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Sciences and Normative Orders: Perspectives from the Earliest Sciences and their Histories

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It has been a great pleasure and privilege to be part of the cluster Normative Orders at Goethe University since the beginning of 2009. During the decade since then, a variety of impressive research from different disciplines focusing on the topic of normative orders from a variety of subjects has been conducted, presented and discussed in an immensely pleasant atmosphere enabled by the framework of this cluster of excellence.[1] For a historian of science, this also prompted some reflections about the status that "sciences" held at various points in time and the normative orders that are found within scientific works as well as the normative orders which were imposed by the sciences of a respective place and time on their surroundings.[2] The latter is also prompted by recent developments concerning the impact (or lack thereof) of scientists on daily life and politics. The following pages are an outcome of my own involvement in the cluster Normative Orders. It touches on several fundamental issues of the history of science as a discipline that have been or are still being intensely debated. I shall not attempt to resolve any of these debates within this paper, and some of my references and arguments are necessarily much simplified. In addition, my use of the term *normative order* may raise hackles with some scholars.[3] Whenever possible, I will refer to literature that I consider helpful to clarify my position – needless to say that the references are a small and subjective selection.

I. Science – some general remarks

The term "science" and its derivatives hold a distinguished place in the modern world. Despite recent turmoil centering around "alternative facts" and (sometimes successful) attempts to disregard scientific results in favor of other factors (e.g. economic gains), the initial statement of the philosopher of science Alan Chalmers in the introduction of his classic *What is this thing called science?* is still valid:

> "Science is highly esteemed. Apparently it is a widely held belief that there is something special about science and its methods. The naming of some claim or line of reasoning

[1] Cf. the series Normative Orders published by Campus.
[2] One result of this research was published in Bawanypeck and Imhausen 2014.
[3] Some sociologists will probably find my approach difficult to reconcile with their positions – which may be a result of different approaches to fundamental concepts not only of a normative order, but also of science. For the latter, I will at least sketch a brief argument in this contribution. For the former, I would like to refer to the literature that came out of the research being undertaken in the Frankfurt Cluster of Excellence, e.g. Fahrmeir and Imhausen 2013 and Forst and Günther 2011.

or piece of research ‘scientific’ is done in a way that is intended to imply some kind of merit or special kind of reliability.”[4]

As Chalmers points out, however, the naïve associations that are connected with the term scientific, e.g. that science is “based on facts” to name only one, are far from evident and, if verified based on historical evidence, can be shown to be definitely missing in certain periods, and are rather difficult to assert in most periods, of which some include events that are considered milestones in the development of scientific knowledge.[5]

From the point of view of a historian of science, the situation becomes murkier still, since we have learned that scientific concepts and methods are not constants nor do they develop in a linear way, but are everchanging.[6] Consequently, in order to understand the scientific cultures of earlier times and civilizations, it is *not* helpful to analyse them based on modern parallels, but instead an emic approach is now favored, that attempts to recreate those concepts as they were understood and used by the former actors.[7] Science is understood in the context of this contribution as “natural science”, the counterpart of the German “Naturwissenschaft”, i.e. those areas of knowledge that are concerned with the description, prediction, and understanding of natural phenomena. Closely linked (and used in the various branches of science as a tool) is mathematics, which will be included.

One bone of contention in the historiography of the natural sciences remains their time(s) of origin. Two extreme points of view can be differentiated. On the one hand there are those who would like to claim science for modern sciences only.[8] The argument made by supporters of this camp is that science as we understand the term today is a modern affair and very different from ancient practices that are often intertwined with gods and other elements of

[4] Chalmers 2013: ix. A similar statement can be found at the beginning of Laudan (1983): „We live in a society which sets great store by science. Scientific 'experts' play a privileged role in many of our institutions, ranging from the courts of law to the corridors of power. At a more fundamental level, most of us strive to shape our beliefs about the natural world in the 'scientific' image. If scientists say that continents move or that the universe is billions of years old, we generally believe them, however counter-intuitive and implausible their claims might appear to be. Equally, we tend to acquiesce in what scientists tell us not to believe.“

[5] The book by Chalmers, which was first published in 1976, has seen numerous reprints and three subsequent editions. Some of the supposed characteristics of science that are analysed by Chalmers have also been studied in more detail by Lorraine Daston, cf. e.g. Daston and Galison 2007, Daston and Stolleis 2008 and Daston and Lunbeck 2011.

[6] For the related field of medicine this has been described in detail as early as 1935 by Ludwik Fleck using the example of syphilis, for an English edition cf. Fleck 1979.

[7] On the distinction of emic and etic and their use in the history of science cf. Jardine 2004. On the implications of this approach for ancient science cf. Imhausen and Pommerening 2016.

[8] There have been various attempts to determine a point in history after which there is science and before which there was not. Another approach to determine this point in time is the argument that science is dependent on discussions about its subject, which usually leads to starting science proper with the works of Plato and Aristotle in ancient Greece and granting Egypt and Mesopotamia a status as proto-scientific cultures. Various historians of science have argued against this point of view cf. most recently Rochberg 2016.

superstition or religious beliefs. One argument raised in this context is that of terminology. Thus, a culture that does not have a term for science, cannot possibly be practicing it. [9] This emergence of modern science has been linked by some historians to what has been called "the scientific revolution" in the early modern period.[10] Others would like to see the starting point even later, arguing that in order to speak of science, an institutional background is needed that includes for example laboratories and scientific instruments.

On the other hand, another extreme point of view can often be found within overviews of scientific disciplines. In this type of book, the author sometimes attempts to find rudiments of his discipline as far back as possible. The motivation for this approach originates either in the wish to claim a long ancestry for a subject or in the wish to claim an ancient culture's contribution to modern science. Thus, even the most remote similarity will be taken as evidence for the subject to have originated in a distant past. In using this approach, it is often pointed out that these earliest attestations are proto-stages of the respective modern concept. In History of Mathematics one prominent example is the history of algebra, in which the role of Egyptian and Mesopotamian mathematics has become the focus of a debate that also led to methodological discussions of appropriate methods in the historical analysis of early mathematics.[11]

Both of these extreme positions are obviously problematic from a historiographic point of view. The debate on how to best describe and analyze the development of sciences from a historical point of view has been a recurring issue in the methodological debates within the field. In its earliest beginnings, the history of science predominantly focused on finding precursors of modern disciplines in earlier times. This is, at least in part, reflected by the existence of the subdisciplines history of mathematics, history of medicine, history of astronomy, history of physics, and others, some of which remain visible as chairs within the scientific

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[9] The restricted modern sense of the term science can be traced back to the 18th century cf. Cresswell 2010: s.v. science.

[10] Needless to say that this description has long been criticized, too, e.g. for its negligence of earlier scientific contributions (cf. Kuhn 1957) and its Eurocentric perspective, ignoring the contribution of Islamicate societies (cf. Saliba 2007)). While many articles and books are about various aspects of the scientific revolution, two monographs that use the expression in its titles are Cohen 1994 and Shapin 1996, the latter starting out with the provocative statement „There was no such thing as the scientific revolution and this is a book about it." (Shapin 1996: 1).

[11] The classical controversy in this respect is Unguru 1975 and its replies van der Waerden 1976 and Weil 1978, answered again by Unguru 1979, after which the argument was presented in more detail in Unguru and Rowe 1981 and 1982, and followed up in Unguru 1979. For an overview of the debate cf. also Rowe 1996. The issue is also part of the collection Classics in the History of Greek Mathematics (Christianidis 2004) and was taken up again in the monograph Fried and Unguru 2001 and as recent as 2016 in the context of a work on the historiography of mathematics in the 19th and 20th centuries (Schneider 2016), where further literature is indicated.

discipline at modern universities. After the problem of anachronism that was part of the earliest methodology was raised, the discipline began to evolve to a historic discipline. However, despite the progress that has been made, remnants of the old approach remain and it is therefore advisable to begin a historic study with an assessment of past historiography. If modern science is not the ideal against which to compare and judge earlier scientific endeavors, the question of what is considered part of the scientific realm of a given time must be addressed.[12]

The argument about the point of origin of science very much depends on the choice of characteristic features of scholarly endeavors one deems necessary to speak of "science". In historiographic literature, this point has been raised multiple times, and has often resulted in characterizations that enable the inclusion of some ancient Egyptian and Mesopotamian scholarship, as may be exemplified by the definition once given by Jens Høyrup (which was not meant to be restricted to science in the sense of natural science but to describe what is called *Wissenschaft* in German), which included socially organized and systematic search for and transmission of coherent knowledge, the aim of intellectual coherence and the character of a continuous endeavor bound together by systematic transmission, and the presence of social organization of some kind.[13] Within the philosophy of science this question is raised in the context of the demarcation problem, which remains unsolved.[14]

Recently the debate about the contribution of the earliest civilizations to the history of science has been reshaped in a new direction by Francesca Rochberg. In her monograph *Before Nature*, Rochberg points out that the concept of nature did not exist in Mesopotamia.[15] Thus, in contrast to the later Greek scholars, who investigated those aspects of the world around us that were not caused by gods and which they designated by the term "nature" (physis), Mesopotamian scholars investigated aspects of a world, in which everything that happened was potentially caused by gods, and moreover, supposedly was the expression of a communication of gods to humans. Therefore, Greek natural philosophy and Mesopotamian scholarship can be referred to – as can specific periods of later times in various places - as different normative orders. In order to produce a meaningful historical analysis,

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[12] Work to be done in this respect remains in several periods and disciplines – compare for example the history of alchemy and its historiography as presented in Moran 2011, Newman 2011 and Principe 2011. Another early example is divination, for the Egyptian case cf. Quack 2010.

[13] Høyrup 1995: x. This definition is also used in Cancik-Kirschbaum 2009 to argue for Mesopotamian science. Obviously these definitions can be found much earlier, too, e.g. in the eleventh edition of the *Encyclopædia Britannica*, where it is defined as "ordered knowledge of natural phenomena and of the relations between them" (William Cecil Dampier Whetham).

[14] Cf. e.g. Laudan 1983, Pigliucci and Boudry 2013 and Gordin 2015, all of which refer to further literature.

[15] Rochberg 2016.

individual contributions have to be assessed according to the normative order in which they were created. As Rochberg further argues, despite the lack of the later fundamental term nature in Mesopotamian scholarship, it must be included in the history of science because of the contributions in various areas of knowledge (e.g. the study of movements of celestial bodies) that were the basis on which later Greek science evolved. And, as should be obvious from the argument that Rochberg put forward, this inclusion must mean an inclusion that describes Mesopotamian scholarship against the framework of its own normative order and not against that of a later scientific culture.[16]

How a historiography of Mesopotamian science along the lines of Rochberg's proposal would look like is probably already available for the area of celestial sciences with the recent work published by historians of Mesopotamian science that is often based on sources from scholars at the Neo-Assyrian court during the reigns of Ashurbanipal and Esarhaddon.[17] The cuneiform tablets from the royal library at Niniveh include numerous documents from which we can learn about the practice of the scholars at the royal court. These include on the one hand reports about their daily work, and on the other hand letters to the respective king that help us to learn about the social position of scholars at the royal court.[18] As has been well studied, the Neo Assyrian king was in permanent contact with an inner group of experts. This was necessary, because in ancient Mesopotamia, gods communicated with the king via signs that could be observed in the sky or on earth.[19] These signs were interpreted, based on scholarly literature, by experts, communicated to the king, and using further experts appropriate action was then taken, e.g. by carrying out a ritual. Within this context scholarly knowledge was collected, written down, transmitted, occasionally reworked and provided the basis for the development of further knowledge.[20] The aim to understand the relation between the various signs motivated the "long-term research project" that produced the astronomical diaries.[21] This was carried out – as far as we can see from the extant

[16] For an earlier discussion of the historiography of Mesopotamian sciences cf. Cancik-Kirschbaum 2010.

[17] Brief overviews that point to further literature can be found in the section on Mesopotamia in Jones and Taub 2018 and Keyser and Scarborough 2018. Cf. also Ossendrijver 2011, Watson and Horowitz 2011, Steele 2016, Hunger and Steele 2019. For other areas of scholarly knowledge cf. e.g. the section „Making Knowledge" in Robson and Radner 2011 and Robson 2019.

[18] These texts were first published in Hunger 1992 and Parpola 1993. These have since been made available online at http://oracc.museum.upenn.edu/saao/corpus (accessed June 5th 2020).

[19] One scholarly work from this context is the Diviners manual, which argues the connection between terrestrial and celestial signs (Reiner 1995: 94). The edition is Oppenheim 1974, further literature includes Williams 2002.

[20] An example from medicine where reworking of former texts is documented is edited in Heeßel 2000.

[21] Editions of the extant sources are published in the series *Astronomical Diaries and Related Texts from Babylonia*, edited by Abraham Sachs and Hermann Hunger. For an overview and contextualization cf. Haubold, Steele and Stevens 2019.

documentation – over a duration of over 800 years, collecting the data that eventually enabled the development of mathematical astronomy.[22]

A parallel argument to that, which Francesca Rochberg put forward, must also be made for the inclusion of ancient Egyptian scholarship. While the ancient Egyptian and Mesopotamian scholars lived at roughly the same times, and some of the content and formal shape of their scholarly output are similar, there are also enough differences discernible that one should argue for separate normative orders in which their knowledge evolved.[23] In ancient Egypt, the focus of the king was the upholding of *maat*, the correct order of the cosmos and its parts. This was the main task that an Egyptian king had to fulfill. If it was carried out correctly, it could be seen in the thriving of the country. Experts that were considered essential in aiding the pharaoh in this task were for example priests who held the knowledge of how and when rituals had to be carried out, in order to maintain the balance with the gods. Another aspect of the upholding of *maat* was the allocation of resources within the Egyptian society. According to the Egyptian conception of the world, each person had his specific role to play within the Egyptian community. According to this role, it was his duty to deliver a certain amount of produce, but also his right to receive a certain amount of resources, e.g. in the form of rations. Experts, i.e. persons that were part of the Egyptian administration, who oversaw the flow of resources in each direction developed sophisticated mathematical procedures to ensure the respective control.

If one transfers the concept of maat to the realm of the human body, a healthy body represented the ideal case to be preserved. Illnesses were considered as imbalances that had to be treated. For this purpose, a whole series of texts are available from which interactions of a doctor, pharmacological substances and magical acts can be reconstructed. Within the history of medicine, Egypt holds a prominent place. Since some papyri, like for example the so-called Edwin Smith Surgical Papyrus, include treatments for wounds that were recognized by modern physicians, in the earliest contributions to the history of Egyptian medicine, the authors selected their source material, which resulted in a collection of those texts that were close to modern medicine. These choices resulted in a corpus of texts that were related to medicine but neglected magical material, which can sometimes be found within the same

[22] Cf. Ossendrijver 2012 and 2016.

[23] Pragmatic differences in the area of mathematics of these two cultures are described in Ritter 1995. The methodology employed in this contribution, to focus on the algorithmic form of mathematical representations in both cultures, enables Ritter to analyse their differences, which is only one of its merits. It has since been used by various researchers to work on sources that are formally similar, e.g. Imhausen 2003, Melville 2004, Miatello 2012, and Ossendrijver 2012. For a critical assessment cf. Høyrup 2008.

papyri.[24] For the domain of ancient Egyptian medicine, this approach resulted in a success story in Egyptology as well as in the History of Medicine. Titles of articles like “The first case of an illness, a diagnosis, a treatment, and so on” indicate on the one hand the high level of competence that was already achieved in ancient Egypt. On the other hand, it also expressed the superior knowledge in modern Western medicine.[25] However, as has already been criticized elsewhere, the early approach neglected those areas of Egyptian healing arts that were not part of modern medicine. Thus the earlier historiography of Egyptian medicine has only used part of the available sources. In a new long-term project, my colleague Tanja Pommerening and I attempt to initiate a new historiography of sciences in ancient Egypt, Mesopotamia, Greece, Rome and also China and India. Based on the observation that our current historiography, despite its developments and changes in methods, remains indebted to the beginnings of the discipline of history of science – and thus does still deliver more of an etic perspective in which areas that have no modern counterpart are neglected or depreciated as wrong tracks that have since been abandoned – we propose a complete reassessment that begins with basic terms of perception and we aim to present as emic a picture of scholarly cultures as possible. In this approach, we also intend to determine those areas of knowledge that were considered privileged by the scholars of those times and cultures themselves and their boundaries and relations. While we expect that those texts that have been part of the scientific corpus from early on will probably remain so, we hope that we will be able to include those sources that were – from an etic perspective – also part of the scholarly literature, but have so far been ignored by an etic historiography.

In this brief, and very much simplified overview, the aspect of normativity has not explicitly been mentioned, but is somewhat always apparent underneath the debates. Obviously, there are some conditions that have to be fulfilled, in order for a collection of knowledge to be eligible for the classification as scientific knowledge.[26] Failing to meet these conditions results in not being admitted to the realm of science. Therefore what can be ascertained

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[24] Cf. for example the *Grundriss der Medizin der Alten Ägypter* (Deines, Grapow, Westendorf 1958-1973). In his first edition of the Edwin Smith Surgical Papyrus, Breasted argues that the surgical content must be differentiated from the „magical hodge-podge, which followed“ (Breasted 1930: xii). For a recent translation that includes those texts cf. e.g. Allen 2005: 72-115.

[25] In this context, a comparison with Mesopotamia is remarkable. From the extant cuneiform sources, it would have easily been possible to write a similar history of Mesopotamian medicine that was free from all magic. But this did not happen. Instead, the history of Mesopotamian medicine was presented with all of its elements, namely a combination of magical, divinatory and medical practices. As a result, the interest of historians of medicine in the Mesopotamian sources was much less.

[26] An aspect that plays a prominent role in this context, but will not be part of my following discussion of Egyptian mathematics is the question of arranging knowledge according to certain principles, i.e. the question of classification and orders. On this cf. Deicher and Maroko 2015 and Pommerening and Bisang 2017.

from this brief look at the history of science, is the impression that at least during specific times and at specific places, there often was a sense of agreement about what belonged to the part of the scientific realm and that for these limited times and geographic areas, certain formal rules existed in specific subjects about what kind of concepts and formalities were allowed.[27] Scientific disciplines therefore can be understood as normative orders themselves, as will be exemplified with a case-study from ancient Egypt in the following section. The term “scientific” has become a marker not only for a certain type of content, but also for quality and often a close link to power of some sort, which has in turn led to competition for this status. The debates in history of knowledge what counts as scientific knowledge are a secondary marker for this competition. Scientific knowledge therefore is privileged knowledge. As has become apparent from the research in history of science, the conditions to qualify as part of this privileged knowledge are neither absolute nor unchangeable, and, rarely expressed explicitly from contemporary scientists of any period.[28] Applying the characteristics of modern science to the ancient corpora has led to a distorted representation of ancient scholarship which recent historians of science have been revising now for several decades. Unfortunately, the old-style historiography has proven very persistent in the realms of the general public, and of (some) modern scientists for whom it is easy to choose their modern approach to the ancient sources. Only those that are interested in the ancient material and spend more time with it will realize the problematic aspects of the former approach. Again, this can be understood – and maybe made transparent – using the concept of sciences themselves as normative orders, as the following example illustrates.

II. TRACING THE BEGINNINGS OF SCIENCES IN TEXTS FROM ANCIENT EGYPT

The earliest scientific texts that originate from ancient Egypt date back well into the second millennium BC. Among these texts, the Egyptian mathematical texts hold a prominent place – mostly for reasons that are of a historiographical nature. From the first discovery of an Egyptian mathematical papyrus during the 19th century, the disciplinary home of such texts in the field of mathematics was undisputed. Some ancient Egyptian medical texts have received a similar historiographical reception. However, since only part of what can be found in Egyptian medical papyri has a relation to modern medicine, and other parts, e.g. the magic

[27] Cf. the results presented in Bawanypeck and Imhausen (2014).

[28] From Mesopotamia, there is evidence for explicit criteria that had to be fulfilled. One more recent example in which a group of scientists have stated in written form what they considered to be the respective criteria is the Manifesto of the Vienna Circle from 1929, cf. Stadler 2015 with references to the wealth of further literature.

elements, did not, the reception was (and remains) more complex than that of the mathematical texts, of which the complete content was accepted as mathematical.
Taking the mathematical texts as an example of ancient Egyptian science, the following description will attempt to sketch its respective normative orders. The plural refers to two separate aspects that can be established for any science, on the one hand, the normative order inside of the scientific subject itself in terms of form and content (internal normative order), and on the other hand the normative order that is imposed by the scientific subject on the culture in which it is practiced (external normative order). The external normative order is most obvious in terms of legal sciences. However, as can be shown with the example of mathematics, it can be claimed as a feature of at least some other sciences as well (one could easily make a similar argument for medicine, astronomy, biology and other disciplines).

The internal normative order of ancient Egyptian mathematics

The internal normative order of a subject begins at its very basics. In the case of mathematics, this is exemplified by the number system.[29] Egyptian mathematics used a non-positional decimal system that utilized a specific sign for each power of ten from 1 (= 10^0) to 1.000.000 (=10^6). In hieroglyphic writing, the number one was represented by a stroke, the number ten by a sign that may be the depiction of a hobble for cattle and the number 100 was indicated by a curled rope. The number 1000 was represented by the white lotus plant, the hieroglyph of a finger indicated the number 10.000, and the number 100.000 was depicted by a tadpole. Finally, the number 1.000.000 was the hieroglyph of the seated god *Heh*.
In order to write any natural number each sign was written as often as it was required for the respective number. Signs of the same kind were arranged symmetrically. Thus, to write the number 4821, the sign for 1000 (the lotus plant) would be written 4 times, the sign for 100 (the curled rope) would be written 8 times followed by twice the sign for 10 (the hobble for cattle) and once the sign for one (i.e. a single stroke). Zeros were expressed by the absence of the sign for this respective power of ten. In hieratic writing, groups of these number symbols developed their own characteristic ligatures.
Fractions developed in ancient Egypt from a basic set of earliest fractions as inverses of integers.[30] As a consequence, the Egyptian notation of fractions did not, like our modern notation, use numerator and denominator, but only marked the integer symbol as fraction –

[29] The most detailed study of the Egyptian numbers system remains Sethe 1916. For a shorter description cf. Imhausen 2016: 18-21.
[30] On the concept of Egyptian fractions cf. Guillemot 1992. For a brief overview of development and historiography cf. Imhausen 2016b: 300-302.

in hieroglyphic writing by placing the hieroglyphic sign for part above it, which was abbreviated to a dot in hieratic writing. The representation of a general fraction was accomplished in Egypt as a series of such inverses, which were written as a series in descending order. Any mathematical content that is extant from ancient Egypt uses this number system.[31]

Egyptian mathematical concepts and techniques were written down, probably for the context of teaching, in form of collections of mathematical problems. These problems indicate at the beginning the type of problem and the respective numerical values associated with it. This is then followed by a procedure to solve this problem. The procedure consists of a series of step-by-step instructions to carry out a sequence of arithmetical operations that in the end will yield the numeric solution of the given problem. This style of representing mathematical knowledge has been named *numeric*, *rhetoric* and *algorithmic* by Jim Ritter and is characteristic not only of Egyptian but also Mesopotamian and early Chinese mathematical texts.[32] Approximately 100 problems are extant within the mathematical papyri from ancient Egypt.[33] While this number (in conjunction with the randomness of preservation) is probably too small to make sophisticated claims about the content of Egyptian mathematics, it is sufficient to sketch the formal normative order of ancient Egyptian mathematical problem texts.[34] The text of the mathematical problems can be divided into up to four sections: introductory part (1), procedure (2), solution (3), and additional elements (4).

The introductory part (1) comprises the type of problem and the respective numerical data. The second section (2) details the procedure in the form of a series of instructions to carry out specific mathematical operations and their results. After the procedure, the numerical solution of the problem is indicated (3). This can be followed (4) by drawings (for problems with a geometrical topic), the written performance of calculations that were indicated in the procedure, or a numerical verification.

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[31] In addition to the representation sketched above, there is also evidence of a multiplicative notation for larger numbers, i.e. multiples of 10.000 and 100.000. Since this is not used in the mathematical texts and attestations in general for this writing are rare, it can be neglected in this context. For this style of number notation cf. Sethe 1916: 8-10.

[32] Ritter 1995: 50. For general introductions on working with Egyptian or Mesopotamian mathematical texts cf. the contributions by Ritter 2016 and Imhausen 2016b. On mathematical problem texts from the Middle Ages cf. Folkerts 1996.

[33] For an overview of these mathematical papyri cf. Imhausen 2016: 63-69.

[34] The problematic aspect about the scope of the content becomes obvious when assigning the individual problems to groups. While some can be assigned easily, others, like for example problems 56-60 of the Rhind mathematical papyrus, that describe the relation between the dimensions of a pyramid and its inclination, seem not as straightforward to place in regard to their context. Problems 56-60 may be placed within a group that is related to construction, however, this group - based on the extant problems - only comprises the mentioned problems as we as problem 14 from the Moscow mathematical papyrus and does not seem to represent adequately the mathematical techniques that were used in this area.

For the detailed analysis of the formal aspects on which the following summary is based cf. Imhausen 2014. For cognate studies of Mesopotamian mathematics cf. Høyrup 2010 and Ritter 1995b. For a comparison of the structures of various Mesopotamian disciplines cf. Ritter 1990, 2004, 2009 and 2010.

Each section can be easily recognized through the choice of verbal forms, specific key phrases, or other formal characteristics. Thus, the introductory section usually begins with the phrase “method of (calculating)” and numerical data are often introduced with “If a scribe tells you…”. The procedure uses imperatives or a specific Egyptian verb-form used to express a necessary consequence, which is used in instructions as well as in announcing intermediate results.[35] The final solution may be expressed as a nominal sentence. The additional elements usually do not use complete sentences, but are mostly only representations of numerical schemes with few additional explanatory words. While despite the use of characteristic formal features a certain variation in the respective sources can be found, the overall impression of a certain formal structure that is adhered to remains, which would enable a researcher to determine a mathematical problem text as such even if the source were extremely fragmentary.[36] Therefore, mathematical problem texts are not only texts that treat a mathematical problem, but they are texts that treat a mathematical problem in a distinct formal representation which was the only form to express mathematical content in ancient Egypt.

As was indicated earlier, mathematical problem texts of ancient Egypt (and also elsewhere) used concrete numerical values in expressing a mathematical problem. While researchers agree that this was the method to teach examples of procedures that could then also be applied to different sets of numerical values, there are two examples in the Rhind mathematical papyrus that express this general application explicitly. In problem 61b two-thirds of a fraction is to be determined. While the problem is phrased in general terms in its introductory section “Calculating two-thirds of a simple fraction” and also ends with a statement of the general validity of the solution “Behold, the calculation is likewise for any simple fraction that may occur”, the introductory section also contains, after the general heading, concrete numerical data of an example of which two-thirds shall be calculated. However, the following procedure then (untypically) does not use the numerical value of the given fraction, but refers to it with a suffix pronoun. The second example of a reference to the general applicability of the procedure can be found at the end of problem 66. The problem calculates a daily ration

[35] On this verbal form, called *sḏm.ḫr.f* in Egyptian grammars see now Clayton 2018. On its use in medical texts in conjunction with the form *sḏm.jn.f* cf. Pommerening 2014: 13-16, for its use in mathematical texts cf. Imhausen 2003: 28.

[36] For examples of these variations and their attestations in the mathematical texts cf. Imhausen 2014. The use of the *sḏm.ḫr.f* in combination with numerical values was used by me to assign the ostracon Turin 57170 to the corpus of mathematical texts, cf. Imhausen 2003: 175. The possibility of allocating a text to the group of mathematical problems very much depends on the rhetoric parts of the text. If only numbers are extant a classification is usually not straightforward, cf. Imhausen 2006.

of fat from a given yearly delivery and concludes “You will calculate likewise concerning everything that was said to you like in this method.”

The external normative order of ancient Egyptian mathematics

Although the available material may not be representative of the content of Egyptian mathematics in its entirety, at least (part of) its context in administration is obvious.[37] These problems can be used as one example that illustrates how mathematics was used to create and execute the normative order of ancient Egyptian working life and management of its resources. Grain, the basic commodity of ancient Egypt, features prominently in these problems.[38] The mathematical texts stipulate how to calculate areas of various shapes, a mathematical technique which was then supposedly used by scribes to calculate the area of a field, which in turn was used to determine the amount of grain that was expected to be harvested. The close connection between mathematics and administration is apparent in the terminology that is used in area calculations. The mathematical *terminus technicus* for the area was derived from the Egyptian word for field. Area calculations can be found in the Rhind and Moscow papyri as well as in one of the mathematical fragments from Lahun.[39]
The grain that was harvested was put into granaries which are found in the context of volume calculations of the mathematical texts. Problems 41-46 of the Rhind Papyrus calculate the volume of granaries of circular or rectangular base. The problems calculate the total capacity of the respective structures and in one case its dimensions from its given capacity.[40] The latter problem raises the question of the degree of practicality of these so-called practical problems, namely the question if the setting isn’t merely invented to provide the frame of a mathematical problem like in the word problems of today’s mathematical education. But even if the actual problems of the mathematical papyri were “only” tools in teaching mathematical concepts, they still inform us about the techniques that would have been used if this problem had to be solved in the context of an actual administrative situation.

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[37] In my analysis of all mathematical problem texts, administrative problems is the largest group with almost 50 examples that can be divided in several subgroups, cf. Imhausen 2003: 93-160. On the connection between Egyptian mathematics and administration cf. also Reineke 1982.
[38] For a more detailed analysis of the mathematical problems related to grain cf. Imhausen 2010.
[39] For a discussion of these problems cf. Imhausen 2003: 65-85. The donation texts on the Edfu temple from Ptolemaic times also include fields and their respective measurements, on these texts and land tenure during these times in general cf. Manning 2003.
[40] Cf. Imhausen 2003: 139-147.

The mathematical control of the production of bread and beer from grain features prominently within the extant mathematical problems. The 21 so-called bread-and-beer-problems constitute about one fifth of all extant problem texts. The most important technical term of these problems is the *psw*-value, which indicates the number of bread loaves (or jugs of beer) that were produced from one *ḥqꜣ.t* (approximately 4.8 liters) of grain.[41] The most basic problem among the bread-and-beer-problems is problem 15 from the mathematical papyrus Moscow. From a given amount of grain (10 *ḥqꜣ.t*) and a given *psw*-value (2) the corresponding number of jugs of beer (20) is calculated.[42] The more sophisticated examples of this group of problems calculate for example changes in quantity if the *psw*-value is changed. Extant sources from administrative contexts document that the concept of the *psw*-value was used not only in mathematical texts but also in actual accounts.[43] The same applies to the two final groups of problems to be mentioned in this context, ration-problems and calculations of work. Ration problems of the mathematical texts include the calculation of equal distributions of resources (loaves of bread) among a group of recipients as well as unequal distributions, where individual recipients receive – based on their different status – a larger share.[44] The concept of recipients who obtain multiples of a basic unit is also attested in an administrative text from the Middle Kingdom. In the list given in Papyrus Berlin 10005, the ordinary worker receives 1/3 of a basic unit as his share, whereas the head of the temple receives 10 times that unit.[45] Finally, the mathematical problems relating to the required produce of ones work are matched by numerous accounts that document the supervision of produce as well as references to this concept in literary texts.[46]

The control of numbers or amounts of goods was conceived as an important aspect of the work of an official, which also led to its representation in the funerary context. Scenes of accounting are depicted in reliefs as well as in models.[47] In addition to the mathematical techniques to calculate the volumes or amounts of grain a sophisticated metrological system was the second tool to implement the control over grain resources.[48] In consequence, much attention had to be placed on the usage of correct measuring instruments. It should come as no surprise therefore that this is also a topic in a wisdom text. Ancient Egyptian wisdom

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[41] On the term psw cf. Verhoeven 1984: 100. On the HoA.t cf. Pommerening 2005.
[42] If one assumes a practical significance of this kind of problems, it must be assumed that these jugs are at least regionally or institutionally standardized (and that loaves of bread also had regular sizes).
[43] E.g. Spalinger 1986.
[44] On ration problems and their application in accounts cf. Imhausen 2003b.
[45] Imhausen 2003b: 13. Cf. also Guillemot 1992: 56 and 59. For further examples of ration texts cf. Imhausen 2003b: 11-12.
[46] Imhausen 2013.
[47] E.g. the models from the tomb of Meketre, chancellor and high steward during the early Middle Kingdom, at Thebes, cf. Winlock 1955.
[48] On metrological units, their developments and relations cf. Pommerening 2005, 2012 and 2020.

texts are the literary genre to express the normative order of behavior. They are written as instructions from a high ranking official to introduce a son or apprentice into the code of conduct in his official position but also in his private life.[49] *The Teaching of Amenemope*, an official who according to his titles and description of his duties ("superintendent of the land", "superintendent of produce, who fixes the grain measure, who sets the grain tax amount of his lord") given in the introduction of the teaching was responsible for the administration of grain, refers to the importance of executing metrological procedures correctly.[50] Within the instructions of this teaching there are 43 lines that refer to the correct execution of metrological practices, e.g. lines 1-4 from the chapter 6 (VII: 12-15):

> "Do not displace the surveyor's marker on the boundaries of the arable land,
> Nor alter the position of the measuring line;
> Do not be covetous for a single cubit of land,
> Nor encroach upon the boundaries of a widow."[51]

The scribe has to ensure that the measuring process is carried out properly and that the measuring tools used in this process (in the example above the measuring line, in other examples weights or grain measures) are correct. In summary, the mathematical (and metrological) implementation enabled the administrators to execute the normative orders that were commanded by the king. As was exemplified with this case of ancient Egyptian mathematics, other sciences at other times can also be shown to be part of an influence if not the creation of normative orders of their societies. And, in conjunction with the change that can be witnessed in the normative orders of societies are changes in the internal normative aspects of sciences.

III. History of Science: Describing and analyzing changes of normative orders

Ancient Egyptian mathematics has been studied in the field of history of mathematics since its first sources were discovered.[52] The first publications appeared in the second half of the

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[49] For an introduction of this kind of texts cf. Burkhard and Thissen 2003: 72-109 and 163-177 and Burkhard and Thissen 2008: 99-140.
[50] For the teaching in general cf. Grumach 1972 and Laisney 2007. An English translation of the text can be found in Simpson 2003: 223-243.
[51] Translation from Simpson 2003: 228.
[52] For a survey of the historiography of Egyptian mathematics in general cf. Imhausen 2003: 8-12 and Imhausen 2016: 2-7. For the historiography of the Rhind Mathematical Papyrus cf. Imhausen 2020. The brief

nineteenth century, when the Rhind Mathematical Papyrus had arrived in the collections of the British Museum and was available for study, at least to those who could read Egyptian.[53] In 1877 the first edition of the Rhind Mathematical Papyrus was published and from then on it was also included in general works of historians of mathematics, e.g. in Moritz Cantors first volume of *Vorlesungen über Geschichte der Mathematik*.[54] Due to the progress in our understanding of ancient Egyptian grammar, the first edition of the Rhind papyrus was replaced in 1923 by the edition of Thomas Eric Peet.[55] An aspect of the Egyptian sources that fascinated Otto Neugebauer and Kurt Vogel, two well-known German historians of mathematics of the early 20th century, was the Egyptian method of handling fractions. Both wrote their dissertations on this subject.[56] Although Neugebauer continued to work with Egyptian sources, his main focus soon became Mesopotamian mathematics and astronomy which proved more satisfying for his approach.[57] The second largest source, the Moscow Mathematical Papyrus, was published in 1930 by Wasili Struve.[58]

Despite the scarcity of sources, a steady stream of publications since has never come to a halt. As was customary until the end of the 20th century, the mathematical content of the Egyptian papyri was compared and measured against their supposed parallels in modern mathematics. This approach was based on the assumption that mathematics is a discipline that developed, if not always linear, at least steadily from its beginnings with few techniques and little knowledge to our modern times with a wealth of mathematical subdisciplines, techniques and a now much larger but still growing understanding of mathematical affairs. The sources of ancient Egypt did not do too well under this approach. Egyptian fraction reckoning, which had fascinated (and still fascinates) modern mathematicians since its discovery, can be used as an example. Otto Neugebauer noticed the characteristic feature of Egyptian fraction reckoning using only inverses and their combinations (sums). In his work he devised a notation system that mirrors this feature by rendering an inverse by the value of the corresponding integer and an overbar to mark it as fraction. Thus 1/5 would be expressed as $\bar{5}$. From the construction of “general fractions” as sums of unit fractions Neugebauer derived a general characterization of Egyptian mathematics as mainly additive, which then became his explanation for the inferior calculation technique found in the Egyptian mathematical

outline given in this article focuses only on those publications that created the foundation of the historiography of ancient Egyptian mathematics.

53 The first edition of the Rhind Mathematical Papyrus is Eisenlohr 1877. The *editio princeps* remains Peet 1923.

54 Cantor 1880: 17-63.

55 Peet 1923.

56 Neugebauer 1926 and Vogel 1929.

57 On Otto Neugebauer and Egyptian mathematics cf. in detail Ritter 2016b.

58 Struve 1930.

texts.[59] The inferiority of Egyptian mathematics was highlighted by Otto Neugebauer through a comparison with its neighboring culture Mesopotamia, which had not only achieved much more advanced mathematical content but also used their superior mathematical techniques to make significant progress in astronomy.[60] The disappointment of Neugebauer with ancient Egyptian mathematics is expressed explicitly in his overview work The Exact Sciences in Antiquity:

> “The fact that Egyptian mathematics did not contribute positively to the development of mathematical knowledge does not imply that it is of no interest to the historian. On the contrary, the fact that Egyptian mathematics has preserved a relatively primitive level makes it possible to investigate a stage of development which is no longer available in so simple a form, except in the Egyptian documents.”[61]

In analyzing the historiographical development of ancient Egyptian mathematics another aspect must be raised, namely the usage of modern concepts and formalisms in describing ancient sources. In this respect, Neugebauers work and his awareness of methodological problems is exemplary as the following qote from the introduction of the same monograph indicates:

> “Indeed, I have consistently tried to keep as close as possible to the source material. (…) And in order to counteract somewhat the impression of security which easily emerges from general discussions I have often inserted methodological remarks to remind the reader of the exceedingly slim basis on which, of necessity, is built any discussion of historical developments from which we are separated by many centuries.”[62]

This careful handling of ancient formal features like for example the notation of Egyptian fractions was soon given up for the sake of the ease with which modern readers would be able to grasp the ancient text.[63] The result was – in history of mathematics - a representation

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[59] Ritter 2016b: 142.
[60] Unsuprisingly, Neugebauers publications on mathematics and astronomy from Mesopotamian sources by far outnumber those from Egyptian sources.
[61] Neugebauer 1969: 72.
[62] Neugebauer 1969: viii.
[63] Cf. e.g. van der Waerden 1961, which is probably the most successful monograph on ancient sciences. At the cost of ignoring ancient formal features and their difference to modern formalities, van der Waerden „explains“ the content of the ancient sources as primitive stages of scientific knowledge which is familiar to most of his readers from their own education and therefore offers them an easier access.

of Egyptian mathematics as a primitive ancestor of (some aspects of) modern mathematics with explicit references to its stranger aspects (e.g. fraction reckoning).[64]

The problematic aspect of this approach can also be described in terms of normative orders. The normative order of ancient Egyptian mathematics was analyzed and described using another normative order, namely that of modern Western mathematics. In those areas, where they were similar, the outcome was satisfactory. In many others, the result did not do justice to the source material or it was completely ignored since it had no relevance.

As in other areas of the history of science, the methodology used in the history of mathematics then developed over time from an anachronistic approach that measured earlier sources according to their likeness and compatibility with modern parallels to attempts at an emic perspective.[65] That is, the goal of the more recent studies is to describe the normative order of Egyptian mathematics on its own terms, that is according to its inherent characteristics, and not based on a comparison with another structure. The future research in Egyptian mathematics therefore will on the one hand attempt to study the internal structures of Egyptian mathematical texts using its formal characteristics as well as the new digital technologies available to facilitate the analysis of texts. On the other hand, the close connection of mathematics to administrative affairs in Egypt should be used to study its development from its beginnings to the end of its documentation in Graeco-Roman times.

VI. What can the history of "science", the focus of past scientific cultures, and the results of past scientific cultures tell and teach us today?

From the introductory section, the case study of ancient Egyptian mathematics and the brief survey of the historiography of Egyptian mathematics, it seems plausible to argue for the perception of sciences at individual points in times and places as distinct normative orders. The task for historians of science is a thick description of these individual normative orders, their changes and developments over time. Studying the history of sciences in detail at various points in time attempting an individual emic perspective results in the realization that the relation between scientific systems and their respective social and cultural environment

[64] It should be noted that research on the subject was also carried out by Egyptologists, which was published in their journals and usually overlooked by historians of mathematics. Despite the usage of modern formalisms in these works, the description and analysis are usually more accurate and less pejorative, cf. e.g. Reineke 1982

[65] For Egyptian mathematics the first publications in this direction are those of Jim Ritter. Despite this development, however, it must be noted that in works on the history of Egyptian mathematics as well as in works on the history of Egyptian medicine, the old-style anachronistic approach is still pursued, often by scholars with a modern mathematical or medical background. Since research in these fields remains attractive to mathematicians and physicians, a volume that explains the current methodology in history of ancient sciences was published (Imhausen and Pommerening 2016).

has always been very close.[66] Sciences not only have an internal normative order, but also provide an external normative order by their application in daily life practices. The historical perspective can, in hindsight, study developments and turns that scientific progress has taken. However, there are so many contingencies that a prediction of a future development is clearly impossible.

The role that history of science can play within modern society and its needs may be the illustration of certain decisions in societies about their sciences and their respective results. Examples from civilizations in the past indicate that periods of bloom often concurred with flourishing sciences supported by rulers, who held sciences in high esteem and provided appropriate material support.

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[66] This is the focus of the Sociology of Scientific Knowledge.

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