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Life Insurance Demand under Health Shock Risk

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Non-Technical Summary

We analyze a family's term life insurance demand in a life cycle portfolio choice model. A major source of risk for a family is the early death of the sole wage earner. To hedge this, the family in our model can contract a term life insurance. Most existing papers studying the life insurance demand consider short term contracts that can be bought or sold continuously which ensures an optimal insurance holding at each point in time. This simplification might crucially affect the results. Therefore, we focus on a more realistic model for the insurance contract. The family can choose between different long-term contracts that differ with respect to their insurance sum. The annual insurance premium includes fees for administrative costs and transaction costs. A belated change of the insurance is costly for the family and only possible as long as the insured person is younger than a specific age and healthy. The wage earner faces stochastic mortality risk with a jump component that we interpret as critical illness. Once the agent suffers from a critical illness, the family cannot change the insurance contract any more, the income of the family reduces, and the mortality risk increases. If the wage earner dies before the maturity of the insurance contract, the remaining family members receive a single, fixed payment of the insurance company. We use a German life table to calibrate the mortality process, German cancer data to calibrate the critical illness shock and data of the German life insurance industry to calibrate the insurance fees. The insurance premiums are calculated such that the contracts are actuarially fair.

The realistically modeled insurance induces new qualitative effects that are important for the optimal decisions over the life cycle. The long-term insurance contract amplifies the effect of negative labor income shocks, since in the undesired case of a negative labor income shock a premature termination of the contract or a reduction of the insurance sum leads to additional losses. In an already bad state, the family has problems to make the premium payments. Families with a lower income volatility have a significantly higher insurance demand. The amplifying effect also reduces the insurance demand of families that are more risk averse. In general, the families increase insurance protection over the life cycle. The long term contract design effect fades away as agents get older, since the contract duration and human wealth uncertainty reduce. Most importantly, young families do not buy any term life insurance. If an older agent suddenly dies, the accumulated financial wealth and contracted insurance ensures that the surviving family member can maintain their consumption level, although consumption growth is reduced. By contrast, an unexpected death in younger years leads to severe problems for the family. To avoid lifetime poverty of the remaining family members, a social security could mitigate those problems by making transfer payments. Of course, a more welfare optimal solution should provide an incentive for families to insure themselves, especially early in life where the potential gap is the biggest. We find that a high level of income, a high labor income volatility, large fees imposed by insurance companies and the presence of health shocks reduce the insurance demand of a family. Our results suggest that a contract with a variable insurance sum which is linked to the actual labor income evolution (like in practice often done in occupational pensions) could avoid the negative amplifying effects of labor income shocks.

To summarize, we find that a realistically modeled insurance contract crucially affects the results in a life cycle portfolio choice model since new qualitative effects appear. Especially, the long-term nature of the contract amplifies the effects of negative income shocks which is bad for young families and should be accounted for by insurance companies.

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Life Insurance Demand under Health Shock Risk

Abstract: This paper studies the life cycle consumption-investment-insurance problem of a family. The wage earner faces the risk of a health shock that significantly increases his probability of dying. The family can buy term life insurance with realistic features. In particular, the available contracts are long term so that decisions are sticky and can only be revised at significant costs. Furthermore, a revision is only possible as long as the insured person is healthy. A second important and realistic feature of our model is that the labor income of the wage earner is unspanned. We document that the combination of unspanned labor income and the stickiness of insurance decisions reduces the insurance demand significantly. This is because an income shock induces the need to reduce the insurance coverage, since premia become less affordable. Since such a reduction is costly and families anticipate these potential costs, they buy less protection at all ages. In particular, young families stay away from life insurance markets altogether.

Keywords: Health shocks, Portfolio choice, Term life insurance, Mortality risk, Labor income risk

JEL-Classification: D14, D91, G11, G22

1 Introduction

For most households, labor income is the essential source to finance lifetime consumption. Therefore, a potential income loss following an early death of the wage earner is a crucial risk. Consequently, a life insurance is of special importance to hedge future consumption of the remaining family members. Following Richard (1975) most studies simplify the insurance decision by including an instantaneous term insurance contract. However, in practice buying life insurance usually involves a long-term commitment and later changes are costly. Furthermore, health checks prevent agents from contracting an insurance if they already have a critical illness. Therefore, we study a life cycle problem where a family has access to a realistic term life insurance that is solely available as a long-term contract which can only be bought or sold at certain lump-sum costs.[1]

Another realistic feature of our model is that the family receives unspanned labor income earned by the head of the household. Since in practice agents cannot borrow against future income, we impose short-sale constraints that are binding, especially at young ages. In addition, we take a liquidity constraint into account so that the family's financial wealth has to stay non-negative at all points in time. Although papers studying life cycle problems without insurance decisions sometimes include unspanned labor income (see, e.g., Munk and Sørensen (2010)), this issue is usually disregarded in papers including insurance decisions. Additionally, we add another layer of incompleteness. In our model, the wage earner faces the risk of suffering from a health shock that we interpret as critical illness. After a health shock the family has no access to the insurance market any more, i.e. cannot buy new insurance or change existing contracts. Furthermore, the wage earner's probability of dying increases significantly. We calibrate the health shock and mortality process to cancer and mortality data.

The combination of these realistic features (long-term insurance contracts, transaction costs, unspanned labor income, short-sale and liquidity constraints, health shocks) distinguishes our model from the related literature discussed in Section 2. This combination generates interesting qualitative effects that are important for the optimal decisions of the family over the life cycle:

[1] Formally, we model the insurance decision as an impulse control problem.

The long-term nature of the insurance contract amplifies the effect of negative labor income shocks, since in the undesired case of a negative labor income shock a premature termination of the contract or a reduction of the insurance sum leads to additional losses. In an already bad state, the family might be worse off due to the stickiness of the insurance contract. Therefore, families with higher income uncertainty have significantly lower insurance demands. The amplifying effect also reduces the demands of families that are more risk averse and face labor income risk.

Most importantly, we find that younger families (head of household less than 30 years old) optimally stay away from life insurance markets and do not buy term life insurance at all. Therefore, an unexpected death in younger years leads to severe problems for the family. Our finding is in line with the low participant rates that are observed empirically.[2] However, it differs significantly from the results in frameworks that model life insurance decisions via an instantaneous contract instead of a long-term contract as in our paper. In these frameworks, the theoretically optimal participation rates are typically much higher.

Furthermore, we find that it is optimal for families to increase insurance protection over the life cycle. This is because the long-term contract design becomes less relevant as agents get older, since the contract duration and the uncertainty about human wealth goes down. Therefore, if an older wage earner suddenly dies, the accumulated financial wealth and existing insurance contracts ensure that surviving family members can maintain their standard of living, although consumption growth must be reduced. To summarize, our results suggest that a high level of income, a high labor income volatility, large fees imposed by insurance companies and the presence of health shocks reduce the insurance demand of a family.

The remainder of the paper is organized as follows. Section 2 provides an overview of the related literature. Section 3 introduces the model setup. Section 4 presents the calibration. Section 5 discusses our benchmark results. Section 6 provides robustness checks. Section 7 concludes.

[2] See, e.g., Hong and Ríos-Rull (2012) and the references therein.

2 Related Literature

The modern portfolio optimization literature starts with Merton (1969) and Merton (1971). In discrete time, Cocco, Gomes, and Maenhout (2005) numerically solve a realistically calibrated life cycle model with deterministic mortality risk. One of their main objects is to study the effect of unspanned labor income that is calibrated to US data. Munk and Sørensen (2010) solve a realistically calibrated life cycle model in continuous time with unspanned labor income, but without mortality risk. Their main focus is on analyzing the effect of unspanned labor income and a stochastic riskfree rate on consumption-investment decisions. They adapt the labor income calibration results from Cocco, Gomes, and Maenhout (2005) to a continuous-time framework and find that *unspanned* labor income significantly effects consumption-investment decisions.

Merton (1975) points out the importance of the risk of dying as a source of risk. Campbell (1980) studies the corresponding optimization problem. He considers a two-period model and introduces an insurance market to allow a family to hedge the risk that the wage earner dies. Analytically, he derives the optimal demand for insurance. Yaari (1965) analyzes the consumption decision of an agent when faced with longevity risk. Richard (1975) analytically solves a life cycle problem with deterministic labor income and a continuous instantaneous term insurance decision. He allows an uncertain time of death with deterministic distribution. Simple forms of an actuarially fair one-period life insurance contract are still widely used in the portfolio optimization literature. Pliska and Ye (2007) mathematically extend the model and analyze the effect of parameter choice on the life insurance demand. Closely related to our work is the paper Huang, Milevsky, and Wang (2008), who also consider the consumption-investment and life-insurance decision of a family with CRRA utility. They focus on the correlation between labor income and asset returns. Their main results are that life-insurance demand is insensitive to changing risk aversion and highly depends on labor income volatility. As the previous literature, they also model life insurance via a short-term contract and do not consider health shocks or stochastic mortality risk.

Recent papers building on the work of Richard (1975) use a continuous-time finite-state Markov chain approach to solve life cycle portfolio problems with insurance decisions analytically. Kraft and Steffensen (2008) focus on the consumption and insurance decision of a single person that faces the risk of dying and disability. Bruhn and Steffensen (2011) consider the consumption-investment and insurance decision of a two- and multi-person household. This strand of literature is able to provides analytical solutions of optimal insurance decisions, but relies on the assumptions that markets are complete and thus is not able to capture crucial realistic features such as unspanned labor income risk.

There are also recent papers studying life cycle problems of families that face mortality risk and can insure themselves via life insurance contracts. Love (2010) focuses on the effect of demographic shocks. In his model, the agent can exogenously get married, get divorced or have children. We adapt his calibration approach to capture the impact of the family size on the utility from consumption. His model involves a simple one-period term life insurance. Hubener, Maurer, and Rogalla (2013) analyze the optimal life insurance demand of retired couples. They model husband and wife via two separate mortality processes, but do not allow for changes in the family status (e.g. divorce). Since they consider only the retirement phase, they especially focus on annuities and disregard labor income. Hong and Ríos-Rull (2012) infer how individuals value consumption in different demographic stages using life insurance holdings by age, sex, and marital status. In particular, they estimate a consumption equivalence scale parameter.

There are also papers on life cycle problems with stochastic mortality risk. In a discrete-time setting, Cocco and Gomes (2012) include stochastic mortality risk in a realistically calibrated life cycle model which they solve numerically. They capture mortality risk by a Lee-Carter type model. The agent can invest in a riskless bond and in a longevity bond that is correlated with shocks in mortality rates. Furthermore, they allow the agent to choose the retirement date endogenously. Huang, Milevsky, and Salisbury (2012) analyze optimal consumption decisions analytically and compare results from a Yaari type model with a model allowing for stochastic mortality risk. In contrast our paper, stochastic mortality risk is modeled as a geometric Brownian motion. Koijen, Van Nieuwerburgh, and Yogo (2013) consider a life cycle problem where the probability of dying can have unsystematic jumps. They develop risk measures for life and health insurance products that pool the effects of several insurance products. In a discrete-time setting, they calculate the corresponding optimal results for their risk measures. They also compare the model implied risk measures with empirically derived values where their focus on agents that are older than 50 years. Their model also involves critical illness jumps. In contrast to our paper, they focus on the insurance implications, but do not consider unspanned labor income or stock market risk.

3 Model Setup

In this section, we present the model setup and describe the optimization problem of the family.

Financial Assets The agent can invest into two financial assets, but faces short-sale constraints. The assets are a risky stock (index) S and a riskfree bond B. The riskfree rate is denoted by r. The dynamics are given by

$$\begin{aligned} dS_t &= S_t \left[(r + \sigma_S \lambda) \; dt + \sigma_S \, dW_t^S \right], \\ dB_t &= B_t r \, dt \end{aligned}$$

with a constant market price of risk λ and stock market volatility σ_S. The process $W^S = \left(W_t^S\right)$ is a standard Brownian motion.

Biometric Risk The sole wage earner faces the risk of a health shock (e.g. cancer) and of a death shock. The state variable A defined by

$$A_t = \begin{cases} 1 & \text{alive and healthy at } t, \\ 2 & \text{alive but unhealthy at } t, \\ 3 & \text{dead at } t, \end{cases}$$

captures the current status of the wage earner. The random age of death is denoted by τ^D and is modeled as doubly stochastic stopping time with intensity $\pi(t, A)$, the so-called hazard rate of death, where $\pi(t, 3) = 0$. Formally, τ^D is the time of the first jump of the jump process $N^D = \left(N_t^D\right)$. The health shock jump process $N^H = \left(N_t^H\right)$ has an only time-dependent intensity $\kappa(t)$ while $A_t = 1$. The health shock is permanent so that agents cannot recover again. Unhealthy agents cannot face another health shock.[3] The time of a health shock is denoted by τ^H. If the agent does not experience a health shock during his lifetime, then τ^H is infinity.

Unspanned Labor Income The family receives an uncertain income stream denoted by Y. Its dynamics are influenced by the health status and age of the wage earner and are given by

$$\begin{aligned} dY_t = \mathbb{1}_{\{A_t=1,2\}} Y_t \left(\mu_Y(t)\, dt + \sigma_Y(t) \left(\rho(t)\, dW_t^S + \sqrt{1-\rho(t)^2}\, dW_t^Y\right)\right) + \mathbb{1}_{\{A_{t^-}=1\}} Y_{t^-}(p^{1,2}(t)-1) dN_t^H \\ + \mathbb{1}_{\{A_{t^-}=1\}} Y_{t^-}(p^{1,3}(t)-1) dN_t^D + \mathbb{1}_{\{A_{t^-}=2\}} Y_{t^-}(p^{2,3}(t)-1) dN_t^D, \end{aligned} \tag{3.1}$$

where $W^Y = \left(W_t^Y\right)$ is a standard Brownian motion, independent of W^S. This income stream is unspanned for two reasons: First, the Brownian motion W^Y cannot be hedged in the financial market. Second, the health shock N^H cannot be fully insured.

Furthermore, $p^{i,j}$ is the fraction of income that remains after a jump from state i to state j. We assume that the agent retires at the prespecified date T_R. The income process has a drift of μ_Y, a volatility of σ_Y and is correlated with the stock via ρ. Before retirement, the family's income is interpreted as labor income, whereas it is a pension after the retirement date. After the death of the wage earner, the income stream can be interpreted as widow's pension indexed by the salary upon death. If a critical illness shock occurs, the decreased income can be interpreted in the sense that the wage earner is forced to reduce work effort or increased medical expenses reduce the net income. Alternatively, it can be interpreted as a transfer income that the family receives from the government.

Insurance The family can buy a term life insurance to hedge the potential income loss resulting from the mortality risk of the wage earner. While the insurance contract is active, the insurance company pays the family a fixed payment I if the wage earner dies. The insurance offers a fixed set of contracts with specific payouts. The set of offered contracts is denoted by

$$\mathcal{I} = \{0, 50\,000, 100\,000, 150\,000, 200\,000, 300\,000, 500\,000, 750\,000, 1\,000\,000, 2\,000\,000\}. \tag{3.2}$$

[3] Our results hardly change if we allow for more than one health shock.

The family must pay a constant insurance premium $\iota(I)$ as long as the contract is active. When the agent changes the insurance sum of the contract, a lump-sum payment η is due. This payment takes the previous insurance sum, the new insurance sum and the age of the agent that determines the mortality pattern into account. The lump-sum payment ensures an actuarially fair new contract, but it also involves a fee. The contract can be changed as long as the wage earner is healthy and younger than T_C. The insurance contract expires at $\min(\tau^D, T_I)$, i.e. at the death of the insured person or at the maturity of the contract, T_I. In the first case, the insurance pays the insurance sum I, in the second case the insurance pays nothing. We assume that $T_C < T_I$, i.e. after T_I there is no insurance available any more.

Technically, the insurance decision can be characterized by an impulse control problem. The family chooses the intervention times ζ_i, $i \in \mathbb{N}$, and the intervention actions ω_i, $i \in \mathbb{N}$. The intervention can take place at time ζ_i if the wage earner is alive ($\zeta_i < \tau^D$), healthy ($A_{\zeta_i} = 1$), and in the insurance market ($\zeta_i \leq T_C$). A feasible intervention action requires that ω_i is chosen such that the new insurance sum is in the set of offered contracts, $I_{\zeta_i} \in \mathcal{I}$. We denote the set of possible interventions at ζ_i by

$$\mathcal{I}_{\zeta_i} = \{0 - I_{\zeta_i^-}, 50\,000 - I_{\zeta_i^-}, \ldots, 2\,000\,000 - I_{\zeta_i^-}\}.$$

Formally, the above statement can be expressed by the condition $\omega_i \in \mathcal{I}_{\zeta_i}$. At an intervention time, the family must pay the lump-sum payment $\eta(\zeta_i, \omega_i, I_{\zeta_i})$ that is a correction payment which makes the insurance contract actuarially fair and involves a fee. A detailed description is postponed to Section 4, see equation (4.4). Following the intervention, the family pays the new yearly premium $\iota(I_{\zeta_i})$ to maintain insurance protection until $\min(\tau^D, T_I)$.

Preferences The family has a fixed time horizon T and a power utility function given by

$$u(x, A) = \frac{\left(\frac{x}{\phi_A}\right)^{1-\gamma}}{1-\gamma}$$

with relative risk aversion γ. Here ϕ_A is a consumption scaling term that depends on the family size and for instance captures that two persons do not need twice as much consumption as a single person for the same utility level.[4] The family maximizes expected utility from intermediate consumption and terminal wealth given by

$$\mathrm{E}_{t,x,y,I,A}\left[\int_t^T \mathrm{e}^{-\delta(u-t)}\,\frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma}\,du+\varepsilon\,\mathrm{e}^{-\delta(T-t)}\,\frac{{X_T}^{1-\gamma}}{1-\gamma}\right]$$

with time preference rate δ and financial wealth X. The constant ε specifies the importance of the bequest motive.

Financial Wealth Dynamics The family chooses consumption c and the fraction θ invested in the risky asset. As long as the wage earner is healthy and young, the family also optimizes the insurance sum decision via the impulse control strategy (ζ_i,ω_i), $i\in\mathbb{N}$. The wealth dynamics follow

$$\begin{aligned}
dX_t &= X_t\left[(r+\theta_t\lambda\sigma_S)\;dt+\theta_t\sigma_S\,dW_t^S\right]+\left[Y_t-c_t-\mathbb{1}_{\{A_t=1,2\,\wedge\,t<T_I\}}\iota(I_t)\right]dt\\
&\quad+\mathbb{1}_{\{A_{t^-}=1,2\,\wedge\,t<T_I\}}I_{t^-}\,dN_t^D,\\
X_{\zeta_i} &= X_{\zeta_i^-}-\eta(\zeta_i,\omega_i,I_{\zeta_i}).
\end{aligned}$$

Optimization Problem As stated above, the family optimizes expected utility from intermediate consumption and terminal wealth. The optimization problem is characterized by several state variables: financial wealth x, labor income y, the health status of the wage earner A and the current insurance choice I. The control variables are the consumption rate c, the proportion of wealth θ invested in risky assets, and the impulse control strategy for the insurance decision (ζ_i,ω_i), $i\in\mathbb{N}$. At time $t=0$ the wage earner is assumed to be 20 years old. The optimization problem is then given by

$$\max_{\{c_s,\theta_s\}_{s\in[0,T)},\{(\zeta_i,\omega_i)\}_{i\in\mathbb{N}}}\mathrm{E}_{0,x,y,I,A}\left[\int_0^T \mathrm{e}^{-\delta u}\,\frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma}\,du+\varepsilon\,\mathrm{e}^{-\delta T}\,\frac{{X_T}^{1-\gamma}}{1-\gamma}\right]$$

[4] Preferences with a consumption scaling parameter are used by Love (2010) and Hubener, Maurer, and Rogalla (2013).

$$
\begin{aligned}
\text{s.t.} \qquad dX_t &= X_t \left[(r + \theta_t \lambda \sigma_S)\, dt + \theta_t \sigma_S \, dW_t^S \right] + \left[Y_t - c_t - \mathbb{1}_{\{A_t = 1,2 \,\wedge\, t < T_I\}} \iota(I_t) \right] dt \\
&\qquad + \mathbb{1}_{\{A_{t^-} = 1,2 \,\wedge\, t < T_I\}} I_{t^-} \, dN_t^D, \\
X_{\zeta_i} &= X_{\zeta_i^-} - \eta(\zeta_i, \omega_i, I_{\zeta_i}),
\end{aligned}
$$

where we impose short-sale constraints, i.e. $\theta_t \in [0,1]$, and liquidity constraints, i.e. consumption has to be chosen in such a way that financial wealth stays positive, $X_t \geq 0$. The value function (indirect utility function) is defined by

$$
J(t,x,y,I,A) = \sup_{\{c_s,\theta_s\}_{s\in[t,T)},\{(\zeta_i,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{t,x,y,I,A} \left[\int_t^T \mathrm{e}^{-\delta(u-t)} \frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma} \, du + \varepsilon \, \mathrm{e}^{-\delta(T-t)} \frac{{X_T}^{1-\gamma}}{1-\gamma} \right].
$$

We split the problem into its impulse control and stochastic control part. Given no intervention at t, but optimal impulse control afterwards, the value function is denoted by

$$
J^*(t,x,y,I,A) = \sup_{\{c_s,\theta_s\}_{s\in[t,T)},\{(\zeta_i \,|\, \zeta_i \neq t,\omega_i)\}_{i\in\mathbb{N}}} \mathrm{E}_{t,x,y,I,A} \left[\int_t^T \mathrm{e}^{-\delta(u-t)} \frac{\left(\frac{c_u}{\phi_{A_u}}\right)^{1-\gamma}}{1-\gamma} \, du + \varepsilon \, \mathrm{e}^{-\delta(T-t)} \frac{{X_T}^{1-\gamma}}{1-\gamma} \right].
$$

In this case, the optimization problem reduces to a stochastic control problem. The corresponding Hamilton-Jacobi-Bellman equation (HJB) is given by

$$
\begin{aligned}
\delta J^* = \sup_{c,\theta} \quad & \Bigg\{ \frac{\left(\frac{c}{\phi_A}\right)^{1-\gamma}}{1-\gamma} + J_t^* \\
& + J_x^* \left[X\,(r + \theta\lambda\sigma_S) + y - c - \mathbb{1}_{\{A=1,2 \,\wedge\, t<T_I\}} \iota(I) \right] \\
& + \frac{1}{2} J_{xx}^* x^2 \theta^2 \sigma_S^2 + \mathbb{1}_{\{A=1,2\}} \left[J_y^* y \mu_Y(t) + \frac{1}{2} J_{yy}^* y^2 \sigma_Y(t)^2 + J_{xy}^* xy \sigma_S \sigma_Y(t) \rho(t) \theta \right] \\
& + \mathbb{1}_{\{A=1\}} \kappa(t) \left[J^*(t,x,p^{1,2}(t)y,I,2) - J^*(t,x,y,I,1) \right] \\
& + \mathbb{1}_{\{A=1\}} \pi(t,A) \Big[J^*(t, x + \mathbb{1}_{\{t \leq T_I\}} I, p^{1,3}(t)y, 0, 3) - J^*\left(t,x,y,I,1\right) \Big] \\
& + \mathbb{1}_{\{A=2\}} \pi(t,A) \Big[J^*\big(t, x + \mathbb{1}_{\{t \leq T_I\}} I, p^{2,3}(t)y, 0, 3\big) - J^*\left(t,x,y,I,2\right) \Big] \Bigg\}
\end{aligned}
$$

with terminal condition $J^*(T,x,y,I,A) = \varepsilon \frac{x^{1-\gamma}}{1-\gamma}$. Here subscripts on J indicate partial derivatives. Finally, we calculate the value function J by maximizing J^* over all possible interventions:

$$
J(t,x,y,I,A) = \sup_{\omega_i \in \mathcal{I}_{\zeta_i}} \quad \Big\{ J^*\big(t, x - \eta(\zeta_i, \omega_i, I_{\zeta_i})), y, I + \omega_i, A\big) \Big\}.
$$

Note that in the case of $\omega_i = 0$ we have a continuation strategy, i.e. the family decides to keep its insurance decision. If this is optimal, then $J(t,x,y,I,A) = J^*(t,x,y,I,A)$. Consequently, there is no lump-sum payment, since we are in the no transaction region and $\eta(\zeta_i, 0, I_{\zeta_i}) = 0$.

4 Calibration

This section describes the model calibration that is also summarized in Table 1.

[INSERT TABLE 1 ABOUT HERE]

Financial Assets We use standard values for the stock market drift ($\mu_S = 0.06$), the stock market volatility ($\sigma_S = 0.2$) and the riskfree rate ($r = 0.02$) that are similar to the values used by Cocco, Gomes, and Maenhout (2005) or Munk and Sørensen (2010), among others.

Biometric Risk Considering mortality risk, we use a Gompertz mortality model with constant parameters x, m, b for the healthy agent and increase the hazard rate of death by a constant term k_1 and an age-dependent term k_2 if the agent becomes unhealthy

$$\pi(t,A) = \begin{cases} \frac{1}{b}\,\mathrm{e}^{\left(\frac{x+t-m}{b}\right)} & \text{for } A_t = 1 \\ \frac{1}{b}\,\mathrm{e}^{\left(\frac{x+t-m}{b}\right)} + k_1 + k_2 t & \text{for } A_t = 2 \\ 0 & \text{for } A_t = 3 \end{cases} .$$

We calibrate the health shock using German cancer data.[5] We weight the gender specific data equally and do not distinguish between genders in the simulation. This yields an overall lifetime risk of getting cancer of 46.75%, a median age at diagnosis of 69 years and an absolute 5-year survival rate of 53.5%. Furthermore, the data provides age and genderspecific cancer incidence rates for 5-year intervals up to an age of 85. We calibrate the health shock rate κ using a gender

[5]We use German data taken from "Cancer in Germany 2007/2008, German Centre for Cancer Registry Data & Robert Koch Institute, 8th Edition 2012".

averaged version of the age specific cancer incidence rates. We assume the following functional form for the health shock rate

$$\kappa(t) = a\,\mathrm{e}^{-\left(\frac{\min(t,65)-b}{c}\right)^2}$$

with constant parameters a, b, c. Since we do not have data for ages higher than 85, for simplicity we assume that cancer rates are constant for agents older than 85 years. We obtain the parametrization $a = 0.02489$, $b = 66.96$ and $c = 29.42$. Figure 1 illustrates the data points and our calibration of κ.

[INSERT FIGURE 1 ABOUT HERE]

For the calibration of the magnitude of the impact of the cancer shock on the mortality intensity (k_1, k_2) we use the absolute average 5-year survival probability. Since there is an age dependency, we split the effect in a constant part k_1 and an age dependent part k_2. We calibrate k_1 and k_2 such that the simulated average 5-year survival probability and simulated death distribution match the empirical ones from the data. Here, we use German mortality data.[6] For the shock impact, the calibration yields $k_1 = 0.048$ and $k_2 = 0.0008$. Considering the Gompertz mortality risk parameters, we set the age at $t = 0$ to $x = 20$, the x-axis displacement to $m = 89.45$ and the growth rate (steepness parameter) to $b = 6.5$. Figure 2 compares our simulated yearly death rates with the empirical death rates in Germany.

[INSERT FIGURE 2 ABOUT HERE]

Our simulation fits the above-mentioned empirical means well: The average time of death is 80.4, the average time of cancer detection is 69.0, and over the lifetime 47.4% of the population face a health shock. The median survival rate at cancer detection is 5 years. Figure 3 shows the histogram of health shocks and death shocks and the corresponding health-state distribution of the wage earner in our simulation.

[INSERT FIGURE 3 ABOUT HERE]

[6]The German mortality data is taken from "Sterbetafel 2009/11, Statistisches Bundesamt, 2013".

Labor Income For the income dynamics (see equation (3.1)), we use the labor income calibration from Munk and Sørensen (2010), which is a continuous-time version of the Cocco, Gomes, and Maenhout (2005) results. They estimate the labor income process using PSID data dependent on the agent's education. Its drift term is modeled as

$$\mu_Y(t) = \begin{cases} \xi_0 + b + 2ct + 3dt^2 & \text{for } t < T_R \\ -(1-P) & \text{for } T_R \le t \le T_R + 1 \\ 0 & \text{for } t > T_R + 1 \end{cases} \cdot$$

In the benchmark calibration, we assume that the wage earner has a high school education and set the corresponding parameters according to Munk and Sørensen (2010). We use a retirement age of 65 ($T_R = 45$), an age- and education-independent real wage increase of $\xi_0 = 0.02$ and an initial income of $Y_0 = 19107$. The drift polynomial is given by $b = 0.1682, c = -0.00323, d = 0.000020$ and the retirement income reduction parameter is $P = 0.68212$. We also assume a different volatility parameter before and after retirement

$$\sigma_Y(t) = \begin{cases} \sigma_Y^w & \text{for } t < T_R \\ \sigma_Y^r & \text{for } t \ge T_R \end{cases},$$

where we use $\sigma_Y^w = 0.2$ and $\sigma_Y^r = 0$ in the benchmark calibration. In the same manner, we fix the correlation parameter

$$\rho(t) = \begin{cases} \rho^w & \text{for } t < T_R \\ \rho^r & \text{for } t \ge T_R \end{cases}.$$

Following Munk and Sørensen (2010), we assume zero correlation ($\rho^w = 0, \rho^r = 0$). If a jump occurs (critical illness or death), the income is reduced. In the critical illness case, we assume that the wage earner loses part of his income because he has to reduce his work effort.[7] We suppose that income decreases by 20%, i.e. $p^{1,2}(t < T_R) = 0.8$. If the critical illness occurs during retirement, the pension is however unaffected, i.e. $p^{1,2}(t \ge T_R) = 1$. In both cases,

[7] In practice, an agent could buy disability insurance, but this usually does not cover all loses. At least potential future wage increases cannot be insured.

the income volatility remains unchanged. In the benchmark calibration, we do not include a social security system which is studied as a robustness check (see Section 6). Hence, we set $p^{1,3} = p^{2,3} = 0$ such that the family has no income if the wage earner dies. Figure 4 depicts the income profile of the wage earner and of the family over the life cycle.

[INSERT FIGURE 4 ABOUT HERE]

Insurance We assume a competitive insurance market, which leads to an actuarially fair insurance. First, we consider the continuation case where no intervention takes place. Then, the family only has to pay the insurance premium. The yearly insurance premium $\iota(I)$ is assumed to be constant over time and is defined by the actuarial fairness criterion at $t = 0$ so that the expected discounted value of the insurance premia, Σ_{agent}, is equal to the the expected discounted value of the payments of the insurance company to the family, Σ_{ins}, i.e.

$$\Sigma_{agent}(0, I) = \Sigma_{ins}(0, I)$$

with

$$\begin{aligned}\Sigma_{agent}(0, I) &= \mathrm{E}_0 \left[\int_0^{T_I} \mathrm{e}^{-rs} \, \frac{\iota(I)}{1 + \psi_{ad} + \psi_{tr}} \mathbb{1}_{\{s<\tau^D\}} \, ds \right] \\ &= \frac{\iota(I)}{1 + \psi_{ad} + \psi_{tr}} \frac{1}{r} \left(1 - \mathrm{E}_0 \left[\mathrm{e}^{-r \min\{\tau^D, I\}} \right] \right),\end{aligned}$$

and

$$\Sigma_{ins}(0, I) = \mathrm{E}_0 \left[\int_0^{T_I} \mathrm{e}^{-rs} \, I \, d\mathbb{1}_{\{s \geq \tau^D\}} \right] = \int_0^{T_I} \mathrm{e}^{-rs} \, I f_D(s) \, ds,$$

where f_D is the probability density function of τ^D. The constant ψ_{ad} captures yearly administrative costs and the constant ψ_{tr} captures deferred acquisition costs that are paid with the yearly premium. Hence, the yearly insurance premium can be expressed as

$$\iota(I) = \frac{Ir(1 + \psi_{ad} + \psi_{tr}) \int_0^{T_I} f_{\tau^D}(s) \, \mathrm{e}^{-rs} \, ds}{1 - \mathrm{E}_0 \left[\mathrm{e}^{-r \min(\tau^D, T_I)} \right]}. \tag{4.3}$$

Next, we consider the intervention case, i.e. a situation where the family increases or decreases the insurance sum I. If an intervention (ζ_i, ω_i) takes place, the family must pay a lump-sum payment $\eta(\zeta_i, \omega_i, I_{\zeta_i})$. This payment is necessary since the insurance premium ι is calculated based on survival patterns at $t = 0$. Therefore, an insurance contract starting at $\zeta_i > 0$ requires an adjusted premium. If the family reduces protection, $\omega_i < 0$, no lump-sum payment is made.[8] If the family increases insurance protection, $\omega_i > 0$, we assume that the insurance company takes previously paid premia into account so that the lump-sum payment is

$$\begin{aligned}\eta(\zeta_i, \omega_i, I_{\zeta_i}) = (1 + \psi_{ad} + \psi_{tr}) \Big(&\Sigma_{ins}(\zeta_i, I_{\zeta_i}) - \Sigma_{agent}(\zeta_i, I_{\zeta_i}) \\ &- \Sigma_{ins}(\zeta_i, I_{\zeta_i} - \omega_i) + \Sigma_{agent}(\zeta_i, I_{\zeta_i} - \omega_i)\Big), \end{aligned} \tag{4.4}$$

where the two last terms depend on the previous insurance sum and capture the retrospective reserve. Note that they vanish in the special case where the family has no previous insurance protection. The variables $\Sigma_{ins}(\zeta_i, I_{\zeta_i})$ and $\Sigma_{agent}(\zeta_i, I_{\zeta_i})$ denote the conditional present values of the payments to the family and to the insurance company:

$$\begin{aligned}\Sigma_{ins}(\zeta_i, I_{\zeta_i}) &= \int_{\zeta_i}^{T_I} \mathrm{e}^{-r(s-\zeta_i)}\, I_{\zeta_i} f_D(s \,|\, \min(\tau^D, \tau^H) > \zeta_i)\, ds, \\ \Sigma_{agent}(\zeta_i, I_{\zeta_i}) &= \frac{\iota(I_{\zeta_i})}{1 + \psi_{ad} + \psi_{tr}} \frac{1}{r} \left(1 - \mathrm{E}_{\zeta_i}\left[\mathrm{e}^{-r(\min(\tau^D, T_I) - \zeta_i)} \mid \min(\tau^D, \tau^H) > \zeta_i\right]\right),\end{aligned}$$

where we condition on the agent being in the insurance market, i.e. being healthy and alive. The continuous premium ι is calculated according to (4.3). Notice that a valid intervention also requires $\zeta_i \leq T_C$.

To summarize, there are two cases: If no intervention takes place, the family must pay the insurance premium $\iota(I)$. If an intervention (ζ_i, ω_i) takes place, an additional lump-sum payment becomes due

$$\eta(\zeta_i, \omega_i, I_{\zeta_i}) = \begin{cases} 0 & \text{if } \omega_i \leq 0, \\ (1 + \psi_{ad} + \psi_{tr}) \Big(\Sigma_{ins}(\zeta_i, I_{\zeta_i}) - \Sigma_{agent}(\zeta_i, I_{\zeta_i}) & \\ \qquad - \Sigma_{ins}(\zeta_i, I_{\zeta_i} - \omega_i) + \Sigma_{agent}(\zeta_i, I_{\zeta_i} - \omega_i)\Big) & \text{else.} \end{cases}$$

We allow the family to choose among insurance contracts with payouts specified by $\mathcal{I}$ (see equation (3.2)). The family is able to change its insurance exposures until the wage earner is

[8] Note that in countries like Germany a term life insurance has usually no repurchase value.

70 ($T_C = 50$). The insurance contract expires at the age of 75 ($T_I = 55$). These ages are typical for German term life insurance contracts.

We set the administrative fee ($\psi_{ad} = 2.99\%$) and the transaction fee ($\psi_{tr} = 5.05\%$) to the average values of the German life insurance market.[9] We discretize the conditional expectations $\mathrm{E}_t\left[\mathrm{e}^{-r\min(\tau^D,T_I)} \mid \min(\tau^D,\tau^H) > t\right]$ and $\int_t^{T_I} \mathrm{e}^{-rs} f_D(s \mid \min(\tau^D,\tau^H) > t)\, ds$, for $t \in \{0, 1, \ldots, T_C\}$ using German mortality data.[10] For intermediate values of t we use linear interpolation.

Preferences We choose standard values from the life cycle portfolio optimization literature for the risk aversion ($\gamma = 4$), the time preference rate ($\delta = 0.03$) and the bequest motive ($\varepsilon = 1$). In the benchmark calibration, the problem starts at time $t = 0$, when the wage earner is 20 years old, and ends at $t = T = 80$, when the wage earner is either 100 years old or dead. Following Love (2010), we calculate the consumption equivalence scaling parameter via

$$\phi = (\alpha_{Adult} + 0.7\alpha_{Child})^{0.7},$$

where α_{Adult} is the number of adults and α_{Child} is the number of children in the household. In the benchmark calibration, we study a family consisting of two adults and one child. Hence, we set the corresponding consumption scaling parameter to $\phi_{1,2} = 2.0043$ if the wage earner is alive and to $\phi_3 = 1.4498$ if the wage earner is dead. We assume that the family starts with a financial wealth level that is twice the initial annual labor income of the wage earner, $X_0 = 2Y_0$.

5 Benchmark Results

This section provides our main results for the model introduced in Section 3 with the calibration presented in Section 4.

[9]Data about the transaction fees and administrative fees on the German insurance market are taken from "map-report no. 807-808".

[10]Mortality data is taken from Life table for Germany ("Sterbetafel 2009/11, Statistisches Bundesamt, 2013").

Average Key Variables over the Life Cycle

[INSERT FIGURE 5 ABOUT HERE]

Figure 5 depicts the average optimal decisions as well as the average financial wealth and income over the life cycle. Similar as in the above mentioned papers, financial wealth is hump-shaped and the portfolio holdings are decreasing over the life cycle. The dark consumption line represents the consumption of the whole family. We see that consumption increases over the life cycle, although the slope decreases significantly after retirement. This is mainly due to two reasons. First, the labor income profile changes at retirement, especially the certainty of the retirement income leads to a flatter consumption path. Second, due to a higher mortality risk at older ages there are more families where the wage earner has already died. Then, the family has no more income and one person less to take care of, which both reduces consumption. The grey line represents the single person equivalent consumption.[11] This line increases almost linearly over the life cycle, which indicates that on average the death of the wage earner does not lead to a reduced utility from consumption for the remaining family members. This may be either due to a high amount of accumulated financial wealth, or a term life insurance contract. As the insurance sum distribution graph shows, it is a combination of both for most families. About 65% of the families buy a term life insurance over the life cycle. At a young age, the families stay away from the insurance market, but start buying insurance at the age of about 30. Furthermore, the families increase the insurance sums over the life cycle and there is no age at which they systematically reduce the insurance exposures. Apparently, agents do not change the insurance contracts after the retirement age of 65, although changes would be possible until the age of 70. This can be explained by the certainty of the retirement income. Consequently, there is no uncertainty with respect to human wealth. Hence, there is no reason to change the insurance decision. The increasing insurance sum over the life cycle might be counterintuitive at first. Since an older agent has on average more financial wealth and more income to hedge the effect of a health shock, one could expect the insurance demand to be lower compared to a

[11] This is a comparable one-person consumption level that is calculated by weighting the consumption of the family with the consumption equivalence scaling parameter $\frac{c_t}{\phi_{A_t}}$.

situation with less income and wealth. However, there are opposing effects. First, for an older agent the contract duration is shorter. Hence, reducing an insurance exposure is relatively more costly than keeping it. Second, uncertainty with respect to human wealth and financial wealth significantly reduces for an older agent. Therefore, it is less likely that he faces a wealth and income state in which the contract is not affordable or too expensive relatively to his financial situation. These two effects dominate and yield an overall increasing insurance demand over the life cycle.

Comparative Statics

[INSERT FIGURE 6 ABOUT HERE]

To analyze the effects of age, income, financial wealth, and the previous insurance contract, we consider the policy functions. Figure 6 depicts the insurance demand for different levels of the labor income. The lines represent different ages for a fixed level of financial wealth and a fixed previous insurance contract with insurance sum 100 000. For all ages, the insurance demand increases in the income level, which has two reasons: First, if income is large, then a higher insurance sum becomes affordable. Second, since an insurance is a hedge against the income loss upon death of the wager earner, a higher income increases this hedging motive. Furthermore, the figure highlights a time-dependence. The black line shows the policy function for a young wage earner at the age of 25. For a reasonable labor income of below 100 000, it is always optimal for the family to ignore any previous insurance contract and to not pay the premium. This leads to a loss of insurance protection and all previous paid premiums are lost as well. We document a large continuation interval ranging from about 175 000 to 450 000, i.e. the family optimally sticks to the current insurance contract and no intervention takes place. The advantage is that the insurance protection maintains, past premiums are not lost and no expensive lump-sum costs must be paid for increasing the insurance sum. For a 50 year old middle-aged agent it is in general optimal to stick to the previous insurance decision, except for very high or low labor income. So the insurance choice at this age is also very robust. For an old retired agent at the age of 68 the optimal insurance sum crucially depends on the pension level. Furthermore, the insurance sum increases due to the certainty of the pension. This means that income volatility as a crucial source of uncertainty is no longer present. Overall, we document a strong dependence of the optimal insurance demand on labor income and time. The increasing insurance demand over time is in line with our previous findings.

[INSERT FIGURE 7 ABOUT HERE]

Figure 7 depicts the corresponding policy functions dependent on financial wealth. Initially, we see that financial wealth has less impact on the insurance decision as income. However, this is not surprising, since the insurance is mainly used to hedge the loss of labor income in the case of death. Overall, the more financial wealth the agent has, the less insurance is optimal. This is intuitive, since with more wealth there is less need for a fixed payout in the case of death. For a middle-aged agent the continuation region includes all reasonable financial wealth levels. The young agent cancels the insurance if he has a huge amount of financial wealth (that is above the average wealth level at this age), whereas the old agent raises the insurance contract if he has less than 1 600 000 wealth. To summarize, the insurance decision is rather insensitive towards the level of financial wealth.

[INSERT FIGURE 8 ABOUT HERE]

The policy functions dependent on the previous insurance decision in Figure 8 document the stickiness of this decision. The diagonal line depicts a situation where the agent keeps his insurance decision, i.e. the previous insurance contract equals the current optimal decision. Below the diagonal, the agent reduces insurance protection, whereas he increases protection above the diagonal. A young agent would cancel any contract, independent of its insurance sum, and thus stays away from the insurance market in the first place. For him, the inflexibility of the contract is very severe, since he is tied to the contract for a potentially long time period or loses a significant amount of money if he cancels or reduces the contract prematurely. The uncertainty of human wealth at a young age amplifies this problem, as contracts are usually downward adjusted when human wealth has deteriorated and the current contract is not affordable any more. A middle-aged agent keeps a contract if the insurance sum is below 500 000. Contracts with higher insurance sum are reduced to this level. The old agent has a continuation region ranging from 200 000 to 750 000, where he sticks to his contract. To summarize, the decisions at every age are pretty stable: Young agents stay away from the insurance market, whereas middle-aged agents usually keep their current positions.

Impact of Critical Illness and Death Shocks

[INSERT FIGURE 9 ABOUT HERE]

Figure 9 depicts the effects of a critical illness shock at the age of 50 (black lines) and a death shock at the age of 60 (grey lines) on the optimal behavior and the financial wealth and income evolution. If the wage earner dies, financial wealth jumps upwards since the family receives an insurance payment. Furthermore, it leads to a negative jump in family consumption due to the reduced number of family members, although there is only a minor change in the level of single person equivalent consumption. However, the family reduces the slope of its consumption path to adjust to the new income situation. Besides, the portfolio holdings are reduced to the classical Merton demands, since there is no labor income any more.

If the agent is first exposed to a critical illness shock, he becomes aware of a high probability of an early death. Consumption is reduced since labor income decreases and the family tries to accumulate financial wealth for the remaining family members. Portfolio holdings are reduced as well. Unfortunately, the agent cannot increase his insurance protection any more. Therefore, at the time of death the average insurance sum is less than 50% of the sum of an agent without previous health shock. Finally, the single person equivalent consumption is adjusted when the health shock occurs, and thus there is no significant decrease at the time of death.

[INSERT FIGURE 10 ABOUT HERE]

Figure 10 illustrates a situation where the wage earner dies at the age of 30. In the first setting there is no previous health shock (grey lines), whereas in the second setting there is a health shock at the age of 25 (dark lines). In this case, the family has only little time to accumulate financial wealth. Furthermore, most families do not buy any insurance at this age, which explains the very low average insurance sum. These two facts together with the early death of the wage earner reduces consumption of the family and also the single person equivalent consumption significantly. Buying a term life insurance contract could easily double the available financial wealth.

6 Robustness Checks

This section presents robustness checks for different labor income volatility, different insurance fees, a calibration without health shock and for a changed family size. We also discuss the main drivers that prevent families from increasing their insurance demands. Furthermore, we give results for a calibration with a social security system that pays a widow's pension after the death of the wage earner.

Labor Income Volatility

[INSERT FIGURE 11 ABOUT HERE]

A term insurance allows a family to (partially) hedge the risk resulting from an early death of the wage owner. Since the death of the wage earner predominately leads to a loss of labor income, the optimal insurance choice crucially depends on the labor income process. A negative feature of an insurance contract is the stickiness of its premia. Consequently, an insurance contract amplifies the effect of negative labor income shock. For instance, if a family is optimally insured and a negative labor income shock occurs, then the family has too much insurance protection given the actual income situation and must cut down on consumption. Alternatively, the family can reduce or terminate the insurance contract yielding to a loss, since term life insurance has no surrender value. In both cases, the effect of a negative labor income shock is stronger if the family has a higher insurance exposure. Therefore, hedging mortality risk comes at the cost of amplifying the effect of a negative labor income shock. In line with these findings, Figure 11 shows that the insurance demand is significantly higher for families with lower income volatility. Besides, these families also buy insurance earlier so that young families are insured as well.

Insurance Structure

[INSERT FIGURE 12 ABOUT HERE]

The structure of a term life insurance contract varies among insurance companies. Especially the fees ψ_{tr}, ψ_{ad}, the date T_C until the insurance decision can be revised and the expiration date T_I can be different. Figure 12 depicts the effect of a change of the fees. We compare the average fees of the benchmark result (grey line) with a regime with high fees (light line) and without fees (dark line). Clearly, the insurance demand decreases if fees are raised. However, although the fees are increased significantly, the decrease in insurance demand is not dramatic. This indicates that the insurance profit, captured by the fees, has a rather small impact on the insurance demand.

Health Shocks

[INSERT FIGURE 13 ABOUT HERE]

A health shock prevents the family from increasing the insurance protection or buying a new contract. Intuitively, one might expect that the family anticipates this restriction and buys more protection at an earlier age. Figure 13 compares the benchmark model with an alternative calibration without health shocks. Surprisingly, our findings document a higher insurance demand in the case without health shocks. The reason is that a health shock can be interpreted as a warning that death becomes more likely. If the wage earner faces a health shock, the family knows that he will die with a high probability in the next few years. Although an early death is clearly negative for the family, the health shock partially resolves uncertainty about the timing of dying, which itself is beneficial. With this new information the family is better able to plan consumption and investment decisions. Therefore, the family reduces consumption in order to accumulate more wealth, which can be seen in Figure 9 and 10. Due to the additional savings, the insurance demand goes down. Notice that in our benchmark calibration labor income is reduced after a health shock, which triggers a decrease in consumption. This is however also true in a calibration where the labor income is not reduced in the critical illness state. Furthermore, one might argue that the increased insurance demand without health shock results from the fact that in this setup there is no state in which the insurance acquisition is forbidden. However, the results also hold when we only consider families that do not face a health shock.

Family Size

[INSERT FIGURE 14 ABOUT HERE]

Figure 14 confirms the intuition that a larger family buys more insurance protection. In our model, this is captured by the consumption scaling parameter ϕ_A. The relative difference in the consumption scaling parameter in state $A = 1, 2$ and $A = 3$ is smaller, the larger the family. Consequently, for a large family more consumption is needed to obtain the same single-person equivalent utility level. If the wage earner dies, the whole income is lost, but the bigger part of consumption remains if the family size is large. This increases the insurance demand.

Risk Aversion

[INSERT FIGURE 15 ABOUT HERE]

Figure 15 shows the impact of the relative risk aversion on the average insurance sum. Apparently, risk aversion has little impact before the age of 30 and after retirement. In between, more risk averse agents demand less insurance. Hence, a more risk averse agent perceives the insurance contract as more risky compared to financial investments (stocks, bonds). However, overall the degree of relative risk aversion has only little impact on the insurance decision.

Social Security System

[INSERT FIGURE 16 ABOUT HERE]

In this paragraph, we add a social security system to the model that pays the family an income after the death of the wage earner. This can be interpreted as a widow's pension and the corresponding income stream is calibrated using data of the German social security system. Therefore, we recalibrate the effects of the wage earner's death on the income process as follows:[12]

$$p^{1,3}(t) = p^{2,3}(t) = \begin{cases} 0.55 \cdot 0.68212(1 - 0.108) & \text{for } t < 40, \\ 0.55 \cdot 0.68212(1 - (43 - t)0.036) & \text{for } 40 \leq t < 43, \\ 0.55 \cdot 0.68212 & \text{for } 43 \leq t < 45, \\ 0.55 & \text{for } t \geq 45. \end{cases} \tag{6.5}$$

Notice that at most a widow receives about 55% of the pension. Besides, before retirement ($t < 45$) there is also an adjustment for for the replacement rate of 0.68212 and deductions. The income jump if the wage earner gets unhealthy $p^{1,2}$ remains unchanged.

Figure 16 depicts the average key variables over the life cycle. Compared to an economy without social security system (see Figure 5), the average income is higher and does not approach zero. Furthermore, the optimal consumption now shows a hump-shaped pattern. The financial wealth and the portfolio holdings are only little affected. The insurance sum distribution highlights that about 90% of the population buy term life insurance over the lifetime, which is a significant increase. However, one of our main results stands: Young families do not participate in term life insurance markets. Figure 17 compares the average insurance demand to the benchmark results without social security system. One might conjecture that a social security system crowds out most of the insurance demand and, consequently, the insurance demand is significantly reduced. Our results however point in a different direction: With a social security system, the insurance demand significantly increases for all ages. This increase can be explained by the changes in characteristics of the income process. First, the human wealth is higher due to the additional payment. Second, human wealth uncertainty reduces since the income loss at death is less pronounced and the widow's pension is deterministic.

[12]Details can be found in the German Social Security Code (SGB).

[INSERT FIGURE 17 ABOUT HERE]

7 Conclusion

This paper studies the insurance demand of a family that is exposed to health shocks and mortality risk. The wage earner receives an unspanned income stream and can buy term life insurance up to the age of 70 as long as he is healthy. We model the available insurance contracts in a realistic way by assuming that they are of long-term nature and that decisions can only be revised at certain costs. The combination of unspanned income as well as the stickiness of the insurance contracts reduces the insurance demand significantly. In particular, young families do not participate in the insurance market at all.

Our results have potentially important policy implications. From a welfare perspective, one might argue that it is beneficial that people buy insurance contracts to avoid poverty. Our results document that long-term contracts prevent families from doing so, since they get locked into these contracts and have difficulties to pay premia if they are in financially dire situations. This finding suggests that families should have access to more flexible insurance contracts. For instance, a contract with a variable insurance sum that is linked to the actual labor income evolution (similar to occupational pensions) could avoid the negative amplifying effect of a labor income shock.

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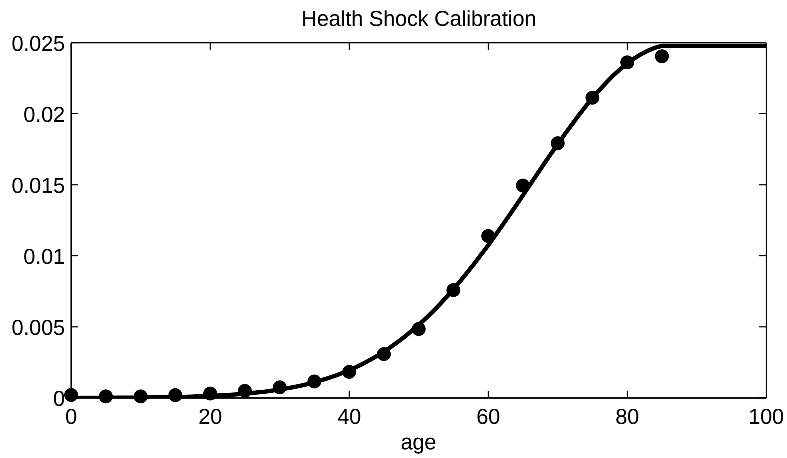

Figure 1: **Health Shock Calibration.** The figure shows empirical gender averaged 5-year cancer detection rates and our fitted curve κ. The empirical realizations (points) are gender averaged values from "Cancer in Germany 2007/2008, German Centre for Cancer Registry Data & Robert Koch Institute, 8th Edition 2012". The parameters for our fitted curve κ (solid line) are given in Section 4.

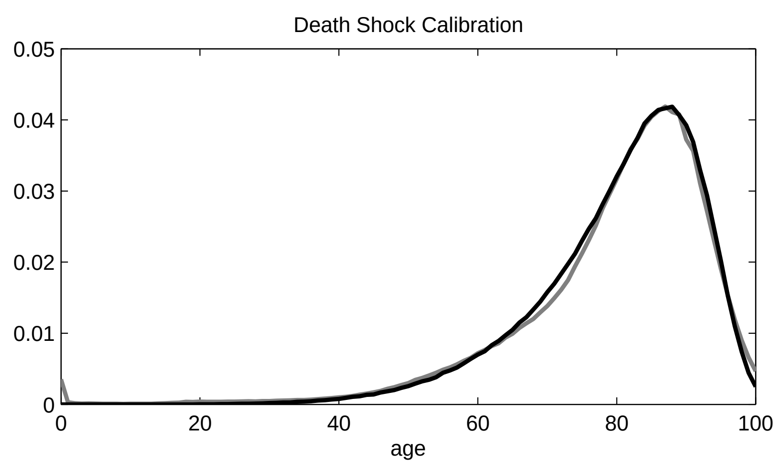

Figure 2: **Death Shock Calibration.** The figure depicts the number of yearly deaths for a normalized population of size 1. The mortality data (grey line) is gender averaged from a life table for Germany ("Sterbetafel 2009/11, Statistisches Bundesamt, 2013"). Our simulated values (dark line) are the averages from 100 000 death shock simulations using the biometric risk calibration of Section 4.

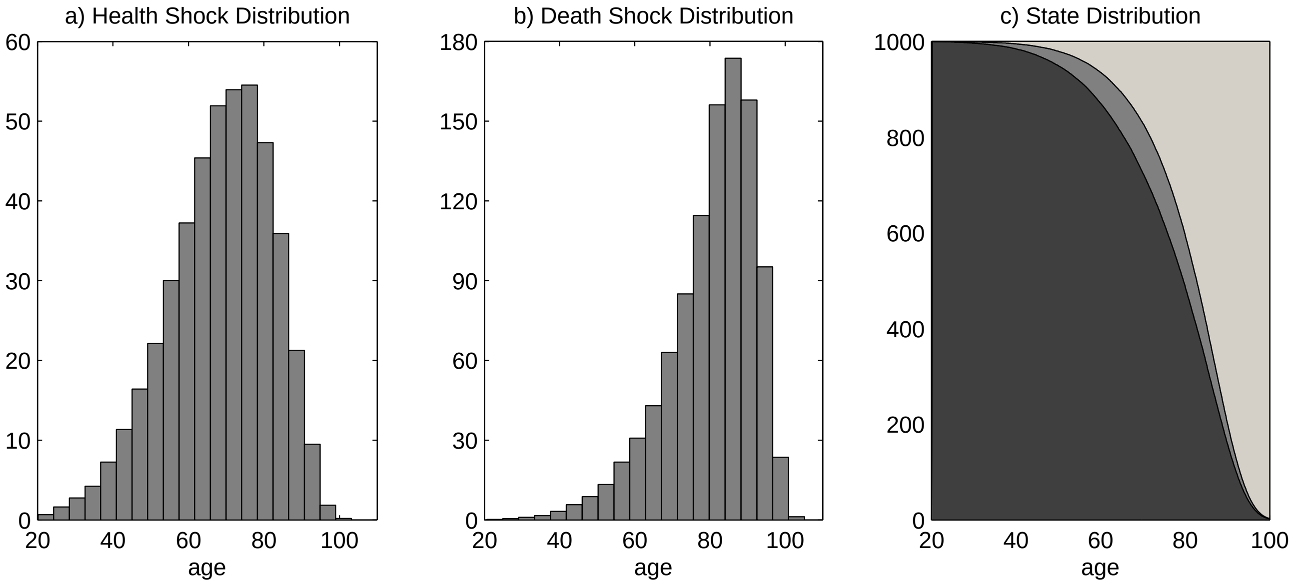

Figure 3: **Simulated Biometric Risk Distribution.** The graphs show the results of 100 000 simulated life cycles with the biometric risk parametrization of Section 4. a) depicts the histogram of the health shocks in our simulation. b) shows the corresponding histogram of death shocks. c) shows the state distribution of the families. The dark area corresponds to families with a healthy wage earner ($A = 1$), the middle grey area represents unhealthy agents ($A = 2$) and the light area marks families with a dead wage earner ($A = 3$).

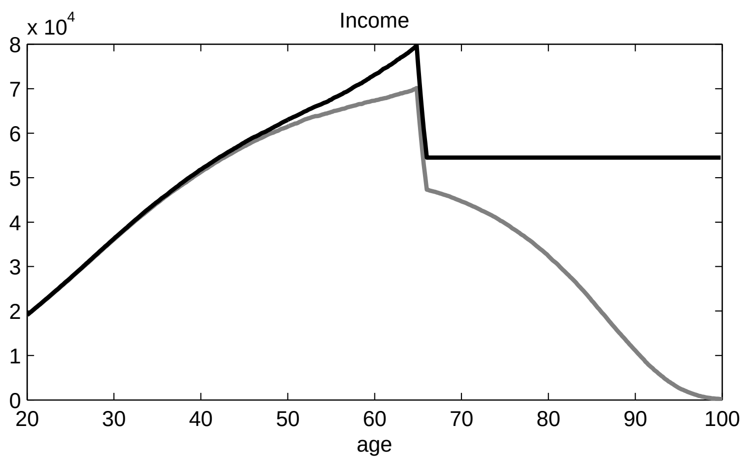

Figure 4: **Expected Income Profile over the Life Cycle.** The figure depicts the expected income of a wage earner conditional on survival (dark line) assuming that he has a high school education. The grey line gives the expected income of the family and incorporates the biometric risk of the wage earner. The labor income and biometric risk parameter values are stated in Section 4 and the lines depict the average values of 100 000 simulations.

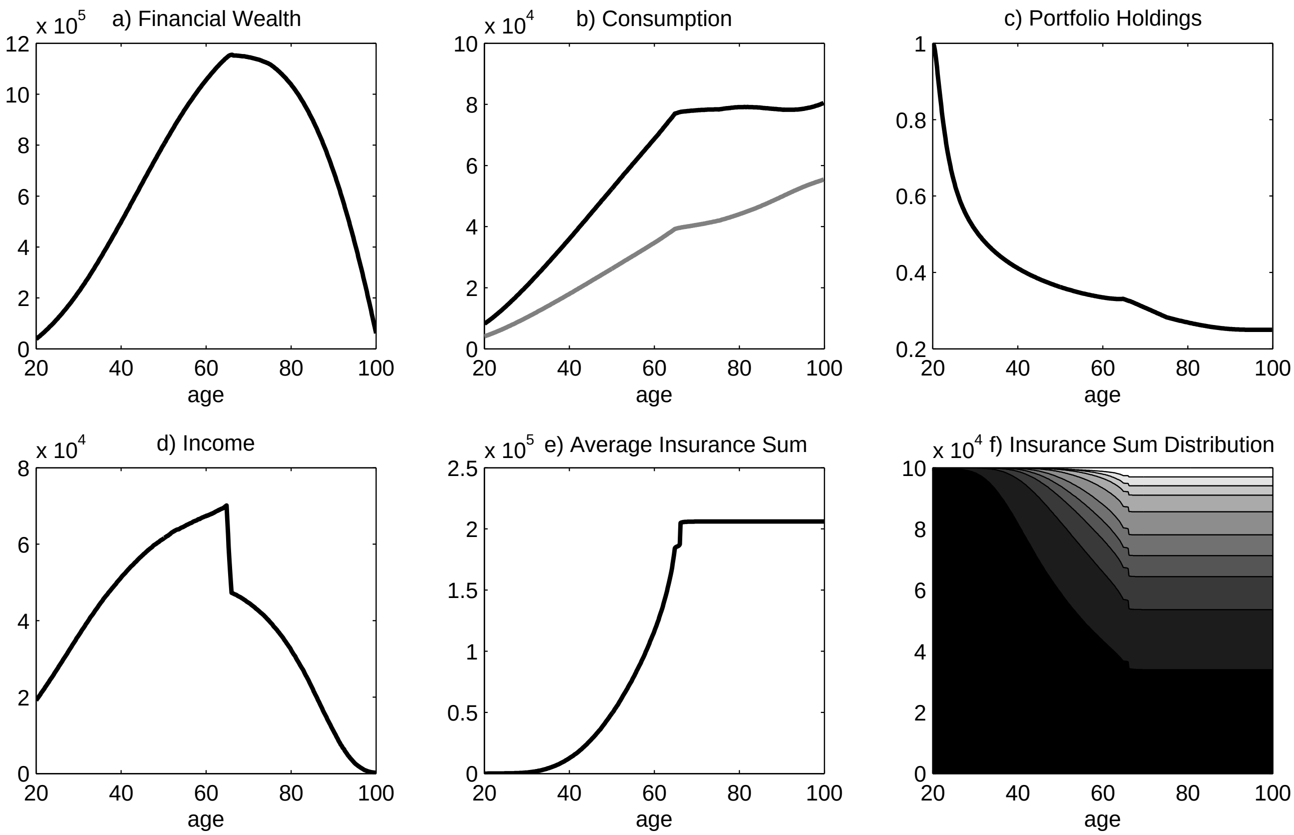

Figure 5: **Average Key Variables over the Life Cycle.** The graphs depict the average optimal control variables as well as the average financial wealth and income evolution over the life cycle based on 100 000 simulations with the benchmark calibration of Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The dark line corresponds to the consumption of the family, where the grey line represents the equivalent level of consumption for a one person household and is scaled with the consumption scaling parameter $\frac{c}{\phi}$. c) shows the optimal average portfolio holdings over the life cycle. d) depicts the average income of the family. e) shows the average insurance sum. f) depicts the distribution of the insurance sum over the life cycle. The darkest area marks families with no insurance contract, the white area families with the highest insurance sum (2 000 000) and the grey areas families with intermediate insurance sums.

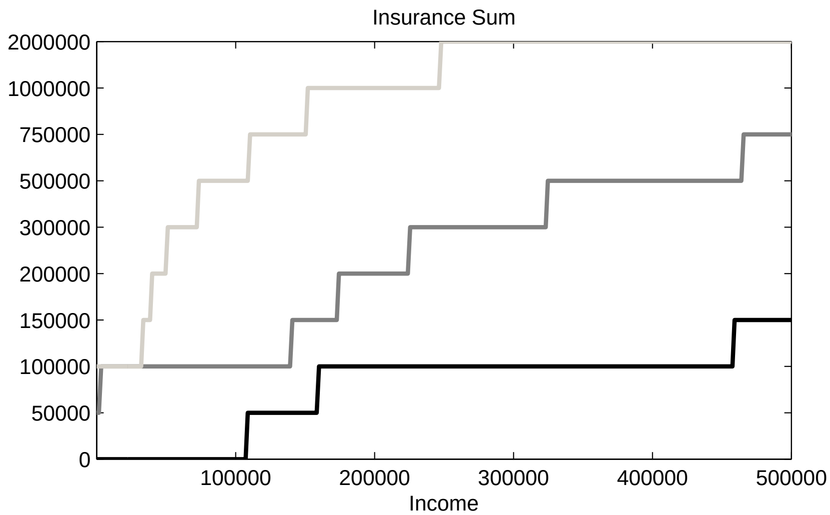

Figure 6: **Insurance Demand Dependent on Income.** This graph depicts the optimal insurance decision dependent on the income of a healthy agent ($A = 1$). The financial wealth is fixed to $x = 800\,000$, the previous insurance choice is fixed to $I = 100\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

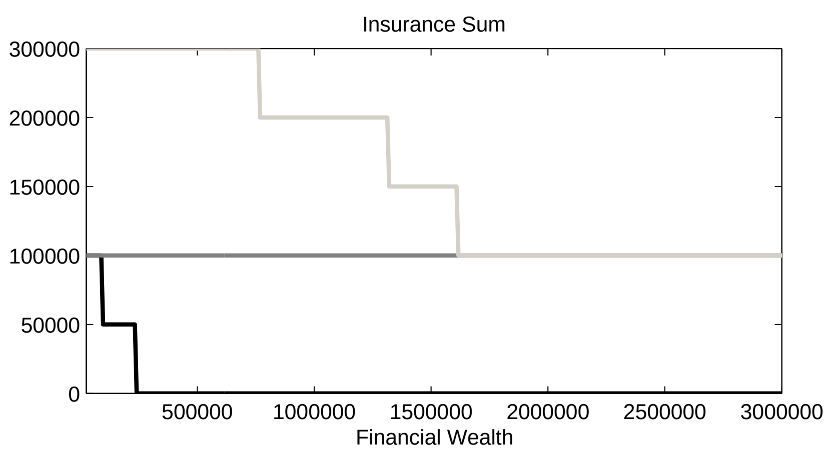

Figure 7: **Insurance Demand Dependent on Financial Wealth.** This graph depicts the optimal insurance decision dependent on the financial wealth of a healthy agent ($A = 1$). The income is fixed to $y = 50\,000$, the previous insurance choice is fixed to $I = 100\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

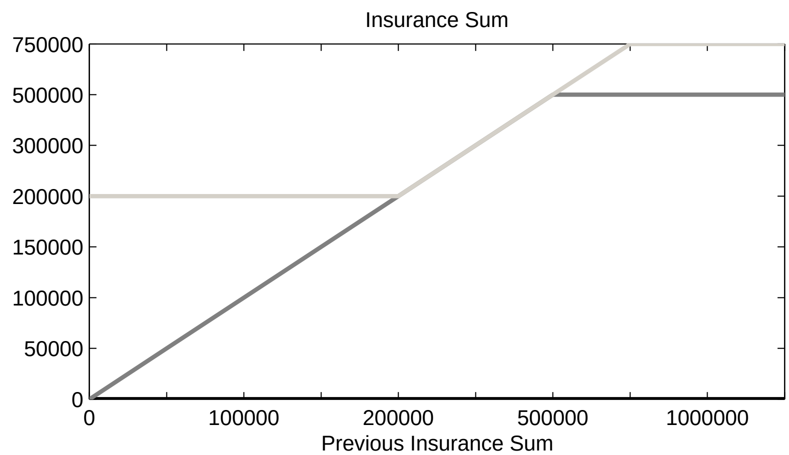

Figure 8: **Insurance Demand dependent on the Previous Insurance Sum.** This graph depicts the optimal insurance decision dependent on the previous insurance sum of a healthy agent ($A = 1$). The financial wealth is fixed to $x = 800\,000$, the income is fixed to $y = 50\,000$ and the age of the agent is fixed to 25 (dark line), 50 (grey line) and 68 (light line). The policy functions are based on the calibration presented in Section 4.

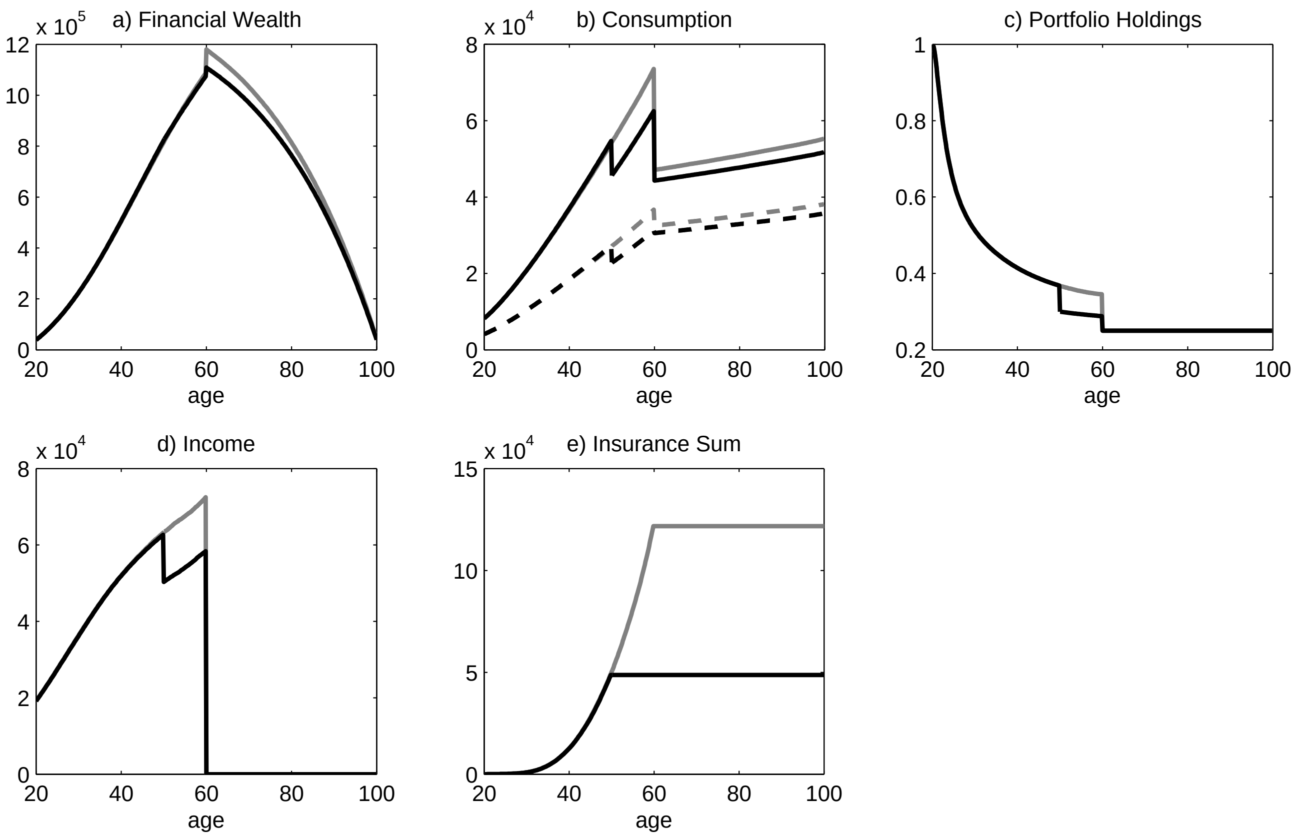

Figure 9: **Optimal Reaction to Critical Illness and Death.** The figure depicts the average financial wealth evolution and average optimal controls based on 100 000 simulations where all wage earners are assumed to die at the age of 60. The black lines correspond to the optimal behavior of a family where the wage earner gets a critical illness at the age of 50, whereas the grey lines represent a family whose wage earner dies without previous critical illness. The calibration equals the one of the benchmark results and is explained in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The solid lines are for the consumption of the family c and the dashed lines for the equivalent single person consumption level $\frac{c}{\phi}$. c) presents the average optimal portfolio holdings, d) gives the average income of the family and e) depicts the average optimal insurance sum over the life cycle.

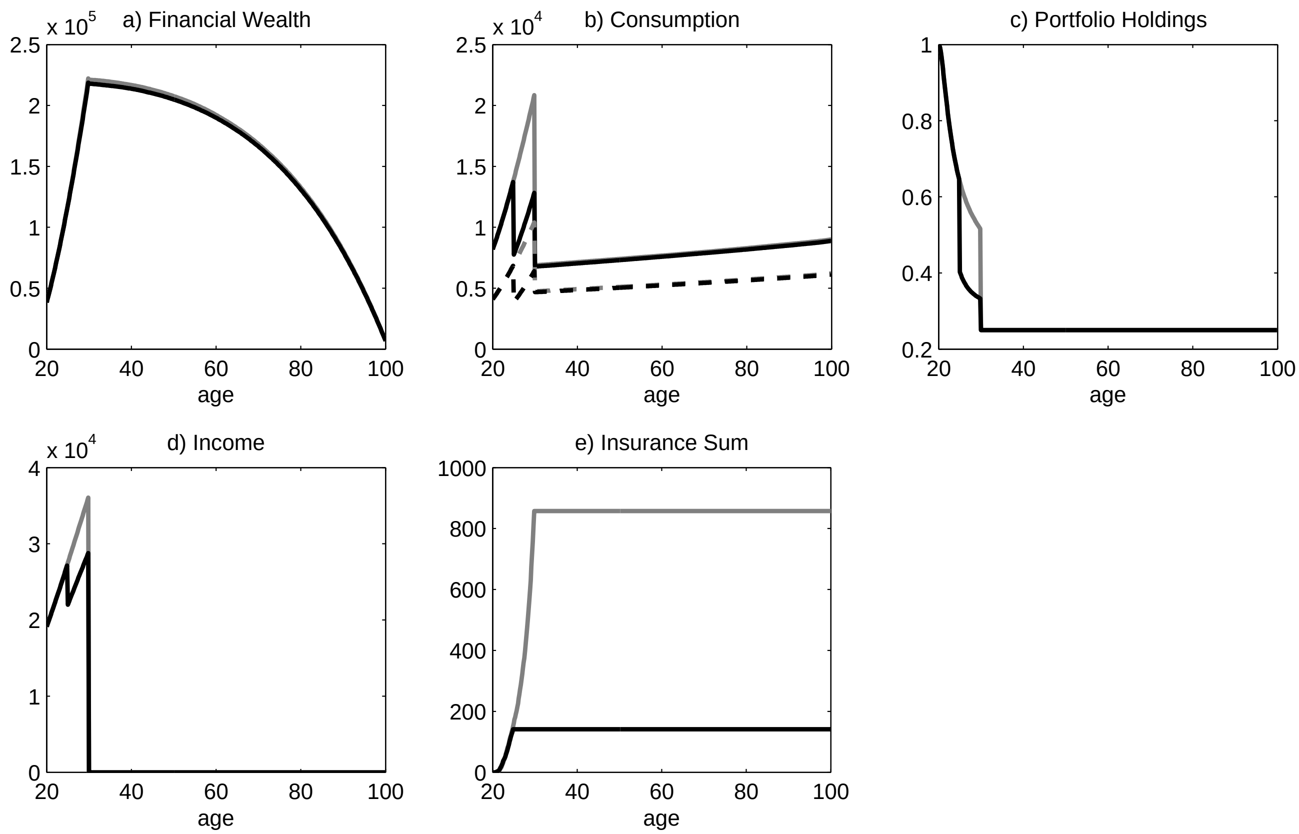

Figure 10: **Optimal Reaction to Early Critical Illness and Death.** The figure depicts the average financial wealth evolution and average optimal controls based on 100 000 simulations where all wage earners are assumed to die at the age of 30. The black lines correspond to the optimal behavior of a family where the wage earner gets a critical illness at the age of 25, whereas the grey lines represent a family whose wage earner dies without previous critical illness. The calibration equals the one of the benchmark results and is explained in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The solid lines are for the consumption of the family c and the dashed lines for the equivalent single person consumption level $\frac{c}{\phi}$. c) presents the average optimal portfolio holdings, d) gives the average income of the family and e) depicts the average optimal insurance sum over the life cycle.

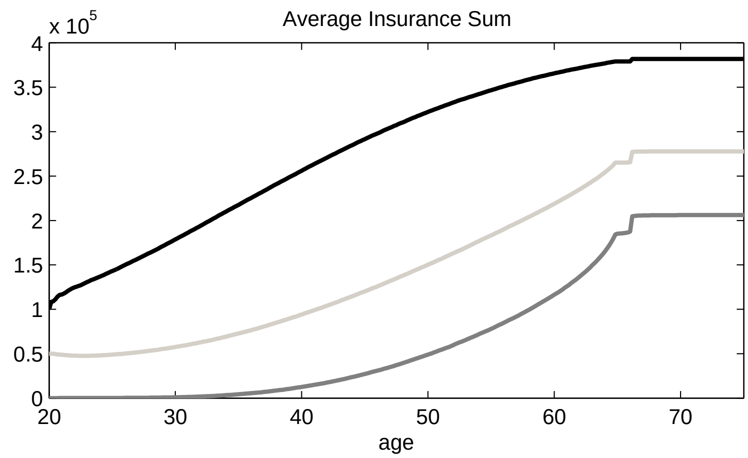

Figure 11: **Insurance Demand for Different Levels of Labor Income Volatility.** The dark line depicts the average insurance sum in a model where the labor income volatility before retirement is reduced to $\sigma_Y^w = 0.1$ and the light line depicts a reduction to $\sigma_Y^w = 0.15$. The grey line represents the benchmark results with a labor income volatility of $\sigma_Y^w = 0.2$. The remaining parameters are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

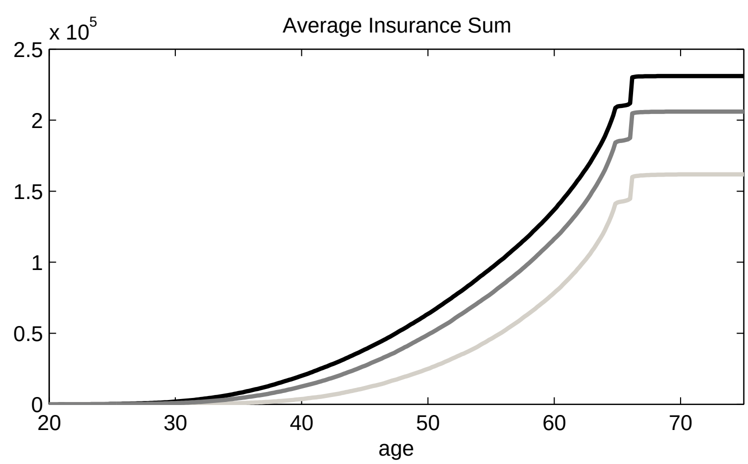

Figure 12: **Insurance Demand for Different Fees.** The light line depicts the average insurance sum in a model where the fees of the insurance company are increased such that the administrative fee is $\psi_{ad} = 12.68\%$ and the transaction fee equals $\psi_{tr} = 13.62\%$. The values are taken from “map-report no. 807-808” and represent the highest fees in the German life insurance market. The dark line is for an actuarially fair insurance without fees ($\psi_{ad} = 0$, $\psi_{tr} = 0$). The dashed line represents the benchmark results with fees of $\psi_a = 2.99\%$ and $\psi_t = 5.05\%$ that correspond to average fees in the German life insurance market. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

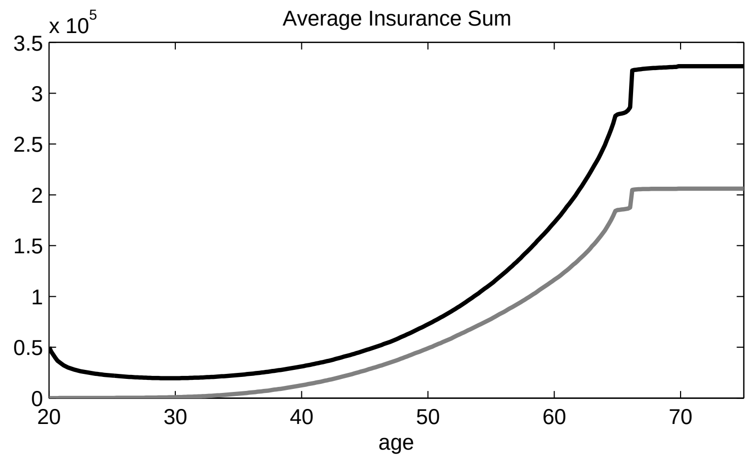

Figure 13: **Insurance Demand Compared to a Model without Health Shocks.** The dark line depicts the average insurance sum for a different biometric risk calibration without health shocks. Thus, the wage earner can only be in two health states ($A = 1, 3$) and the family can contract and increase term life insurance contracts as long as the wage earner is alive and $t < T_C$. The health shock rate and magnitude are set to zero ($\kappa = 0$, $k_1 = 0$, $k_2 = 0$) and the mortality model has a standard Gompertz structure. The parameter calibration is changed to $b = 8.9$, $m = 85.47$ to get a similar death shock distribution as in the benchmark model with critical illness shocks. The grey line represents the benchmark results including health shocks and with the original mortality parameters. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

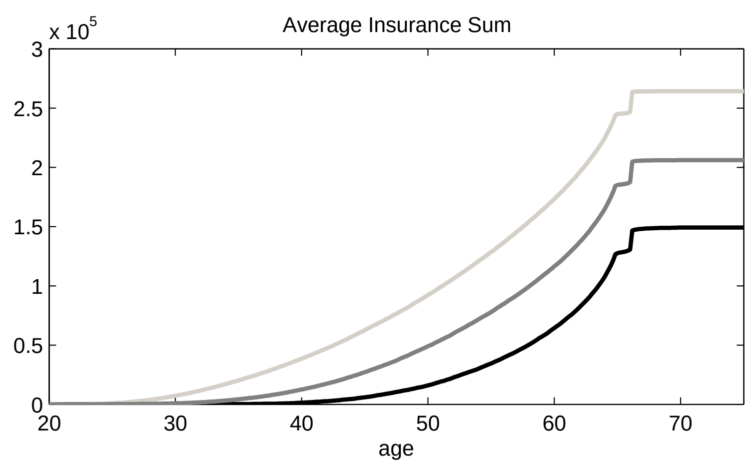

Figure 14: **Insurance Demand for Different Family Sizes.** The dark line depicts the average insurance sum over the life cycle for a family without children. The consumption scaling parameter are changed to $\phi_{1,2} = 1.6245$ and $\phi_3 = 1$. The light line gives the insurance demand with three children and the corresponding parameters are $\phi_{1,2} = 2.6850$, $\phi_3 = 2.2078$. The grey line represents the benchmark results for a family with one child and consumption scaling parameters are $\phi_{1,2} = 2.0043$ and $\phi_3 = 1.4498$. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

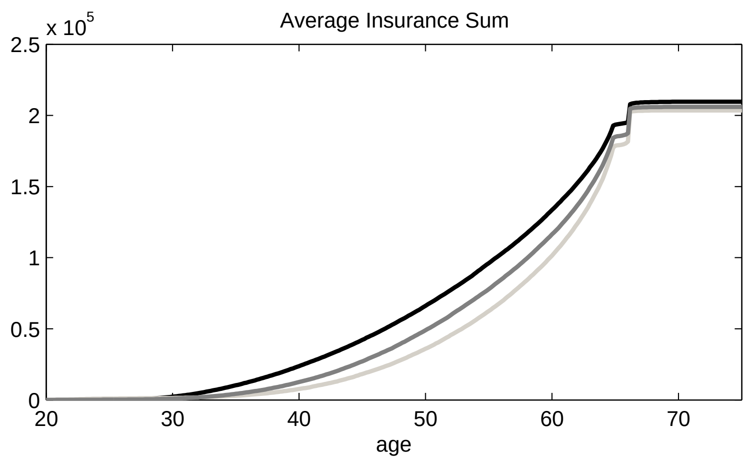

Figure 15: **Insurance Demand for Different Risk Aversions.** The figure depicts the average insurance sum over the life cycle for different values of the relative risk aversion. The dark line shows results for a low level of relative risk aversion ($\gamma = 3$), the grey line corresponds to the benchmark case with $\gamma = 4$ and the light line presents a more risk averse agent with $\gamma = 5$. The remaining parameters are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

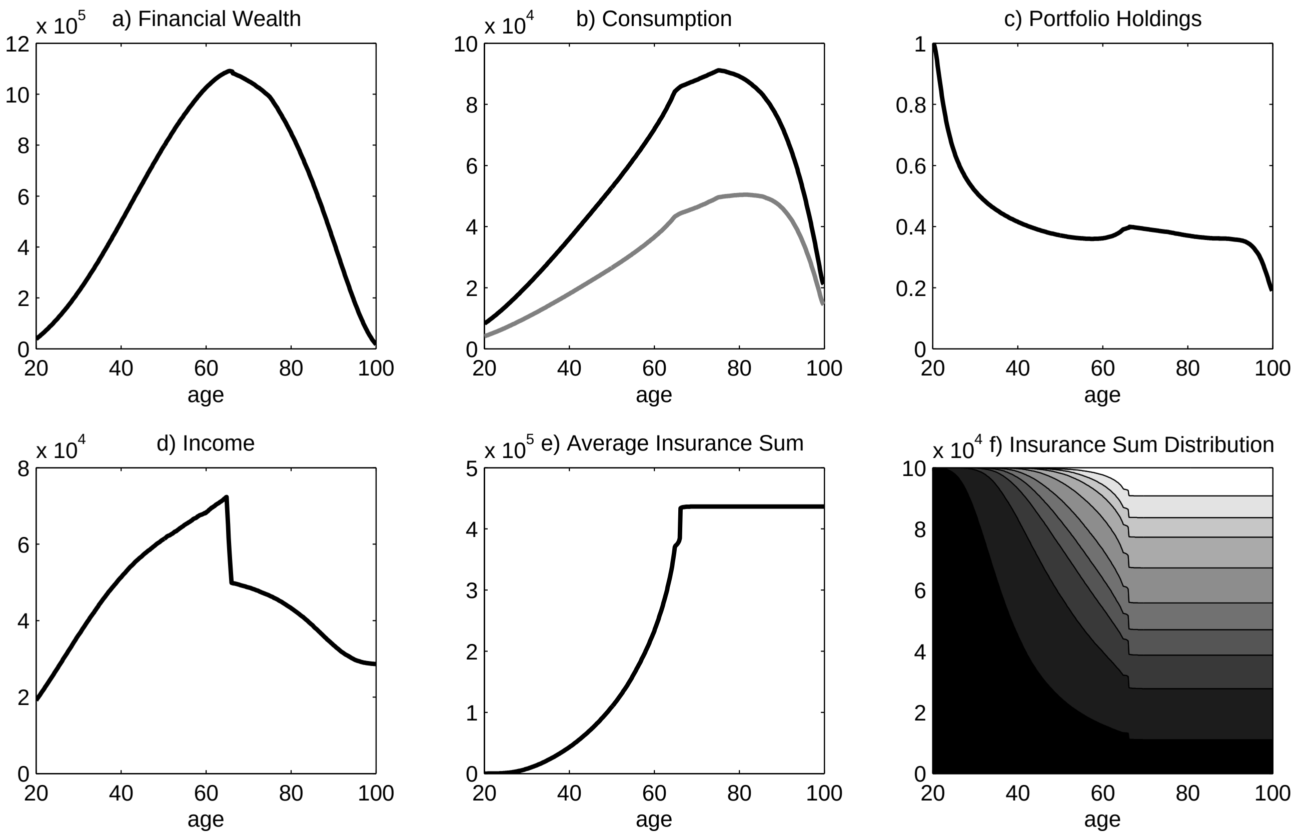

Figure 16: **Average Key Variables over the Life Cycle with Social Security.** The graphs depict the average optimal control variables as well as the average financial wealth and income evolution over the life cycle based on 100 000 simulations with a social security system as given in (6.5). The remaining calibration is described in Section 4. a) shows the average financial wealth evolution over the life cycle. b) depicts the average optimal consumption over the life cycle. The dark line corresponds to the consumption of the family, where the grey line represents the equivalent level of consumption for a one person household and is scaled with the consumption scaling parameter $\frac{c}{\phi}$. c) shows the optimal average portfolio holdings over the life cycle. d) depicts the average income of the family. e) shows the average insurance sum. f) depicts the distribution of the insurance sum over the life cycle. The darkest area marks families with no insurance contract, the white area families with the highest insurance sum (2 000 000) and the grey areas families with intermediate insurance sums.

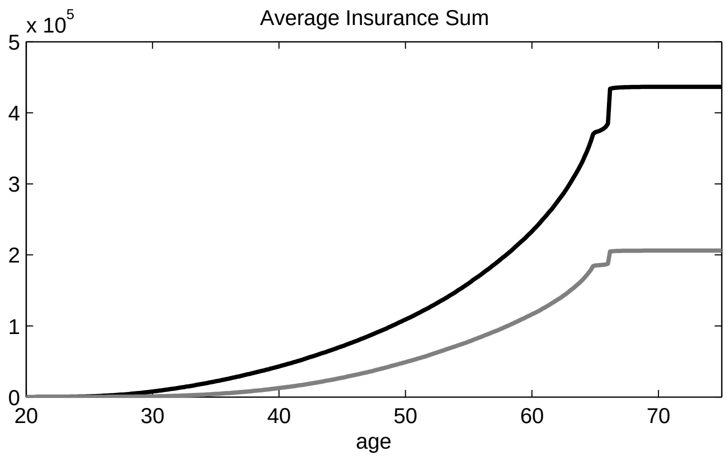

Figure 17: **Insurance Demand with Social Security System.** The figure depicts the average insurance sum over the life cycle for a different income reduction at death $p^{1,3}, p^{2,3}$. The dark line presents results of a model with social security system as given in (6.5). The grey line corresponds to the benchmark results without social security system $p^{1,3} = p^{2,3} = 0$. The remaining parameter are calibrated as stated in Section 4. The results are based on 100 000 simulations for each model.

	Financial Market	
μ_S	Stock drift	0.06
σ_S	Stock volatility	0.2
r	Bond drift	0.02
	Preferences	
δ	Time preference rate	0.03
γ	Relative risk aversion	4
ε	Weight of the bequest motive	1
α_{Adult}	Number of adults in the household	2
α_{Child}	Number of children in the household	1
T	Time horizon of the family	80
X_0	Initial financial wealth	38 214
	Mortality Risk	
x	Age of the wage earner at $t = 0$	20
m	X-axis displacement	89.45
b	Steepness parameter	6.5
k_1	Constant impact of a health shock	0.048
k_2	Age-dependent impact of a health shock	0.0008
	Health Shock Risk	
a	Scaling parameter	0.02489
b	X-axis displacement	66.96
c	Steepness parameter	29.42
	Income	
ξ_0	Age and education independent wage increase	0.02
b	Education dependent wage increase	0.1682
c	Education and age dependent wage increase parameter	−0.00323
d	Education and age dependent wage increase parameter	0.00002
P	Replacement ratio	0.68212
T_R	Retirement time	45
Y_0	Initial income	19 107
σ_Y^w	Volatility while working	0.2
σ_Y^r	Volatility during retirement	0
ρ^w	Correlation with the stock while working	0
ρ^r	Correlation with the stock during retirement	0
$p^{1,2}(t < T_R)$	Income level after a health shock while working	0.8
$p^{1,2}(t \geq T_R)$	Income level after a health shock during retirement	1
$p^{1,3}$	Income level at death without previous health shock	0
$p^{2,3}$	Income level at death with previous health shock	0
	Insurance	
ψ_{ad}	Administrative fee	0.0299
ψ_{tr}	Transaction fee	0.0505
T_C	Latest time for changing the insurance contract	50
T_I	Contract maturity	55

Table 1: **Benchmark Calibration Parameters.**

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