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Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks based on correlation functions with scattering interpolating operators both at the source and at the sink

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We present first results of a recently started lattice QCD investigation of antiheavy-antiheavy-light-light tetraquark systems including scattering interpolating operators in correlation functions both at the source and at the sink. In particular, we discuss the importance of such scattering interpolating operators for a precise computation of the low-lying energy levels. We focus on the $\bar{b}\bar{b}ud$ four-quark system with quantum numbers $I(J^P) = 0(1^+)$, which has a ground state below the lowest meson-meson threshold. We carry out a scattering analysis using Lüscher's method to extrapolate the binding energy of the corresponding QCD-stable tetraquark to infinite spatial volume. Our calculation uses clover u, d valence quarks and NRQCD b valence quarks on gauge-link ensembles with HISQ sea quarks that were generated by the MILC collaboration.

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1. Introduction

We report on a recently started lattice QCD project in which we aim to study possibly existing heavy-heavy-light-light tetraquark resonances. In the following, we focus on the $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$, which is theoretically simpler compared to other tetraquark candidates because it is QCD-stable. This tetraquark is the counterpart of the $\bar{c}\bar{c}ud$ tetraquark T_{cc} recently discovered by LHCb [1, 2].

In the past couple of years, several independent lattice QCD studies of $\bar{b}\bar{b}qq$ and $\bar{b}\bar{c}qq$ systems (q denotes a light u, d or s quark) were published. These computations employed either exclusively local four-quark interpolating operators [3–7] or local and scattering four-quark interpolating operators, but the latter only at the sink [8, 9].

In the work presented here, we include scattering interpolating operators both at the source and at the sink. This allows a more precise determination of finite-volume energy levels not only for bound states, but also for scattering states. This is particularly important for $\bar{Q}\bar{Q}qq$ systems, where bound states and scattering states are very close, or where bound states do not exist but resonances might exist. An example is a future full lattice QCD investigation of a possibly existing $\bar{b}\bar{b}ud$ tetraquark resonance with $I(J^P) = 0(1^-)$ proposed in Ref. [10].

2. Interpolating operators

To study the $\bar{b}\bar{b}ud$ four-quark system with quantum numbers $I(J^P) = 0(1^+)$, we use local interpolating operators

$$O_1 = O_{[BB^*](0)} = \sum_{\mathbf{x}} \bar{b}\gamma_5 d(\mathbf{x})\, \bar{b}\gamma_j u(\mathbf{x}) - (d \leftrightarrow u), \tag{1}$$

$$O_2 = O_{[B^*B^*](0)} = \epsilon_{jkl} \sum_{\mathbf{x}} \bar{b}\gamma_k d(\mathbf{x})\, \bar{b}\gamma_l u(\mathbf{x}) - (d \leftrightarrow u), \tag{2}$$

$$O_3 = O_{[Dd](0)} = \sum_{\mathbf{x}} \bar{b}^a \gamma_j C \bar{b}^{b,T}(\mathbf{x})\, d^{a,T} C\gamma_5 u^b(\mathbf{x}) - (d \leftrightarrow u), \tag{3}$$

and scattering interpolating operators

$$O_4 = O_{B(0)B^*(0)} = \bigg(\sum_{\mathbf{x}} \bar{b}\gamma_5 d(\mathbf{x})\bigg)\bigg(\sum_{\mathbf{y}} \bar{b}\gamma_j u(\mathbf{y})\bigg) - (d \leftrightarrow u), \tag{4}$$

$$O_5 = O_{B^*(0)B^*(0)} = \epsilon_{jkl}\bigg(\sum_{\mathbf{x}} \bar{b}\gamma_k d(\mathbf{x})\bigg)\bigg(\sum_{\mathbf{y}} \bar{b}\gamma_l u(\mathbf{y})\bigg) - (d \leftrightarrow u). \tag{5}$$

Here, C denotes the charge conjugation matrix, and upper indices a and b are color indices. For more details we refer to our previous work [8].

3. Lattice setup

We use $2 + 1 + 1$-flavor HISQ gauge-link ensembles generated by the MILC collaboration [11] as summarized in Table 1.

Ensemble	a [fm]	$N_s^3 \times N_t$	$m_\pi^{(\text{sea})}$ [MeV]	$m_\pi^{(\text{val})}$ [MeV]	N_conf
a12m310	0.1207(11)	$24^3 \times 64$	305.3(4)	310.2(2.8)	1053
a12m220S	0.1202(12)	$24^3 \times 64$	218.1(4)	225.0(2.3)	1020
a12m220	0.1184(10)	$32^3 \times 64$	216.9(2)	227.9(1.9)	1000
a12m220L	0.1189(09)	$40^3 \times 64$	217.0(2)	227.6(1.7)	1030
a09m310	0.0888(08)	$32^3 \times 96$	312.7(6)	313.0(2.8)	1166
a09m220	0.0872(07)	$48^3 \times 96$	220.3(2)	225.9(1.8)	657

Table 1: Gauge-link ensembles (a: lattice spacing; N_s, N_t: number of lattice sites in spatial and temporal direction; $m_\pi^{(\text{sea})}$, $m_\pi^{(\text{val})}$: pion mass corresponding to light sea and light valence quarks; N_conf: number of gauge-link configurations used for computations).

We use a mixed-action setup with Wilson-clover u and d valence quarks [12, 13]. For the b valence quarks we use lattice NRQCD [14].

Correlation functions are computed with point-to-all propagators if there is a local operator at the source. If there is a scattering operator at the source, we use stochastic timeslice-to-all propagators combined with the one-end trick (see e.g. Ref. [15]). Moreover, we use APE smearing for the gauge links and Gaussian smearing for the quark fields.

For the analysis of the correlation matrices we employ two independent methods: solving standard generalized eigenvalue problems (GEVP) as well as the Athens Model Independent Analysis Scheme (AMIAS) [16].

4. Effective masses from "exclusively local" versus "local and scattering" interpolating operators

Based on previous lattice QCD computations [3, 5, 7, 8] we expect the ground state around 100 MeV below the BB^* threshold (the lowest meson-meson threshold in this channel) representing the QCD-stable tetraquark. The first and second excitations in the finite spatial volume should be meson-meson scattering states resembling BB^* and B^*B^* close to the respective thresholds.

The left plot in Figure 1 shows effective masses from a GEVP using only local interpolating operators O_1, O_2 and O_3 (i.e. corresponding to a 3×3 matrix). The plateaus exhibit a strong discrepancy with the expectation discussed in the previous paragraph. The right plot in Figure 1 shows effective masses from a GEVP using both local interpolating operators O_1, O_2 and O_3 as well as scattering interpolating operators O_4 and O_5 (i.e. corresponding to a 5×5 matrix). These effective masses are consistent with the expectation. Thus, Figure 1 demonstrates that scattering operators are essential for a precise determination of scattering states.

5. Scattering analysis

To determine the mass of the $\bar{b}\bar{b}ud$ tetraquark in infinite volume (for a given ensemble, i.e. at given m_π and nonzero a), we proceed as in our previous work [8]:

(1) Compute the two lowest energy levels in the finite spatial volume (see section 4).

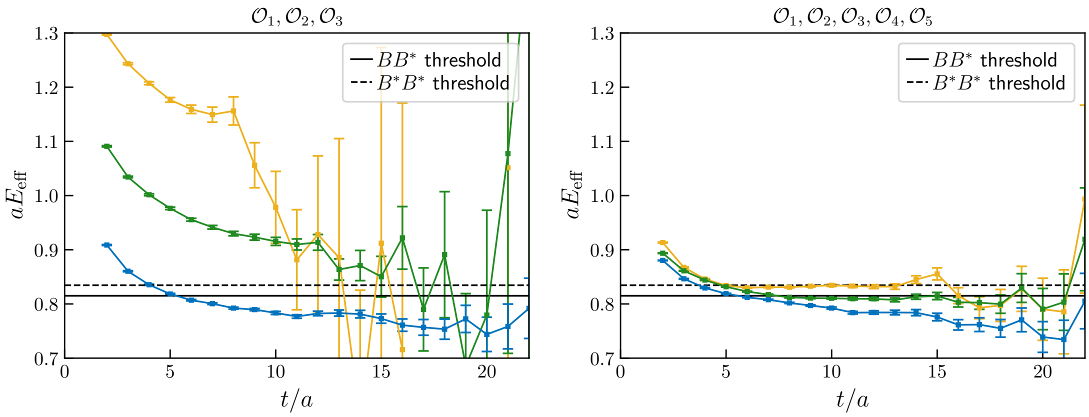

Figure 1: Effective energies from a GEVP for ensemble a09m310. **(left)** 3×3 matrix, only local interpolating operators O_1, O_2 and O_3. **(right)** 5×5 matrix, both local interpolating operators O_1, O_2 and O_3 and scattering interpolating operators O_4 and O_5.

(2) Compute the corresponding phase shifts $\delta_0(k_0)$, $\delta_0(k_1)$ using Lüscher's finite-volume method [17].

(3) Parameterize $\delta_0(k_0)$, $\delta_0(k_1)$ using the effective-range expansion,

$$k \cot(\delta_0(k)) = \frac{1}{a_0} + \frac{r_0}{2} k^2 \tag{6}$$

with fit parameters a_0 and r_0.

(4) The mass of the $\bar{b}\bar{b}ud$ tetraquark (and the energy of the first excitation) in infinite spatial volume corresponds to a pole in the scattering amplitude

$$T_0(k) = \frac{1}{\cot(\delta_0(k)) - i}. \tag{7}$$

The position of the pole can be obtained via Eqs. (6) and (7).

The dark-gray data points in the left plot of Figure 2 represent lattice QCD finite-volume energy levels (ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a \approx 0.12\,\text{fm}$ and $m_\pi \approx 220\,\text{MeV}$ (ensembles a12m220S, a12m220, a12m220L). The orange curves correspond to the two lowest finite volume energy levels as functions of the spatial extent L, computed with Lüscher's finite-volume method using the effective-range expansion (6). There are rather small differences between the finite-volume and infinite-volume energy levels. We attribute this to the large binding energy, $\Delta E_0 \approx O(100\,\text{MeV})$. Scattering analyses are, however, expected to be more important for smaller binding energies (e.g. for the $\bar{b}\bar{b}su$ system with $J^P = 1^+$) and essential for tetraquark resonances (e.g. for the $\bar{b}\bar{b}ud$ system with $I(J^P) = 0(1^-)$ [10]).

The right plot of Figure 2 shows an extrapolation of the tetraquark binding energy in the light u/d quark mass based on the six ensembles listed in Table 1. The preliminary result at the physical pion mass $m_{\pi,\text{phys}} = 135\,\text{MeV}$ is

$$\Delta E_0(m_{\pi,\text{phys}}) \approx (-103 \pm 8)\,\text{MeV}, \tag{8}$$

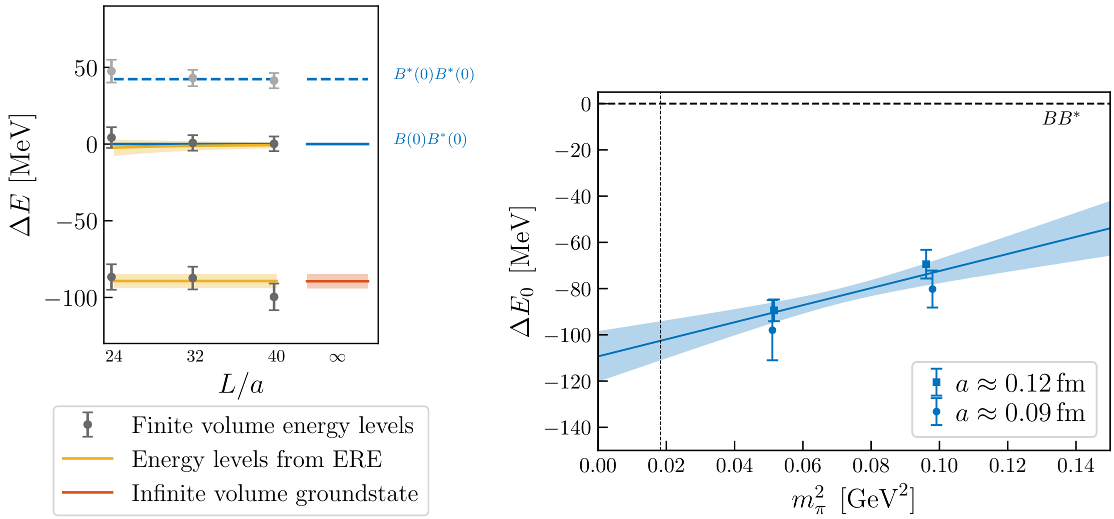

Figure 2: (left) Lattice QCD finite volume energy levels (dark gray: ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a \approx 0.12$ fm and $m_\pi \approx 220$ MeV (ensembles a12m220S, a12m220, a12m220L) together with a fit based on the effective range expansion (6). **(right)** Extrapolation of the tetraquark binding energy in the light u/d quark mass to the physical point.

where only the statistical uncertainty is shown. This binding energy is slightly smaller than, but consistent with, previous lattice results [3, 5–8].

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