[8] that the minimal mass for the first purely gravitationally bound neutralino cloud is $10^{-13}\,(150\ {\rm GeV}/M_{\widetilde{\chi}})^3\ M_\odot$. The huge difference to our result stems from the assumption that chemical and kinetic decoupling happened simultaneously, which we proved to be wrong. We find instead that the very first bino objects have to have masses above $10^{-6} M_\odot$. This result is very robust with respect to the MSSM parameters. These bino clouds are very different from possible axion mini-clusters with typical masses[9] around $10^{-12} M_\odot$.

According to hierarchical structure formation these very first CDM clouds are supposed to merge and form larger objects. Large scale structure simulations show structure formation on all accessible scales down to the resolution of the simulation[10]. However, the dynamic range of todays simulations is not sufficient to deal with the very first CDM objects, so the fate of these objects is an open issue. A cloudy distribution of CDM in galactic halos would have important implications for direct[11] and indirect[12] searches for dark matter.

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