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# Baryon Number Conservation and Statistical Production of Antibaryons

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## Abstract

The statistical production of antibaryons is considered within the canonical ensemble formulation. We demonstrate that the antibaryon suppression in small systems due to the exact baryon number conservation is rather different in the baryon-free ( $B = 0$ ) and baryon-rich ( $B \geq 2$ ) systems. At constant values of temperature and baryon density in the baryon-rich systems the density of the produced antibaryons is only weakly dependent on the size of the system. For realistic hadronization conditions this dependence appears to be close to  $B/(B + 1)$  which is in agreement with the preliminary data of the NA49 Collaboration for the  $\bar{p}/\pi$  ratio in nucleus-nucleus collisions at the CERN SPS energies. However, a consistent picture of antibaryon production within the statistical hadronization model has not yet been achieved. This is because the condition of constant hadronization temperature in the baryon-free systems leads to a contradiction with the data on the  $\bar{p}/\pi$  ratio in  $e^+e^-$  interactions.

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## 1. Introduction

Among the first models of multiparticle production in high energy interactions were statistical models [1, 2, 3]. In the last decade a significant development of these models and the extension of the area of their applicability took place. The main reason for this is a surprising success of the statistical approach in reproducing new experimental data on hadron multiplicities in nuclear (A+A) [4] and elementary ( $e^+e^-$ , p+p, p+ $\bar{p}$ ) collisions [5, 6]. One of the important results of the analysis of hadron yield systematics at high energies (SPS and higher) done within the statistical models is the approximate independence of the temperature parameter  $T = 160 \div 190$  MeV from the system size and collision energy [7]. This result can be attributed to the statistical character of the hadronization process.

The statistical models are based on the key assumption that all microscopic states of the system allowed by the conservation laws are equally probable. Calculations within these models are straightforward when the mean number of particles of interest is large and consequently it is enough to fulfill the conservation laws in the average sense, i.e., for the macroscopic state. This is achieved by the introduction of parameters: the temperature  $T$  and chemical potentials  $\mu_i$ , which control correspondingly the average values of the system energy density and its material content (e.g., baryon number, strangeness and electric charge). In this case (grand canonical ensemble, *g.c.e.*) the mean particle multiplicities are just proportional to the volume  $V$  of the system. The particle density and the ratio of the multiplicities of two different particles often used for the comparison with the data are volume independent.

This simple volume dependence is however not valid any more for a small system in which the mean particle multiplicity is low. In this case (canonical ensemble, *c.e.*) the material conservation laws should be imposed on each microscopic state of the system. This condition introduces a significant correlation between particles which carry conserved charges. The correlation reduces the effective number of degrees of freedom and consequently leads to the *c.e.* suppression of the 'charged' particle multiplicity when compared with the result of the calculations done within *g.c.e.* A 'neutral' particle (a particle which does not carry conserved charges) does not feel the *c.e.* suppression in the small system, its mean multiplicity remains proportional to the volume. Therefore, a different dependence on the system volume is expected within statistical models for 'charged' and 'neutral' particles. The magnitude of the *c.e.* suppression of the 'charged'-'anticharged' pair creation increases with the mass of the lightest hadron needed to compensate the particle charges. Therefore, a strong *c.e.* suppression may be expected for antibaryon production as the mass of the lightest baryon is  $m \cong 938$  MeV which is much larger than the value of the temperature parameter found in the hadronization models. The *c.e.* suppression is still rather essential for strange particle production (see, e.g., [8]).

The above expectation seems to be *violated* by the preliminary data on antiproton production presented recently by the NA49 Collaboration [9]. The  $\bar{p}/\pi$  ratio is found to be approximately the same for p+p interactions and central Pb+Pb collisions at 158 A·GeV (*antiproton scaling*). Thus the hadronization volume increases but the effect of the *c.e.* suppression is not observed. The *c.e.* suppression can be expected in p+p interactions because of the key difference between antiproton and pion: the antiproton carries baryon number in addition to the electric charge carried by both particles. In order to

compensate the electric charge of a produced particle it is enough to create an additional charged pion. As the pion mass is much smaller than the nucleon mass, a significantly stronger *c.e.* suppression is expected for antiproton production than for the pion production, consequently it should lead to a strong violation of the experimentally observed scaling. Thus the crucial question is whether the *antiproton scaling* can be understood within the statistical model of hadron production in which the condition of exact baryon number conservation is imposed. We note that the antiproton multiplicities in high energy collisions were shown to approximately agree with the predictions of statistical models [4, 5, 6]. However different versions of the models with different parameters were used to fit various sets of data. Thus the question whether consistent description of the antiproton data in the statistical model is possible is still opened.

The importance of the exact treatment of the material conservation laws within statistical models of strong interactions was first pointed out by Hagedorn [10] (see also Refs. [11, 12]). Subsequently a complete treatment has been developed (see, e.g., [13] and references therein) and applied to analyze the hadron yields in elementary collisions [5, 6]. In this letter we derive explicit analytical formulae to study the role of the exact material conservation laws within the statistical model of hadronization and we use them to discuss the *antiproton scaling* observed experimentally in Pb+Pb collisions at 158 A·GeV [9]. We also discuss the data on the  $\bar{p}/\pi$  ratio in  $e^+e^-$  interactions [14, 15] within the statistical hadronization model.

## 2. Model formulation

Let us consider the system of baryons 'b' and antibaryons 'a' with total baryon number  $B$  as the Boltzmann ideal gas in the volume  $V$ , at temperature  $T$ . The *c.e.* partition function is

$$Z(T, V, B) = \sum_{N_b^{(1)}, N_a^{(1)}}^{\infty} \dots \sum_{N_b^{(j)}, N_a^{(j)}=0}^{\infty} \dots \delta_K \left[ B - \sum_j (N_b^{(j)} - N_a^{(j)}) \right] \quad (1)$$

$$\times \prod_j \frac{(\lambda_b^{(j)} z_j)^{N_b^{(j)}}}{N_b^{(j)}!} \frac{(\lambda_a^{(j)} z_j)^{N_a^{(j)}}}{N_a^{(j)}!},$$

where the index  $j$  runs over all (non-strange) baryon states  $N, \Delta, N^*, \dots$ , and the single baryon (antibaryon) partition function reads

$$z_j = z_j(T, V) = \frac{g_j V}{(2\pi)^3} \int d^3 k \exp[-(k^2 + m_j^2)^{1/2}/T] = \quad (2)$$

$$= \frac{g_j V}{2\pi^2} T m_j^2 K_2(m_j/T) \equiv V f_j(T).$$

The baryon mass and the baryon degeneracy factor are denoted here by  $m_j$  and  $g_j$ , respectively. Auxiliary parameters  $\lambda_b^{(j)}$  and  $\lambda_a^{(j)}$  are introduced in order to calculate the mean number of baryons and antibaryons and they are set to unity in the final formulae. By expressing  $\delta_K$  as

$$\delta_K(n) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-in\phi},$$

Eq. (1) becomes

$$\begin{aligned}
Z(T, V, B) &= \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iB\phi} \prod_j \sum_{N_b^{(j)}=0}^{\infty} \sum_{N_a^{(j)}=0}^{\infty} \frac{(\lambda_b^{(j)} z_j e^{i\phi})^{N_b^{(j)}}}{N_b^{(j)}!} \frac{(\lambda_a^{(j)} z_j e^{-i\phi})^{N_a^{(j)}}}{N_a^{(j)}!} = \\
&= \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iB\phi} \exp \left[ \sum_j z_j (\lambda_b^{(j)} e^{i\phi} + \lambda_a^{(j)} e^{-i\phi}) \right]. \quad (3)
\end{aligned}$$

This form of the *c.e.* partition function allows one to derive the mean numbers of baryons and antibaryons

$$\langle N_b^{(j)} \rangle = \left( \frac{\partial \log Z}{\partial \lambda_b^{(j)}} \right)_{\lambda_b=\lambda_a=1} = z_j \frac{Z(T, V, B-1)}{Z(T, V, B)}, \quad (4)$$

$$\langle N_a^{(j)} \rangle = \left( \frac{\partial \log Z}{\partial \lambda_a^{(j)}} \right)_{\lambda_b=\lambda_a=1} = z_j \frac{Z(T, V, B+1)}{Z(T, V, B)}. \quad (5)$$

For  $\lambda_b = \lambda_a = 1$  the partition function (3) can be presented as the modified Bessel function

$$Z(T, V, B) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-iB\phi} \exp(2z \cos \phi) = I_B(2z), \quad (6)$$

where  $z \equiv \sum_j z_j$ . This yields final expressions for the mean number of baryons and antibaryons

$$\langle N_b^{(j)} \rangle = z_j \frac{I_{B-1}(2z)}{I_B(2z)}, \quad \langle N_a^{(j)} \rangle = z_j \frac{I_{B+1}(2z)}{I_B(2z)}. \quad (7)$$

As the exact baryon number conservation is imposed on each microscopic state it is evidently fulfilled also by the average values (7):

$$\langle N_b \rangle - \langle N_a \rangle \equiv \sum_j \langle N_b^{(j)} \rangle - \sum_j \langle N_a^{(j)} \rangle = B, \quad (8)$$

as indeed can be easily seen from the identity  $I_{n-1}(x) - I_{n+1}(x) = 2nI_n(x)/x$  [16]. Eq. (7) is valid for all combinations of  $B$  and  $z$  values. For a specific case of  $B = 0$  in the nucleon-antinucleon gas (i.e., no resonances included) our results (7) are reduced to the result of Rafelski and Danos [12].

The *c.e.* expressions for the mean number of baryons and antibaryons can be further simplified for the two limiting cases:  $z \ll 1$  (small systems) and  $z \gg 1$  (large systems). Using the representation of  $I_n$  as the infinite series [16]

$$I_n(2z) = \sum_{k=0}^{\infty} \frac{z^{n+2k}}{k!(n+k)!},$$

one obtains for small systems

$$\langle N_b^{(j)} \rangle \cong B \frac{z_j}{z} + \frac{z_j \cdot z}{B+1} + o(z_j \cdot z^3), \quad \langle N_a^{(j)} \rangle \cong \frac{z_j \cdot z}{B+1} + o(z_j \cdot z^3). \quad (9)$$

The dependence  $\langle N_a \rangle \propto V^2/(B+1)$  is therefore observed from Eq. (9) for the antibaryon yield in small systems. Such a dependence can be intuitively understood from the kinetic picture of the baryon–antibaryon pair creation and annihilation. Let’s consider the time dependence  $N_a(t)$  (the time averaging of  $N_a(t)$  should reproduce our statistical average  $\langle N_a \rangle$ ). At each time moment the value of  $N_a(t)$  equals 0 or 1 (configurations with  $N_a(t) \geq 2$  can be safely neglected as  $\langle N_a \rangle \ll 1$ ). The kinetics of the evolution of  $N_a(t)$  is defined by the frequency  $\omega$  of the baryon–antibaryon pair creation and the life–time  $\Delta t$  of the produced baryon–antibaryon pair:  $\langle N_a(t) \rangle = \omega \Delta t$ . As a baryon–antibaryon pair is locally produced in any point of the system we have  $\omega \propto V$ . Being produced as baryon–antibaryon pair the antibaryon can be then locally annihilated by any baryon existing in the system. Therefore,  $\Delta t \propto 1/n_B$  ( $n_B$  is the density of baryons).  $n_B = 1/V$  for  $B = 0$  (as only one baryon exists in the volume  $V$ ), and  $n_B = (B+1)/V$  if  $B$  baryons are present in the system before the baryon–antibaryon pair creation. These lead to the dependence of the antibaryon number on  $V$  and  $B$  as given by Eq. (9).

For large systems ( $z \gg 1$ ) the *c.e.* becomes equivalent to the *g.c.e.* where the partition function and the average number of baryons and antibaryons are calculated as

$$Z(V, T, \mu_B) = \sum_b \exp\left(\frac{\mu_B b}{T}\right) Z(T, V, b) = \exp\left(ze^{\mu_B/T} + ze^{-\mu_B/T}\right), \quad (10)$$

$$\langle N_b^{(j)} \rangle = z_j \exp(\mu_B/T), \quad \langle N_a^{(j)} \rangle = z_j \exp(-\mu_B/T). \quad (11)$$

Here  $\mu_B$  is a baryon chemical potential which using Eq. (8) is defined for  $B = \langle b \rangle \geq 0$  as:

$$\exp(\mu_B/T) = \frac{B}{2z} + \sqrt{1 + \left(\frac{B}{2z}\right)^2}. \quad (12)$$

Note that the function  $f_j = f_j(T)$  introduced in Eq. (2) has the physical meaning of the density of the  $j$ -th baryon and antibaryon in the *g.c.e.* formulation for  $\mu_B = 0$ . Using the uniform asymptotic expansion of the modified Bessel functions at  $n \rightarrow \infty$  [16]

$$I_n(nx) \cong \frac{1}{\sqrt{2\pi n}} \frac{\exp(n\eta)}{(1+x^2)^{1/4}} \left[1 + o\left(\frac{1}{n}\right)\right]; \quad \eta \equiv \sqrt{1+x^2} + \ln \frac{x}{1+\sqrt{1+x^2}}, \quad (13)$$

results (11) and (12) for the *g.c.e.* are also easy to obtain from the *c.e.* (7) in the thermodynamical limit  $V \rightarrow \infty, B \rightarrow \infty$  with  $B/V \equiv \rho_B = \text{const}(V)$ .

In order to remove a ‘trivial’ linear dependence of the particle multiplicities on the system volume it is convenient to make a comparison between the particle ratios from the model and the experimental data. In the statistical model the multiplicity of any ‘neutral’ meson state  $M$  is just proportional to the volume ( $\langle N_M \rangle = z_M = V f_M(T)$ ) for both small (*c.e.*) and large (*g.c.e.*) systems. Therefore, the system volume dependence of the ratio  $\langle N_a \rangle / \langle N_M \rangle$  at fixed temperature is the same as for the antibaryon density. From Eqs. (7) and (11) the antibaryon densities for the *c.e.* and *g.c.e.* are equal to

$$\frac{\langle N_a^{(j)} \rangle}{V} \Big|_{ce} \equiv n_a^{(j)} \Big|_{ce} = f_j \frac{I_{B+1}(2z)}{I_B(2z)} = f_j \frac{I_{B+1}(xB)}{I_B(xB)} \cong f_j \frac{x}{2} \frac{B}{B+1}, \quad (14)$$

$$\frac{\langle N_a^{(j)} \rangle}{V} \Big|_{gce} \equiv n_a^{(j)} \Big|_{gce} = f_j \exp(-\mu_B/T) = f_j \frac{x}{1+\sqrt{1+x^2}}, \quad (15)$$

where  $f \equiv \sum_j f_j$ ,  $x \equiv 2z/B = 2f/\rho_B$ , and the last approximation in Eq. (14) is valid for small systems only. Note that introducing the variable  $x$  we have transformed the finite size  $V$ -dependence of the *c.e.* density (14) into its dependence on the baryon number  $B$ . Eqs. (14,15) give us the primary thermal density for all individual antibaryon states  $j$ . Each non-strange resonance (anti)baryon state decays finally into (anti)nucleon plus meson(s). Therefore, the total (primary plus resonance decay) antinucleon density equals to the total thermal antibaryon density,  $n_a = \sum_j n_a^{(j)}$  and is given by Eqs. (14,15) with the substitution of  $f_j$  by a sum  $f = \sum_j f_j$ .

For the purpose of the following discussion we define a canonical suppression factor

$$F_{cs} \equiv \frac{(n_a)_{ce}}{(n_a)_{gce}}. \quad (16)$$

It quantifies the antinucleon suppression due to the exact baryon number conservation. We note also that the suppression factor  $F_{cs}$  (16) is the same for any individual antibaryon state.

### 3. Discussion

The results derived in the previous section are used here to discuss antibaryon production in high energy collisions.

In the  $B = 0$  case the baryon and antibaryon densities are equal and Eqs. (14) and (15) yield

$$n_a^{(j)}|_{ce} = n_b^{(j)}|_{ce} = f_j \frac{I_1(2z)}{I_0(2z)} \cong f_j z, \quad n_a^{(j)}|_{gce} = n_b^{(j)}|_{gce} = f_j, \quad (17)$$

where the approximation for the *c.e.* density is valid for small system only. The canonical suppression factor (16) for  $B = 0$  is equal to

$$F_{cs}^0 = \frac{I_1(2z)}{I_0(2z)} \cong f V. \quad (18)$$

The approximation in Eq. (18) is valid for small system only ( $z \equiv fV \ll 1$ ).

The behavior of the canonical suppression factor  $F_{cs}^0$  (18) is shown by the solid lines in Fig. 1 for  $T = 160$  MeV, 170 MeV and 180 MeV, assuming that  $f$  is the sum of  $f_j$  over all non-strange baryons. The lines start from  $V = 5 \text{ fm}^3$ , which is approximately equal to the estimate of the hadronization volume for  $e^+ + e^-$  interactions at  $\sqrt{s} = 29$  GeV [5]. One observes (see Fig. 1) a strong *c.e.* suppression of the (anti)baryon density. For  $T = 160$  MeV the (anti)baryon density increases by a factor of 10 from its value at  $V = 5 \text{ fm}^3$  to its  $V \rightarrow \infty$  *g.c.e.* limit. For the small systems the (anti)baryon density increases approximately linearly with  $V$ , i.e., the (anti)baryon multiplicity for the small systems is proportional to  $V^2$ . The *c.e.* suppression becomes less pronounced and the volume region with linear increase of the (anti)baryon density is reduced for increasing temperature.

Let us now turn to the antibaryon production in baryon rich system. In the analysis of data on particle multiplicities in p+p, p+A and A+A collisions one usually assumes that all participating nucleons in the collisions (wounded nucleons) take part in the statistical

hadronization of the system. It means that in the analysis of the NA49 results on antiprotons from p+p interactions to central Pb+Pb collisions at 158 A·GeV we should study statistical systems with  $2 \leq B \leq 400$ . It was found that the mean multiplicity of pions per wounded nucleon increases (at the SPS collision energies) only by about 20% when going from p+p interactions to central Pb+Pb collisions [17]. The pion to baryon ratio in the statistical model is determined by two parameters: the temperature and baryon density. Thus as the temperature is found to be constant ( $T = 175 \pm 15$  MeV) we conclude that the baryon density at hadronization in nuclear collisions at 158 A·GeV is also approximately constant.

Therefore, for the comparison with the NA49 results we study the evolution of the antibaryon density with increasing net baryon number  $B$  at  $T = \text{const}$  and  $\rho_B = \text{const}$ . The *c.e.* suppression factor (16) is found at these conditions from Eqs. (14,15)

$$F_{cs}^B = \frac{1 + \sqrt{1 + x^2}}{x} \frac{I_{B+1}(xB)}{I_B(xB)}; \quad x \equiv \frac{2f}{\rho_B}. \quad (19)$$

Its  $B$ -dependence is plotted in Fig. 2 for several different values of the parameter  $x$ . Note that our assumption  $T = \text{const}$  and  $\rho_B = \text{const}$  for statistical hadronization at different values of  $B$  can be substituted by a weaker one,  $x = \text{const}$ . From Fig. 2 one observes that the *c.e.* suppression of antibaryon density becomes stronger at high baryon density (i.e., small  $x$ ). For  $x < 1$  the *c.e.* suppression  $F_{cs}^B$  (19) becomes close to its  $x \rightarrow 0$  limit:

$$F_{cs}^B = \frac{B}{B + 1}. \quad (20)$$

Eq. (20) shows that the strongest *c.e.* suppression of the antibaryon density is for the  $B = 2$  (nucleon–nucleon interactions) case and it leads to the suppression factor of  $2/3$ . This moderate effect of *c.e.* suppression is in strong contrast with the large *c.e.* suppression (i.e.,  $F_{cs}^0 \ll 1$ ) in the baryon–free system. A mathematical reason of this very different behavior for  $B = 0$  and  $B \geq 2$  (with  $\rho_B = \text{const}(V)$ ) is due to the fact that in the latter case both the order of the modified Bessel functions and their arguments are dependent on  $B$  (i.e., on  $V$ ) whereas in the  $B = 0$  case only the argument increases with  $V$ .

The presence of non-zero baryon number  $B > 0$  has a twofold effect on antibaryon production. First, it suppresses the production of antibaryons: the additional factors  $\exp(-\mu_B/T) = x/(1 + \sqrt{1 + x^2}) < 1$  and  $1/(B + 1) < 1$  appear respectively in the 'large' and 'small' systems for the antibaryon density in comparison with the  $B = 0$  case. On the other hand, the *c.e.* suppression effect due to the exact baryon number conservation becomes smaller: at fixed  $T$  and  $V$  the following inequality is always valid,  $F_{cs}^B > F_{cs}^0$ . For fixed  $B > 0$  the *c.e.* suppression of antibaryons becomes smaller when  $\rho_B$  decreases and it disappears completely (i.e.,  $F_{cs}^B \rightarrow 1$ ) in the limit  $\rho_B \rightarrow 0$  (and respectively  $V \rightarrow \infty$  in order to keep the  $B$  value fixed). This is because the total number of baryon–antibaryon pairs becomes large due to large  $V$ . Note that in this case the last approximation in Eq. (14) is no more valid. Instead one should use the large argument asymptotic of the modified Bessel functions.

Thus for  $B \geq 2$  systems at constant  $x = 2f/\rho_B$  the *c.e.* suppression factor  $F_{cs}^B$  (19) ranges between  $2/3$  and 1 for  $x \ll 1$  and between  $(1 - 1/4x)$  and 1 for  $x \gg 1$ .

Previous analyses of hadron production at the CERN SPS indicate large baryon densities at hadronization. The typical values of  $T \cong 170$  MeV and  $\mu_B \cong 250$  MeV found for Pb+Pb collisions [4] lead to the estimate  $x = 2f/\rho_B = sh^{-1}(\mu_B/T) \approx 0.5$ . As seen from Fig. 2 the *c.e.* suppression factor  $F_{cs}^B$  (19) for this 'small' value of  $x$  is close to its limiting pattern  $B/(B+1)$  (20). In Fig. 3 the NA49 results [9] on the  $\bar{p}/\pi$  ratio in p+p and Pb+Pb collisions at 158 A·GeV are compared with this limiting pattern. From this comparison we conclude that the model of statistical production of antiprotons at hadronization in baryon-rich system correctly reproduces the observed *antiproton scaling*.

Let us return again to the case of the baryon-free system. The statistical model calculations for  $e^+e^-$  [5] and  $p+\bar{p}$  [6] interactions include large *c.e.* suppression effects. As discussed in the introduction we assume that the hadronization temperature reflects a universal property of the hadronization process and therefore should be collision energy independent. In the case of  $e^+e^-$  [5] interactions the hadronization volume is small and therefore one expects approximate proportionality to  $V^2$  of the multiplicity of nucleon-antinucleon pairs but only a linear increase with  $V$  of the pion multiplicity. Therefore, the  $\bar{p}/\pi$  ratio calculated within the model increases linearly with increasing pion multiplicity. However experimental data [14, 15] contradict this expectation of the statistical model. The  $\bar{p}/\pi$  ratio which is plotted in Fig. 4 as a function of pion multiplicity for  $e^+e^-$  interactions at different energies,  $\sqrt{s} = 14 \div 91$  GeV, is approximately constant.

Within the discussed statistical hadronization model one can try to solve the problem by assuming an increase of the temperature  $T$  with decreasing volume  $V$ . The function  $f(T)$  strongly increases with  $T$  which allows to compensate the *c.e.* suppression effect (to keep the (anti)nucleon density,  $n_a \cong f^2(T)V$ , constant) for moderate (of about 10 MeV) changes of  $T$ . This indeed is observed in the fit results of the statistical model for the  $e^+e^-$  and  $p+\bar{p}$  data: the increase of the volume is always accompanied with the decrease of the temperature parameter [5, 6]. Thus one may argue that the hadronization condition  $T = const$  has to be substituted by a different criterion which should explain the decrease of the temperature with increasing size of the system in  $e^+e^-$  and  $p+\bar{p}$  interactions. The constant energy per particle was recently discussed as a chemical freeze-out condition [18]. The detailed study of this question is, however, outside of the scope of the present paper. We note only that the statistical production of heavy particles (e.g.,  $J/\psi$  mesons [19]) is very sensitive to the temperature parameter. Their yields, therefore, can be used to clarify the problem.

#### 4. Summary

The role of baryon number conservation in the calculations of antibaryon multiplicity within statistical model of hadronization was investigated. We derived explicit analytical formulae for the antibaryon multiplicity in baryon-free and baryon-rich small and large systems. This formalism was further used to discuss *antiproton scaling* observed experimentally in A+A collisions. The statistical model with constant hadronization temperature correctly reproduces the weak dependence of the  $\bar{p}/\pi$  ratio on the system size in p+p and nuclear collisions at the CERN SPS energy. A description of the ratio of  $J/\psi$  mesons to pions within the statistical hadronization model requires also a constant temperature parameter in p+p and A+A collisions at the CERN SPS. However, the same model with  $T = const$  does not give a natural explanation of the approximate indepen-



dence of the  $\bar{p}/\pi$  ratio of collision energy in  $e^+e^-$  interactions. Therefore, a consistent description of hadron production within the statistical hadronization model has not yet been achieved.

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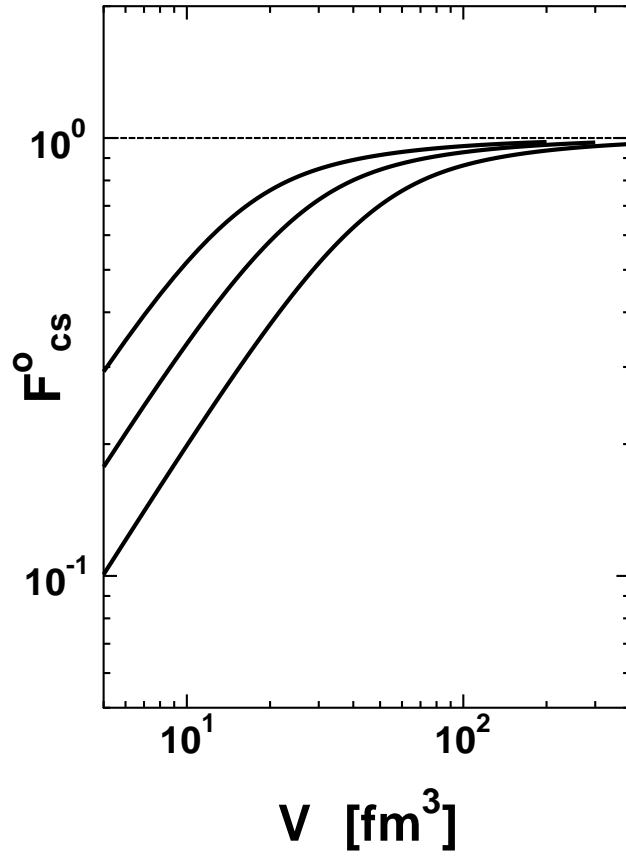


Figure 1: The solid lines show the *c.e.* suppression factor  $F_{cs}^0$  (16) for  $T = 160$  MeV, 170 MeV and 180 MeV (from bottom to top) for  $B = 0$ .

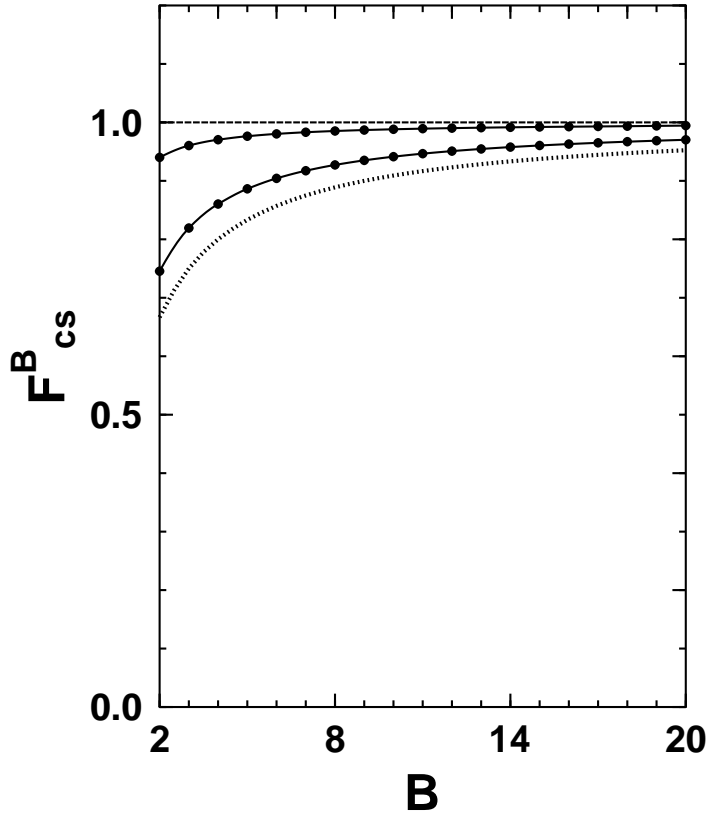


Figure 2: The finite size  $B$ -dependence of the antibaryon production in baryon rich ( $B \geq 2$ ) systems at different values of the variable  $x$  ( $x \equiv 2z/B = 2f/\rho_B$ ). The solid lines show the *c.e.* suppression factor  $F_{cs}^B$  (19) for  $x=1$  and  $x=5$  (from below to above). The lower dotted line corresponds to the limiting  $B/(B+1)$  behavior (20).

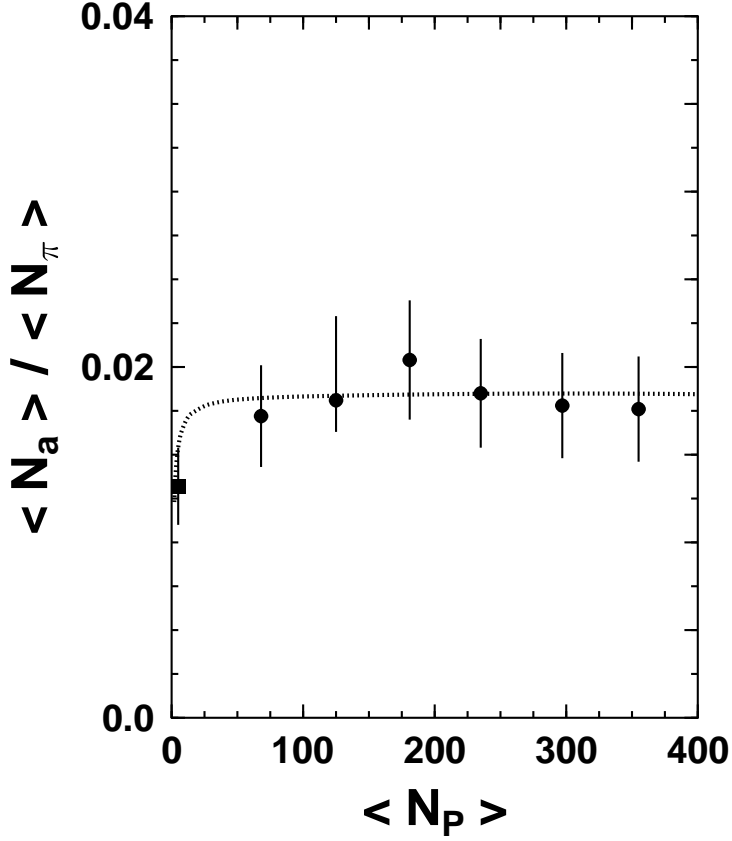


Figure 3: The NA49 data [9] on the antiproton to pion ratio ( $\langle N_\pi \rangle \equiv (\langle N_{\pi^+} \rangle + \langle N_{\pi^-} \rangle)/2$ ) in p+p (square) and centrality selected Pb+Pb (dots) collisions at 158 A·GeV are plotted as a function of the mean number of wounded nucleons,  $\langle N_P \rangle$ . The dependence of the ratio on  $\langle N_P \rangle$  expected within the statistical model,  $\langle N_P \rangle / (\langle N_P \rangle + 1)$ , is shown by dotted line, the function is normalized to the experimental data.

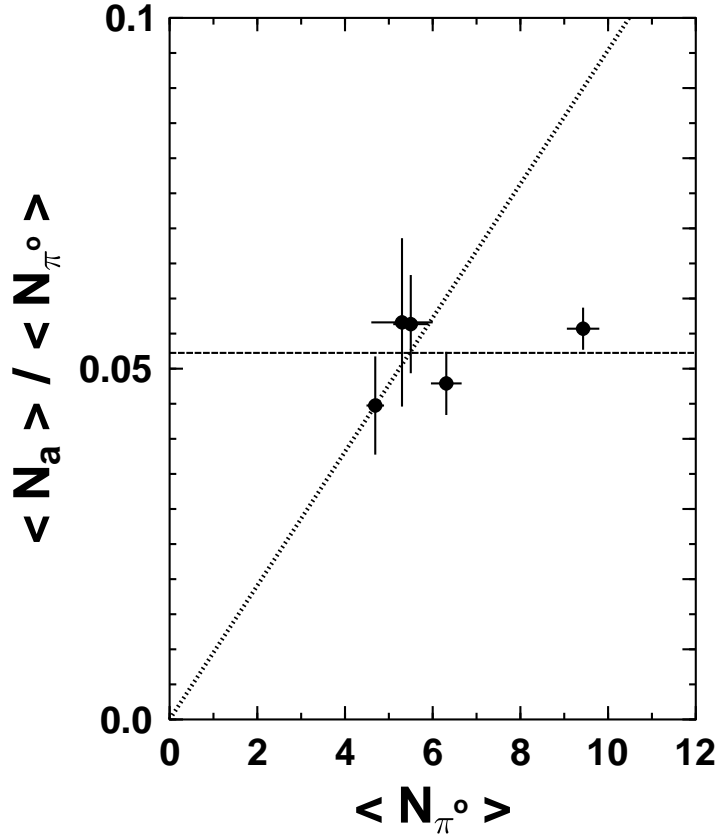


Figure 4: The antiproton [14] to  $\pi^0$  [15] ratio in  $e^+e^-$  annihilation at  $\sqrt{s} = 14, 22, 29, 35$  and  $91$  GeV is plotted as a function of mean multiplicity of  $\pi^0$  mesons,  $\langle N_{\pi^0} \rangle$ . In order to increase the significance of the data the antiproton multiplicity was calculated as average of proton and antiproton multiplicities. The dotted line indicates the proportional dependence of the ratio on  $\langle N_{\pi^0} \rangle$  expected within statistical model for small systems. The horizontal dashed line is shown for the reference.