Essays in Empirical Macroeconomics

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List of Original Working Papers

This thesis consists of the following working papers:

a) Bick, Alexander (2009):
Threshold Effects of Inflation on Economic Growth in Developing Countries

b) Bick, Alexander and Dieter Nautz (2008):
Inflation Thresholds and Relative Price Variability: Evidence from U.S. Cities[1]

c) Bick, Alexander (2009):
Fertility, Female Labor Force Participation and Child Care

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Overview

This dissertation consists of three chapters. The first two chapters investigate the real effects of inflation and the third chapter the role of child care for fertility and female female labor supply.

Chapter 1 introduces a generalized panel threshold model to analyze the relation between inflation and economic growth for a sample of developing countries. It is demonstrated that allowing for regime intercepts can be crucial for obtaining unbiased estimates of both, inflation thresholds and its marginal effects on growth in the various regimes. The empirical results confirm that the omitted variable bias of standard panel threshold models can be statistically and economically significant.[2]

Chapter 2, which is joined work with Dieter Nautz, investigates the impact of inflation on relative price variability (RPV) as a further important channel of the real effects of inflation. With a view to the recent debate on the Fed's implicit lower and upper bounds of its inflation objective, the econometric model introduced in Chapter 1 is used to explore the inflation-RPV linkage in U.S. cities.

Chapter 3 investigates the relationship between fertility, female labor supply and child care in the context of a life cycle model for Germany. A particular emphasis is placed on the differences between West and East Germany. Counterfactual policy experiments mimicking recent policy reforms on maternal leave and the provision of subsidized child care are conducted with a structurally estimated version of the model.

[2] In a joint paper with Dieter Nautz and Stephanie Kremer, which is not enclosed in the dissertation, the threshold model is extended to a dynamic panel setting.

Deutsche Zusammenfassung

Einleitung Die Dissertation besteht aus drei Kapiteln, die sich in zwei Themengebiete einteilen lassen.

Im ersten Themengebiet werden die realen Effekte von Inflation mittels eines Schwellenwert-Modells untersucht. In Kapitel 1 wird eine Verallgemeinerung des Schwellenwert-Modells von Hansen (1999) eingeführt, womit anschliessend der nicht-lineare Einfluss von Inflation auf das Wirtschaftswachstum geschätzt wird. Im zweiten Kapitel, das in Ko-Autorenschaft mit Dieter Nautz entstand, wird diese Methodik auf den Zusammenhang von Inflation und der Variabilität relativer Preise angewandt. In einem weiteren dieser Dissertation nicht beigefügten Artikel wird die Analyse aus Kapitel 1 bezüglich der Wirkung von Inflation auf Wachstum im Rahmen eines dynamischen Panel Schwellenwert-Modells vertieft und weitergeführt.

Das zweite Themengebiet der Dissertation und gleichzeitig dritte Kapitel untersucht den Zusammenhang von Fertilität, Frauenerwerbstätigkeit und Kinderbetreuung im Rahmen eines Lebenszyklus-Modells für Deutschland, mit besonderem Augenmerk auf die Unterschiede zwischen West- und Ostdeutschland.

Kapitel 1 diskutiert eine Verallgemeinerung des von Hansen (1999) eingeführten Schwellenwert-Modells, angewandt auf den nicht-linearen Zusammenhang zwischen Inflation und Wachstum.

Wirtschaftliches Wachstum und niedrige Inflationsraten sind zwei der zentralen Ziele makroökonomischer Politik. Der Zusammenhang dieser beiden Variablen war Gegenstand vieler empirischer Arbeiten in der jüngeren

Vergangenheit, jedoch ohne eindeutige Ergebnisse. Fisher (1993) identifizierte als erster einen nicht-linearen Zusammenhang, wobei niedrige Inflationsraten einen positiven und hohe Inflationsraten einen negativen Wachstumseffekt haben. Bruno and Easterly (1998) bestätigten den negativen Effekt, bezweifelten jedoch den wachstumsförderenden Effekt von niedriger Inflation. Die Studie von Khan and Senhadji (2001) unterstützt dieses Ergebnis. Sie schätzen einen Inflations-Schwellenwert von 11% für Entwicklungsländer, wobei Inflationsraten unterhalb des Schwellenwerts keinen signifikanten Einfluss auf das Wirtschaftswachstum haben, während hingegen Inflationsraten oberhalb von 11% einen signifikanten wachstumsmindernden Effekt haben. Mittels einer Verallgemeinerung des Schwellenwert-Modells von Hansen (1999) präsentiere ich neue Ergebnisse für den nicht-linearen Zusammenhang von Inflation und Writschaftswachstum.

Hansen (1999) entwickelt ein Schwellenwert-Modell und die dazugehörige asymptotische Theorie für Panel Daten mit individuellen Effekten, welches es erlaubt, sowohl die Schwellenwerte zu schätzen als auch auf deren statistische Signifikanz zu testen. Aufgrund der notwendigen Bereinigung um den individuellen Effekt, ist es nicht möglich in jedes der durch die Schwellenwerte getrennten Regime eine Konstante aufzunehmen, wie es in Schwellenwert-Modellen für Zeitreihen- oder Querschnittsdaten üblich ist. Da in Hansen (1999) die Entwicklung der Methodik und Asymptotik an sich im Vordergrund steht, geht er auf dieses Problem nicht ein. Jedoch läßt sich einfach für Unterschiede in den regimespezifischen Konstanten kontrollieren, in dem in jedes Regime, bis auf eines, eine Konstante eingeführt wird. Diese Konstanten werden bei der Bereinigung der individuellen Effekte somit nicht eliminiert und das von Hansen (1999) eingeführte Schätzverfahren bleibt bei geringfügigen Modifikationen weiterhin anwendbar. Wie im Folgenden dargestellt werden soll, kann diese Verallgemeinerung sowohl theoretisch als auch in empirischen Anwendungen entscheidenden Einfluss auf die Schätzergebnisse haben. Ohne Beschränkung der Allgemeinheit des Ergebnisses zeige ich für ein vereinfachendes Beispiel, dass es im Falle von Schwellenwert-Modellen zu verzerrten Schätzern der linearen Effekte in den einzelnen Regimen kommen kann, wenn Unterschiede in den regimespezifischen Konstanten vorliegen. Dies ist insofern nicht überraschend, da jede in einer Schätzung vernachlässigte Variable, die sowohl mit der zu erklärenden als auch einer der erklärenden Variablen korreliert ist, zu einer verzerrten Schätzung dieser Regressionskoeffizienten führt, dem sogenannten "Omitted Variable Bias". Die Verzerrung des Schätzers der linearen Effekte ist proportional zu der Größe des vernachlässigten Unterschiedes in den regimespezifischen Konstanten, da die Orthogonalität der Regressoren in den verschiedenen Regimen nicht bewahrt wird. Im Rahmen des Schwellenwert-Modells von Hansen (1999) zieht dies jedoch noch weitere Konsequenzen nach sich, da sowohl die Schätzung des Schwellenwerts als auch der Test auf statistische Signifikanz direkt von den Schätzern der regimespezifischen linearen Koeffizienten abhängt. Am Beispiel des Zusammenhangs zwischen Inflation und Wachstum zeige ich die Bedeutung der korrekten Spezifizierung des Modells von Hansen (1999) auf.

Untersuchungsgegenstand ist ein Panel von 40 Entwicklungsländern für die Zeitperiode 1960 bis 2004, wobei wie üblich in der empirischen Wachstumstheorie Fünf-Jahres-Durchschnitte gebildet werden, um Konjunkturschwankungen zu glätten. Das Original-Modell von Hansen (1999) schätzt einen signifikanten Schwellwert in Höhe von 19%. Während Inflationsraten unterhalb dieses Schwellenwertes einen positiven Wachstumseffekt auf dem 10% Signifikanzniveau aufweisen, lässt sich zwar ein negativer, jedoch statistisch nicht signifikanter Effekt von Inflationsraten größer als 19% finden. Unter Berücksichtigung einer regimespezifischen Konstanten, wird ebenfalls ein signifikanter Schwellenwert geschätzt, der jedoch nur bei 12% liegt. Im Vergleich zur ersten Schätzung sind nun beide regimespezifischen linearen Effekte auf dem 1% Niveau signifikant und doppelt so hoch wie in der vorherigen Schätzung. Die regimespezifische Konstante ist ebenfalls auf dem 1% Niveau signifikant. Aus Sicht der Geldpolitik hat dies drastische Implikationen: Im Vergleich zur ursprünglichen Schätzung sind die wachstumsfördernden Effekte zweimal so groß, allerdings nur bis zu einem um 7% geringeren Schwellenwert. Des Weiteren verringern Inflationsraten oberhalb des Schwellenwerts das Wirtschaftswachstum in signifikanter Weise. Diese Argumentation trifft auch zu, wenn als konservative Annahme die unteren Grenzen der 95% Konfidenzintervalle der Schwellenwert-Schätzer gewählt werden.

Abschliessend lässt sich festhalten, dass die von mir vorgeschlagene Verallgemeinerung des Modells in empirischen Anwendungen zu wichtigen Unterschieden in den Schätzergebnissen und Politikempfehlungen führen kann.

Kapitel 2 setzt sich ebenso wie das erste mit den realen Effekten von Inflation auseinander und ist in Ko-Autorenschaft mit Dieter Nautz entstanden. Untersuchungsgegenstand ist jedoch der nicht-lineaere Einfluss auf die Variablität relativer Preise (RPV).

Aus theoretischer Sicht wird ein Zusammenhang von Inflation und RPV mit Menu-Kosten und Unsicherheiten über das aktuelle Preisniveau begründet, wobei in beiden Fällen Inflation RPV erhöht und somit den Informationsgehalt der nominalen Preise verringert. Diese theoretischen Zusammenhänge wurden in einer Vielzahl von Studien für verschiedene Länder bestätigt, siehe zum Beispiel Aarstol (1999). Jedoch gibt es auch Ausnahmen, die einen negativen oder gar keinen Einfluss von Inflation auf RPV auffinden, vgl. Lastrapes (2006). Während sich diese Studien auf einen linearen Zusammenhang zwischen Inflation und RPV beschränken, zeigen Caglayan and Filiztekin (2003) für die Türkei und Caraballo et al. (2006) für Spanien und Argentinien Nicht-Linearitäten in Form von Schwellenwerten auf, welche allerdings exogen gesetzt wurden. In diesem Papier wenden wir das Schwellenwert-Modell von Hansen (1999) samt der in Kapitel 1 beschriebenen Verallgemeinerung auf den Zusammenhang zwischen Inflation und RPV in den USA an. Unser besonderes Interesse an der möglichen Existenz von Schwellenwerten für die USA ist begründet in der allgemein verbreiteten Vermutung, dass die US-Amerikanische Notenbank eine implizite Unter- und Obergrenze ihres Inflationsziels hat, siehe Thornton (2006). Während Schwellenwerte in der Beziehung zwischen Inflation und RPV nicht die zunehmende Bedeutung von Inflationszielen in der Zentralbankpraxis erklären können, sollte die Identifizierung solcher Schwellenwerte jedoch nützliche Informationen über die Lage und Breite eines Inflationszielbands liefern.

Die empirische Analyse wird für 14 Metropolregionen in den USA für den Zeitraum von 1998 bis 2005 durchgeführt, wobei die im zwei Monatsrhythmus veröffentlichen Daten des US-Amerikanischen Konsumenten Preis Indizes (CPI) zugrunde gelegt werden. Inflation ist definiert als die jährliche Veränderungsrate des CPI und RPV als die Standardabweichung der jährlichen Inflationsraten der Subkategorien des CPI um die jährliche Inflationsrate, jeweils gewichtet nach der Bedeutung der einzelnen Subkategorie im CPI. Im Gegensatz zu den vorherigen Studien liefert die Schätzung mittels des um regimespezifische Konstanten erweiterten Schwellenwert-Modells von Hansen (1999) zwei Schwellenwerte bei 1.6% und 4.2%, die ebenso statistisch signifikant sind wie die regimespezifischen Konstanten und linearen Effekte. Inflationsraten unterhalb von 1.6% sind mit einem negativen Einfluss auf RPV verbunden, wonach ein weiteres Absinken der Inflation Richtung Deflation ein Ansteigen von RPV in diesem Regime bedeutet. Dies lässt sich durch die Existenz von nach unten rigiden, nominalen Preisen und Löhnen erklären. Inflationsraten zwischen den beiden Schwellenwerten von 1.6% und 4.2% haben ebenso einen negativen Effekt der allerdings deutlich schwächer ausfällt. Inflationsraten grösser als 4.2% erhöhen RPV wie dies von den klassischen Theorien vorhergesagt wird, während die RPV verringernden Aspekte von Inflation abgeklungen sind. Die absolute Stärke des Effekts ist vergleichbar mit dem für das Inflationsregime unterhalb von 1.2%.

Diese Ergebnisse stellen zunächst einen Bezug zu den Ergebnissen vorheriger Studien dar, in denen sowohl negative als auch positive Effekte von Inflation aufgezeigt werden, die jedoch abhängig vom Inflationsniveau sind. Des Weiteren lässt sich sagen, dass die optimale Inflationsrate im Bezug auf ihre Wirkung auf RPV zwischen 1.6% und 4.2% bzw. zwischen 1.8% und 2.8% liegt, wenn als konservative Annahme die Ober- bzw. Untergrenze der 95% Konfidenzintervalle des unteren bzw. oberen Schwellenwertes als Bezugspunkte gewählt werden. Diese Aussage beruht auf der Annahme, dass nicht das Niveau der RPV minimiert sondern nur der Einfluss von Inflation auf RPV minimal sein sollte, vgl. Woodford (2003).

Kapitel 3 untersucht im Rahmen eines Lebenszyklus-Modells die Bedeutung von bezahlter Kinderbetreuung für das Fertilitäts- und das Arbeitsangebotsverhalten verheirateter Frauen in Deutschland.

Während für über neunzig von hundert Kindern in der Altersgruppe von drei bis sechseinhalb Jahren in Deutschland ein subventionierter Kinderbetreuungsplatz zur Verfügung steht, ist nur für drei von hundert Kindern im Alter von null bis zwei Jahren in Westdeutschland ein subventionierter Kinderbetreuungsplatz vorhanden, in Ostdeutschland jedoch für mehr als dreißig von hundert Kindern. Nach einem erst kürzlich erlassenen Gesetzentwurf der Bundesregierung sollen die Kinderbetreuungsangebote im Westen bis zum Jahre 2013 auf das ostdeutsche Niveau angehoben werden. Ziel dieser Inititative ist unter Anderem, die Erwerbsbeteiligung von Müttern mit jungen Kindern sowie die Geburtenrate zu erhöhen. Da diese Maßnahme mit vermehrten öffentlichen Ausgaben verbunden ist, besteht das Interesse, die Erfolgsaussichten dieser Initiative zu beurteilen. Dies versuche ich im Rahmen eines Lebenszyklus-Modells mit endogenen Fertilitäts-, Arbeitsangebots- und Kinderbetreuungsentscheidungen.

Unter Benutzung des Sozio-ökonomischen Panels konstruiere ich für die Jahre von 1983 für Westdeutschland bzw. 1989 für Ostdeutschland bis 2006 Beziehungs-, Geburten- und Arbeitsangebothistorien für Frauen sowie deren Nutzung von Kinderbetreuung. Ähnlich wie Francesconi (2002) beziehe ich in meine Studie nur Frauen ein, die in einer festen Partnerschaft leben. Falls eine Frau Kinder hat, muss der aktuelle Lebenspartner auch der Vater sein. Dieses Auswahlkritierum lässt sich damit begründen, dass die meisten ökonomischen Theorien für Fertilität und das weibliche Arbeitsangebot für Frauen in stabilen Beziehungen entwickelt wurden. Es gilt jedoch anzumerken, dass die Ergebnisse dieser Studie sich somit nicht auf die Gesamtheit der (weiblichen) Bevölkerung übertragen lassen, falls unbeobachtbare Eigenschaften, die zu stabilen Partnerschaften führen, mit den Geburts- und Arbeitsangebotsentscheidungen korreliert sind. Des Weiteren liegt das Interesse in diesem Kapitel bezüglich des weiblichen Arbeitsangebots und der Nutzung von Kinderbetreuung auf unterschiedlichen Lebensabschnitten eines Kindes, was sich in der Definition der zugrunde gelegten Zeitperioden und Variablen widerspiegelt.

Für die Beobachtungsstichprobe lassen sich für Westdeutschland folgende Fakten dokumentieren. Die Betreuungskosten in subventionierten Einrichtungen sind zwei- bis viermal günstiger als in nicht-subventionierten Einrichtungen. Jedoch ist für Kinder im Alter von null bis zwei Jahren nur eine geringfügige Anzahl von Betreuungsplätzen verfügbar, hingegen in der Altersgruppe von drei bis sechseinhalb Jahren für jedes Kind. Die Nutzung von Kinderbetreuung ist für beide Altersgruppen ungefähr so hoch wie die Anzahl der bereitgestellten, subventionierten Plätze. Allerdings wird in etwa die Hälfte der Kinder im Alter von null bis zwei Jahren, die sich in Betreuung befinden, in nicht-subventionierten Einrichtungen betreut, ausschliesslich oder zusätzlich zu subventionierter Betreuung. Des Weiteren lässt sich feststellen, dass Kinderbetreuung ebenso von nicht-erwerbstätigen Müttern genutzt wird, jedoch keine Voraussetzung für Frauen ist, um zu arbeiten. Die große Mehrheit der erwerbstätigen Frauen mit Kindern im Alter von null bis zwei Jahren nutzt nämlich keinerlei kostenpflichtige Kinderbetreuungsangebote. Die Erwerbsbeteilung von Müttern steigt mit dem Alter der Kinder stetig an, besonders stark während die Kinder im nicht-schulpflichtigen Alter sind. Im Vergleich zu Westdeutschland ist subventionierte Kinderbetreuung in Ostdeutschland preisgünstiger und in wesentlich größerem Umfang vorhanden, insbesondere in Form von Ganztagsangeboten. Dementsprechend ist auch die Nutzung von bezahlter Kinderbetreuung im Osten wesentlich höher als im Westen und nicht-subventionierte Betreuung spielt in beiden Altersgruppen eine vernachlässigbare Rolle. Die Frauenerwerbstätigenrate weist bezogen auf das Kindesalter in Ostdeutschland einen ähnlichen Verlauf auf wie in Westdeutschland, liegt jedoch konstant um 10 bis 20 Prozentpunkte höher als im Westen. Abschliessend lässt sich festhalten, dass in Ostdeutschland sowohl Vollzeit-Erwerbstätigkeit als auch Vollzeit-Kinderbetreuungsnutzung über den jeweiligen Teilzeitraten liegt, während dies im Westen genau umgekehrt ist.

Das anschliessend entwickelte Lebenszyklus-Modell mit endogenen Fertilitäts-, Arbeitsangebots- und Kinderbetreuungsentscheidungen berücksichtigt die eingeschränkte Verfügbarkeit subventionierter Betreuung sowie die Akkumulation von Arbeitsmarkterfahrung. Das Modell wird mit den Daten des westdeutschen Teils der Beobachtungsstichprobe mittels einer vereinfachten simulierten Methode der Momente geschätzt. Obgleich nicht alle Dimensionen der Daten durch das Modell erklärt werden können, lässt sich doch von einem soliden Fit des Modells sprechen. Anschliessend werden in einem Politikexperiment mit dem geschätzten Modell Daten simuliert, wobei jedoch die westdeutschen Rahmenbedingungen bezüglich Verfügbarkeit und Preise subventionierter Betreuung sowie des Einkommens mit den ostdeutschen Rahmenbedingungen ersetzt werden. Mit diesem Experiment soll geklärt werden, zu welchem Grade die Unterschiede im Erwerbsverhalten zwischen ost- und westdeutschen Müttern durch die Unterschiede der Rahmenbedingungen erklärbar sind. Das Ergebnis dieses Experiments besagt, dass sich das unterschiedliche Verhalten nicht durch die Rahmenbedingungen erklären lässt. Dies kann einerseits auf eine Fehlspezifikation des Modells hindeuten oder auf unterschiedliche Präferenzen zwischen west- und ostdeutschen Frauen, die in einem gemeinsamen Modellrahmen nicht abbildbar sind. In einem weiteren Experiment wird die Betreuungskapazität in Westdeutschland soweit ausgedehnt, dass jedem Kind ein Ganztagsbetreuungsplatz zur Verfügung steht. Dies führt zu einem starken Anstieg der Nutzung von Kinderbetreuung, was sich als Überschussnachfrage nach subventionierten Betreuungsplätzen interpretieren lässt, während das Arbeitsangebot und die Geburtenrate nur in sehr geringem Maße ansteigen.

Diese Ergebnisse bewegen sich jedoch auch im Rahmen anderer Studien, vgl. Wrohlich (2006) und Hank and Kreyenfeld (2003), was wiederum die Plausibilität des Modellrahmens unterstützt. In einem abschliessenden Experiment wird noch die Wirkung des im Januar 2007 eingeführten Elterngeldes untersucht, was eine Verringerung des Arbeitsangebots von Müttern mit Kindern im Alter von null bis zwei Jahren mit sich bringt, allerdings ohne längerfristige Auswirkungen auf die Erwerbsbeteiligung. Abschliessend werden noch einige Verbesserungsvorschläge des Modells diskutiert.

Chapter 1

Threshold Effects of Inflation on Economic Growth in Developing Countries

1.1 Introduction

A central objective of macroeconomic policies is to foster economic growth and to keep inflation on a low level. In recent years there has been substantial empirical work on the relationship between inflation and growth, yet the results have been mixed. Fisher (1993) was the first to identify a non-linear relationship where low inflation rates have a positive impact on growth which turns negative as inflation increases. Bruno and Easterly (1998) confirm the finding of a negative effect for high inflation rates but doubt the growth-enhancing effect of low inflation. In line with this result, Khan and Senhadji (2001) estimate a threshold of 11% for developing countries where inflation rates above this threshold are associated with a significant negative effect on growth, while inflation rates below 11% do not have any significant impact.

This paper sheds new light on the inflation-growth nexus introducing a natural extension of Hansen's (1999) panel threshold model by accounting for regime intercepts. The empirical results confirm the importance of including a regime intercept from a statistical and economical perspective. Once the regime intercept is included, the threshold, up to which inflation is growth enhancing, decreases substantially and, more importantly, the negative impact of inflation above the threshold becomes significant.

The paper is structured as follows. The next Section reviews the panel threshold model by Hansen (1999). Section 3 discusses the role of regime intercepts. Section 4 introduces the data and presents the estimation results for the inflation-growth nexus. Finally, Section 5 concludes.

1.2 The Panel-Threshold-Model

In this Section the panel threshold model developed by Hansen (1999, 2000) is presented. First, a single threshold model is discussed and afterwards the case of multiple thresholds. The outline of this Section follows Hansen (1999).

1.2.1 The Single Threshold Model

The model applies to balanced panels with n individuals and T time periods with a scalar dependent variable y_{it}, a scalar threshold variable q_{it} and a k-dimensional vector of exogenous regressors x_{it} where i indexes the individual and t indexes time. Thus the single threshold model takes the following form:

$$y_{it} = \alpha_i + \beta_1' x_{it} I(q_{it} \leq \gamma) + \beta_2' x_{it} I(q_{it} > \gamma) + \varepsilon_{it}, \tag{1.1}$$

where $I(\cdot)$ is an indicator function and α_i is an individual specific fixed effect. Another representation of equation (1.1) is given by

$$y_{it} = \alpha_i + \beta' x_{it}(\gamma) + \varepsilon_{it}, \tag{1.2}$$

where $x_{it}(\gamma) = \begin{pmatrix} x_{it} I(q_{it} \leq \gamma) \\ x_{it} I(q_{it} > \gamma) \end{pmatrix}$ and $\beta = (\beta_1' \;\, \beta_2')'$. The error term ε_{it} is independent and identically distributed with zero mean and finite variance σ^2. This assumption excludes the possibility of lagged dependent variables being part of the regressor vector x_{it}. Neither all variables of x_{it} need to have different slope coefficients for the two regimes, nor has q_{it} to be an element of x_{it}. The threshold divides the observations into two regimes depending on whether the threshold variable q_{it} is smaller or larger than the threshold γ. β_1 and β_2 are the regime dependent regression slopes.

Estimation of a Single Threshold

The individual effect α_i is removed by applying the classical fixed effects transformation. If the data and errors are first stacked for an individual and afterwards over all individuals (X^* and Y^*), the OLS estimator of β is obtained by

$$\hat{\beta}(\gamma) = \left(X^*(\gamma)'X^*(\gamma)\right)^{-1} X^*(\gamma)'Y^*. \tag{1.3}$$

The vector of regression residuals is $\hat{\varepsilon}^*(\gamma) = Y^* - X^*(\gamma)\hat{\beta}(\gamma)$ and the sum of squared errors can be written as

$$S_1(\gamma) = \hat{\varepsilon}^*(\gamma)'\hat{\varepsilon}^*(\gamma) = Y^{*\prime}\left(I - X^*(\gamma)'\left(X^*(\gamma)'X^*(\gamma)\right)^{-1} X^*(\gamma)'\right)Y^*. \tag{1.4}$$

As proposed by Hansen (2000) the least squares estimate of γ is obtained from minimizing (1.4), i.e.

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmin}}\; S_1(\gamma). \tag{1.5}$$

Knowing $\hat{\gamma}$, the slope coefficient estimate is easily obtained by $\hat{\beta} = \hat{\beta}(\hat{\gamma})$. The residual vector is $\hat{\varepsilon}^* = \hat{\varepsilon}^*(\hat{\gamma})$ and the residual variance is defined as

$$\hat{\sigma}^2 = \frac{1}{N(T-1)}\hat{\varepsilon}^{*\prime}\hat{\varepsilon}^* = \frac{1}{N(T-1)}S_1(\hat{\gamma}). \tag{1.6}$$

Testing for a Threshold

Once the threshold level is determined, it has to be checked whether the threshold effect is statistically significant. The null hypothesis of no threshold effect in equation (1.1) is simply:

$$H_0: \quad \beta_1 = \beta_2. \tag{1.7}$$

Under the null the threshold is not identified implying that classical tests have non-standard distributions. For fixed effects estimation Hansen (1996) proposed a bootstrap to simulate the asymptotic distribution of the likelihood ratio test. Under the null equation (1.1) boils down to

$$y_{it} = \alpha_i + \beta_1' x_{it} + \varepsilon_{it}, \tag{1.8}$$

and can be rewritten after removing the individual specific mean as

$$y_{it}^* = \beta_1' x_{it}^* + \varepsilon_{it}^*. \tag{1.9}$$

Estimating equation (1.9) with OLS yields $\tilde{\beta}_1$, the residuals $\tilde{\varepsilon}_{it}^*$ and the sum of squared errors $S_0 = \tilde{\varepsilon}_{it}^{*\prime} \tilde{\varepsilon}_{it}^*$. The likelihood ratio test of H_0 is based on the test statistic

$$F_1 = \frac{S_0 - S_1(\hat{\gamma})}{\hat{\sigma}^2}, \tag{1.10}$$

where $\hat{\sigma}^2$ is the residual variance defined in (1.6). The asymptotic distribution of F_1 is non-standard and depends in general upon moments of the sample and thus critical values cannot be tabulated. However with the bootstrap procedure proposed by Hansen (1996) asymptotically valid p-values can be constructed. The regression residuals are grouped by individual $\hat{\varepsilon}_i^* = \{\hat{\varepsilon}_{i1}^*, \hat{\varepsilon}_{i2}^*, \ldots, \hat{\varepsilon}_{iT}^*\}$ and $\{\hat{\varepsilon}_1^*, \hat{\varepsilon}_2^*, \ldots, \hat{\varepsilon}_n^*\}$ is used as the empirical distribution. In the bootstrapping procedure the regressors x_{it} and the threshold variable q_{it} are taken as given and held constant. A sample of size n is drawn with replacement from the empirical distribution to obtain a bootstrap sample under H_0 which is then used to estimate the model under H_0 and H_1 to calculate F_1. This procedure has to be repeated a large number of times and the percentage of draws for which the simulated statistic exceeds the actual yields the bootstrap estimate of the asymptotic p-value for F_1 under H_0. If the p-value is below the critical value imposed, the null of no threshold effect is rejected.

Confidence Intervals for the Threshold Estimate and Slope Coefficients

In the presence of a threshold effect, i.e. $\beta_1 \neq \beta_2$, Hansen (2000) has shown that $\hat{\gamma}$ is consistent for the true value γ_0 of γ and follows a highly non-standard asymptotic distribution. Therefore the confidence intervals for γ are obtained by using the likelihood ratio statistics which is given by

$$LR_1(\gamma) = \frac{S_1(\gamma) - S_1(\hat{\gamma})}{\hat{\sigma}^2}. \tag{1.11}$$

The null hypothesis $H_0: \ \gamma = \gamma_0$ is rejected for large values of $LR_1(\gamma_0)$. Valid asymptotic confidence intervals are formed by an asymptotic distribution for $T \to \infty$ or $N \to \infty$, see Hansen (2000). One of the technical assumptions needed to obtain this asymptotic distribution is

$$(\beta_2 - \beta_1) \to 0 \text{ as } n \to 0, \tag{1.12}$$

which means that the difference in the slops between the two regimes is 'small' relative to sample size. Practically this implies that the asymptotic approximation of the distribution of the likelihood ratio is more likely to hold if $\beta_2 - \beta_1$ is small. However, large threshold effects will be estimated quite precisely. The inverse of the distribution function $c(\alpha) = -2\ln(1 - \sqrt{1-\alpha})$ allows to calculate the critical values, e.g. the 5% critical value is 7.35 and the 1% critical value is 10.59. The null $H_0: \ \gamma = \gamma_0$ is rejected at the asymptotic level α if $LR_1(\gamma_0)$ exceeds $c(\alpha)$. Asymptotic confidence intervals for γ are formed by the 'non-rejection-region' of the confidence level $1-\alpha$. This region consists of those values of γ for which $LR_1(\gamma) \leq c(\alpha)$, i.e. where the null $H_0: \ \gamma = \gamma_0$ is not rejected.

Although the estimator $\hat{\beta} = \hat{\beta}(\hat{\gamma})$ depends on the threshold estimate $\hat{\gamma}$, Hansen (2000) shows that the inference on β can proceed as if the threshold estimate $\hat{\gamma}$ were the true value. This implies that $\hat{\beta}$ is asymptotically normal with the covariance matrix V which can be estimated by

$$\widehat{V} = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}^{*}(\hat{\gamma}) x_{it}^{*}(\hat{\gamma})' \right)^{-1} \hat{\sigma}^2.$$

1.2.2 Multiple Thresholds

Estimation of Multiple Thresholds

As Hansen (1999) shows, the model can be easily extended to multiple thresholds. Once the double threshold model is introduced, one can easily expand the analysis to further thresholds. Thus, the focus will be on the double threshold model which takes the form

$$y_{it} = \alpha_i + \beta_1' x_{it} I(q_{it} \leq \gamma_1) + \beta_2' x_{it} I(\gamma_1 < q_{it} \leq \gamma_2) + \beta_3' x_{it} I(\gamma_2 < q_{it}) + \varepsilon_{it} \tag{1.13}$$

with $\gamma_1 < \gamma_2$. Since for given (γ_1, γ_2) (1.13) is linear in slopes, OLS estimation is applicable. As in the single threshold model, the sum of squared residuals $S(\gamma_1, \gamma_2)$ is straightforward to calculate, and is minimized by the joint least squares estimates of (γ_1, γ_2). A grid search over (γ_1, γ_2) in order to obtain the minimized value of $S(\gamma_1, \gamma_2)$ is computationally quite expensive since approximately $(nT)^2$ regressions have to be run. Following the multiple change point literature (see e.g. Bai (1997) or Bai and Perron (1998)) sequential estimation is consistent. Thus in a first step, the single threshold model is estimated, resulting in the threshold estimate $\hat{\gamma}_1$ which is the minimizer of $S_1(\gamma)$. Fixing the first-stage estimate $\hat{\gamma}_1$, the second-stage criterion is

$$S_2^r(\gamma_2) = \begin{cases} S(\hat{\gamma}_1, \gamma_2) & if\ \hat{\gamma}_1 < \gamma_2 \\ S(\gamma_2, \hat{\gamma}_1) & if\ \gamma_2 < \hat{\gamma}_1 \end{cases} \tag{1.14}$$

and the second-stage threshold estimate is given by

$$\hat{\gamma}_2^r = \underset{\gamma_2}{\operatorname{argmin}}\ S_2^r(\gamma_2). \tag{1.15}$$

Since in the first-stage estimation the presence of an additional regime was omitted, Bai (1997) has shown that the estimate $\hat{\gamma}_1$ is asymptotically not efficient whereas the second-stage estimate $\hat{\gamma}_2^r$ is. Therefore Bai (1997) suggests a third-stage estimation to obtain an asymptotically efficient estimator to improve $\hat{\gamma}_1$. Fixing the second-stage criterion, the third-stage criterion is

$$S_1^r(\gamma_1) = \begin{cases} S(\gamma_1, \hat{\gamma}_2^r) & if\ \gamma_1 < \hat{\gamma}_2^r \\ S(\hat{\gamma}_2^r, \gamma_1) & if\ \hat{\gamma}_2^r < \gamma_1 \end{cases}. \tag{1.16}$$

and the asymptotically efficient third-stage estimate for γ_1 is obtained by

$$\hat{\gamma}_1^r = \underset{\gamma_1}{\operatorname{argmin}}\ S_1^r(\gamma_1). \tag{1.17}$$

Testing for the Number of Thresholds

If the null of no threshold in the single threshold model (equation (1.1)) is rejected, one moves on to estimate equation (1.13). Having done so, one needs to discriminate between one or two thresholds. Using $S_2^r(\hat{\gamma}_2^r)$ with the variance estimate $\hat{\sigma}^2 = S_2^r(\hat{\gamma}_2^r)/N(T-1)$ an approximate likelihood ratio test of one versus two thresholds can be obtained by

$$F_2 = \frac{S_1(\hat{\gamma}_1) - S_2^r(\hat{\gamma}_2^r)}{\hat{\sigma}^2}. \tag{1.18}$$

The null of one threshold is rejected in favor of two thresholds for large values of F_2. Again the sampling distribution is approximated by a bootstrap procedure where the threshold variable q_{it} and the regressors x_{it} are fixed in the repeated bootstrap samples. The bootstrap errors are drawn from the residuals obtained from the least squares regression of equation (1.13), i.e. under the alternative hypothesis. The residuals are grouped by individual $\hat{\varepsilon}_i^* = \{\hat{\varepsilon}_{i1}^*, \hat{\varepsilon}_{i2}^*, \ldots, \hat{\varepsilon}_{iT}^*\}$ and the sample $\{\hat{\varepsilon}_1^*, \hat{\varepsilon}_2^*, \ldots, \hat{\varepsilon}_n^*\}$ is used as the empirical distribution. n draws with replacement are taken from the empirical distribution. Let $\varepsilon_i^\sharp$ denote a generic $T \times 1$ draw. The dependent variable $y_{it}^\sharp$ is generated under the null hypothesis of one threshold using the equation

$$y_{it}^\sharp = \hat{\beta}_1' x_{it} I(q_{it} \leq \hat{\gamma}) + \hat{\beta}_2' x_{it} I(q_{it} > \hat{\gamma}) + \varepsilon_{it}^\sharp, \tag{1.19}$$

which depends on the least square estimates $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\gamma}$ from the single threshold model. The test statistic F_2 can be calculated from the bootstrap sample. Repeating this procedure a large number of times yields the bootstrap p-value.

Confidence Intervals for the Threshold Estimates

In analogy to the multiple change-points model (see Bai (1997)), the confidence intervals for the two threshold parameters are constructed in the same way as in the single threshold case and are given by

$$LR_2^r(\gamma) = \frac{S_2^r(\gamma) - S_2^r(\hat{\gamma}_2^r)}{\hat{\sigma}^2}$$

and

$$LR_1^r(\gamma) = \frac{S_1^r(\gamma) - S_1^r(\hat{\gamma}_1^r)}{\hat{\sigma}^2}$$

where $S_2^r(\gamma)$ and $S_1^r(\gamma)$ are defined in (1.14) and (1.16), respectively. Finally, the asymptotic $1-\alpha$ confidence regions for the threshold estimates are the set of values of γ with $LR_2^r(\gamma) \leq c(\alpha)$ and $LR_1^r(\gamma) \leq c(\alpha)$, respectively.

As already mentioned, allowing for multiple thresholds is a straightforward extension of what has been described in subsection 1.2.2. Allowing for an arbitrary number of thresholds, the number of statistically significant thresholds is determined by the sequential testing sequence. If the null of at least $K-1$ thresholds is rejected and the null of at most K thresholds cannot be rejected, the number of thresholds is obtained, namely K threshold levels and $K+1$ regimes.

1.3 Regime Intercepts

The role of regime intercepts will be discussed in the context of a single threshold model, though it is straightforward to introduce them in a model with multiple thresholds. The elimination of the individual specific effect in Equation (1.1) with the standard fixed-effects transformation implies for the identification of slope coefficients β_1 and β_2 that the elements of x_{it} are neither time-invariant nor adding up to a vector of ones. This latter case applies to regime intercepts which are usually included in each regime in threshold models in pure cross-sectional or time-series contexts. Even in the presence of fixed-effects it is possible to control for differences in the regime intercepts by including them in all but one regime as in the following extension of equation (1.1):

$$y_{it} = \mu_i + \beta_1' x_{it} I(q_{it} \leq \gamma) + \delta_1 I(q_{it} \leq \gamma) + \beta_2' x_{it} I(q_{it} > \gamma) + \varepsilon_{it}. \quad (1.20)$$

This formulation assumes that the difference in the regime intercepts, represented by δ_1, is not individual specific but the same for all cross-sections. Since equation (1.20) has neither been considered by Hansen (1999) nor any of the numerous studies, e.g. Adam and Bevan (2005), Lensink and Hermes (2004) or Nautz and Scharff (2006), applying his methodology, it seems worthwhile to briefly discuss the role of regime intercepts for the estimation results in the Hansen (1999) framework.

In case a regime intercept is included, as in specification (1.20), the slope estimates for each regime are identical to those from a regression using only observations from the respective regime which reflects the orthogonality of the regressors $I(x_i \leq x_m)$ and $x_i I(x_i > x_m)$.[1] Omission of any variable correlated with at least one regressor and the dependent variable causes biased estimates, but regime intercepts are a particularly interesting case. First, the bias can be clearly interpreted. Estimating equation (1.1) in the presence of a regime intercept in the data generating process results in a bias proportional to $\hat{\delta}_1$ because the orthogonality of the regressors is not preserved anymore. Second, availability of regime intercepts as regressors is not an issue since they are as easily constructed as the regime-dependent exogenous regressors for a given threshold.

[1]The exact algebraic expressions for the coefficient estimates of both specifications are given in the Appendix A.1.

Biased estimates of the regression slopes have further consequences in the panel threshold model because the threshold estimates are also obtained by least squares, compare Equation (1.5). Only by coincidence, these estimates will be the same for specifications (1.1) and (1.20) if a regime intercept is present in the data generating process. Moreover, unbiased estimates of β_1 and β_2 are crucial for the test of the significance of a threshold which is based on the null hypothesis of equality of the two coefficients, compare Equation (1.7).

Eventually, the setup in Hansen (1999) has to be extended to allow for regime intercepts as in equation (1.20). First, the null hypothesis to test for the significance of the threshold (Equation (1.7)) has to be extended by $\delta_1 = 0$. Second, the derivation of the asymptotic distribution of the threshold estimate now relies on the additional technical assumption that $\delta_1 \rightarrow 0$ as $N \rightarrow \infty$. It means that the difference in the intercepts between the two regimes is 'small' relative to sample size which is completely analogous to the assumption regarding the slope coefficients, compare Equation (1.12). Third, the proof in the appendix in Hansen (1999) now relies on the following two expressions taking the regime intercept as an additional regressor into account: $\theta' = ((\beta_2 - \beta_1)' \quad -\delta_1)$ and $z'_{it} = (x'_{it} \quad 1)\, C$.

1.4 The Inflation-Growth Nexus

The relationship between inflation and growth is investigated for a balanced panel of 40 developing countries through the period from 1960 to 2004. As it is standard in the empirical growth literature, the results on the determinants of long-term economic growth will be based on five-year averages. The equation of interest is given by

$$\Delta \ln gdp_{it} = \mu_i + \beta_1 \tilde{\pi}_{it} I(\tilde{\pi}_{it} \leq \gamma) + \delta_1 I(\tilde{\pi}_{it} \leq \gamma) + \beta_2 \tilde{\pi}_{it} I(\tilde{\pi}_{it} > \gamma) + \phi' w_{it} + \varepsilon_{it}, \tag{1.21}$$

representing a single threshold model that already includes a regime intercept. The dependent variable is the growth rate of GDP per capita. The inflation variable $\tilde{\pi}$ serves as the regime-dependent regressor and threshold variable and is a semi-log transformation of inflation with $\tilde{\pi}_{it} = \pi_{it} - 1$, if $\pi_{it} < 1$ and $\tilde{\pi}_{it} = \ln \pi_{it}$, if $\pi_{it} \geq 1$. Inflation rates smaller one are rescaled for the sake of continuity. Using inflation levels in growth regressions implies that the marginal effect of inflation on economic growth is independent of the average level of inflation whereas the log model has the more plausible implication that multiplicative inflation shocks will have identical effects. The control variables are selected in accordance with the empirical growth literature, see e.g. Islam (1995) or Khan and Senhadji (2001), and passed the robustness tests in Levine and Renelt (1992), and Sala-i-Martin (1997). w_{it} contains investment as a share of GDP (*igdp*), population growth (*dpop*), the log of initial income per capita of the previous period (*initial*) as well as the growth rate and standard deviation of terms of trade (*dtot*, *sdtot*).

Table 1 presents the results for both specifications, i.e. without (column 1) and with (column 2) regime intercepts. The upper panel shows that in both cases the null hypothesis of no threshold can be rejected at the 5% significance level, while the presence of one threshold cannot be rejected. Inclusion of a regime intercept decreases the threshold estimate (middle panel) from 19% to 12% and the lower bound of the 95% confidence interval from 11.8% to 5.3%. The most striking point is that in absence of a regime intercept, inflation rates below the threshold of 19% have a significant positive effect (0.407) on growth only on the 10% significance level, while the negative impact (-0.232) for inflation rates above 19% is not statistically significant at all, compare the lower panel. In contrast, allowing for differences in the regimes' intercepts doubles the magnitude (0.785, -0.531) and establishes significance at least on the 5% level of the marginal impacts of inflation on growth in both regimes. The regime intercept $\hat{\delta}_1$ itself is also significant on the 5% level. Most of the regime-dependent coefficients are consistent with the implications of standard growth theory and are very similar for both specifications. The results from the specification with a regime intercept are in line with those by Khan and Senhadji (2001), despite that, similarly to Fisher (1993), low inflation rates (less than 12%) are associated with a significant positive effect on growth.[2]

[2]The results of Khan and Senhadji (2001) are not exactly comparable to those presented here for two reasons. First, they use an unbalanced panel of more than 100 developing countries from 1960 to 1998. Second, they introduce continuity at the threshold which, though not explicitly stated, is nothing else but a nonlinear restriction on regime intercepts:

$$\Delta \ln gdp_{it} = \mu_i + \beta_1(\tilde{\pi}_{it} - \gamma)I(\tilde{\pi}_{it} \leq \gamma) + \beta_2(\tilde{\pi}_{it} - \gamma)I(\tilde{\pi}_{it} > \gamma) + \phi' w_{it} + \varepsilon_{it}.$$

Note that the setup in Hansen (1999), with and without regime intercepts, implies a

Table 1.1: Inflation-Growth Nexus in Developing Countries

	No regime intercepts	Regime intercepts
Test for the number of thresholds: p-value		
H_0 : *No threshold (K=0)*	0.013	0.025
H_0 : *At most one threshold (K=1)*	0.252	0.642
Threshold estimates and confidence intervals		
$\hat{\gamma}$	19.16%	12.03%
95% confidence interval	[11.82%, 20.48%]	[5.29%, 20.48%]
Coefficient estimates from Equation (1.21)		
Regime-dependent regressors		
$\hat{\beta}_1$	0.407*	0.785***
	(0.214)	(0.281)
$\hat{\delta}_1$		-1.985**
		(1.000)
$\hat{\beta}_2$	-0.232	-0.531**
	(0.146)	(0.245)
Regime-independent regressors		
initial	-3.353***	-3.341***
	(0.563)	(0.567)
igdp	0.031	0.021
	(0.041)	(0.042)
dpop	-0.814***	-0.646**
	(0.306)	(0.307)
dtot	0.014	0.002
	(0.028)	(0.028)
sdtot	-0.054**	-0.052**
	(0.020)	(0.020)

Notes: Standard errors are given in parentheses, */**/*** indicate the 10%/5%/1% significance level. Similarly to Hansen (1999), each regime has to contain at least 5% of all observations. 1000 bootstrap replications were used to obtain the p-values to test for the number of thresholds. By construction, the confidence intervals for the threshold estimates can be highly asymmetric.

From a policy perspective, choosing the correct specification, i.e. controlling for differences in the regime intercepts, has important implications. First, the point estimate and lower bound of the confidence interval from which onwards inflation is harmful for growth are both substantially lower. Second, the detrimental impact for inflation rates above the threshold turns significantly and doubles in magnitude. Third, keeping inflation below the threshold has a stronger beneficial effect.

1.5 Conclusion

This paper revisits the relationship between inflation and economic growth for developing countries using a generalization of Hansen's (1999) panel threshold model. Regime intercepts are introduced and the potential bias of omitting these readily available regressors for both, regression slope and threshold estimates, is discussed. The regime intercept is significant in the inflation-growth nexus and affects the results in important ways.[3]

Appendix

A.1 Coefficient Estimates

Without loss of generality assume that $T = 1$; $\mu_i = \mu \; \forall \; i = 1, \ldots, N$; x is scalar and $x_1 < x_2 < \ldots < x_m < x_{m+1} < \ldots < x_N$ and the threshold is known at $\gamma = x_m$ s.t. (1.1) and (1.20) boil down to

$$y_i = \tilde{\mu} + \tilde{\beta}_1 x_i I(x_i \leq x_m) + \tilde{\beta}_2 x_i I(x_i > x_m) + \varepsilon_i \qquad (1.22)$$

and

$$y_i = \mu + \beta_1 x_i I(x_i \leq x_m) + \delta_1 I(x_i \leq x_m) + \beta_2 x_i I(x_i > \gamma) + \varepsilon_i. \qquad (1.23)$$

discontinuity at the threshold and refers to balanced panels.

[3] Appendix A.2 presents the results for Hansen's (1999) original application without and with regime intercepts. Similar to the inflation-growth nexus, a threshold estimate is changed and one of the regime slope coefficients changes it's sign once regime intercepts are included.

The coefficient estimates for specification (1.23) are given by

$$\begin{pmatrix} \widehat{\mu} \\ \widehat{\beta}_1 \\ \widehat{\delta}_1 \\ \widehat{\beta}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{N-m}\sum_{i=m+1}^{N} y_i - \widehat{\beta}_2 \frac{1}{N-m}\sum_{i=m+1}^{N} x_i \\ \frac{\frac{1}{m}\sum_{i=1}^{m} x_i y_i - \frac{1}{m}\sum_{i=1}^{m} x_i \frac{1}{m}\sum_{i=1}^{m} y_i}{\frac{1}{m}\sum_{i=1}^{m} x_i^2 - [\frac{1}{m}\sum_{i=1}^{m} x_i]^2} \\ \frac{1}{m}\sum_{i=1}^{m} y_i - \widehat{\beta}_1 \frac{1}{m}\sum_{i=1}^{m} x_i - \widehat{\mu} \\ \frac{\frac{1}{N-m}\sum_{i=m+1}^{N} x_i y_i - \frac{1}{N-m}\sum_{i=m+1}^{N} x_i \frac{1}{N-m}\sum_{i=m+1}^{N} y_i}{\frac{1}{N-m}\sum_{i=m+1}^{N} x_i^2 - [\frac{1}{N-m}\sum_{i=m+1}^{N} x_i]^2} \end{pmatrix} \tag{1.24}$$

and can be expressed for specification (1.22) in the following way

$$\begin{pmatrix} \widehat{\widehat{\mu}} \\ \widehat{\widehat{\beta}}_1 \\ \widehat{\widehat{\beta}}_2 \end{pmatrix} = \begin{pmatrix} \widehat{\mu} \\ \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{pmatrix} + \begin{pmatrix} \frac{\sum_{i=m+1}^{N} x_i^2 [(\sum_{i=1}^{m} x_i)^2 - m\sum_{i=1}^{m} x_i^2]}{(\sum_{i=m+1}^{N} x_i)^2 \sum_{i=1}^{m} x_i^2 + \sum_{i=m+1}^{N} x_i^2 [(\sum_{i=1}^{m} x_i)^2 - N\sum_{i=1}^{m} x_i^2]} \\ \frac{\sum_{i=1}^{m} x_i [(\sum_{i=m+1}^{N} x_i)^2 - (N-m)\sum_{i=m+1}^{N} x_i^2]}{(\sum_{i=m+1}^{N} x_i)^2 \sum_{i=1}^{m} x_i^2 + \sum_{i=m+1}^{N} x_i^2 [(\sum_{i=1}^{m} x_i)^2 - N\sum_{i=1}^{m} x_i^2]} \\ \frac{-\sum_{i=m+1}^{N} x_i [(\sum_{i=1}^{m} x_i)^2 - m\sum_{i=1}^{m} x_i^2]}{(\sum_{i=m+1}^{N} x_i)^2 \sum_{i=1}^{m} x_i^2 + \sum_{i=m+1}^{N} x_i^2 [(\sum_{i=1}^{m} x_i)^2 - N\sum_{i=1}^{m} x_i^2]} \end{pmatrix} \widehat{\delta}_1 \tag{1.25}$$

where $\widehat{\beta}_1$, $\widehat{\beta}_2$ and $\widehat{\delta}_1$ are taken from (1.24). Note that in the presence of a fixed effect, $\widehat{\mu}$ and $\widehat{\widehat{\mu}}$ would correspond to the estimate of the average fixed effect.

A.2 Hansen's (1999) Application

Hansen (1999) investigates whether the presence of financing constraints implies that a firm's cash flow will be positively related to its investment rate.[4] As discussed in Section 1.3, regime intercepts (δ_1 and δ_2, fourth row) are added to the specification in Hansen (1999, equation 22):

$$\begin{aligned} I_{it} = \mu_i + \phi' w_{it} &+ \beta_1 CF_{it-1} I(D_{it-1} \leq \gamma_1) \\ &+ \beta_2 CF_{it-1} I(\gamma_1 < D_{it-1} \leq \gamma_2) \\ &+ \beta_3 CF_{it-1} I(\gamma_2 < D_{it-1}) \\ &+ \delta_1 I(D_{it-1} \leq \gamma_1) + \delta_2 I(\gamma 1 < D_{it-1} \leq \gamma_2) + \varepsilon_{it}, \end{aligned}$$

[4] The dataset is available on Bruce Hansen's webpage, as is the GAUSS code for estimation and inference.

which represents a double threshold model for illustration where $q_{it} = D_{it-1}$ and $x_{it} = CF_{it-1}$. The vector of control variables is given by

$$w'_{it} = (Q_{it-1}, Q^2_{it-1}, Q^3_{it-1}, D_{it-1}, Q_{it-1}D_{it-1}).$$

Let I_{it} be the ratio of investment to capital; Q_{it} be the ratio of total market value to assets; CF_{it-1} be the ratio of cash flow to assets; and D_{it} be the ratio of long-term debt to assets.

The estimation output is presented in Table 1.2. For both specifications, i.e. without (column 1) and with (column 2) regime intercepts, the null of two thresholds cannot be rejected (upper part of Table 1). The threshold estimates and corresponding confidence intervals are presented in the middle part. While the lower threshold estimate, when controlling for differences in the regime intercepts, is identical to the one presented in Hansen (1999), the upper threshold is by a magnitude larger. Sticking to his interpretation this implies a larger range of firms which is characterized by 'low debts'. For 'high debt' firms whose debt to asset ratio is above one, cash flow is now even negatively related to investment, see the bottom part of Table 3. A possible interpretation could be that these firms need to use cash flows primarily to serve debt before pursuing further investments. The regime intercepts themselves are highly significant and have a substantial impact on the results, underpinning the importance to include them.

Table 1.2: Hansen's (1999) Application

	No regime intercepts	Regime intercepts
Test for the number of thresholds: p-value		
H_0 : *No threshold (K=0)*	0.000	0.000
H_0 : *At most one threshold (K=1)*	0.017	0.000
H_0 : *At most two thresholds (K=2)*	0.723	0.123
Threshold estimates and confidence intervals		
$\widehat{\gamma}_1$	0.0157	0.0157
95% confidence interval	[0.0139, 0.0181]	[0.0145, 0.0181]
$\widehat{\gamma}_2$	0.5362	1.0006
95% confidence interval	[0.5305, 0.5629]	[0.9109, 1.0006]
Coefficient estimates of regime-dependent regressors from Equation (A.2)		
Regime-dependent regressors		
$\widehat{\beta_1}$	0.063*** (0.006)	0.057*** (0.007)
$\widehat{\beta_2}$	0.098*** (0.006)	0.100*** (0.006)
$\widehat{\beta_3}$	0.039*** (0.012)	-0.044** (0.022)
$\widehat{\delta}_1$	-	-0.083*** (0.011)
$\widehat{\delta}_2$	-	-0.088*** (0.010)

Notes: Standard errors are given in parentheses, */**/*** indicate the 10%/5%/1% significance level. The estimation setup is identical to the one described in Hansen (1999). For brevity only the regime dependent-regressors are displayed.

Chapter 2

Inflation Thresholds and Relative Price Variability: Evidence from U.S. Cities

(with Dieter Nautz)

2.1 Introduction

There is a growing consensus that inflation has real effects on the economy through its impact on the variability of relative prices (RPV) which in theoretical models are typically generated by menu costs or imperfect information about the price level.[1] In both types of model, inflation increases RPV and, thus, distorts the informativeness of nominal prices. These models on the real effects of inflation have been very influential for recent macroeconomics. In particular, in standard New-Keynesian DSGE models, increased relative price variability is "the root of all evil" caused by inflation, see Green (2005, p.132).

In line with these theoretical predictions, several studies have provided evidence in favor of a positive impact of inflation on RPV for various countries, see e.g. Parsley(1996), Debelle and Lamont (1997), Jaramillo (1999), Aarstol (1999), Chang and Cheng (2000), Konieczny and Skrzypacz (2005),

[1] Given inflation, price adjustments or menu costs increase RPV by making it optimal for (heterogenous) firms to change their prices infrequently even if their real prices erode, see e.g. Rotemberg (1983). In incomplete information models introduced by Lucas (1973), noisy price information leads to misperceptions of relative price changes and inefficient supply.

and Nautz and Scharff (2005). Yet, there are notable exceptions: Following Lastrapes (2006), the established relationship between U.S. inflation and RPV broke down in the mid-eighties while Reinsdorf (1994) found that the relation is even negative during the disinflationary early 1980s. In the same vein, Fielding and Mizen (2000), and Silver and Ioannidis (2001) show that RPV decreases in inflation for several European countries.

A common feature of empirical contributions on the inflation-RPV linkage is that they restrict the attention to *linear* relationships. However, the mixed evidence provided by the empirical literature suggests that the relationship between inflation and RPV is more complex. In particular, the marginal impact of inflation on RPV may differ for high and low inflation regimes. For example, Jaramillo (1999) finds that U.S. inflation's impact on RPV is stronger when it is below zero. Further evidence in favor of threshold effects of inflation on RPV is provided by Caglayan and Filiztekin (2003) for Turkey and Caraballo, Dabús and Usabiaga (2006) for Spain and Argentina. In all these contributions, however, both the number and the location of inflation thresholds are not estimated but imposed exogenously.

This paper sheds more light on the empirical relevance of inflation thresholds for RPV by employing a modified version of the panel threshold model introduced by Hansen (1999) to recent price data from U.S. cities. This enables us to estimate the number of inflation thresholds, the threshold levels, as well as the marginal impact of inflation on RPV in the various regimes. Although our sample focuses on the recent low inflation period, the panel data provides us with a sufficient variation of inflation rates in a range which should be of particular interest for assessing the current low inflation environment.

Threshold models nest the linear case, such that they can be viewed as a first, natural step to generalize the standard inflation-RPV equations. Of course, one may think of alternative non-linear specifications, see e.g. Fielding and Mizen (2008) who investigate the inflation-RPV linkage with non-parametric methods. Our particular interest in the empirical relevance of inflation thresholds is stirred by the recent discussion about the acceptable range of inflation. Although the Federal Reserve has never officially stated a target range of inflation, most analysts believe that the Fed has *implicit* upper and lower limits of its inflation objective, compare e.g. Thornton (2006). Threshold effects of inflation are not required for explaining the increasing role of inflation targets.[2] However, in view of the important role of the inflation-RPV linkage for the inflation transmission mechanism, the identification of inflation thresholds could provide useful information about the appropriate location and width of an inflation targeting band.

The remainder of the paper is structured as follows. Section 2 introduces the data and presents results from a linear panel regression. Section 3 applies the threshold model to the inflation-RPV linkage revealing regime dependent effects of inflation. Section 4 summarizes our main results and offers some conclusions.

2.2 Inflation and RPV in U.S. Cities

2.2.1 The Data Set

Our empirical analysis uses price data of the eight major CPI subcategories published by the Bureau of Labor Statistics (BLS) for a panel of 14 U.S. cities. Due to data availability the sample starts in January 1998 and ends in August 2005.[3]

The frequency and the timing of the CPI publication differs across cities: for eleven cities data is released every second month. Only for three cities (Chicago, Los Angeles, and New York), price data is available on a monthly basis. Since the estimation of Hansen's (1999) panel threshold model requires a balanced panel, we took only the data of every second month for these three cities. Specifically, we selected the observations of the odd months because this choice implied that the number of observations in our sample from odd and even months is exactly the same.[4] After these data

[2]Inflation targets may help anchoring inflation expectations and increase the transparency and accountability of the central bank. Mishkin and Westelius (2006) show that the announcement of an inflation targeting band can be interpreted as an inflation contract ameliorating the inflation bias of discretionary policy. Explicit inflation targeting bands or critical values of inflation are used by many central banks, including the Bank of England and the European Central Bank, to facilitate the communication of monetary policy.

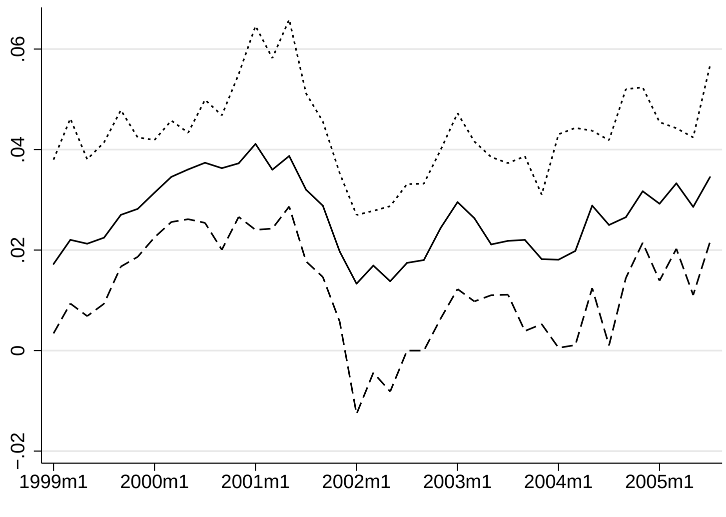
[3]We use the CPI-U index representing the expenditures by all urban consumers which can be downloaded from http://www.bls.gov/cpi/home.htm. The eight subcategories are food and beverages, housing, apparel, transportation, medical care, recreation, education and communication, and other goods and services. We selected January 1998 as starting point because before data for Atlanta, Seattle and Washington data were only published twice a year and in addition two new major groups were introduced in the CPI-U.

[4]Note that the lack of synchronicity in the data is not a problem because both, the traditional linear equation and the threshold model will contain no lagged variables. Data

adjustments we are left with $40 \times 14 = 560$ observations of yearly inflation rates, a sufficient sample size for applying panel threshold models.

U.S. inflation has been low and stable over the last years. Since 1999 the average inflation rate across U.S. cities has fluctuated around 2.7%. Figure 2.1 further displays the minimum and the maximum of the city-specific inflation rates indicating that inflation in U.S. cities exceeded 6% and went even below zero at least for some cities in some periods. This illustrates that inflation differentials between U.S. cities have been modest but far from negligible. Typically, inflation rates varied in a range of 3 to 4 percentage points.[5] Figure 2.2 reveals more information about the distribution of city inflation rates from a time-less perspective. Note that our sample provides us with a sufficient variation of inflation rates. In particular, 25% of the observed inflation rates were below 1.88% or above 3.50%, respectively.

Figure 2.1: Inflation Rates Across U.S. Cities

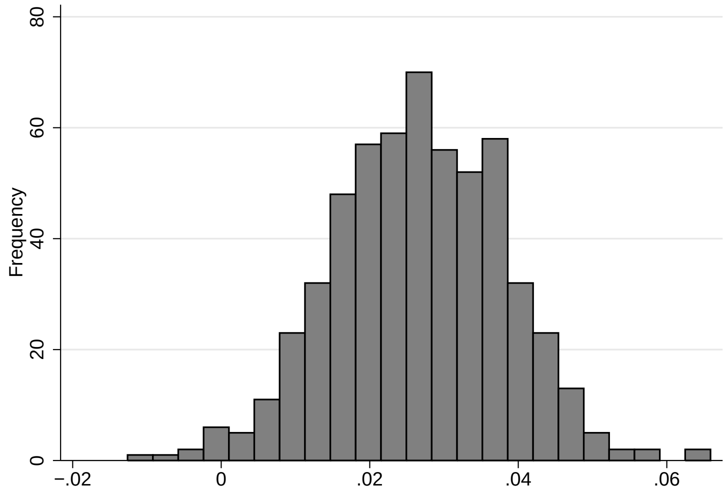


Notes: Minimum, mean and maximum of yearly CPI-U inflation rates of 14 U.S. cities from 1999.01 to 2005.08. Source: BLS.

for Atlanta, Detroit, Houston, Miami, Philadelphia, San Francisco and Seattle is released only in even months; for Boston, Cleveland, Dallas and Washington only in odd months.

[5]The persistence of inflation differentials between U.S. cities has been relatively low. Therefore, inflation differentials between cities did not lead to significant price level divergence.

Figure 2.2: Distribution of City Inflation Rates in the U.S.



Notes: Yearly CPI-U inflation rates of 14 U.S. cities from 1999.01 to 2005.08. Source: BLS.

2.2.2 Relative Price Variability

Following the empirical literature, we define relative price variability (RPV_{it}) for city $i = 1, \ldots, 14$ in period $t = 1, \ldots, 40$ as

$$RPV_{it} = \sqrt{\sum_{j=1}^{8} w_j(\pi_{ijt} - \pi_{it})^2} \tag{2.1}$$

where $\pi_{ijt} = \ln P_{ijt} - \ln P_{ijt-6}$ is the yearly inflation rate for subcategory $j = 1, \ldots, 8$ and P_{ijt} is the level of the corresponding price index. $\pi_{it} = \sum_{j=1}^{8} w_j \pi_{ijt}$ denotes the inflation rate for city i and w_j refers to the weight of the j-th subcategory in the aggregate index such that $\sum_{j=1}^{8} w_j = 1$.[6] Silver and Ioannidis (2001) introduce the coefficient of variation as an alternative measure of relative price variability. However, this RPV measure is not applicable in our sample because it includes inflation rates below zero.

[6]In the following, the RPV measure will take into account that subcategory weights are adjusted on a yearly basis. The subcategory weights can also be downloaded from http://www.bls.gov/cpi/home.htm. They are only available as averages over all cities covered in the CPI.

2.2.3 The Linear Relation Between Inflation and RPV

Following the empirical literature on inflation and RPV, we begin our analysis with a linear panel regression of RPV on aggregate inflation with city-specific fixed effects α_i:

$$RPV_{it} = \alpha_i + \beta\pi_{it} + \varepsilon_{it}. \tag{2.2}$$

In line with Lastrapes (2006), the coefficient of inflation in the linear RPV equation (2.2) is clearly insignificant for recent U.S. data.[7] In a widely used alternative specification, RPV is regressed on $|\pi_{it}|$, the absolute value of inflation, see e.g. Parks (1978) and Jaramillo (1999). In a low inflation environment, where some inflation rates are actually below zero (compare Figures 1 and 2), this could make a difference.

Table 2.1: The Linear Relation Between Inflation and RPV

	$\hat{\beta}$	R^2
$RPV_{it} = \alpha_i + \beta\pi_{it} + \varepsilon_{it}$ (2.2)	-0.025 (0.03)	0.00
$RPV_{it} = \alpha_i + \beta\lvert\pi_{it}\rvert + \varepsilon_{it}$ (2a)	-0.016 (0.02)	0.00

Notes: Standard errors are given in parentheses. Inflation and relative price variability (RPV) for 14 U.S. cities; Sample period 1999.1 - 2005.08; data source: BLS.

For our data, however, the results presented in Table 2.1 (equation 2a) reveal that this plausible non-linearity in inflation's impact on RPV is not supported by the data.

[7] As a consequence, Lastrapes (2006) suggests to include all individual prices in a linear VAR to estimate the cross-sectional distribution of impulse responses of these prices to e.g. monetary shocks.

2.3 Inflation Thresholds and the Inflation-RPV Linkage

2.3.1 The Threshold Model

In this section, we investigate whether the linear model (2) is misspecified because the marginal impact of inflation on RPV depends on the inflation level. In contrast to recent work by e.g. Caglayan and Filiztekin (2003) for Turkish provinces, and Caraballo, Dabús and Usabiaga (2006) for Spain and Argentina, we do not impose the number and the locations of the different inflation regimes a priori. Rather, we employ a modified version of Hansen's (1999) panel threshold model that enables us to test for the number of thresholds and to estimate the threshold values, i.e. the critical inflation levels where the impact of inflation on RPV changes.

Specifically, we consider the following threshold model for the inflation-RPV linkage:

$$\begin{aligned} RPV_{it} = \alpha_i + \sum_{k=0}^{K-1} (\delta_{k+1} + \beta_{k+1}\pi_{it}) I(\gamma_k < \pi_{it} \leq \gamma_{k+1}) \\ + \beta_{K+1}\pi_{it} I(\gamma_K < \pi_{it} \leq \gamma_{K+1}) + \varepsilon_{it}, \end{aligned} \tag{2.3}$$

where $\gamma_0 = -\infty$, $\gamma_{K+1} = \infty$ and I is the indicator function. Equation (2.3) allows for K inflation thresholds and, thus, $K+1$ regimes. In each regime, the marginal impact of inflation (β_k) on RPV may differ. Given the observed inflation differentials across cities, it is an additional feature of the panel threshold model that different cities are allowed to be in different inflation regimes.

It is worth noting that the panel threshold model (2.3) generalizes the original setup in Hansen (1999) by allowing for regime dependent intercepts (δ_k). According to Bick (2009), ignoring intercepts can lead to biased estimates of both, the thresholds and the corresponding marginal impacts.

2.3.2 The Number of Inflation Thresholds

In a first step, we applied Hansen's (1999) sequential testing procedure for determining the number of inflation thresholds. Following Hansen (1999), we require that each regime contains a minimum number of observations.

Column 2 of Table 2.2 shows the results obtained for the 5% rule predominantly applied in empirical applications of the threshold model. For our data set, the 5% rule implies that inflation thresholds may range from 0.81% to 4.44%. The results indicate a clear rejection of a linear relation ($K = 0$) between RPV and inflation in favor of a double threshold model. Specifically, the null hypothesis of a single inflation threshold ($K = 1$) in the inflation-RPV equation can be rejected at the 1% significance level, while the hypothesis of a double threshold ($K = 2$) cannot be rejected at the 10% significance level.

Table 2.2: Test Procedure Establishing the Number of Thresholds

$$RPV_{it} = \alpha_i + \sum_{k=0}^{K-1} (\delta_{k+1} + \beta_{k+1}\pi_{it}) I(\gamma_k < \pi_{it} \leq \gamma_{k+1}) + \beta_{K+1}\pi_{it} I(\gamma_K < \pi_{it} \leq \gamma_{K+1}) + \varepsilon_{it}$$

	5 % Rule	10 % Rule
No threshold (H_0: K=0)		
F_1	37.75	36.48
p-value	0.00	0.00
(10%, 5%, 1% critical values)	(11.48, 13.13, 16.17)	(11.16, 12.78, 16.30)
One threshold (H_0: K=1)		
F_2	18.61	18.31
p-value	0.00	0.01
(10%, 5%, 1% critical values)	(11.24, 12.53, 16.46)	(10.61, 12.48, 16.72)
Two thresholds (H_0: K=2)		
F_3	10.64	11.68
p-value	0.15	0.07
(10%, 5%, 1% critical values)	(11.48, 12.99, 15.94)	(10.82, 12.16, 14.78)

Notes: $\gamma_0 = -\infty$, $\gamma_{K+1} = \infty$. The sequential test procedure indicates that the number of thresholds is $K = 2$. 1000 bootstrap replications were used to obtain the p-values. Following Hansen (1999), each regime is required to contain at least 5% or 10% of all observations, respectively.

This conclusion appears very robust with respect to different assumptions concerning the minimum number of observations in each regime. In particular, we found that the 5% constraint is not binding, implying that adopting the less restrictive 1% rule (where feasible inflation thresholds range from 0.00% to 5.24%) leads to identical results. This already indicates that the evidence in favor of a regime-dependent influence of inflation on RPV is not driven by a few outliers. We also performed the test adopting the unusually restrictive 10% rule. In this case, the range of feasible inflation thresholds shrinks to $[1.22\%, 4.12\%]$ which leads to slightly different values of the test statistics, see Column 3 of Table 2.2. Yet, the main result of the test remains unaffected. In particular, regardless of the minimum number of observations contained in each regime, Table 2.2 strongly suggests that the inflation-RPV linkage is characterized by two inflation thresholds and, thus, three different inflation regimes.

2.3.3 A Double Threshold Model for the Relation Between Inflation and RPV

In view of the evidence in favor of two inflation thresholds, we estimated the following double threshold model:

$$\begin{aligned} RPV_{it} = \alpha_i &+ (\delta_1 + \beta_1 \pi_{it})\, I(\pi_{it} \leq \gamma_1) \\ &+ (\delta_2 + \beta_2 \pi_{it})\, I(\gamma_1 < \pi_{it} \leq \gamma_2) + \beta_3 \pi_{it} I(\gamma_2 < \pi_{it}) + \varepsilon_{it} \end{aligned} \tag{2.4}$$

Table 2.3 reports both the estimates obtained under the 5% and the 10% rule. Results for the 1% rule are not presented since they are identical to those received for the 5% rule.

The upper part of the table shows the results for the two inflation thresholds. The results for the lower inflation threshold are virtually unaffected by the applied rule. Both, the point estimate for the threshold (1.672%) and the corresponding 95% confidence intervals are very similar. Note that the confidence interval does not contain 2.00%, probably the most popular number for inflation targets. The second inflation threshold estimated for the 5% rule (4.274%) exceeds the upper limit for feasible thresholds under the 10% rule (4.12%). As a consequence, the point estimate for the second threshold decreases under the 10% rule. Yet, the main conclusions about the threshold's location are very robust: according to the 95% confidence intervals the upper threshold is clearly above 2.8% and certainly below 4.4%. Note that observations of all three regimes do not belong exclusively to a small subset of cities, see Table 2.4. Therefore, the established non-linearity of the inflation-RPV linkage does indeed result from a *regime*-dependent marginal impact of inflation and cannot be captured by city-specific inflation coefficients.

The estimates ($\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$) for the marginal impact of inflation in the three inflation regimes are shown in the lower part of Table 2.3. In contrast to the results obtained for the linear specification, the threshold model

Table 2.3: A Double Threshold Model for the Inflation-RPV Linkage

$$RPV_{it} = \alpha_i + (\delta_1 + \beta_1 \pi_{it})\, I(\pi_{it} \leq \gamma_1) + (\delta_2 + \beta_2 \pi_{it})\, I(\gamma_1 < \pi_{it} \leq \gamma_2) + \beta_3 \pi_{it} I(\gamma_2 < \pi_{it}) + \varepsilon_{it}$$

	5 % Rule	10 % Rule
Threshold estimates		
$\widehat{\gamma}_1$	1.672	1.672
95% confidence interval	[1.586, 1.803]	[1.586, 1.820]
$\widehat{\gamma}_2$	4.274	3.648
95% confidence interval	[2.852, 4.385]	[2.824, 4.102]
Regime dependent inflation coefficients:		
$\widehat{\beta}_1$	-0.595^{**} (0.13)	-0.593^{**} (0.13)
$\widehat{\beta}_2$	-0.198^{**} (0.05)	-0.189^{**} (0.07)
$\widehat{\beta}_3$	0.548^{*} (0.23)	0.712^{**} (0.14)
Regime dependent intercepts:		
$\widehat{\delta}_1$	0.028^{**} (0.01)	0.036^{**} (0.01)
$\widehat{\delta}_2$	0.026^{**} (0.02)	0.035^{**} (0.01)
R^2	0.095	0.091
Observations in regime 1	105	105
Observations in regime 2	415	344
Observations in regime 3	40	111

Notes: $^{**},^{*}$ indicate significance at the 1%, 5% level, standard errors in parentheses. Each regime consists of at least 5% and 10% of all observations, respectively.

reveals that inflation has a significant impact on RPV. However, both magnitude and sign of the inflation coefficient depend on the level of inflation. In the low inflation regime, i.e. when inflation is below 1.672%, the marginal impact of inflation on RPV is significantly negative (-0.59). Thus, a further decline of inflation would increase RPV significantly. According to e.g. Akerlof, Dickens and Perry (1996) or Jaramillo (1999), this effect of inflation rates close to zero may point to the presence of nominal downward wage and price rigidities. In fact, in the intermediate inflation regime, when inflation is low but well above zero, the impact of inflation on RPV is significantly weaker. In the high inflation regime, the marginal impact of inflation is positive under both the 5% ($\widehat{\beta}_3 = 0.548$) and the 10% ($\widehat{\beta}_3 = 0.712$) specification. When inflation exceeds an upper threshold, it seems that RPV-increasing

Table 2.4: U.S. Cities and Inflation Regimes

	Low Regime	Medium Regime	High Regime
Atlanta	14 (14)	25 (20)	1 (6)
Boston	2 (2)	28 (21)	10 (17)
Chicago	11 (11)	28 (27)	1 (2)
Cleveland	14 (14)	26 (23)	0 (3)
Dallas	9 (9)	27 (23)	4 (8)
Detroit	7 (7)	32 (29)	1 (4)
Houston	10 (10)	27 (19)	3 (11)
Los Angeles	0 (0)	39 (28)	1 (12)
Miami	8 (8)	30 (25)	2 (7)
New York	1 (1)	38 (34)	1 (5)
Philadelphia	5 (5)	30 (26)	5 (9)
San Francisco	14 (14)	16 (9)	10 (17)
Seattle	9 (9)	30 (25)	1 (6)
Washington	1 (1)	39 (35)	0 (4)
Total	105 (105)	415 (344)	40 (111)

Notes: The Table shows how often a city appears in the various inflation regimes estimated for the inflation-RPV linkage under the 5% rule, compare Table 2.3. The respective numbers under the 10% rule are given in parantheses.

aspects of inflation (including e.g. menu costs and imperfect information about the price level) become eventually dominant while RPV-decreasing aspects of inflation have faded out.

2.4 Concluding Remarks

The impact of inflation on relative price variability (RPV) is a major channel for real effects of inflation. This paper focused on the recent low-inflation period using price data from a panel of U.S. cities from 1999 through 2005. For this sample, we found that the common linear inflation-RPV equation has to be rejected in favor of a double threshold model with surprisingly small 95% confidence intervals for both inflation thresholds. Partly reconciling the mixed evidence provided by the empirical literature, the estimated inflation coefficients reveal that there are both, positive and negative effects of inflation on RPV. Inflation increases RPV only if it exceeds a critical value which is estimated to range from about 2.8% to 4.4%. By contrast inflation decreases RPV for inflation rates close to zero, or more precisely below 1.67%. The weakest impact of inflation on RPV is found for the intermediate regime, when inflation is still low but well above zero.

Even central banks with a strong commitment to price stability are not really interested in zero inflation rates.[8] Typically, central banks prefer a more sophisticated notion of price stability. According to e.g. Blinder, Canetti, Lebow and Rudd (1998, p.98), "one prominent definition of 'price stability' is inflation so low that it ceases to be a factor in influencing people's decisions." Therefore, given the crucial importance of relative prices for economic decisions, an acceptable band of inflation rates should ensure the smallest impact of inflation on the variability of relative prices.

The recent literature on the importance of price rigidities revealed that there are notable differences in the frequency of price adjustments and implied durations between sectors. Golosov and Lucas (2007), and Klenow and Kryvtsov (2007) demonstrated that idiosyncratic shocks are an important factor for the price setting of firms. In accordance with Woodford (2003), the efficient level of RPV in an economy with multiple sectors is typically not zero, since it reflects relative price changes driven by fundamentals. Therefore, the optimal rate of inflation need not drive the level of RPV to zero. It is the marginal effect of inflation on RPV that has to be minimized. From this perspective, our empirical results may shed light on the location and width of an appropriate inflation targeting band. In particular, the inflation thresholds in the inflation-RPV nexus suggest that U.S. inflation should range between 1.8% and 2.8%.

The repercussions of the introduction of an explicit targeting band on inflation's impact on relative prices are not obvious. The analysis of highly disaggregated price data indicates that the relation between inflation and the price setting of firms has been underresearched, see e.g. Golosov and Lucas (2007). In particular, the Calvo and Taylor sticky-price models predominantly used in current New-Keynesian DSGE models generate only poor predictions for the persistence and volatility of inflation, see Bils and Klenow (2004). Recent evidence on the price setting of firms seem to support the relevance of inflation thresholds. According to Nakamura and Steinsson (2007), aggregate inflation plays no role for the frequency and the size

[8]Several arguments point to the difficulties implied by inflation rates too close to zero. For example, positive inflation rates may ameliorate problems caused by the zero-bound for nominal interest rates, see e.g. Adam and Billi (2006).

of price changes during the low-inflation period 1998-2005. However, from 1988 to 1997, when average inflation exceeded 2.8%, the impact of aggregate inflation is significant and plausibly signed.

Generalizing the traditional linear inflation-RPV regressions, the current paper employed Hansen's (1999) panel threshold model to allow for a more complex relation between inflation and RPV. In the current model, the marginal impact of inflation jumps to a new value whenever inflation exceeds a threshold. Of course, allowing for a more gradual change of the inflation coefficient might lead to a more realistic view on the inflation-RPV linkage. Therefore, following Strickholm and Teräsvirta (2006), incorporating elements of smooth transition into a threshold model could be a natural extension and is left for future research.

Chapter 3

Fertility, Female Labor Supply and Child Care

3.1 Introduction

At the Barcelona summit in 2002 the European Union member states agreed on increasing the supply of subsidized child care slots such that for at least 90% of all children from age three until school entry age and for one third of the children younger than three subsidized child care is provided. The aim of this policy is to encourage labor force participation of mothers of young children and to foster fertility. Since then, child care policies have gained increased attention by the public and politicians in Germany which is a particular interesting case. While in Germany the provision rate of subsidized child care for children from age three onwards is far above 90%, in the former Western territorial only for three out of hundred children younger than three subsidized child care is available – which is among the lowest provision rates in Europe but still representative for a lot of continental and southern European countries – whereas for more than one third in the former Eastern territorial.[1] In line with the 2002 Barcelona announcements, the federal government of Germany has passed a law that by 2013 this West-East gap has to be closed. Since an increase in subsidized child care implies higher public expenditures, it is of an inherent interest whether the intended goals of higher maternal labor force participation and fertility rates can be

[1]When talking about West and East Germany, or the West and the East, I always refer to the territory that comprised the Federal Republic of Germany (FRG) and German Democratic Republic (GDR) prior to reunification.

indeed achieved by this policy.

I approach this question by setting up a dynamic and structural life cycle model with endogenous fertility, female labor supply, subsidized and non-subsidized child care enrollment choices which will be estimated with data from the German Socioeconomic Panel (GSOEP) for West Germany. Due to the differences in the provision rates between West and East Germany but the otherwise mostly identical institutional setup, Germany can serve as an ideal object for studying this question.

Before estimating the model, I document West-East differences in the provision of subsidized child care, enrollment in child care and maternal labor force participation. While the estimated model provides a solid fit for West Germany, policy experiments show that the differences in the provision rates of subsidized child care between the West and East – even after controlling for the differences in the income level and child care prices – are not sufficient to explain the differences in maternal labor force participation and child care enrollment. On the one hand, this might be partly attributable to shortcomings of the current version of the model. On the other hand, the results seem to be plausible since they are not at odds with those from other studies. The usefulness of the model for conducting further policy experiments of interest is illustrated by evaluating the 2007 reform of the maternal leave benefit. Overall, despite some shortcomings the model seems to be promising for providing quantitative answers to urgent policy questions concerning fertility and female labor supply.

This paper contributes to the specific literature on female labor supply decisions and the general literature on structural modeling of married females's behavior, see Blundell and MaCurdy (1999), by explicitly allowing for child care choices. It is most strongly related to Wrohlich (2006), Francesconi (2002) and Greenwood et al. (2003). Wrohlich (2006) estimates a structural model on female labor supply and child care enrollment for Germany using data from the GSOEP, but not within a life-cycle context and without a fertility choice. In contrast, Francesconi (2002) studies life-cycle fertility with endogenous wages through the accumulation of experience. Child care as a substitute for maternal time is however not modeled within his framework. He focuses on females living in a stable relationship and having the same partner throughout the observation period, which is inline with many economic theories of household production, female labor supply and fertility. This feature as well as the set of choice variables and their discrete nature from these two papers belonging to the empirical microeconomics literature is included in my approach. Other elements are closer to the macro literature. In particular, I abstract from any preference heterogeneity and preferences shocks, and impose strong structural form assumptions guided by the setup in Greenwood et al. (2003). Compared to their paper, the structure of the life cycle is richer in my model, though still much more stylized than in Francesconi (2002), but I neither use an equilibrium concept nor model marriage and divorce.

The structure of the paper is as follows: In Section 3.2, I describe the data set, and how the sample is selected and constructed. Section 3.3 documents facts about the provision and prices of subsidized child care, female labor force participation and child care enrollment choices in West and East Germany. I introduce the model in Section 3.4 and the set of externally determined parameters in Section 3.5. Section 3.6 discusses the estimation strategy and results. In Sections 3.7 and 3.8 I conduct the policy experiments and lay out the dimensions along which the model setup could be improved. Finally, Section 3.9 concludes.

3.2 Data

In this Section I introduce the data set and briefly discuss the selection and construction of the sample for my analysis of female labor force participation, child care and fertility. Details can be found in the appendix.

3.2.1 German Socio-Economic Panel (GSOEP)

The GSOEP is an annual household panel, comparable in scope to the American PSID. It is a representative longitudinal study of private households with the first survey having been conducted in 1984 for West Germany and in 1990 for East Germany. New samples were added in 1994, 1998, 2000, 2002 and 2006.[2] The variables I construct from the GSOEP include female cohabitation, participation and birth histories, child care enrollment choices and income profiles. The data I use are drawn from the 1984 to 2007 waves

[2]Detailed information about the GSOEP are provided on the corresponding webpage `http://www.diw.de/english/soep/26636.html`.

and span the years 1983 to 2006 since the variables on labor force participation and income refer to the year prior to the interview.

3.2.2 Sample Selection

As in Francesconi (2002), only females living in a continuous relationship with the same partner are included in the sample since many economic theories of household production, female labor supply, and fertility are meant to describe behavior of this group only.[3] This might introduce a selection bias, if the unobservables affecting marriage stability are also correlated with the fertility and participation decisions.[4] In this paper, I will use marriage interchangeably with cohabitation because the interest is less on the legal status but rather on living in a relationship in one household. Females with multiple relationships contribute only with their most recent one which has to be still intact at the last interview.[5] Among mothers, only those are included in the sample that have all children within the current marital spell, while for childless females the requirement is that they were already in that spell prior to age forty and thus had (at least theoretically) the possibility to give birth to a child.

Since the objective of the paper is to compare West and East Germany, the sample is split up accordingly. Females are assigned to West or East Germany by their location in 1989 or, if this information is not available, by the sample region at their first interview. Females who moved between the two parts of the country during the observation period are dropped to ensure that all females in the sample faced the same environment in terms of child care provision throughout the whole sample period. Finally, the analysis is restricted to females born between 1955 and 1975. The time span of female birth years was chosen that large to have a sufficient final sample size.[6]

[3]For a survey on fertility theories for married females see e.g. Jones et al. (2008). Recent contributions also model fertility, marriage and divorce jointly, e.g. Regalia and Rios-Rull (2001) or Greenwood et al. (2003).

[4]The direction of the sample selection bias with respect to labor force participation and fertility can go in either direction. For a detailed discussion, see Francesconi (2002) pp 347f.

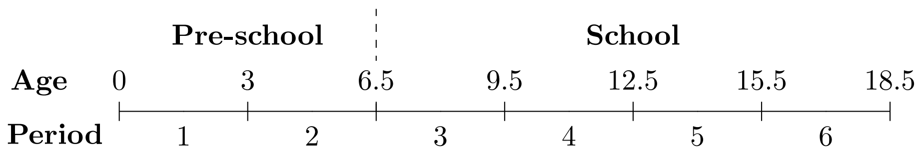
[5]Of course, this does not preclude that relationships might break up after the last interview.

[6]The exact selection criteria and the fraction of females affected by each criterion are shown in Appendix A.1.

3.2.3 Sample Construction

Female labor force participation profiles along children's age constitute the core of my analysis. Similarly to Apps and Rees (2005), my focus centers less on the labor force participation status in each month of a child's life but rather on the stages during a child's adolescence, see Figure 3.1.

Figure 3.1: A Child's Life from Birth to Adulthood



The first two periods comprise the pre-schooling years with age six and a half as the mean age at school entry. The third period refers to grades one to three in elementary school (Grundschule), while the fourth period comprises fourth grade of elementary school plus grades five and six of secondary school (Förderstufe or Unterstufe). After the fifth period, teenagers can already graduate (Hauptschulabschluss) and start an apprenticeship or continue to attend school until they reach adulthood at the end of period six. All periods have a length of three years with the exception of period two which has a mean of three years.[7]

For each period the female labor supply and child care enrollment status is constructed similar to Francesconi (2002): I assign 0 to each month in which the female does not work, one half to each month in which she works part-time and one to each month in which she works full-time.[8] Next, the mean over all months in a period is taken from which the period labor supply status is defined. Values below 0.25 correspond to not working, values between 0.25 and 0.75 to part-time working, and values above 0.75 to full-time working. This assignment implies that a female working part-time in each month of the period and one not working in the first half of a period but full-time in the second half have the same period labor supply status, namely part-time working. As already mentioned before, this

[7]Some more details on the first two periods are given in Appendix A.2.1.

[8]The monthly labor force participation status is based on the retrospective information for the previous year at each interview months. Further details are provided in Appendix A.2.2.

reflects how much a female has worked throughout certain stages during her children's adolescence. The same procedure is applied to define the child care enrollment status.[9] The period income is defined as the sum of all labor income, including side jobs and self-employment, pensions, unemployment benefits, compensation for further training or education, and any additional payments as 13$^{\text{th}}$ and 14$^{\text{th}}$ salary, vacation and Christmas pay or any further boni received during the period.

Having defined the periods and variables, one further issue has to be addressed. If a female has more than one child, the life periods of siblings only overlap perfectly for twins or triplets. Females observed the last time prior to the end of their fertile period might get another child. Therefore, the analysis of females with one child (two/three children) will be based on the decisions of females after their first (second/third) birth until they are not observed anymore or get a second (third/fourth) child. Hence, a female who gives birth to two children during the observation period contributes to facts about females with one child until the second child is born and to facts about females with two children afterwards. Moreover, only periods that are observed over their full length, i.e. neither interrupted by another birth nor left or right censored through the first or last interview, are included to avoid biased averages if there are trends in labor participation or child care enrollment within a period.

Recall that childless females are only included in the sample if they are observed to reach at least age forty. I therefore assign the first three years of childless females after turning forty to the first period, the next three and half years to the second period and so forth.

3.2.4 Sample Size

Table 3.1 shows the number of observations for each period for West and East Germany grouped by the number of children, e.g. for West Germany 389 females with one child that is younger than three and 181 females with one child of age three to six and half are observed. Since there are not sufficient females with zero or four and more children, the analysis on labor force participation and child care enrollment will focus on females with one

[9] Information on child care enrollment is only available for the interview date. In Appendix A.2.3 I describe how I impute the child care enrollment status for the remaining months of a year.

Table 3.1: Distribution of Observations per Region and Number of Children by Age

	West					East				
Ages	**0**	**1**	**2**	**3**	**4+**	**0**	**1**	**2**	**3**	**4+**
< **3**	67	389	450	125	39	8	92	100	23	2
< **6.5**	36	181	326	98	27	4	66	112	22	2
< **9.5**	14	128	270	84	30	3	62	136	28	1
< **12.5**	0	108	208	58	15	0	69	150	25	1
< **15.5**	0	85	126	38	7	0	67	145	20	0
< **18.5**	0	63	105	22	7	0	69	110	16	0

Note: For childless females< **3** corresponds to female ages 40 to 42, < **6.5** to 43 and 46.5 and so forth. Since the first birth cohort of females included in the sample was born in 1945 and the last observations are from 2006, childless females could only be observed for three periods.

to three children.

The sample used to calculate the fertility rate and distribution comprises all selected females who are at least of age 40 at their last interview, even if they only have incomplete periods and thus do not contribute to the set of stylized facts about labor supply and child care enrollment. Since the timing of birth will not be part of my investigation, females who have not yet completed their fertile period, assumed to end at the age of forty, are not included. Similarly, fertility outcomes of East German females are not investigated because the majority of them has made a part of their fertility decisions prior to reunification and thus in a completely different economic and institutional setup. Eventually, there are 1112 West German females left over for the fertility analysis.

3.3 Stylized Facts

I first present facts on the child care market in West and East Germany which serve as exogenous inputs in my model. Afterwards, I present facts on female labor force participation and child care enrollment for West and East Germany from which a subset will be used as moments the model has to match.

3.3.1 Child Care Market

The market for child care in Germany is divided into a subsidized and non-subsidized sector with the former mainly made up by daycare centers and the latter by nannies which either provide care at their or the families' homes.

Table 3.2: Child Care Fees

		Subsidized West	Subsidized East	Non-Subsidized
Baseline price				
Part-time, Ages 3 to 6.5		2946.77	2737.42	8525.53
Markups				
Full-time	(+)	1718.63	789.06	6000.42
Ages 0 to 2	(+)	984.52	613.55	—
Per Sibling in Care	(−)	1431.01	701.14	—

Child Care Fees Table 3.2 shows the parental fees for subsidized and non-subsidized child care.[10] The per child, parental fee for a subsidized part-time slot for children aged three to six and a half in West Germany amounts to 2946.77 € for the whole period. If the slot is full-time, the fee increases by 1718.63 €. Slots for children aged zero to two cost 984.52 € more than for a child of age three to six and a half. Finally, a discount of 1413.01 € is granted per sibling in care.[11] With the exception of the discount, child care fees are substantially lower in the East than in the West, in particular for full-time child care. Non-subsidized child care is estimated to be two to four times as expensive as subsidized child care which seems plausible as around 75% of the costs per slot are covered by the subsidy, see Kolvenbach

[10] The details on how the fees were estimated are given in Appendix A.3.1. All prices are in real 2007 terms and have been computed based on price level changes for unified Germany. The price level in East Germany is on average lower than in the West but official statistics do not exist, see Kosfeld et al. (2008). Hence the real differences in income and prices between the two parts of the country are overestimated here.

[11] In addition to the sibling discount, the fees for subsidized child care increase with the household income. Moreover, between 5% to 10% of the children are fully exempted from fees. These features have been ignored here but will be incorporated in future work.

et al. (2004). There is neither a significant markup for children younger than three nor a sibling discount or West-East difference for non-subsidized child care.

Table 3.3: Provision Rates for Subsidized Child Care

	West	**East**
Ages 0 to 2		
Part-time	4.3	38.8
Full-time	1.7	41.1
Total	*6.0*	*79.9*
Ages 3 to 6.5		
Part-time	71.8	2.4
Full-time	23.7	97.6
Total	*95.4*	*100.0*

Subsidized Slot Provision Table 3.3 shows the provision rates of subsidized part- and full-time child care slots in both parts of the country broken further down by age.[12] The differences in the provision rates between West and East Germany and the two age groups originate from very distinctive objectives of the former German Democratic Republic (GDR) and the Federal Republic of Germany (FRG) which have persisted until today. In the GDR the intention was to enable mothers of young children to work which is reflected in a provision rate of subsidized child care slots for 79.9 out of 100 children aged zero to two with around a half of them being full-time and that all children aged three to six and a half have access to a subsidized slot of which 97.6% are full-time. In contrast, the FRG offered subsidized child care to provide affordable pre-school education for children from age three onwards. Accordingly, for children aged zero to two hardly any subsidized child care is provided – only for 4.3 out of hundred a part-time and for 1.7 a full-time slot – whereas for nearly every child above from age three to six

[12] The definition of a subsidized part- and full-time child care slot corresponds to the definition of part- and full-time child care enrollment as outlined in Section 3.2.3. The details on how these rates were constructed are outlined in Appendix A.3.2.

and a half with around 3/4 being only part-time. Although aggregate statistics on queuing for subsidized child care slots are not available, the supply of subsidized child care slots in Germany is usually considered to be fixed, at least in the short to medium run, rather than an equilibrium outcome equating demand for subsidized child care at the regulated, fixed prices, see Kreyenfeld et al. (2002). Wrohlich (2008) estimates the excess demand for both parts of the country to be close to zero for children from age three onwards but far above zero for the younger age group.[13] Put differently, some females might face a constrained choice set when deciding on child care enrollment.

3.3.2 Female Labor Force Participation and Child Care Enrollment

My analysis focuses on female labor force participation profiles. Cohort and time effects, and the timing and spacing of births are not part of the investigation. I first present facts about overall female labor force participation and child care enrollment. Afterwards, I distinguish between part- and full-time rates.[14]

Subsidized and Non-Subsidized Child Care Child care enrollment in West Germany (6.3%) for children *aged zero to two* is slightly above the provision rate (6.0%), which can be attributed to the large usage of non-subsidized child care, see Table 3.4. 40.4% of the children enrolled are enrolled in non-subsidized child care, either exclusively or in addition to subsidized child care.[15] In contrast, in the East 60.3% of the children are enrolled in child care, i.e. 54 percentage points more than in the West, which is first even below the provision rate of 79.9% and second nearly entirely in subsidized child care.

[13] The numbers in Wrohlich (2008) are not directly comparable to those discussed here due to the different period length and sample selection.

[14] The fraction of females with one, two and three children is neither constant over time nor the same between West and East Germany, see Table 3.1. Therefore, I weight the labor force participation and child care enrollment rates for both parts of the country by the fraction of West German females with one, two and three children, conditional on having children, which are 0.234, 0.565 and 0.201. This has only a small quantitative but no qualitative impact on the presented facts.

[15] Due to changes in the survey questionnaire only the overall enrollment rate as well as the fraction enrolled in subsidized child care can be calculated for the whole time period, see also Appendix A.2.3.

Table 3.4: Subsidized and Non-Subsidized Child Care

	West	East
Ages 0 to 2		
Provision Rate Subsidized Care	6.0	79.9
Enrollment Rate (Subsidized and Non-Subsidized Care)	6.3	60.3
Fraction Enrolled in Non-Subsidized Care	40.4	1.8
Ages 3 to 6.5		
Provision Rate Subsidized Care	95.4	100.0
Enrollment Rate (Subsidized and Non-Subsidized Care)	95.3	93.3
Fraction Enrolled in Non-Subsidized Care	0.8	1.6

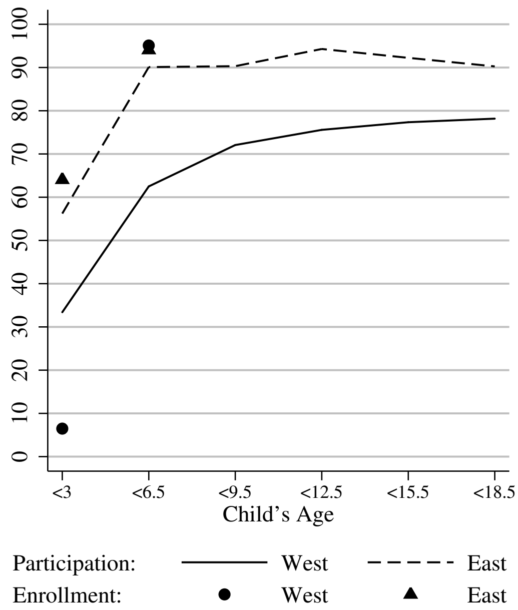
Table 3.5: Child Care Enrollment Rate
Conditional on Maternal Labor Force Participation Status

	West	East
Ages 0 to 2		
Not working	2.9	30.7
Working	13.9	87.3
Ages 3 to 6.5		
Not working	93.4	68.0
Working	96.7	98.2

The picture looks entirely different for children *aged three to six and a half.* Enrollment increases to over 90%, is very similar for the West and the East, and in the West non-subsidized child care ceases to be important also in relative terms.

Child Care Enrollment by Maternal Labor Force Participation Table 3.5 shows the child care enrollment rate conditional on the maternal labor force participation status. On the one hand, child care is strongly used by *non-working females*, with the exception of children aged zero to two in the West. On the other hand, child care is not a prerequisite for mothers to work. For example, only 13.9% of the *working* females with children aged zero to two in West Germany use child care, while the remaining 86.1% in this group rely on non-paid child care provided by family members or friends. Still, *working* females use more child care than *non-working* females.

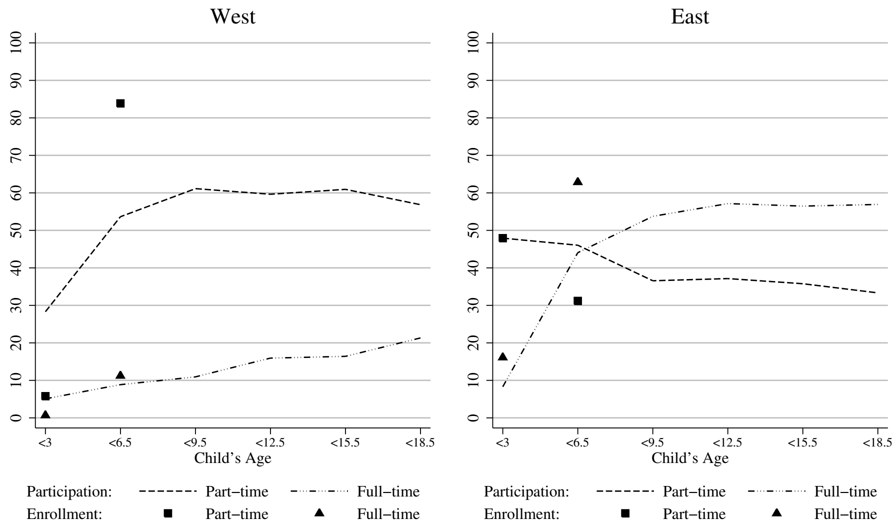
Figure 3.2: Labor Force Participation and Child Care Enrollment Rates



Labor Supply Figure 3.2 presents female labor force participation profiles by the child's age. While the shape is similar for the West and the East, the profile is shifted upwards for East German females. The gap originates when children are of *age zero to two* (56% vs. 33%), increases further when children are of *age three and six and half* and starts to shrink once they enter school. The participation rate in the East remains around 90% from *age three onwards* whereas it increases constantly in the West and reaches 78% when the child is between *15.5 and 18.5*.

Part- vs. Full-time As already shown in Table 3.3, a further difference in the provision of subsidized child care slots between West and East Germany is the fraction of full-time slots. In the East more than one half of the subsidized child care slots for children aged zero to two and nearly all for children aged three to six and a half are full-time, whereas in the West only around one fourth for both age groups. Similar differences can be found for part- and full-time child care enrollment and labor force participation, see Figure 3.3.

Figure 3.3: Part- and Full-time
Labor Force Participation and Child Care Enrollment Rates



In West Germany *part-time* child care enrollment and labor force participation dominate, whereas in the East the *full-time* rates are larger with the exception for children aged zero to two. Furthermore, the shape of the labor force participation profile in the West is mainly driven by the *part-time* and in the East by the *full-time* labor force participation rate, see figures 3.2 and 3.3.[16,17]

3.3.3 Summary Key Facts

The facts documented in this Section about labor force participation of married females with children and their child care enrollment decisions can be summarized as follows:

West Germany

1) Subsidized child care is two to four times as cheap as non-subsidized child care, but only provided for very few children aged zero to two whereas for nearly all children aged three to six and half.

2) Enrollment rates in child care match up with the provision rates while non-subsidized child care is only important for children aged zero to two.

3) Child care is used by non-working females but is not a prerequisite for females to work.

4) The labor force participation rate grows strongly while children are of pre-school age and less afterwards.

West vs. East Germany

1) Subsidized child care in East Germany is cheaper than in the West and more subsidized slots per 100 children, particularly full-time, are provided.

2) Enrollment in the East is higher than in the West and even below the provision rates leaving hardly any role for non-subsidized child care.

[16] Similar to child care enrollment conditional on the labor force participation status as shown in Table 3.5, full-time child care enrollment is larger for children of females full-time working compared to part-time working females, but again full-time child care enrollment is not a prerequisite for full-time labor force participation.

[17] Kreyenfeld (2001) and Wrohlich (2006) document similar patterns for West and East Germany. Kreyenfeld (2001) shows them for all mothers independent of the relationship status from the 1997 Mikrozensus, a representative cross-section of the German population. Similar to my paper, Wrohlich (2006) focuses on females in relationships and uses the 2001 to 2003 waves from the GSOEP. Both studies restrict their attention to mothers with children up to age ten.

3) In the East, the labor force participation rate is always higher than in the West and already remains stable after children turned three.

4) In East Germany the full-time labor force participation and child care enrollment rates are larger than the corresponding part-time rates, whereas the opposite is true for the West.

In the next Section, I develop a model that is aimed to capture the stylized facts for West Germany. Having an estimated version of it, I ask to which degree, ceteris paribus, the differences in the provision rates of subsidized child care slots and the prices for subsidized child care explain the differences in labor force participation profiles.

3.4 The Model

I now present a life cycle model for married females featuring endogenous fertility, labor supply with accumulation of experience and child care enrollment decisions. The model borrows some of the elements in Wrohlich (2008), Francesconi (2002) and Greenwood et al. (2003).

Demographics A female lives for six periods. At the beginning of her life, she is exogenously matched with a man and chooses how many children to have. Both the husband and children stay with the female throughout her whole life. The period length corresponds to three years such that each period corresponds to certain period in a child's life, see Figure 3.1.[18]

Endowments Each female and husband are indexed by productivity levels ϵ and ϵ^* representing the stochastic part of each spouses' market wages. Asterisks refer to values for the husband. Both spouses are assigned an initial productivity in period one which evolves over time according to an AR(1) process:

$$\begin{aligned} \epsilon_t &= \rho\epsilon_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \\ \epsilon_t^* &= \rho^*\epsilon_{t-1} + \varepsilon_t^* \text{ with } \varepsilon_t^* \sim N(0, \sigma_{\varepsilon^*}^2) \end{aligned} \tag{3.1}$$

[18]For period two the overlap is not exact since the mean duration in the data is three and a half years.

In the first two periods, while the children are not yet in school females can enroll them in subsidized and/or in non-subsidized child care. Both types of child care are perfect substitutes but subsidized child care is not available for every female. I assume that access to subsidized child care, denoted as a_t, is determined by a lottery from which each mother draws and which will be described in more detail further below.

Preferences The female is the household's decision maker and her utility function consists of two parts. The first is given by

$$U_t^I = \delta_0 \frac{(\psi c_t)^{1-\gamma_0}}{1-\gamma_0} + \delta_1 \frac{n^{1-\gamma_1}}{1-\gamma_1} \frac{m_t^{1-\xi}}{1-\xi} + \delta_2 \frac{(1-l_t-m_t)^{1-\gamma_2}}{1-\gamma_2}. \tag{3.2}$$

A female is endowed with one unit of (non-sleeping) time which she can split between working l_t, spending time m_t with her n children or enjoying leisure $1 - l_t - m_t$. The interaction of the number of children with maternal time introduces a quantity-quality trade-off. The function ψ translates household consumption into the consumption realized by a female using the OECD equivalence scale – also called the Oxford scale – with $\psi = (1.7 + 0.5n)^{-1}$. Equation (3.3) states the budget constraint:

$$c_t = y_t(l_t, x_t, \epsilon_t) + y_t^*(t, \epsilon_t^*) - p_{cc}(n, t, cc_{s,t}, cc_{ns,t}) + \varpi(n, t, l_t). \tag{3.3}$$

$y_t(l_t, x_t, \epsilon)$ is the female's net income depending on her current productivity, accumulated experience which evolves according to

$$x_t = x_{t-1} + l_{t-1} \tag{3.4}$$

and labor supply l_t. The husband is assumed to be working full-time and receives a net income $y_t^*(t, \epsilon_t^*)$.[19] $p_{cc}(n, t, cc_{s,t}, cc_{ns,t})$ denotes the child care costs which may vary nonlinearly with the number of children, time period, amount and type of demanded child care, i.e. subsidized $cc_{s,t}$ and non-subsidized $cc_{ns,t}$ care. The exact forms for the income process and child care prices are specified further below. Finally, $\varpi(n, t, l_t)$ represents public transfers related to the female's labor force participation status, age (equal to the period) and number of children.

Children of pre-school age cannot be left alone. If the female does not

[19] Only 2% of the husbands in the sample are at some point not in the labor force.

take care of them, e.g. because she is working, someone else has to do it. Recall that only 13.9% of the females with the children younger than age zero to two who work use child care, see Table 3.5. I denote the time children neither spend with the mother, nor in subsidized or non-subsidized child care, by

$$h_t = 1 - m_t - cc_{s,t} - cc_{ns,t} \text{ for } t \leq 2. \tag{3.5}$$

Given that most working females with children aged zero to two used non-paid child care, the fact that they switch to using paid child care when their children are of age three to six and a half, see Table 3.5, implies the existence of some costs of non-paid child care – as long as one does not assume that non-paid child care ceases to be available once children turn three. I formalize this by introducing a cost for non-paid child care for periods one and two:[20]

$$U_t = \begin{cases} U_t^I - \omega h_t^2 & \forall\, t \leq 2 \\ U_t^I & \forall\, 3 \leq t \leq 6 \end{cases}. \tag{3.6}$$

The weight ω of the cost function is related to the children's ages and the number of children:

$$\omega = \phi_0 \phi_1^{t-1} n^{\phi_2}. \tag{3.7}$$

ϕ_1 attaches a weight to the cost function. Similar to paid child care, I allow the costs of non-paid child care to vary with the age (ϕ_1) and the number of children (ϕ_2).

Choice Variables In line with the stylized facts, all choices are discrete. Labor supply l, subsidized cc_s and non-subsidized child care cc_{ns} can take on three values: 0 for non-working/no child care, $\frac{1}{4}$ for part- and $\frac{1}{2}$ for full-time work/care. If the (non-sleeping) time endowment would be 16 hours, then part-time work/care would correspond to four and full-time work/care to eight hours. I assume that child care facilities are only open during the first half of the day, i.e. in the morning and early afternoon which implies that

$$cc_s + cc_{ns} \leq \frac{1}{2}. \tag{3.8}$$

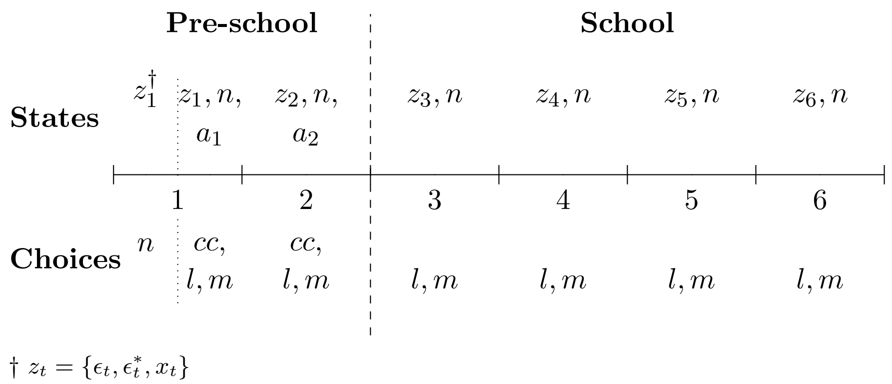
[20] It might be reasonable to consider such costs also for children in elementary school or even of older ages but I refrain from it since the focus of this paper is on pre-school child care.

Similarly, I assume that working opportunities also only exist during the first half of a day. A female can also spend time with her children in the late afternoon and evening such that $m_t \in \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$ but not while she is at work or her children are in child care or in school (s_t):

$$m_t \leq \begin{cases} 1 - \max\{l_t, cc_{s,t} + cc_{ns,t}\} & \forall\, t \leq 2 \\ 1 - \max\{l_t, s_t\} & \forall\, 3 \leq t \leq 6 \end{cases} . \tag{3.9}$$

Dynamic Problem Figure 3.4 presents the timing of events during a female's life which is defined by the stages of her children's adolescence, see also Figure 3.1. The term z_t combines the productivity states of both spouses ϵ_t and ϵ_t^* and the female's experience level x_t, with $x_1 = 0$. The first

Figure 3.4: Life Cycle



period is split up in two stages with different states and decisions. In the first stage, the initial productivity levels are assigned and the female chooses the optimal number of children.[21] By then she does not know whether she will have access to subsidized child care in period one. The first stage value function in period one is

$$\widehat{V}(\epsilon_1, \epsilon_1^*, x_1) = \max_n \; \{E_{a_1} V(1, \epsilon_1, \epsilon_1^*, x_1, n, a_1),\; n = 0, 1, 2, ..., N\} . \tag{3.10}$$

Since the children stay with a female throughout her entire life, n becomes a state variable. In the second stage, a lottery determines whether a female has access to subsidized child care a_1, which can take on three values: 0, $\frac{1}{4}$ and

[21] Fertility is perfectly controllable, for a model and the consequences of uncertain fertility see e.g. Choi (2009).

$\frac{1}{2}$ referring to no, a part- or full-time subsidized child care slot. Afterwards, the female decides on her labor supply l_1 and those with $n > 0$ children, on how much time to spend with them m_1 and on their enrollment in subsidized child care $cc_{s,1}$ – possibly restricted by a_1 – and non-subsidized child care $cc_{ns,1}$. The second stage value function in period two is

$$V(1, \epsilon_1, \epsilon_1^*, x_1, n, a_1) = \max_{m,l,cc_s \leq a_1, cc_{ns}} U(\cdot) + \beta E_{\epsilon,\epsilon^*,a_2} V(2, \epsilon_2, \epsilon_2^*, x_2, n, a_2)$$
$$\text{subject to (3.3), (3.4), (3.8) and (3.9).} \tag{3.11}$$

$U(\cdot)$ is given by Equation (3.6) and β is the discount factor. At the beginning of period two, the new productivity levels ϵ_2 and ϵ_2^* realize according to the AR(1) process specified in Equation (3.1) and access to child care a_2 is drawn from a new lottery. The set of choice variables in period two is identical to the second decision stage in period one. The value unction in period two is

$$V(2, \epsilon_2, \epsilon_2^*, x_2, n, a_2) = \max_{m,l,cc_s \leq a_2, cc_{ns}} U(\cdot) + \beta E_{\epsilon,\epsilon^*} V(3, \epsilon_3, \epsilon_3^*, x_3, n, 0)$$
$$\text{subject to (3.3), (3.4), (3.8) and (3.9).} \tag{3.12}$$

From period three onwards, children attend school and females cannot use child care anymore ($a_t = 0$ for $t \geq 3$). She only decides on how much to work and how much time to spend with their children, the value function in period t is

$$V(t, \epsilon_t, \epsilon_t^*, x_t, n, 0) = \max_{m,l} U(\cdot) + \beta E_{\epsilon,\epsilon^*} V(t+1, \epsilon_{t+1}, \epsilon_{t+1}^*, x_{t+1}, n, 0) \ \forall\ 3 \leq t \leq 5$$
$$\text{subject to (3.3), (3.4) and (3.9).} \tag{3.13}$$

In the last period, the value function is given by

$$V(6, \epsilon_6, \epsilon_6^*, x_6, n, 0) = \max_{m,l} U(\cdot) \text{ subject to (3.3) and (3.9).} \tag{3.14}$$

Maternal Leave In Germany, every mother who has been working until the birth of a child has the right to return to her pre-birth employer at her pre-birth wage within three years after birth.[22] To account for this

[22]Since in the model life starts with the birth decision, there is no pre-birth labor supply

regulation, I introduce a further state variable q_t:

$$q_t = \begin{cases} 0 & \text{for } t = 1,\ t \geq 3 \\ 0 & \text{for } t = 2,\ x_2 > 0 \text{ or } n = 0 \\ \epsilon_1 & \text{for } t = 2,\ x_2 = 0,\ n > 0 \end{cases} . \tag{3.15}$$

The effect of taking the maternity leave in period one, i.e. $l_1 = 0$ and thus $x_2 = 0$, materializes in period two where the income is based on the maximum from $\{\epsilon_1, \epsilon_2\}$.[23,24] The period three productivity level is then determined by

$$\epsilon_3 = \begin{cases} \rho \max\{\epsilon_1, \epsilon_2\} + \varepsilon_3 & \text{if } q_2 = \epsilon_1,\ l_2 > 0 \\ \rho\epsilon_2 + \varepsilon_3 & \text{else} \end{cases} .$$

Income Process A female's offered wage is given by a classical Mincer (1974) earnings equation with returns to experience. As a normalization x_t is multiplied by two ($\tilde{x}_t = 2x_t$) such that part-time work increases $\tilde{x}$ by $\frac{1}{2}$ and full-time work by one.

$$\ln Y_t = \eta_0 + \eta_1 \tilde{x}_t + \eta_2 \tilde{x}_t^2 + \epsilon_t. \tag{3.16}$$

Labor income taxes are levied according to the function τ which together with labor supply result in the following net incomes:

$$y_t(l_t, x_t, \epsilon_t) = \begin{cases} 0 & \text{for } l = 0, \text{ i.e. non-working} \\ \tau\left(\frac{e^{Y_t}}{2}\right) & \text{for } l = \frac{1}{4}, \text{ i.e. part-time work} \\ \tau\left(e^{Y_t}\right) & \text{for } l = \frac{1}{2}, \text{ i.e. full-time work} \end{cases} . \tag{3.17}$$

Since the husbands is assumed to be working full-time, he accumulates only full-time experience such that his income is given by:

$$\ln Y_t^* = \eta_0^* + \eta_1^*(t-1) + \eta_2^*(t-1)^2 + \epsilon_t^* \tag{3.18}$$

and therefore all females can go on maternal leave. This assumption is supported by the data for the first birth, prior to which 94% of the West and 98% of the East German females in the sample work.

[23] Females working part- or full-time in period one "return" by construction at their pre-birth income ϵ_1.

[24] Studies focusing on maternity leave policies are Erosa et al. (2008) or Bernal and Fruttero (2008).

The husband's net income y_t^* is then determined by applying the tax code τ to $e^{Y_t^*}$.[25]

Child Care Costs The functional form for the child care costs corresponds to the structure of fees as reported in Table 3.2. They accrue per child if a source of care is used and are associated with a fixed cost ($\zeta_{j,0}$), a mark up for full-time child care ($\zeta_{j,1}$) and period one ($\zeta_{j,2}$). A per child discount is granted for every additional sibling enrolled ($\zeta_{j,3}$),

$$p_{cc}(t, n, cc_{s,t}, cc_{ns,t}) = \sum_{i=s,ns} n\mathrm{I}(cc_j > 0) \left[\zeta_{j,0} + \zeta_{j,1}\mathrm{I}\left(cc_{j,t} = \frac{1}{2}\right) + \zeta_{j,2}I(t = 1) - \zeta_{j,3}(n - 1)\right]. \tag{3.19}$$

3.5 Taking the Model to the Data

All non-preference parameters are obtained directly from the data which I describe now in more detail.

Income Process Since nearly all men in the data are working full-time, their period incomes can be easily calculated such that it is straightforward to estimate the deterministic (Equation (3.18)) and stochastic (Equation (3.1)) part of the male income process. For females it is more difficult to construct a period income from the retrospective monthly incomes because of the variation in hours worked for part-time employment.[26] I circumvent this problem by assuming that females face the same wage process as their husbands but take into account that females are on average younger than their spouses, 2.9 years in the West and 2.5 years in the East. This age difference corresponds approximately to one model period. I therefore shift the income process for males by one period to obtain the one for females. The logic behind that is the following. A female that has worked full time in all periods, i.e. $\tilde{x}_t = t - 1$, should receive the same (deterministic) wage

[25] In the model, the age of the female and her husband are the same, which is not true in the data. This age difference is captured by allowing for gender specific coefficients in the income process and will be discussed in more detail below.

[26] Conditional on having an appropriate period income, a Heckman (1979)-style selection model could be estimated to account for the selection into employment.

a male had in the period before because of the age difference:

$$\ln Y_t = \eta_0^* + \eta_1^*(\underbrace{t-1}_{\tilde{x}_t} - 1) + \eta_2^*(\underbrace{t-1}_{\tilde{x}_t} - 1)^2 + \epsilon_t \tag{3.20}$$

Equation (3.20) can then be reformulated to obtain the coefficients of the female income process:

$$\ln Y_t = \underbrace{\eta_0^* - \eta_1^* + \eta_2^*}_{\eta_0} + \underbrace{[\eta_1^* - 2\eta_2^*]}_{\eta_1}\tilde{x}_t + \underbrace{\eta_2^*}_{\eta_2}\tilde{x}_t^2 + \epsilon_t \tag{3.21}$$

In the model in which males and females have the same age, at a given period females have therefore a lower mean wage and face larger returns to experience than their spouses if $\eta_2^* < 0$. I allow for a discriminatory gender wage gap in mean income not driven by the age difference, using the full-time wages of both sexes at the last interview date prior birth:[27]

$$\eta_0 = \eta_0^* - \eta_1^* + \eta_2^* + \Delta_{gender}. \tag{3.22}$$

The last missing piece of the income process concerns the stochastic part (Equation (3.1)) where I follow Attanasio et al. (2008) and use the male estimates for the females. For the numerical solution of the model, the AR(1) process for productivity (Equation (3.1)) is discretized using the method proposed by Tauchen (1986) into 20 states. The initial productivity levels are assigned according to the corresponding stationary distribution.

Table 3.6 and Figure 3.5 summarize the estimation results on the income process.[28] The gender wage gap, mean income, returns to experience and the standard deviation of the stochastic part of income are larger in West Germany providing additional sources for the difference in female labor force participation and child care enrollment between West and East Germany on top of the distinctive provision and prices of subsidized child care.

[27] By then 75% of the females are working full-time such that selection into full-time employment is much less of a problem.

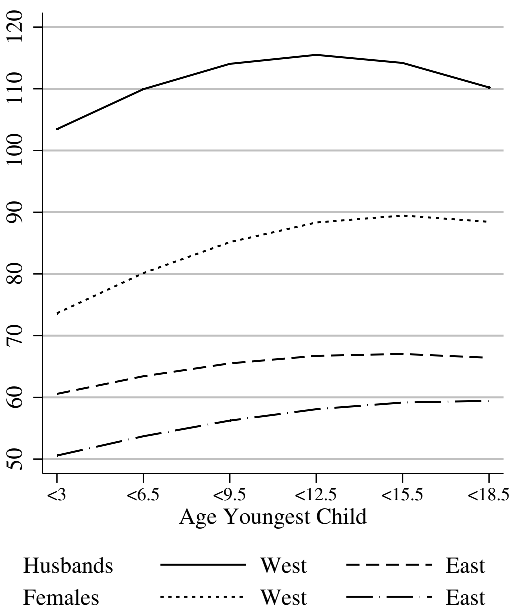
[28] The difference between η_0 and η_0^* without the gender wage gap is -0.085 for the West and -0.06 for the East. Not controlling for age, increases the pre-birth gender wage gap by -0.075 for the West and -0.063 for the East. Thus, using the same experience profile for men and females but shifting it by one period as done in Equation (3.20) and Equation (3.21) provides an accurate estimate of the pre-birth gender income difference due to the age difference of spouses.

Table 3.6: Income Process

	West	**East**
Gender wage gap		
Δ_{gender}	-0.255	-0.120
Deterministic part (3.18)/(3.16)		
$\eta_0^* \ / \ \eta_0$	11.547 / 11.207	11.011 / 10.831
$\eta_1^* \ / \ \eta_1$	0.073 / 0.097	0.053 / 0.067
$\eta_2^* \ / \ \eta_2$	-0.012 / -0.012	-0.007 / -0.007
Stochastic part (3.1)		
$\rho^*, \ \rho$	0.818	0.819
$\sigma_{\varepsilon^*}, \ \sigma_\varepsilon$	0.350	0.315

Note: η_0 is calculated as in Equation (3.22) and η_1, η_2 as in Equation (3.21).

Figure 3.5: Income Profiles (in 1000 €)



Child Care Similar to Wrohlich (2006), the slot provision rates shown in Table 3.3 are used directly as success probabilities for drawing access to a part- or full-time slot.[29] The parameters of the child care cost function Equation (3.19) can be directly linked to the estimates from Table 3.2.

Taxes and Transfers Taxes are kept as simple as possible and will be proportional to income with an estimated tax rate of 0.36.[30] The transfers include child benefits which are paid each period depending on the total number of children, see Table 3.7. Finally, non- and part-time working

Table 3.7: Child Benefits

Number of Children	**Total Benefits**
1	3178.52 €
2	7172.09 €
3	13108.90 €
4	19775.58 €

mothers receive in period one a maternity benefit of 1583.91 €. The transfers and taxes are the same for both parts of the country.

Schooling hours Once children enter school, maternal time is restricted by the time children spend there as specified in Equation (3.9). I set $s_t = \frac{1}{2} \; \forall \; t \geq 3$, i.e. children are assumed to attend full-time schooling, which partially accommodates for the model feature that females are not allowed to use child care after period two. Though elementary schools usually open only in the morning, i.e. are part-time, subsidized child care within or outside the school exist. While from the GSOEP actual enrollment in these offers is available, there are no aggregate statistics on the provision of care provided in schools in the early afternoon. To keep the focus on pre-school

[29]This is the a strong assumption since the assignment of slots is of course not fully random. Single females usually have privileged access. From that perspective, the provision rates used here constitute an upper bound on the true access probabilities. Furthermore, working females and those with already a child enrolled in a subsidized institution have higher chances of getting a slot. However, the magnitude of this effect cannot be extracted from the aggregate statistics on slot provision and I therefore ignore this issue.

[30]Implementing a more realistic tax system featuring progressive and joint taxation is left for future work.

child care, I do not treat this issue.

3.6 Estimation

In this Section, I first discuss the estimation methodology, the data moments to be matched, and show the final parameter estimates. Afterwards I present the model fit and conduct several comparative statics exercises to gain some insights about the role of the preference parameters.

3.6.1 Methodology

Three preference parameters are fixed and the remaining eight are estimated jointly by minimizing the square differences between data and model moments. The discount factor is chosen to reflect a 4% yearly interest rate as in Kydland and Prescott (1982), $\beta = \left(\frac{1}{1.04}\right)^3$, and the curvature of the utility of consumption γ_0 is set to one, i.e. I assume log utility.[31] Since one of the four weights can be normalized, δ_0 is set to one. The estimation is a simplified simulated methods of moments procedure with the identity matrix used for weighting the moments:

1) Pick an initial guess for the parameters to be estimated.
2) Solve the model recursively, obtain optimal policy functions, simulate life cycles for a large number of females, here 100000, and calculate model equivalents to data targets.
3) Calculate the objective function, i.e. the sum of the square distance between data and model moments.
4) If the improvement in the objective function is
   (a) below a pre-specified tolerance, exit.
   (b) above a pre-specified tolerance, update the parameter values, return to step two and iterate until convergence.[32]

[31] Estimation of these kind of models is far from being trivial. The best estimation results were obtained with $\gamma_0 = 1$. In future work I will try to estimate γ_0 as well.

[32] The parameters are updated via an asynchronous parallel pattern search algorithm, see Gray and Kolda (2005), and Kolda (2005). The corresponding software (APPSPACK) is freely available on the web (`https://software.sandia.gov/appspack/version5.0/index.html`) and was run

3.6.2 Data Moments

The model is estimated for West German data with the following set of data moments:

- the fraction of females with one, two and three children:[33]

  3 moments

- the fraction of mothers not-, part- and full-time working by number of children and period:

  54 moments = 3 fractions × 3 number of children × 6 periods

- the fraction of children not-, part- and full-time enrolled in child care by number of children and period:

  18 moments = 3 fractions × 3 number of children × 2 periods

In total, there are 75 moments to match and eight model parameters to be estimated.

3.6.3 Parameter Estimates

Table 3.8 shows the fixed (only consumption) and estimated parameter values.[34] A brief comment on two parameters is worth mentioning. First, $\phi_1 > 1$ implies that the costs of non-paid child care are larger for children aged three to six and a half relative to children aged zero to two. A possible interpretation could be some sort of benefits of paid over non-paid child care that gain more importance as the children age, e.g. being together with other children. Second, $0 < \phi_2 < 1$ indicates that the costs of non-paid child care are concave in the number of children, i.e. organizing non-paid child care for two children is more but not twice as costly as for one child.

in parallel mode on the Deutsche Bank/E-Finance Lab House of Finance Servercluster. I am indebted to the cluster administrator Alexander Zeiss for installing APPSPACK for me on the cluster and his help to get started with Linux.

[33] The maximum number of children is restricted to be three.

[34] There is no guarantee that the estimated parameters refer to a global minimum of the objective function. To check for other local minima, a thorough search of the parameter space was performed and no better fit was found.

Table 3.8: Preference Parameters

	Value
Consumption[†]	
δ_0	1.00
γ_0	1.00
Children	
δ_1	0.04
γ_1	0.94
ξ	0.85
Leisure	
δ_2	0.43
γ_2	0.47
Non-paid child care	
ϕ_0	0.17
ϕ_1	2.61
ϕ_2	0.43

[†]Parameters are not estimated.

3.6.4 Model Fit

To evaluate the model fit, I first compare the set of stylized facts presented in Section 3.3 for West Germany to the corresponding model moments. Afterwards, I confront the set of targeted moments from the data with the model.

Aggregate Moments Table 3.9 presents the enrollment rates in subsidized and non-subsidized child care for West Germany. The model generates only one third of the observed enrollment rates in child care for children aged zero to two (6.3 vs. 1.8%) and one half of the fraction enrolled in non-subsidize care (40.4 vs. 21.0%). In contrast, the fit for children from age three onwards is much better and the huge difference in the enrollment rates between the two age groups is captured as well.

Table 3.9: Model Fit – Subsidized and Non-Subsidized Child Care

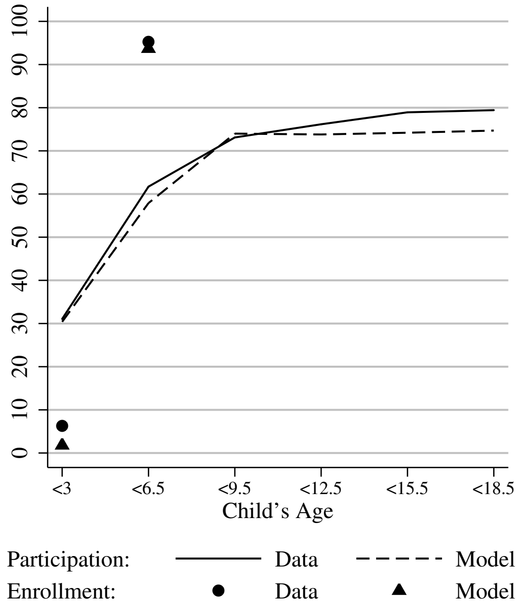
	Data	**Model**
Ages 0 to 2		
Provision Rate Subsidized Care	6.0	6.0
Enrollment Rate (Subsidized and Non-Subsidized Care)	6.3	1.8
Fraction Enrolled in Non-Subsidized Care	40.4	21.0
Ages 3 to 6.5		
Provision Rate Subsidized Care	95.4	95.4
Enrollment Rate (Subsidized and Non-Subsidized Care)	95.3	93.7
Fraction Enrolled in Non-Subsidized Care	0.8	1.8

Table 3.10: Model Fit
Child Care Enrollment Rate Conditional on Maternal Labor Force Status

	Data	**Model**
Ages 0 to 2		
Not working	2.9	0.0
Working	13.9	4.6
Ages 3 to 6.5		
Not working	93.4	86.3
Working	96.7	98.0

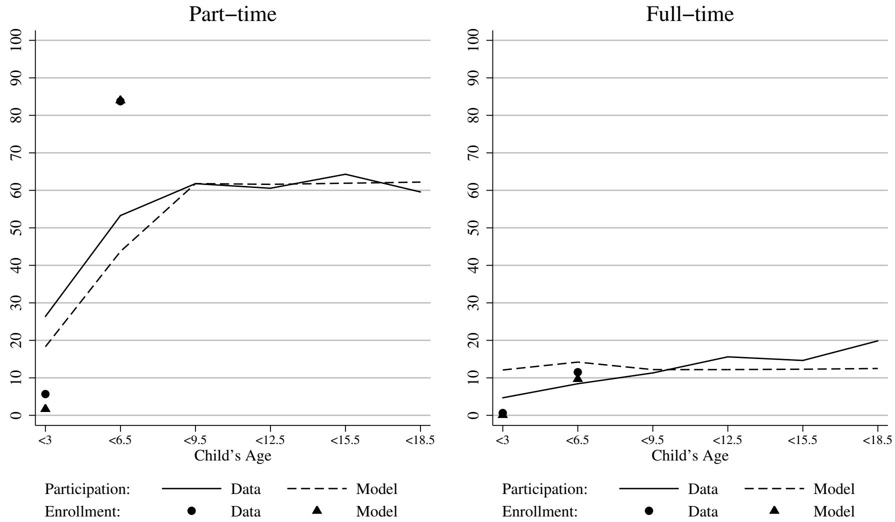
Child Care Enrollment rates conditional on the maternal labor force status are shown in Table 3.10. As in the data, the vast majority of the working females with children aged zero to two in the model is not using any paid child care but relies on non-paid child care. Further, most of the non-working females with children aged three to six and a half uses paid child care. The estimate of ϕ_1 being larger than one indicates that the increase in the child care enrollment rate between the two age groups for non-working mothers cannot be only generated by the higher provision rate and lower prices of subsidized child care slots as well as increasing spousal income.

Figure 3.6: Model Fit
Labor Force Participation and Child Care Enrollment Rates



Female labor force participation profiles are displayed in Figure 3.6. Though the overall fit looks very well, the part-time labor force participation rate in the model is less than in the data while the children are of pre-school age and the increase of full-time labor force participation over time is not matched, see Figure 3.7.

Figure 3.7: Model Fit – Part- and Full-time
Labor Force Participation and Child Care Enrollment Rates



Targeted Moments Table 3.11 shows the fractions of females with $n = \{0, 1, 2, 3\}$ children in the data and the model, which have been among the set of target moments, and the implied fertility rate. There are no

Table 3.11: Model Fit – Fertility

	Fraction with n children				**Fertility**
	0	**1**	**2**	**3**	**Rate**
Data	13.9	20.2	48.7	17.3	1.69
Model	0	35.7	30.2	34.1	1.98

childless females in the model in contrast to 13.9% in the data. Part of this mismatch might be attributed to physical constraints preventing some females to get children, from which I abstracted in the model by assuming perfectly controllable fertility choices. In total, the fraction of females with $n \leq 1$ children is very close between data (34.1) and model (35.7). The fraction of females with three children in the model is nearly twice as large as in the data.[35] This adds up to a 0.29 higher fertility rate in the model than in the data.

Figure 3.8 shows the part- and full-time labor force participation and child care enrollment rates grouped by the number of children which comprises the second set of targeted moments. The model does not generate the observed decrease in the part-time labor force participation rate for females with one child after period three and the full-time labor force participation profile is exactly reversed. In addition, though consistent with the mistake for full-time labor force participation, full-time child care enrollment for children aged three to six and a half is too high. The fit for females with two and three children is much better, in particular for labor force participation. Part-time enrollment for children aged zero to two is below actual enrollment for $n = 2$ and $n = 3$. The part-time child care enrollment rate for three children is too high while the children are of age three to six and a half whereas full-time child care enrollment is too low compared to the data.

3.6.5 Comparative Statics

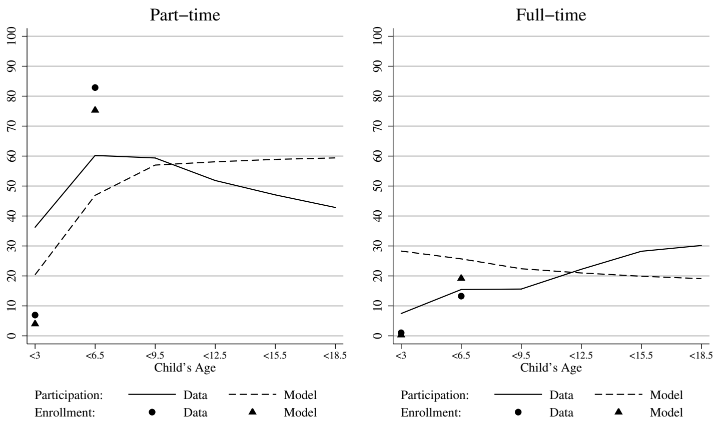
Setup To understand the role of the preference parameters, in this section I undertake several comparative statics exercises. Every parameter change has two effects. First, conditional on the number of children labor participation and child care enrollment decisions are affected and second, the birth decision might be altered. In this case, labor force participation and child care enrollment decisions might differ due to the change in the number of children. I therefore split the analysis in two parts by looking first at changes in the labor force participation and child care enrollment rate without changing the number of children of a female. In a second step, the impact on the fertility rate is investigated.

I increase each parameter by 5% leaving all remaining ones at their estimated or pre-specified values, cf. Table 3.8, and calculate the difference between the new and baseline labor force participation/child care enrollment/fertility rate. For the set of parameters of the first part of the utility function (Equation (3.2)), I only report results for the utility weights because the qualitative effects are similar to those for the curvature parameters

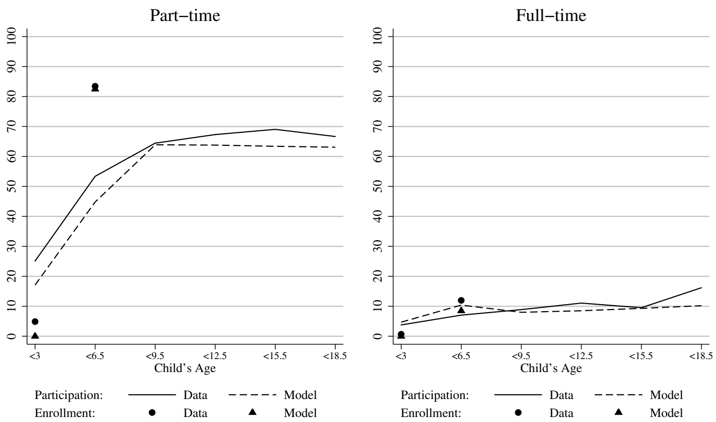
[35] In future work, I will relax the restriction that females can get a maximum of three children to ensure that $n = 3$ is not a corner solution.

Figure 3.8: Model Fit – Part- and Full-time
Participation and Enrollment Rates by Number of Children

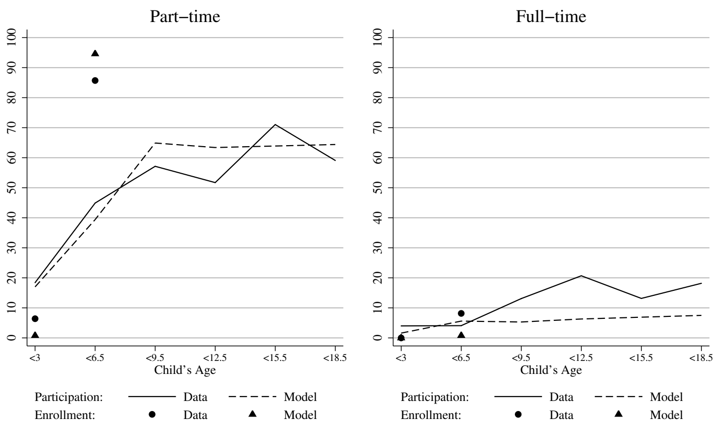
One child



Two children



Three children



while the quantitative differences are only small.

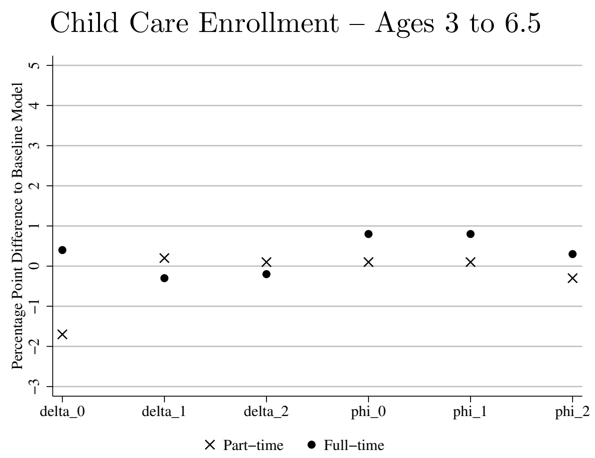
Female Labor Force Participation and Child Care Enrollment In this part of the exercise the number of children for each female is fixed at the value prior to a parameter change. Figure 3.9 presents the percentage point differences between a set of model statistics at the new and baseline preference parameters for pre-school children of age three to six and a half (two upper panels) and for school children between 12.5 and 15.5 (lower panel) which are representative for the remaining periods.

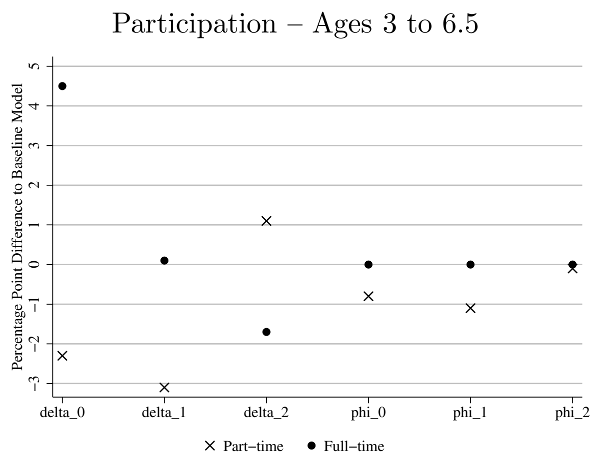
Consumption (δ_0) Raising the utility from consumption by 5%, increases full-time labor force participation rate compared to the baseline setup by 4.5 percentage points for females with children aged three to six and a half (middle panel) and by 1.7 percentage points for females with children aged 12.5 to 15.5 (lower panel). This increase stems from females switching from part- to full-time labor force participation and from previously non-employed females that start to work part-time. A small fraction of the newly full-time working females starts to use full-time child care (upper panel). Part-time child care enrollment decreases however by more than that because females save the costs from paid child care by substituting it with non-paid child care.

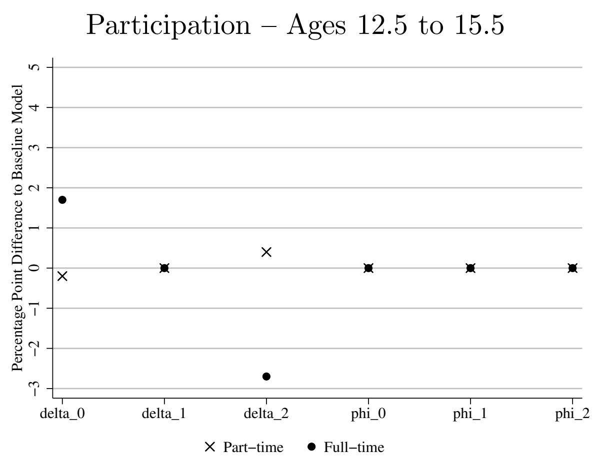
Children (δ_1) Receiving a higher utility from spending time with the children leads to a drop in part-time labor force participation and a substitution from full-time towards part-time child care enrollment for the age group three to six and a half because females simply spend more time with their children. In the older age group, 12.5 to 15.5, there is no effect on female labor force participation since females already spend all time the children are not in school with them. Hence, there is no margin to adjust. The lower accumulation of experience for the females that stop to work in the new setup reduces the incentives to work as the husband's incomes further increase. The effect is however not strong enough to result in a lower labor force participation rate.

Leisure (δ_2) In all periods females switch from full- to part-time labor force participation and from part-time labor force participation out of the labor force as the weight on leisure increases. Along with the reduction in labor force participation, but to a much smaller degree, full-time child care is substituted with part-time child care. Those females do however not spend more time with their children. They have decreased their labor supply and

Figure 3.9: Comparative Statics Exercise - Fixed Number of Children







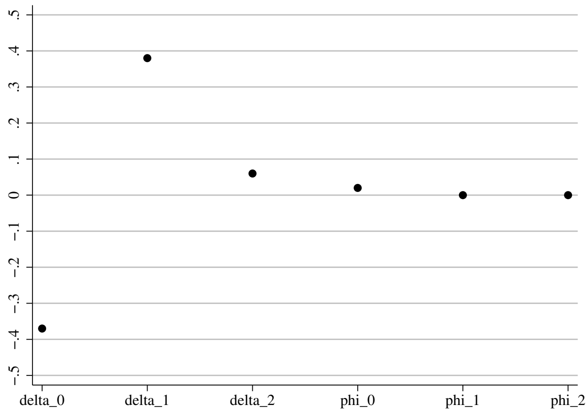
compensate the consumption decrease by substituting paid with non-paid child care.

Non-Paid Child Care A larger weight on the costs of non-paid child care (ϕ_0) decreases the part-time labor force participation rate of females with children aged three to six and a half by 0.8 percentage points and increases child care enrollment. Full-time working females substitute part- with full-time child care and in addition, more females use part-time child care. Increasing the costs of non-paid child care for children from age three to six and a half, ϕ_1, has qualitatively and quantitatively a similar effect as increasing ϕ_0. The parameter ϕ_2 allows the costs of non-paid child care to be increasing in the number of children and the larger value of this parameter lets females of more than one child substitute part- by full-time child care. Finally, note that for none of these three parameters an increase induces a different participation behavior once children entered school since the costs of non-paid child care are assumed to affect the females utility only while the children are of pre-school age. The only feasible channel via the accumulation of experience is not strong enough to result in a lower labor force participation rate also later in life.

Summary Increasing a subset of the preference parameters by 5% and holding the fertility decision constant at the pre-increase level changes the behavior of only a small fraction of females in the directions one would a priori except. A higher preference for consumption increases the labor force participation rate while the opposite is true for a higher weight on leisure. Females indeed spend more time with their children if this gains more importance in the utility function. Higher costs of non-paid child care result in a lower labor force participation rate and higher child care enrollment rate.

Fertility Rate Having analyzed the impact of a variation of the parameters holding the birth decision fixed, Figure 3.10 shows the changes in the fertility rate induced by a parameter increase. A higher weight on *Consumption* (δ_0) reduces the fertility rate since children are costly, whereas a higher weight on the time spend with *Children* (δ_1) is associated with an increase by a similar magnitude. At the larger value of *Leisure* (δ_2) females increase their leisure by spending less time with their children which they compensate by having an additional child resulting in a small increase in the fertility rate, the quantity-quality trade-off. The cost of *Non-Paid Child*

Figure 3.10: Comparative Statics Exercise – Fertility Rate



Care have only an impact on fertility via the weight ϕ_0. The increase of the weight lets a small fraction of females spend more time with their child instead of using non-paid child care. However, conditional on spending more time with the children, the benefits of an additional child outweigh the costs in form of reduced consumption.

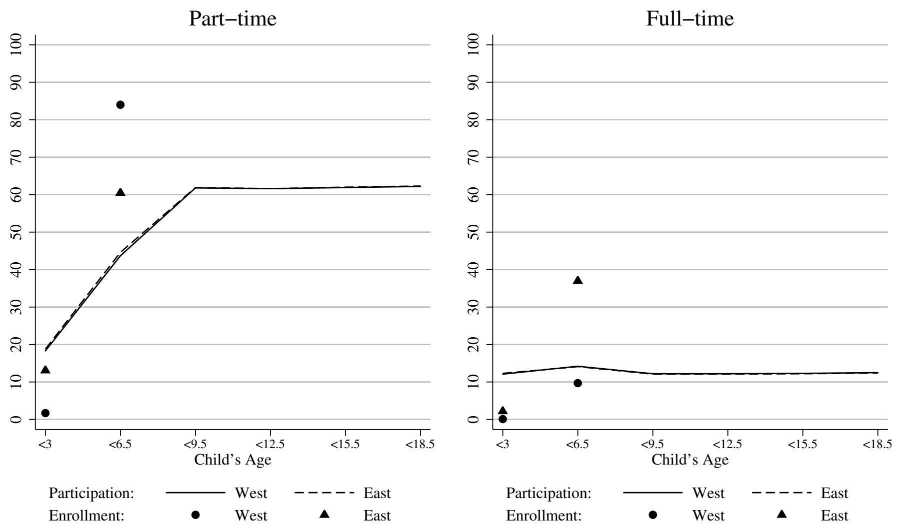
3.7 Experiments

3.7.1 East Germany

In Section 3.3 I documented differences between West and East Germany regarding the economic environment (provision and prices of subsidized child care, income process) and the behavior (labor force participation, child care enrollment). Which fraction of the differences in the behavior can be attributed to the differences in the economic environment? To answer this question, I simulate data using the model with the parameters estimated with West German data but replace the West with the East German economic environment.

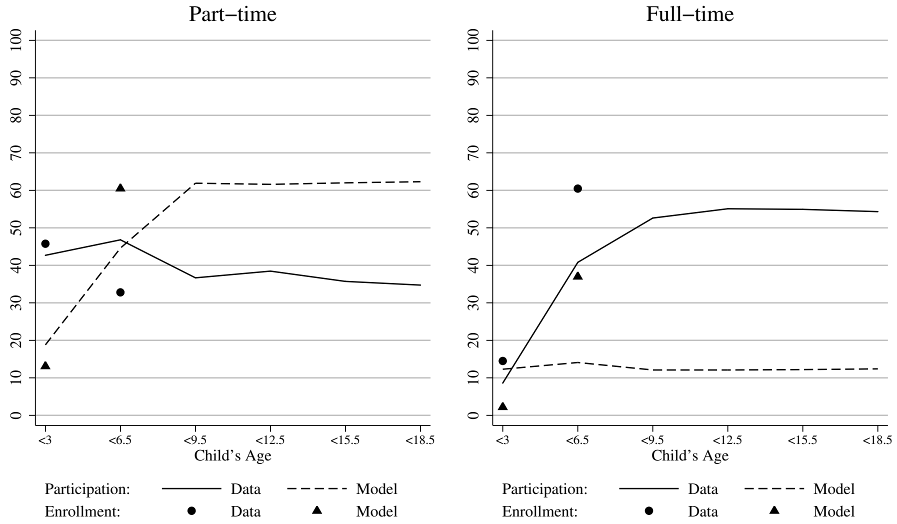
Figure 3.11 compares labor force participation and child care enrollment rates from the simulated data with the West and East German environment. With the exception of the child care enrollment rates for children aged three to six and a half, the differences between the "West" and "East" are only small. This implies that the model in it's current form is not able to explain the actual East German data, see Figure 3.12.

Figure 3.11: Model West vs. Model East
Labor Force Participation and Child Care Enrollment Rates



Neither the labor force participation nor the child care enrollment rates from the simulated data with the East German environment (labeled as "Model") come close to the data, which is also true for the rates grouped by number of children (not shown). Note that this is not driven by the endogenous fertility choice. Forcing females to have the same number of children as in the West German setup does not alter the results qualitatively. Since in the current experiment, three exogenous forces were changed – the income processes (for males and females), prices and provision rates of subsidized child care – it is difficult to pin down the contribution of each force. Therefore, in the experiment in the next subsection I only change one of them. Let me remark that this result does not necessarily imply a failure of the model. Alesina and Fuchs-Schündeln (2007) find that East Germans are much more in favor of government redistribution and state interventions than West Germans, which they attribute to the impact of communism on shaping people's preferences. This could also be case for the preferences over consumption, children, leisure and non-paid child care in the two parts of the country.

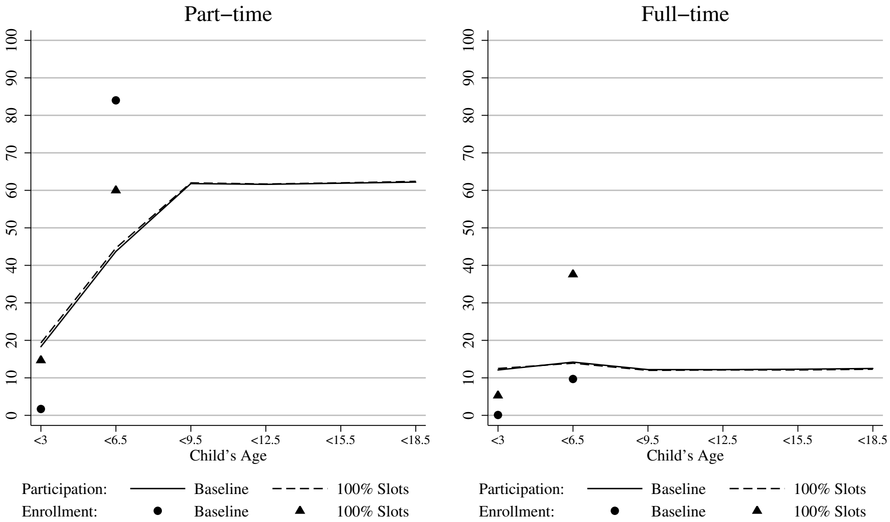
Figure 3.12: Model East vs. Data East
Labor Force Participation and Child Care Enrollment Rates



3.7.2 100% Subsidized Child Care Provision

To make the experiment as transparent as possible, I assume that each female has access to a subsidized full-time child care slot while the child care prices and income remain unchanged at the West German values. The results are very similar as if I had replaced the West by the East German provision rates. Figure 3.13 compares the outcome from the baseline model as discussed in Section 3.6.4 with the new setup of unrestricted access to subsidized full-time child care. The large increase in the child care enrollment rate and the substitution of part- with full-time child care demonstrates that there is indeed an excess demand for subsidized child care. However, females simply substitute their own time spend with the children and/or non-paid child care with paid child care whereas the effect on the labor force participation rate is very small. For females with children aged zero to two the labor force participation rate increases by 1.4 percentage points, i.e. an increase of 4.6%, and for females with children aged three to six and half by 0.6 percentage points, i.e. an increase of 1%. Still, these numbers are not

Figure 3.13: 100% Subsidized Child Care Provision
Labor Force Participation and Child Care Enrollment Rates



far away from the estimates in Wrohlich (2006), although some caution is needed because her figures refer to West and East Germany. She reports for a similar policy experiment that the labor force participation rate of females with children aged zero to two would rise by 2.8 percentage points, which in her case implies an increase of 10%, and by 1.5 percentage points for all females with children up to age ten.[36]

Table 3.12: 100% Subsidized Child Care Provision – Fertility

	Fraction with n children				**Fertility**
	0	**1**	**2**	**3**	**Rate**
Baseline	0.0	35.7	30.2	34.1	1.98
07 Leave Benefits	0.0	35.9	26.3	37.8	2.02

[36]Domeij and Klein (2009) find a much larger response for the female labor force participation rate for a similar experiment in the context of an overlapping generations model with exogenous fertility for Germany compared to the results here and in Wrohlich (2006).

Finally, Table 3.12 shows the change in fertility resulting from the provision of subsidized full-time child care slots for all children. 0.2% get one child less and 3.7% an additional child which increases the fertility by 0.04 children per female. Hank and Kreyenfeld (2003) estimate the effect of child care provision rates on the county level for West Germany on first and second birth risks and do not find any significant impact of higher child care provision rates on fertility, letting the small increase in the fertility rate from this experiment appear plausible. To which extent the costs of increasing the provision rate of subsidized full-time child care slots outweigh the social benefits from the increase in the labor force participation and fertility rate, is out of the current scope of this paper.[37]

3.7.3 The 2007 Maternal Leave Benefits

Since January 2007, females can receive for up to twelve months after birth a monthly maternity leave benefit of 67% of their pre-birth monthly net income or 1800 € – whatever is less. In this experiment, I replace the initial maternity leave benefit of 1583.91 € granted to all females with children aged zero to two either not or part-time working with the new policy. As net income I use one third of the after tax wage a female is offered in that period. Not surprisingly, part- and full-time labor force participation rates for females with children aged zero to two decreases, see Figure 3.14. Child care enrollment is hardly affected. The labor force participation rate is even higher by 1.1 percentage points for females with children aged three to six and half. Additionally, the fertility rate increases by 0.04 children per female because having more children becomes cheaper for females that were already under the old policy not working. From the perspective of a policy maker, the new maternity leave benefit increases fertility and labor force participation at least for one period.

3.8 Model Improvements

There are of course a lot of dimensions along which the model could be improved. I want to point out those of them which seem to me the most relevant and would not come along with a drastic change in the overall setup.

[37]For a treatment of this question, the reader is referred to Domeij and Klein (2009).

Figure 3.14: 2007 Maternity Leave Benefits
Labor Force Participation and Child Care Enrollment Rates

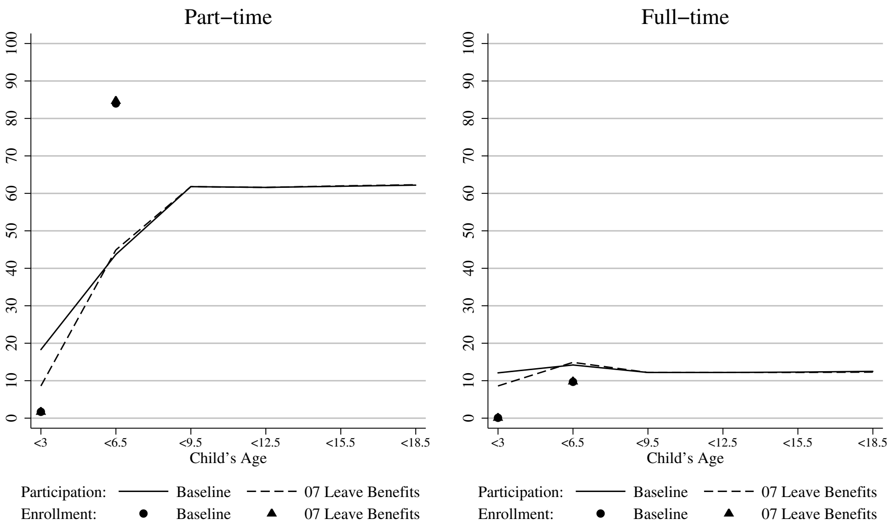


Table 3.13: 2007 Maternity Leave Benefits – Fertility

	Fraction with n children				Fertility
	0	**1**	**2**	**3**	**Rate**
Baseline	0.0	35.7	30.2	34.1	1.98
07 Leave Benefits	0.0	33.8	30.2	36.0	2.02

Slot Assignment The assignment rule for subsidized child care slots implies that any rejected slot is not filled with another potential applicant despite the excess demand for subsidized child care. An allocation rule could be constructed which lets only females draw from the lottery who will also use a slot eventually. The actual child care provision rates could still be used to determine the overall supply of slots while the success probabilities would depend on the pool of applicants and thus be at least as big as the provision rates. Such a setup would imply a deviation from a pure life cycle setting and require the introduction of a rational expectations equilibrium for the lottery participation. Going one step further, working females and those with more children usually have a prioritized access to child care which could also be captured by splitting up the lottery in several rounds.

Non-Paid Child Care All females are assumed to face the same function translating usage of non-paid child care in disutility. This assumption might however downplay the role of paid child care as a substitute for maternal time to enable females to work. The comparative statics exercise on the weight of non-paid child care costs ϕ_0 showed that an increase results in a higher usage of paid child care and lower labor force participation. Thus, introducing heterogeneity in this weight could make labor force participation easier for females with lower costs and induce females with higher costs to use paid child care. Allowing for heterogeneity in these costs does not seem implausible. In reality, for some females the costs might be infinity if no relative is available that could take care of the children during the day while for others they could be close to zero if they live under same roof with their own or spouse's retired parents.

Early School Years For children between age six and a half and 12.5, the assumption of full-time schooling is not compatible with real schooling hours. I made this assumption to accommodate that I do not allow for child care enrollment in these age groups which in turn was necessary because information on the availability of subsidized child care for this age group is only available if provided outside but not within schools. This assumption is responsible for the flat profile of the labor force participation rate for females of children in this age group while there is a smoother increase for West Germany in the data. In addition, it could be argued that children in this age group cannot be left alone such that for the usage of non-paid child care still the costs would need to be incurred, in particular if schooling is not to be assumed full-time anymore.

Last School Years For children from age 12.5 onwards I also assume that children attend full-time schooling which is in line with reality. The only effect of schooling is to restrict the time females can spend with their children. Since the children are in their teenage years by then and start to develop more of an own life, it might be reasonable to restrict the time females can spend with their children even further. This could potentially also lead to a stronger increase in full-time labor force participation in these periods since at the current estimates females spend all spare time of their children with them.

Life Span East German females are on average at their first/second/third birth 3.6/3/2 years younger than West German females. This age difference might also contribute to the different labor force participation behavior, in particular because pension payments upon retirement depend on contributions made throughout the life. This could be taken into account by making the terminal value function dependent on accumulated experience and be different for West and East Germany.

3.9 Conclusion

In this paper, I documented facts about labor force participation and child care enrollment decisions of married females in Germany. A particular emphasis was made on pointing out differences between West and East Germany. I developed a stylized life-cycle model on female labor supply that features endogenous fertility and takes child care choices explicitly into account. The model was estimated on data for West Germany and provides a solid fit although not all relevant moments were matched successfully. The model was not able to explain the differences between West and East Germany. Since a counterfactual policy experiment in which subsidized full-time child care slots were provided for all children, delivered results close to those in other studies, it is not clear whether the failure to explain the West-East differences is due to a model miss-specification or different preferences between West and East German females. Finally, I outlined some dimensions along which to improve the model.

Appendix

A.1 Sample Selection

The following list contains all applied selection criteria and their exact definition.

General criteria

1) Women born between 1955 and 1975, for those with children the child identifier has to be known.

2) Either born in Germany or born in another country as German citizen.

3) Only moves within West Germany for West German females or within East Germany for East German females.

Mothers

4) Females for which it can be identified whether a child has been born within a relationship.

    - Identification feasible if child born
        (a) in the year before the first interview or later.
        (b) earlier than one year before the first interview.
            i. Case 1
                - The female is married at the first interview.
                - The marriage started before the first interview and the actual starting date is known.
                - The child is born within the marriage defined as from the year before the marriage started onwards.
            ii. Case 2
                - The female is in a relationship, married or cohabiting, at the first interview.
                - The child is born more than one year
                    * before the first interview was conducted or
                    * before the marriage started, in case the relationship status at the first interview is married.

- The last observation is in 2000 or later because for each child the father's identifier is known if the father participated actively in the SOEP at least until 2000.

5) Births only in relationships.

  - As in Todd and Wolpin (2006) births in the year before the start of the relationship are treated as if they happened within the relationship.
  - For Case 2 of the previous criterion, criterion 5 is satisfied if the partner at the first interview is also the father of all children a female has given birth to prior to the first interview.

6) All births only in one relationship.

7) Relationship still intact at last interview.

Childless females

8) At age 39, i.e. prior to age 40, in a relationship that is still intact at last interview.

Table 3.14 shows the number of females satisfying each criterion.

Table 3.14: Sample Selection in Detail

West Germany

Criterion	**General**	**Mothers**	**Childless40up**
1	6610	.	.
2	4949	.	.
3	4909	2896	424
4	.	2868	.
5	.	2267	.
6	.	2229	.
7	.	1929	.
8	.	.	177
9	.	1873	169

East Germany

Criterion	**General**	**Mothers**	**Childless40up**
1	1763	.	.
2	1725	.	.
3	1565	1153	58
4	.	1112	.
5	.	780	.
6	.	770	.
7	.	653	.
8	.	.	24
9	.	645	22

A.2 Sample Construction

A.2.1 Period Definition

Ages 0 to 2 I exclude the month of birth and the subsequent two months to account for the mandatory maternity leave which outlaws females to work in the first eight weeks after the child is born. Depending on when the child is born within a month this implies an exclusion of the first eight to 13 weeks of a child's life. Thus, it is guaranteed that only the months in which it is legally allowed to work contribute.[38] Hence, by construction this period has a duration of 2.75 years.

Ages 3 to 6.5 The second period lasts from the month in which the child turns three until school entry. According to a cut-off rule, which is very similar across all German states, children who are at least six in July of a given year have to enter mandatory schooling. There are exceptions permitting a child to enter school one year earlier or later. Because of some peculiarities in the timing of the survey the age at school entry cannot always be determined exactly and has to be constructed. I therefore assume that for all children the cut-off date determines school entry but allow children to enter school earlier if this is known from the survey. Even if the exact entry age would be known, the length of the period is heterogenous among children because school starts only once a year. The mean duration in the data is 3.5 years.

A.2.2 Labor Force Participation

The GSOEP provides for every participant an employment spell history and reports the starting and end month of each spell. The following spell types exist:

1) Full-time Employment
2) Short-time Hours (Kurzarbeit)
3) Part-time Employment
4) Vocational Training

[38] Excluding only the month in which a child is born and the next month would correspond to the first five to nine weeks.

5) Unemployed
6) Retired
7) Maternity Leave
8) School, College
9) Military, Community Service
10) Housewife, Husband
11) Second Job
12) Other
13) First Job Training, Apprenticeship
14) Continuing Education, Retraining
15) Minijob (up to 400 Euro)
16) Gap (Missing Data)

Spells can overlap, e.g. a person could be working part-time and attend college at the same time. While the spell history is obtained from the retrospective monthly information, at each interview date the current labor force status is also asked for. Similar to the spell history, more than one answer is feasible. In these cases, the GSOEP provides a hierarchial order to obtain the labor force status which I apply here as well. In brief, full-time work dominates part-time work which dominates non-working.[39] The labor supply categories used in this paper are made up in the following way:

- Full-time work:

  1, 4, 9, 13, 14. The latter four categories are included because they are usually associated with a salary and require full-time labor force participation.

- Part-time work:

  2, 3, 11, 15

- Non-working:

  5, 6, 7, 8, 10, 12, 15

[39] For details see http://www.diw.de/documents/dokumentenarchiv/17/60055/pgen.pdf.

A.2.3 Child Care Enrollment

The GSOEP does not have the categories of subsidized and non-subsidized child care as I use them. Prior to 1995, it was only asked for enrollment in child care. From 1995 onwards a distinction between daycare centers and nannies was made. Since daycare centers are usually subsidized and nannies not, I use this definition to distinguish between subsidized and non-subsidized child care. Between 1995 and 1999 the distinction between day-care centers and nannies was exclusive and from 2000 onwards non-exclusive. Furthermore, for care provided by nannies from 2004 onwards part- and full-time can not be distinguished anymore. I therefore only calculate the following two variables. First, child care enrollment comprising subsidized (daycare centers) and non-subsidized (nannies) child care for all years which can be part- or full-time. Second from the year 1995 onwards the fraction of children enrolled in non-subsidized child care (nannies) from all children enrolled in child care (daycare centers and/or nannies).

Information on the child care enrollment status for each child is only available at the interview date. Therefore, the child care enrollment status has to be imputed for the other months of the year which will be based on the following reasoning: Since school starts at the same time for all children, the oldest cohort in a daycare center usually leaves the daycare center together at the same time of the year, i.e. at the end of the first half of the year. Therefore the majority of entries into daycare centers occurs at the beginning of the second half of the year. Hence, the child care enrollment status in the first half of a year is a good predictor for the status in the second half of the previous year. Similarly, the child care enrollment status in the second half of a year is a good predictor for the child care enrollment status in first half of the next year. In the following the detailed imputation procedure will be described. The child care enrollment status in the interview month is assumed to be the same in all other months of a half year. The second half of a year and the first half of the next year have the same status in case no new information is available. To make that point more clearly, if the interview month is in the first half of the year, which is the case for more than 90% of the interviews, I use this child care enrollment status also for the second half of the previous year if no interview has been conducted in the second half of the previous year. Analogously, if the interview month is in the second half of the year I use this child care enrollment status also for the first half of the next year if no interview is conducted in the first half of the next year. Finally, in all other cases the last known child care enrollment status is used until one of the two described situation occurs. Although this reasoning applies more to child care provided in daycare centers, I use the same imputation rule for child care provided by nannies.

A.3 Stylized Facts

A.3.1 Child Care Fees

Information on child care fees is available in the GSOEP only in the years 1987, 1996, 2002 and 2005. Moreover, 1987 cannot be used because of the missing distinction between nannies and daycare centers and in 2005 fees are only reported for daycare centers. Because of these rare number of years, the child care prices used in the model will be obtained from the full GSOEP sample and not only the selected sample.

The per child fees reported in Table 3.2 are determined by a regression on a set of corresponding dummy variables and were run separately for West and East Germany for subsidized child care but not for non-subsidized child care. In the regression for subsidized child care cross-sectional weights were used. For non-subsidized child care I refrained from doing so due to the low number of observations. In both regressions only those children, for which a positive fee was reported, were included.

A.3.2 Provision Rate of Subsidized Child Care

The slot provision rates are calculated from the data provided by Statistische Bundesamt (Statistik der Jugendhilfe, various years). They are only available for a subset of years (West Germany: 1986, 1990; East Germany: 1991; Both parts: 1994, 1998 and 2002) and change over time. Tables 3.16 shows the annual averages over the years 1983 to 2006, the period for which the monthly labor supply status from the GSOEP is available. These averages are constructed for the two age groups (zero to two, three to six and half) and the two regions (West and East) as follows: Years before the earliest observation of the slot provision rates, i.e. 1983 to 1985 in the West and 1989 to 1990 in the East, will be assigned the same value as the first observation of the slot provision rate (1986/1991). Similarly, years after the last observation, i.e. 2003 to 2006, will be assigned the same value as the last observation (2002). For the years between two observations the mean of the corresponding two observations will be used. The overall provision rates are then obtained as the mean over all years. From 1994 onwards the provision rates can be further distinguished by part- and full-time from which the fraction of full-time slots from all slots, the full-time share, will be calculated. As for the overall provision rate, the full-time share before the first and after the last observed data points are extrapolated and between two observation interpolated. The annual provision rate of part- and full-time slot is then given by the provision rate of slots times the fraction of part- or full-time slots from all slots. The mean over all these years then finally gives the average provision rate of part- and full-time slots.

These rates are used to construct the success probabilities for the slot lottery. If a females would have only one draw from the slot lottery at age zero and age three, the provision rates could be immediately used as model input. There is however no way to determine how often mothers apply for a slot within a period whereas in the model they only have a single draw for each age group. I therefore transform the observed provision rates into period equivalents in the following way: As already described for the imputation of the child care status, the majority of entries into daycare centers happens once a year. In addition, new information on the child care enrollment status is available once a year. Therefore I assume that a female can apply once per year for a slot. If she gets a slot, she can send her child there until it reaches age three and has to apply again for a slot or goes to school. Put differently, in each year a female can draw once from the lottery and a successful draw implies that the slot is open for the remainder of the period, i.e. until age three is reached or the child enters school. Once a full-time slot is drawn, the female does not have to redraw until the end of the period. Drawing a part-time slot implies that the female can redraw but success is then defined only as drawing a full-time slot because she already has access to a part-time slot for the rest of the period. Since a model period corresponds to three years I assume that within a period there is a maximum of three draws which leads to the set of possible access histories displayed in the left panel of Table 3.15.

Consider the case that a female would always use as much subsidized child care as she can get access to. In line with the definition for period

Table 3.15: Access to Subsidized Child Care

Access in Year			**Period Mean**	**Access Status**	**History Probability**
1	**2**	**3**			
No	No	No	0	No	$(1-P_P-P_F)^3$
No	No	Part	1/6	No	$(1-P_P-P_F)^2 P_P$
No	No	Full	1/3	Part	$(1-P_P-P_F)^2 P_F$
No	Part	Part	1/3	Part	$(1-P_P-P_F)P_P(1-P_F)$
No	Part	Full	1/2	Part	$(1-P_P-P_F)P_P P_F$
No	Full	Full	2/3	Part	$(1-P_P-P_F)P_F$
Part	Part	Part	1/2	Part	$P_P(1-P_F)^2$
Part	Part	Full	2/3	Part	$P_P(1-P_F)P_F$
Part	Full	Full	5/6	Full	$P_P P_F$
Full	Full	Full	1	Full	P_F

child care enrollment status in each year no slot is assigned a 0, part- and full-time slots with $\frac{1}{2}$ and 1. The mean over the whole period - the three years - would be given in column 4 in Table 3.15 whereas column 5 corresponds to the associated child care enrollment status for each possible access history using the same thresholds as before (0.25 and 0.75). Since I assume that a females does not have to use the slot she has drawn access to for some part of the period or at all, columns 4 and 5 state the period access status as opposed to the period enrollment status. Column 6 displays the probability of observing a specific access history. P_P and P_F are the probabilities of drawing a part- or full-time slot in a given year and correspond to the observed slot provision rates which differ by region and age. Finally, the probability for having access to no, a part- or full-time slot over the whole period is equal to the sum of the history probabilities that are associated with the respective period access status. For example, the probability to have no slot as defined by the period access status would be the sum over the two first histories ([**No, No, No**]; [**No, No, Part-time**]) and equal to $(1-P_P-P_F)^3 + (1-P_P-P_F)^2 P_P$. Table 3.16 presents the annual, i.e. observed, slot provision rates and the period provision rates after the transformation. E.g. while there are 1.2 part-time and 40.6 full-time slots per 100 children up to age two in East Germany, the probability for an East German female that she has access to a part-time slot over the whole period where the child is between zero and two is 38.8% and 41.1% for a full-time slot. Note that by construction, the period provision rates have to be larger than the annual/observed provision rates (for children aged zero to two in West Germany this also the case for the non-rounded numbers).

Table 3.16: Annual and Period Provision Rates of Subsidized Child Care Slots

	West			**East**		
	Annual		Period	Annual		Period
Ages 0 to 2						
Part-time	0.5		4.3	1.2		38.8
		⇒			⇒	
Full-time	1.7		1.7	40.6		41.1
Ages 3 to 6.5						
Part-time	62.3		71.8	1.8		2.4
		⇒			⇒	
Full-time	14.6		23.7	95.9		97.6

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