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Contingent Contracts in Banking: Insurance or Risk Magnification?*

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Abstract

We examine whether the economy can be insured against banking crises with deposit and loan contracts contingent on macroeconomic shocks. We study banking competition and show that the private sector insures the banking system through such contracts, and banking crises are avoided, provided that failed banks are not bailed out. When risks are large, banks may shift part of them to depositors. In contrast, when banks are bailed out by the next generation, depositors receive non-contingent contracts with high interest rates, while entrepreneurs obtain loan contracts that demand high repayment in good times and low repayment in bad times. As a result, the present generation overinvests, and banks generate large macroeconomic risks for future generations, even if the underlying productivity risk is small or zero. We conclude that a joint policy package of orderly default procedures and contingent contracts is a promising way to reduce the threat of a fragile banking system.

Keywords: Financial intermediation, macroeconomic risks, state-contingent contracts, banking regulation.

JEL Classification: D41, E4, G2

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1 Introduction

Motivation and Contribution
The literature and, as a real-world example, the recent financial crisis provide ample evidence that the costs of banking crises, in terms of GDP losses, may be very large. One reason for this is that traditional contractual arrangements in banking expose the banks to the risks associated with macroeconomic shocks, which may in turn lead to a crisis, as already observed by Hellwig (1998). Such shocks can be exogenous or they can arise when many banks undertake investments with correlated risks and thereby increase economy-wide aggregate risk.

A promising way to solve this problem might be to induce banks to make deposit and loan contracts contingent on macroeconomic events, such as GDP growth, or other contractible macroeconomic indicators that are highly correlated with the financial health of the banking sector, such as the average bank equity of competing banks. In this paper, we examine whether the banking system can be insured against crises by the private sector through contingent loan and deposit contracts.\(^1\) Our analysis is both a positive and normative exercise. On the positive side, we examine what happens when banks compete with contingent deposit and loan contracts. On the normative side, we explore how different regulatory approaches towards insolvent banks affect the scope of private insurance against banking crises in such circumstances. The contribution of the paper is to identify the properties of double-sided Bertrand Competition with contingent contracts on both sides. This opens up both insurance possibilities and the opposite—the amplification of risk—or even the creation of new risks, depending on the regulatory approach towards banking crises. While, at least since Merton (1977) and Kareken and Wallace (1978), it is well-known that governmental approaches towards failing banks affects the banks’ risk-taking incentives, there are new phenomena: the risk amplification and creation consequences of double-sided banking competition with contingent deposit and loan contracts.

\(^1\)How contracts can be made dependent on macroeconomic risk by defining and maintaining standardized macroeconomic indices has been examined and discussed extensively in the context of sovereign GDP-linked bonds by Shiller (Shiller (2003) and recently by Benford, Ostry and Shiller (2018). In our concluding section, we will comment on the range of possible macroeconomic indicators in our model.
Model and Results
Specifically, consider a simple model in which banks alleviate agency problems in financial contracting. Banks compete for funds and offer credit contracts to potential borrowers. We allow for macroeconomic shocks affecting the average productivity of investment projects. There are two overlapping generations of agents. We focus on the behaviour of the first generation while the second generation may be burdened by bailout costs.

We distinguish between bailout and failure, depending on whether insolvent banks are bailed out or have to go bankrupt. Bailout occurs through the second generation. The main conclusions are as follows: Suppose that the regulator commits to bankruptcy for insolvent banks. Then, financial intermediation with contingent contracts yields an efficient risk allocation. If macroeconomic shocks are small, depositors and entrepreneurs are offered non-contingent deposit and loan contracts. All macroeconomic risk is borne by entrepreneurs. The inside funds of entrepreneurs act as a buffer for macroeconomic risks. If macroeconomic shocks are large, banks write state-contingent contracts for depositors and debtors. Part of the macroeconomic risk is shifted to consumers, since entrepreneurs cannot bear the entire risk. Consumers and entrepreneurs together insure the banking sector, and banking crises are avoided.

The risk situation changes completely if bank deposits are guaranteed. Thus, future generations provide funds to pay back banks’ obligations to the previous generation to prevent them from becoming insolvent. With bailout, competing banks try to generate a profitable (positive intermediation margin) and a non-profitable (negative intermediation margin) state of the world. In the good state—with high productivity of investment projects—, they request high loan interest rates from entrepreneurs. To motivate entrepreneurs to invest rather than to save, banks request very low repayments in the bad state—with low productivity of investment projects. Deposit rates are non-contingent, since deposits are insured by the next generation.

Competition among banks for the creation of a profitable state pushes deposit rates up to the maximal amount entrepreneurs can repay in that state. As a result, banks create a state of the world with high repayment obligations to depositors, but with very low pay-back requirements for entrepreneurs. This creates large risks for future
generations, even if the underlying productivity risk is small or zero. This is called the “risk-magnification effect”.

We also show that when banks choose between contingent or non-contingent contracts, they will always offer non-contingent deposit contracts but contingent loan contracts to maximize risk creation.

As a consequence of the risk-magnification effect, the present generation receives higher interest rates on savings than in a situation with bank failures. This induces overinvestment among the current generation at the expense of future generations. Specifically, output from investment projects over two generations is higher under bank failure than under bailout when the second generation has to bailout the first.

Implications

Overall, the analysis reveals that it is possible for the private sector to insure the banking sector against a crisis. If it is possible for insolvent banks to go bankrupt, the negative macroeconomic shock is absorbed by the fluctuations of wealth in the private sector. Hence, large-scale defaults of banks and thus banking crises are avoided.

However, if the preconditions regarding the commitment of the government to refrain from large-scale bailouts are not given, risk allocation not only changes compared to the case with non-contingent banking contracts, but the situation may, in fact, worsen. Suppose that banks can expect to be bailed out by the government, so that tax payers insure the banking sector. In the recent crisis, for instance, even banks not protected by explicit deposit insurance (investment banks or “shadow banks”) have been bailed out. In such cases, contingent contracts will exacerbate the consequences of negative events, generating additional risk. The overinvestment and booms of today would be followed by even more cataclysmic crashes tomorrow.

Thus, contingent contracts are neither an all-purpose panacea nor do they guarantee a decline in the likelihood and severity of banking crises, as the creation of additional risk will set in if other policy conditions are not met. It will be therefore essential to develop orderly default procedures for large banks, such that failing banks do not themselves threaten the stability of the banking system, as became evident when Lehman Brothers collapsed. This would make insurance against systemic crises viable and would make the use of default procedures unnecessary in the first place. Thus, a joint policy package
of orderly default procedures and contingent contracts is a promising way to reduce—or even eradicate—the threat of a fragile banking system.

2 Relation to the Literature

Our paper is related to the current discussion on regulatory issues regarding financial intermediaries. First, our model may explain that competition of financial intermediaries with contingent deposit and loan contracts under a bailout system increases the underlying productivity risk or creates risk even if there is no productivity risk, as banks compete to create profitable states of the world. The usual regulatory discussion has focused on the behavior of single institutions (see e.g. Dewatripont and Tirole (1994)), or on the incidence of aggregate risk on the banking system without contingent contracts (Blum and Hellwig 1995, and Gehrig 1997). The former literature has pointed out and tested (e.g. Keeley 1990) that a low charter value increases a bank’s incentive to take on risk. Our model shows that this risk-taking incentive for bank managers is complemented by the risk-magnification effect we introduce in this paper. Even if the underlying productivity risk is small or zero, competition among banks, with contingent contracts and under a bailout approach, yields large macroeconomic risks for future generations.

Second, the idea that financial contracts could and should be conditioned on macroeconomic indicators has been around for some time (see e.g. Hellwig (1998) and Shiller (2003)). It has been pointed out by Hellwig (1995, 1998) that it is unclear why the terms of the deposit contracts are not made contingent on aggregate events, such as fluctuations in the gross domestic product. Hellwig (1998) offers three explanations for this phenomenon: lack of awareness among contractors, moral hazard in banking, and transaction costs together with the market-making role of financial intermediaries. Hellwig (1998) points out that the absence of such terms in deposit contracts may be a manifestation of excessive risk-taking. Our model indicates that bailouts of banks in a crisis will not induce contingent deposit contracts, even if they become feasible, but will lead to contingent loan contracts with very large differences in state-dependent repayments. As a consequence, the possibility to write deposit and loan contracts contingent on macroeconomic shocks creates larger aggregate risks than the underlying
fundamental risk. State-dependent deposit contracts only occur for large productivity shocks and for a regulatory scheme that induces bankruptcy of insolvent banks.

Third, there are empirical parallels to our results. Inflation-indexed or Forex-related loan and deposit contracts in Latin America and Southeast Asia appear to have contributed to macroeconomic instability, and hollowed out the banking system through defaults. This suggests that contracts contingent on macroeconomic factors may trigger banking crises, or contribute to them. Our argument is that financial intermediation with deposit and loan contracts contingent on macroeconomic shocks can imply banking instability when such schemes are offered competitively under bailout schemes.²

Fourth, at a more general level, our investigation indicates that new contractual opportunities, i.e. the possibility to make deposit and loan contracts contingent on macroeconomic shocks, may increase both aggregate credit and financial instability. Our exercise complements the important insights of Shin (2009), i.e. that securitization enables credit expansion through higher leverage of the financial system as a whole, while the impact on financial stability is ambiguous.

Fifth, the performance of the banking system changes entirely if there is a credible commitment by the regulator to let insolvent banks go bankrupt. Then, productivity shocks are fully absorbed by the fluctuation of income or risk-neutral agents who enter into contingent loan contracts. This insures the banking system and risk averse depositors. Banks never default in equilibrium. The risk allocation is efficient. This can be viewed as a private deposit insurance scheme. Historically, countries have embarked on public deposit insurance schemes. In the US, for instance, federal deposit insurance started under the (Glass-Steagall) Banking Act of 1933 which created the Federal Deposit Insurance Corporation (FDIC) that is in charge of insuring deposits at commercial banks. A thorough discussion of this scheme can be found in Pennacchi (2009). Calomiris and Jaremski (2016) provide a comprehensive historical account of the economic and political theories of deposit insurance. They conclude that public deposit insurance generally tends to increase systemic risk rather than reduce it. We complement this literature and show that public deposit insurance generates the risk-magnification effect.

²Gersbach (2013) provides a survey of insurance approaches against banking crises.
The paper is organized as follows. The next section describes the model. In the third section, we derive the equilibrium in the intermediation market without the presence of macroeconomic shocks. In section four, we introduce temporary productivity shocks, state-contingent deposit and loan contracts, and regulatory schemes. In sections five and six, we examine small and large productivity shocks under different regulatory schemes. Section seven presents our conclusions.

3 The Model

We consider a two-period model ($t = 1, 2$). Later, when we consider regulatory policies such as bailouts, we will introduce more than one generation of agents living for two periods to guarantee credible deposit insurance by taxing future generations.

The generation under consideration consists of a continuum of agents, indexed by $[0,1]$, and living for two periods. There are two classes of agents in each generation. A fraction $\eta$ of individuals consists of potential entrepreneurs. The rest, $1 - \eta$, of the population are consumers. Potential entrepreneurs and consumers differ insofar that only the former have access to investment technologies. There is one physical perishable good that can be used for consumption or investment. Each individual in each generation receives an endowment $e$ of the good when young and none of it when old.

Each entrepreneur has access to a production project that converts time 1 goods into time 2 goods. The required funds for an investment project are $F := e + I$. Hence, an entrepreneur must borrow $I$ units of the good in order to undertake the investment project. The class of entrepreneurs is not homogeneous. We assume that entrepreneurs are indexed by a quality parameter $q$ uniformly distributed on $[\bar{q} - 1, \bar{q}]$, $1 < \bar{q} < 2$, in the population of entrepreneurs. If an entrepreneur of type $q$ obtains additional resources $I$ and decides to invest, he realizes gross investment returns in the next period of

$$q(I + e).$$ (1)

$\bar{q}$ is the aggregate indicator of the productivity of investment projects in period 1. If $\bar{q}$ is
uncertain the generation under consideration faces macroeconomic risk. For simplicity, we assume that potential entrepreneurs are risk neutral and are only concerned with consumption in their old age, i.e., they do not consume when young.

Consumers consume in both periods. They have utility functions $u(c_1, c_2)$ defined over consumption in the two periods. The variables $c_1, c_2$ are the consumption of the consumers when young and old, respectively. Consumers are risk-averse. If an agent can transfer wealth between the two periods at a riskless real interest rate, denoted by $r$, the solution of the agent’s intertemporal consumption problem generates the saving function, denoted by $s\{r\}$. We follow the standard assumptions in the OG literature that the substitution effect (weakly) dominates the income effect, i.e., savings are an increasing function of the interest rate. We drop the time index whenever convenient.

The rationale for the underlying banking model we are using, the consequences of monitoring banks and the nature of contracts that arise are developed in Gersbach and Uhlig (2006), who abstract from macroeconomic shocks. A detailed description of this banking model is given in Appendix B. In the main text, we summarize the two core assumptions and the two properties of this banking model that will be used in the current investigation.

- **Assumption 1**: Banks cannot observe quality $q$.
- **Assumption 2**: Banks can secure the liquidation value.
- **Property 1**: An entrepreneur with quality $q$ will invest if he obtains a loan.
- **Property 2**: The contracts banks are offering are debt contracts (deposit and loan contracts).

With these two assumptions and the simplification to assume zero monitoring costs for banks, we concentrate on the consequences when banks compete with deposit and loan contracts contingent on macroeconomic shocks.

We deliberately assume that banks have zero equity capital, which makes the insurance of the system most difficult.\(^3\) For all our arguments, it will be sufficient that two banks

\(^3\)When banks have positive but not a large amount of equity capital, insurance in the case of bank failure works as presented in the paper. When banks are bailed out, the incentives to generate risk
compete. Hence, we assume that there are two banks, indexed by $j$ ($j = 1, 2$), which finance entrepreneurs.

First, we discuss the nature of contracts offered by banks indexed by $j = 1, 2$. Bank $j$ can sign deposit contracts $D(r^d_j)$, where $1 + r^d_j$ is the repayment offered for one unit of resources. Loan contracts of bank $j$ are denoted by $C(r^c_j)$, where $1 + r^c_j$ is the repayment required from entrepreneurs for one unit of funds. If macroeconomic risk is present, we allow contracts to be conditioned on the realization of $\eta$ or on the resulting GDP in period 1. In such cases, state-contingent deposit or loan contracts can be written.

Note that the availability of production technologies from period 1 to 2 allows depositors and entrepreneurs of each generation to trade among themselves. Generations are connected by financial intermediaries which represent the sole long-living institution. A new generation is only affected by the preceding generation when banks have accumulated either profits or losses. In the former case, a generation may buy the shares of the banks. As we focus on Bertrand Competition, the profits and the price of bank shares will be zero. Thus, this case is trivial and can be disregarded. In the latter case, a generation may be forced by regulation or may wish to rescue banks since otherwise a collapse will impose high costs. This will be the focus of our analysis. Losses of banks will only occur if aggregate risk is present and, hence, there is uncertainty about $\eta$.

Finally, we assume that banks are owned by entrepreneurs who are risk-neutral, and we specify the objectives of banks. We assume that banks maximize expected profits and hence internalize losses that accrue to depositors, if their claims cannot be fully served. We shall focus on expected profits and not on return on equity. We do this for two reasons. First, the case of return on equity maximization is equal to the case of expected profit maximization with bailout, as shareholders have zero returns in the event of losses. Hence, our results will automatically cover the case of equity return maximization. Second, expected profit maximization is a realistic scenario since bank remains, but the risk generated for future generations is reduced by the equity buffer. Details are available upon request.

$\text{As we focus on Bertrand Competition, an extension to more than two banks is straightforward.}$

$\text{In this model, intergenerational trade does not improve autarky for all generations. In particular,}$

$\text{insuring depositors against the macroeconomic risk by taxing future generations will make these future}$

$\text{generations worse off.}$

$\text{If banks are owned by depositors, the objective function of banks is more subtle and one has to}$

$\text{add a risk premium since different bank strategies are associated with different risks and insurance}$

$\text{opportunities for depositors. See Gersbach et al (2015).}$

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managers may suffer a utility loss in the event of default. Such utility losses may occur because there are non-pecuniary penalties associated with default. The non-pecuniary utility loss may occur because career opportunities decline and/or the reputation is being destroyed.\textsuperscript{7} Utility losses in the event of default could also occur if bankers are financed through wages and with bonuses that vary with profits.

4 Equilibrium without Macroeconomic Shocks

4.1 The Game

We begin with a discussion of the case where no macroeconomic shocks are present as this will be proved useful in understanding the results presented later in this paper. Obviously, deposit and loan contracts will have a length of one period as no transformation of maturities needs to take place. We examine the following four-stage intermediation game for the generation under consideration.

Period 1

1. Banks offer deposit contracts to consumers and entrepreneurs.

2. Banks offer credit contracts to entrepreneurs.

3. Consumers and entrepreneurs decide which contracts to accept. Resources are exchanged. Entrepreneurs start producing, which is subject to macroeconomic risk.

Period 2


The game is a multi-stage game with observed actions. That is, actions at each stage are chosen simultaneously, and players know the actions in all previous stages when they enter the next stage. As there is a continuum of consumers and entrepreneurs,\textsuperscript{7} This may be actively promoted by bank regulators, if they punish failures by bank managers.
they are assumed to be contract takers. Banks are the only strategic players that set deposit and loan interest rates.

It is useful to start with the loan application decision of an entrepreneur with quality \( q \), given that he observes \( r^d_j \), \( r^c_j \) of banks. Entrepreneurs are contract-takers and thus make loan application decisions with the assumption that they will not be rationed at banks that offer the highest deposit rate.\(^8\) If entrepreneurs seeking loans were rejected, they would choose to save \( s \) at the banks that offer the highest deposit rate. In all equilibria in this paper, the entrepreneurs applying for loans are not rationed and thus their expectations are correct.\(^9\)

If an entrepreneur obtains a loan, he also has an incentive to invest. The reason is the assumption, detailed in Appendix B, according to which monitoring technologies are efficient enough at reducing the private benefits of entrepreneurs if they do not invest.

If an entrepreneur applies for a loan at the bank offering the lowest loan rate, because of limited liability his terminal wealth or consumption \( W(q) \) will amount to

\[
W(q) = \max\{q(e + I) - I(1 + \min\{r^c_j\}), 0\}.
\]

We note that an entrepreneur defaults if \( q(e + I) < I(1 + \min\{r^c_j\}) \). Since banks can secure the liquidation value, the entrepreneur has zero wealth in such cases. If he does not apply, he obtains \( e(1 + \max\{r^d_j\}) \) by saving his endowments. Thus, there exists a critical quality parameter, denoted by \( q^* \), and given by

\[
q^*(\min\{r^c_j\}, \max\{r^d_j\}) = 1 + \frac{I\min\{r^c_j\} + e\max\{r^d_j\}}{e + I},
\]

which motivates entrepreneurs with \( q \geq q^* \) to apply for loans and entrepreneurs with \( q < q^* \) to save. Equation (3) determines the marginal entrepreneur.

\(^8\)As only those banks will obtain deposits and will be active, it is intuitive that entrepreneurs seeking loans only apply at banks that offer the highest deposit rate. We could relax the assumption by modeling entrepreneurs as contract takers at any bank, which, however, complicates the analysis considerably.

\(^9\)If a bank does not have enough deposits to lend to all candidate borrowers, loans are rationed. In such a case, we assume that the loan applicants at the said bank are rationed with the same probability, such that loan volume and deposits are balanced. Other rationing schemes might be considered, where rejected entrepreneurs go to another bank in order to apply for loans. In an extended version of the model, we show that the results are robust for different rationing schemes. The main argument is that more sophisticated rationing schemes tend to lower the profits of banks that deviate from an equilibrium. Details are available upon request.
Note that we have assumed that banks can ensure investment and can verify output conditional on investment. Thus, they are not concerned about low-quality entrepreneurs since such entrepreneurs would have less consumption than with saving endowments and thus will not apply for a loan. Still, entrepreneurs may default if they cannot repay the loan. In such cases, banks obtain the output of the entrepreneur as the liquidation value.

Banks are assumed to maximize expected profits. The assumption is justified in detail in Section 3. Hence, conditional on granting a credit to an entrepreneur with quality level $q$ and receiving funds from savers profits per credit of a bank $j$ amount to

$$G_j = \min\{q(e + I), I(1 + r_j^c)\} - I(1 + r_j^d). \quad (4)$$

If the entrepreneur does not default, profits per credit are

$$G_j = I(1 + r_j^c) - I(1 + r_j^d) = I(r_j^c - r_j^d) = I\Delta_j. \quad (5)$$

$\Delta_j$ is the intermediation margin of bank $j$. In order to derive the intermediation equilibrium, we make the following technical assumptions:

**Assumption 1**

$$(1 - \eta) s \{\bar{q} - 1\} < \eta I.$$  

**Assumption 2**

$$(1 - \eta)s\{0\} + \eta e(1 - (\bar{q} - 1)) < \eta (\bar{q} - 1) I.$$  

The first assumption implies that savings are never sufficient to fund all entrepreneurs. Since the deposit rate $r_j^d$ cannot exceed $\bar{q} - 1$ without causing losses for banks, and since we have assumed that the savings of consumers are weakly increasing in the deposit rate, Assumption 1 is a sufficient condition that savings are lower than the funds needed to finance all entrepreneurs.

Assumption 2 states that investments exceed savings at zero deposit and loan interest rates. At such interest rates, we have $q^* = 1$ and hence entrepreneurs with $q \in [1, \bar{q}]$ apply for loans, while entrepreneurs with $q \in [\bar{q} - 1, 1)$ save their endowments. As no entrepreneur would apply for loans at interest rate $r_j^c > \bar{q}$, Assumption 2 implies that savings and investments can be balanced at positive interest rates.

A subgame perfect Nash equilibrium among banks is a tuple
\[
\left\{ \left\{ r_{d,j}^* \right\}_{j=1,2} \right\}, \left\{ r_{c,j}^* \right\}_{j=1,2}
\]

so that

- entrepreneurs make optimal credit application and saving decisions, as contract takers, i.e. entrepreneurs with \( q \geq q^* \), apply for loans and entrepreneurs with \( q < q^* \) save, where \( q^* \) is given by Equation (3);
- no bank has an incentive to offer different deposit or loan interest rates.

Therefore, the strategy spaces of banks are deposit and loan rates.\(^\text{10}\) In order to complete the description of the game, we assume that failing banks are not bailed out.\(^\text{11}\)

### 4.2 The Equilibrium

The following proposition is proved in the Appendix:

**Proposition 1**

There exists a unique equilibrium of the intermediation game with

(i) \( r^* = r_{c,j}^* = r_{d,j}^* \) for \( j = 1, 2 \),

(ii) \( r^* \) is determined by

\[
(1 - \eta) s \{ r^* \} + \eta e \left( 1 + r^* - (\bar{q} - 1) \right) = \eta \left( \bar{q} - (1 + r^*) \right) I,
\]

(iii) \( q^* = 1 + r^* \),

(iv) bank profits are zero.

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\(^\text{10}\)Interest rates on deposits and loans are usually constrained in such a way that repayments of debtors in Stage 4 are non-negative.

\(^\text{11}\)We note, however, that the equilibrium below continues to exist if failing banks are bailed out, but uniqueness is not necessarily guaranteed.
We note that Condition (ii) is the savings (left-hand side) and investment balance (right-hand side). The critical quality of the entrepreneur who is indifferent between saving and investment is equal to interest rate factor $1 + r^*$. Hence, all entrepreneurs with quality levels $q \in [1 + r^*, \bar{q}]$ will apply for loans.

Hence, the intermediation game yields the competitive outcome in which savings and investments are balanced and in which there is a common interest rate for loans and deposits. We can conclude from Proposition 1 that intermediation margins are zero in equilibrium, and savings and investments are balanced.

Note that in our model the incentive of banks to corner one side of the market, in order to obtain monopoly rents on the other side, does not destroy the perfect competition outcome. Suppose a bank offers a deposit rate slightly above $r^*$ in order to attract all depositors. If this bank raises $r_c$ in order to exploit its monopoly power among entrepreneurs, a portion of them will switch market sides. This, however, causes large excess resources for the deviating bank, inducing a loss greater than the excess returns from the remaining entrepreneurs. The excess resources resulting from market side switching is only one of several arguments why Walrasian outcomes can arise. For our purpose, it is important that competitive outcomes occur.

In equilibrium, all entrepreneurs with projects whose quality levels $q$ are equal to or above $1 + r^*$ will obtain funds and invest.

Aggregate income in period 2 is denoted by $Y$

$$Y = \eta(I + e) \cdot \left\{ \frac{(\bar{q})^2 - (1 + r^*)^2}{2} \right\} - e.$$  

Note that we have assumed $\bar{q} \leq 2$. If $\bar{q} > 2$, the pool of entrepreneurs has such high quality that loan demand is very high and equilibria with positive intermediation margins may exist.

Formally, aggregate income is given by

\[
Y = \left(1 - \eta \right) s \{ r^* \} (1 + r^*) + \eta \left( (1 + r^*) - (\bar{q} - 1) \right) (1 + r^*) e
\]

wealth of investors at the end

wealth of not investing entrepreneurs at the end

\[
+ \eta \int_{1+r^*}^{\bar{q}} (q(e + I) - I(1 + r^*)) dq - e
\]

wealth of investing entrepreneurs at the end

total wealth at the beginning

Using the equation in point (ii) in Proposition 1, yields the expression of \( Y \) in Equation (6). Aggregate income in period 2 is the output generated by investments in period 1 minus the initial wealth at the beginning of period 1. Note that banks do not need to put up equity to perform their intermediary function as they can fully diversify their lending activities.

5 Temporary Productivity Shocks, Contracts, and Regulation Schemes

In this section, we consider the possibility of aggregate productivity shocks. In period 1, \( \bar{q}_t \) is assumed to be \( \bar{q}^h \) with probability \( p \) (good state) \((0 < p < 1)\) or \( \bar{q}^l \) with probability \( 1 - p \) (bad state). The distribution of the entrepreneurs’ qualities varies accordingly. We assume \( \bar{q}^l < \bar{q}^h \) and \( z = \bar{q}^h - \bar{q}^l \) denotes the size of the shock. \( \bar{q}^e = p \cdot \bar{q}^h + (1-p) \bar{q}^l \) is the average productivity of the possible qualities. At the time when financial contracts are written, macroeconomic events are not known.

We maintain the assumptions that savings and investment can be balanced at positive interest rates for any of the following constellations. In particular, we assume that the boundary conditions in Assumptions 1 and 2 from the last section hold for both shock scenarios \( \bar{q}^l \) and \( \bar{q}^h \).

Equilibria of the intermediation game will now crucially depend on the regulator’s approach to banking crises. A banking crisis occurs in our model when one or both banks, and thus the entire banking system, is unable to repay depositors. We distinguish two
polar cases when banking crises occur: bailout and failure. If the regulator commits to failure, banks that are unable to satisfy depositors go bankrupt. If the regulator commits to bailout, he will tax future generations in order to save banks.\footnote{While we focus on polar cases of regulatory approaches toward banking crises, there are intermediate scenarios where the regulator taxes the current generation to bail out banks. This case is discussed in the Conclusion.}

With bailout, we assume that banks expect losses to be recovered precisely so that they will have zero profits in the future. If banks incur no losses in period $t$, they will anticipate zero profits due to Bertrand Competition. The assumption ensures that we can define an equilibrium of the financial intermediation game for a particular period.

The focus of our paper is to compare two regulatory schemes for banking crises when banks compete with contingent deposit and loan contracts.\footnote{The regulatory schemes could be endogenized in the following way. Suppose that the current generation can determine the regulatory approach toward banking crises. If the costs to establish a new banking system after the failure of the existing one are negligible, the current generation will always choose failure when faced with the case of a banking crisis. If the costs are prohibitively high and the current and future generations are taxed in the same way to pay for the set-up costs of new banks, existing banks will be saved.}

With stochastic aggregate productivity shocks, banks can offer state-contingent contracts in period $t - 1$. We use $C(r_{j}^{ch}, r_{j}^{cl})$ to denote the credit contract offered by bank $j$. $r_{j}^{ch}$ and $r_{j}^{cl}$ denote the interest rates demanded from borrowers in the good state and in the bad state respectively. Similarly, $D(r_{j}^{dh}, r_{j}^{dl})$ denotes deposit contracts with deposit rates $r_{j}^{dh}$ and $r_{j}^{dl}$, depending on the realization of macroeconomic shocks. We maintain the assumption that banks are risk neutral.\footnote{Since entrepreneurs as owners of banks are risk neutral, the assumption follows naturally.}

Since consumers are risk averse, they prefer a riskless interest rate over a lottery $\{r_{j}^{dh}, r_{j}^{dl}\}$ with the same expected interest rate. We assume that the consumers’ intertemporal preferences and their attitudes towards risk generate the saving function, now denoted by $s\{r_{j}^{dh}, r_{j}^{dl}\}$, which is assumed to be strictly increasing in each of its arguments.

The expected deposit rate is denoted by $r_{j}^{de} = p r_{j}^{dh} + (1-p)r_{j}^{dl}$. Similarly, the expected interest rate on loans is given by $r_{j}^{ce} = p r_{j}^{ch} + (1-p)r_{j}^{cl}$. To simplify notation, we use the following convention. An entrepreneur is characterized by his quality in the good state, $q \in [\eta^h - 1, \eta^h]$, or by his quality in the bad state, $q - z \in [\eta^l - 1, \eta^l]$ or by his
average quality, denoted by $q^e$, and given by

$$q^e = p \cdot q + (1 - p)(q - z). \quad (8)$$

The critical entrepreneur is denoted by $q^{e*}$ and depends on $(r^{ch}_j, r^{cl}_j, r^{dh}_j, r^{dl}_j)$. An entrepreneur with an expected quality $q^e$ and associated quality $q$ in the good state faces the following choices:

Applying for a credit yields expected wealth

$$\mathbb{E}[W(q)] = p \left\{ \max \{ q(I + e) - I(1 + r^{ch}_j), 0 \} \right\}$$
$$+ (1 - p) \left\{ \max \{ (q - z)(I + e) - I(1 + r^{cl}_j), 0 \} \right\}. \quad (9)$$

Saving funds yields expected wealth

$$e \left( p(1 + r^{dh}_j) + (1 - p)(1 + r^{dl}_j) \right) = e(1 + r^{de}_j).$$

Potential entrepreneurs are risk neutral. Thus, the comparison of the expected wealth from investing and saving determines the critical quality level above which entrepreneurs choose to invest. We note that in the bad state, the project returns may be insufficient to pay back the loan. In the following section, we examine the intermediation game in period $t - 1$, depending on the size of the shock.

### 6 Bank Failure

We first investigate the equilibria when insolvent banks go bankrupt.

#### 6.1 Small Productivity Shocks

We first consider the case where shocks are so small that funded and investing entrepreneurs are always able to pay back. The upper limit for small shocks such that this assumption holds will be given in the next proposition. In this case, the critical entrepreneur, in terms of expected quality, would be given by

$$q^{e*} = 1 + \frac{I \min \{ r^{ce}_j \} + e \max \{ r^{de}_j \}}{e + I}. \quad (10)$$
such that entrepreneurs with \( q^e \geq q^e^* \) apply for loans, while entrepreneurs with \( q^e < q^e^* \) save their endowments.\(^{17}\) Note that \( q^e^* \) is associated with a critical quality value in the good state, denoted by \( q^* \) and defined by

\[
q^e^* = p q^* + (1 - p)(q^* - z).
\]

We first derive the equilibrium when the regulator commits to failure. In the case of failure, depositors know that banks can never pay back a promised deposit rate if the lending rate is lower in the same state of the world. Hence, we restrict our analysis to \( r_{dj}^{\text{dh}} \leq r_{dj}^{\text{ch}} \) and \( r_{dj}^{dl} \leq r_{dj}^{cl} \), as banks have no incentive to offer deposit rates \( r_{dj}^{ch} < r_{dj}^{dh} \), since it would not be credible.

Provided funds are received and credit is granted to the entrepreneurs, expected profits per credit of bank \( j \), when there is no bailout, amount to

\[
\mathbb{E}[G_j] = pI(r_{dj}^{\text{ch}} - r_{dj}^{\text{dh}}) + (1 - p)I(r_{dj}^{\text{cl}} - r_{dj}^{\text{dl}})
\]

\[
= I(r_{dj}^{ce} - r_{dj}^{de}).
\]

The critical entrepreneur in equilibrium is denoted by \( q_{j}^{e^*} \). We obtain

**Proposition 2**

*Suppose that the regulator commits to failure. Then, there exists a unique equilibrium of the intermediation game if*

\[
z \leq \frac{e(1 + r^f)}{p(e + I)},
\]

*where \( r^f \) is determined by *

\[
(1 - \eta) s\{r^f, r^f\} + \eta e \left(1 + r^f - (q^e - 1)\right) = \eta \left(q^e - (1 + r^f)\right) I.
\]

*The equilibrium is given by *

\[
(i)
\]

\[
r^f = r_{j}^{ch} = r_{j}^{cl} = r_{j}^{dl} = r_{j}^{dh}, \quad j = 1, 2.
\]

\[
(ii)
\]

\[
q_{j}^{e^*} = 1 + r^f.
\]

\(^{17}\)Note that \( \min \{r_{ce} \} \) is restricted to the set of banks that offer the highest deposit rate, as entrepreneurs seeking loans will only apply at these banks.
At the equilibrium interest rates, the critical entrepreneur does not default:

\[(q^*_f - zp)(e + I) \geq I(1 + r^f).\]  \hspace{1cm} (12)

Banks make zero profits in both states of the world.

The proof is given in the Appendix. We first note that no entrepreneur defaults at the equilibrium interest rates as long as the size of the macroeconomic shock fulfills the condition of the proposition \(z \leq \frac{e(1 + r^f)}{p(e + I)}\). Second, we observe that the equilibrium interest rates, the critical entrepreneur, and the upper bound of the shock are fully determined by the exogenous variables.

Proposition 2 implies that financial intermediation, with a commitment to bankruptcy of insolvent banks by the regulator, yields the following intra-generational allocation of risks for the generation under consideration. Risk-neutral entrepreneurs can bear the entire macroeconomic risk, since they can repay the same interest rate in both states as the macroeconomic shock is below the critical size. The productivity shock is fully absorbed by the fluctuation of the entrepreneurs’ income, which insures the banking system. Banks never default in equilibrium. The risk allocation ensuing from Proposition 2 is efficient in the sense that the consumption allocation is Pareto efficient. As risk-neutral entrepreneurs bear the entire risk and insure risk-averse consumers,\(^{18}\) any other allocation of consumption in the first and second period would make at least one class of agents (consumers, saving entrepreneurs or investing entrepreneurs) worse off.\(^{19}\)

Finally, we observe that Proposition 1 can be viewed as a special case of Proposition 2 when we set \(p = 1\). In this case, all expressions of Proposition 2 collapse into those of Proposition 1.

### 6.2 Large Productivity Shocks

We complete our analysis with the examination of the case in which the shock is large. If the shock is sufficiently large, this makes complete insurance of depositors impossible

\(^{18}\)This can be viewed as a private deposit insurance scheme. A thorough discussion of public deposit insurance schemes can be found in Pennacchi (2009). Calomiris and Jaremski (2016) provide a comprehensive historical account of the economic and political theories of deposit insurance.

\(^{19}\)Formal details are available upon request.
in the failure regime. The essential condition is that the wealth of entrepreneurs is insufficient to insure depositors, i.e., \( z > \frac{e(1+r^f)}{p(e+I)} \), where \( r^f \) is determined by Proposition 2. We obtain

**Proposition 3**

Suppose that the regulator commits to failure and that \( z > \frac{e(1+r^f)}{p(e+I)} \). Then there exists an equilibrium of the intermediation game with:

(i) 
\[ r^h = r_j^{ch} = r_j^{dh}, \quad j = 1, 2. \]

(ii) 
\[ r^l = r_j^{cl} = r_j^{dl}, \quad j = 1, 2. \]

(iii) 
\[ r^h = r^h(r^l) := \frac{I(1 + r^l) + (e + I) \left\{zp - 1 - (1 - p)r^l\right\}}{p(e + I)}. \]

(iv) \( r^l \) is smaller than \( r^h \) and is determined by
\[
(1 - \eta) \cdot s \left\{ r^h(r^l), \ r^l \right\} + \eta e \left( q^{e^*}_f - (\bar{q}_e - 1) \right) = \eta (q^e - q^{e^*}_f) \cdot I, \quad \text{with}
\]

(v) 
\[ q^{e^*}_f = 1 + pr^h + (1 - p)r^l = \frac{I(1 + r^l)}{e + I} + zp. \]

Banks make zero profits in both states of the world.

The proof is given in the Appendix.\(^{20}\)

Several remarks are in order. First, the three endogenous variables, interest rates \( \{r^h, r^l\} \) and the critical entrepreneur \( q^{e^*}_f \) are determined by three conditions: savings/investment balance, indifference of critical entrepreneur between savings and investment and the condition that the critical entrepreneur and all other entrepreneurs just do not default in the bad state. As shown in the proof, this yields the three conditions (iii), (iv) and (v). Condition (iv) is the savings and investment balance since quality levels above \( q^{e^*}_f \) apply for loans. Condition (iii) is the value of the interest rate

\(^{20}\)Establishing uniqueness is extremely cumbersome. Details on how to prove that other equilibria do not exist are available upon request.
in the good state such that an entrepreneur with $q^*_f$ is indeed indifferent between investing and saving. Condition (v) expresses that the output of the critical entrepreneur in the bad state is equal to his repayment obligation in that same state.

Second, deviations by an individual bank that either cause default or positive profits for the critical entrepreneur in the bad state are not profitable.

Third, from the proof, we observe that $r^h > r^l$ for sufficiently large productivity shocks and that $r^h - r^l$ is monotonically increasing in the size of the shock. Larger shocks require larger spreads, as otherwise the critical entrepreneur would default or savings and investment would not balance.

Fourth, as $r^h > r^l$ for sufficiently large productivity shocks, banks offer state-contingent deposit and loan contracts. Part of the macroeconomic risk is shifted to depositors. This prevents the aggregate risk from being shifted to future generations.

Fifth, there is room for further improvements in risk allocation by repackaging deposit contracts into two securities. Risk-neutral entrepreneurs who save could hold very risky contracts, and could bear the entire macroeconomic risk. Risk-averse consumers could be offered less risky or even risk-free contracts. This contract arrangement would further improve intra-generational risk allocation, as the risk would be shifted entirely to risk-neutral agents. Such an allocation would be Pareto efficient.

7 Bailouts

We suppose now that the regulator commits to bailouts. In particular, we assume that future generations will be taxed to bail out banks. In this case, banks might be tempted to request a particularly high interest rate on loans in the good state and a low interest rate in the bad state. It is instructive to first show that for this reason, the efficient risk allocation, as expressed in Proposition 2, can no longer be an equilibrium.

Proposition 4

Suppose that the regulator commits to bailouts. Then, the intra-generational risk allocation under failure is not an equilibrium.

The proof is given in the Appendix. The intuition is as follows: A bank can profitably
deviate by offering a slightly higher deposit rate, a slightly higher loan rate in the good state, and an appropriately chosen lower loan rate in the bad state. Then, the bank will obtain all deposits and all entrepreneurs while savings and loans remain balanced. By raising the loan rate in the good state more than the deposit rate, the bank makes positive profits in this state. Losses in the bad state do not matter, as the bank is bailed out.

In the next proposition, we establish the equilibrium of the game. The critical entrepreneur who is indifferent between saving and applying for a loan in the case of bailouts is denoted by \( q^e_w \).

**Proposition 5**

Suppose \((\bar{q}^e - 1 - p)e + (\bar{q}^e - 2p)I \leq 0\) and that the regulator commits to bailouts. Then, there exists a unique equilibrium with

(i)

\[
r^w = r^{ch}_j = r^{dh}_j = r^{dl}_j, \quad j = 1, 2,
\]

(ii)

\[
r^{cl}_j = -1, \quad j = 1, 2.
\]

(iii) \( r^w \) is determined by

\[
(1 - \eta)s\{r^w, r^w\} + \eta e\left(q^e_w - (\bar{q}^e - 1)\right) = \eta(\bar{q}^e - q^e_w)I,
\]

with

\[
q^e_w = 1 + \frac{I\left(p r^w - (1 - p)\right) + e r^w}{e + I}.
\]

Banks make zero profits in the good state and aggregate losses

\[
(1 - \eta)s\{r^w, r^w\}(1 + r^w) + e\eta(q^e_w - (\bar{q}^e - 1))(1 + r^w)
\]

in the bad state.
The proof is given in the Appendix. The intuition for this result is as follows. Under bailout, banks wish to create a profitable state, i.e., a state of the world where \( r_j^{ch} - r_j^{dh} \) is large, while being unconcerned about losses in the other state. In the good state, competition drives profits to zero and we have \( r_j^{ch} = r_j^{dh} \). To be able to demand high interest rates from entrepreneurs in one state of the world, banks do not require any repayment in the bad state. This motivates entrepreneurs to apply for loans. The condition in Proposition 5 is fulfilled as long as the expected upper level of the productivity is not too high and the probability of the good state is not too low. The condition on \( \bar{q}^e \) in the proposition essentially requires that the expected productivity of entrepreneurs is not too large, i.e. \( \bar{q}^e \) is not too large. Otherwise, banks could profitably deviate by offering higher deposit rates and loan rates in the good state. Obviously, Proposition 5 is extreme since banks are able to write contracts with entrepreneurs, demanding negative interest rates in one state of the world. If we restrict the set of contracts to non-negative interest rates, our results are qualitatively the same, but the potential losses for future generations decrease. In the bad state, banks will demand \( r_j^{cl} = 0 \).

An important implication of Proposition 5 is that bailing out banks in the bad state is accompanied by bailing out firms as well. Entrepreneurs pay no interest on their loan (if \( r_j^{cl} = 0 \)) or do not have to pay anything (if \( r_j^{cl} = -1 \)) and hence can still make profits in the bad state. There are various cases where bailout guarantees for banks and hidden subsidies to entrepreneurs have contributed to the emergence of banking crises (see e.g. Krugman (1999) for the Asian crisis). Our analysis suggests that this will naturally arise when banks compete with contingent contracts under a bailout regime, even if moral hazard of entrepreneurs has been eliminated since banks offer large spreads in contingent loan interest rates.

Proposition 5 holds independently of the size of the shock, provided \( \bar{q}^e \) fulfills the aforementioned condition. Thus, even if the macroeconomic risk is small, future generations face large aggregate risks.

Moreover, Proposition 5 even holds if the distinction between the good state and the
bad state is caused by a sunspot variable, but is not reflected by real quality differences of projects of entrepreneurs. This case occurs if there are sunspot random variables with the probability distribution \((p, 1 - p)\), upon which banks write contingent deposit and loan contracts, but \(q^d = q^h\).

Proposition 5 shows that banks generate risk for future generations. Hence, we use the term “risk-magnification effect” rather than the well-known “risk-shifting effect” to describe the equilibrium outcome in Proposition 5, as risk is generated even if there is no underlying real risk. We summarize these observations in the following corollary:

**Corollary 1**

Suppose \((q^e - 1 - p)e + (q^I - 2p)I \leq 0\) and that the regulator commits to bailouts.

(i) If macroeconomic events are sunspot, i.e. \(z = 0\), Proposition 5 continues to hold. In particular, financial intermediation generates real risk for future generations.

(ii) In the bad state, future generations face losses equal to the savings of the last generation (if \(r_{cl,j} = -1\)) or equal to interest rate payments (if \(r_{cl,j} = 0\)).

The logic and the proof of the results in Proposition 5 also reveal that when banks can offer contingent or non-contingent contracts, they will choose the contracts in Proposition 5, since nothing prevents banks in the proof of Proposition 5 to choose contracts with \(r_{ch,j} = r_{cl,j}\) and \(r_{dh,j} = r_{dl,j}\) and thus non-contingent contracts. We summarize this observation in the following corollary:

**Corollary 2**

If banks can choose between contingent and non-contingent contracts, they choose

- non-contingent deposit contracts with deposit rates according to Proposition 5,
- contingent loan contracts with loan interest rates according to Proposition 5.

### 8 Comparison

In the next proposition, we compare the interest rates and investment levels for both regulatory schemes when productivity shocks are small.\(^{22}\)

\(^{22}\)A similar comparison can be performed for large productivity shocks.
Proposition 6
Suppose that the interest rate elasticity of savings is strictly positive. The comparison between bailout and failure in the case of small productivity shocks yields

\( r^w > r^f \),

\( q^w < q^f \).

The proof is given in the Appendix. Proposition 6 implies that under the bailout regime, the current generation invests more compared to the bank failure regime, and depositors receive more attractive interest rates. Since entrepreneurs do not need to pay back in one state of the world under bailout, a larger percentage of entrepreneurs choose to invest rather than save compared to the failure regime.

Proposition 6 can be interpreted as a lending boom under bailout, as aggregate credit expands. The result complements the seminal work on lending booms (Dell’ Ariccia and Marquez (2008)) who show that lending standards may endogenously decline, which, in turn, may increase aggregate surplus, but also the risk of financial instability. In our model, intertemporal aggregate output is lower in the bailout regime. Specifically, aggregate output over two generations is higher under bank failure than under bailout when the second generation has to bailout the first generation. This is obvious in the simplest case when the interest elasticity of savings is zero. Then we have \( q^w = q^f \) and, hence, expected aggregate output in the first generation is the same in both regimes. In the next period, however, savings and investment are lower in the bailout regime than in the failure regime, when the bad state has occurred in the generation before. In the good state, output is identical in both regimes. Hence, expected aggregate output over two generations is smaller in the bailout regime than in the failure regime.

\[ 23 \] To prevent the decline in aggregate output, the regulator could fix deposit rates at the level \( r^f \) from the outset. Such an ex ante deposit rate ceiling would not, however, eliminate the risk magnification incentive of banks, since banks would still like to create a profitable and an unprofitable state of the world on the loan side.

\[ 24 \] The general proof is tedious. Two effects occur. First, entrepreneurs of low quality (i.e. entrepreneurs with \( q^w < q^f \)) invest in the first generation under the bailout regime, but not under the failure regime. Second, bailout reduces investment of entrepreneurs with higher quality levels than \( q^f \) in the second generation. Accordingly, aggregate output over two generations is higher under the failure regime than under the bailout regime. Details are available upon request.
The analysis in this section points to interesting political economic considerations regarding banking regulation and bailout procedures. We mention some obvious conflict of interest, leaving a thorough analysis for future research. First, the generation of current depositors benefit in two ways from bailouts. They receive higher interest rates than in the failure regime, as \( \rho^w > \rho^f \), and can depend on the next generation to bear a substantial fraction of bailout costs. The next generation will be harmed by taxation if it has to resolve banking crises and pay the depositors back.

Second, entrepreneurs, on average, benefit from a failure regime, as their loan interest rates will be lower as long as this gain is not outweighed by contributions to the rescue funds. It is likely, as the last global banking crisis has shown, that the number of current depositors is so large that bailout will be the rule and a substantial share of costs will be borne by future generations through higher government debt.

9 Conclusions

We have examined the incidence of macroeconomic shocks in a model of financial intermediation under different bailout schemes. Our analysis indicates that the combination of allowing banks to fail, along with contingent deposit and loan contracts, tends to yield an efficient intra-generational risk allocation. Together with a large number of further issues to be considered in banking regulation, our results may help to design an overall second-best banking regulation scheme.

The current framework should allow a number of useful extensions. For instance, it may be useful to consider a wider range of macroeconomic shocks. In particular, one could condition contracts on other contractible macroeconomic events that are highly correlated with the financial health of the banking sector. For instance, one might try to use an index that measures the average default rate of entrepreneurs, or the level of aggregate bank capital that would occur if the good state were assumed. In our model, all these variants of macroeconomic indicators would yield the same results.

It is also useful to consider contingent bailout schemes. For example, one may conjecture that with small shocks, the regulator is expected to stay out, while with large

\[ \text{See Dewatripont and Tirole (1994), Hellwig (1998), and Allen and Santomero (1998).} \]
macroeconomic shocks, the regulator is expected to step in. Such contingent government bailout schemes would preserve the incentives of banks to generate profitable states of the world. Hence, contingent bailout would, at best, alleviate the risk-magnification effect.

Another useful extension is to consider bailouts within a generation when entrepreneurs (and consumers) may be taxed to bail out depositors. This implies that the current generation has to provide rescue funds in case of bank default. This could force banks to require a lower loan rate for the good state and may lessen the moral hazard problem, but would not eliminate it. As long as lump-sum taxation is used, the qualitative nature of our results, as to the risk-magnification effect, does not change. If bailout schemes are anticipated by agents, however, their decision problems have to be adapted before a welfare analysis can be conducted. This subject is left for future research.
10 Appendix A:

Proof of Proposition 1:

We first show the existence of the equilibrium. The boundary condition of Assumption 2 ensures that at least one solution exists. For sufficiently high interest rates, investments are zero, and hence the left-hand side of the equation in point (ii) of the proposition for $r^*$ is greater than the right-hand side. For $r^* = 0$, the boundary condition ensures that the right-hand side is greater than the left-hand side. The intermediate value theorem establishes that at least one solution exists, since both sides are continuous in $r$.

Moreover, the left-hand side of the implicit equation for $r^*$ in Proposition 1 is monotonically increasing in $r^*$. In contrast, the right-hand side is decreasing in $r^*$. Hence, the solution is unique.

Loan application decisions of entrepreneurs are optimal, given $r^d = r^c = r^*$. Profits of banks per credit contract are zero (see Equation (5)).

Changing one interest rate, while leaving the other at $r^*$, is never profitable for a bank. Consider a change of $r^d_j$. Profits are either negative if $r^d_j > r^*$, or a deviating bank obtains no resources if $r^d_j < r^*$. Consider a change of $r^c_j$. Profits are negative if $r^c_j < r^*$ since the interest rate margin is negative, or the deviating bank does not obtain loan applications if $r^c_j > r^*$, as entrepreneurs seeking loans go to competing banks offering better terms. If entrepreneurs were rejected at competing banks, they would save according to our rationing schemes.

Suppose, however, that bank $j$ offers slightly better conditions for depositors, $r^d_j = r^* + \epsilon$, with some $\epsilon > 0$, and tries to exploit its monopolistic power on the lending side, i.e., the bank changes both interest rates. Since bank $j$ would obtain all deposits, entrepreneurs can only receive loans at this bank. Hence, profits of the deviating bank, denoted by $\pi_j$, amount to

$$
\pi_j = \eta(\bar{q} - q^*)I(1 + r^c_j) - \eta\epsilon(\bar{q}^* - (\bar{q} - 1))(1 + r^* + \epsilon)
- (1 - \eta)s \{r^* + \epsilon\} (1 + r^* + \epsilon),
$$

(13)
where
\[ q^* = 1 + \frac{Ir_j^e + e(r^* + \epsilon)}{e + I} \]
and
\[ r_j^c > r^* + \epsilon. \]

As \( q^* > 1 + r^* \), bank \( j \) has excess resources. The amount of excess resources is
\[ (1 - \eta)s \{ r^* + \epsilon \} + \eta e \left( q^* - (\bar{q} - 1) \right) - \eta(\bar{q} - q^*)I, \]
which, however, can neither be invested nor used in the next period, since the good is perishable. We obtain
\[
\frac{\partial \pi_j}{\partial r_j^c} = \eta \left\{ (\bar{q} - q^*)I - \frac{I}{e + I}I(1 + r_j^c) \right\} - \eta e \frac{I}{e + I}(1 + r^* + \epsilon)
\]
\[
= \frac{\eta I}{e + I} \left\{ (\bar{q} - 1)(e + I) - 2Ir_j^c - I - e(1 + 2r^* + 2\epsilon) \right\}
\]
\[
\leq \frac{\eta I}{e + I} \left\{ (\bar{q} - 2)(e + I) \right\}.
\]

Therefore, \( \frac{\partial \pi_j}{\partial r_j^c} \) is negative if \( \bar{q} \leq 2 \).

Hence, profits are negative for \( r_j^c = r^* + \epsilon \) because of excess resources and because profits are decreasing for \( r_j^c \geq r^* + \epsilon \) with the loan interest rate. Thus, bank \( j \) makes losses by offering \( r_j^d = r^* + \epsilon \) and a lending rate \( r_j^c \geq r^* + \epsilon \). Finally, it is obvious that setting \( r_j^d = r^* + \epsilon \) and \( r_j^c < r^* + \epsilon \) is not profitable because profits are negative.

Uniqueness follows through similar observations. First, if both banks chose the interest rates \( \bar{r}^c = \bar{r}^d < r^* \), loan demand would exceed savings, and both banks would make zero profits. By setting \( r_j^d = \bar{r}^d + \epsilon \) and \( r_j^c = \bar{r}^c + 2\epsilon < r^* \), bank \( j \) would generate positive profits. Second, if both banks chose \( \bar{r}^c = \bar{r}^d > r^* \), both would make losses due to the excess resources, and a bank \( j \) would be better off by choosing \( \bar{r}_j^c = \bar{r}_j^d = r^* \) and making zero profit. Finally, no interest rate constellation with \( r^d < r^c \) can be an equilibrium. A bank can profitably deviate by setting \( r^d + \delta_1, (\delta_1 > 0) \) and \( r^c - \delta_2, (\delta_2 > 0) \), where \( \delta_1 \) and \( \delta_2 \) are arbitrarily small and can be selected such that no excess resources are generated.
Proof of Proposition 2:

We observe that, given \( r_{j}^{ce} \) and \( r_{j}^{de} \), and hence a given critical entrepreneur \( q_{j}^{e*} \) and a given profit per credit, banks can offer risk-averse depositors the highest utility by setting \( r_{j}^{dh} = r_{j}^{dl} \). Therefore, Bertrand Competition will lead to \( r_{j}^{dh} = r_{j}^{dl} = r_{j}^{de} \). Moreover, banks are forced to offer \( r_{j}^{ce} = r_{j}^{de} \). Raising \( r_{j}^{de} \) slightly and increasing \( r_{j}^{ce} \) to obtain monopoly profits from entrepreneurs is not profitable for the same reasons as outlined in Proposition 1. \( r_{j}^{dh} = r_{j}^{dl} = r_{j}^{de} = r_{j}^{ce} \) and the repayment conditions \( r_{j}^{dh} \leq r_{j}^{ch} \) and \( r_{j}^{dl} \leq r_{j}^{cl} \) imply \( r_{j}^{ch} = r_{j}^{cl} = r_{j}^{dh} = r_{j}^{dl} \).

This equilibrium interest rate is denoted by \( r^{f} \) and determined by the saving and investment balance. Finally, we need to verify that banks are able to pay back in both states of the world, since otherwise, their deposit rates would not be credible. In the bad state the repayment condition is given by

\[
(q^{*} - z)(e + I) = (q_{j}^{e*} - zp)(e + I) \geq I (1 + r^{f}).
\]

Using \( q_{j}^{e*} = 1 + r^{f} \), this implies

\[
z \leq \frac{e(1 + r^{f})}{p(e + I)}.
\]

Proof of Proposition 3:

a) We construct the equilibrium in the following way. In the bad state, the interest rate \( r^{f} \) is determined by the requirement that the critical entrepreneur can simply pay back what he owes. We must have

\[
(q^{*} - z)(e + I) = I (1 + r^{f}).
\]

Using \( q^{e} = pq + (1 - p)(q - z) \)
and thus
\[ q_f^{\ast} = p q^{\ast} + (1 - p)(q^{\ast} - z) \]
leads to
\[ q^{\ast} - z = q_f^{\ast} - zp. \]

We obtain
\[ (q_f^{\ast} - zp)(e + I) = I(1 + r^l). \]  
(15)

Inserting \( q_f^{\ast} = 1 + pr^h + (1 - p)r^l \), which follows from Equation (10), yields
\[ r^h = \frac{I(1 + r^l) + (e + I) \{ zp - 1 - (1 - p)r^l \}}{p(e + I)} \]
which corresponds to (iii). (v) follows by solving Equation (15) for \( q_f^{\ast} \). Conditions (iii), (iv) and (v) determine \( \{r^h, r^l\} \) and \( q_f^{\ast} \).

b) For sufficiently large productivity shocks, we always have \( r^h > r^l \).

Using (iii), \( r^h > r^l \) is equivalent to
\[ p(e + I)r^l < p(e + I)r^h = I(1 + r^l) + (e + I) \{ zp - 1 - (1 - p)r^l \} \]
\[ er^l < I + (e + I)(zp - 1). \]  
(16)

For a given \( r^l \), \( q_f^{\ast} \) is increasing in \( z \). In order to fulfill the savings/investment balance in (iv), an increase in \( z \) leads to a decline in \( r^l \). Hence, for sufficiently high \( z \), Equation (16) is fulfilled. Therefore, \( r^h > r^l \) for sufficiently large productivity shocks.

c) Expected profits of banks are zero. Suppose bank \( j \) offers deposit interest rates \( r^h \) and \( r^l + \epsilon \), for some small \( \epsilon > 0 \). Since bank \( j \) obtains all deposits, it could change the individually optimal interest rates on loans. In order to avoid an excess resource problem, bank \( j \) needs to ensure that enough entrepreneurs want to apply for credits. Therefore, \( q^e \) should not rise above \( q_f^{\ast} = 1 + pr^h + (1 - p)r^l \).

If the deviating bank wishes to achieve \( q^e = q_f^{\ast} \), i.e.
\[ q_f^{\ast} = 1 + \frac{I r^e + e \left( pr^h + (1 - p)(r^l + \epsilon) \right)}{e + I} = 1 + pr^h + (1 - p)r^l, \]

26 Although we have assumed \( s(r^h, r^l) \) is strictly increasing in \( r^h \) and \( r^l \), uniqueness is not guaranteed. A sufficient condition for uniqueness is that \( r^h(r^l) \) in (iii) is non-decreasing in \( r^l \), which requires \( \frac{1 - p}{p} \leq \frac{I}{e} \).
we obtain
\[ r^{ce} = pr^h + (1-p)r^l - \frac{e \varepsilon (1-p)}{I} < pr^h + (1-p)r^l < r^{de} = pr^h + (1-p)(r^l + \varepsilon). \]

Accordingly, expected profits per credit amount to
\[ \mathbb{E}[G_j] = p(r_{ch}^j - r_{dh}^j)I + (1-p)(r_{cl}^j - r_{dl}^j)I \]
\[ = I (r^{ce} - r^{de}) \leq 0. \]

Hence, the deviation does not benefit bank \( j \). Similar reasoning for any other potential deviation establishes that \( \{ \{ r_{ch}^j = r_{ch}^k \}_{j=1,2}, \{ r_{dl}^j = r_{cl}^k \}_{j=1,2} \} \) is an equilibrium.

**Proof of Proposition 4:**

Consider the risk allocation of Proposition 2. We show that a bank \( j \) can deviate and be better off by offering the following interest rates:
\[ r_{dh}^j = r_{dl}^j = r^f + \epsilon, \]
\[ r_{ch}^j = r^f + \delta, \]
\[ r_{cl}^j = r^f - \frac{p\delta}{1-p}, \]
where \( \delta > \epsilon > 0 \). Bank \( j \) would obtain all deposits since \( r_{de}^j > r^f \). The critical entrepreneur amounts to
\[ q_f^{es} = 1 + \frac{Ir^f + e(r^f + \epsilon)}{e + I} = 1 + r^f + \frac{e\epsilon}{e + I}. \]

Hence, for sufficiently small \( \epsilon \), savings and investments are almost balanced. Since \( r_{dh}^j < r_{ch}^j, r_{cl}^j > r_{dl}^j \), bank \( j \) will not be able to pay back depositors in the bad state. However, since banks are bailed out, their profit in the bad state will be zero. Hence, expected bank profits per credit in this case amount to
\[ \mathbb{E}[G_j] = p \cdot I (\delta - \epsilon). \quad (17) \]

For a sufficiently small amount of \( \epsilon \), excess resources from depositors are negligible. However, by choosing \( \delta > \epsilon \) and making \( \delta \) sufficiently large, expected profits will be
large. Hence, the profitable deviation of bank \( j \) eliminates the existence of the efficient intra-generational risk allocation equilibrium.

**Proof of Proposition 5:**

We first observe that \( r^w \) is uniquely determined. The left-hand side of the implicit equation for \( r^w \) in Proposition 5 is increasing in \( r^w \), since \( s\{r^w, r^w\} \) and \( q^e^* \) are monotonically increasing in \( r^w \). By contrast, the right-hand side is decreasing in \( r^w \). The corresponding boundary condition ensures that a unique solution exists.

The most promising deviation of bank \( j \) would be\(^{27}\)

\[
\begin{align*}
  r^{dh}_j &= r^{dl}_j = r^w + \epsilon,  \\
  r^{cl}_j &= -1.  
\end{align*}
\]

(18)

(19)

The bank would obtain all resources and would try to maximize expected profits by choosing an interest rate \( r^{ch}_j \) \((r^{ch}_j \geq r^w + \epsilon)\). Entrepreneurs expect to obtain loans at the deviating bank \( j \) only. Expected profits are given by

\[
\mathbb{E}[\pi_j] = p \{ \eta(q^e - q^*) I(1 + r^{ch}_j) - \eta e(q^* - (q^e - 1))(1 + r^w + \epsilon) \\
- (1 - \eta) s\{r^w + \epsilon, r^w + \epsilon\}(1 + r^w + \epsilon) \},
\]

(20)

with \( q^* = 1 + \frac{I(p r^{ch}_j - (1 - p)) + e(r^w + \epsilon)}{e + I} \).

We obtain

\(^{27}\)It is straightforward but tedious to verify that no other potential deviation is profitable.
\[
\frac{\partial E[\pi_j]}{\partial r_j^{ch}} = \frac{p \eta I}{e + I} \left\{ (\bar{q}^e - 1)(e + I) - I \left\{ pr_j^{ch} - (1 - p) \right\} - e(r^w + \epsilon) - pI(1 + r_j^{ch}) - ep(1 + r^w + \epsilon) \right\}
\]
\[
= \frac{p \eta I}{e + I} \left\{ (\bar{q}^e - 1)(e + I) - I(2pr_j^{ch} + 2p - 1) - e(p + r^w(1 + p) + \epsilon(1 + p)) \right\}
\]
\[
\leq \frac{p \eta I}{e + I} \left\{ (\bar{q}^e - 1 - p)(e + I) + I(1 - p) \right\}
\]
\[
= \frac{p \eta I}{e + I} \left\{ (\bar{q}^e - 1 - p)e + (\bar{q}^e - 2p)I \right\}.
\]

(21)

Note that we have used that \(r_j^{ch}, r^w\) and \(\epsilon\) are non-negative to obtain the inequality. The assumption of the proposition implies that the last expression is not positive. Hence, the deviation is not profitable if the assumption of the proposition holds.\(^{28}\)

**Proof of Proposition 6:**

We compare the savings and investment balance in both cases. Suppose that \(r^w \leq r^f\). As \(0 < p < 1\), this implies that

\[
q_w^{es} < 1 + \frac{I r^f + e r^f}{e + I} = 1 + r^f = q_f^{es}.
\]

Hence, using Proposition 2, we obtain

\[
(1 - \eta) s\{r^f, r^f\} + \eta e \left( q_w^{es} - (\bar{q}^e - 1) \right) < \eta (\bar{q}^e - q_w^{es}) I.
\]

The strict inequality is reinforced when \(r^f\) is lowered to \(r^w\) because savings will (weakly) decline. This is, however, a contradiction to the savings and investment balance in the bailout case. Hence, we obtain \(r^w > r^f\). Moreover, \(r^w > r^f\) implies that \(q_w^{es} < q_f^{es}\), as the interest elasticity of savings is \(> 0\) in order to balance savings and investments.

\(^{28}\)Uniqueness can be established by first establishing that any constellation with \(r_j^{cl} > -1\) cannot be an equilibrium. Second, in any equilibrium, loan and deposit rates in the good state have to be identical.
Let us briefly state the underlying agency conflicts that provide the rationale for the occurrence of financial intermediation in our paper. The depositors face the following informational asymmetries. The quality $q$ is known to entrepreneurs, but not to depositors. Moreover, depositors cannot verify whether an entrepreneur invests. To alleviate such agency problems in financial contracting, financial intermediation can act as delegated monitoring (see Diamond (1984)). Bank activities are characterized by two features: First, banks cannot observe the quality of an entrepreneur ex-ante, but can verify output conditional on investment at low or zero costs. The assumption is justified by the possibilities that banks have to secure the repayments if entrepreneurs invest. Monitoring in order to secure repayments takes different forms: inspection of firms’ cash flow when customers pay, and efforts to collateralize assets if they have been created in the process of investing and selling products to customers, for instance. If the final products of an entrepreneur’s project are physical goods, such as houses or machines, standard banks can secure repayment conditional on investment at very low costs.

Second, entrepreneurs can have large private benefits if they do not invest, but banks are able to reduce these benefits by monitoring. The monitoring can take many forms. For instance, standard banks can collateralize parts of the credit, or may release the funds sequentially to the entrepreneur, depending on his investment behavior. Such efforts can reduce the private benefits of entrepreneurs who do not invest. The simplest monitoring function is given as follows: If a bank $j$ offers a loan $I$ to an entrepreneur and monitors by paying a resource cost $m, m \geq 0$, it can secure a repayment of $\gamma I$ with $0 < \gamma \leq 1$. If $\gamma$ is sufficiently high, such that $q(e + I) - (1 + r^c)I \geq e + (1 - \gamma)I$, where $r^c$ is interest on loans, an entrepreneur with quality $q$ will invest if he obtains a loan. We assume that monitoring technologies are efficient enough at reducing the private benefits of entrepreneurs, such that entrepreneurs applying for loans will always invest. For simplicity, we also assume that monitoring outlays per credit contract are negligible for a bank. Our analysis, however, is also applicable to the case where banks can completely alleviate agency problems in contracting by investing a fixed amount.
per credit contract in monitoring. In this case, the interest rate spread will be positive and will cover the costs of monitoring in equilibrium.\textsuperscript{29} For simplicity of presentation, we assume in this paper that such fixed monitoring costs are zero.

We next justify the use of debt contracts in financing entrepreneurs, either unconditional contracts or contracts conditional on macroeconomic shocks. A theoretical justification is given in Gersbach and Uhlig (2006). They abstract from monitoring, as we do in this paper. They show that banks enter into a Bertrand-like Competition for the different types of investing borrowers in such games. This makes it impossible for a lender to cross-subsidize among them. In any pure strategy equilibrium, only debt contracts will be offered.\textsuperscript{30} As the argument can easily be extended to banks with monitoring technologies\textsuperscript{31}, we assume directly that banks compete with debt contracts.

\textsuperscript{29}A further extension could allow banks to compete on monitoring intensity, which may increase risk magnification when banks choose a low intensity of monitoring (see e.g. Gehrig and Stenbacka (2004)).

\textsuperscript{30}Moreover, in the optimal contract entrepreneurs must invest their endowments if they apply for loans. Otherwise, shirking would become attractive and would deter banks from lending.

\textsuperscript{31}The monitoring technology simply allows banks to reduce the cost of shirking and increases the share of investing entrepreneurs.
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