CRIME AND EDUCATION IN A MODEL OF INFORMATION TRANSMISSION

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ABSTRACT: We model the decisions of young individuals to stay in school or drop out and engage in criminal activities. We build on the literature on human capital and crime engagement and use the framework of Banerjee (1993) that assumes that the information needed to engage in crime arrives in the form of a rumour and that individuals update their beliefs about the profitability of crime relative to education. These assumptions allow us to study the effect of social interactions on crime. In our model, we investigate informational spillovers from the actions of talented students to less talented students. We show that policies that decrease the cost of education for talented students may increase the vulnerability of less talented students to crime. The effect is exacerbated when students do not fully understand the underlying learning dynamics.

Keywords: Human capital, The economics of rumours, Social interactions, Urban economics

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1 Introduction

Many developing countries and poor areas in developed countries are plagued by high crime rates and low levels of education. Young people seem to be particularly vulnerable

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to crime engagement. Oftentimes, once crime has started it spreads in an epidemiological way through a community. We here suggest a theory of juvenile crime that is motivated by the idea that the further people are from receiving a return on educational investments they have made, and the more likely they are to be surrounded by other young criminals, the more they will be willing to engage in crime. It allows us to investigate the effect that educational policies have on the diffusion of crime among young people.

Following Becker (1968), economic theory sees crime as an occupational choice or investment opportunity. A person compares the streams of payoffs from crime versus other occupations or investments in human capital such as going to school to obtain a good job later. Lochner (2004) builds a dynamic model of education and crime engagement and explains the decreasing age–petty crime pattern. The more individuals have invested in education, the larger the opportunity cost of crime. Hence, older people who have accumulated more human capital or are closer to graduation will be less prone to engaging in crime. But, crime is also a social phenomenon. The first economic models of social interactions and crime were developed by Sah (1991) and Glaeser et al. (1996). The former develops a model in which the decision of a person to commit crime reduces the probability of other offenders to be arrested. The latter develop a model in which the individuals decision about crime depends on their neighbors' decisions about criminal activities. Particularly relevant for our theory are Calvó-Armengol and Zenou (2004) and Calvó-Armengol et al. (2009) who investigate effects of social networks on crime and education.

There is strong evidence supporting the ideas of Becker and Lochner (see Levitt 1998; Mocan and Rees 2005). But there is also evidence showing that social interactions are important determinants of crime engagement. Ludwig et al. (2001) and Kling et al. (2005) show that a neighborhood’s wealth has an effect on incidence of youth crimes. Particularly important to our paper is the evidence found by Case and Katz (1991) who show that in low-income Boston neighborhoods the behavior of peers appears to affect youth behaviors in a manner suggestive of contagion models. Another important piece of evidence is provided by Luallen (2006) who shows that reducing school incapacitation increases crime rates among youngsters. More recently, Damm and Dustmann (2014) and Rotger and Galster (2019) find a positive relation between the share of criminals in a neighborhood and later convictions of young males. Using individual data, Drago and Galbiati (2012) and Corno (2017) disentangle (contagious) endogenous social network from exogenous social network effects and show that criminal behavior is learned.

Taken together, the literature shows that the causal link between crime and low levels of human capital is quite complex (Card and Giuliano 2013). However, there seems to be agreement that fostering education is a good way to fight crime (Card 1999; Deming 2011; Machin et al. 2011) and that interactions in school might foster crime specially when there is school segregation (Billings, Deming and Ross 2019).

We investigate the interaction between educational policies and juvenile crime. We assume that everybody is rational, but that information on the opportunity to become a criminal is not readily available. Rather it is transmitted through an information diffusion process in society: people who have become criminals meet students and students learn about the possibility to become a criminal rather than going to school. Our assumption is in line with the evidence cited above. We investigate the nature of the information transmission process between criminals and students and carry out an
investigation on the policies that reduce the cost of education such as scholarships, meals or transport subsidies, better teachers and materials.

In our model people are rational, they are young and go to school but they can drop out and engage in crime. Going to school costs some effort or money. Individuals will only drop out if they believe crime is more profitable than staying in school. Some of the students are more talented, thus they have lower costs, while others are less talented, and have higher costs of going to school. Talent (or ability) is private information.

We introduce social interactions using a model of a rumour process à la Banerjee (1993). There is aggregate uncertainty: crime may pay or not and, because of differences in the opportunity costs of crime engagement, the pay off of engaging in crime depends on whether you are talented or not. Three cases are possible: (i) crime can be profitable for talented and less talented individuals, (ii) only for the less talented or (iii) for neither of them. Initially, everybody knows that dropping out to engage in crime is feasible, but everybody also believes that the expected net return of engaging in crime is too low for dropping out from school to be worth it (case (iii)). However, the information individuals have about the profitability of crime changes over time and, thus, their belief about that profitability. There is a rumour process by which individuals may learn that some other individual dropped out from school to engage in crime ruling out case (iii) above. When individuals meet other individuals, they only learn whether the other has engaged in crime or not but not how profitable crime is. Nor do they learn whether the individual is talented or less talented. When individuals listen to the rumour they also form their beliefs about the likelihood of being in case (i) or (ii) above and decide whether to stay in school or become a criminal. These probabilities change in time because the time that passes before a given student meets a criminal for the first time provides crucial information about the probability that crime is profitable. This is so because the speed of the rumour transmission depends on the number of criminals, which in turn depends on the profitability of crime.

But whether an individual engages in crime upon hearing the rumour also depends on how much time he has spent in school and on how much of the costs needed to finish high school he has already incurred. The two mechanisms imply that there is a point in time after which students will not be tempted any more to become criminals. This time occurs earlier for the talented than for the less talented. Hence, the less talented are more vulnerable to crime engagement. However, the rumour process is responsible for the fact that the difference between these stopping times do not depend only on differences in individual types (talented and less talented) but also on externalities from talented to less talented students.

These spillover effects give rise to our main result which is relevant for policy considerations. Consider a policy reducing the cost of schooling for talented students (for instance, a meritocratic scholarship program). Of course, the objective of the policy is to reduce the number of students that drop out from school. This policy directly

1 There is a broader literature on information diffusion, such as Banerjee (1992) and Scharfstein and Stein (1990), who develop models of herd behavior. In those models information goes through a process of word-of-mouth learning and they are thought to explain financial runs, behavior facing new products, etc. In the context of social economics, Jackson and Yariv (2011) reviewed the literature on the influence of social networks on diffusion processes in different realms, such as disease contagion, technology adoption, vote decisions, etc.

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reduces the vulnerability to crime of talented students. Ideally it should also reduce the vulnerability of less talented students; however this is not guaranteed.

To understand the effect on less talented students, the way the rumour about crime spreads at any time afterwards is crucial. Individuals update their beliefs of the profitability of crime by taking into account the time that passes until they meet a criminal for the first time. Older rumours are a signal that crime is less profitable; this is the effect that appears in Banerjee (1993). Banerjee (1993) carries out comparative statics showing that the parameters may have ambiguous effects on the adoption of the investment opportunity (in our case this will be engaging in crime). Our analysis decomposes this effect to understand why policies that target the reduction of education costs of talented students may have the effect of increasing vulnerability of the less talented. With the policy, talented students stop engaging in crime earlier and after this moment only less talented students engage in crime. This has three effects: first, there are less students contributing to the dispersion of the rumour. Second, as a consequence, the dispersion of the rumour is slower. Third, less talented students know that the speed is reduced because the cost of schooling of the talented is lower and not because a change in the fundamentals of the profitability of crime. The first two effects play in favor of a reduction of less talented students that become criminals in any posterior moment. The last goes in the opposite direction. Consequently, a policy reducing the cost of education of talented students may increase or decrease the vulnerability of less talented students depending on the strength of each of the two effects. We show that it is possible that the net effect is an increase in the vulnerability of less talented students.

This implies that meritocratic policies may hurt less talented students and should be accompanied by policies that neutralize this effect. We also show that when students do not fully understand the learning dynamics, the effect on the less talented students become exacerbated, and we hence conclude that meritocratic policies, such as those studied by Angrist et al. (2016), Bettinger et al. (2016) and Marx and Turner (2015) should be accompanied by information campaigns.

2 Model setting

We consider a population of students given by the interval $[0,1]$ with equal life length $T$. We denote $s$ the length of schooling of a student. After graduation, students earn an income of $W$ in each moment of the rest of their lives. Education is costly; the instantaneous cost of education (in terms of effort, tuition etc.) is $e$. There are two types of students: a proportion $q$ of the students have high costs, $e_{\bar{e}}$, and a proportion $1 - q$ of the population have low costs, $e_c$. Leaving problems of access to credit markets aside (a topic that is beyond the scope of this paper), notice that we can refer for simplicity to high-cost students as ‘less talented’ and low-cost students as ‘talented’. To simplify the model, we assume that the discount rate is equal to zero.

Education is a riskless project. Its value depends on the moment of life of a person. At any moment in time $t < s$ the instantaneous continuation value of education is

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2 One can argue that education may also be a risky project. However, the existence of institutions like minimum wages, that are common in both developed and developing countries, make the education project less risky than the crime project. Moreover, in those contexts in which education is riskier than crime, rumours about criminal projects may be more pervasive. Hansen
\[ R(t) = \frac{(T - s)W - (s - t)e}{T - t}. \] (1)

We have then \( R(t) \) and \( R(t) \) for \( \bar{e} \) and \( e \), respectively. The idea of \( R(t) \) is that students must study a proportion \( s/T \) of life in order to obtain a degree and to earn \( W \) in each moment of the rest of their lives. Students hence first have to invest the cost of education to obtain its benefits afterwards. Clearly, the value of education increases in \( t \). The sunk-cost nature of education will be a crucial feature in our model. We will simply refer to \( R(t) \) as the value of education.

To understand our assumptions consider a simple benchmark where becoming a criminal is a riskless project with instantaneous returns \( a_0 \). Then if \( a_0 < R(0) \) there is no crime. If \( R(0) < a_0 < R(0) \), the less talented (high-cost) students commit crime during their entire life and talented (low-cost) students commit no crime and the total number of criminals is \( q \). And, if \( R(0) < a_0 \), all students commit crime during their entire life, and in this case the total number of criminals is 1.

It makes much more sense, however, to consider that being a criminal is a risky project. Consider that its returns are \( a \) with probability \( p \), \( b \) with probability \( r \), and \( d \) with probability \( 1 - p - r \), where \( a > b > 0 > d \). Everybody knows that there is an opportunity to engage in crime but nobody knows ex ante the return.

Note that we are assuming that the gross returns to education \( (W) \) and the net returns to crime \( (a, b and d) \) are the same to all individuals independent on whether they are talented or not. Given our additivity and linearity assumptions about the valuation individuals give to education or crime what is important are the net returns. In this case, net returns to education vary with the talent of the individual while net returns to crime do not vary. Our results would not change if we allow returns to education for talented to be higher than for non-talented; but they will probably change if we assume that returns to crime depend on the type of individual. We make the following assumptions about the interaction between education, crime and different types of students.

**Assumption 1.** \( (T - s)W - s\bar{e} > 0 \).

**Assumption 2.** \( pa + rb + (1 - p - r) d < 0 \).

**Assumption 3.** \( W \geq a \).

**Assumption 4.** \( R(0) < b < R(0) < a \).

Assumption 1 says that education pays for the less talented students and hence also for the talented one. Assumption 2 is a somewhat stronger assumption: it says that the ex ante expected value of crime is negative such that without further information neither type of student would engage in crime. It allows us to focus attention on crime due to social interactions. Together, Assumptions 1 and 2 imply that at \( t = 0 \) the entire

and Machin (2002) present empirical evidence showing that the establishment of minimum wage actually causes a decrease in crime rates. At the cost of further complication but without much benefit in terms of economic insights one could assume that each of the states \( a, b \) and \( d \) were lotteries themselves, with \( a \) the best lottery and \( d \) the worst.
population is attending school. Assumptions 3 and 4 are about the crime–education
decisions during schooling time. Assumption 3 says that the riskless reward of education
is larger or equal to the largest payoff of crime. As a consequence, nobody becomes a
criminal after $s$, when all educational efforts are sunk. Assumption 4 says that, at $t = 0$,
crime is profitable for the talented type only if the true state of the world is $a$. Crime is
profitable for the less talented type if the true state of the world is either $a$ or $b$.

The previous assumptions deserve further discussion. Assumptions 1 and 2 to-
gether say that, ex ante, education pays more than crime. This is in line with pre-
vious evidence on gang earnings showing that risks of criminal activities more than
offset the wage premium with respect to legal earnings (Levitt and Venkatesh 2000).
Assumption 3 gathers the findings of Lochner and Moretti (2004), that high school
graduation significantly reduces engagement in crime.\footnote{This is so because wages after graduation are much higher than wages with no graduation.}

At $t = 0$, all the population is attending school. Without additional information,
under Assumption 2 nobody would engage in crime. We assume between $t = 0$ and $dt$
a proportion $x$ of the population (randomly chosen) learns the true state of the world,
which is either $a$, $b$ or $d$. These students then choose whether to drop out of school (and
commit crime) or to attend school (and exert effort). If the true state of the world is $a$
both types would engage in crime, if it is $b$ only less talented ones would do so and if
it is $d$ nobody will do (Assumptions 2 and 4). Any student that leaves school to engage
in crime after learning the true state would not want to go back to school at any later
moment because he knows that crime is profitable for him.

After the first students learn the true state of the world, nobody will do so again.
Instead other students will only have information observed indirectly from their peers.
With probability $y < 1$ each student meets another agent between $t$ and $t + dt$ ($dt > 0$).\footnote{Together, the no unitary probability of meeting and agent ($y < 1$) and the assumption that
agents meet in a non-instantaneous lapse of time ($dt > 0$) ensure that all agents will not meet all
other agents in a very short lapse of time.}
The agent may be either a criminal or a student. The student learns whether the agent
is a criminal or not, but he does not learn the true state of the world or the agent’s
type (talented or less talented as a student). If the person he meets is not a criminal he
would not know whether this is someone who has met a criminal before and has decided
not to engage in crime; thus, meeting non-criminals does not convey any information
about the profitability of crime. We will let $m$ denote the event in which the student
meets a criminal for the first time. When a student meets a criminal for the first time,
he can choose whether to commit crime or not. This reflects the idea that crime is an
occupational choice that becomes available only through social interaction. In order to
adopt crime, one needs to have contact with other people who are criminals, because
there are no formal channels through which one can take up this type of career.\footnote{We exclude that one can become a criminal without having any contact with other criminals,
as we are interested in crime, education and social interaction through information diffusion and
not in the isolated decision of an individual to commit crime (which has been thoroughly studied
by Becker and other scholars building on his work).} Upon meeting a criminal, a student is then confronted with the choice of staying in school or
engaging in a very different type of career. We will suppose that a student that engages
in crime after $m$ will continue being a criminal for the rest of his life.
Assumption 5. If a student commits crime once, he stays a criminal for the rest of life – that is there is no way back to schooling once it has been interrupted.

Our final assumption concerns the information students take into account in their updating process.

Assumption 6. Students know the distribution of types and the date \( t = 0 \) in which the rumour started.

Under Assumption 6, the process of information transmission about crime becomes a *rumour* process in the sense of Banerjee (1993). Criminals become a source of the rumour on crime and, thus, the probability of hearing the rumour (meeting a criminal) increases with the number of criminals. Anybody’s decision whether or not to engage in crime thus creates information externalities. Under our assumptions, especially Assumptions 1 and 2, nobody will invest in crime unless somebody learns that someone else has already committed crime. Rumour begins if the true state of the world is either \( a \) or \( b \). If \( a \), a proportion \( x \) of people will become criminals between \( t = 0 \) and \( dt \). If \( b \), a proportion \( qx \) will do so. If the true state is \( d \), nobody will.

3 Students’ behavior

We now turn to the analysis of students’ behavior. From Assumption 4 we already know that any student who, in \( t = 0 \), learns that the true state is \( a \) would engage in crime; if the true state is \( b \) only the less talented would do so. From Assumption 3 we also know that, in \( t = s \) and after, nobody would want to engage in crime any more. The question now is behavior \((0, s)\) for those individuals who meet a person who has engaged in crime and, thus, learns that the true state is either \( a \) or \( b \). This depends on his assessment about the expected return of doing so. In this section we show that as time passes the incentives to drop out of school change and that there are three critical points in time for understanding this decision. These are \( \tau^* \), \( \tau \) and \( \overline{\tau} \). The latter two are the moments in time when talented and less talented students respectively stop engaging in crime; \( \tau^* \) is the moment in time at which less talented students would stop engaging in crime if the probability of state \( a \) were zero. We also characterize these periods and derive some properties that are useful to understand consequences of policies.

From equation (1) we obtain \( \tau^* \) equating \( R(\tau^*) \) to \( b \); from Assumption 4, by continuity of \( R(t) \) we know that \( \tau^* > 0 \).

We now turn to determining \( \tau \) and \( \overline{\tau} \) which requires some additional notation. In what follows, it is important to keep in mind that, when someone meets a person who has engaged in crime he immediately knows that the true state is not \( d \) because otherwise the rumour had not started. So the expected value of engaging in crime depends only on \( a \), \( b \) and the probabilities assigned to each of these two events. The behavior of students depends on their beliefs about these probabilities which, as we show, change with time.

Let \( p(t) \) be the probability of the true state being \( a \) given that the student meets a criminal for the first time between \( t \) and \( t + dt \) and suppose that for a given \( t \)

\[
EC(t) \equiv p(t)a + (1 - p(t))b > R(t).
\]
Then, a student who meets a criminal between $t$ and $t + dt$ will engage in crime. The probability $p(t)$ is estimated using Bayes' rule:

$$p(t) = \frac{\pi}{\pi + \frac{\text{Prob}[m|b,t]}{\text{Prob}[m|a,t]}(1 - \pi)}$$

with $\pi = \frac{p}{p + r}$. $\text{Prob}[m|s,t]$ is the probability that in state $s \in \{a, b\}$ a student meets a criminal for the first time between $t$ and $t + dt$. The ratio of $\text{Prob}[m|b,t]$ and $\text{Prob}[m|a,t]$ determines $p(t)$; this ratio will be crucial for our analysis and we hence define it formally.

**Definition 1.** $z(t) \equiv \frac{\text{Prob}[m|b,t]}{\text{Prob}[m|a,t]}$.

We also define, $z^*(t)$, the net gain of engaging in crime when it is profitable, relative to the net loss when it is not profitable:

**Definition 2.** $z^*(t) \equiv \frac{\pi(a - \overline{R}(t))}{(1 - \pi)(\overline{R}(t) - b)}$.

Because $z^*(t)$ depends on the type of student which affects $\overline{R}(t)$:

$$z^*(t) \equiv \frac{\pi(a - \overline{R}(t))}{(1 - \pi)(\overline{R}(t) - b)} \quad \text{and} \quad \overline{z}^*(t) \equiv \frac{\pi(a - \overline{R}(t))}{(1 - \pi)(\overline{R}(t) - b)}.$$

Notice also that $\overline{z}^*(t)$ is defined in the interval $(\tau^*, T]$ while $z^*(t)$ is defined in $[0, T]$.

It can then readily be shown that inequality (2) holds for talented students if $z(t) < z^*(t)$ and for less talented students if $z(t) < \overline{z}^*(t)$. This analysis is summarized in the following result.

**Result 1.** The behavior of students is as follows:

1. A less talented student who meets a criminal for the first time between $t$ and $t + dt$ before $\tau^*$, engages in crime.
2. A less talented student who meets a criminal for the first time between $t$ and $t + dt$ after $\tau^*$, engages in crime if and only if $z(t) \leq z^*(t)$.
3. A talented student who meets a criminal for the first time between $t$ and $t + dt$, engages in crime if and only if $z(t) \leq \overline{z}^*(t)$.

We have shown that the crucial element in the decision to become a criminal or not when hearing the rumour is the relative probability of meeting a criminal in each of the two states of the world. Recall that the rumour of crime only begins if the condition in equation (2) holds at $t = 0$ – that is, both types must be vulnerable to crime at $t = 0$. The updating of fully rational students uses the age of the rumour to calculate the number of criminals in each state of the world. The decision rule stated in Result 1 identifies a critical level for $z(t)$ after which talented and less talented students continue going to school; the critical level depends on the costs that each individual faces to complete school.

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7 Function $\overline{z}^*(t)$ is not defined for $t < \tau^*$ because before $\tau^*$ crime is more profitable than education for less talented students in both $a$ and $b$ states of the world.
From Result 1, it becomes clear that both types of students are vulnerable to crime. Since $z^*(t) < \bar{z}(t)$ we can already say that whenever a talented student would drop out to engage in crime a less talented student would also do so. Less talented students are more vulnerable than talented students since they are likely to become criminals during a longer period of life. To understand the dynamics of the diffusion process and, in the next step, the effect of different policies, we need to investigate the properties of $z(t)$, $z^*(t)$ and $\bar{z}^*(t)$. Result 2 states the properties of $z^*(t)$ and $\bar{z}^*(t)$.

**Result 2.** The functions $z^*(t)$ and $\bar{z}^*(t)$ are both monotonically decreasing in $t$ and convex.

Proofs are in Appendix B. According to Result 2 the profitability of crime decreases with time for both types, in particular because the education cost is continuously sunk at each moment of time. The result holds in the respective domain of each function; that is, for $z^*(t)$ in $t \in [0, T]$, and for $\bar{z}^*(t)$ in $t \in (\tau^*, T]$.

The analysis of $z(t)$ is more challenging and it requires the use of additional notation and definitions. First, we need to distinguish between two regimes, the first is that in which talented students engage in crime and the second that in which they do not. These two regimes are important because dynamics of the diffusion of information about the profitability of crime differ between the two as a consequence of having different number and types of individuals engaging in crime. Second, we need to show that there is a moment in time in which the system transits from the first to the second regime. Then we need to study these dynamics. We borrow these from Banerjee (1993).

**Definition 3.** We distinguish between Regime 1, where $z(t) \leq z^*(t)$ and Regime 2, where $z(t) > z^*(t)$.

Note that the process must start off in Regime 1 (i.e. the condition in equation (2) must hold at $t = 0$), otherwise there will be no uncertainty about crime. If the process were to begin in Regime 2, everybody who meets a criminal would know the criminal is a less talented student. Formally, the process must start off when $z(t) \leq z^*(t)$, which at $t = 0$ boils down to

$$q \leq \frac{\pi(a - R(0))}{(1 - \pi)(R(0) - b)}.$$  

We now show that there is a moment in time, which we call $\tau$ (and which we already mentioned at the beginning of this section), in which $z(\tau) = z^*(\tau)$; i.e. a moment in time in which the system switches from Regime 1 to Regime 2. For that purpose we need to show how $z(t)$ changes with $t$ and for that it is useful to write $z(t)$ in an alternative form that depends on the proportion of population that has committed a crime and the proportion of population that has not met a criminal in a given $t$.

**Definition 4.** For $i = a, b$, we define:

1. $N(i, t) \equiv$ the proportion of the population that has committed crime until time $t$ in state $i$.
2. $P(i, t) \equiv$ the proportion of the population that has not met a criminal until time $t$ in state $i$. 

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Using Definition 4 and the fact that

\[ \text{Prob}[m|b, t] = yN(b, t)P(b, t) \]

and

\[ \text{Prob}[m|a, t] = yN(a, t)P(a, t), \]

we can then write:

\[ z(t) = \frac{yN(b, t)P(b, t)}{yN(a, t)P(a, t)}. \]  (3)

Notice that \( P(a, 0) = 1 - x, P(b, 0) = 1 - x, N(a, 0) = x, N(b, 0) = xq, \) and that \( z(0) = q. \)

In Regime 1, the dynamics of \( N(i, t) \) and \( P(i, t) \) are given by

\[ \frac{dP(i, t)}{dt} = -yN(i, t)P(i, t). \]  (4)

\[ \frac{dN(i, t)}{dt} = yN(i, t)P(i, t). \]  (5)

In Regime 2, the dynamics of \( N(i, t) \) and \( P(i, t) \) are given by

\[ \frac{dP(i, t)}{dt} = -yN(i, t)P(i, t), \]  (6)

\[ \frac{dN(i, t)}{dt} = qyN(i, t)P(i, t). \]  (7)

The intuition for the difference is of course that in Regime 2 only less talented students (a proportion \( q \) of the total population) may become criminals at any interval \([t, t + dt] \) after \( \tau. \)

Furthermore, for an economy that has always been in Regime 1 holds:

\[ P(a, t) = 1 - N(a, t), \]  (8)

\[ P(b, t) = 1 - x(1 - q) - N(b, t). \]  (9)

For an economy that has made its first transition to Regime 2 at moment \( \tau \) holds:

\[ q[P(a, \tau) - P(a, t)] = N(a, t) - N(a, \tau), \]  (10)

\[ q[P(b, \tau) - P(b, t)] = N(b, t) - N(b, \tau). \]  (11)

Equations (8) and (9) evaluated in \( t = 0 \) together with equations (10) and (11) evaluated in \( t = \tau \) provide the initial conditions for the differential equations (4)–(7), respectively. These differential equations differs depending on the true state of the world. In both, Regime 1 and Regime 2, we have that \( P(a, t) < P(b, t) \) and \( N(a, t) > N(b, t). \) This means that the proportion of individuals that engage in crime is higher in state \( a \) than in state \( b; \) consequently, in a given \( t, \) the number of individuals that have not heard the rumour
is smaller in state $a$ than in state $b$. With these things established the following result can readily be shown (see the formal proof in Banerjee 1993).

**Lemma 1 (The Banerjee effect).** The ratio $z(t)$ increases monotonically in $t$ and is unbounded.

The lemma states that the older the rumour (i.e. the larger $t$), the stronger the belief of students that the true state is $b$. The later a student meets a criminal for the first time, the more likely he believes the benefits of crime are low.

From equations (4)–(11) one can see that since the dynamics in both regimes differ, $z(t)$ will have different forms in each regime. For $t < \tau$, in Regime 1, $z(t)$ is defined by (4), (5), (8) and (9); we will let $z^1(t)$ represent this part of $z(t)$. For $t \geq \tau$, in Regime 2, $z(t)$ is defined by (6), (7), (10) and (11); we will let $z^2(t)$ represent this part of the function. $z^1$ depends on $x$ and $q$; $z^2$ depends on $x$, $q$ and the parameters that determine $z^*$ (since $\tau$ is determined by the equality $z^* = z^1$). Indeed $\tau$ defines the initial conditions for (and determines the actual path followed by) $z^2$. We can then define formally the function $z(t)$ as follows:

$$z(t) = \begin{cases} 
z^1(t), & \text{if } t \leq \tau, \\
z^2(t), & \text{if } t > \tau. 
\end{cases}$$

Explicit expressions for $z^1(t)$ and $z^2(t)$ can be easily obtained using (3), (4)–(11). For our analysis we will need the explicit expression for $z^2$ which can be expressed in terms of $P(i, \tau)$, $N(i, \tau)$ and $(t - \tau)$, as follows:

$$z^2 = \frac{\frac{dN(b, t)}{dt}}{\frac{dN(a, t)}{dt}} = \frac{qyN(b, t)P(b, t)}{qyN(a, t)P(a, t)} = \frac{\frac{dN(b, \tau)}{dt}g(P(b, \tau), t - \tau)}{\frac{dN(a, \tau)}{dt}f(P(a, \tau), t - \tau)}.$$  (13)

The specific forms $z^2$ and $z^1$ appear in the Appendix A where it will be clear that $g$ and $f$ are two specific functions.

The following lemma shows that $\tau$ exists.

**Lemma 2.** Provided that $q \leq \frac{\pi(a - R(0))}{(1 - \pi)(R(0) - b)}$, there will be an instant $\tau$ at which there will be a transition from Regime 1 to Regime 2.

We have so far established that the beliefs on the true state of the world converge to $b$ (a consequence of Lemma 1), that the value of education relative to crime is increasing over time (Result 2), and that $\tau$ exists (Lemma 2). It is important to note that the rumour on crime goes beyond $\tau$. Consider a student who meets a criminal between $t$ and $t + dt$ after $\tau$. Although he knows that talented students are no longer vulnerable, he also knows that there are criminals who had been talented students, but met a criminal before $\tau$. At $\tau$, non-talented students are still vulnerable to crime.

Following a similar reasoning we can show how the less talented students behave. Indeed, from Results 2 and 1 we have that $z(t) - z^*(t)$ is monotonically increasing. Therefore there exists a $\tau$ such that $z(\tau) = z^*(\tau)$. Since an increasing amount of education cost is sunk over time, there is a moment in time in which no student becomes a criminal any more. After $\tau$, education is more valuable than crime for both types. The
total number of criminals thus reaches its maximum at $\tau$. After $\tau$, some students will still meet criminals (who can be of either type) but they will not engage in crime. As we have argued before, the time at which talented students stop engaging in crime, $\tau$, is strictly shorter than the time at which less talented students do so, $\bar{\tau}$. In a nutshell, less talented students are more vulnerable to crime than talented students since their cost of schooling is larger ($\bar{\sigma} > \sigma$). Figure 1 depicts the solutions we presented above.

There is one more characteristic of $z(t)$ that we must consider; this is stated in the following result.

**Lemma 3.** At $t = \tau$, there is a downward kink in $z(t)$, that is, for $t$ very near to $\tau$, the slope of $z(t)$ is larger for $t \leq \tau$ than for $t > \tau$.

There is a kink at $\tau$ because beyond this point, talented students who have not yet met a criminal will never engage in crime and are not vulnerable to crime any more. At the kink the speed of the rumour decreases, which has important consequences for the policy effects we present below. However, there is a second effect, which has also important consequences for policy effects related to the fact that less talented students (the only ones who are still vulnerable) understand the reason for the reduction in the speed of the rumour and take this into account in their beliefs. This effect is explained and exploited in the following section.

## 4 Policy: The effect of changing the costs of education

In the previous section we have investigated how crime spreads in a society and how it affects education. The main implications of the model are: first, for both talented and less talented students there is a moment in time in which crime stops being profitable but this is earlier for talented than for less talented (i.e. $\tau < \bar{\tau}$); second, at $\tau$, there is a downward kink in $z(t)$, which reflects the fact that from this moment on the stock of talented students that spread the crime rumour does not increase any more. In this section we present our main result which shows that policies reducing the cost of
education may have surprising effects and that this is a direct result of the downward kink in $z(t)$ at $\tau$. Examples of such policies are reductions of tuition fees, food for school programs, improvements in school infrastructure or teachers. These measures can be made dependent on performance of a student, which makes them contingent on their type (talented vs. less talented).

First, consider policies such that $e$, $\bar{e}$ or both are reduced. These reductions have a direct effect on the vulnerability to crime of the targeted type of student, but there is also an indirect effect through the transmission of information about crime profitability among students of different types. The direct effect of reducing $e$ ($\bar{e}$) shifts $\tau$ ($\bar{\tau}$) to the left. That is, the point in time at which no more students of a given type will engage in crime occurs earlier when their cost of attending and succeeding at school decreases. Put differently, students become less vulnerable to crime.

The indirect interaction, i.e. effects between different types, are much more subtle, but they only go from talented students to less talented students. To understand this statement notice first that both the talented and the less talented students carry out the same type of comparison between costs and benefits of engaging in crime. More precisely, they both have the same uncertain benefit of engaging in crime and they assign the same probability distribution to the states $a$, $b$, $d$. Also, wages upon graduation are assumed to be the same for both types. The opportunity costs of engaging in crime, however, are type dependent: talented students have lower costs of going to school than less talented ones, which explains why $\tau$ is to the left of $\bar{\tau}$.

For any $t$ smaller than $\tau$, any change in the parameters affects equally the expected benefits both types of students assign to crime. This implies that among criminals, the proportion of talented and less talented types is constant, reflecting the respective proportions in the entire population. There are interaction effects here, but they do not depend on the types. This changes at $t = \tau$. Here, no further talented student engages in crime, while less talented students who meet a criminal continue to do so, implying that the proportion of less talented types among criminals increases. Hence, when one wants to understand the interaction effects between different types, it suffices to investigate how a shift in $\tau$ will affect the behavior of less talented students. This is stated in the following proposition.

**Proposition 1.** Effects of a decrease in $e$ (the costs of education for talented students): (i) $\tau$ shifts to the left, i.e. talented students become less vulnerable to crime; (ii) the effect of a reduction in $e$ on $\bar{\tau}$ is ambiguous, in particular a reduction of $e$ may result in an increase of $\bar{\tau}$, i.e. less talented students may become more vulnerable to crime.

To understand the second part of the proposition recall that students vulnerability to crime depends on $z(t)$: in particular, if $z(t)$ increases because of an intervention, students become less vulnerable, and if $z(t)$ decreases they become more vulnerable. When there is a policy that reduces vulnerability of talented students this has two different effects that go in opposite directions. First, it reduces the stock of individuals that have dropped out from school to engage in crime (the talented stop dropping out earlier) and thus reduces the probability of meeting someone who has engaged in crime by a less talented student. The result is that opportunities to engage in crime by less talented students are reduced. At the same time, when a less talented student meets a criminal he knows that the rumour took more time to arrive for two reasons: first, less
individuals have engaged in crime and second, this is because crime is not profitable any more for talented students but it may still be profitable for him (a less talented student). This means that even if the opportunities to engage in crime are reduced the rumour about crime can last longer.

A subsidy scheme for more talented students will cause an instantaneous reduction of the number of criminals in both states of the world. After the point in time in which talented students cease to be vulnerable to crime, the rumour starts to slow down. The total effect on the less talented students’ vulnerability depends on their interpretation about the determinants of the time the rumour took to arrive to them. They can assign this to the profitability of crime, or to the opportunity cost of engaging in crime of the talented. Without the subsidy, the relative weight of the profitability of crime is higher than that of the opportunity cost of the talented; with the subsidy, the relative weight they give to the profitability explanation can be lower.

To formally see this effect consider two situations, one without the subsidy and one with the subsidy; let $\tau'$ and $\tau''$ be the two moments in time in which talented and less talented students stop engaging in crime without the subsidy and $\tau''$ and $\tau'''$ with the subsidy. We know from Lemma 3 that at the moment in time in which talented students are not vulnerable to crime any more, there is a kink in $z(t)$; this implies that at $\tau''$ the slope to the right of $z(t)$ with and without a subsidy is different. Without a subsidy the right derivative of function $z(t)$ at $\tau''$ is equal to

$$z(\tau'')[2(P(b, \tau'') - P(a, \tau'')) + x(1 - q)].$$

(14)

When there is a subsidy, the right derivative of $z(t)$ at $\tau''$ is

$$z(\tau'')[1 + q](P(b, \tau'') - P(a, \tau'')) + x(1 - q)]$$

(15)

which is smaller than the previous one. This means that the introduction of a subsidy scheme makes state $a$ more likely at $\tau''$. Equations (14) and (15) correspond to the left and right limits, respectively, of the derivative of $z(t)$ evaluated in $\tau''$; the general expressions appear in the proof of Lemma 3 in the Appendix.

From $\tau''$ onwards this effect coexists with the Banerjee effect (Lemma 1), which makes the beliefs about $b$ being the true state of the world increase with time. The two effect hence go in opposite directions.

Another way to formally see these two effects is by looking at the derivative of $z(t)$ in Regime 2, that is after the time talented students cease to be vulnerable to crime, with respect to $\tau$. Deriving the function $z'^2$ in equation (13) we obtain

$$\frac{\partial z'^2}{\partial \tau} = \left[ \frac{\partial^2 N(b, t)}{\partial t^2} - \frac{\partial^2 N(a, t)}{\partial t^2} \right]$$

$$+ \left[ \frac{\partial g}{\partial P(b, t)} \frac{\partial P(b, t)}{\partial t} - \frac{\partial g}{\partial P(a, t)} \frac{\partial P(a, t)}{\partial t} - \frac{\partial f}{\partial P(a, t)} \frac{\partial P(a, t)}{\partial t} \right] .$$

The first term in the right-hand side corresponds to the subsidy effect that increases the belief of state $a$ being the true state of the world. The second term gathers
the interaction of the first effect with the Banerjee effect in each state of the world. Notice that the size of the interaction depends on \( t \). Less talented students will become more (less) vulnerable to crime if at \( t = \bar{t} \) the first effect is stronger (weaker) than the second effect.

Two different societies that differ in \( e \) only differ in their dynamics after the lower \( \tau \). The differences are a consequence of changing the initial conditions for \( z^{r2} \) and on changes in the dynamics of \( z(t) \) after \( \tau \). Precisely, what one wants to know is whether, for \( t > \tau \), \( z'' \) is to the left or to the right of \( z' \) or whether they cross. If they cross one also wants to know whether \( z'' \) crosses \( z' \) from above or from below. If \( z'' \) is to the right (left) of \( z' \) this would mean that reducing the cost of talented students increases (reduces) the vulnerability of less talented students. In other words if \( z'' \) is to the right (left) of \( z' \) a reduction in \( e \) would bring the undesirable effect of increasing \( \tau \); in the other case reducing \( e \) would have a positive externality since it would also reduce \( \tau \). A full comparison of \( z'(t) \) and \( z''(t) \) is in general not possible. However, the following example shows that for two sets of parameters in which the only difference is the value of \( a \), a change in \( e \) of the same size induces changes in \( t \) of different signs.

**Example 1.** Consider the following values for the parameters of our model: \( y = 1, x = 0.1, q = 0.6, \tau = 10, W = 69, a = 55, b = 15, \pi = 0.6 \) and \( T = 34 \). The effort of the less talented students is \( e = 153 \) and the effort of the talented students is \( e = 100 \). With this information, all less talented students that hear the rumour about crime before \( \tau \approx 5.44 \) choose to become criminals. Similarly, all talented students that hear the rumour before \( \tau \approx 4.07 \) will do so. Let us consider a policy that reduces the costly effort of education for talented students. It reduces \( e \) to 35. This policy reduces the vulnerability to crime of talented students to 1.58 and makes less talented students more vulnerable to crime increasing \( \tau \) to 5.51. This example is depicted in Figure 2.

Now consider an alternative situation in which crime pays more, such that \( a = 64 \). In this case the initial \( \tau \) is 5.39; the initial \( \tau \) is 5.96. When the policy that reduces \( e \) from 100 to 35 is implemented, the vulnerability of talented students decreases to 2.68 but less talented students become less vulnerable to crime; \( \tau \) decreases from 5.96 to 5.93. The example is in Figure 3.
The next proposition identifies a sufficient condition under which there is no ambiguity.

**Proposition 2.** Comparative statics with respect to student heterogeneity ($\overline{e} - e$). For sufficiently low levels of student heterogeneity, a decrease in the cost of education of talented students makes less talented students more vulnerable to crime.

**Proof.** Consider a situation in which $\overline{e} - e$ is small. Consequently $\tau - \overline{\tau}$ is also small. Consider two levels of cost of education for talented students $e'$ and $e''$, such that $e' > e''$. The corresponding moments in time in which talented students stop engaging in crime are $\tau'$ and $\tau''$, and they satisfy $\tau' > \tau''$. We also have two functions for $z(t)$; let these functions be $z'(t)$ and $z''(t)$. These two functions are exactly the same for any $t \leq \tau''$ and differ for $t > \tau''$. Consider a $t$ such that $\tau' > t > \tau''$. Since $\tau' > t > \tau''$, $t$ belongs to Regime 1 when $e = e'$ and to Regime 2 in the second case. From Lemma 3, we have that there is downward kink at $\overline{\tau}$. Therefore, since functions $z(t)$, $z'(t)$ and $z''(t)$ are continuous, for $t$ near enough to $\overline{\tau}$" we have that $z'(t) > z''(t)$ for $t > \overline{\tau}$. Since $\overline{\tau}'$ is near $\overline{\tau}$, $\overline{\tau}''$ is near $\overline{\tau}''$ and $z''(t)$ is downward sloping we then have that $\overline{\tau}' < \overline{\tau}''$. □

We now consider how rationality affects behavior. We introduce bounded rationality by assuming that students do not understand the dynamics of information about crime. When a boundedly rational student meets someone who has engaged in crime, he will not take into account the time in which the rumour has started when updating his believes about the profitability of crime. Thus, he will believe that $z(t) = q$, always. A comparison with the cutoff values for boundedly rational students with those of fully rational students, as derived in the preceding section, establishes the following proposition.

**Proposition 3.** A society with fully rational students will be less vulnerable to crime then an otherwise identical society with boundedly rational students.

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The updating process of unboundedly rational students makes them believe that \( z(t) \) is increasing in \( t \) and everywhere above \( q \). Since \( \bar{z}'(t) \) and \( \bar{z}''(t) \) are both decreasing in \( t \), this implies that \( \tau < \tau_{br} \) and \( \bar{\tau} < \bar{\tau}_{br} \). This situation is depicted in Figure 4.

The significance of this conclusion relates to the content of interventions targeted to high school students that focus on providing information about the returns to education. The literature about the effects of giving information to young people about the benefits of investing in human capital is large and constantly growing (Lavecchia et al. (2016)). However, virtually all empirical research about providing information to high school students concentrate on information about the returns to education (Jensen 2010; Nguyen, 2008; Oreopoulos and Dunn 2013; Oreopoulos and Petronijevic 2013; Kaufman 2014; Dinkelman and Martinez 2014; McGuigan et al. 2016; Hoxby and Turner 2013; Bonilla et al. 2017). Up to this point there are no results about giving information about alternatives to education like criminal activities. Conventional reasoning may say that the underestimation of the returns to schooling is parallel or just the other side of the coin of an overestimation of the returns to criminal activities. However, the social dynamics of crime and education can be very different. Our paper shows that the form of social dynamics of both activities should be consider when thinking on information policies targeting human capital acquisition by young people. Including information about crime and about social dynamics of crime is complimentary to giving information about the returns to education.

5 Concluding remarks

Subsidizing education on the basis of merits unambiguously makes talented students less vulnerable to crime, but the learning dynamics between more and less talented students may have the surprising effect that the less talented become more, not less vulnerable to crime after the introduction of such a subsidy scheme. This effect is exacerbated when students have limited understanding about the learning dynamics. While our theory is abstract in nature and the finely grained empirical evidence that we would need to support our results seem unavailable, it lends itself to the interpretation that meritocratic educational programs should be accompanied by information
campaigns. These campaigns are already in use in many places (e.g. Jamaica) and tend to concentrate on the benefits of education (mainly returns, the labor market and opportunities of higher education funding) but they do not include alternatives projects to education. Moreover, most of the time the campaigns are not tailored to particular individuals and are very general in that they do not take into account the costs (not only economic but also psychologic) that different individuals may face when trying to complete an education project. Our theory suggests that education campaigns should include much more contextual information that would allow individuals from different backgrounds see the benefits of education for themselves (in contrast to the average benefits). Additionally our theory also says that the timing of informational campaigns is important: campaignings that target youth earlier in life will be more effective that those that target them later.

REFERENCES


Appendix A

The form of $z^r_1$ and of $z^r_2$

The forms $z^r_1$ is

$$z^r_1 = \frac{dN(b, t)}{dN(a, t)}$$

where

$$\frac{dN(b, t)}{dt} = \frac{y(1-x)(1-x(1-q))xqe^{(1-x(1-q))t}}{(1-x + xq e^{(1-x(1-q))t})^2}$$

and

$$\frac{dN(a, t)}{dt} = \frac{yx(1-x)e^t}{(1-x + xe^t)^2}.$$

The form of $z^r_2$ is

$$z^r_2 = \frac{dN(b, τ)}{dN(a, τ)}g(P(b, τ), t - τ)$$

where

$$\frac{dN(b, τ)}{dt} = \frac{yq(1-x)(1-x(1-q))xqe^{(1-x(1-q))τ}}{(1-x + xq e^{(1-x(1-q))τ})^2},$$

$$\frac{dN(a, τ)}{dt} = \frac{yx(1-x)e^τ}{(1-x + xe^τ)^2},$$

$$g(P(b, τ), t - τ) = \frac{(1-x(1-q) - (1-q)P(b, τ))e^{(1-x(1-q)−(1-q)P(b, τ))(t-τ)}}{[qP(b, τ) + (1-x(1-q) - P(b, τ))e^{(1-x(1-q)−(1-q)P(b, τ))(t-τ)}]^2}$$

and

$$f(P(a, τ), t - τ) = \frac{(1-(1-q)P(a, τ))e^{(1-(1-q)P(a, τ))(t-τ)}}{[qP(a, τ) + (1-P(a, τ))e^{(1-(1-q)P(a, τ))(t-τ)}]^2}.$$
Appendix B

Proofs

(1) Proof of Result 2:

Since \( z^*(t) \) is continuous and differentiable then to show the result it is enough to analyze the signs of the first two derivatives of \( z^*(t) \).

We first need to know that

\[
R'(t) = R(t) + \frac{e}{1 - t} > 0,
\]

\[
R''(t) = 2 \frac{R(t)}{(1 - t)^2} > 0.
\]

Taking the first derivative of \( z^*(t) \), we obtain that

\[
z^*'(t) = - \frac{R'(t)}{R(t) - b} \left( \frac{\pi}{1 - \pi} + z^*(t) \right) < 0.
\]

Taking the second derivative

\[
z^{**}(t) = \left( \frac{\pi}{1 - \pi} + z^*(t) \right) \left[ 2 \left( \frac{R(t)}{R(t) - b} \right)^2 - \frac{R''(t)}{R(t) - b} \right] > 0.
\]

(2) Proof of Lemma 2:

Since \( q \leq \frac{\pi(a - R(0))}{(1 - \pi)(R(0) - b)} \), the rumour on crime starts off. Let us first consider the case of strict inequality. Once a talented student meets a criminal between \( t \) and \( t + dt \), he updates beliefs on the state of the world and takes decisions following the rule in Result 2. Since \( z(0) < z^*(0) \), the crime is profitable for talented students. Those students meeting criminals between \( t = 0 \) and \( dt \) will become criminals. This will be the behavior of talented students for any time interval \([t, t + dt]\) provided that \( z(t) < z^*(t) \).

From Result 2, \( z^*(t) \) is monotonically decreasing with time and from Result 1, \( z(t) \) is monotonically increasing with time. Therefore, the difference \( z(t) - z^*(t) \) monotonically increases with time. At \( t = 0 \), it is negative, then it becomes zero and finally it becomes positive. Let \( \tau \) be the moment in time at which \( z(\tau) - z^*(\tau) = 0 \). For any \([t, t + dt]\) after \( \tau \) the talented student that meets a criminal will stay in school. At \( \tau \) there will be a transition from Regime 1 and Regime 2. Now let us consider the case of strict equality. In this case the talented students that know the true state of the world are indifferent between staying at school and becoming criminals. Those talented students that hear the rumour between \( t = 0 \) and \( dt \) are also indifferent between school and crime. However, since \( z(t) - z^*(t) \) is monotonically increasing, all talented students that hear the rumour in any interval \([t, t + dt]\) for which \( t > 0 \) will stay at school. In this case, \( \tau = 0 \).

(3) Proof of Lemma 3:

To show that there is a downward kink at \( \tau \), we have to show that

\[
\lim_{t \to \tau^-} z'(t) > \lim_{t \to \tau^+} z'(t).
\]

Indeed,

\[
\lim_{t \to \tau^-} z'(t) = \lim_{t \to \tau^-} \left[ z(t) \left[ 2(P(b, t) - P(a, t)) + x(1 - q) \right] \right] = z(\tau) \left[ 2(P(b, \tau) - P(a, \tau)) + x(1 - q) \right]
\]

\[
\lim_{t \to \tau^+} z'(t) = \lim_{t \to \tau^+} \left[ z(t) \left[ 2(P(b, t) - P(a, t)) + x(1 - q) \right] \right] = z(\tau) \left[ 2(P(b, \tau) - P(a, \tau)) + x(1 - q) \right]
\]

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and

\[
\lim_{t \to \tau^+} z'(t) = \lim_{t \to \tau^+} z(t) \left[ 2q(P(b, t) - P(a, t)) + (1 - q)(P(b, \tau) - P(a, \tau)) + x(1 - q) \right] = z(\tau)[(1 + q)(P(b, \tau) - P(a, \tau)) + x(1 - q)].
\]

Since \( P(b, \tau) > P(a, \tau) \) and \( 2 > 1 + q \), then there is a downward kink in \( \tau \).