The Valuation of Employee Stock Options – How Good is the Standard?*

Peter Raupach
Goethe University Frankfurt am Main

January 24, 2003

Abstract

This study contributes to the valuation of employee stock options (ESO) in two ways: First, a new pricing model is presented, admitting a major part of calculations to be solved in closed form. It incorporates a vesting period and independent, forced terminations of the contract. Designed with a focus on good replication of empirics, the model fits with given exercise characteristics better than earlier models. In particular, it is able to account for the correlation of the time of exercise and the stock price performance at exercise, suspected of being crucial for the option value. The impact of correlation is weak, however, whereas the average rate of cancellations turns out to play a central role. The second contribution of this paper is an examination to what extent the current accounting standard of ESO valuation is subject to discretion and potential misspecification. Given my model is true, the standard is a good proxy. Yet, one important input for the standard valuation process is often unobservable at least for outsiders. Using my model as an example how the accountant's belief on that input could interact with the truth, I show that there is wide latitude in the determination of prices.

JEL classification: G13; J33; M41; M52

Keywords: Employee stock options; Executive stock options; Barrier options; Exercise Behavior; Fair value accounting

1 Introduction

Firms use employee stock options (ESO) in order to align the interests of employees to the long-term interests of shareholders. The question of how good stock options perform in this discipline is a true challenge for theorists. Ignoring one side of the coin – incentives, the more difficult side – I focus on the cost of ESO to shareholders. When shareholders grant ESO or similar incentive instruments to their employees, they want to know how much they have to pay for incentive. The costs may become substantial in practice; for instance, a sample of 239 German IPOs shows 43 firms with a ratio of outstanding ESO to outstanding shares above 0.1.[1]

*I would like to thank Jennifer Carpenter and Mark Vargus for the kind provision of data and useful hints. I am indebted to Steven Huddart for a statistical analysis and to Mark Wahrenburg and Gunter Löffler for useful discussions and comments. Seminar participants at Goethe University provided useful comments.

[1]Private sample, unpublished.

While the assumption that shareholders are unrestricted in trading stocks and treasury notes seems reasonable, one cannot ignore that employees must neither sell their ESO nor hedge them. It may be a consequence of these restrictions that most grantees exercise their options considerably earlier than standard option pricing theory suggests (relying on unrestricted option holders). Possibly, risk-averse option holders decide to forego a part of an option's time value in favor of secure cash obtained by early exercise.

No matter what the reason is, all valuation models have to pay regard for early exercise either way. Literature presents two main types of models. *Rational* models try to explain *why* an option holder might exercise options early. In contrast, *heuristic* models just specify *when* they usually exercise. Formally, heuristic models specify the joint distribution of price process and time of exercise. When outsiders are interested only in the cost of stock options, a heuristic model is sufficient. They need to know how likely what cash flows become due under which condition.

Since there are no market prices for ESO, some other observable characteristics of the history of option exercises must be utilized to see how good they correspond to their model counterparts. Rational models may give a deeper insight than heuristic ones. The latter, in contrast, can be designed easier to fit well with real exercise patterns while keeping things reasonably simple. Setting up a heuristic pricing model boils down to the following:

- Choose some characteristics of exercise behavior (like the mean time of exercise), serving as empirical benchmarks for the fit of a model.
- Estimate the characteristics.
- Model the exercise behavior.
- Fit the model under the physical probability measure.
- Compute prices under a corresponding risk-neutral measure.

The contribution of this paper is twofold: *First*, a heuristic pricing model for plain call options with a vesting period is presented. The simple structure allows to solve essential parts of the formula in closed form. The model adapts to a set of empirical characteristics of exercise better than other models known from the literature. Conform to the results of Carpenter [3], the average rate of option cancellations (one of the characteristics) has by far the strongest price impact. *Second*, I use the model to get a picture to what extent the current standard of ESO valuation, SFAS 123, is subject to discretion and potential misspecification.

1.1 Previous Research

Let me begin with explaining some terms. By *termination* I mean the end of the option contract by *any* reason. *Exercise* denotes the end of a contract with a payoff greater than zero, while *cancellation* is a termination with zero payoff, with no regard to the reasons. I use *forfeiture* as a synonym for cancellation. *Premature* means "not at expiry", no matter what is optimal.

Obviously, the value of an ESO grant shrinks if some option holders forfeit them, possibly since they leave the firm before vesting or while the option is out of the money. The value also declines when they exercise options at share prices different from the optimal killing price.[2] The current standard method for ESO valuation, SFAS 123, reflects

[2]Confer Barone-Adesi and Whaley [2].

non-optimal exercise as follows: The dividend-adjusted Black/Scholes option price is calculated with a maturity equal to the expected lifetime of the option, given it vests. In order to correct for the forfeiture of options, the result is multiplied by the probability of the option being vested.

By these adjustments, the SFAS method picks a certain exercise strategy with some arbitrariness: Ignoring the (weak) concavity of the Black/Scholes price in time, the SFAS price is correct[3] if options terminate at some independent random time – in the money or not, vested or not. The distribution of the termination time has to produce only the given probability of termination before vesting and the appropriate expected value under vesting. On the one hand, independency is rather implausible for a number of reasons, as Rubinstein [20] argues. On the other hand, the relation between price path and exercise decision can have a large impact on the value of options. For illustration, compare a world of option holders deciding on exercise at pure random (roughly conforming to SFAS) with a world of utility-maximizing risk-neutral option holders, to be completely free in their decision. Given there are no forfeits before vesting and given some representative stock-related parameters[4], a risk-neutral holder would exercise a ten-year ESO – at the killing price, like standard theory predicts – on average after 7.5 years. Given further, the independent termination time in the SFAS world ended up at the same average, the corresponding price would be 13% below. Evidently, a world of risk-neutral, unbiased option holders is far from reality. But at the level of information demanded by SFAS 123, this world is not less arbitrary than that of independency.

Several authors have modeled the *rationales* behind the crucial relation between exercise decision and stock price path, as I call it, by a rational behavior of restricted option holders and some sort of "game" they are exposed to. For instance, Kulatilaka and Marcus [16], Huddart [9], Rubinstein [20], or Hall and Murphy [7] assume that a representative risk-averse option holder decides between holding the option or exercising it and investing the proceeds in the riskless asset.

Of course, rational models are indispensable when incentives to employees are examined. Yet, the less-demanding heuristic approach is justifiable for the sole purpose of valuation, too: Not explaining the exercise behavior thoroughly, instead supposing some probability law of exercise that accounts well for empirical observations. The SFAS 123 method obviously follows this "reduced-form" approach. Jennergren and Näslund ([11] and [12]) incorporate early exercise by introducing an external, independent stopping time as a proxy for option holders resigning or getting fired. If stopped, the option is liquidated at its current intrinsic value. If not, the option considered in [12] pays off only at expiry (like a European option), which allows for a nearly closed pricing formula. The model is therefore an example for the concept of independent termination, to be close to the SFAS methodology as discussed above. The American counterpart is considered by the same authors in [11]: Given the option is not stopped, the risk-neutral holder can freely decide on exercise. The barrier model presented in this paper adopts independent stopping from Jennergren and Näslund, yet the part of "free" decisions differs from that model.

Rubinstein [20] notes that it is difficult to estimate relevant input factors reliably. His estimate of the option value gives a rather radical lower bound but is based on few (and reliable) factors. Such simple estimates are easier to be compared between different firms.

Carpenter [3] compares the heuristic model of Jennergren and Näslund [11], called *extended American model*, with a three-parameter rational model. In a first step, both of

[3] Under some assumptions on the remaining risk of imperfect hedges; cf. sect. 2.3.

[4] See sect. 3.1.

the models are calibrated in order to reproduce a number of statistical values on exercise patterns. Information on stock price paths and option exercises is obtained from a sample of ESO grants in 40 firms. The following benchmarks are used for the goodness of fit: 1. the mean lifetime of an option (given exercise); 2. the normalized mean stock price at exercise; 3. the mean cancellation rate as an average over the time from grant to maturity. The cancellation rate thus is a mix of forfeitures before vesting, after vesting and expirations out of the money. Several parameters form a set of conditions under which the exercise characteristics should be reproduced by the model: the length of vesting period, the mean stock return under the real-world probability measure, volatility, dividend rate and the normalized mean stock price at expiry[5]. In a second step, the best-fitting parametrizations of the models are used to forecast the exercise characteristics on behalf of each firm's specific stock price parameters. In either step of investigation the extended American model appears to perform as well as the dynamic one. The extended American model gives prices strikingly similar to that of the SFAS approach, thus supporting the appropriateness of the SFAS statement 123.

1.2 Adapting for Correlation

At this point I pick up the thread. Although the characteristics of exercise considered by Carpenter [3] are certainly relevant, the correlation between exercise time and stock price performance at exercise is well worth a look.

Suppose for the moment that all ESO holders behave like unrestricted rational investors. They decide to exercise the options if the stock price hits the downward-sloping curve of killing prices, which is well-known from the theory of American options. Given a sample of exercise events for a dividend-paying stock, time and stock price of exercise will be in a strictly negative relation. For example, in the representative setting from Carpenter [3] a correlation of -0.86 is obtained.[6] In the rational model of Hall and Murphy [7], risk-averse employees exercise options along downward-sloping lines of prices, too. Real-world samples of exercise, however, show a positive correlation. In the Carpenter dataset, it amounts to 0.14, while S. Huddart reports a correlation of about 0.2 for a sample of over 50,000 ESO holders from seven companies.[7] It seems that the time/price pairs at exercise form a "cloud" of a very different shape, signalling inefficiency from the viewpoint of an unrestricted rational investor. A switch from negative to positive correlation, leaving other characteristics constant, should reduce the option's value since a higher level of late payoffs (at high discounts) will not offset the shrinkage of early payoffs (at low discounts). Since ESO runtimes are quite long, the change of discount factors could be significant.

Correlation may also play a role in the following problem: Recall that, instead of maturity, the SFAS method enters the mean lifetime of an option (given it vests) into the Black/Scholes formula. The time is estimated under real-world probabilities, but used to compute a risk-neutral expectation. While a change of measure leaves an independent stopping time unchanged, it will alter the distribution of times to be correlated with stock price at exercise. Suppose that the correlation is strongly positive [negative]. The change

[5] I will argue in sect. 3.1 that a single point is not the best way to account for abnormal market returns.

[6] Parameters are quoted in sect. 3.1.

[7] Unpublished, private correspondence; for a description of the sample, see Huddart and Lang [10] or Heath et al. [8]. Instead of time from grant to exercise, the time left to expiration was used, yielding a correlation of -0.21. It is roughly, but not precisely the same as a constant minus exercise time since the maturities vary.

from the physical measure to the risk-neutral one will usually diminish the expected performance at exercise. Given the correlation is about the same under both measures, the expected time of exercise plausibly should decrease [increase] accordingly. In such settings, the SFAS model uses an overestimated [underestimated] expected exercise time, leading to higher [lower] prices.

In order to get a better fit with empirics, I modify the model of Jennergren and Näslund [11] as follows. Two independent events trigger an option exercise. First, and just like in the earlier models, an independent random stopping event may enforce that the option is paid off at the inner value – be it in the money or not. An unvested option pays zero in case of being stopped. Second, an employee is supposed to exercise her (vested) options if the stock price hits some deterministic, exponentially growing target. Formally, it is nothing but the trigger of a barrier option, giving reason to call the whole setting a *barrier model*. Its exercise-related parameters are the barrier's height and growth rate plus the constant intensity (hazard rate) of stopping.

Assuming a single barrier to be representative for all option holders seems merely restrictive. One could instead imagine a portfolio of ESOs held by a group of employees with different barriers. Yet, I stick to a single barrier since numerical examples show that it is almost negligible whether the barrier is diversified or not. The simple structure of exercise decisions allows to solve parts of the formula analytically. A double integral is left, to be treated with standard numerical methods.

For comparison, I will refer to Carpenter's empirical values from [3]. Compared with the extended American model, the barrier model suits better not only to the three characteristics used there, but allows to adapt for the correlation between exercise time and stock price at exercise as well. Since the extended American model cannot produce a positive correlation, it only makes sense to compare prices when correlation is ignored through adapting. If so, the characteristics from [3] yield a barrier model price, which is 9% below that of the extended American model.

I argued why correlation could affect the value of an ESO. Correlation, however, is just one determinant of the model among four. I cannot calibrate any of the characteristics leaving the others untouched, since there are only three model parameters. In order to find out what characteristic should be measured with particular diligence, I vary each characteristic by a small proportion of its standard deviation, seeing how much the newly-adapted price has changed. In this sense, the average cancellation rate is by far the most important parameter, followed by the mean stock price at exercise, and the exercise time. The correlation has the weakest impact, though strong enough that it should not be ignored.

Calibrating the barrier model with the empirical characteristics yields a mean stopping rate of about 9 %. It is, in contrast, impossible to get a good fit without independent stopping. In other words, a pure strategy of exercising at an exponential barrier is unable to explain the exercise pattern well. It conforms to Carpenter's result that idiosyncratic stopping events play an important role. Furthermore, possibly one half or more of the stopping events seem not to be caused by a fluctuation of staff: Boldly inferring from top executives to all ESO grantees, about 3% of option holders leave their company through one year by reasons that are expected to trigger an immediate option exercise.[8]

[8] Some empirical evidence on management turnover is collected in the appendix, sect. 5.6.

1.3 Discretion in Implementing SFAS 123

With this paper, I seek to provide a valuation model which is not too complicated but flexible. In addition, I use the model to investigate how prices according to the SFAS method could relate to a true value. Encouraged by the flexibility of the barrier model, I venture to take it for the truth, looking at the "errors" of the SFAS 123 prices with regard to the prices of the "true" barrier model.

The valuation method of SFAS 123 integrates, besides market-based parameters, two characteristics of exercise: the probability that an option vests, and the mean lifetime of an option under condition that it vests. Given a parameter set for the barrier model, I compute the implicit SFAS input parameters and compare the SFAS price with the "true" one. For a wide range of barrier model parameters, the error is rather small. The strong link relies, however, on the assumption that the probability of vesting can be measured properly, which is doubtful. The typical outsider, who must rest on standard filings, cannot separate whether an option was cancelled before or after vesting or simply because the option expired out of the money. From public data, cancellations are usually determined by comparing an executive's option holdings with the number of exercised options. Since only the aggregated number of holdings is reported, cancellations usually cannot be addressed to grants. The best of what can be estimated on a reliable basis is an average of the mean cancellation rate over the whole lifetime of an option.[9] The option's mean lifetime – the second input of the SFAS method – is often unobservable either.

An accountant having only the average cancellation rate at disposal, asked to value an ESO after SFAS 123, has to split up the overall probability of cancellation into parts before vesting, after vesting, and at expiration – by any assumption. Since there is no explicit rule how to estimate the probability of vesting, every model could be accepted that connects a given cancellation rate with some probability of vesting either way.

I use the barrier model as an example how the connection between cancellation rate and probability of vesting could be established. Assuming that some cancellation rate and mean exercise time have been observed, I select all model parameter triplets as candidates for the connecting model if only they produce the correct cancellation rate. Similar, models delivering the right cancellation rate and mean exercise time are identified to be candidates for the "truth" of exercise behavior. The "truth" and "the accountant's belief" do not need to coincide, resulting in proportional price discrepancies between -22% and $+10\%$.

Although I think that the barrier model provides a better picture of the truth, the advantage of simplicity of the SFAS method is not to be neglected. Under the barrier model, the SFAS 123 method could be regarded as a reliable proxy if the inputs were defined with more precision. The lack of specifying how the probability of vesting should be estimated is a loophole, which can be closed at low effort. The simplest way would be to specify how the probability of vesting should be gained from other figures.

[9] See Carpenter [3, section 5]

.

2 The Barrier Model

2.1 Assumptions

Suppose the share price S follows the stochastic differential equation

$$dS_t = S_t \mu \, dt + S_t \sigma \, dW_t \tag{1}$$

with a standard Brownian motion on a complete probability space $[\Omega, \mathcal{F}, \mathbf{P}]$. Let μ and σ be constant. The stock pays dividends at a constant rate of δ. These values are assumed not to be under control of the option holder through the lifetime of an option. The money market account pays out a constant yield rate r. Shareholders, who authorize ESO grants, are assumed to be unrestricted both in holding stock and investing in the money market account. Moreover, they can trade continuously.

The employee holds a plain, not dividend-protected[10] call on S with strike price K, starting at time 0 and expiring in T. It is not exercisable until V, $0 \leq V \leq T$, and fully vested afterwards. Other exercise constraints like time windows around earnings announcements are neglected. Although they are common practise, none of the models considered here account for features like time windows either, and a comparison will not be biased strongly. Equation (1) excludes that any dilution of the share price takes place through the lifetime of the option. It is presumed that the price has been adjusted for the option grant before it starts.

The case of independent stopping is excluded at the moment and considered in section 2.3. I specify the exercise behavior as follows: Every option holder is supposed to have chosen some target of stock price performance in advance. If the target is hit, she exercises all options at once. The target is a function of type

$$b(t) = B \exp \{\alpha (t - V)\}, \quad V \leq t \leq T, \tag{2}$$

with constants $B > K$, $\alpha \geq 0$. The option holder exercises all and immediately in t if $S_t \geq b(t)$. I will refer to b as the *barrier*. Because the holder has to wait until vesting, the option is exercised in V (and not before) if $S_V \geq b(V)$. Given the price has never hit the barrier between V and T, the option matures like a European call.

In the sequel, I will consider *one single* barrier function. Such a single line is flawed as the joint distribution of exercise time and stock price is degenerate onto a zigzag line in $[0, T] \times [0, \infty)$, which seems unrealistic (see figure 1). I will attempt to replace the unique barrier by a "portfolio" of barriers in the appendix 5.5. However, the generalization is skipped since the implications are negligible.

2.2 Pricing without Independent Stopping

It is convenient to cut the option payoff into three options, each paying out only along one part of the zigzag line in figure 1. Let be

$$\tau^* := \inf \{s \geq V : S_s \geq b(s)\} \quad \text{or} \quad \tau^* := \infty \text{ if never hit.} \tag{3}$$

Consider the following events:

$$\begin{aligned} \mathcal{V} &:= \{S_V \geq b(V)\} \quad \text{(exercise at vesting)} \\ \mathcal{B} &:= \{S_V < b(V), \tau^* \leq T\} \quad \text{(barrier is hit)} \\ \mathcal{E} &:= \{S_V < b(V), \tau^* > T\} \quad \text{(termination at expiry).} \end{aligned}$$

[10] Protection against dividends is desirable from a perspective on agency conflicts but rarely observed.

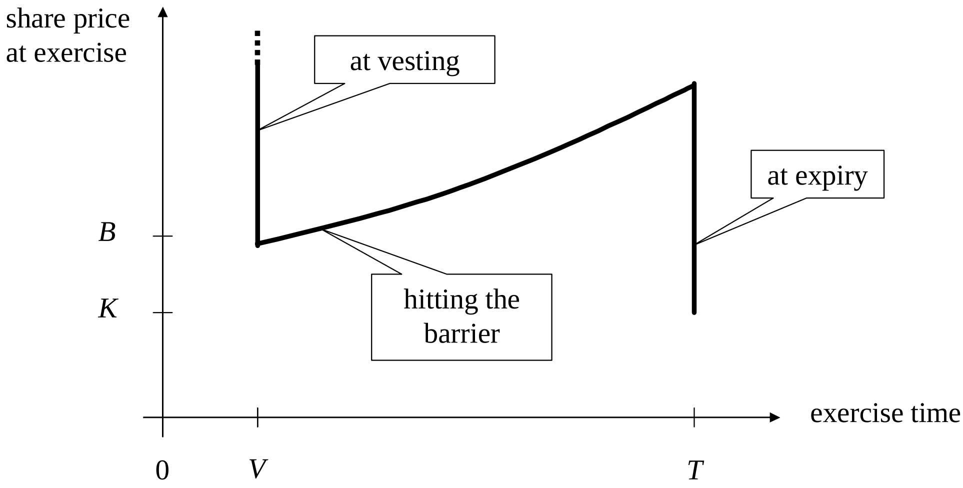

Figure 1: Support of the joint distribution of exercise time and the price at exercise; the vertical line on the left consists of exercises immediately after vesting; the flat line growing from B at time V until T represents hits of the barrier, leading to an option exercise; the vertical line on the right are exercises in the money at expiry.

Then the random option lifetime τ of the option can be written as

$$\tau = \begin{cases} V & : \mathcal{V} \\ \tau^* & : \mathcal{B} \\ T & : \mathcal{E} \end{cases}$$

The random value τ is, the same as τ^*, a stopping time of the augmented filtration $\mathbb{F}^S := \left(\mathcal{F}_t^S\right)_{t\geq 0}$ generated by S.[11] It meets the prerequisites for the definition of American contingent claims in the sense of Musiela and Rutkowski [18, chapter 8.1]. Such contingent claims can be priced in the absence of arbitrage as the expectation of the discounted payoff under the unique equivalent martingale measure $\mathbf{Q}$.

Given the call option is terminated at τ, the option payoff equals $\pi(\tau) := [S_\tau - K]^+$. Its value in $t = 0$ is denoted by P and fulfils

$$P = \mathbf{E_Q} e^{-r\tau} \pi(\tau) = \mathbf{E_Q} e^{-r\tau} [S_\tau - K]^+$$

Here, $\mathbf{Q}$ is the $\mathbf{P}$-equivalent measure under which $\ln S$ is an arithmetic Brownian motion with volatility σ and constant drift $r - \delta - \sigma^2/2$. Now I consider parts of the contingent claim paying only in the events $\mathcal{V}$, $\mathcal{B}$ and $\mathcal{E}$:

$$\begin{aligned} \pi_1(\tau) &: = [S_\tau - K]^+ I_{\mathcal{V}} = [S_V - K]^+ I_{\mathcal{V}} \text{ (exercise at vesting)} \\ \pi_2(\tau) &: = [S_\tau - K]^+ I_{\mathcal{B}} = [b(\tau^*) - K]^+ I_{\mathcal{B}} \quad \text{(barrier is hit)} \\ \pi_3(\tau) &: = [S_\tau - K]^+ I_{\mathcal{E}} = [S_T - K]^+ I_{\mathcal{E}} \quad \text{(exercise at expiry)} \end{aligned} \tag{4}$$

Clearly, $\{\mathcal{V}, \mathcal{B}, \mathcal{E}\}$ is a disjoint decomposition of the sure event, implying $\pi(\tau) = \pi_1(\tau) + \pi_2(\tau) + \pi_3(\tau)$. When the prices P_1, P_2, and P_3 of $\pi_1(\tau)$, $\pi_2(\tau)$, and $\pi_3(\tau)$ (i.e., the corresponding expectations) are determined, the option price is found as their sum. The expectations are evaluated by different techniques: Part $\pi_1(\tau)$ is nothing but a European

[11] See Karatzas and Shreve [15, chapter 1.2.]

call option maturing at V, with an additional hurdle at the height of $B = b(V)$. Such options are well-known and solved straightforward. Part $\pi_2(\tau)$ is similar to a barrier option but a little more complicated because the barrier implies exercises not before V. It is possible to perform parts of the integration by conditioning on $\mathcal{F}_V^S$, leaving a one-dimensional integral to be solved numerically. Part $\pi_3(\tau)$ seems, at first glance, to be a European call capped at $b(T)$, the final value of the barrier. The law of S_T under $\mathcal{E}$, however, is different from that of paths simply running from time 0 to T since all paths hitting the barrier are filtered out in $\pi_3(\tau)$. A one-dimensional numerical integral is left for P_3, too. For a detailed analysis and pricing formulae for P_1, P_2, and P_3, the reader is referred to the appendix.

2.3 Independent Stopping

This step expands the model by external, or independent, events of termination of the option. They are intended to subsume events like liquidity shocks of the option holder, dismissal, sudden disability, and things like that. Following Jennergren and Näslund [11], there is a random time $\varphi \geq 0$, to be independent of $\mathcal{F}_T^S$ and exponentially distributed with a constant intensity λ, called the *stopping rate*. The case $\varphi \leq \tau$ means that an external event enforces the ESO contract to be terminated and paid off at its intrinsic value, be it zero or not. If the option's life is stopped before vesting, the proceeds are zero. Formally, I set

$$\pi_{\text{stop}}(t) := \begin{cases} 0 & : \ t < V \\ [S_t - K]^+ & : \ \text{else} \end{cases} \quad , \ t \leq T \ ,$$

and define $\pi_{\text{stop}}(\tau \wedge \varphi)$ to be the option payoff with independent stopping. Note that, by independence, a change of measure on $\mathcal{F}_T^S$ has no influence on the distribution of φ. As a preparation for the next, consider $\pi_{\text{stop}}(\tau \wedge \varphi)$ under the measure $\mathbf{P}(\cdot\,|\,\varphi)$. The law of S and τ is not affected by this condition, which means that $\mathbf{Q}(\cdot\,|\,\varphi)$ is the equivalent martingale measure of $\mathbf{P}(\cdot\,|\,\varphi)$. The payoff $\pi_{\text{stop}}(\tau \wedge \varphi)$ for fixed φ is the same as the non-stopped $\pi(\tau)$ from section 2.2 if the expiration term T was replaced by $T \wedge \varphi$. In order to give emphasis to the impact of T on the payoff $\pi(\tau)$, I write it $\pi(\tau, T)$ henceforth, and its price $P(T)$ accordingly. It follows that under $\mathbf{P}(\cdot\,|\,\varphi = t)$

$$\text{price}_{\varphi=t}(\pi_{\text{stop}}(\tau \wedge \varphi)) = \mathbf{E}_{\mathbf{Q}} e^{-r(\tau\wedge t)} \pi(\tau, T \wedge t) = P(T \wedge t) \ . \tag{5}$$

Returning to the unconditional measure, the contingent claim $\pi_{\text{stop}}(\tau \wedge \varphi)$ cannot be hedged perfectly by holding shares and the riskless asset because $\tau \wedge \varphi$ is obviously not a stopping time of $\mathbb{F}^S$, which destroys the integral representability of $\pi_{\text{stop}}(\tau \wedge \varphi)$. Clearly, a sole arbitrage argument is unable to derive a unique price. I assume, as the preceding authors, that there is no premium for the additional risk arising from independent stopping. Section 5.4 in the appendix gives a formal justification based on a diversification argument. The contingent claim $\pi_{\text{stop}}(\tau \wedge \varphi)$ is therefore priced just like a perfectly hedgeable option at its expected present value. This immediately leads to the price formula. Let $\lambda e^{\lambda t}$ be the density of φ. With $\mathbf{Q}(\varphi \geq T) = e^{-\lambda T}$, I get from (5)

$$\begin{aligned} P_{\text{stop}} &= \mathbf{E}_{\mathbf{Q}} \text{price}_{\varphi}(\pi_{\text{stop}}(\tau \wedge \varphi)) \\ &= \mathbf{E}_{\mathbf{Q}} e^{-r(\tau\wedge\varphi)} \pi(\tau, T \wedge \varphi) \\ &= \mathbf{E}_{\mathbf{Q}} e^{-r\tau} \pi(\tau, T) I_{\{\varphi \geq T\}} + \mathbf{E}_{\mathbf{Q}} e^{-r(\tau\wedge\varphi)} \pi(\tau, \varphi) I_{\{\varphi < T\}} \\ &= e^{-\lambda T} P(T) + \int_V^T P(t) \lambda e^{\lambda t} \, dt \end{aligned} \tag{6}$$

The integral must be computed numerically since $P(t)$ is determined by numeric integration either. The calculations altogether lead to a two-dimensional integral, which needs (potentially) much time to be computed. However, smooth integrands make the algorithm converge fast[12].

3 Comparison of the Models

3.1 Calibration

The question how an ESO pricing model should be calibrated is quite delicate since there are no market prices available. Following the approach of [3] and previous authors, I try to reconcile certain characteristics of the probability law of S, τ and φ with their empirical counterparts. The choice of characteristics may have large impact on prices, but the same is true for the probability measure that is chosen to compute the corresponding characteristics in the model. It may be useful to take a conditional measure in order to account for, say, an atypical market environment. Let $\mathbf{M}$ be that probability measure, the specific shape of which I will discuss below. Consider the following model characteristics of exercise:

- the mean lifetime of an option given that it is exercised, denoted after introduction of $\kappa := \tau \wedge \varphi$ by
$$\widehat{\kappa} := \mathbf{E_M}\left[\kappa \,|\, \text{exercise at positive payoff}\right] ;$$
- the mean stock price at the time of exercise, normalized by the strike price – likewise under condition of exercise, denoted by
$$\widehat{S_\kappa} := \mathbf{E_M}\left[S_\kappa \,|\, \text{exercise at positive payoff}\right] ;$$
- the average cancellation rate, i.e., the average over $[0, T]$ of the expected rate of forfeiture, given the option was not terminated before. Note that holders can forfeit their options by stopping before vesting, by premature stopping of an underwater option or by expiration out of the money. The cancellation rate is nothing but a hazard rate, known from the credit risk literature or from reliability theory.[13] Formally and in continuous time,
$$\widehat{c} := \int_0^T \frac{\mathbf{M}\left(\kappa \in dt\,,\, \text{zero payoff at } t\right)}{\mathbf{M}\left(\kappa \geq t\right)} .$$
- the correlation of exercise time and stock price at exercise, given exercise (at a positive payoff):
$$\widehat{\rho} := \operatorname{corr}_{\mathbf{M}(\cdot\,|\,\text{exercise})}\left(S_\kappa, \kappa\right)$$

The variables $\widehat{\kappa}$, $\widehat{S_\kappa}$, $\widehat{c}$ have already been suggested by Carpenter [3]. For reasons explained in section 1.2, I add $\widehat{\rho}$, which causes no additional effort in the provision of data. The corresponding empirical values will be labeled by a tilde: $\widetilde{\kappa}$ etc.

[12] The routines are written in C++ code. They value an option within 2 seconds on a 400 MHz personal computer.

[13] Confer Barlow and Proschan [1], for example.

characteristics (firm-specific)	characteristics (average) value	notation	standard deviation
mean time of exercise	5.83	$\widetilde{\kappa}$	2.25
mean stock price perf. at exercise	2.75	$\widetilde{S_\kappa}$	1.42
mean cancellation rate	7.3 %	$\widetilde{c}$	7.1 %
correlation of κ and S_κ	0.14	$\widetilde{\rho}$	0.14[14]
volatility σ_i	31 %	σ	(10 %)
proportional dividend rate δ_i	3 %	δ	(2 %)
vesting period V_i	1.96	V	(1.03)
mean stock price perf. at expiry $S_{10,i}$	327	S_{10}	(225)

Table 1: Summary statistics on exercise of ESO and stock price related information from the sample in Carpenter [3]; values in brackets are not referred to.

Carpenter [3] analyses a sample of ESO exercises from 40 firms, indicated here by i. The above exercise/forfeiture characteristics plus firm-specific parameters like volatility σ_i, dividend rate δ_i, mean length of the vesting period V_i and the mean stock price return S_{10}/S_0 from grant to expiration of the ESO have been calculated as firm-wide averages over grants. All contracts have been running over 10 years. The firm-specific averages of every variable form the final sample. For lack of own data, I will refer to this sample, parts of which are at my disposal by courtesy of J. Carpenter. For comprehensive descriptive statistics, see Carpenter [3, table 1]. The excerpt used here is found in table 3.1. I will refer to these values as a benchmark throughout the empirical part of this paper, trying to adapt the model to $\left(\widetilde{\kappa}, \widetilde{S_\kappa}, \widetilde{c}, \widetilde{\rho}\right) = (5.83,\ 2.75,\ 0.073,\ 0.14)$.

The remaining input parameters are taken from the same source: the mean annual stock return $\mu = 15.5$ %, the riskless return rate $r = 7\,\%$ and a time to expiration of $T = 10$. It is assumed that $S_0 = 1$ and that all ESO are granted at the money.

Since on the one hand every sample is at least biased by random and, on the other hand, several of the observed values are correlated, it may be a good idea to account for anomalies by an appropriate choice of $\mathbf{M}$. The simplest approach is to ignore them, taking averages just as they are. In other words, the characteristics are computed under the physical measure $\mathbf{M} = \mathbf{P}$, and S_{10}, the mean stock price performance at expiry, is ignored. As a sophisticated alternative, one could condition every exercise decision on the stock price path the option holder really witnessed. In this case, $\mathbf{M}$ should be close to the measure that assigns probability N^{-1} to each of N observed paths and zero to the rest.[15] This approach exploits a maximum of information (and needs high computational effort) and must be based on more detailed data.

Carpenter takes a way "in between": Her approach accounts for atypical stock returns by setting $\mathbf{M} = \mathbf{P}\left(\,\cdot\,|S_{10}\right)$. The physical measure is taken under the condition that every path ends in the mean stock price performance at expiry, which is 3.27 here. It means that all functionals are computed under some Brownian bridge.[16] In my opinion, an adverse effect outweighs the advantages: Under the Brownian bridge, ending with a deterministic positive return above K, not a single option expires out of the money. All probability of

[14] Obtained from a bootstrap algorithm.

[15] In a continuous-time model, $\mathbf{M}$ must be absolutely continuous with regard to $\mathbf{P}$ in order to enable inference from the model onto the model characteristics. Since a normalized counting measure of a finite number of paths is not integrable under the Wiener measure, an absolutely continuous measure "close" to the counting measure would have to be used.

[16] See Karatzas and Shreve [15].

cancellation must therefore go back to *premature* termination of underwater or unvested options – in contrast to the sample, where 15 % of the firms have a negative mean return.[17] The impact of underwater expiration is considerable. For example, I apply the extended American Model from Carpenter [3, section 3.3] to the setting of table 3.1. Carpenter reports that an annual stopping rate of $\lambda = 11\,\%$ is necessary to produce a cancellation rate of $\widehat{c} = 7\,\%$ under the Brownian bridge. In contrast, when $\mathbf{M} = \mathbf{P}$, as specified for my model, setting $\lambda = 5.6\,\%$ leads to $\widehat{c} = 7\,\%$. The stopping rates correspond to prices of 0.2664[18] ($\lambda = 11\,\%$) and 0.3214 ($\lambda = 5.6\,\%$), which is a difference of 21 %.

Notation The vector $\left(\widehat{\kappa}, \widehat{S_\kappa}, \widehat{c}, \widehat{\rho}\right)$ is denoted by $\widehat{\theta}$. Its corresponding empirical counterpart is $\widetilde{\theta}$. A subscript like $\widehat{\theta}_{1101}$ indicates a sub-vector of $\widehat{\theta}$, with elements eliminated that correspond to zero.

For lack of grant-specific exercise data I will set $\mathbf{M} = \mathbf{P}$, which means that the empirics are compared to the characteristics under the physical measure. In order to obtain a good fit of statistical and model characteristics, I seek to minimize a quadratic distance. The least value of

$$
\begin{aligned}
\mathrm{dist}_{1111} \; &: = C_1 \left(\widetilde{\kappa} - \widehat{\kappa}\right)^2 + C_2 \left(\widetilde{S_\kappa} - \widehat{S_\kappa}\right)^2 + C_3 \left(\widetilde{c} - \widehat{c}\right)^2 + C_4 \left(\widetilde{\rho} - \widehat{\rho}\right)^2, \qquad (7)\\
\mathrm{dist}_{1110} \; &: = C_1 \left(\widetilde{\kappa} - \widehat{\kappa}\right)^2 + C_2 \left(\widetilde{S_\kappa} - \widehat{S_\kappa}\right)^2 + C_3 \left(\widetilde{c} - \widehat{c}\right)^2, \\
\mathrm{dist}_{1101} \; &: = C_1 \left(\widetilde{\kappa} - \widehat{\kappa}\right)^2 + C_2 \left(\widetilde{S_\kappa} - \widehat{S_\kappa}\right)^2 + C_4 \left(\widetilde{\rho} - \widehat{\rho}\right)^2,
\end{aligned}
$$

is searched by varying B, α and λ. The coefficients C_1 to C_4 are set equal to one over the empirical variance of the underlying characteristic. Doing so, I assign equal "importance" to each of them. When only a subset of characteristics like $\widetilde{\theta}_{1101}$ is considered, the corresponding coefficient switches to zero.

3.2 Results

I compare the prices of the extended American model and the barrier model under different specifications. One aspect of specification are the characteristics to be relevant for a good fit, i.e., the distance. Three types of distance are investigated: the full term (dist_{1111}), without correlation (dist_{1110}), and without the cancellation rate (dist_{1101}). The other aspect of specification refers to the freedom of choice for the model parameters B, α and λ. The parameters B and α are always free to be optimized. The cancellation rate is either fixed at zero (attempt 1), at 3% (attempt 2), or free for optimization like B and α (attempt 3). On behalf of attempt 1, I check whether independent stopping is essential for a good fit. Attempt 2 was motivated by a practitioner's rule-of-thumb, claiming that high-level employees fluctuate at a mean rate of 3 %.[19] The stopping rate in the extended American model, as the only parameter, is clearly free for optimization.

Comparing the model prices with those of the SFAS method is one objective of this paper but not easily achieved since the inputs of the SFAS method are only loosely connected with $\widehat{\theta}$, as described in section 1.3. A *unique* SFAS price cannot therefore be derived from a given $\widetilde{\theta}$. Instead, I compute SFAS prices *as if the model was true*: Given

[17] Note that 15% here is the proportion of *firms* with paths expiring underwater on average. I expect the proportion to be even higher in a disaggregated sample of *option grants*.

[18] Despite thorough tests, I cannot resolve a contradiction between a price of 26.6 from my own computations and that of 29 reported by Carpenter [3].

[19] See appendix, sect. 5.6 for some empirical support based on the turnover of top executives.

values for B, α and λ, the SFAS inputs relevant for exercise behavior are derived from the barrier model. The same procedure is repeated for the extended American model.

Following Carpenter, I interpret the "expected option life" as the expected time until termination (including cancellation) given that it vests, i.e. $\mathbf{E_P}[\kappa \mid \kappa \geq V]$. Its value is computed under the barrier model and inserted into the Black/Scholes formula as maturity. The Black/Scholes price is multiplied then by the probability of survival over the vesting period, which equals $e^{-\lambda V}$ here. The expected option lifetime is usually larger than $\widehat{\kappa}$; as a proxy, it exceeds $\widehat{\kappa}$ by up to 25 %. The ratio depends mainly on the starting point B of the barrier – the lower B, ceteris paribus, the more the expected option lifetime exceeds $\widehat{\kappa}$.

Table 3.2 presents prices for different types of distance and different specifications of λ. Every block of rows summarizes models that are fitted for a common type of difference but with a varying parametrization of λ. P is the model price, whereas P_{SFAS} denotes the corresponding price after SFAS 123, given the model is true. Confer also figure 2, a summary of the model prices.

First, I look how good the models fit. The extended American model seems to produce only negative correlations[20]. When dist_{1111} or dist_{1101} are applied, both including correlation, the positive target value $\widetilde{\rho} = 0.14$ forces the extended American model to make unrealistic "compromises" in other characteristics, which results in extreme prices. Correlation should therefore not be a criterion for fit of the extended American model. Under dist_{1110}, the only type that admits a "fair" comparison by neglecting $\widehat{\rho}$, the optimal stopping rate $\lambda = 8.1\,\%$ is significantly lower than that of 11 % reported by Carpenter. I attribute the difference to the choice of $\mathbf{M}$: depending on whether $\mathbf{P}$ is conditioned on the stock price at expiration or not (confer section 3.1).

Next, I check whether the concept of a barrier option alone is flexible enough to achieve a good fit, or, in other words, whether independent stopping is negligible. If λ is fixed at 0, the distance is the worst among all specifications of the barrier model. The outcomes of correlation are good, whereas a cancellation rate of $\widehat{c} \approx 4\,\%$ (now caused only by underwater expirations) is much too low. The prices under condition $\lambda = 0$ are the highest of all, coming close to the standard optimal American call price[21], which amounts to 0.392.

The barrier model is now adapted "freely", i.e. B, α and λ are subject to optimization. The barrier model fits best with the observed characteristics under all distances. Because there are more free parameters, a better fit is no big surprise. Less obviously, and remarkably, even under dist_{1110}, when correlation is cut out, the barrier model produces a *positive* correlation of 30.2 %. I judge the right sign of a "spontaneous" correlation – with due care – as a signal that some aspects of real-world exercise behavior are captured well by the barrier model.

The parametrization with a fixed $\lambda = 3\,\%$ gives a fit in between. It provides no further insight except that it probably would be a doubtful practise to estimate the stopping rate by fluctuation rates. The calibrated stopping rates of the barrier model, $\lambda = 9.8\,\%$ (dist_{1111}) or $\lambda = 9.6\,\%$ (dist_{1110}), are similar to that of the extended American model. Its high value – compared with typical rates of staff fluctuation[22] – confirms the conjecture from the bad fit of the setting with $\lambda = 0$: Some kind of externally driven terminations seem to play an important part in exercise patterns.

The *prices* of the barrier model (see also figure 2) suggest robustness regarding the

[20] Unproved; it was checked for $\lambda \in [0, 20\,\%]$, everything else kept constant.

[21] reflecting the vesting period

[22] See appendix, sect. 5.6.

model + parametrization	type of distance	fitted parameters			prices		characteristics				distance
		B	α	λ	P	P_{SFAS}	$\widehat{\kappa}$	$\widehat{S_\kappa}$	$\widehat{c}$	$\widehat{\rho}$	dist_{iiii}
barrier mod., $\lambda = 0$	1111	1.77	16.1 %		.3687	.3338	5.91	2.37	3.9 %	13.8 %	.305
barrier mod., $\lambda = 0.03$	1111	1.87	16.8 %		.3379	.3114	5.94	2.42	5.2 %	13.9 %	.148
barrier mod., λ free	1111	2.29	16.6 %	9.8 %	.2748	.2654	6.05	2.52	7.5 %	14.1 %	.037
extended Amer. mod.	1111			19.9 %	.1960	.1920	4.92	2.24	13.7 %	−4.9 %	.928
barrier mod., $\lambda = 0$	1110	3.67	−12.2 %		.3828	.3362	6.10	2.35	6.8 %	−98.2 %	.100
barrier mod., $\lambda = 0.03$	1110	3.53	−8.5 %		.3491	.3137	6.11	2.41	7.3 %	−73.0 %	.074
barrier mod., λ free	1110	1.63	66.1 %	9.6 %	.2682	.2647	5.92	2.64	7.3 %	30.2 %	.008
extended Amer. mod.	1110			8.1 %	.2944	.2784	6.23	2.53	8.2 %	−29.8 %	.072
barrier mod., $\lambda = 0$	1101	1.79	17.2 %		.3705	.3364	6.10	2.43	3.8 %	14.1 %	.065
barrier mod., $\lambda = 0.03$	1101	1.90	17.4 %		.3388	.3131	6.08	2.46	5.1 %	14.1 %	.053
barrier mod., λ free	1101	4.89	0.0 %	16.7 %	.2168	.2202	5.93	2.59	10.3 %	14.1 %	.015
extended Amer. mod.	1101			20.0 %	.1952	.1912	4.91	2.24	13.8 %	−4.8 %	2.102
empirical target of optimization							**5.83**	**2.75**	**7.3** %	**14** %	
particular target for comparison with ext. American model (dist_{1110})										−29.8 %	
barrier mod., λ free	*1111*	*290*	*0.9 %*	*6.7 %*	*.3084*	*.2865*	*6.08*	*2.45*	*7.5 %*	*-29.7 %*	*.058*

Table 2: The extended American Model and three parametrizations of the barrier model. Either model is calibrated in order to fit best with given observable characteristics of exercise: $\widehat{\kappa}$, the mean time of exercise; $\widehat{S_\kappa}$, the mean stock price performance at exercise; $\widehat{c}$, the mean cancellation rate; $\widehat{\rho}$, the correlation between exercise time and performance at exercise. The empirical target characteristics are found in row 3 from below; the corresponding model values above.
Each block of rows summarizes models optimized under one type of distance between empirics and model. "1111" includes the fit of $\widehat{S_\kappa}$, $\widehat{S_\kappa}$, $\widehat{c}$, and $\widehat{\rho}$. "1110" ignores $\widehat{\rho}$, and "1101" ignores $\widehat{c}$. In the barrier model under "$\lambda = 0$" and "$\lambda = 0.03$", only the level of the barrier $B = b(V)$ and its growth rate α are subject to optimization, under "free λ" the stopping rate is optimized as well. In the extended American Model, λ is the only exercise-related parameter. The price P is computed under the model with the fitted parameters, P_{SFAS} is the price after SFAS 123, given the model is true: the European Black/Scholes price with a maturity set equal to the expected lifetime of the option, given it vests (computed under the model), adjusted for the probability of cancellations before vesting $(1 - \exp\{-\lambda V\})$. The last column contains the distance achieved by the model. In the setting "1101" a numerical condition $\lambda \leq 20\%$ has become binding for the extended American model. The last row shows the outcome of a fit with the characteristics as above, except correlation, set equal to the model outcome of the extended American model under dist_{1110}.

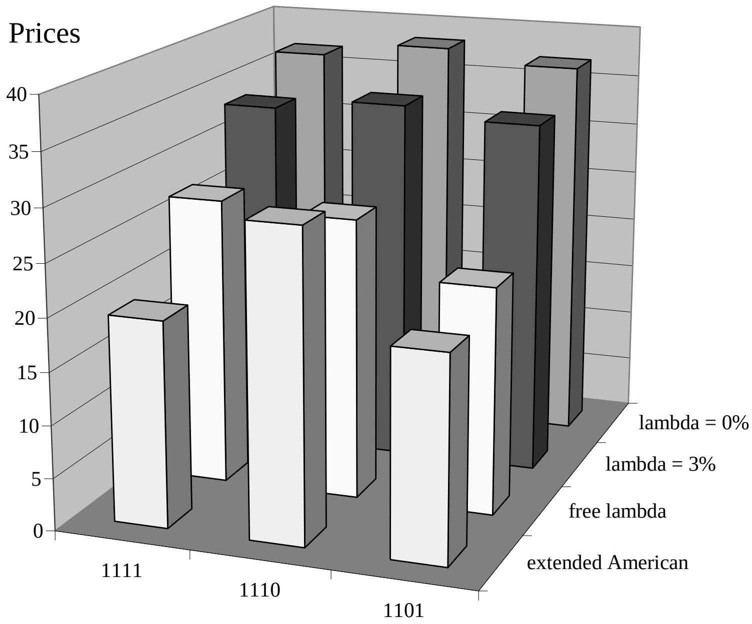

Figure 2: Prices of the extended American model and three parametrizations of the barrier model. Each model is calibrated in order to fit best with given observable characteristics of exercise. Different notions of distance between empirics form the x-axis. Here, "1111" includes the mean values of exercise time, stock price performance at exercise, cancellation rate, and the correlation; "1110" excludes correlation; "1101" excludes the cancellation rate. Under "$\lambda = 0\%$" and "$\lambda = 3\%$", only level and growth rate of the barrier are subject to a minimization of distance, whereas under "free λ" the stopping rate is also adapted.

choice of distance as long as the cancellation rate is involved. The range of prices coming from different distances is below 3.8 % for $\lambda = 0$ and $\lambda = 3\,\%$, and equals 2.5 % for free λ (without dist_{1101}). The exceptional low price under dist_{1101} signals a lack of stability when $\widetilde{c}$ is ignored. Therefore, I will drop dist_{1101} in the further.

Under each distance, the barrier model yields a considerable decrease in prices through an increase in λ (for more details, see section 3.2.2). The overall price level of the barrier model for free λ is below that of the extended American model under dist_{1110}, which amounts to 0.2976. The prices deviate by $-6.6\,\%$ (dist_{1111}) and $-8.9\,\%$ (dist_{1110}). I attribute the shrinkage of value to the "inefficiency" coupled with positive correlation, as mentioned in sect. 1.2. Let me illustrate the impact by a reference model with negative correlation: The last row of table 3.2 shows a fit of the barrier model with the original empirical benchmarks for $\widehat{\kappa}$, $\widehat{S_\kappa}$, and $\widehat{c}$, but with $\widetilde{\rho} = -29.8\,\%$, which is the outcome of $\widehat{\rho}$ for the extended American model under dist_{1110}. The barrier model then yields a price of 0.3084, now even higher than that of the extended American model.[23] Hence, the lower correlation significantly increases the price.

[23] At first glance, an "inefficient" exercise strategy, like that of the barrier model, cannot give a higher value than that of the optimal policy, to be applied in the extended American model. Note that, however, λ is much lower in the barrier model's parametrization since the barrier already shortens the average option lifetime considerably. An equal λ in both models clearly leads to lower prices for the barrier model.

3.2.1 SFAS Prices and Discretion

Figure 3 compares the model prices with their corresponding SFAS prices. The input characteristics of exercise for SFAS 123 (the probability of vesting p_{vest}, and the mean lifetime given vesting $\widetilde{\kappa}_{\text{vest}}$) are hard to be observed. Like Carpenter [3], I obtain the inputs from the fitted models, implicitly assuming the models were true. All SFAS prices are below their model counterparts (with one exception in case "free λ under dist_{1110}", a non-robust case). While the discount is around 10 % for $\lambda = 0$, it reduces to 3 % for the free λ, and to 5.6 % for the extended American model. Above I suspected the real-world mean exercise time of being too high to be a realistic (risk-neutral) maturity for the Black/Scholes formula. An overstated time could lead to higher prices. Since I observe *lower* SFAS prices in most cases, my conjecture is not confirmed.

A more elaborate check compares barrier model prices and SFAS prices for parameters on a grid G over the ranges $B \in [1.1, 5.0]$, $\alpha \in [0.0, 0.4]$, and $\lambda \in [0.0, 0.2]$. It turns out that the proportional "error" lies between -10% and $+3\%$.

It may be reasonable to tolerate an error of 10% (a hypothetical one, given the model is true) in favor of keeping things simple. However, it is doubtful whether the SFAS method could be fed with correct inputs – even if holders did behave in accordance with the barrier model. People who must rely on public data cannot estimate the probability of vesting p_{vest} directly because they will not be able to separate cancellations before vesting from those after. Some model or assumption is needed to infer p_{vest} and $\widetilde{\kappa}_{\text{vest}}$ from other characteristics. Since there is no rule how the SFAS inputs should be obtained, there is room for arbitrariness. To what extent could an accountant influence the SFAS price by a deliberate design of the estimation method for the inputs? Computing the SFAS price *given the barrier model is true* means that (by the barrier model's fitting) four exercise figures are processed in order to get p_{vest} and $\widetilde{\kappa}_{\text{vest}}$. To my knowledge, such a degree of precision is not common.

I want to investigate the latitude left to the accountant by an example, assuming that only a part of information is used in order to estimate the SFAS inputs. Suppose option holders behave in accordance with a certain (unknown) parametrization of the barrier model, leading to a publicly observable cancellation rate $\widetilde{c} = 0.073$ and a mean time of exercise $\widetilde{\kappa} = 5.83$.[24] The accountant now seeks to get her estimates of p_{vest} and $\widetilde{\kappa}_{\text{vest}}$ in accordance with $\widetilde{c}$ and $\widetilde{\kappa}$. The mean lifetime $\widetilde{\kappa}_{\text{vest}}$ is set equal to $\widetilde{\kappa}$. The accountant forms, based on the observation of $\widetilde{c}$, a "belief" about the true exercise behavior in the shape of some (possibly different) parametrization of the barrier model. She picks a barrier model that produces a correct $\widetilde{c} = 0.073 \pm 0.001$, and obtains from it the implicit $p_{\text{vest}} = \exp\{-\lambda V\}$. Yet, the belief does not need to reproduce $\widetilde{\kappa}$ likewise. This way, I demand informativeness at a level of widely accepted ad-hoc statements like "The probability of cancellation is assumed to spread evenly over the options runtime" (leading to $p_{\text{vest}} := (T - V)/T\,\mathbf{P}(\text{cancellation})$) or "...to be distributed with a constant hazard rate" (leading to $p_{\text{vest}} := \exp\{-V\widetilde{c}\}$).

With p_{vest} and $\widetilde{\kappa}_{\text{vest}}$, the accountant is ready to calculate the SFAS price. Let both, truth and the accountant's belief, be points in the grid G introduced above. Since truth and belief do not necessarily coincide, any possible true parametrization yielding $\widetilde{c} \approx 0.073$ and $\widetilde{\kappa} \approx 5.83$ may face some arbitrary belief yielding the correct $\widetilde{c}$ as well. Given $\widetilde{c}$, $\widetilde{\kappa}$, and the grid, the true model prices range from 0.275 to 0.326, whereas the SFAS prices corresponding to the accountant's belief take values in [0.253, 0.301]. Assuming every combination of truth and belief to be admissible yields proportional discrepancies between

[24] Selected from table 3.1.

-22% and $+10\%$. Since it is not clear whether all points in G are sufficiently realistic, a more restrictive scenario pairs the beliefs with the (only) truth of the parametrization $B = 2.29$, $\alpha = 16.6\,\%$, and $\lambda = 9.8\%$, which is the model's free fit under dist_{1111}. Then, mispricing lies between -8% and 9.5%. Note that the example is restricted to a universe of barrier models, suggesting that the accountant is given a wider latitude in reality.

The degree of discretion seems to be serious, leading me to assert that some precision should be added to the accounting standard. When p_{vest}, the rate of options being vested, is intransparent (a common situation, as I see it), a rule should specify how to estimate p_{vest} from the average cancellation rate $\widetilde{c}$ – from a value that outsiders can verify. One obvious way of implementation is a rule-of-thumb like "Set $p_{\text{vest}} := \exp\{-\widetilde{c}V\}$". Even if such a rule is systematically biased, at least the comparative power of SFAS option values should improve. Under the barrier model for instance, where $\lambda > \widehat{c}$, the rule was even not so bad since it increases the SFAS price relative to the "precise" SFAS price, which uses $\exp\{-\lambda V\}$. This way, the rule-of-thumb would compensate parts of the undervaluation of the SFAS method.

Alternatively, the procedure chosen to link between $\widetilde{c}$ and p_{vest} could remain at discretion of the accountant, but in this case evidence should be demanded whether the procedure conforms with characteristics beyond $\widetilde{c}$ as well. In terms of the above example, the accountant would be ordered to verify if her belief on the barrier model produces the correct $\widetilde{\kappa}$. Yet, the valuation process became more complicated, giving away the main advantage of the SFAS method – simplicity.

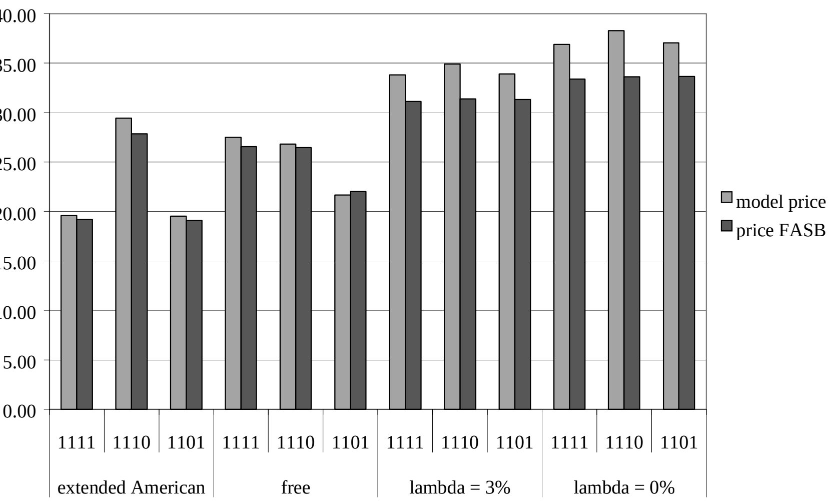

Figure 3: Model prices vs prices according to SFAS 123 under the assumption the models were true: The implicit probability of forfeiture before vesting and the mean stopping time, given that the option vests, are used as an input for the SFAS price calculation.

3.2.2 Value Drivers

The price sensitivity to the model parameters is investigated graphically. I evaluate the price as a function of B, λ and α, respectively, each for a representative number of pairs of the remaining parameters.

The price depends nonlinearly on the height B of the barrier at time V (figure 4). While the curves show strong and monotonous growth roughly up to $B = 200$, the price may even decrease beyond this value, though weakly – presumably, since an exponential barrier may crudely substitute the optimal killing price of American options. The shape of the curves does not alter substantially when λ and α are changed – only the absolute height of curves is affected. The sensitivity of the price to a change of λ, the continuous

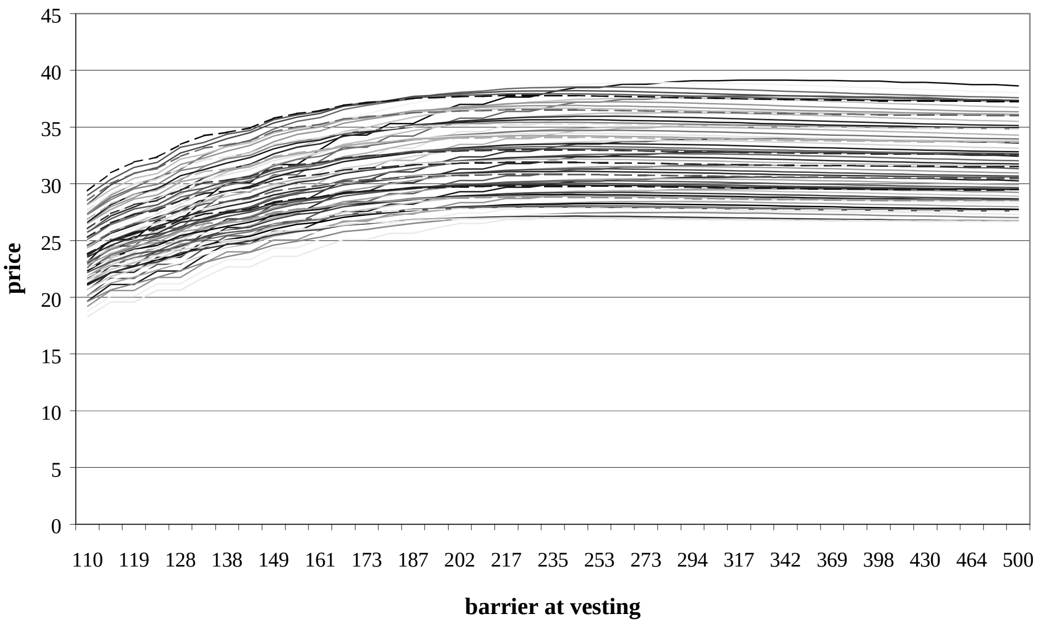

Figure 4: Prices of the barrier model as a function of B, the height of the barrier at time V, for selected pairs $(\alpha, \lambda) \in [0, 0.4] \times [0, 0.1]$, where α is the growth rate of the barrier and λ is the continuous stopping rate. The abscissa is logarithmic.

stopping rate, (figure 5) is strong, decreasing and very weakly concave for all pairs of the remaining parameters.[25] Furthermore, the steepness of the function roughly corresponds to a linear function of the absolute level of prices at some fixed λ. Compared to the other parameters, α is a weak value driver. The sensitivity is still the strongest for low B (figure 6). To sum up, the price can be called a "tame" function of the parameters (B, α, λ) since it is smooth. The parameters λ and B are more important for the price than α.

Since four characteristics of exercise are taken into account under dist_{1111}, whereas only three parameters can be calibrated in order to fit the model, I clearly cannot control a single characteristic leaving the other ones unchanged. It is therefore not obvious how a fitted model (seen as a map $\widetilde{\theta} \mapsto (B, \alpha, \lambda)$) responds to changes in $\widetilde{\theta}$. Ideally, the question should be treated with the help of a representative sample of characteristics. For lack of such data, I will present comparative statics. Taking as a reference point the optimal parametrization from the setting with free λ under dist_{1111}, the price change is measured when each component of θ alters. The change of price is expressed in units of every characteristic's standard deviation σ_j from table 3.1 in order to see which factors drive the option value "in practise". I move each characteristic j in steps of size $0.2\,\sigma_j$ from $-\,0.6\,\sigma_j$ to $+0.6\,\sigma_j$. The range is narrow. First, to avoid problems with nonlinearity;

[25] The relation admits quadratic interpolation. A parabola, pinned at the prices for $\lambda \in \{0;\ 5\,\%;\ 10\,\%\}$, yields proportional errors less than 10^{-3} within $\lambda \in [0, 10\,\%]$. The error is around 10^{-2} for linear approximation.

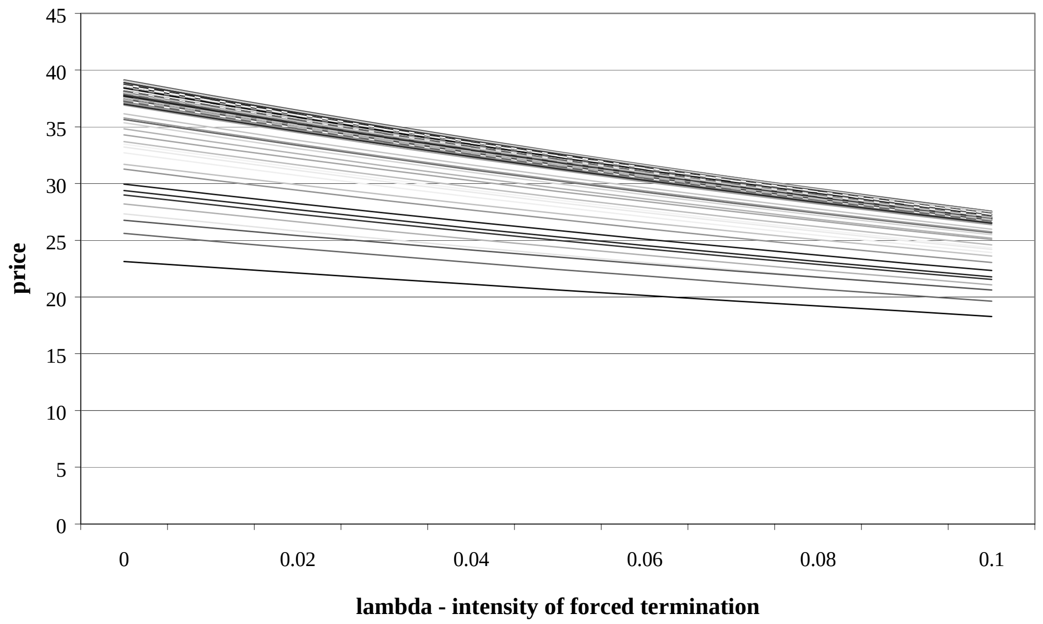

Figure 5: Prices of the barrier model as a function of the stopping rate λ for selected pairs $(B,\alpha) \in [110,500] \times [0,0.4]$. B is the height of the barrier at time V, the growth rate of the barrier is denoted by α.

second, because the characteristics of $\widetilde{\theta}$ will be correlated in practise[26], whereas isolated variations of characteristics could lead to unrealistic combinations if values from outer quantiles of the marginal distributions occur.

The following table presents the sensitivity of the price to an increase of each $\widetilde{\theta}_j$ by one standard deviation. For a detailed summary, see table 5.6 in the appendix.

characteristic $\widetilde{\theta}_j$	$\Delta\ \widetilde{\theta}_j$ **(one stand. deviation)**	$\Delta P/P$
mean time of exercise $\widetilde{\kappa}$	2.25	$-\,6.6\,\%$
mean stock price perf. at exercise $\widetilde{S_\kappa}$	1.42	$-\,14.6\,\%$
mean cancellation rate $\widetilde{c}$	7.1 %	$-\,50.4\,\%$
correlation $\widetilde{\rho}$ of κ and S_κ	0.14	$-\,3.2\,\%$

The cancellation rate has by far the strongest impact, followed by the stock price performance at exercise and exercise time. Note that the price *de*creases when the time of exercise rises – as opposed to the Black/Scholes model used in SFAS 123. Deferring the average time of exercise does not mean realizing a larger part of time value in general. Here, the postponement of exercise is achieved by raising the barrier, accompanied by more intensive independent stopping to keep $\widetilde{S_\kappa}$ down. A portion of profitable payoffs from stopping at the barrier is therefore replaced by payoffs at independent stops, containing a considerable amount of options cancelled out of the money.

The influence of correlation is weak, it should be noticed yet that it is one objective of this paper to clarify whether correlation is an important issue *at all*. The answer is that a model of similar flexibility as the barrier model should not *ignore* correlation, while a crude estimate of $\widetilde{\rho}$ seems to be sufficient. Moreover, it is remarkable that $\widetilde{S_\kappa}$ – not relevant in SFAS 123 – has a price impact two times stronger than $\widetilde{\kappa}$, which is close to the

[26]See Carpenter [3, table 1].

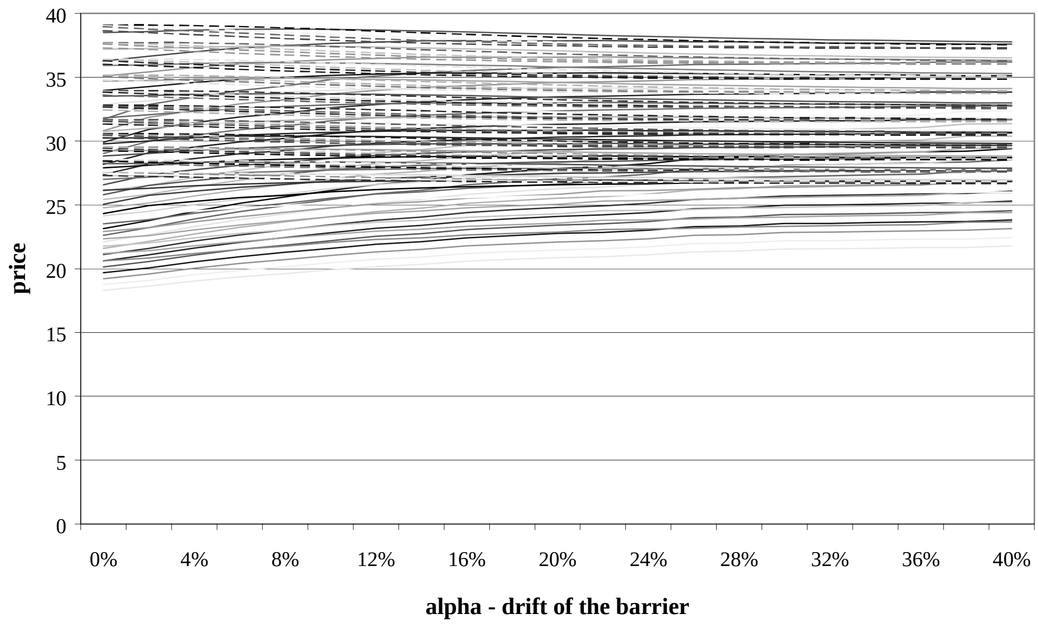

Figure 6: Prices of the barrier model as a function of the barrier's growth rate α for selected pairs $(B, \lambda) \in [110, 500] \times [0, 0.1]$. B is the height of the barrier at time V, λ is the continuous stopping rate. The lower B, the steeper the curves are.

second SFAS input $\widetilde{\kappa}_{\text{vest}}$, the mean lifetime given vesting. The significance of $\widetilde{S_\kappa}$, however, might be weaker when the characteristics are computed under a measure $\mathbf{M}$ that accounts for a bullish/bearish market (cf. sect. 3.1).

4 Conclusion

This paper deals with the valuation of employee stock options from an external perspective such as that of shareholders or analysts. Unlike shareholders who are assumed to be able to freely take any position in shares or debt, option holders are not allowed to hedge them. This leads to option exercise patterns deviating substantially from that of standard theory. To account for this fact, I follow the approach of Carpenter [3]. Observing certain characteristics of an exercise pattern, I specify a model's parameters such that it best reproduces the observations. The model incorporates vesting periods as well as forced termination of the option. A grantee is supposed to exercise her option if the stock price passes a deterministic threshold, which may grow exponentially. Another source of forced terminations of the option life is some independent, exponentially distributed random time. If stopped, the current intrinsic value is paid off. Parts of the calculation are solved in closed form, leaving a smooth function to be integrated in two dimensions.

The model is specified by the intensity of independent stopping and height plus drift of the barrier triggering exercise. It adapts better to a representative set of empirical characteristics than the extended American model from Carpenter [3]. Clearly, the advantage of higher flexibility involves the risk of lower robustness especially with regard to the characteristics chosen as a criterion of the model's fit. It turns out that the model *is* robust as long as the annual cancellation rate of options, one of the criteria, is included. Hence, even if the barrier model is seen to be too complicated for external reporting, it appears to be applicable at least for internal purposes, for instance when members of a compensation committee want to estimate the cost of an ESO program.

Besides practical application, a more theoretical contribution of this paper is the investigation of the influence of the correlation between exercise time and the stock price at exercise. Correlation is interesting because it largely deviates in practise from theoretical values. Since the model is able to incorporate correlation appropriately, it provides an opportunity to study the impact of correlation, given the model was true. The effect of correlation exists, but it is weak. In general, the model prices are slightly lower than those of the extended American model. Comparative statics show that the annual cancellation rate is the most important value driver, followed by the stock price performance at exercise.

Since cancellations are such important for the option value, I use my model to investigate how precisely the standard valuation approach accounts for options being canceled. Suppose the barrier model were true and all inputs for the SFAS method were measured reliably, the SFAS prices would be rather stable and slightly lower than barrier model prices. There is no evidence for suspecting SFAS 123 to be an unreliable proxy – at such a favorable level of data provision.

The SFAS approach uses the probability of forfeiture before vesting as an input, whereas an aggregated cancellation rate over different grants is often the only reliable information about cancellations. Using the barrier model in a double role as the "truth" and "the accountant's belief" – utilized to estimate the SFAS input from the cancellation rate – I observe a wide latitude of discretion.

5 Appendix

In order to value the option when independent stopping is excluded ($\lambda = 0$), the payoff $\pi(\tau)$ is decomposed into $\pi_1(\tau) := [S_\tau - K]^+ I_{\mathcal{V}}$ (exercise at vesting), $\pi_2(\tau) := [S_\tau - K]^+ I_{\mathcal{B}}$ (barrier is hit), and $\pi_3(\tau) := [S_\tau - K]^+ I_{\mathcal{E}}$ (exercise at expiry). Each payoff is valued separately.

5.1 Part I: Immediately After Vesting – $\pi_1(\tau)$

This contingent claim is nothing but a European call with maturity V, strike K, and an additional exercise hurdle of height B. The hurdle option can be decomposed into a European call with strike B and a digital option paying out $B - K$ iff $S_V \geq B$. Both are well-known, and the price of $\pi_1(\tau)$ amounts to

$$P_1 = e^{-rV}\mathbf{E}_{\mathbf{Q}}\pi_1(\tau) = e^{-\delta V} S_0 \Phi(d_1) - e^{-rV} K \Phi(d_2)$$

where $\Phi(\cdot)$ denotes the standard normal distribution function and

$$d_{1,2} := \frac{\ln(S_0/B) + \left(r - \delta \pm \frac{1}{2}\sigma^2\right) V}{\sigma\sqrt{V}}$$

5.2 Part II: Exercise at the Barrier – $\pi_2(\tau)$

In a first step, I condition the expected value on $\mathcal{F}_V^S$:

$$P_2 = \mathbf{E}_{\mathbf{Q}} e^{-r\tau}\pi_2(\tau) = \mathbf{E}_{\mathbf{Q}}\mathbf{E}_{\mathbf{Q}}\left[e^{-r\tau^*}\left(b(\tau^*) - K\right) I_{\mathcal{B}} \middle| \mathcal{F}_V^S\right]$$

The random time τ^*, restricted to $\mathcal{B}$, is $\sigma(S_t, V \le t \le T)$-measurable. It is therefore a stopping time on $\mathcal{B}$ for the augmentation of the filtration $(\sigma(S_s, V \le s \le t))_{V \le t \le T}$. Since the stock price path has the strong Markov property, I can replace the condition $\mathcal{F}_V^S$ by $\sigma(S_V)$:

$$P_2 = \mathbf{E_Q E_Q}\left[e^{-r\tau^*}(b(\tau^*) - K) I_{\mathcal{B}} \middle| S_V\right] \tag{8}$$

Consider now the inner conditional expectation at some fixed S_V. It turns out to be similar to a special type of barrier option[27]. The distribution of τ^* under $\mathbf{Q}(\cdot| S_V)$ is essential for computation. Under $\mathbf{Q}(\cdot| S_V)$, the process $Y_t := \ln(S_t/B)$, $t \ge V$, is a Brownian motion with a constant drift $r - \delta - \sigma^2/2$ and a starting point $-\ln(B/S_V)$. The condition $S_t \ge b(t)$ of hitting the barrier from (3) can be rewritten with $C := \ln(B/S_V)$ and the Brownian motion $Z_t := Y_t - \alpha(t - V) + C$ to

$$Z_t \ge C \tag{9}$$

Hence, τ^* is the hitting time of Z for a constant barrier C, where Z starts at time V in 0, running at a constant drift $\beta := r - \delta - \alpha - \sigma^2/2$. The distribution of τ^* has a well-known density h^*, which amounts to[28]

$$h^*(t, C)\ dt := \mathbf{Q}(\tau^* \in dt| C) = \frac{C}{\sigma\sqrt{2\pi}(t - V)^{\frac{3}{2}}} \exp\left\{-\frac{(C - \beta(t - V))^2}{2\sigma^2(t - V)}\right\} dt$$

I now return to the unconditional measure $\mathbf{Q}$. Taking into account that

$$C \sim N\left(\ln(B/S_0) - \beta V, \sigma\sqrt{V}\right),$$

the density h of τ^* under $\mathbf{Q}$ is now determined by integration over C: Set $q := \ln(B/S_0) - \alpha V$. Then

$$h(t) = \frac{1}{\sigma\sqrt{2\pi V}} \int_0^\infty \exp\left\{-\frac{(x - q)^2}{2\sigma^2 V}\right\} h^*(t, x)\ dx\ .$$

After some lengthy substitutions, the density can be rewritten to

$$h(t) = (a(t) + d(t)) \exp\{g(t)\} \tag{10}$$

where

$$\begin{aligned} \psi(t) &: = \frac{q}{\sigma}\sqrt{\frac{t - V}{tV}}\ , \quad g(t) := -\frac{(q - \beta t)^2}{2t\sigma^2} \\ a(t) &: = \Phi(\psi(t)) \frac{q}{\sigma\sqrt{2\pi t^3}}\ , \quad d(t) := \frac{\sqrt{V}}{2\pi t\sqrt{t - V}} \exp\left\{-\frac{\psi^2(t)}{2}\right\} \end{aligned}$$

For further computation, I introduce

$$H(\gamma) := \int_V^T \exp\{-\gamma t\} h(t)\ dt\ ,\ \gamma \in \mathbb{R}\ , \tag{11}$$

which is applied when returning to the price of $\pi_2(\tau)$:

$$\begin{aligned} P_2 &= \mathbf{E_Q} e^{-r\tau^*}(b(\tau^*) - K) I_{\mathcal{B}} = \int_V^T e^{-rt}\left(Be^{\alpha(t-V)} - K\right) h(t)\ dt \\ &= Be^{-\alpha V} H(r - \alpha) - KH(r)\ . \end{aligned}$$

[27] In the terminology of Rich [19], it is an *up-and-out call* with a *rebate* equal to the barrier minus strike.

[28] See Rich [19].

Computation The function H must be evaluated numerically. All parts of the integrand are bounded, except d, which is unbounded at $t \downarrow V$. The peak is eliminated by a further substitution: h is split up by resolving the brace in (10) into $a(t)\exp\{g(t)\}$ and $d(t)\exp\{g(t)\}$. The first part is evaluated as before; for the second part I replace t by $s := (t-V)^{1/2}$, arriving at

$$\begin{aligned}
I(\gamma) \quad : \quad &= \int_V^T \exp\{g(t)-\gamma t\}\, b(t)\, dt \\
&= \int_0^{\sqrt{T-V}} 2s \exp\left\{g\left(s^2+V\right)-\gamma\left(s^2+V\right)\right\} b\left(s^2+V\right)\, ds \\
&\vdots \\
&= \int_0^{\sqrt{T-V}} \frac{\sqrt{V}}{\pi\left(s^2+V\right)} \exp\left\{-\frac{q^2s^2+\left(q-\left(s^2+V\right)\right)^2}{2\sigma^2\left(s^2+V\right)}-\gamma\left(s^2+V\right)\right\} ds
\end{aligned}$$

which has a bounded and equicontinuous integrand that suits for numerical integration. Now,

$$H(\gamma) = I(\gamma) + \int_V^T \exp\{g(t)-\gamma t\}\, a(t)\, dt$$

which is used in (11) as before.

5.3 Part III: Exercise at expiration – $\pi_3(\tau)$

The remaining term collects cases in which the barrier was not met before T. At first glance, $\pi_3(\tau)$ seems equivalent to a European call capped at $b(T)$. But this is not correct since some of the paths of Z that would mature at a height within $[K, b(T)]$ do not do so because they hit b before. The distribution is biased downwards. It is more convenient now to turn over from Z to a similar process $X_t := \ln(S_t/S_0) - \alpha t$, $t \in [0,T]$, which has the same drift β as Z but a starting point zero. Define $M := \sup_{V\le t\le T} X_t$. Recalling $q = \ln(b(0)/S_0)$, the condition of not hitting the barrier changes to

$$X_t < q,\ V \le t \le T, \quad \text{or, equivalently,} \quad M < q.$$

For the integral, the distribution of $S_T I_{\mathcal{E}}$ or, similar, that of $X_T I_{\mathcal{E}}$ is needed. I condition the probability on X_V, which is equivalent to S_V regarding the generated σ-algebras:

$$\mathbf{Q}(M<q, X_T \le z) = \mathbf{E_Q}\,\mathbf{Q}\left[M<q, X_T\le z \middle| X_V\right].$$

Note that $\mathcal{E}$ implies $S_V < B$, which is $X_V < q$. An application of the reflection principle and Girsanov's theorem (cf. Musiela and Rutkowski [18, sect. B3]) yields for the conditional probability

$$\begin{aligned}
&\mathbf{Q}\left[M<q, X_T\le z \middle| X_V = x\right],\ x<q \\
= \ &\mathbf{Q}\left[M-x<q-x, X_T - x\le z-x \middle| X_V = x\right],\ x<q \\
= \ &\Phi\left(\frac{z-x-\beta(T-V)}{\sigma\sqrt{T-V}}\right) - \exp\left\{\frac{2\beta(q-x)}{\sigma^2}\right\}\Phi\left(\frac{z+x-2q-\beta(T-V)}{\sigma\sqrt{T-V}}\right)
\end{aligned}$$

This has to be integrated over X_V, where $X_V \sim N\left(\beta V, \sigma\sqrt{V}\right)$:

$$\begin{aligned}
&\mathbf{Q}(M<q, X_T\le z) \\
= \ &\frac{1}{\sigma\sqrt{2\pi V}} \int_{-\infty}^{q} \mathbf{Q}\left[M<q, X_T\le z \middle| X_V = x\right] \exp\left\{-\frac{(x-\beta V)^2}{2\sigma^2 V}\right\} dx\,. \qquad (12)
\end{aligned}$$

For integration over the payoff $\pi_3(\tau)$, the density $l(z)$ of X_T is needed. I obtain it by differentiation of (12)

$$\begin{aligned}
l(z)\ dz &= \mathbf{Q}(M<q, X_T\in dz) \ = \ dz\frac{d}{dz}\mathbf{Q}(M<q, X_T\le z)\\
&= \frac{dz}{\sigma\sqrt{2\pi V}}\int_{-\infty}^{q}\exp\left\{-\frac{(x-\beta V)^2}{2\sigma^2 V}\right\}\frac{dx}{\sigma\sqrt{2\pi(T-V)}}\times\\
&\quad\times\left[\exp\left\{-\frac{(x+\beta(T-V)-z)^2}{2\sigma^2(T-V)}\right\}-\right.\\
&\quad\left.-\exp\left\{\frac{2\beta(q-x)}{\sigma^2}\right\}\exp\left\{-\frac{(z+x-2q-\beta(T-V))^2}{2\sigma^2(T-V)}\right\}\right]\\
&\quad\vdots\\
&= \left[f(z)\,\Phi\left(\frac{q-\mu_1}{\sigma_1}\right)-g(z)\,\Phi\left(\frac{q-\mu_2}{\sigma_1}\right)\right]\ dz
\end{aligned}$$

where

$$\begin{aligned}
\mu_1 :=&\quad \frac{V}{T}z\,,\ \ \mu_2 := \frac{V}{T}(2q-z)\ ,\\
U :=&\quad T-V\,,\ \ \sigma_1 := \sigma\sqrt{\frac{V}{T}U}\ ,\\
f(z) :=&\quad \frac{1}{\sigma\sqrt{2\pi T}}\exp\left\{\frac{1}{2\sigma^2 U}\left(\frac{V}{T}z^2-(\beta U-z)^2-V\beta^2 U\right)\right\}\ ,\\
g(z) :=&\quad \frac{1}{\sigma\sqrt{2\pi T}}\exp\left\{\frac{1}{2\sigma^2 U}\left(\frac{V}{T}(z-2q)^2-\beta^2 VU+4\beta qU-(z-2q-\beta U)^2\right)\right\}\ .
\end{aligned}$$

Finally, using $S_T = S_0\exp\{\alpha T + X_T\}$, the price P_3 of the part "exercise in T" amounts to

$$\begin{aligned}
P_3 &= \mathbf{E}_{\mathbf{Q}}e^{-rT}[S_T-K]^+ I_{\mathcal{E}}\\
&= e^{-rT}S_0\int_{\ln(K/S_0)-\alpha T}^{q}\left(e^{\alpha T}e^z-\frac{K}{S_0}\right)l(z)\ dz
\end{aligned}$$

which can be solved numerically in a straightforward manner.

5.4 The Unhedgeable Risk of Independent Stopping

In this section I present a sufficient condition, under which my assumption of section 2.3 not to price the unhedgeable risk of independent stopping is justified. Suppose the ESO is not granted to a single person but to a large group of N employees, holding the Nth part each. By assumption, the risk of stopping is idiosyncratic to each of them. Formally, there is a whole number of i.i.d. random times φ_i, the entirety of which being independent of $\mathcal{F}_T^S$. This implies the identity of $\mathbf{P}$ and $\mathbf{Q}$ on $\sigma(\varphi_i, i\in\mathbb{N})$. By independence, holding the portfolio of claims

$$C_{\text{portf}} := \left\{\frac{1}{N}\pi_{\text{stop}}(\tau\wedge\varphi_i)\,,\ \text{due in }\tau\wedge\varphi_i\,\middle|\ i=1,\ldots,N\right\}$$

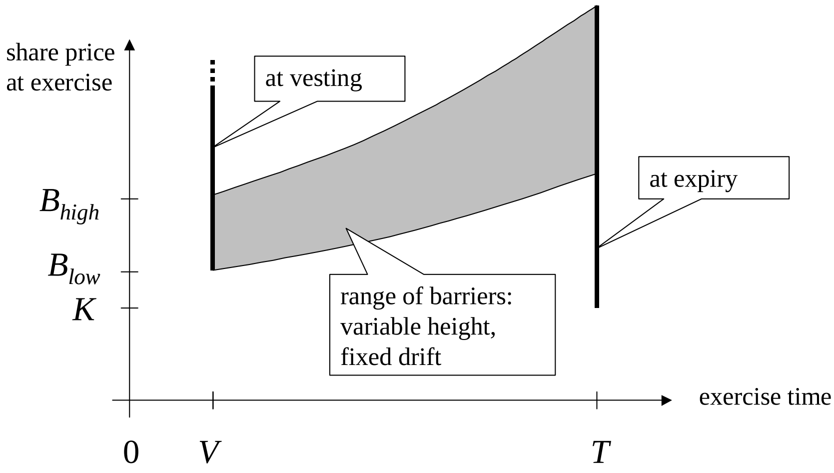

Figure 7: Joint distribution of exercise time and price at exercise if barriers are dispersed; the vertical line on the left consists of exercises immediately after vesting; the line on the right of exercises in the money at expiry; the area in between arises from exercises at various barriers, growing at the same proportional rate and starting from a height between B_{high} and B_{low}, to be equally distributed.

is nearly the same for large N as holding a continuum of contingent claims

$$C_{\text{cont}} := \{ \mathbf{P}(\varphi \in dt)\ \pi_{\text{stop}}(\tau \wedge t),\ \text{due in } \tau \wedge t \,|\ t \in [V, T] \}$$

which is easily proved by the Strong Law of Large Numbers.[29] The price of C_{portf} then almost equals that of C_{cont}, yielding with (5) and the independence of φ and $\mathcal{F}_T^X$

$$\lim_{N \to \infty} \text{price}(C_{\text{portf}}) = \int_0^\infty \text{price}(\pi_{\text{stop}}(\tau \wedge t))\ \mathbf{P}(\varphi \in dt)\ = \mathbf{E} e^{-r(\tau \wedge \varphi)} \pi(\tau, T \wedge \varphi)\ , \quad (13)$$

which is a generalization of (6). This result can be justified by CAPM-like arguments, too.

5.5 Dispersion of the Barrier

In figure 1 it was seen that the joint distribution of τ and S_τ is degenerate to a zigzag line of Lebesgue measure zero. While people exercise their options quite often immediately after vesting or at expiry, leading to jumps in the marginal distribution of κ, a single barrier for the time between vesting and expiry seems less plausible. Instead, I follow the idea of a group of option holders, each choosing some *individual* barrier. For simplicity, only the height B is subject to variation, while the proportional growth rate α remains constant (confer figure 7). I consider a special setting: B is equally distributed on some interval $[B_{\text{low}}, B_{\text{high}}]$ such that $K < B_{\text{low}}$. By assumption, it is independent of all random variables introduced so far. Let be $\Delta := B_{\text{high}} - B_{\text{low}}$ and $B_{\text{center}} := 1/2\,(B_{\text{high}} + B_{\text{low}})$. The option is priced as the average price over different barrier functions by the same argument as in section 5.4. Given that $P_{B,\alpha}(T)$ denotes the price for an individual barrier, one obtains

$$P_{\text{dispersed}} = \Delta^{-1} \int_{B_{\text{low}}}^{B_{\text{high}}} P_{B\,,\,\alpha}(T)\ dx\,.$$

[29] Given prices are additive and ESOs are infinitely divisible.

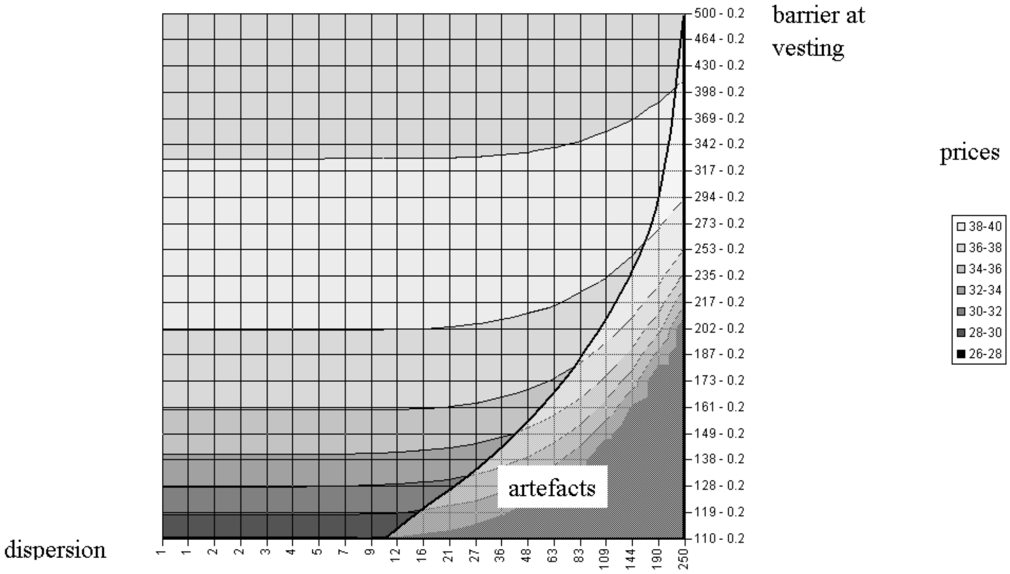

Figure 8: Option prices as a function of the dispersion and the general level of the barrier. Areas of equal lightness are areas of roughly the same price. Dispersion is measured by the width Δ of the barrier's range $[B_{\text{low}}, B_{\text{high}}]$. "Barrier at vesting" denotes the midpoint B_{center} of the interval. In the area "artefacts", the condition $B_{\text{low}} > K$ is hurt. Other parameters: $\alpha = 20\%$, $\lambda = 3\%$

Keeping B_{center} fixed, I check how much $P_{\text{dispersed}}$ is affected by an increase of Δ. The impact is weak. For illustration, consider the option price as a function of Δ and B_{center} for $\alpha = 20\,\%$, $\lambda = 3\,\%$. It is drawn in contour lines in figure 8. The flat lines on the left half of the field show that the impact of Δ is negligible at a range from 0 to about 50. Note that the range refers to $B = b(V)$. When $\alpha = 20\,\%$, this means a range from 0 to 248 at expiry. Other characteristics show a low sensibility, too. An analysis for $\alpha = 0\,\%$ or $\alpha = 40\,\%$ provides similar results. To sum up, a model with a dispersed barrier looks more aesthetic but gives neither new insight nor essential price differences.

5.6 Some Evidence on Management Turnover

Hadlock and Lumer [6] report an annual rate of 3.8% for CEOs from a sample of 259 U.S. firms. Kaplan [14] compares the CEO turnover in large U.S. and Japanese firms, resulting in rates of 2.2% (Japan) and 2.9% (U.S.) when CEOs entering the supervisory board are left out. I assume that they may continue to hold their options. Kang and Shivdasani [13] find 3.1% p.a. for Japanese firms when the turnover is corrected for executives remaining on the board. The U.S. sample of Denis, Denis and Sarin [5] yields a weighted mean rate of 7.5%, yet it is not corrected in the above sense. The same problem holds for the rate of 9.2% from Mikkelson and Partch [17], where CEO turnover in unacquired U.S. firms is measured over ten years. Dahya, McConnell and Travlos [4] report forced CEO turnover at rates between 2.7% and 5% from a dataset of 470 industrial firms in the U.K.

References

[1] Richard E. Barlow and Frank Proschan. *Statistical Theory of Reliability and Life Testing*. Holt, Rinehart and Winston, 1975.

[2] G. Barone-Adesi and R. E. Whaley. Efficient analytic approximation of american option values. *Journal of Finance*, 42:301–320, 1987.

[3] Jennifer N. Carpenter. The exercise and valuation of executive stock options. *Journal of Financial Economics*, 48:127–158, 1998.

[4] Jay Dahya, John J. McConnell, and Nickolaos G. Travlos. The cadbury committee, corporate performance, and top management turnover. *Journal of Finance*, 57(1):461–483, 2002.

[5] David J. Denis, Diane K. Denis, and Atulya Sarin. Ownership structure and top executive turnover. *Journal of Financial Economics*, 45:193–221, 1997.

[6] Charles J. Hadlock and Gerald B. Lumer. Compensation, turnover, and top management incentives: Historical evidence. *Journal of Business*, 70(2):153–186, 1997.

[7] Brian J. Hall and Kevin J. Murphy. Stock options for undiversied executives. *Journal of Accounting and Economics*, 33:3–42, 2002.

[8] Chip Heath, Steven Huddart, and Mark Lang. Psychological factors and stock option exercise. *Quarterly Journal of Economics*, 114(2):601–627, 1999.

[9] Steven Huddart. Employee stock options. *Journal of Accounting and Economics*, 18:207–231, 1994.

[10] Steven Huddart and Mark Lang. Employee stock option exercises. an empirical analysis. *Journal of Accounting and Economics*, 21:5–43, 1996.

[11] L. Peter Jennergren and Bertil Näslund. A comment on "valuation of executive stock options and the FASB proposal". *Accounting review*, 68(1):179–183, 1993.

[12] L. Peter Jennergren and Bertil Näslund. A class of option with stochastic lives and an extension of the black-scholes formula. *European Journal of Operational Research*, 91:229–234, 1995.

[13] Jun-Koo Kang and Anil Shivdasani. Firm performance, corporate governance, and top executive turnover in japan. *Journal of Financial Economics*, 38(1):29–58, 1995.

[14] Steven N. Kaplan. Top executive rewards and firm performance: A comparison of japan and the united states. *Journal of Political Economy*, 102(3):510–546, 1994.

[15] I. Karatzas and S. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, 1988.

[16] Nalin Kulatilaka and Alan J. Marcus. Valuing employee stock options. *Financial Analyst Journal*, 50:46–56, 1994.

[17] Wayne H. Mikkelson and M. Megan Partch. The decline of takeovers and disciplinary managerial turnover. *Journal of Financial Economics*, 44:205–228, 1997.

[18] Marek Musiela and Marek Rutkowski. *Martingale Methods in Financial Modelling*. Springer, 1998.

[19] Don R. Rich. The mathematical foundations of barrier option-pricing theory. *Advances in futures and options research*, 7:267–311, 1994.

[20] M. Rubinstein. On the accounting valuation of employee stock options. *Journal of Derivatives*, 3(1):8–24, 1995.

target characteristics				**fitted parameters**			**prices**		**achieved characteristics**				**distance**
$\widetilde{\kappa}$	$\widetilde{S_\kappa}$	$\widetilde{c}$	$\widetilde{\rho}$	B	α	λ	P	P_{SFAS}	$\widehat{\kappa}$	$\widehat{S_\kappa}$	$\widehat{c}$	$\widehat{\rho}$	dist_{1111}
4.48	275	0.073	0.140	175	0.144	0.072	28.91	26.82	4.95	214	0.076	0.142	0.231
4.93	275	0.073	0.140	189	0.155	0.081	28.51	26.74	5.33	226	0.076	0.142	0.151
5.83	**275**	**0.073**	**0.140**	**229**	**0.167**	**0.098**	**27.48**	**26.55**	**6.05**	**252**	**0.075**	**0.141**	**0.037**
6.28	275	0.073	0.140	259	0.162	0.103	27.04	26.55	6.38	265	0.074	0.141	0.007
6.73	275	0.073	0.140	310	0.143	0.106	26.72	26.66	6.69	278	0.073	0.140	0.001
7.18	275	0.073	0.140	396	0.105	0.105	26.72	27.01	6.98	290	0.070	0.139	0.022
5.83	190	0.073	0.140	190	0.156	0.071	29.58	27.66	5.49	230	0.070	0.138	0.105
5.83	218	0.073	0.140	199	0.162	0.078	29.03	27.38	5.66	237	0.072	0.139	0.023
5.83	247	0.073	0.140	211	0.167	0.088	28.30	26.99	5.85	244	0.073	0.140	0.000
5.83	**275**	**0.073**	**0.140**	**229**	**0.167**	**0.098**	**27.48**	**26.55**	**6.05**	**252**	**0.075**	**0.141**	**0.037**
5.83	303	0.073	0.140	259	0.157	0.110	26.47	26.01	6.26	261	0.077	0.142	0.128
5.83	332	0.073	0.140	374	0.094	0.127	24.92	25.14	6.49	273	0.081	0.143	0.269
5.83	360	0.073	0.140	662	-0.014	0.128	24.76	25.27	6.68	283	0.080	0.142	0.451
5.83	275	0.030	0.140	174	0.176	-0.007	37.70	34.16	6.13	241	0.033	0.141	0.076
5.83	275	0.045	0.140	185	0.180	0.025	34.33	31.68	6.10	245	0.047	0.141	0.061
5.83	275	0.059	0.140	201	0.179	0.060	30.93	29.15	6.08	248	0.061	0.141	0.048
5.83	**275**	**0.073**	**0.140**	**229**	**0.167**	**0.098**	**27.48**	**26.55**	**6.05**	**252**	**0.075**	**0.141**	**0.037**
5.83	275	0.087	0.140	303	0.116	0.140	23.95	23.87	6.03	256	0.089	0.141	0.026
5.83	275	0.101	0.140	501	0.000	0.166	21.81	22.17	5.93	257	0.100	0.140	0.018
5.83	275	0.116	0.140	1816	-0.263	0.176	21.09	21.50	5.81	256	0.111	0.137	0.021
5.83	275	0.073	0.056	244	0.120	0.093	28.02	26.85	6.06	249	0.075	0.057	0.044
5.83	275	0.073	0.084	240	0.134	0.096	27.80	26.72	6.07	251	0.075	0.085	0.042
5.83	275	0.073	0.112	235	0.149	0.097	27.64	26.64	6.06	251	0.075	0.113	0.039
5.83	**275**	**0.073**	**0.140**	**229**	**0.167**	**0.098**	**27.48**	**26.55**	**6.05**	**252**	**0.075**	**0.141**	**0.037**
5.83	275	0.073	0.168	222	0.187	0.099	27.36	26.50	6.05	253	0.075	0.169	0.034
5.83	275	0.073	0.196	216	0.212	0.100	27.15	26.38	6.04	255	0.075	0.197	0.030
5.83	275	0.073	0.224	208	0.246	0.101	26.97	26.30	6.03	256	0.075	0.225	0.026

Table 3: Comparative statics for the barrier model. The model's fit under dist_{1111} with the empirical target characteristics from table 3.2 is the reference point (bold letters). Either characteristic changes ceteris paribus in steps of 1/5 of its standard deviation from three steps below the reference point up to three above. Given a modified target set of characteristics, the model parameters are now fitted again. The column "achieved characteristics" summarizes the corresponding model characteristics after calibration. P denotes the model price, P_{SFAS} is the price according to SFAS 123, given the model is true. Notation of characteristics: $\widehat{\kappa}$, the mean time of exercise; $\widehat{S_\kappa}$, the mean stock price performance at exercise; $\widehat{c}$, the mean cancellation rate; $\widehat{\rho}$, the correlation of $\widehat{\kappa}$ and $\widehat{S_\kappa}$. Notation of model parameters: B, the value of the barrier at vesting time V; α, the barrier's growth rate; λ, the intensity of independent stopping.